# Title

## Subtitle

Rebecca Wei

Northwestern University

Date/Event

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  Objects: algebra maps $f : A \to A$
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$$\begin{aligned} {}_f\delta_g(\phi)(a_1 \otimes \ldots \otimes a_n) =& \epsilon_\phi \Big( f(a_1) \cdot \phi(a_2, \ldots, a_n) + \\ & + \sum_{1 \le i \le n-1} (-1)^i \phi(a_1, \ldots, a_i a_{i+1}, \ldots, a_n) + \\ & + (-1)^n \phi(a_1, \ldots, a_{n-1}) \cdot g(a_n) \Big) \\ \epsilon_\phi =& (-1)^{|\phi|+1} \end{aligned}$$

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$$\begin{tikzcd} A \arrow[rr, bend left=60, "f"] \arrow[rr, "g"] \arrow[rr, bend right=60, "h"'] \arrow[rr, phantom, shift left=3, "\Downarrow\phi_1"] \arrow[rr, phantom, shift right=3, "\Downarrow\phi_2"] & & A \arrow[r, squiggly, "compose"] & A \arrow[rr, bend left=60, "f"] \arrow[rr, bend right=60, "h"'] \arrow[rr, phantom, "\phi_1 \cup \phi_2 \Downarrow"] & & A \end{tikzcd}$$

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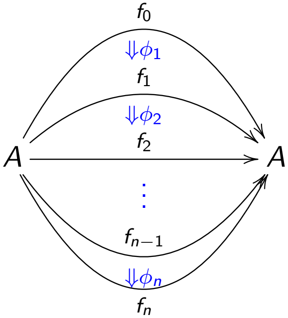


A morphism from $f_0$ to $f_n$ in Bar(Hoch(A))

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$$\begin{tikzcd}[column sep=huge]
A \arrow[r, bend left=60, "f_0"] \arrow[r, bend left=30, "f_1"] \arrow[r, "f_2"] \arrow[r, bend right=30, "f_{n-1}"'] \arrow[r, bend right=60, "f_n"'] \arrow[r, phantom, bend left=45, "\Downarrow\phi_1"] \arrow[r, phantom, bend left=15, "\Downarrow\phi_2"] \arrow[r, phantom, bend right=15, "\vdots"] \arrow[r, phantom, bend right=45, "\Downarrow\phi_n"] & A \arrow[r, bend left, "g_0"] \arrow[r, bend right, "g_1"'] \arrow[r, phantom, "\Downarrow\psi"] & A \arrow[r, mapsto, "\bullet"] & A \arrow[r, bend left=60, "g_0 f_0"] \arrow[r, bend right=60, "g_1 f_n"'] \arrow[r, phantom, "\psi\{\phi_1,\ldots,\phi_n\}\Downarrow"] & A
\end{tikzcd}$$

# A more categorical appropach (or somehow otherwise motivate $B(n)$)

Fix an algebra, $A$.
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In this context, braces give multilinear maps:

$$\begin{tikzcd}
Bar(Hoch(A)) \otimes Bar(Hoch(A)) \arrow[r, "\bullet"] \arrow[dr, dotted, "\bullet"'] & Hoch(A) \\
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Then, (Bar(Hoch(A)), $\bullet$) is an algebra in DGCocats.
But we have more...

# A more categorical appropach, (or somehow otherwise motivate $B(n)$)

Fix algebras, $A_0, A_1, \ldots, A_n$.
Define a dg cocategory $B(A_0 \to A_1 \to \ldots \to A_n \to A_0)$
where, for n=0, $B(A_0 \to A_0) := Bar(Hoch(A_0))$.

# A sheafy-cyclic object in DGCocat

Fact: We have a functor