# Title

## Subtitle

Rebecca Wei

Northwestern University

Date/Event

# A more categorical appropach (or somehow otherwise motivate $B(n)$)

Fix an algebra, $A$.
Define a dg category, $Hoch(A)$:

# A more categorical appropach (or somehow otherwise motivate $B(n)$)

Fix an algebra, $A$.
Define a dg category, $Hoch(A)$:
  Objects: algebra maps $f : A \to A$
  Morphisms: $Hoch(A)(f, g) = (C^\bullet(A, {}_f A_g), {}_f\delta_g)$

# A more categorical appropach (or somehow otherwise motivate $B(n)$)

Fix an algebra, $A$.
Define a dg category, $Hoch(A)$:

Objects: algebra maps $f : A \to A$

Morphisms: $Hoch(A)(f, g) = (C^\bullet(A, {}_fA_g), {}_f\delta_g)$

$$
\begin{aligned}
{}_f\delta_g(\phi)(a_1 \otimes \ldots \otimes a_n) =& \epsilon_\phi \Big( f(a_1) \cdot \phi(a_2, \ldots, a_n) + \\
& + \sum_{1 \le i \le n-1} (-1)^i \phi(a_1, \ldots, a_i a_{i+1}, \ldots, a_n) + \\
& + (-1)^n \phi(a_1, \ldots, a_{n-1}) \cdot g(a_n) \Big) \\
\epsilon_\phi =& (-1)^{|\phi|+1}
\end{aligned}
$$

# A more categorical appropach (or somehow otherwise motivate $B(n)$)

Fix an algebra, $A$.
Define a dg category, $Hoch(A)$:
  Objects: algebra maps $f : A \to A$
  Morphisms: $Hoch(A)(f, g) = (C^\bullet(A, {}_fA_g), {}_f\delta_g)$
  Composition: cup product on cochains

# A more categorical appropach (or somehow otherwise motivate $B(n)$)

Fix an algebra, $A$.
Define a dg category, $Hoch(A)$:

Objects: algebra maps $f : A \to A$

Morphisms: $Hoch(A)(f, g) = (C^\bullet(A, {}_fA_g), {}_f\delta_g)$

Composition: cup product on cochains

$$\begin{tikzcd} A \arrow[r, bend left=60, "f"] \arrow[r, "g"'] \arrow[r, bend right=60, "h"'] \arrow[r, phantom, shift left=3, "\Downarrow\phi_1"] \arrow[r, phantom, shift right=4, "\Downarrow\phi_2"] & A \arrow[r, squiggly, "\mathit{compose}"] & A \arrow[r, bend left=60, "f"] \arrow[r, bend right=60, "h"'] \arrow[r, phantom, "\phi_1\cup\phi_2\ \Downarrow"] & A \end{tikzcd}$$

# A more categorical appropach (or somehow otherwise motivate $B(n)$)

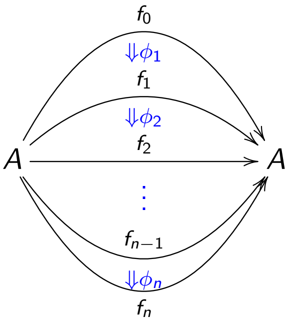
Fix an algebra, $A$.
Define a dg category, $Hoch(A)$:

$$Bar(Hoch(A)) \otimes Bar(Hoch(A)) \xrightarrow{\bullet} Hoch(A)$$

# A more categorical appropach (or somehow otherwise motivate $B(n)$)

Fix an algebra, $A$.
Define a dg category, $Hoch(A)$:

$$Bar(Hoch(A)) \otimes Bar(Hoch(A)) \xrightarrow{\bullet} Hoch(A)$$




A morphism from $f_0$ to $f_n$ in Bar(Hoch(A))

# A more categorical appropach (or somehow otherwise motivate $B(n)$)

Fix an algebra, $A$.
Define a dg category, $Hoch(A)$:

$$Bar(Hoch(A)) \otimes Bar(Hoch(A)) \xrightarrow{\bullet} Hoch(A)$$

$$\begin{tikzcd}[column sep=huge]
A \arrow[r, bend left=70, "f_0"] \arrow[r, bend left=35, "f_1"] \arrow[r, "f_2"] \arrow[r, bend right=35, "f_{n-1}"'] \arrow[r, bend right=70, "f_n"'] \arrow[r, phantom, shift left=10, "\Downarrow\phi_1"] \arrow[r, phantom, shift left=5, "\Downarrow\phi_2"] \arrow[r, phantom, shift right=5, "\vdots"] \arrow[r, phantom, shift right=12, "\Downarrow\phi_n"] & A \arrow[r, bend left, "g_0"] \arrow[r, bend right, "g_1"'] \arrow[r, phantom, "\Downarrow\psi"] & A \arrow[r, mapsto, "\bullet"] & A \arrow[r, bend left=50, "g_0 f_0"] \arrow[r, bend right=50, "g_1 f_n"'] \arrow[r, phantom, "\psi\{\phi_1,\ldots,\phi_n\} \Downarrow"] & A
\end{tikzcd}$$

# A more categorical appropach (or somehow otherwise motivate $B(n)$)

Fix an algebra, $A$.
Define a dg category, $Hoch(A)$:

In this context, braces give multilinear maps:

$$\begin{tikzcd}
Bar(Hoch(A)) \otimes Bar(Hoch(A)) \arrow[r, "\bullet"] \arrow[dr, dotted, "\bullet"'] & Hoch(A) \\
& Bar(Hoch(A)) \arrow[u, two heads]
\end{tikzcd}$$

Then, (Bar(Hoch(A)), •) is an algebra in DGCocats.