$$p(x) = \int p_\theta(x \mid z) p(z) dz$$
$$\log p_\theta(x) \to \max_\theta$$
$$\geq \mathbb{E}_{q_\phi(z|x)} \log \frac{p_\theta(x,z)}{q_\phi(z \mid x)}$$

картинка про структуру вычислений

$$q(z \mid x) = \text{Bernoulli}(\mu(x))$$
$$[u < \mu] = H(\mu - u)$$

- Discrete VAE, Semi-supervised learning
- GANs for discrete data
- Hard Attention, Control flow manipulation

$$L(\phi) = \mathbb{E}_{q_\phi(z)} f(z) \to \max_\phi$$
$$\nabla L(\phi) = \mathbb{E}_{q_\phi(z)} \nabla_\phi f(z) + [\nabla_\eta \mathbb{E} f_\eta(z)]\Big|_{\eta=\phi}$$

- reparametrization trick
- REINFORCE

$$g : \mathbb{E} g = \nabla_\phi L(\phi)$$

Continuous Relaxation
Gumbel-Max Trick

$$z \sim \text{Cat}(\pi_1, \ldots, \pi_k) \quad z = y$$
$$y = \arg\max_{1 \leq j \leq k} \{\log \pi_j + \gamma_j\} \quad \gamma_j \sim \text{Gumbel}(0,1)$$
$$\gamma_j = -\log(-\log u_j)$$
$$\text{argmax} \mapsto \text{softmax}$$
$$\arg\max(x) = \lim_{\tau \to 0} \text{softmax}(x/\tau)$$
$$\xi = \log \pi + \gamma \quad \gamma_i \sim \text{Gumbel}(0,1)$$
$$z = \arg\max(\xi) \quad \tilde{z} = \text{softmax}_\tau(\xi)$$
$$\tilde{z} = \text{softmax}\left(\frac{\log \pi + \gamma}{\tau}\right) \quad \tau \leq \frac{1}{k-1} \text{ — нет мод внутри}$$

- Gumbel-Softmax
- The Concrete distribution

$$\nabla_\phi \mathbb{E}_{q_\phi(z)} f(z)$$
$$\nabla_\phi f(\tilde{z}(\gamma, \phi)) \quad \gamma \sim \text{Gumbel}(0, 1)$$

- Biased
- Hyperparameters

$$\nabla_\phi \mathbb{E}_{q_\phi(z)} f(z) = \mathbb{E}_{q_\phi(z)} f(z) \nabla_\phi \log q_\phi(z)$$
$$\underbrace{f(z)}_{scalar} \underbrace{\nabla_\phi \log q_\phi(z)}_{vector} \quad z \sim q_\phi(z)$$

$$\mathbb{E}_{q(\phi(z)}(f(z) - b(z))\nabla_z \log q_\phi(z) + \underbrace{\mathbb{E}_{q_\phi(z)} b(z) \nabla_\phi \log q_\phi(z)}_{\nabla_\phi \mathbb{E}_{q_\phi(z)} b(z)}$$

$$\mathbb{E}_{q_\phi(\xi)} f(\arg\max \xi) = \mathbb{E}_{q_\phi(z)} f(z)$$
$$\mathbb{E}_{q_\phi(\xi)}[f(\arg\max \xi) - b(\xi)] \nabla \log q_\phi(\xi) + \nabla_\phi \mathbb{E}_{q_\phi(\xi)} b(\xi)$$
$$b(\xi) = f(\text{softmax}_\tau(\xi))$$

$$\nabla \log q_\phi(z)$$
$$\nabla_\phi \mathbb{E}_{q_\phi(\xi)}(\tilde{z}) = \nabla_\phi \int q_\phi(z) f(\tilde{z}) dz =$$
$$= \nabla_\phi \int f(\tilde{z}) q_\phi(\xi) \sum_{k=1}^{K} q(z = k \mid \xi) d\xi =$$
$$= \nabla_\phi \int \sum_{k=1}^{K} q(\xi, z = k) f(\tilde{z}) d\xi =$$
$$= \nabla_\phi \sum_{k=1}^{K} \left[\int q_\phi(z = k) q_\phi(\xi \mid z = k) f(\tilde{z}) d\xi \right] =$$
$$= \sum_{k=1}^{K} q_\phi(z = k) \nabla_\phi \log q_\phi(z = k) \int q_\phi(\xi \mid z = k) f(\tilde{z}) d\xi +$$
$$+ \sum_{k=1}^{K} q_\phi(z = k) \nabla_\phi \int q_\phi(\xi \mid z = k) f(\tilde{z}) d\xi =$$
$$= \mathbb{E}_{q(\phi(z)} \mathbb{E}_{q_\phi(\xi \mid z = k)} f(\tilde{z}) \nabla \log q_\phi(z) + \mathbb{E}_{q_\phi(z)} \mathbb{E} q_\phi(\xi \mid z = k) f(\tilde{z})$$

$$\mathbb{E}_{q_\phi(z)}\mathbb{E}_{q(\xi|z)}f(\tilde{z})\nabla_\phi \log q_\phi(z) =$$
$$= \nabla_\phi \mathbb{E}_{q_\phi(\xi)} f(\tilde{z}) - \mathbb{E}_{q_\phi(z)}\nabla_\phi \mathbb{E}_{q_\phi(\xi|z)} f(\tilde{z})$$

$$\xi_i(z=k) = \begin{cases} -\log(-\log u_i)), & \text{if } i = k \\ -\log(-\log u_i / q_\phi(z=i) - \log u_k), & \text{if } i \neq k \end{cases}$$
$$u_j \sim U[0,1]$$

$$\tilde{z} \mid z = \mathrm{softmax}_\tau(\xi \mid z)$$

$$g = [f(z) - f(\tilde{z} \mid z)]\nabla_\phi \log q_\phi(z) +$$
$$+ \eta \nabla_\phi f(\tilde{z}) - \eta \nabla_\phi f(\tilde{z} \mid z),$$
$$\text{where}$$
$$\xi = \log \pi + \gamma, \quad \xi \mid z = \text{formula}$$
$$z = \arg\max \xi, \quad \tilde{z} \mid z = \mathrm{softmax}_\tau(\xi \mid z)$$
$$\tilde{z} = \mathrm{softmax}_\tau \xi$$

REBAR

$$\mathbb{E}g = \nabla_\phi \mathbb{E}_{q_\phi(z)} f(z)$$
$$\mathbb{D}g_i = \mathbb{E}g_i^2 - (\mathbb{E}g_i)^2 \to \min_{\tau,\eta}$$
$$\sum \mathbb{E}g_i^2 \to \min_{\tau,\eta}$$
$$\mathbb{E}\|g\|^2 \to \min_{\tau,\eta}$$

Следующий шаг (RELAX): замена $\eta f(z)$ на $h_\eta(z)$