VAE

$$x \in \mathbb{R}^D, z \in \mathbb{R}^d, x = (x_1, \dots, x_n)$$
$$p(x, z \mid \theta) = \prod_{i=1}^{n} p(x_i, z_i \mid \theta) =$$
$$= \prod_{i=1}^{n} p(x_i \mid z_i, \theta) p(z_i) =$$
$$= \prod_{i=1}^{n} \mathcal{N}(x_i \mid \mu + \omega z_i, \sigma^2) \mathcal{N}(z_i \mid 0, I)$$
$$p(X \mid \theta) \to \max_{\theta}$$

$$p(X \mid \theta) = \prod_i p(x_i \mid \theta) = \prod_i \int p(x_i \mid z, \theta) p(z) dz$$

E-step $q(z) = p(z \mid x, \theta) = \prod p(z_i \mid x_i, \theta)$
M-step $\mathbb{E}_{q(z)} \log p(x, z \mid \theta) \to \max_\theta$

E'-step $j \sim U\{1, \dots, n\}, q(z_i) = p(z_i \mid x_i, \theta)$
M'-step $\theta_{t+1} - \theta_t = \varkappa_t (\nabla_\theta \log p(x_j, \hat{z} \mid \theta)), \quad \hat{z} \sim q(z_i)$

$$\log p(X \mid \theta) \geq \mathcal{L}(\theta, \phi) = \int q(z \mid \phi) \log \frac{p(x, z \mid \theta)}{q(z \mid \phi)} dz =$$
$$= \sum_{i=1}^{n} \Bigg(\int q(z_i \mid x_i, \phi) \log p(x_i \| z_i, \theta) dz_i -$$
$$- \int q(z_i \mid \phi) \log \frac{q(z_i \mid \phi)}{p(z_i)} dz_i \Bigg)$$

reparametrization trick
encoder
NF

$$\sum \int q_T(z_T \mid x_i, \phi) \log p(x_i \mid z_T, \theta) dz_T -$$

$$- \int q_T(z_T \mid x_i, \phi) \frac{\log q_T(z_T \mid x_i, \phi)}{\log p(z_T)} dz_T =$$

$$= \sum_{i=1}^{n} \int q_0(z_0 \mid x_i, \phi) \log p(x_i \mid f_T(\dots f_1(z_0) \dots) dz_0 -$$

$$- \int q_0(z_0 \mid x_i, \phi) \log \frac{q_0(z_0 \mid x_i, \phi)}{p(f_T(f_{T-1}(\dots f_1(z_0) \dots)} dz_0 +$$

$$+ \sum_{t=1}^{T} \int q_0(z_0 \mid x_i, \phi) \log \left| \frac{\partial f_t}{\partial z_{t-1}} \right| dz_0 \to \max_{\theta, \phi, f}$$

$$f_t(z_{t-1}) = z_{t-1} + u_t \sigma(w_t^T z_{t-1} + b_t)$$

$$\left| \frac{\partial f}{\partial z_{t-1}} \right| = \left| I + \sigma'(w_t^T z_{t-1} + b_t) u_t \omega_t^T \right| =$$

$$= \left| 1 + \sigma'(w_t^T z_{t-1} + b_t) \omega_t^T u_t \right|$$