Discrete Differentiation Filters

- Frequency Sampling method

(1) Write a Matlab program that uses the frequency sampling method to design a $(2k+1)$-point discrete differentiation filter $H(F) = j2\pi F$ (k is an input parameter and can be any integer). (25 scores)

The transition band can be assigned to reduce the error (unnecessary to optimize). The impulse response of the designed filter should be shown. The Matlab code should be emailed to me.

(Ans)

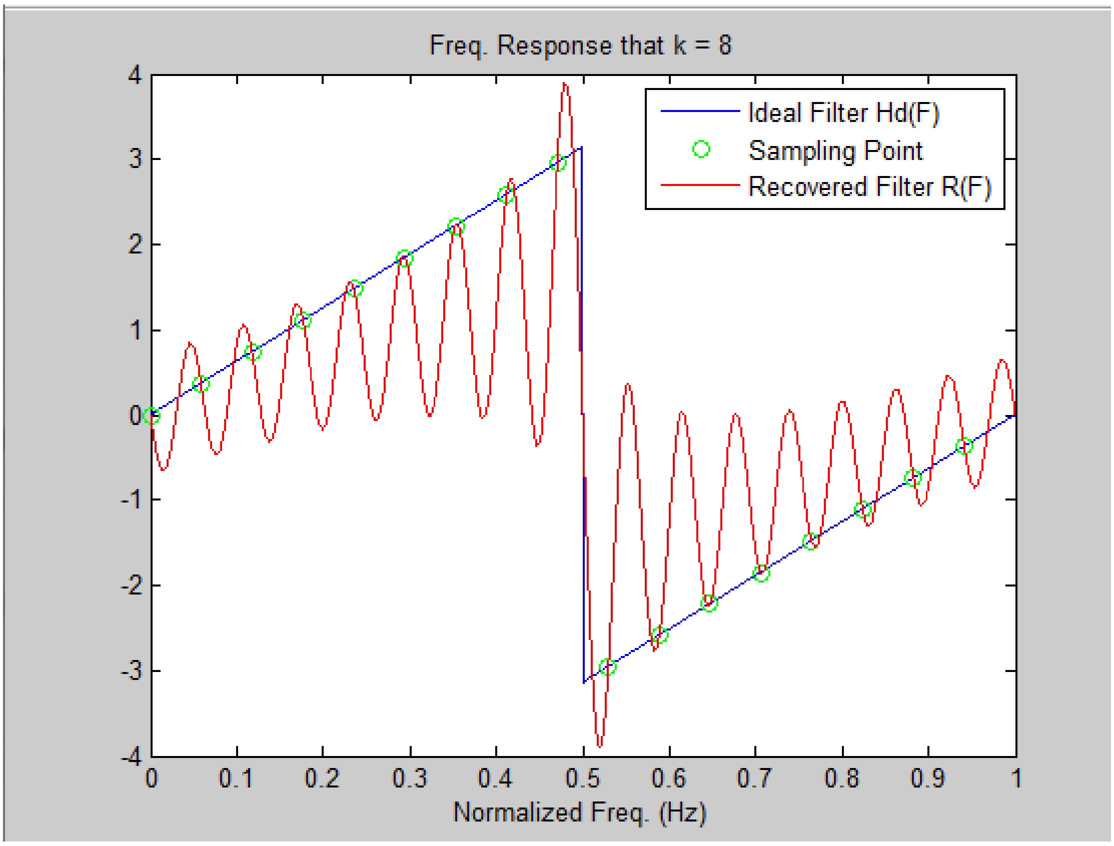
[1]

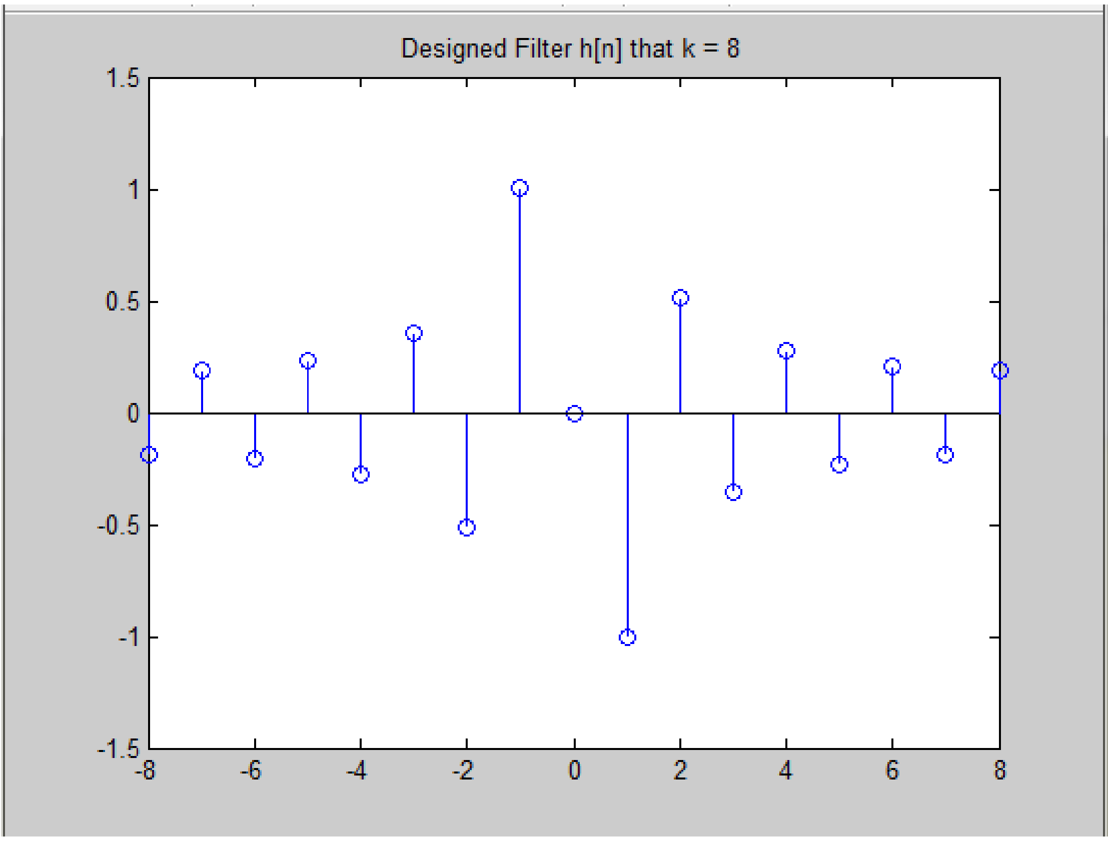
Freq_sampling_diff.m

```matlab
clear all;
close all;
clc;
%% Parameter
k = 8;
df = 0.0001;

F = [0 : df : 0.5 , -0.5 : df : 0-df]; %Normalized Freq.

N = 2*k + 1;  % 17
dn = length(F) / N;
n = floor(1:dn:length(F));  % sampling points

nn = -k:1:k;  % -k ~ 0 ~ k, N points in total
ff = 0:df:1;
sample_F = ff(n);
%% (Step1) Ideal Diff filter Hd(f)
H = 1i * 2 * pi * F;  % F = -0.5 ~ 0.5  only imaginary part

%% (Step2)
R = H(n);   % F = 0 ~ 1 (normalized)
r = ifft(R);  % n = 0 ~ 16
%% (Step3)
rn = fft(r, length(F));  % r[n]  n= -8 ~ 8

%% (Step4)
h = [ r(k+2:end) , r(1:k+1) ]; %h[n] Time-domain Shift  n = 0 ~ 16

```

```
%% Ploting
figure(1);
plot(0:df:1, imag(H), 'b');

hold on;
plot(sample_F, imag(R), 'go');
plot(0:df:1, imag(rn), 'r');
hold off;
title(['Freq. Response that k = ', num2str(k)]);
xlabel('Normalized Freq. (Hz)');
legend('Ideal Filter Hd(F)', 'Sampling Point', 'Recovered Filter R(F)');
figure(2);
stem(nn, real(h));
title(['Designed Filter h[n] that k = ', num2str(k)]);
```

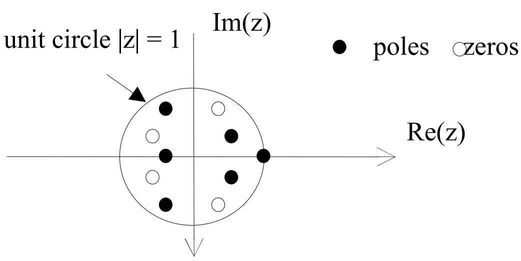
[2]





(2) What are the advantages of the <u>minimum phase filter</u>? (10 scores)

(Ans)



There are two advantages of minimum phase filter:

1. <u>The **inverse impulse response** is also **stable**</u>

   If the impulse response is $H(z)$, the poles of its inverse $1/H(z)$ is the zeros of $H(z)$ which are **all inside the unit circle**.

2. <u>**Energy** of **impulse response** concentrates on $\boldsymbol{n = 0}$</u>

   For example:

$$z^{-1}(z-a) = 1 - az^{-1} \xrightarrow{Z^{-1}} x[0] - a\,x[1]$$

$$\frac{z}{z-a}=\frac{1}{1-az^{-1}}\xrightarrow{z^{-1}}x[0]+a\,x[1]+a^{2}\;x[2]+a^{3}\;x[3]+\cdots$$

Because the zeros and the poles are all inside the unit circle, **|*a*| is less than 1**. So that the energy concentrates on $n = 0$.

(3) Write two concepts (from different speakers) that you learn from the oral presentation on 4/24. (10 scores)

(Ans)

- **<u>Pulse Shaping</u>**

1.使輸出訊號符合帶限通道 (Band-limited channel)
2.減少多路徑信號的反射符號間干擾(ISI)

常使用 SRRC filter 來作 Pulse Shaping
因為他的 Time domain 的 side-lob 比 sinc 小很多、main-lob 的下降幅度又夠大，大到足以防止 ISI
而他的 freq.domain 又是一個低通濾波器，使輸出訊號滿足 band-limited channel

- **Surf 演算法**

1.特徵點偵測：特色是從以往的 Hessian Matrix => boxed type convolution filter(一種行列式)，用以增快運算效率
2.特徵點描述：利用小波轉換來完成
3.特徵點比對：用來測定伴隨者與機器之間的距離，作為調整速度的參考

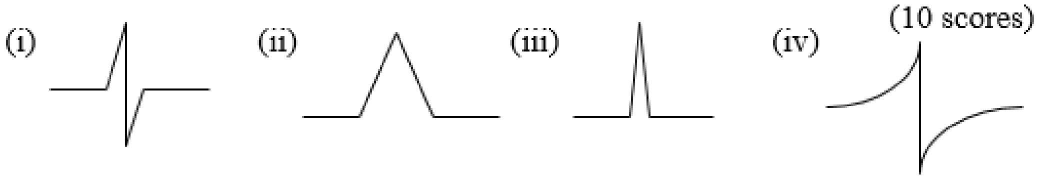
1. 影像校正與座標轉換 => 深度資訊補充
2. 偵測車格 => Sobel Operator
3. 模糊控制

- Face Hallucination/Super Resolution:

一種 **低解析度**影像 => **高解析度**影像 的技術
常用 co-occurrence model algorithms
是一種利用機器學習的方法，將一大群低解析度與高解析度影像當作 Training Data，讓機器去觀察低解析度影像與高解析度影像的對應關係。接著用這套對應關係去將輸入的低解析度影像去作高解析度化，亦即增加他的解析度。
另外還需要注意的地方，就是 Training Data 與 Input Data 的影像大小要

相等

(4) The following figures are the impulse responses of some filters. Which one is suitable for ridge detection when the SNR is low? Also illustrate the reasons. (10 scores)



(Ans)

Figure **(iv)** is the most suitable.

Although filter (ii) and filter (iv) are edge detection filter, the filter **(iv)** is more **smooth** and can be more **tolerant to noise**, so that it could work well though SNR is low.

(5) Suppose that the smooth filter is $h[n] = 0.04$ for $|n| \leqq 7$, $h[n] = 0.02$ for $8 \leqq |n| \leqq 17$, and $h[n] = 0$ otherwise. What is the efficient way to implement the convolution operation $y[n] = x[n] * h[n]$? (10 scores)

(Ans)

(6) Suppose that $x[0] = 0.5$, $x[1] = -0.85$, $x[2] = 0.03$, $x[3] = 0.18$, and $x[n] = 0$ otherwise. (a) Please find the cepstrum of $x[n]$ (Hint: the Matlab command 'roots' may be helpful for polynomial decomposition). (b) Please convert $x[n]$ into the minimum phase form. (20 scores)

(Ans)

Assume $X(z) = 0.5 - 0.85z^{-1} + 0.03z^{-2} + 0.18z^{-3}$, from MATLAB:

```
>> roots([0.5 -0.85 0.03 0.18])
```

ans =

1.5000

0.6000

-0.4000

We can obtain **z**

The Z-transform of the cepstrum is

$\hat{X}(z) = \log X(z) =$ **log0.5 + log(1-1.5z^{-1}) + log(1-0.6z^{-1})+log(1+0.4z^{-1})**

From the Taylor expansion $\log(1-x) \approx -x$, the inverse Z-transform of $\hat{X}(z)$ will become:

$$\hat{x}(n) = \begin{cases} \log 0.5 & , n=0 \\ -(\frac{0.6^n}{n} + \frac{1.5^n}{n}) & , n>0 \\ \frac{(-0.4)^{-n}}{n} & , n<0 \end{cases}$$

(b) 手寫

The Z transform of x[n] is

$$X(z) = 0.5\,\frac{(z-1.5)(z-0.6)(z+0.4)}{z^{-3}}$$

Since one of the zero **1.5** is outside the unit circle, we modify X(z) into

(7) (a) Why the cepstrum is more suitable for dealing with the multipath problem than the equalizer? (b) Why the Mel-cepstrum is more suitable for dealing with the acoustic signal than the original cepstrum? (15 scores)

(Ans)

(a)

(b)

The frequency bands of the MFC are equally spaced on the mel scale, which approximates the human auditory system's response more closely than the linearly-spaced frequency bands used in the normal cepstrum.