Walsh Transform

(1) Write a Matlab program that can generate 2^k-point Walsh transform in sequency ordering where k can be any positive integer.

$$W = \text{Walsh}(k)$$

The Matlab program should be mailed to me. (20 scores)

[ANS]

2^k-point Walsh transform

Walsh2k.m | Main.m | +

```matlab
clear all; clc;
% Main
%% 2k-point Walsh transform
size = 8; W8 = Walsh2k(size)
size = 16; W16 = Walsh2k(size)
size = 7; W = Walsh2k(size)  % Must has error message


```

Walsh2k.m | Main.m | +

```matlab
function W = Walsh2k(size)
k=log2(size);
if(mod(k,1)~=0)
    disp('The input number must be 2^k, while k is an integer.')
    W = 'Illegal';
    return
end
W2=[1  1; 1 -1]; % 2-point Walsh transform
if k==1  % Size=2
    return;
end
W = W2;
```

```matlab
%% Correct Order of Walsh
% Observing that the sign-changes of initial W is
% 0 (size-1) 1 (size-2) 2 (size-3) 3 ......
% Thus, by proper spliting, we could easily form a correct
% 2k-point Walsh transform

for n = 2:k
    W = kron(W,W2);  % K = kron(A,B) returns the Kronecker tensor product of matrices A and B
    V1 = W(1:2:2^n,:);
    V2 = W(2^n:-2:1,:);
    W = [V1;V2];
end
end
```

1. Write a MATLAB function W=Walsh2(size) generate the size×size Walsh transform matrix by providing the input of the parameter :size

2. Set $k = \log_2(\text{size})$

$$\mathbf{W}_1 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$n = 1$: 2-point Walsh transform matrix $\mathbf{W}_1 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$n = 2$: 4-point Walsh transform matrix
(where $\otimes$ is the **Kronecker tensor product**)

① $$\mathbf{W}_n = \mathbf{W}_{n-1} \otimes \mathbf{W}_1 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

② The **odd** rows of $\mathbf{W}_n$:

$$\mathbf{V}_1 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix}$$

The **even** rows of $\mathbf{W}_n$ (reverse order of rows):

$$\mathbf{V}_2 = \begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

③ Combines $\mathbf{V}_1$, $\mathbf{V}_2$ to get $\mathbf{W}_n$

$$\mathbf{W}_n = \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$ **% Wn = [V1;V2]**

$n = 3, 4, \ldots$: 2^n-point Walsh transform

```
W=kron(W,W2);
V1=W(1:2:2^n,:);
V2=W(2^n:-2:1,:);
W=[V1;V2];
```

repeat until $n = k$.

The reason why it works is according to the observation that the sign-changes of the initial Wn is 0 (size-1) 1 (size-2) 2 (size-3) 3 ......

Thus, by proper splitting, we could easily form a correct 2k-point Walsh transform

(2) In addition to the linear complexity, what is the other important advantage of the sectioned DFT convolution? (10 scores)

[ANS]

First of all ,the time complexity of "Section DFT conv." is O(N).

Second, 在硬體架構上，使用 "Section DFT convolution"能簡化設計，我們只要設計固定點數的 DFT 處理器即可，且 Forward and Reverse DFT 皆適用。

基於硬體架構上的好處，我們能得到一個" Input-independent"的 Process

(3) If the number of additions for the 2^k-point Walsh transform is A, what is the number of additions for the 2^{k+1}-point Walsh transform ? (10 scores)

[ANS]

(4) Compared to the Fourier transform, what are the advantages and the disadvantages of (a) the Walsh transform and (b) the number theoretic transform? (20 scores)

[ANS]

(a)

Advantages:

- Real calculations
- No **multiplication** is required
- Many properties are similar to those of the DFT

Disadvantages:

- 與 DFT 相比，收斂比較慢。自然界的訊號需要用較多的係數來表示
- Walsh transform 比較適合作 Spectrum Analysis，但不太適合作 convolution

(b)

Advantage:

・ shifting and addition are only for NTT

・ The NTT and INTT are exact inverse.

・ INNT can be calculated from NTT.

・ If N (transform length) is a power of 2, then the radix-2 FFT butterfly algorithm can be used for efficient calculation for NTT. (fast algorithm)

Disadvantage:

・ The output value of convolution cannot be greater than (M-1).

(5) (a) Determine the smallest value of α to construct the 5-point number theoretic transform (modulus 11). (b) Write the forward and the inverse 5-point number theoretic transform matrices (modulus 11) using the smallest value of α. (20 scores)

[ANS]

(6) (a) What is the results of CDMA if there are three data [1 0 1], [1 0 0], [0 1 1] and these three data are modulated by the 2^{nd}, 3^{rd}, and 6^{th} rows of the 16-point Walsh transform?

(b) Is it better to use the Haar transform and the number theoretic transform for CDMA? Why? (20 scores)

[ANS]

(a)

In practical, we'll transform basis into [1,-1,1]、[1,-1,-1]、[-1,1,1]。

- The **2nd row** of the sequence ordering 16-point Walsh transform is

W1 = [1 1 1 1 1 1 1 1 -1 -1 -1 -1 -1 -1 -1 -1]

Convert [1, 1, 0] to **[1, 1, -1]**, and modulated by W2:

M1 = [1 1 1 1 1 1 1 1 -1 -1 -1 -1 -1 -1 -1 -1;
1 1 1 1 1 1 1 1 -1 -1 -1 -1 -1 -1 -1 -1;
-1 -1 -1 -1 -1 -1 -1 -1 1 1 1 1 1 1 1 1]

- The **3nd row** of the sequence ordering 16-point Walsh transform is

W3 = [1 1 1 1 -1 -1 -1 -1 -1 -1 -1 -1 1 1 1 1]
Convert [1, 0, 0] to [1, -1, -1], and modulated by *W*2:
*M*3 = [1 1 1 1 -1 -1 -1 -1 -1 -1 -1 -1 1 1 1 1;
-1 -1 -1 -1 1 1 1 1 1 1 1 1 -1 -1 -1 -1;
-1 -1 -1 -1 1 1 1 1 1 1 1 1 -1 -1 -1 -1]

- The **6th row** of the sequence ordering 16-point Walsh transform is

W6 = [1 1 -1 -1 -1 -1 1 1 -1 -1 1 1 1 1 -1 -1]
Convert [0,1,1] to [-1,1,1], and modulated by *W*6:
M6 = [-1 -1 1 1 1 1 -1 -1 1 1 -1 -1 -1 -1 1 1;
1 1 -1 -1 -1 -1 1 1 -1 -1 1 1 1 1 -1 -1;
1 1 -1 -1 -1 -1 1 1 -1 -1 1 1 1 1 -1 -1]

(b)

No, the **zeros** in the Haar transform are not robust for preventing the interference noise.