N-point FFT Implementations

(1) Write the Matlab program to compute the FFT of two *N*-point real signals using only one N-point FFT. (20 scores)

[fx, fy] =fftreal(x, y)

The Matlab file should be mailed to me.

[ANS]

HW4_Main.m | fftreal.m | +

```
% HW4 Main
clc; clear all;
%% Discrete Fourier Transform for two Real Inputs
x = rand(1,10);
y = rand(1,10);
XX = fft(x);
YY = fft(y);
[fx,fy]=fftreal(x,y);

%% Detection
exp1 = sum(abs(XX-fx));
exp2 = sum(abs(YY-fy));

if exp1<0.001 && exp1<0.001
    disp('the FFT is also legal by using only one N-point FFT');
else
    disp('Not legal');
end
```

HW4_Main.m | fftreal.m

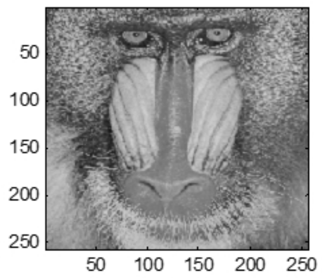
```matlab
% Discrete Fourier Transform for two Real Inputs
function [fx,fy]=fftreal(x,y)

z = x + 1i*y;
fz = fft(z);
N = length(fz);

fzz(1) = fz(1);
fzz(N:-1:2) = fz(2:N);
fx = (fz + conj(fzz))/2;
fy = (fz - conj(fzz))/2/1i;

end
```

(2) What are the conditions that the normalized root mean square error (NRMSE) cannot reflect the similarity of (a) two image signals and (b) two vocal signals? (15 scores)

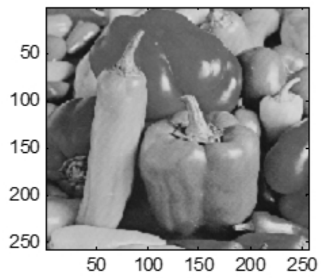
[ANS]

(a) Two image signals

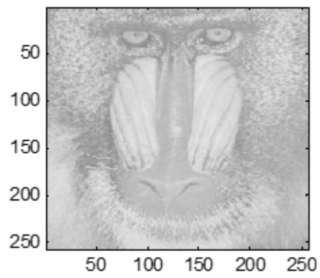
Case 1 : Pictures of the **same structure** but **different pixel value**(same shape but different intensity)



圖一



圖二



圖三

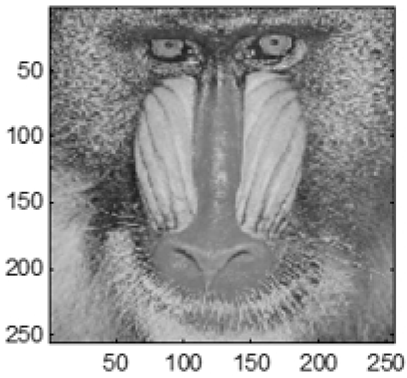
圖三 = 圖一 $\times 0.5 + 255.5 \times 0.5$

照理來說，圖一和圖三較相近

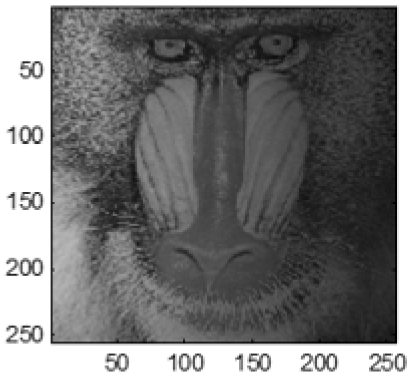
然而，圖一和圖二之間的 NRMSE 為 0.4411

圖一和圖三之間的 NRMSE 為 0.4460

Case 2 : The Shadow Effect



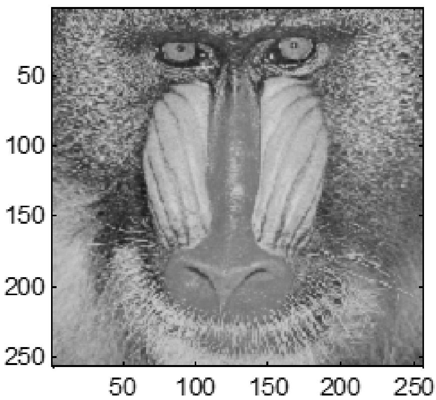
圖四



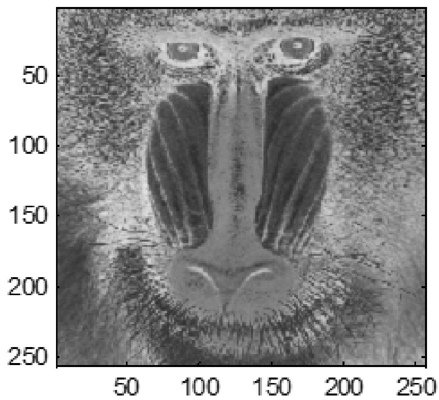
圖五

NRMSE = 0.4521 (大於圖一、圖二之間的 NRMSE)

Case 3 : The Negative of the photo (底片)



圖六



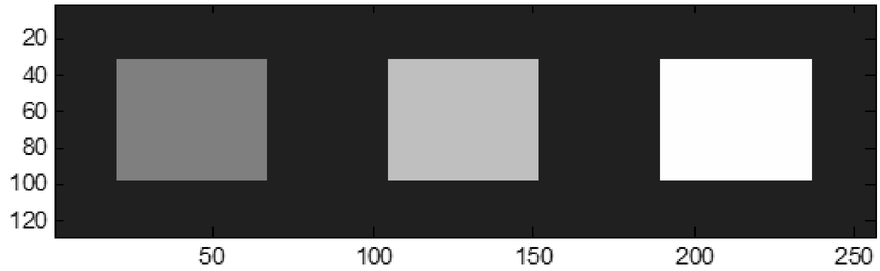
圖七

圖七 = 255 − 圖六

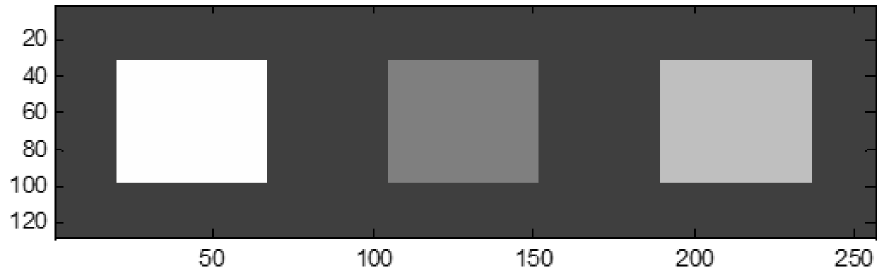
NRMSE = 0.5616 (大於圖一、圖二之間的 NRMSE)

Case 4 : Same shape but different intensity (similar condition to Case 1)

圖八



圖九



NRMSE = 0.4978 (大於圖一、圖二之間的 NRMSE)

(b) Two vocal signals

Case 1：Different frequency but similar feeling for people's ear

$\cos(2\pi \times 500\,Hz)$ v.s $\cos(2\pi \times 501\,Hz)$

人耳對於微幅的頻率差異感受不深

Case 2：Different Phase but similar feeling for people's ear.

$\cos(2\pi \times 500\,Hz)$ v.s $-\cos(2\pi \times 500\,Hz)$

The Phase Difference of these two signals are 180° ,but not so obviously different for people's ear.

Case 3：Different Delay but similar feeling for people's ear.

微幅的延遲人耳聽起來差不多，但 NRMSE 值不會很高。

Case 4：Small Amplitude Difference but similar feeling for people's ear.

微幅的振幅差異人耳聽起來差不多，但 NRMSE 值不會很高。

(3) Implement the following operation with the least number of multiplications.

(15 scores)

$$\begin{bmatrix} y[1] \\ y[3] \\ y[5] \\ y[7] \end{bmatrix} = \begin{bmatrix} 0.9808 & 0.8315 & 0.5556 & 0.1951 \\ 0.8315 & -0.1951 & -0.9808 & -0.5556 \\ 0.5556 & -0.9808 & 0.1951 & 0.8315 \\ 0.1951 & -0.5556 & 0.8315 & -0.9808 \end{bmatrix} \begin{bmatrix} z[4] \\ z[5] \\ z[6] \\ z[7] \end{bmatrix}$$

[ANS]

(4) Determining the numbers of real multiplications for the (a) 91-point DFT, (b) 165-point DFT, and the (c) 363-point DFT. (15 scores)

[ANS]

(a) 91-point DFT

91 = 7*13

(b) 165-point DFT

165 = 3*5*11

(c) 363-point DFT
363 = 3*11*11

(5) Suppose that the 1-D edge detection operation is:

$x_e[n] = x[n] * h[n]$ $\quad$ $h[1] = -h[-1] = -1$ $\quad$ $h[2] = -h[-2] = -0.4$

$h[3] = -h[-3] = -0.2$ $\quad$ $h[4] = -h[-4] = -0.1$ $\quad$ $h[n] = 0$ otherwise

Design an efficient way to implement the above edge detection operation.

(10 scores)

[ANS]

(6) Suppose that length($x[n]$) = 1000. What is the best way to implement the convolution of $x[n]$ and $y[n]$ if

(a) length($y[n]$) = 500, (b) length($y[n]$) = 50,

(c) length($y[n]$) = 8, and (d) length($y[n]$) = 3? (25 scores)

[ANS]

(a) length(y[n]) = 500

(b) length(y[n]) = 50

(c) length(y[n]) = 8

(d) length(y[n]) = 3