

參考網頁ytm計算器的計算公式,code裡有用到一個套件 scipy。接著驗算現值pv

例子說明

```
sp.irr([-820, 30.0, 30.0, 30.0, 30.0, 30.0, 30.0, 30.0, 30.0, 30.0, 30.0, 1030.0])
期初投入820, 每期拿回利息=30, 最後一期拿回本金+利息=1030
820 = 30/(1 + 0.05376) + 30/(1 + 0.05376)^2 + ... + 1030/(1 + 0.05376)^10
```

$$P_{t} = \frac{1}{(1+y_{t})^{t}}$$

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$$P_{t}^{-\frac{1}{t}} = 1 + y$$

$$P_{t}^{-\frac{1}{t}} = 1 + y_{t} \qquad \Rightarrow \Upsilon_{t} = -\frac{1}{t} \Omega_{n} P_{t}$$

$$\Rightarrow y_t = P_t^{-\frac{1}{t}} - 1$$

Discrete:

$$(1+f_{\bar{x}\bar{j}})^{\bar{j}-\bar{\lambda}} = \frac{(1+y_{\bar{j}})^{\bar{j}}}{(1+y_{\bar{k}})^{\bar{\lambda}}}$$

$$\Rightarrow f_{\bar{\lambda}\bar{j}} = \left(\frac{(1+y_{\bar{j}})^{\bar{j}}}{(1+y_{\bar{i}})^{\bar{\lambda}}}\right)^{\frac{1}{\bar{j}-\bar{\lambda}}} - 1 \qquad \Rightarrow f_{\bar{\lambda}\bar{j}} = \left(\frac{p_{\bar{\lambda}}}{p_{\bar{j}}}\right)^{\frac{1}{\bar{j}-\bar{\lambda}}} - 1$$

$$(1+f_{\bar{x}\bar{j}})^{\bar{j}-\bar{\lambda}} = \frac{(1+y_{\bar{j}})^{\bar{j}}}{(1+y_{\bar{\lambda}})^{\bar{\lambda}}} = \frac{(1+y_{\bar{j}})^{\bar{j}}}{(1+y_{\bar{\lambda}})^{\bar{\lambda}}} = \frac{\rho_{\bar{\lambda}}}{(1+y_{\bar{\lambda}})^{\bar{\lambda}}} = \frac{\rho_{\bar{\lambda}}}{\rho_{\bar{j}}}$$

$$\Rightarrow f_{\lambda j} = \left(\frac{P_{\lambda}}{P_{j}}\right)^{\frac{1}{j-\lambda}} - 1$$

Continuous:

$$e^{F_{x_{\overline{j}}} \cdot (j-\lambda)} = \frac{e^{-Y_{x} \cdot \lambda}}{e^{-Y_{\overline{j}} \cdot j}} \qquad e^{F_{ij} \cdot (j-\lambda)} = \frac{P_{\lambda}}{P_{j}}$$

$$e^{F_{ij}(j-\lambda)} = e^{-Y_j \cdot j - Y_i \cdot \lambda}$$

$$e^{F_{ij}(j-\lambda)} = e^{Y_j \cdot j - Y_i \cdot \lambda}$$

$$\Rightarrow F_{\lambda j} = \frac{1}{j-\lambda} (Y_j \cdot j - Y_j \cdot \lambda)$$

3 given price

$$e^{F_{ij}\cdot(j-\lambda)}=\frac{P_{\lambda}}{P_{i}}$$

$$e^{F_{n,j}(j-\lambda)} = e^{Y_j \cdot j - Y_k \cdot \lambda}$$
 $\Rightarrow F_{n,j} = \frac{1}{j-\lambda} \cdot \mathcal{N}_n \frac{P_n}{P_j}$

Spot rate & Forward rate公式推導

```
In [21]: # spot rate calculator
    decimal.getcontext().rounding = "ROUND_HALF_UP"

    t = int(input('Duration of spot rate (years): '))
    p = float(input('Price of {} year unit zero-coupon bond: '.format(t)))

y = p ** (-1/t) -1
Y = (-1/t) * math.log(p)

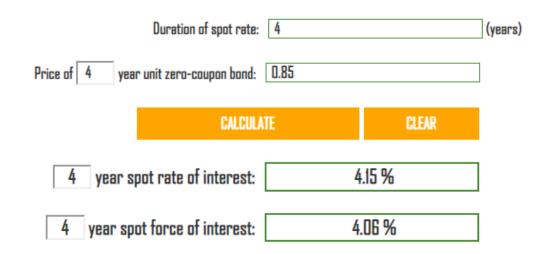
y = Decimal(100*y).quantize(Decimal('0.00'))
Y = Decimal(100*Y).quantize(Decimal('0.00'))

print('{} year spot rate of interest: '.format(t) + str(y) + '%')

print('{} year spot force of interest: '.format(t) + str(Y) + '%')

Duration of spot rate (years): 4
Price of 4 year unit zero-coupon bond: 0.85
4 year spot rate of interest: 4.15%
4 year spot force of interest: 4.06%
```

Enter the following details:



程式碼驗證spot rate

```
In [14]: # forward rate calculator
          decimal.getcontext().rounding = "ROUND HALF UP"
          t = int(input('Time due for the beginning of forward rate (years): '))
          r = int(input('Duration of forward rate (years): '))
          Pt = float(input('Price of {} year unit zero coupon bond: '.format(t)))
          Pt r = float(input('Price of {} year unit zero coupon bond: '.format(t+r)))
          v = (Pt/Pt r)**(1/r) - 1
          Y = (1/r) * math.log(Pt/Pt r)
          y = Decimal(100*y).quantize(Decimal('0.00'))
          Y = Decimal(100*Y).quantize(Decimal('0.00'))
          print('{} year forward rate of interest beginning {} years from now: '.format(r,t) + str(y) + '%')
          print('{} year forward force of interest beginning {} years from now: '.format(r,t) + str(Y) + '%')
         Time due for the beginning of forward rate (years): 5
         Duration of forward rate (years): 4
          Price of 5 year unit zero coupon bond: 0.83
          Price of 9 year unit zero coupon bond: 0.6743
          4 year forward rate of interest beginning 5 years from now: 5.33%
         4 year forward force of interest beginning 5 years from now: 5.19%
                          Enter the following details:
                                         Time due for the beginning of forward rate: 5
                                                                                                  (years)
                                                    Duration of forward rate: 4
                                                                                                   (years)
                                         Price of 4 year unit zero coupon bond: 0.83
                                         Price of 9 year unit zero coupon bond: 0.6743
                                                                CALCUALTE
                                                                                        CLEAR
               4 year forward rate of interest beginning 5 years from now:
                                                                                 5.33 %
```

5.19 %

4 year forward force of interest beginning 5 years from now:

程式碼驗證 forward rate