EX. For a 15% BEY, a 10-year bond with a coupon rate of 10% paid semiannually sells for

$$P = 5 \times \frac{1-[1+0.15/2]^{-2\times 10}}{0.15/2} + \frac{100}{[1+0.15/2]^{2\times 10}} = 74.5138$$

So, 15% is the yield to maturity if the bond sells for 74.5138

$\begin{cases} \text{par bonds} \\ \text{premium bonds} \\ \text{discount bonds} \end{cases}$ coupon = F · coupon rate

EX. A bond with a 10% coupon rate paid semiannually, with clean price 111.2891. The maturity date is 3/1 1995, and the settlement date is 7/1 1993, There are 60 days between 7/1 1993 and the next coupon date 9/1 1993. (30/360)

Ans: $AI = \frac{10}{2} \times (1 - \frac{\overset{w}{\downarrow}}{}\!\!\frac{60}{180}) = \frac{10}{3}$

$$\text{clean price} + AI = \sum_{i=0}^{n-1} \frac{C}{(1+\frac{r}{m})^{w+i}} + \frac{F}{(1+\frac{r}{m})^{w+n-1}}$$

$$\Rightarrow 111.2891 + \frac{10}{3} = \sum_{i=0}^{4-1} \frac{5}{(1+\frac{r}{2})^{\frac{1}{3}+i}} + \frac{100}{(1+\frac{r}{2})^{\frac{1}{3}+4-1}}$$

Accrued Interest

- The yield to maturity is the r when PV is the full price

$$\text{clean price} + AI = \sum_{i=0}^{n-1} \frac{C}{(1+\frac{r}{m})^{w+i}} + \frac{F}{(1+\frac{r}{m})^{w+n-1}}$$

$$AI = C \times \frac{\text{number of days from last coupon payment to the settlement date}}{\text{number of days in the coupon period}} = C \times (1-w)$$

Price Volatility

- Volatility measures how bond prices respond to interest rate changes

$$\Rightarrow \frac{\frac{\partial P}{\partial y}}{P}$$

Macaulay Duration : $MD \triangleq \frac{1}{P} \sum_{i=1}^{n} \frac{C_i}{(1+y)^i} \cdot i$

MD of a level-coupon bond : $MD = \frac{1}{P} \left[\sum_{i=1}^{n} \frac{iC}{(1+y)^i} + \frac{nF}{(1+y)^n} \right]$

Modified Duration : modified duration $\triangleq -\frac{\partial P}{\partial y} \cdot \frac{1}{P} = \frac{MD}{1+y}$

EX. modified duration = 11.54 with yield of 10% , if 10% → 10.1%

the approximate percentage price change will be ?

percent price change $\approx$ - modified duration × yield change

$$-11.54 \times (0.1\%) = -1.154$$

Effective Duration : $\frac{P_- - P_+}{P_0 (y_+ - y_-)}$

Convexity : convexity (in periods) $\triangleq \frac{\partial^2 P}{\partial y^2} \frac{1}{P}$

Coupon Rate : 票面利率

YTM : 當期殖利率 = 全年票息 ÷ 當期債卷價格