COMP250 REVIEW

Lecture 1

Algorithm: a sequence of instructions or operations for manipulating data to produce some result

Grade School Arithmetic:

1. Addition of 2 numbers:

$$\begin{array}{rrrrr} & a[3] & a[2] & a[1] & a[0] \\ + & b[3] & b[2] & b[1] & b[0] \\ \hline r[4] & r[3] & r[2] & r[1] & r[0] \end{array}$$

- Use arrays to store each number
- For each column, add r[i] = a[i] + b[i] and carry value to next column
- The result array needs to have 1 more slot than max(a, b)

2. Multiplication of 2 numbers
- Slow: a*b = add a times the number b
- Fast:
  - Create 2D array and compute all rows in advance
  - For each column, sum of the single digits in the row and add the carry
  - Could instead use a running sum and add as we compute the rows

3. Division
- Slow: use num = q * b + r. Subtract num by b, and add 1 to q, until r is smaller than b.
- Fast: long division

Lecture 2

Logarithms:

$$\log_b a = \log_b c * \log_c a$$
$$a^{\log_b c} = c^{\log_b a}$$

Modulo:
- Quotient remainder theorem: a = b*q + r
- Remainder: a mod b = r
- Convention: find positive remainder

$$-3 \; mod \; 5 = 2$$
$$-3 = (-1) * 5 + 2$$

- However, in JAVA:

$$-3 \; mod \; 5 = -3$$

- In j=Java defined as:

$$a = \frac{a}{b} * b + a\%b$$

- Addition:

$$(a + b) \; mod \; c = \big((a \; mod \; c) + (b \; mod \; c)\big) \; mod \; c$$

- Multiplication:

$$(a * b) \; mod \; c = \big((a \; mod \; c) * (b \; mod \; c)\big) \; mod \; c$$

Shift elements using mod:
- left shift 2 spots:

$$index = (i - 2 + arr.length) \; \% \; arr.length$$

- right shift 2 spots:

$$index = (i + 2) \; \% \; arr.length$$

Lecture 3

Base representation:

$$(132)_5 = 1 * 5^2 + 3 * 5^1 + 2 * 5^0 = 42$$

Base conversion: m, from decimal to base b
- divide m by b and prepend the remainder of division
- m = previous quotient

Binary Arithmetic:
- Carry when your sum is greater or equal to your base
- carry = # of times you can pull out base from sum

$$carry = (a_i + b_i + carry) \; \% \; base$$

For all rows, and assign carry to r[n]

Lecture 3

Primitive type:
- predefined by the language
- named by a reserve keyword

In Java:
- Integer: byte, short, int, long
- Real num: float, double
- Boolean
- Char

- One can represent 2^n values with n bits
- One needs $\log_2 x$ (round up) to represent x different values
- One needs $floor((\log_2 m) + 1)$ to represent a positive integer M

Type	Keyword	Size	Values
Very Small Integer	byte	8-bits	$[-128, 127]$
Small Integer	short	16-bits	$[-2^{15}, 2^{15}-1]$
Integer	int	32-bits	$[-2^{31}, 2^{31}-1]$
Big Integer	long	64-bits	$[-2^{63}, 2^{63}-1]$
Low Precision Reals	float	32-bits	-
High Precision Reals	double	64-bits	-
True/False	boolean	1-bit	[true, false]
One character	char	16-bits	-

Char
- single quotes
- single character
- A character set: an ordered list of character, **each character corresponds to a number** (i.e. Unicode)

Type Casting:
- Convert back and forth between variables of different types
- Explicit cast is not necessary when going from int -> double, but necessary in reverse!
- In general, widening (going to more bits) does not need explicit type conversion
- Char is more narrow than short, short can store negative values while char cannot, though they are of the same size

Lecture 5

Packages
- A group of classes: each class is a package member
- A class: a group of methods
- A method: an ordered group of commands

File and folder naming rules in Java:
1. Name of class must match name of file
2. Folder path must match exactly the package name. Each period in package name is a subfolder

Using a package member outside its package:
- write full path
- import the specific class of the package
- import entire package

- Java automatically imports java.lang and current package
- Note: import line is not necessary! It simply saves the trouble of writing the full path

Objects and Classes
- class is a blueprint/template for a type of object
- object: an instance of some class

A class:
- Has attributes/fields
- Constructor (method to create an object)
- Other methods

If you write your own constructor, you **no longer have access** the default constructor!

Inheritance: Every class' constructor implicitly calls the **super()** constructor. If the constructor is overwritten in the super class, the subclass must have a corresponding **super(with parameters)**

Nested Class
- Define class within another class (outer class)
- Benefits: organize, encapsulation (control over data), readable and maintainable code

Modifiers
- keywords that you add to class/method/variable's definition to change their meaning

	Class	Package	Subclass (same pkg)	Subclass (diff pkg)	World
public	+	+	+	+	+
protected	+	+	+	+	
no modifier	+	+	+		
private	+				

+ : accessible
blank : not accessible

- Only when a member is visible, it can be inherited
- Outer classes can only be declared **public** or **package private**

Encapsulation:
- Wrapping data and code acting on that data in one unit. Better control data
- Make all fields private
- Provide getters and setters

Non-access modifiers:
- static
- final
- abstract

Static:
- fields, methods, and nested classes
- Associated with the entire class and not to a specific instance
- Independent from one specific instance of the class
- **Static fields: class variables**
- Non-static method belongs to an instance of the class
- **Non-static fields: instance variable**

Final:
- Variables, methods, and classes
- Value can never be changed after initial assignment
- Final fields must be initialized

Abstract:
- methods and classes

UML Diagram
3 sections:
1. Class name
2. Attributes/Fields
3. Methods

+ if attributes/fields or methods are public and – if they are private
Underline if method or variable is static
#: protected
Italics: abstract class

Local Variables and Fields
- local variables are declared inside a method or block
- fields (class and instance variables) are declared inside a class but outside a method

Differences:
- Scope: where they can be accessed
- local variables: only accessible within method or block of declaration
- class variables: accessible from any method or block within the class
- instance variables: requires existence of an object

Access:
- local variables: cannot have access modifiers. Cannot access local variables from other classes or methods

- Fields: access modifiers, can be accessed from methods within the class and from other classes if public

Lecture 6

Inheritance
- a subclass inherits all **public (or protected) fields and methods** from its superclass. Constructors are the only thing not inherited

Object Class
- the only class without a superclass, every class is implicitly a subclass of Object

In the subclass:
- Declaring a same field: hiding the inherited field
- Writing a **non-static** method with same signature: **overriding**. The non-static method in the **subclass** is called.
- Writing a **static** method with the same signature: **hiding**. The static method in the **superclass** is called.

Overloading
- Two or more methods in the same class with same name but different parameters

Overriding
- Two (non-static) methods with same signature and return type

Constructor
- Default constructor
- Java automatically inserts a call to the no-argument constructor of superclass

Super
- Access members of superclass
- Access overridden methods in the superclass

Modifiers and Inheritance
- A final class cannot be extended!
- A final method cannot be overridden!

Lecture 7

toString()
- toString() for **Object** class returns a string consisting of name of class, @, and the hashcode (unsigned hexadecimal representation of hashcode)
- toString() in **String** class returns the object itself

Equals()
- In Object class, equals() is true if and only if they are the same object with the same address
- In String class, equals() is true if both String objects have the same sequence of characters
- In ArrayList class, equals() is true if both lists have same size and corresponding, identical pairs of elements

Clone()
- In ArrayList: clone() returns a shallow copy. Only 1 arraylist

Type Conversion
- Implicit upcasting is allowed
- Explicit downcasting: results in run-time error if type casted is of the wrong type
- **Casting does NOT change the object itself, it just labels it differently**

Instanceof
- test whether an object is an instance of the specified type
- returns: true or false
- if the class is a subclass of the specified class: returns true
- a method to prevent error when downcasting

Lecture 8

Polymorphism

- JVM calls the **appropriate** method for object that is referred to in each variable. It does not call the method that is defined by the variable's type

```
Dog snoopy = new Beagle();
```

- At **compile-time**: the compiler uses `bark()` in `Dog` class to validate statement
- During **run-time**: JVM invokes `bark()` from `Beagle` class since `snoopy` is referring to a `Beagle` object

Abstract
- If you want a class to contain a particular method, but would like the implementation to be specified by the subclass
- **declared without implementation**

Abstract Methods
- the class containing it must be abstract
- every subclass of the current class must either override (implement) the abstract method or declare Itself as abstract

Abstract Classes
- declared with `abstract` keyword
- can have abstract and non-abstract methods
- cannot be instantiated
- can have constructors (called when instance of subclasses are created) and static methods
- can have `final` methods, will force subclass not to change body of method

Lecture 9

Arrays
- O(1)

List (array list, linked list, etc.)
Array List:
1. get(i):
   - check i range
   - return a[i]
2. set(i,e):
   - check i range
   - a[i] = e
3. add(i,e):
   - check if array is full, if full, create bigger array and copy all elements forward
   - shift all elements after i one spot down the array
   - insert element at i and increase size by 1
4. remove(i)
   - shift all elements after i one spot up
   - decrease size by 1

Note: add(i,e) allows adding to i == size, but set(i,e) does not allow this!

ArrayList class
- Grows the size of array by 50% when it is full and a new element is added

ArrayList object
- a private field for size of arraylist
- a private field that references an array object

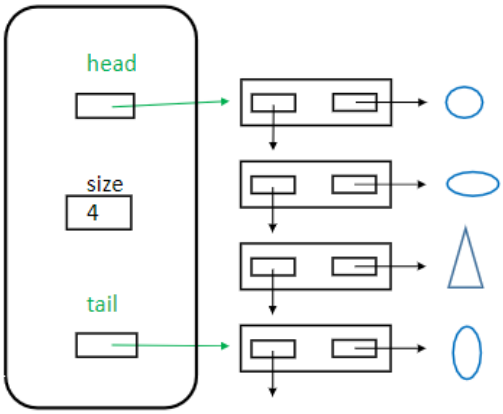
Lecture 10

ArrayList: slots are in consecutive locations in memory, but objects can be anywhere
LinkedList: nodes and objects can be anywhere in memory

Singly Linked List:
- consists of a sequence of nodes, reference to the first (head) and last (tail) node
- each node object points to the next node, and points to the object

1. addFirst(e):
   - create new node
   - nextnode points to current head
   - head points to the new node
   - increase size by 1
2. removeFirst():
   - head points to the next
   - decrease size by 1
3. addLast(e):
   - create new node
   - tail.next points to the new node
   - tail points to the new node
   - increase size by 1
4. removeLast():
   - create a tmp to point to head
   - iterate tmp until it tmp.next is tail
   - tail point to what tmp is pointing
   - decrease size by 1

ArrayList and LinkedList
- In linked list, addFirst(e) and removeFirst() does not depend on the number of elements
- In arraylist they do, because of shifting



1 + 4 + 4 = 9

SLinkedList SNode Shape

Lecture 11

Doubly Linked List
- Each node has a reference to the next node and previous node
- Motivation: able to access both previous and next nodes

1. removeLast():
   - tail points to previous
   - size decreases by 1
2. remove(i):
   - get the node by traversing either from bottom or top
   - node.next.prev points to node.prev
   - node.prev.next points to node.next

- edge case: null next field and null prev fields in singly and doubly linked lists
- Dummy Nodes: avoid edge cases by adding two nodes (dummyHead and dummyTail) and the beginning and end of DLL

Time Complexity

	ArrayList	LinkedList	DLL
addFirst	O(N)	O(1)	O(1)
removeFirst	O(N)	O(1)	O(1)
addLast	O(1)*	O(1)	O(1)
removeLast	O(1)	O(N)	O(1)
remove(i)	O(N)	O(N)	O(N)

*if array is full

Java Enhanced for loop
- iterates through the list

Lecture 12

Sorting
- arranging items in a ordered list
- O(n^2): selection, bubble, insertion
- O(n*logn): heap, merge, quick

Bubble Sort
- simplest sorting algorithm
- Idea: repeatedly iterate through the list and swap adjacent elements if they are in the wrong order
- finishes when no sorting takes place in an iteration
- worst case: O(n^2)
- best case: O(n)

```
sorted = false
i = 0
while (!sorted) {
   sorted = true
   for j from 0 to list.length - i -2 {
      if(list[j] > list[j+1]) {
         swap(list[j], list[j+1])
         sorted = false
      }
   }
   i++
}
```

Selection Sort
- Idea: consider the list as if it was divided into two parts, one sorted and the other one unsorted
- Procedure:
   - select smallest element in unsorted part
   - swap element to the beginning of list
   - change where the sorted and unsorted parts are divided and repeat process
- worst case: O(n^2)
- best case: O(n^2)

```
for delim from 0 to N-2 {
   min = delim
   for i from delim+1 to N-1 {
      if(list[i] < list[min]) {
         min = i
      }
   }
   if(min != delim) {
      swap(list[min], list[delim])
   }
}
```

Repeat until list is all sorted (~N times)

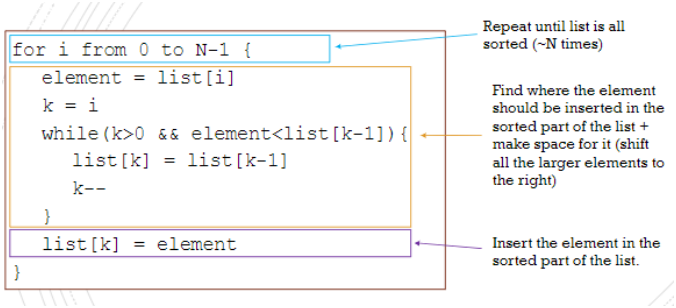
Find the index of the min element in the unsorted part of the list

Swap the min element in the first position of the unsorted part of the list.

Insertion Sort
- Idea: consider the list as if it was divided into two parts, one sorted and the other unsorted
- Procedure:
   - select first element of unsorted part

- insert the element at correct position
- change where the sorted and unsorted parts are divided

- Inserting: similar to adding element to array, shift elements ahead to make hole, and fill hole
- best case: O(n) list already ordered and no shifting is needed
- worst case: O(n^2) slowest when sorted in reverse order



Lecture 13

Abstract Data Type (ADT)
- defines a data type by the values of data and operation on the data
- ignores details of implementation

Stack ADT
- push and pop
- Last in first out (LIFO)

	Push(e)	Pop()
Array list*	addLast(e)	removeLast()
SLL	addFirst(e)	removeFirst()
DLL	Either row above	

*Java arraylist does not have addfirst or removelast methods

Algorithm: decide if parentheses are matched.

```
while (there are more tokens) {
   token = get next token
   if token is a left parenthesis
        push(token)
   else {                              // token is a right parenthesis
            if stack is empty
               return false
            else {
               pop left parenthesis from stack
               if popped left parenthesis doesn't match the right parenthesis
                    return false
            }
        }
}
return stack.empty   // true if stack is empty, false if not.
```

Lecture 14

Queue ADT
- enqueue(e), dequeuer
- first in first out (FIFO)

	Enqueue(e)	dequeue()
ArrayList	addLast(e)	removeFirst()
SLL	addLast(e)	removeFirst()
DLL	Either row above	

- with ArrayList it is slow due to shifting
- even with expanding array it is still bad

Circular Array

$$tail = (head + size - 1) \% length$$

- Increasing array length:
  - Head stays at same index
  - Head moves to slot 0

Note:
Since Java might return a negative number for mod operations, we could do:

$$tail = (head + size - 1 + length) \% length$$

To ensure tail is valid

```
enqueue( element ){
    if ( queue.size == queue.length)  {
        // increase length of array

       create a bigger array tmp[ ]   // e.g. 2*length
       for i = 0  to queue.length  - 1
            tmp[i] = queue[ (head + i) %  queue.length ]
       head = 0
       queue = tmp
    }
    queue[ (head + size) % queue.length] = element            (*)
    queue.size = queue.size + 1
}
```

Note that we don't have a tail variable here. Instead, it can be computed anytime with:
tail = (head + size – 1) % length

- in the new array, head is slot 0 of new array

```
dequeue( ){    // check that queue.size > 0
     element = queue[head]
     queue.size = queue.size – 1
     head = (head+1) % length
     return  element
}
```

- Using two stacks, we can create a queue

Java API
- API = application program interface

Java `interface`
- reserved word
- like a class, but only method signatures are defined

Lecture 15

Interfaces
- public or package-private (default)
- all **methods** are by default public and abstract
- all **fields** are by default public, static, and final
- interfaces cannot be instantiated
- An interface `extends` another interface and cannot extend another class

Syntax of interface
- `interface` instead of `class`
- implicitly abstract, does not need `abstract` keyword
- all methods are implicitly abstract

Inheritance
- `implements` instead of `extends`
- specifies what a class must do, not how
- a class can **implement one or more interfaces**, to achieve multiple inheritance
- if a class implements the interface, but does not implement all methods specified in the interface, then the class must be `abstract` class

Interface VS Abstract
- Abstract:
  - not all methods have to be abstract
  - `abstract` keyword must be added to class declaration
  - can contain implemented methods and instance variables
  - useful when some general methods should be implemented and specified by subclasses

- Interface:
  - all methods are abstract by default (no keyword)
  - interfaces are implicitly abstract
  - no methods can be implemented and only constants (final static fields) can be declared
  - interfaces are useful in situation where all properties should be implemented

Generics in Java
- A generic type is a class or interface that is parametrized over types

Usage of Interfaces:
- Interfaces define new data types
- We can create variables of these data types and assign the variables any instance created from the classes that implemented the interface.
- In the case of a method parameter, whenever an object of the interface type is required, any instance of any of the classes that implemented the interface can be used

Comparable
- comparable defines a natural ordering
- used to define an ordering on objects of user-defined class
- contains only one method: compareTo(object)

Lecture 16

Iterable and Iterator
- Objects of type `Iterable` are representations of a series of elements that can be iterated over (e.g. a specific ArrayList)
- Objects of type `Iterator` allows you to iterate through objects that represent a collection (a series of elements)

Java `Iterable` Interface
- A class that implements `Iterable` needs to implement the iterator() method. The iterator() method returns: an object of type `Iterator` that can be used to iterate through elements of that class
- A class that implements the `Iterator` needs to implement the methods hasNext() and next()
- The iterator() method returns an iterator to the start of the collection. You can traverse the collection using hasNext() and next().

How to Implement `Iterable` Interface
- Generally, when we write a class that implements the interface `Iterable`, we also write a class implementing the interface `Iterator`. Often, such class is defined as an inner class.
- The reason is: to implement `Iterable`, we need to implement iterator(). Since iterator() needs to return a `Iterator` type object, we need a class to create such an object.
- However, iterators cannot reset and start over again. The only way to restart iteration is to call iterator() method to obtain a new iterator that points to the head of the provided list.

Lecture 17

The `Class` class
- the compiler translates the .java file to a .class file
- a “class descriptor”, created during runtime by JVM, is an instance of the class `Class`.
- instances represent classes and interfaces in a running Java application

getClass():

- returns the **run-time** class of the calling object

getSuperclass():
- method from class `Class`
- returns class representing the superclass of the calling class

Memory Allocation
- Heap: used by java runtime to allocate memory to Object and JRE classes. Objects are stored in Heap.
- Stack: used for execution of a thread. Threads contain method specific values and references to other objects in the heap that are getting referred from the method.

Stack (LIFO data structure)
- stores methods
- each method block has all the local values, as well as **references** to other objects that are being used by the method
- after a method terminates, its block will be erased
- the values stored in each block are accessible only from that particular method
- local variables and method parameters

Heap
- stores objects
- no specific order in reserving blocks
- objects created in heap space has global access and can be referenced from anywhere of the application
- Garbage Collection runs on heap memory to free memory used by objects that doesn't have any reference
- object instances and fields

Permanent Generation:
- contains all data required by JVM to describe the classes and methods used at runtime
- methods and static fields

Lecture 18

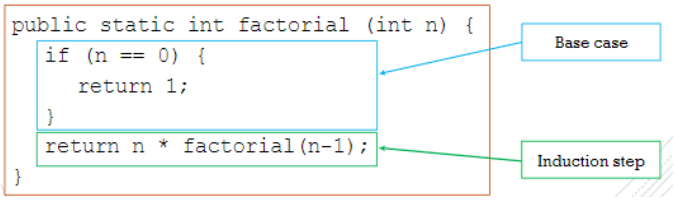
Recursive Definition
- Base clause: basic element of the set
- Inductive clause: how to generate new elements of the set from old ones
- Final clause: states that no other element is part of set

(Weak) Mathematical Induction
- Base case: show all properties hold for initial elements of set
- Inductive: assume property holds for some element n, and show the property holds for any element generated from n
- Conclusion: property holds for all elements

(Strong) Mathematical Induction
- Prove property holds for all n

Lecture 19

Factorial



Fibonacci

```
public static int fibonacci (int n) {
   if(n==0 || n==1) {
      return 1;
   }
   return fibonacci(n-1)+fibonacci(n-2);
}
```

Reverse list

```
public static void reverse(List list) {
   if(list.size()==1) {
      return;
   }
   firstElement = list.remove(0); // remove first element
   reverse(list); // now the list has n-1 elements
   list.add(firstElement); // appends at the end of the list
}
```

Sorting a list

```
public static void sort(List list) {
   if(list.size()==1) {
      return;
   }
   minElement = removeMinElement(list);
   sort(list); // now the list has n-1 elements
   list.add(0, minElement); // insert at the beginning of list
}
```

Tower of Hanoi

```
tower(n, start, finish, other) { // e.g. tower(5,A,B,C)
   if(n==1) {
      move from start to finish.
   } else {
      tower(n-1, start, other, finish)
      tower(1, start, finish, other)
      tower(n-1, other, finish, start)
   }
}
```

- Anything recursion can do, iteration can do
- Anything iteration can do, recursion can do

Lecture 20

DEC to BIN

- Euclid's algorithm
- recursively call method on $\frac{n}{2}$

Power
- recursively call method on $n - 1$
- Complexity: $O(log_2 n)$
- Better implementation:

```
power( x, n) {
   if (n == 0)
      return 1;
   else if (n == 1)
      return x;
   else{
      tmp = power(x, n/2);
         if (n%2==0)
            return tmp*tmp;     // one multiplication
         else
            return tmp*tmp*x    // two multiplications
   }
}
```

Binary Search
- Idea:
  - first compare key with middle element
  - if key is greater, search second half and discard first half
  - if key is smaller, search first half and discard second half
  - if key equals middle element, return index
- Complexity: $O(\log_2 n)$

Implementation
- keep track of left and right indices denoting section of list needs to be searched

Iterative implementation:

```
binarySearch(list, key){
   left = 0                          } initialize left and right
   right = list.size() - 1           }
   while(low <= high){ // until there are elements to search
      mid = (left + right)/2     // compute mid
      if(list[mid]==key)         // compare element with key
         return mid
      else {
         if(key<list[mid])
            right = mid -1       // update right
         else
            left = mid + 1       // update left
      }
   }
   return -1            // key not in list
}
```

Recursive implementation:

```
binarySearch(list, key, left, right){
if(low <= high){
      mid = (left + right)/2
      if(list[mid]==key)
         return mid
      else {
         if(key<list[mid])              Add recursive calls
            binarySearch(list, key, left, mid-1)
         else
            binarySearch(list, key, mid+1, right)
      }
   }
   return -1
}
```

Lecture 21

Merge Sort
- divide and conquer
- Idea:
  - partition list into two halves
  - sort each half recursively
  - merge the sorted half **maintaining order**

```
mergesort(list){
      if (list.size() == 1)                          Base case
            return list
      else {
         mid = (list.size() - 1) / 2                 Partition
         list1 = list.getElements(0,mid)
         list2 = list.getElements(mid+1, list.size()-1)
         list1 = mergesort(list1)                    Recursive sort
         list2 = mergesort(list2)
         return merge(list1, list2)                  Merge
      }
}
```

Where merge(list1, list2) is implemented as:

```
merge(list1, list2){
      list = ...initialize with empty list...
      while (!list1.isEmpty() && !list2.isEmpty()){
            if (list1.get(0) < list2.get(0))
                  list.addlast(list1.removeFirst())
            else
                  list.addlast(list2.removeFirst())
      }                                              Pick elements to add until one of the two lists is empty
      while (!list1.isEmpty())
            list.addlast(list1.removeFirst())
      while (!list2.isEmpty())                       Then add the remaining elements
            list.addlast( list2.removeFirst())
}
```

- Complexity: $O(n \log_2 n)$

Quick Sort
- divide and conquer
- Idea:
  - pick an element as the pivot
  - partition list by placing all elements smaller than the pivot to its left, and all elements larger than the pivot to the right
  - sort left and right parts recursively
  - repeat until there is nothing left to sort

Picking the pivot:
- always pick first element

- always pick last element
- pick random element
- pick median as pivot

Implementation
- compare elements with pivot
- if element is smaller then move wall and swap place with element left of wall
- stop when the right index is reached
- call quicksort on left and right parts until base cases are reached

```
quickSort(list, leftIndex, rightIndex) {
     // Base case:
     if(leftIndex >= rigthIndex) {
      return; // done!
     } else { // recursive step:
      i ← placeAndDivide(list, leftIndex, rightIndex)
      // i = index where the pivot is placed
      quickSort(list, leftIndex, i-1)
      quickSort(list, i+1, rightIndex)
     }
}
```

- Implementation of placeAndDivide():

```
placeAndDivide(list, leftIndex, rightIndex) {
     // pick the right most element
     pivot ← list.get(rigthIndex)
     // place the wall to the left
     wall ← leftIndex -1
     // go through all elements and compare them to the pivot
     for(int i=leftIndex; i< rigthIndex; i++) {
          if(list.get(i)<pivot) {
               wall++; // move wall
               swap list.get(i) list.get(wall)// move element behind wall
          }
     }
     swap list.get(rigthIndex) list(wall+1) // move pivot next to wall
     return wall+1;
}
```

Merge Sort VS Quick Sort
- Merge sort typically uses extra lists, hurts performance for big lists

Lecture 22

Linear Data Structures:
- array
- linked list

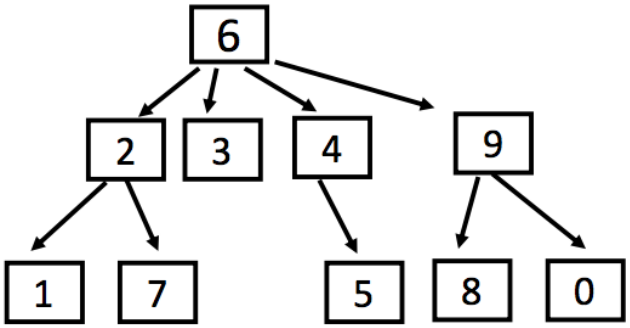
Non-Linear Data Structures
- tree
- graph

Tree Terminology:
- root: highest node
- directed edge: ordered pair of nodes (from, to)
- sibling: has same parent
- internal nodes: non-empty file directories, points to some other node
- external nodes, leaves: files or empty directories
- path: a sequence of nodes
- length of a path: number of edges in the path (number of nodes – 1)
- ancestor: higher node in a path
- descendent: lower node in a path
- depth or level: length of the path from the root to the node
- height: the **maximum** length of a path from the node to a leaf

- Every node except root is a child, and has exactly 1 parent
- A tree with n nodes has $n-1$ nodes

Recursive definition of a rooted tree:
- consists of subtrees

Represented using lists:



$(6(217)3(45)(980))$

Lecture 23

Tree Traversal
- Recursive: (Depth first)
  - Preorder traversal: root, left, right
  - Postorder traversal: left, right, root
- Non-Recursive:
  - Stack: depth first with different order: root, right, left

```
treeTraversalUsingStack(root){
    initialize empty stack s
    s.push(root)
    while s is not empty {
      cur = s.pop()
      visit cur
      for each child of cur
          s.push(child)
    }
}
```

  - Queue: breadth first, root, left to right

```
treeTraversalUsingQueue(root){
    initialize empty queue  q
    q.enqueue(root)
    while q is not empty {
        cur = q.dequeue()
        visit cur
        for each child of cur
            q.enqueue(child)
    }
}
```

Lecture 24

Binary Tree
- each node has at most two children
- Max number of nodes and height: $n = 2^{h+1} - 1$
- Min number of nodes and height: $n = h + 1$
- Traversal:
  - preorder: root, left, right
  - postorder: right, left, root
  - inorder: left, root, right

Prefix, infix, postfix expressions:
  - prefix: root, left, right
  - infix: left, root, right
  - postfix: left, right, root

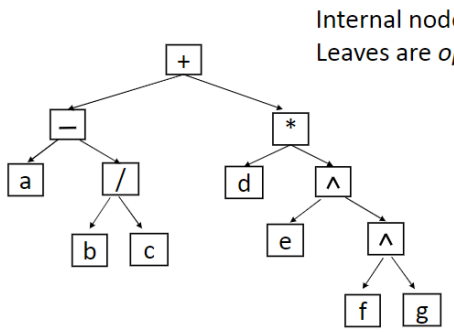
Prefix: polish notation
Postfix: reverse polish notation (RPN)

Algorithm: Use stack to evaluate postfix expression
- push when token is a base expression
- pop two elements and push in the evaluated integral element when token is an operator

```
s = empty stack
cur =  first token of expression list
while (cur != null){
    if ( cur  is a base expression )
        s.push( cur )
    else{                                  // cur is an operator
        operand2  =  s.pop()
        operand1  =  s.pop()
        operator  = cur.element        //  for clarity only
        s.push( evaluate( operand1,  operator,  operand2 )   )
    }
    cur = cur.next
}
```

Expression Tree:
- operators are internal nodes
- operands are leaves

a – b / c + d * e ^ f ^ g ≡ (a – (b / c)) + (d ∗ (e ^ (f ^ g)))



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Lecture 25

Binary Search Tree (BST)
- keys are comparable, unique (no duplicates)
- for each node, all descendents in left subtree are less than the node, and all descendents in the right subtree are more than the node
- an inorder (left, root, right) traversal on a BST visits the nodes in the natural order defined by the key
- a new key is always a leaf

1. find(root, key):
   - recursive
   - if key < root.key, call find(root.left, key)
   - else, call find(root.right, key)
2. findMin(root):
   - keep traversing left until leftmost node is reached
3. findMax(root):
   - keep traversing right until a rightmost node is reached
4. add(root, key):
   - compare at each node
   - if smaller, call add(root.left, key)
   - if larger, call add(root.right, key)
   - when null reached, add
   - does nothing when the same key is already present
5. remove(key):
   - searches for the key by calling remove recursively until a match
   - if left or right of the current node is null, set the left or right child to be the new root, return the root of this subtree
   - if neither is not null, then the value of the root is the value of the **smallest element in the right subtree**
   - the right subtree is then the subtree with the minimum element removed

	Best case	Worst case
findMin()	O(1)	O(n)
findMax()	O(1)	O(n)

find(key)	O(1)	O(n)
add(key)	O(1)	O(n)
remove(key)	O(1)	O(n)

Lecture 26

Priority Queue (ADT)
- assume a set of comparable elements
- heap is a good implementation

Complete Binary Tree
- Binary tree of height h such that every level less than h is full, and all nodes at level h are as far to the left as possible

Min Heap
- Complete binary tree with unique comparable elements, such that **each node's element is less than its children's elements**

1. add(element): ("upheap")
   - create new node at next available leaf position
   - while current element is smaller than direct parent, swap places
2. removeMin(): ("downheap")
   - replace root with last element
   - compare with the **smaller** of two childs and swap if necessary
3. remove(element):
   - remove element and replace slot with last element
   - compare with parent:
     - if smaller than parent: call upheap()
     - if larger than parent: call downheap()

Parent child relations in **array implemented** heap structure:
- 0th index is not used
- $parent = \frac{child}{2}$
- $left = 2 * parent$
- $right = 2 * parent + 1$

- Implementation of add(element):

```
add(element ){
   size = size + 1        // number of elements in heap
   heap[ size ] = element  // assuming array
                           //  has room for another element
   i = size

  //  the following is sometimes called "upHeap"

   while ( i > 1  and  heap[i] < heap[ i/2 ]){
      swapElements( i, i/2 )
      i = i/2
   }
}
```

Lecture 27

Building a Heap
- upheap() elements in array, add element to last and check with parent to see if child is smaller than parent, if so, swap.
- Repeat until i = 1 (reached root)

- implementation of upheap():

```
buildHeap(){
   //  assume that an array already contains size elements
   for (k = 2;  k <=  size;  k++)
        upHeap( k ) }
}

upHeap(k){
     i = k
     while (i > 1) and ( heap[i] <  heap[i / 2] ){
          swapElement(i, i/2)
          i = i/2
     }
}
```

- Best Case: $O(n)$. Elements already satisfy the heap parent-child ordering constraint, no swaps
- Worst Case: $O(n \log_2 n)$. Original list is from large to small.

- Implementation of removeMin():

```
removeMin( ){
    tmpElement = heap[1]          //  heap[0] not used.
    heap[1] = heap[size]
    heap[size] = null
    size = size - 1
    downHeap( size )              //   next slide
    return  tmpElement
}
```

- Implementation of downHeap:

```
downHeap( maxIndex ){

  i = 1
  while ( 2*i <= maxIndex){          // if there is a left child
    child = 2*i
    if child < size {                // identify smaller child
      if (heap[child + 1] < heap[child])
          child = child + 1
    }
    if (heap[child] < heap[ i ]){    //  Swap if necessary.
      swapElements(i , child)
      i = child
    }
    else return                      //  Avoid infinite loop.
  }
}
```

Heapsort

- build heap with n elements, then call down heap n times
- each time swapping the first and last elements, then calling downheap considering a smaller heap $(n-1)$
- finally reverse the heap array to obtain a sorted array

```
heapsort(list){

  heap = buildheap(list)

  for i = 1 to n − 1 {
    swapElements( heap[1], heap[n + 1 - i])
    downHeap( 1, n - i)
  }
  return reverse(heap)
}
```

- we could build a "max heap" and remove the maximum element each time to avoid **reverse** at the end
- best case: $O(n\ log_2 n)$
- worst case: $O(n\ log_2 n)$

Lecture 28

Map
- set of pairs {x, f(x)}

Map (ADT)
- set of (key, value) pairs
- for each key, there is at most one value
- each (key, value) pair is called an entry

1. put(key, value):
   - map key and place value
   - If map previously contained a mapping for the key, old value is replaced by the specified value, and the previous value is returned. Otherwise, return null.
2. get(key)
   - returns the value corresponding to the mapping of the key. Returns null if no entry is found.
3. remove(key)
   - removes the entry for the key, if it is present, and returns the corresponding value. Returns nul if the map contained no mapping for the key.

Data Structure for Maps:
1. ArrayLists, SLL, or DLL
2. Comparable keys:
   ArrayList:
   - get(key) : $O(\log_2 n)$
   - put(key, value): $O(n)$
   - remove(key): $O(n)$
   BST: (depends on tree, assume balanced here)
   - put(key, value): $O(log_2 n)$
   - get(key): $O(log_2 n)$
   - remove(key): $O(log_2 n)$
   minHeap:
   - put(key, value): $O(log_2 n)$
   - get(key): $O(n)$
   - remove(key): $O(log_2 n)$
3. Keys are unique positive integers in small range:
   Array:
   - array of type V (value): O(1) access
4. General Case:
   - keys might not be comparable
   - define a mapping of keys to large range positive integers (i.e. code)

Java Object.hashcode()
- object's (base) address in JVM memory (unique)
- default definition:

$$obj1.hashcode() == obj2.hashcode()$$

Is equivalent to:

$$obj1 == obj2$$

Java String.hashcode()
- for each string, defines an integer

$$s.hashcode() = \sum_{i=0}^{n-1} s[i] * x^{n-1-i}$$

Where $n = s.length$ and $x = 31$

- Java uses **Horner's Rule** for efficient polynomial evaluation $O(n)$

```
h = 0
for (i = 0; i < s.length; i++)
    h = h*31 + s[i]
```

Lecture 29

Hashing

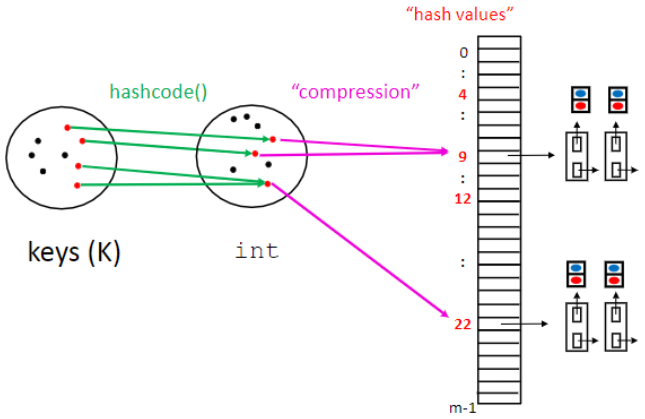
- Keys are hashed into hashcodes using hashcode()
- hashcodes are compressed into hashvalues using %

Compression: $i \rightarrow |i| \ mod \ m$
- where m is the length of the array

Hash Function: hashcode and compression

Collision:
- when 2 or more **hashcodes** map to the same **hash value**
- when 2 or more **Keys K** map to the same **hashcode**, and mapping to the same **hash value**

Solution:
- Hash Table (or Hash Map): each array slot holds a SLL of entries
- each array slot + linked list is called a bucket, there are m buckets



Load Factor of Hash Table

$$= \frac{number\ of\ entries}{number\ of\ buckets, m}$$

- typically load factor below 1
- In Java HashTable and HashMap classes, default maximum load factor is 0.75

Good Hash: no collisions
Bad Hash: many collisions

Performance of Hash Maps
- If load factor is less than 1 and if hash function is good

1. put(key, value): $O(1)$ in practice
2. get(key): $O(1)$ in practice
3. remove(key): $O(1)$ in practice
4. contains(value): need hash table traversal $O(n + m)$
5. getKeys(): need hash table traversal $O(n + m)$
6. getValues(): need hash table traversal $O(n + m)$

Java HashMap <K,V> class
- specify initial number of buckets and load factor in constructor
- Hash Function: Use key's hashcode(), take absolute value, and compress it by taking mod of the number of buckets

Java HashSet <E> class
- similar to HashMap, but there are no values. Use it to store a set of objects of some type
- add(e), contains(e), remove(e), etc.

Lecture 30

Graphs

Terminology
- Directed graph: a set of vertices, and a set of **ordered** pairs of these vertices called edges
- Undirected graph: a set of vertices and a set of **unordered** pairs called edges
- In degree: # of incoming edges to the vertex
- Out degree: # of outgoing edges from the vertex
- Path: a sequence of edges such that end vertex of one edge is the start vertex of the next edge. **No vertex may be repeated except first and last!**
- Cycle: a path such that the last vertex is the same as the first vertex
- Direced Acyclic Graph: directed graph with no cycles

Graph ADT

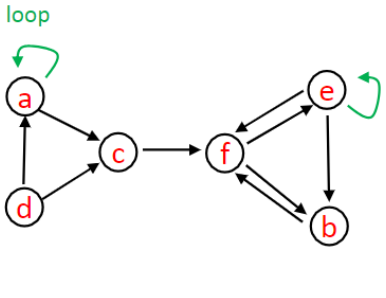
Implementation
- graphs are a generalization of trees, but does not have a root vertex
- outgoing edges: children of a vertex in a tree
- incoming edges: parent (s)

1. Adjacency List (for edges)
- shows the outgoing edges for the vertices

Implementation of a Graph class in Java:
- **Vertex** class: adjacency list (of outgoing edges) using ArrayList and the element itself
- **Edge** class: Vertex the edge is pointing to and the weight, a double, of the edge

- **HashMap** object containing all Vertices as values with keys (could be a String, etc.)

2. Adjacency Matrix
- a square matrix



	a	b	c	d	e	f
a	1	0	1	0	0	0
b	0	0	0	0	0	1
c	0	0	0	0	0	1
d	1	0	1	0	0	0
e	0	1	0	0	1	1
f	0	1	0	0	1	0

Adjacency List VS Adjacency Matrix
1. Sparse graph: 10000 vertices and 20000 edges. Save space
- Use adjacency list: matrix has 10000^2 slots, while list uses 20000 objects
2. Dense graph: 10000 vertices and 20000000 edges. Save space
- Use adjacency matrix: matrix has 10000^2 but list has 20000000 objects
3. Check adjacency as quickly as possible.
- Use adjacency matrix. $O(1)$ access
4. Insert new vertex
- Use adjacency list. For matrix, you need to add new row and new row in array, and copy everything over

Lecture 31

Graph Traversal (Recursive)
- need to specify starting vertex
- visits all nodes that are "reachable" by a path from starting vertex

Depth-First:
- order of nodes visited depends on order of nodes in the adjacency list

```
depthFirst_Graph(v){
  v.visited = true
  for each w such that (v,w) is in E   // w in v.adjList
    if  ! (w.visited)                  // avoids cycles
       depthFirst_Graph(w)
}
```

Breadth-First: (Queue)
- implement using a queue
- for each child vertex, if not visited, visit and add to queue
- for the first node, add to queue and set state to visited

Graph Traversal (Non-Recursive)
- Use stack or queue

Depth-First: (Stack)
- Last pushed in vertex is popped and visited

```
graphTraversalUsingStack(v){
  initialize empty stack s
  v.visited = true
  s.push(v)
  while (!s.empty) {
    u =  s.pop()
    for each w in u.adjList{
      if (!w.visited){              // The only new part.  Why?
         w.visited = true
         s.push(w)
      }
    }
  }
}
```

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Breadth-First: (Queue)
- Vertices are visited in enqueuer order

```
graphTraversalUsingQueue(v){
  initialize empty queue q
  v.visited = true
  q.enqueue(v)
  while (! q.empty) {
    u =  q.dequeue()
    for each w in u.adjList{
      if (!w.visited){
         w.visited = true
         q.enqueue(w)
      }
    }
  }
}
```

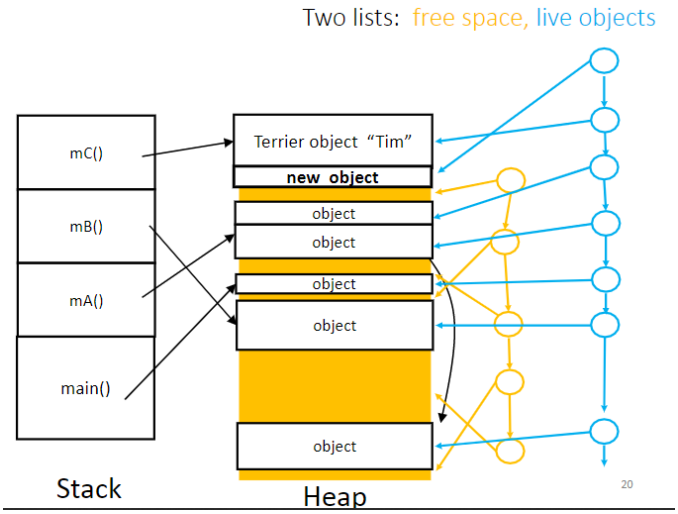
Lecture 32

Garbage Collection
- Object is not referenced = garbage
- Live object: referenced either from call stack variable or from an instance variable in a live object
- JVM maintains a linked list of all objects, stores Object.hashCode() of each object

Garbage Collection: Mark and Sweep
1. build a graph and identify live objects
  - vertices are reference variables in call stack and the objects in heap
  - edges are references (arrows)

  - for each reference variable on call stack, traverse graph and mark each object reached "visited"

2. remove garbage
- for objects not visited, JVM has another list (free space list) to keep track, and marks these objects as available space



- new objects can be added, where there is a big enough gap in free space
- garbage collection is needed again when there is no gap big enough for the new object. Objects can also be moved around if heap space is too fragmented
- program temporarily stops to do garbage collection, not good for real time applications

Graph Traversal Applications
- the set of web pages on the world wide web define a graph. Webpage are vertices and hyperlinks are edges.
- Google traverses this graph by following the links and retrieves as many web pages as it can find
- Google then builds a graph data structure:
  - vertices are web pages
  - edges are hyperlinks within the web pages
  - keys for the vertex map are the URL's

Google PageRank: Importance of V
- which set of pages link to v, and how important are these pages
- how many other pages does each w point to

Naïve PageRank:

$$R(v) = \sum_{incoming\ edges\ (w,v)\ to\ v} \frac{R(w)}{N_{out}(w)}$$

- Rank of v is determined by summing up all the in/out degree ratios of all the w pointing to v.
- Since importance is relative, initialize R(w) and calculate R(v), then set the new R(v) as the initial rank. R(v) is a vector containing ranks of all web pages

Google Search Engine:
- web crawler downloads all reachable web pages
- build/updates the graph
- compute page rank for each web page
- build a map: maps keywords (keys) to URLs of web pages containing keyword (values)

User:
- enter keywords

Google Search Engine:
- for each key, get value
- for multiple keys, compute intersection of lists of web pages
- output resulting list, in order of PageRank

Lecture 33

Recurrence Relation
- a recurrence relation is a sequence of numbers where the n-th term depends on previous terms
- $t(n)$: time to execute a recursive algorithm as a function of the input size n

Reverse a list:

$$t(n) = c + t(n-1)$$
$$t(n) = c + c + t(n-2)$$
$$t(n) = c + c + c + t(n-3)$$
$$\dots$$
$$t(n) = c(n-1) + t(1)$$
$$t(n) = cn + t(0)$$

- $t(0)$ is the base case

Tower of Hanoi

$$t(n) = c + 2t(n-1)$$
$$t(n) = c(2^n - 1) + 2^n\, t(0)$$

- $n = k$

Binary search

$$t(n) = c + t\left(\frac{n}{2}\right)$$
$$t(n) = c\ log_2 n + t(1)$$

- assume $n = 2^k$

Geometric Series Sums:

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$$1 + 2 + 4 + 8 + \cdots + 2^{n-1} = \frac{1}{1-2} - \frac{2^n}{1-2} = 2^n - 1$$

$$1 + x + x^2 + x^3 + \cdots + x^{n-1} = \frac{x^n - 1}{x - 1}$$

Lecture 34

Merge Sort

$$t(n) = cn + 2t\left(\frac{n}{2}\right)$$

- cn from copying array

$$t(n) = c\, n\, log_2 n + n\, t(1)$$

- $n = 2^k$
- Total calls $f(n)$ to mergesort is $2n - 1$

$$1 + 2 + 4 + \cdots + n = 2^{k+1} - 1 = 2n - 1$$

- this is also the case for adding n elements to an empty arraylist and resizing $log_2 n$ times
- also the case for putting n elements into empty hash table and rehasing $log_2 n$ times

Quick Sort

- best case: same as merge sort

$$t(n) = cn + 2t\left(\frac{n}{2}\right)$$

- cn from partition based on pivot
- best case when two sublists have same size

$$t(n) = c\, n\, log_2 n + n\, t(1)$$

- worst case: one sublist has only 1 element

$$t(n) = cn + t(n-1)$$

$$t(n) = c\frac{n(n+1)}{2}$$

Reduce chance of unbalanced partition for quicksort:
- Median of three: take median of the first, last, and median of the list and set as pivot

Lecture 35

Formal Definition of Limit:

- A sequence $t(n)$ has a limit if, for any $\epsilon > 0$, there exists an n_0 such that for any $n \geq n_0$, $|t(n) - t_\infty| < \epsilon$

Bounded Definition:

- Let $t(n)$ and $g(n)$ be two functions, where $n \geq 0$. Then $t(n)$ is asymptotically bounded above by $g(n)$ if there exists n_0 such that, for all $n \geq n_0$, $t(n) \leq g(n)$

Definition of Big O: (upper bound)

- Let $t(n)$ and $g(n)$ be two functions, where $n \geq 0$. $t(n)$ is $O(\, g(n)\,)$ if there exist two positive constants n_0 and c such that, for all $n \geq n_0$, $t(n) \leq c\, g(n)$

Lecture 36

Definition of Big Omega (Ω)**:** (asymptotic lower bound)

- Let $t(n)$ and $g(n)$ be two functions, where $n \geq 0$. We say $t(n)$ is $\Omega(\, g(n)\,)$, if there exist two positive constants n_0 and c such that, for all $n \geq n_0$. $t(n) \geq c\, g(n)$

- The two statements are equivalent:

$$f(n)\ is\ O(g(n))$$
$$g(n)\ is\ \Omega(f(n))$$

Definition of Big Theta (Θ):

- Let $t(n)$ and $g(n)$ be two functions of $n \geq 0$. We say $t(n)$ is $\Theta(g(n))$ if $t(n)$ is both $O(g(n))$ and $\Omega((g(n))$

Lecture 37

- Some $t(n)$ does not have a "simple" $g(n)$ such that $t(n)$ is $\Theta(g(n))$

The time for an algorithm to run depends on:
- constant factors, implementation dependent
- size of input
- values of input

For any algorithm:
- $t_{best}(n)$ is the runtime on the best case input(s)
- $t_{worst}(n)$ is the runtime on the worst case input(s)

Operations/Algorithms for Lists		$t_{best}(n)$	$t_{worst}(n)$
add, remove, find an element (array list)		$\Theta(1)$	$\Theta(n)$
add, remove, find an element (doubly linked list)		$\Theta(1)$	$\Theta(n)$
insertion sort		$\Theta(n)$	$\Theta(n^2)$
selection sort	best = worst	$\Theta(n^2)$	$\Theta(n^2)$
binary search (sorted array list)		$\Theta(1)$	$\Theta(\log n)$
mergesort		$\Theta(n \log n)$	$\Theta(n \log n)$
quick sort		$\Theta(n \log n)$	$\Theta(n^2)$

- $O(\,)$ is an asymptotic upper bound
- $\Omega(\,)$ is an asymptotic lower bound

Determining Complexity from Limits

- Case 1:

If

$$\lim_{n\to\infty} \frac{t(n)}{g(n)} = 0$$

Then $t(n)$ is $O(g(n))$, $t(n)$ is not $\Omega(g(n))$, $t(n)$ is not $\Theta(g(n))$

- Case 2:

If

$$\lim_{n\to\infty} \frac{t(n)}{g(n)} = \infty$$

Then $t(n)$ is not $O(g(n))$, $t(n)$ is $\Omega(g(n))$, $t(n)$ is not $\Theta(g(n))$

- Case 3:
If

$$\lim_{n\to\infty} \frac{t(n)}{g(n)} = 0 < \alpha < \infty$$

Then $t(n)$ is $O(g(n))$, $t(n)$ is $\Omega(g(n))$, $t(n)$ is $\Theta(g(n))$

Complexity Order

$$O(n!) \geq O(2^n) \geq O(n^2) \geq O(n\log_2 n) \geq O(n)$$
$$\geq O(\log_2 n) \geq O(1)$$