ECSE 543A
NUMERICAL METHODS IN ELECTRICAL ENGINEERING

Assignment 1

Posted: 15-September-2020 Due: 14-October-2020

(**Note:** Please comment your program listings and explain how they are structured in your report.)

1. (a) Write a program to solve the matrix equation $Ax=b$ by Choleski decomposition. A is a real, symmetric, positive-definite matrix of order n.

   (b) Construct some small matrices (n = 2, 3, 4, …,10) to test the program. Remember that the matrices must be real, symmetric and *positive-definite*. Explain how you chose/created the matrices.

   (c) Test the program you wrote in (a) with each small matrix you built in (b) in the following way: invent an x, multiply it by A to get b, then give A and b to your program and check that it returns x correctly.

   (d) Write a program that reads from a file a list of network branches (J_k, R_k, E_k) and a reduced incidence matrix, and finds the voltages at the nodes of the network. Use the code from part (a) to solve the matrix problem. Explain how the data is organized and read from the file. Test the program with a few small networks that you can check by hand. Compare the results for your test circuits with the analytical results you obtained by hand. Cleary specify each of the test circuits used with a labeled schematic diagram.

2. Take a regular N by N finite-difference mesh and replace each horizontal and vertical line by a 10 kΩ resistor. This forms a linear, resistive network.

   (a) Using the program you developed in question 1, find the resistance, R, between the node at the bottom left corner of the mesh and the node at the top right corner of the mesh, for N = 2, 3, …, 15. (You will probably want to write a small program that generates the input file needed by the network analysis program. Constructing the incidence matrix by hand for a 225-node network can be tedious.)

   (b) In theory, how does the computer time taken to solve this problem increase with N, for large N. Are the timings you observe for your practical implementation consistent with this? Explain your observations.

   (c) Modify your program to exploit the sparse nature of the matrices to save computation time. What is the half-bandwidth b of your matrices? In theory, how does the computer time taken to solve this problem increase now with N, for large N. Are the timings you observe for your practical sparse implementation consistent with this? Explain your observations.

   (d) Plot a graph of R versus N. Find a function $R(N)$ that fits the curve reasonably well and is asymptotically correct as N tends to infinity, as far as you can tell.

3. Figure 1 shows the cross-section of an electrostatic problem with translational symmetry: a coaxial cable with a square outer conductor and a rectangular inner conductor. The inner conductor is held at 110 volts and the outer conductor is grounded.

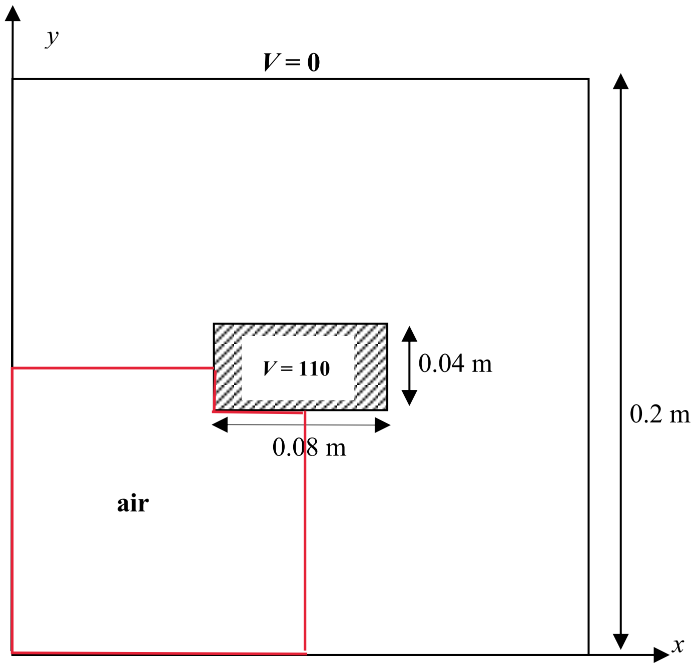

Figure 1.

(a) Write a computer program to find the potential at the nodes of a regular mesh in the air between the conductors by the method of finite differences. Use a five-point difference formula. Exploit at least one of the planes of mirror symmetry that this problem has. Use an equal node-spacing, h, in the x and y directions. Solve the matrix equation by successive over-relaxation (SOR), with SOR parameter ω. Terminate the iteration when the magnitude of the residual at each free node is less than 10^{-5}.

(b) With $h = 0.02$, explore the effect of varying ω. For 10 values of ω between 1.0 and 2.0, tabulate the number of iterations taken to achieve convergence, and the corresponding value of potential at the point $(x, y) = (0.06, 0.04)$. Plot a graph of number of iterations versus ω.

(c) With an appropriate value of ω, chosen from the above experiment, explore the effect of decreasing h on the potential. Use values of $h = 0.02, 0.01, 0.005$, etc, and both tabulate and plot the corresponding values of potential at $(x, y) = (0.06, 0.04)$ versus $1/h$. What do you think is the potential at (0.06, 0.04), to three significant figures? Also, tabulate and plot the number of iterations versus $1/h$. Comment on the properties of both plots.

(d) Use the Jacobi method to solve this problem for the same values of h used in part (c). Tabulate and plot the values of the potential at $(x, y) = (0.06, 0.04)$ versus $1/h$ and the number of iterations versus $1/h$. Comment on the properties of both plots and compare to those of SOR.

(e) Modify the program you wrote in part (a) to use the five-point difference formula derived in class for non-uniform node spacing. An alternative to using equal node spacing, h, is to use smaller node spacing in more "difficult" parts of the problem domain. Experiment with a scheme of this kind and see how accurately you can compute the value of the potential at $(x, y) = (0.06, 0.04)$ using only as many nodes as for the uniform case $h = 0.01$ in part (c).