MIME – 262 Properties of Materials in Electrical Engineering
Fermi function, doping, conductivity
11/20/2013

Nov 20th's lecture is about Fermi function. What is the Fermi function?

Fermi function gives the probability of finding an electron at a given energy level
So basically, Fermi function f (energy level) = probability

Among different energy levels, there is also a special energy level called the Fermi energy level

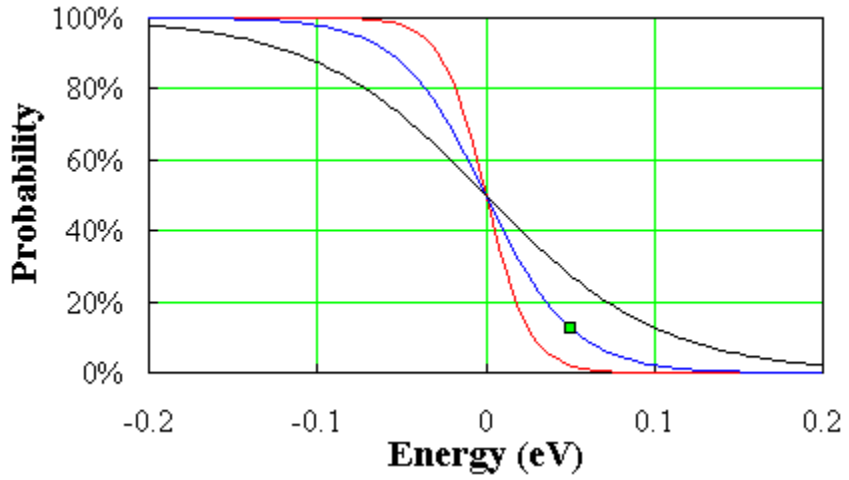
Fermi energy level is the energy level where the probability of finding an electron is 50%
So basically, Fermi function f (Fermi energy level) = 50%

To illustrate this in a clearer manner, consider the table and a graph below

Table: Let each box be allowable locations for electrons to occupy with respect to different energy levels
Note each u (up) and d (down) correspond to spin configurations of the electron e^-

0.2 eV	e^- (u)			
0.1 eV	e^- (u)			
0 eV	e^- (u)	e^- (d)		
-0.1 eV	e^- (u)	e^- (d)	e^- (u)	
-0.2 eV	e^- (u)	e^- (d)	e^- (u)	e^- (d)

Graph: Probability of finding electron (%) vs. different energy levels (eV), basically plot of Fermi function
Note each lines are with respect to different temperatures $T_1 < T_2 < T_3$



What to understand from the table and the graph:

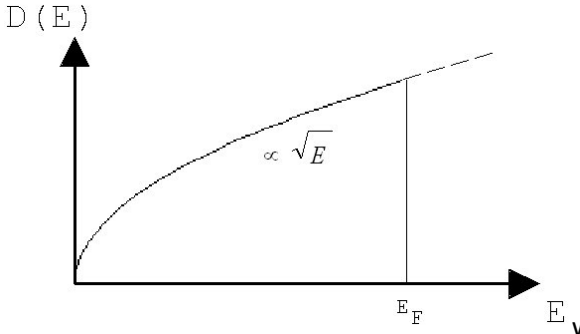
- Looking at the graph, at 0 eV the probability of finding an electron is 50%. This is the Fermi level and this also corresponds to 0 eV on the table, since half of the boxes are filled by electrons

    - From this fact one can derive the following equation #electrons at a given energy level = #allowable locations at the given energy level x probability of finding electron at the given energy level
- The Fermi function becomes steeper as temperature goes down
    - At 0K, the Fermi function becomes a negative step function at the Fermi level

In this lecture the term “Density of states” is introduced
Density of states = # of allowable places for electron to reside at a given energy level E
Hence, density of states is a function of energy level which outputs # of allowable places for electron
Density of states D(E) = # of allowable places for electron

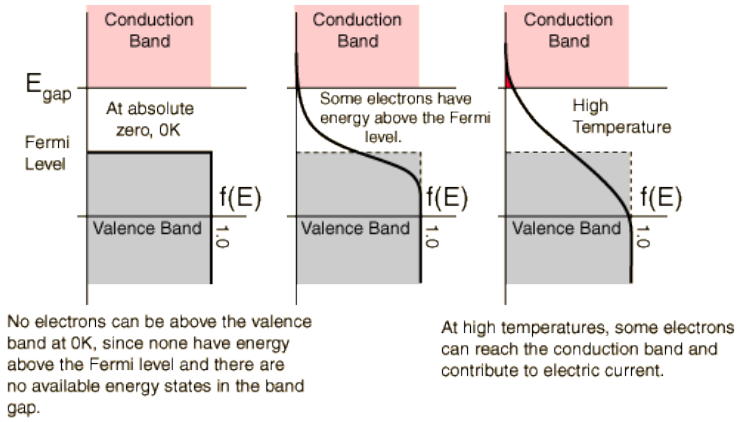
So basically, we could have basically stated the previous equation:
#electrons at a given energy level = #allowable locations at the given energy level x probability of finding electron at the given energy level
as
# of electrons at energy level E = Density of states at E x Fermi function f(E)

In our table example above, D(E) = 4 regardless of whatever E
However in reality, D(E) looks more like this:



where E corresponds to different energy levels
We could learn from this graph that density of states is proportional to $E^{0.5}$

It is also possible to make connection of this concept to the band model.



No electrons can be above the valence band at 0K, since none have energy above the Fermi level and there are no available energy states in the band gap.

At high temperatures, some electrons can reach the conduction band and contribute to electric current.

Observation regarding band model and Fermi function:

The Fermi function (viewed vertically) serves as a proof that there are very few electrons in the conduction band (assuming T > 0K) and that it is abundant of electrons in the valence band. One should also remember that the conduction band is called the conduction band because it enables space for electrical conduction. Electrical conduction occurs due to the fact that electrons have a lot of space to move around. On the other hand in the valence band, the electrons have very small amount of space for movement, and hence, is very difficult to have electrical conduction.

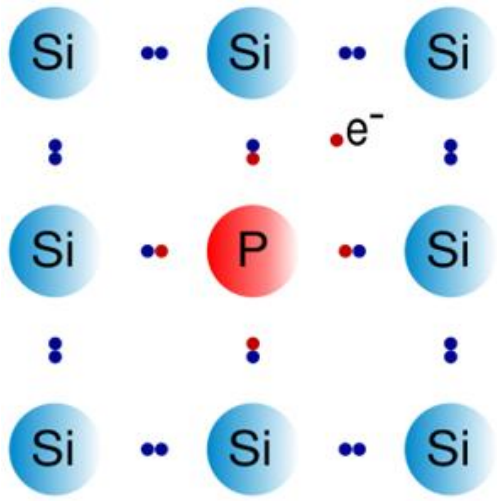
Next, the concept "doping" is introduced

Doping adds impurity to a pure Silicon matrix to make donors or acceptors to allow semi conductive ability

Semiconductors that haven't been doped are called **intrinsic semiconductors**, and semiconductors that have been doped are called **extrinsic semiconductors**.

To illustrate, consider doping purely semiconductor material Silicon Si. We will talk about extrinsic semiconductors.

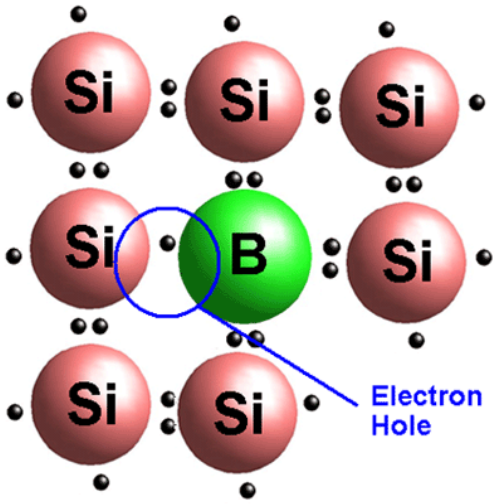
This is what happens if Phosphorus is doped into Silicon



Given each atoms want 8 valence electrons, Silicon has 4 valence electrons and Phosphorus has 5 valence electrons at its neutral state (+/- 0eV), when Silicon and Phosphorus form covalent bond, there will be 1 extra electron. This electron can move around freely, thus allowing for conduction.

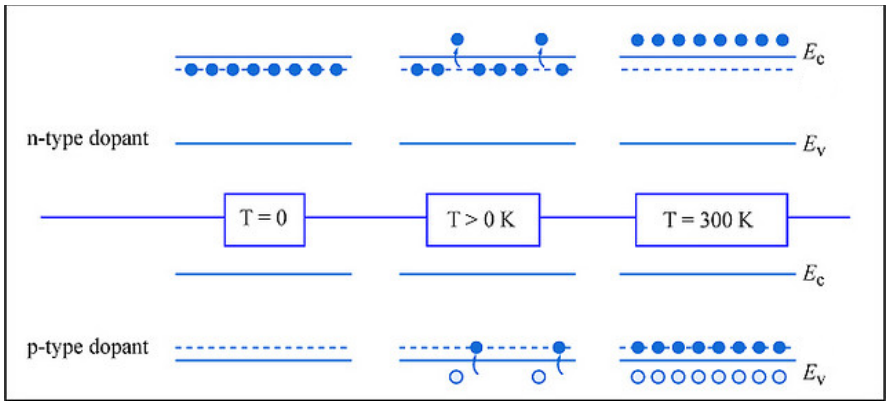
This doping has added net negative charge due to the abundant electron. Hence, Phosphorus is known as a **donor** for adding electron, and is known as a **n-type (negative-type) dopant.**

This is what happens if Boron is doped into Silicon



Again, given Silicon has 4 valence electrons and Boron has 3 valence electrons at their electrically neutral state, the covalent bonding of Boron into Silicon results in 1 less electron required to have a full 8 electron valence shell. The location where there is a missing electron is known as the "**hole**," and the important thing is that the electron nearby could move to this hole, where electron movement defines conduction. Since adding Boron has introduced holes in which electrons can move to, and since it results in net positive charge, Boron is known as an **acceptor**, and a **p-type** (**positive-type**) **dopant**. Note that when an electron moves to a hole, it creates a hole at where the electron used to reside in. Hence, it could be visualized as the holes moving too.

We could observe this in relation to the band model:

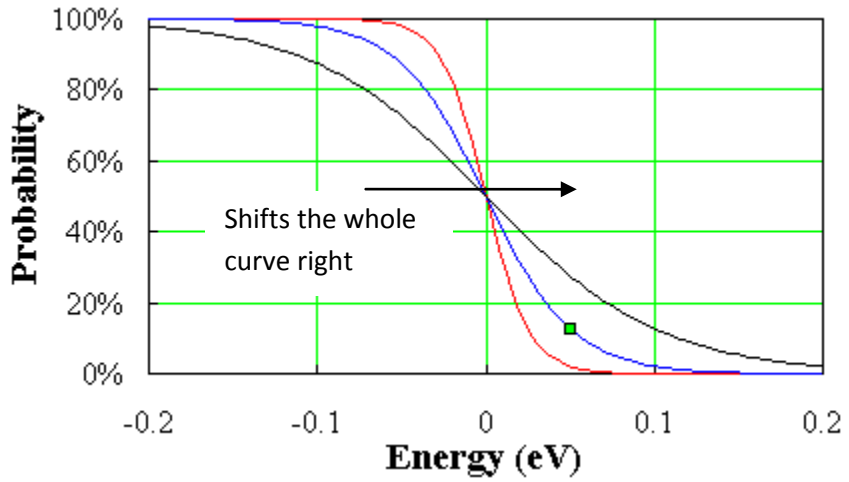


E_c and E_v correspond to conduction band and valence band energy levels respectively.
This graph conveniently shows how donors (n-type) and acceptors (p-type) interact within the band at different temperatures.

For the n-type dopant, the electrons loosely attached to the Phosphorus (in our example) are at energy level just lying below Ec. Hence they could go into the conduction band with a small amount of energy. Remember that there is very few # of electrons in the conduction band making it possible for electrons to move around a lot, which directly results in conduction.

For the p-type dopant, there are electron holes at energy levels just above Ev. Hence, the electrons within the valence band could go into those holes with a small amount of energy. This creates holes in the valence band, making more space for electrons to move within the valence band, which directly results in conduction.

Relationship between Fermi function and adding donors
How does adding donors affect the Fermi function?
We account for increase in donors by shifting F(E) right, *increasing the Fermi level in the process*



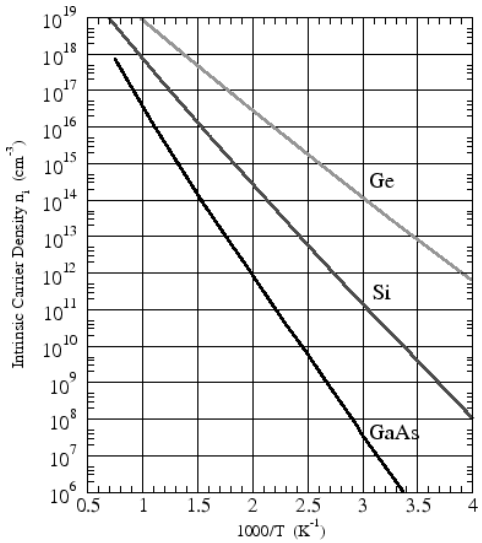
Next, we will talk about conductivity σ within intrinsic semiconductors (no dopant)
σ is proportional to $n_e\mu_e + n_h\mu_h$
where μ_e is the electron mobility, μ_h is the hole mobility
n_e is the # of electron per volume, n_h is the # of holes
In specific terms, $\sigma = e(n_e\mu_e + n_h\mu_h)$

Note that in metals: $\sigma = e(n_e\mu_e)$ because there are no holes.

Let's give specific mathematical terms for n_e and n_h.
$n_e = N_c e^{\frac{-(E-EF)}{kT}}$
$n_h = N_v e^{\frac{-EF}{kT}}$

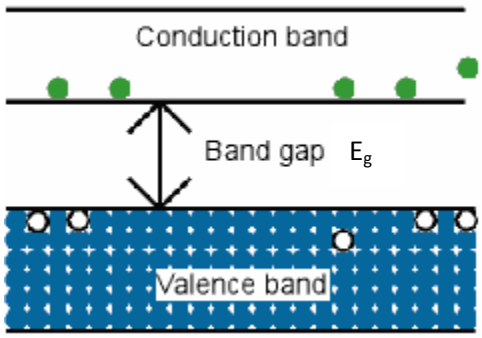
Hence, upon a bit of mathematical manipulation, we could infer that:
n_e is proportional to $\frac{1}{T}$
It is proportional at slope $\frac{Eg}{2}$, where E_g is the energy level gap between conduction band energy level E_c and valence band energy level E_v.
$E_g = E_c - E_v$

This can be illustrated in the graph below:



Why is this the case though?

Increasing temperature allows for the electrons in the valence band to gain E_g to go to the conduction band. Temperature is basically required for electrons in the valence band to go to the conduction band. Therefore, lower the temperature, less electrons in the valence band there are, because less electrons can gain energy over the band gap energy.



Electrons in the valence band which gain energy over Eg can go to the conduction band.

This results in two things:

- Charge carriers in conduction band
- Holes in valence band

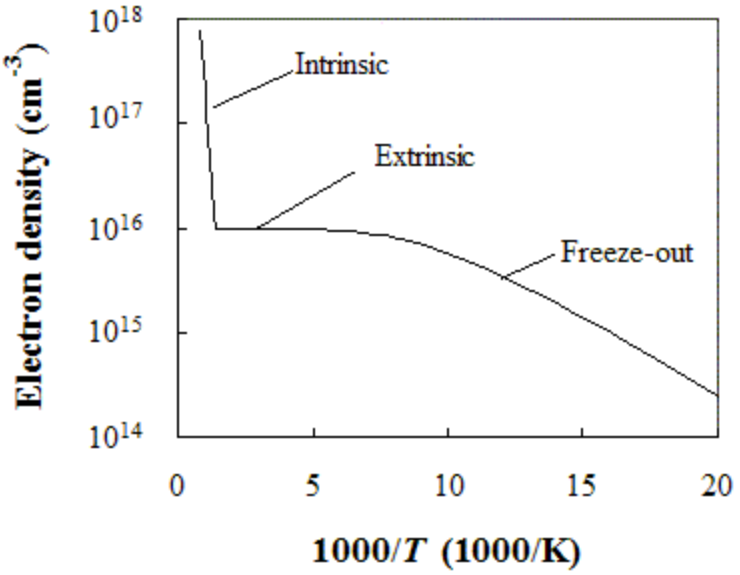
Both of these carriers and holes contribute to be electrical conductivity, this underlying the conductivity equation $\sigma = e(n_e\mu_e + n_h\mu_h)$ qualitatively

In intrinsic Silicon where number of carriers and holes are equal as explained above:

$n_e = n_h = (\mathrm{NvNc})^{\frac{1}{2}} e^{\frac{-Eg}{2kT}}$

Next, we will talk about the carriers within extrinsic semiconductors

First, look at the graph below:



This graph shows carrier density vs. inverse temperature.

There are three sections where the curve behaves in the following regions:

- Intrinsic (electrons within valence shell)
- Extrinsic (electrons arising from n-type dopant)
- Freeze-out

In extrinsic semiconductors, there are two sources of carriers. Here begins the explanation:

First, the reader should remember that abundant electrons from a n-type dopant and holes from a p-type dopant have very small energy requirements to go into the valence band and the conduction band, respectively. Hence, at some mediocre temperatures, there will be no change in electron density because at those temperatures, some electrons aren't limited in terms of their energy to move over the energy barrier. This gives rise to the extrinsic region of the curve.

For the extrinsic region, we have mentioned the electrons from dopant. In the intrinsic region, the electrons residing in the valence band travels to the conduction band due to the high temperature, giving rise to increasing number of electrons within the conduction band.

In the freeze-out region of the curve, most guess-ably the lack of temperature limits the high-energy electrons from the n-type dopant and obviously the electrons in the valence shell to travel to the conduction band.

Please re-read the previous pages if you cannot understand this explanation of carriers in extrinsic semiconductors.