Hand-In Exercise: Admittance Controller

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1 System Modeling

Below is figure that shows the DH parameter of UR5 robot arm in the position where all the joint angles at zero.

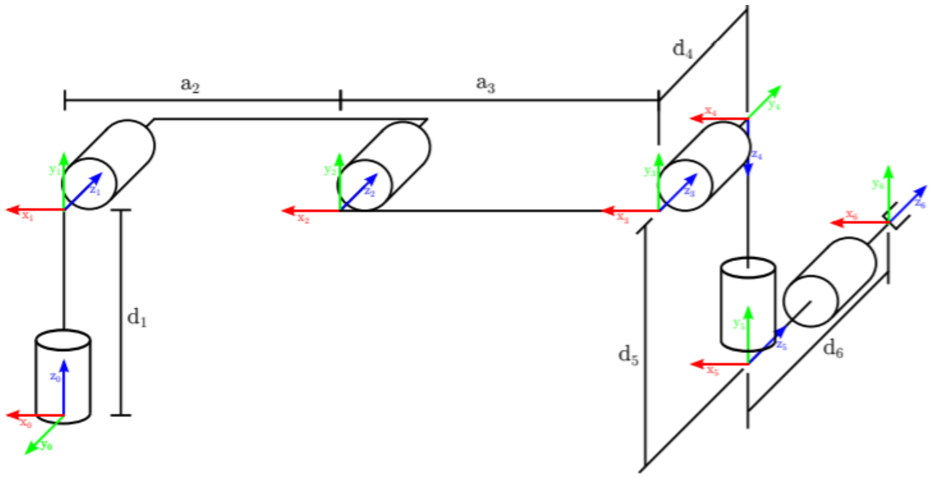

Figure 1: UR5 all joint angles at zero

Table 1: DH parameters

Joint	a	α	d	θ
1	0	$\pi/2$	0.089159	$\theta 1$
2	-0.425	0	0	$\theta 2$
3	-0.392254	0	0	$\theta 3$
4	0	$\pi/2$	0.10915	$\theta 4$
5	0	$-\pi/2$	0.09465	$\theta 5$
6	0	0	0.0823	$\theta 6$

As with any 6-DOF robot, the homogeneous transformation from the base frame to the gripper can be defined as follows:

$$T_6^0(\theta 1, \theta 2, \theta 3, \theta 4, \theta 5, \theta 6) = T_1^0(\theta 1)T_2^1(\theta 2)T_3^2(\theta 3)T_4^3(\theta 4)T_5^4(\theta 5)T_6^5(\theta 6) \tag{1}$$

1.1 Robot Dynamics including External Forces

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + F\dot{q} + g(q) = \tau - J^T(q)h_e \tag{2}$$

Where $B(q)$ is the inertial tensor, $C(q,\dot{q})$ is the centripetal component, F is the joint friction term and $g(q)$ is the gravitational component. τ is the applied torque at each joint and $J^T(q)h_e$ is the force and torques applied at the end effector; the jacobian is used to change the reference frame to base frame.

2 Admittance Control

The equation is chosen so as to impose a dynamic behavior for the position displacement under a 3×1 force vector f_0 at the object

$$M_p \Delta \ddot{p}_{dc} + D_p \Delta \dot{p}_{dc} + K_p \Delta p_{dc} = f_0 \tag{3}$$

where M_p, D_p and K_p are 3×3 positive matrices representing the mass, the damping. and stiffness characterizing the impedance.

Let M_ϵ, D_ϵ and K_ϵ denote 3×3 positive matrices representing the inertia, rotational damping, and rotational stiffness. The equation for orientation displacement under a 3×1 moment vector μ_0 with respect to the origin of Σ_0 is given by

$$M_\epsilon \Delta^c \dot{\omega}_{dc} + D_\epsilon \Delta^c \omega_{dc} + K'^c_\epsilon \epsilon_{dc} = {}^c\mu_0 \tag{4}$$

where $\Delta^c \dot{\omega}_{dc} = {}^c\omega_d - {}^c\omega_c$ is the 3×1 angular velocity vector of Σ_d relative to Σ_c and K'_ϵ is an equivalent stiffness which is related to K_ϵ as

$$K'_\epsilon = 2(\eta_{dc} I + S({}^c\epsilon_{dc}))K_\epsilon \tag{5}$$

2.1 Control Law

Admittance control means that the output position of the robot arm is offset from the original set target point under the action of external force. The goal is to simulate a second-order system, assuming that the position of the end point of the robotic arm is our control variable. Where in Mp,Dp,Kp each inertia,damping, stiffness coefficients, analogous to the oscillator model of the spring mass damper system. By adjusting these three parameters, the dynamic properties of the robot can be changed.

2.2 Gain Selection

The dynamics are given by second order systems. Hence, it is characterized by the a natural frequency ω_n and a damping ratio ζ. For a system to be critically damped the damping ratio must equal 1. With this knowledge, a proportional gain can be selected based on how much movement is desired a a given force, which is calculated by $K_p \Delta p_{dc} = f$ and $K_o \Delta \epsilon_{dc} = \tau$. Then the damping and mass gain can be tuned until a desired respond is found. In a real application the natural frequency should be considered to avoid any unwanted behaviour from vibrations or other periodic disturbances but in simulation this is not a problem.

$$\zeta = \frac{k_D}{2\sqrt{m k_p}} \tag{6}$$

$$\omega_n = \sqrt{\frac{k_p}{m}} \tag{7}$$

Following this methodology the gains for our implementation were selected as the following for positional and rotational control respectively.

$$[M_p, D_p, K_p] = [diag(0.3, 0.3.0.3), diag(5.55, 5.55, 5.55), diag(13, 13, 13)] \tag{8}$$

$$[M_\epsilon, D_\epsilon, K_\epsilon] = [diag(1, 1, 1), diag(3.5, 3.5, 3.5), diag(2, 2, 2)] \tag{9}$$

2.3 Implementation

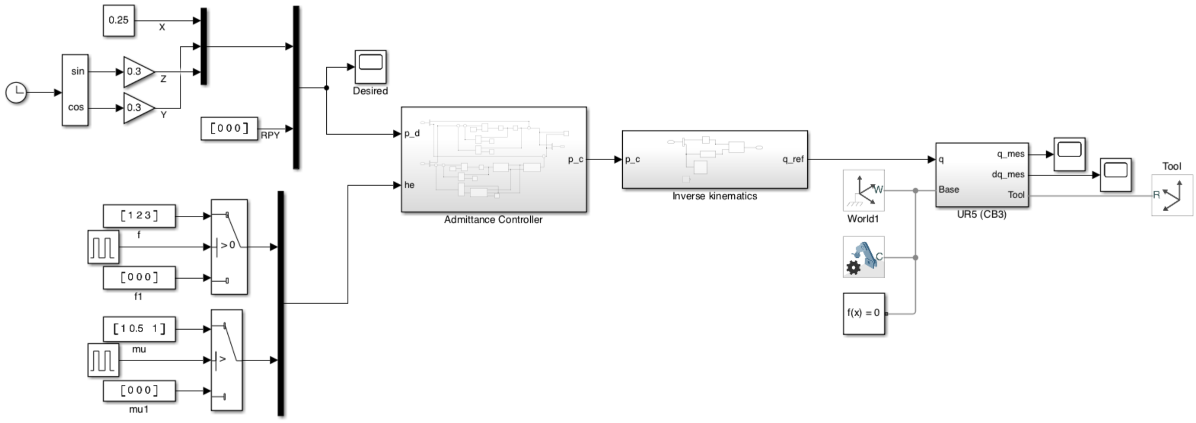

Figure 2: Simulink Model

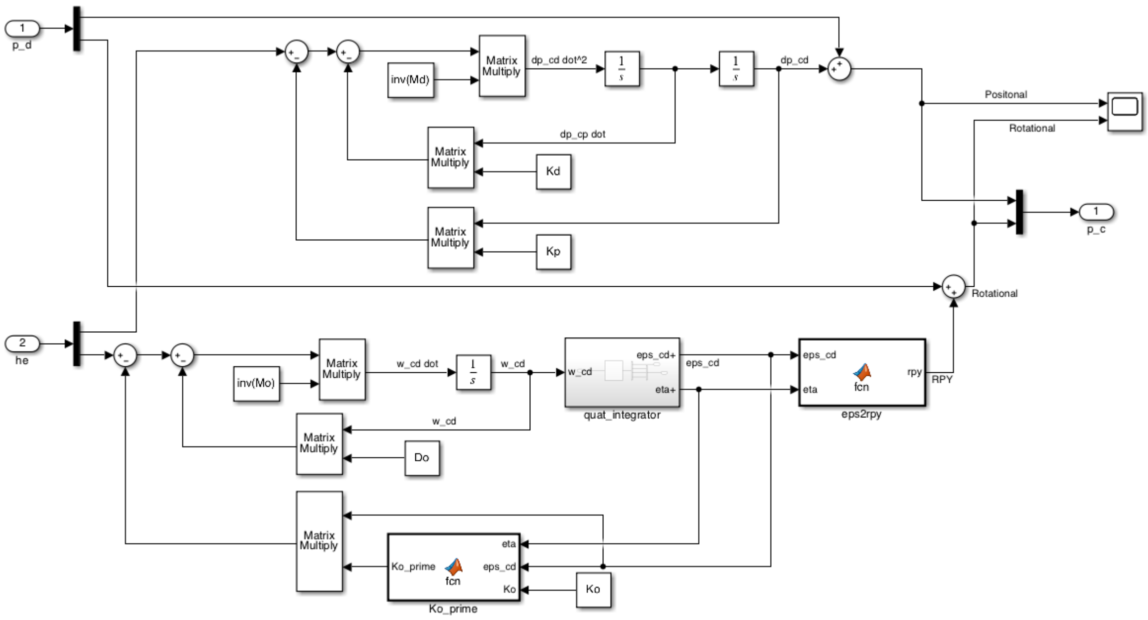

Figure 3: Admittance Controller

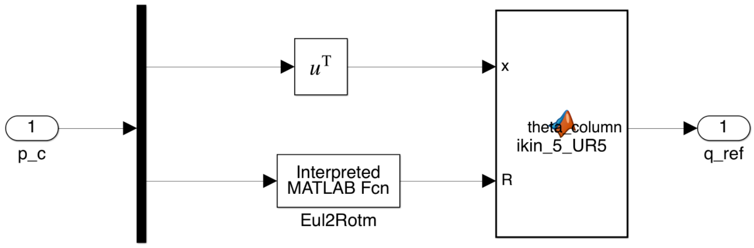

Figure 4: Inverse Kinematics

3 Simulation

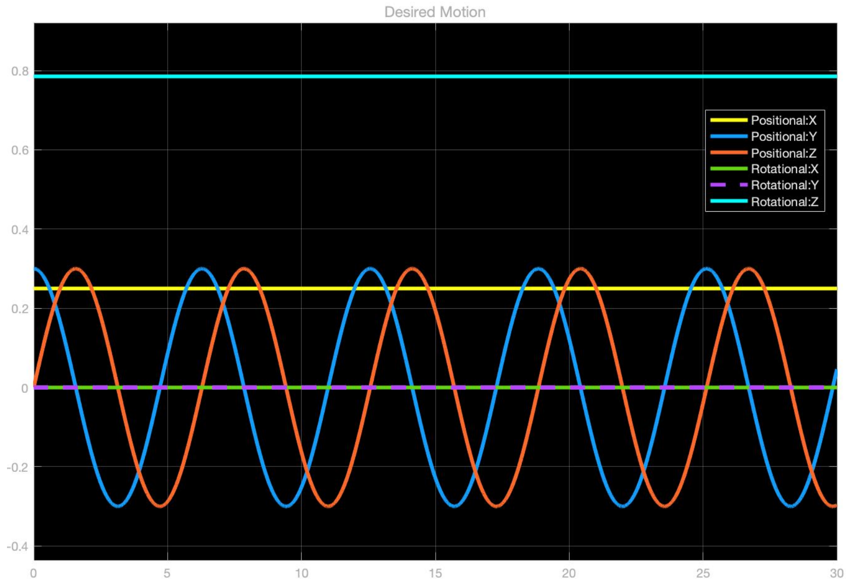

Figure 5: The scope output of the desired motion include both position and orientation

Figure 5 contains the desired motion of the robot during the 30 second test period. The motion described shows the robot moving the end effector in a circle with a radius of 0.3 m in the plane $x = 0.25$ m while keeping the orientation constant with no rotation in the x and y axes and a slight turn along the z axis for better visual interpretation.

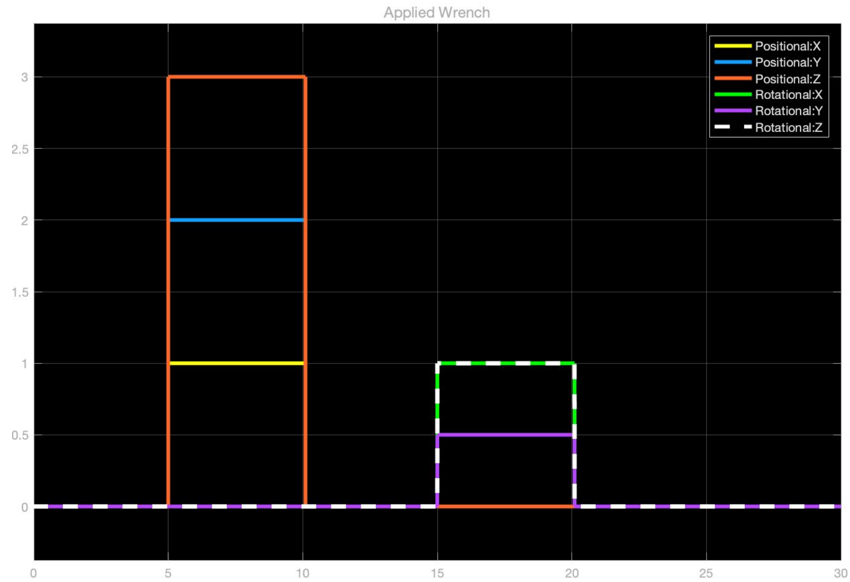

Figure 6: The applied forces and torques.

Figure 6 contains the forces and torques applied to robot during the test period.

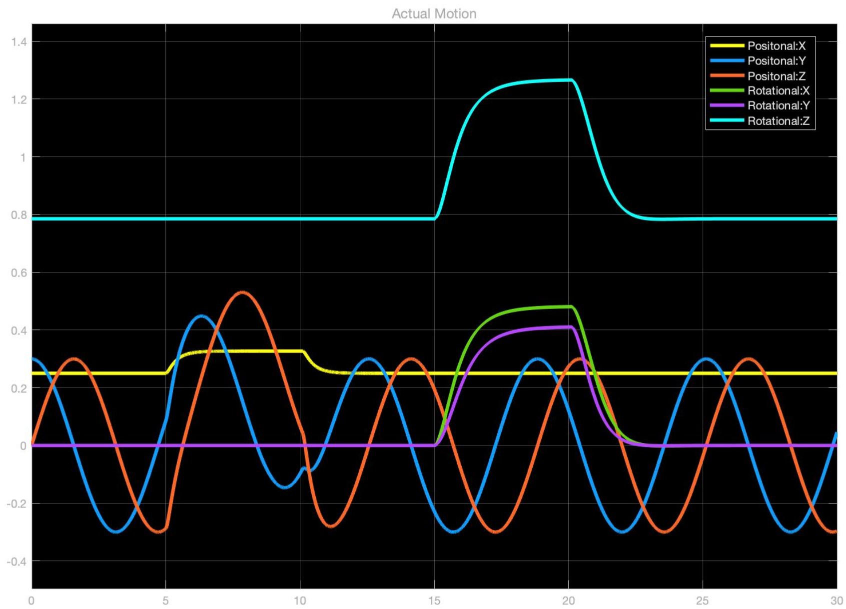

Figure 7: Resulting positional and rotation values once forces and torques were applied

Figure 7 contains the resulting motion of the robot during the 30 second test period with forces and torques applied. With the selected gains it can be seen that the positional admittance was significantly lower than the rotational as the forces only deflect the trajectory slightly.

4 Bibliography

Caccavale, F., Natale, C., Siciliano, B., & Villani, L. (2000). Quaternion-Based Impedance Control for Dual-Robot Cooperation. Robotics Research, 59–66.