# ECON 1190: Econometrics 2: Slides 3: Regression Review

Claire Duquennois

# Review: Simple Linear Regression

# Reg Review: Related random variables

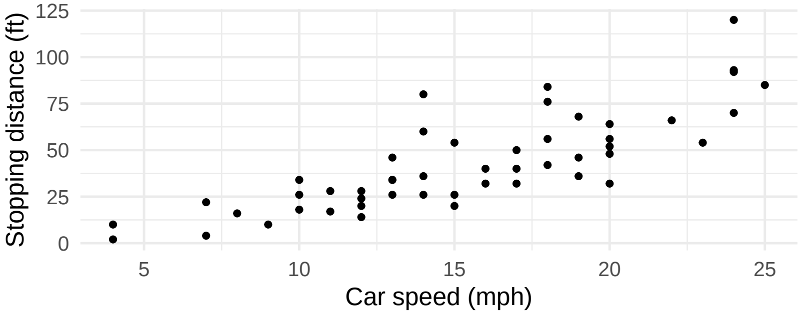
Values of two (or more) random variables might be related:

- (X,Y)=(height, weight) of a person
- (X,Y)=(sqft, bedrooms) of a home
- (X,Y)=(price, quantity demanded) of a product
- (X,Y)=(speed, stopping distance) of a car

How can we understand these relationships?

# Reg Review: Line of best fit

```
library(ggplot2)
#here I use the cars data which is a part of base R
my_plot_stats<-ggplot(data = cars, aes(x =speed, y = dist))+
    geom_point(size=1)+
  labs(x = "Car speed (mph)", y = "Stopping distance (ft)")+
  theme_minimal()
my_plot_stats
```

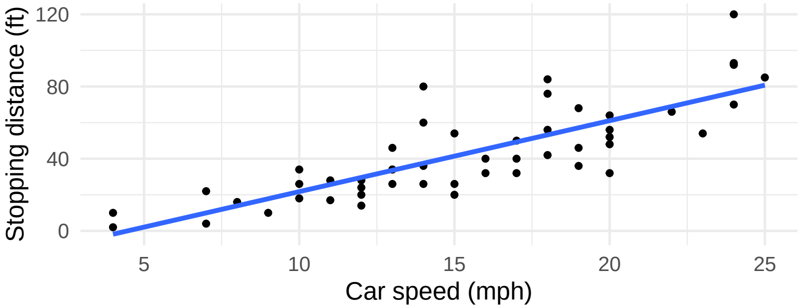



Let's draw a line: $Y = \beta + 0 + \beta_1 X$ and select $\beta_0$ and $\beta_1$ to "fit' ' the data as *closely* as possible.

# Reg Review: Line of best fit

Usually we do this with OLS (Ordinary Least Squares): minimizing the sum of squared residuals.

```
library(ggplot2)
my_plot_stats2<-ggplot(data = cars, aes(x =speed, y = dist))+
  geom_point(size=1)+
  geom_smooth(method='lm', se = FALSE)+
  labs(x = "Car speed (mph)", y = "Stopping distance (ft)")+
  theme_minimal()
my_plot_stats2
```



# Reg Review: Conditional expectation

The line gives the **Conditional expectation**: $E[Y|X] = \beta_0 + \beta_1 X$

- Example: $E[stoppingdistance|speed]$
- the line gives us a unique expected value for any speed

What are the values of $\beta_0$ and $\beta_1$?

```
cars_reg<-lm(dist~speed, cars)
cars_reg
```

```
##
## Call:
## lm(formula = dist ~ speed, data = cars)
##
## Coefficients:
## (Intercept)        speed
##     -17.579        3.932
```

So Distance $= -17.6 + 3.9$Speed

- when speed is 15mph, we predict a stopping distance of 41.

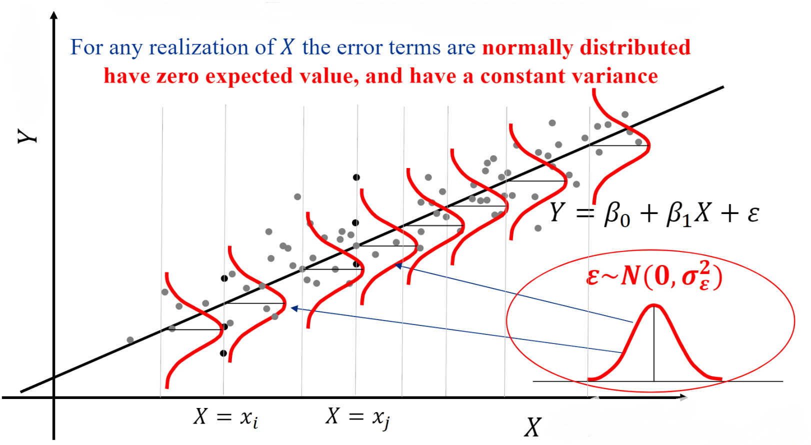
# Reg Review: The errors

For an observation $i$, the actual observed outcome $Y_i$ will differ from the expected outcome $\hat{Y}_i$ given its $X_i$ because of random unknown factors we call the error ( $\epsilon_i$).

- for a specific observation $i$ we can represent the actual outcome $Y_i$ as: $Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$
- its predicted outcome $\hat{Y}_i$ is: $\hat{Y}_i = \beta_0 + \beta_1 X_i$
- the error, or **residual**, is $\epsilon_i = Y_i - \hat{Y}_i$

These errors are normally distributed, with $E[\epsilon] = 0$ and a constant variance.

# Reg Review: The errors



# Reg Review: Interpretation

Model: $Dist = \beta_0 + \beta_1 Speed$

```r
reg1<-lm(dist~speed, cars)
summary(reg1) #For a more detailed summary that includes standard errors
```

```
##
## Call:
## lm(formula = dist ~ speed, data = cars)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -29.069  -9.525  -2.272   9.215  43.201
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -17.5791     6.7584  -2.601   0.0123 *
## speed         3.9324     0.4155   9.464 1.49e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 15.38 on 48 degrees of freedom
## Multiple R-squared:  0.6511, Adjusted R-squared:  0.6438
## F-statistic: 89.57 on 1 and 48 DF,  p-value: 1.49e-12
```

$\beta_1 = 3.9$ (se=0.42). How should we interpret this coefficient?

# Reg Review: Interpretation

3 elements you want to touch upon (the three S'):

1) **Sign**- is the coefficient you are discussing positive or negative? Does the sign of the coefficient match your priors or is it surprising?
2) **Size**- What is the magnitude of the coefficient? Is the effect of $x$ on $y$ economically meaningful or not? Make your interpretation informative to your audience, by being precise.
3) **Significance**- Is the estimate statistically significant? Can we reject that the true coefficient is equal to zero? With what confidence level?

# Reg Review: Interpretation

Model: $Dist = \beta_0 + \beta_1 Speed$

$\beta_1 = 3.9$ (se=0.42)

Each additional mile per hour of speed predicts an increase in the stopping distance of 3.9 feet. This relationship is highly statistically significant. We can reject the null of no relationship at the 99% confidence level.

# Reg Review: Scaling

Generally good to use intuitive units of measurement.

```r
reg_USunits<-lm(dist~speed, cars)
reg_USunits
```

```
##
## Call:
## lm(formula = dist ~ speed, data = cars)
##
## Coefficients:
## (Intercept)        speed
##     -17.579        3.932
```

The European in me does not like the regression where distance is measured in feet (whose feet? not my foot) and miles per hour... my heart belongs to the metric system.

- You can rescale when you interpret the coefficients
- Or before running the regression

# Reg Review: Scaling

```
cars$speed_kmh=cars$speed*1.61  #1 mile=1.61 km
cars$dist_m=cars$dist*0.3 #1 foot=0.3 meters

reg_metric1<-lm(dist_m~speed, cars)
reg_metric1
```

```
##
## Call:
## lm(formula = dist_m ~ speed, data = cars)
##
## Coefficients:
## (Intercept)        speed
##      -5.274        1.180
```

Scaling $Y$ by $c = 0.3$: all coefficients get multiplied by $c$

# Reg Review: Scaling

```
cars$speed_kmh=cars$speed*1.61  #1 mile=1.61 km
cars$dist_m=cars$dist*0.3 #1 foot=0.3 meters

reg_metric1<-lm(dist_m~speed, cars)
reg_metric1
```

```
##
## Call:
## lm(formula = dist_m ~ speed, data = cars)
##
## Coefficients:
## (Intercept)        speed
##      -5.274        1.180
```

Scaling $Y$ by $c = 0.3$: all coefficients get multiplied by $c$

```
reg_metric2<-lm(dist_m~speed_kmh, cars)
reg_metric2
```

```
##
## Call:
## lm(formula = dist_m ~ speed_kmh, data = cars)
##
## Coefficients:
## (Intercept)    speed_kmh
##     -5.2737       0.7327
```

Scaling $X_1$ by $c = 1.61$: $\beta_1$ gets divided by $c$

# Reg Review: Multivariate regression

# Reg Review: Multivariate regression

What if we have more than 1 explanatory variable?

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_k X_k + \epsilon$$

- Most of what we discussed is similar.
- Key difference: when interpreting the coefficient for one variable we are "keeping all other variables fixed"

Some models also become more complicated:

- Categorical variables
- Interaction terms
- Quadratic specifications
- Log specifications

# Reg Review: Categorical variables

I make a variable (*Man*):*Man*=1 for men, 0 for women.

I estimate the model: $\text{Earnings} = \beta_0 + \beta_1 \text{Man} + \epsilon$

```
data<-read.csv("CPSSW8.csv")

data$man<-NA
data$man[data$gender=="male"]<-1
data$man[data$gender=="female"]<-0

model<-lm(earnings~man, data)
model
```

```
##
## Call:
## lm(formula = earnings ~ man, data = data)
##
## Coefficients:
## (Intercept)          man
##      16.338        3.748
```

- $\hat{\beta}_0 = 16.3$, the estimate for the omitted category (Women)
- $\hat{\beta}_1 = 3.7$, the estimated difference between Men and the omitted category.

# Reg Review: Categorical variables

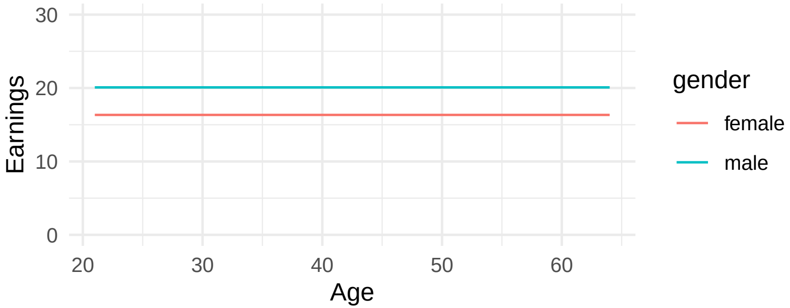
For men the variable Man=1, so predicted earnings are

$$\widehat{\text{Earnings}} = 16.3 + 3.7(1) = 20$$

For women the variable Man=0, so predicted earnings are

$$\widehat{\text{Earnings}} = 16.3 + 3.7(0) = 16.3$$

This simple regression on a categorical variable essentially gives us the mean earnings of men and women.



# Reg Review: Categorical variables

Now, I estimate the model: Earnings $= \beta_0 + \beta_1 \text{Man} + \beta_2 \text{Age} + \epsilon$

```
model2<-lm(earnings~man+age, data)
model2
```

```
##
## Call:
## lm(formula = earnings ~ man + age, data = data)
##
## Coefficients:
## (Intercept)          man          age
##      8.8466       3.8302       0.1806
```

So $\hat{\beta}_0 = 8.8$ and $\hat{\beta}_1 = 3.8$ and $\hat{\beta}_2 = 0.18$

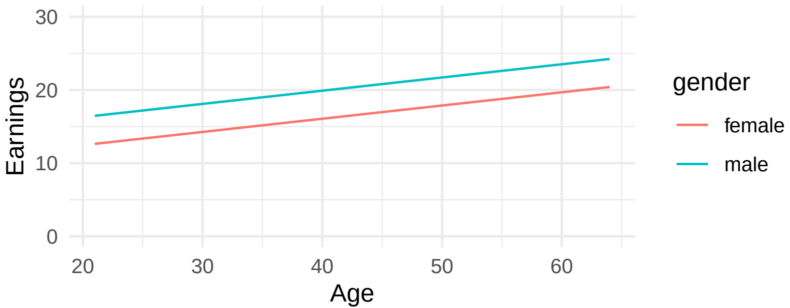
# Reg Review: Categorical variables

For men the variable Man=1, so predicted earnings are

$$\widehat{\text{Earnings}} = 8.8 + 3.8(1) + 0.18 \times \text{Age} = 12.6 + 0.18 \times \text{Age}$$

For women the variable Man=0, so predicted earnings are

$$\widehat{\text{Earnings}} = 8.8 + 3.8(0) + 0.18 \times \text{Age} = 8.8 + 0.18 \times \text{Age}$$



# Reg Review: Interaction terms

Now, I estimate the model:
$\text{Earnings} = \beta_0 + \beta_1 \text{Man} + \beta_2 \text{Age} + \beta_3 \text{Age} \times \text{Man} + \epsilon$

```
model3<-lm(earnings~man+age+age*man, data)
model3
```

```
##
## Call:
## lm(formula = earnings ~ man + age + age * man, data = data)
##
## Coefficients:
## (Intercept)          man          age      man:age
##     11.3334      -0.6057       0.1206       0.1074
```

So $\hat{\beta}_0 = 11.3$ and $\hat{\beta}_1 = -0.6$ and $\hat{\beta}_2 = 0.12$ and $\hat{\beta}_3 = 0.11$

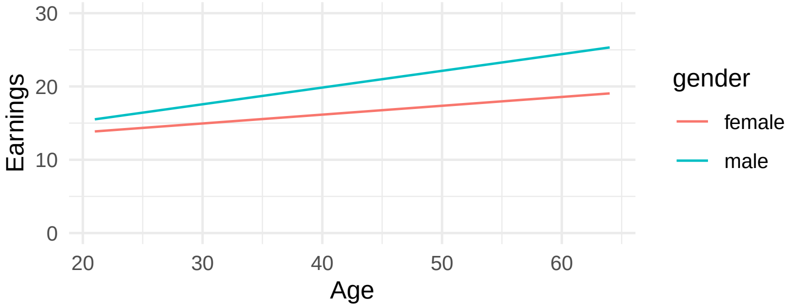
# Reg Review: Interaction terms

For men the variable Man=1, so predicted earnings are

$$\widehat{\text{Earnings}} = 11.3 + (-0.6)(1) + 0.12 \times \text{Age} + 0.11(1) \times \text{Age} = 10.7 + 0.23 \times \text{Age}$$

For women the variable Man=0, so predicted earnings are

$$\widehat{\text{Earnings}} = 11.3 + (-0.6)(0) + 0.12 \times \text{Age} + 0.11(0) \times \text{Age} = 11.3 + 0.12 \times \text{Age}$$



# Reg Review: Non-linear specifications: Quadratic

We are interested in the relationship between age and sleep.

- Do you think this relationship is linear? Who sleeps a lot?

If the data is "curved' ', specify a quadratic by adding a squared term to the specification.

$$sleep = \beta_0 + \beta_1 age + \beta_2 age^2 + u$$

```
sleep75$age2<-sleep75$age*sleep75$age
regquad<-lm(sleep~age+age2, sleep75)
regquad
```

```
##
## Call:
## lm(formula = sleep ~ age + age2, data = sleep75)
##
## Coefficients:
## (Intercept)          age         age2
##   3608.0297     -21.4904       0.3012
```

# Reg Review: Non-linear specifications: Quadratic

When interpreting a variable that includes a quadratic, the marginal effect of the variable is not linear

- how an additional year of age affects sleeps depends on how old you are.
- To see this, take the derivative of sleep with respect to age:

$$\frac{dsleep}{dage} = \beta_1 + 2 * \beta_2 * age = -21.5 + 2 \times 0.3 \times age = -21.5 + 0.6 \times age.$$

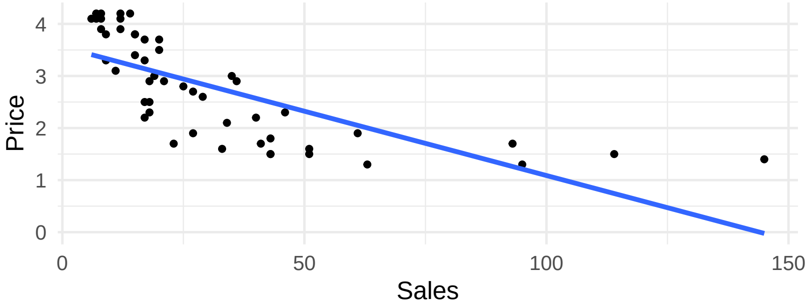
Getting older means less sleep until you are 35. Then more.

When interpreting the marginal effect:

- specify the X at which you are interpreting at,
- give a sense of the effect of a unit increase in X at different key points of the distribution of X.

# Reg Review: Log variables

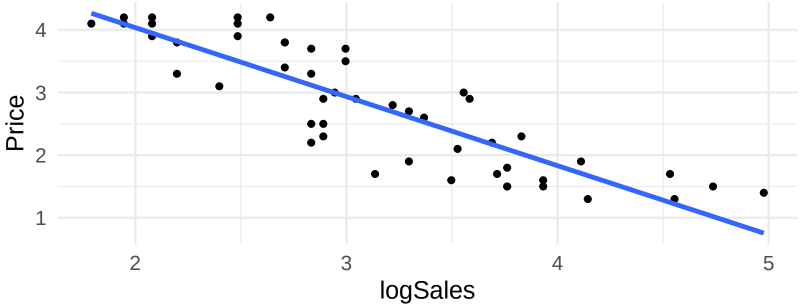
What if your data is not linear, or quadratic?




Does this line do a good job of fitting the data?

# Reg Review: Log variables

Using log's to transform a variable can greatly improve the fit of your model.




When you do this, you need to then adjust your interpretation accordingly.

# Reg Review: Log adjusted interpretation

Linear:

- $y = \beta_0 + \beta_1 x$
- Interpretation: $\Delta y = \beta_1 \Delta x$

Logarithmic:

- $y = \beta_0 + \beta_1 log(x)$
- Interpretation: $\Delta y = \beta_1 \frac{\%\Delta x}{100}$

Exponential:

- $log(y) = \beta_0 + \beta_1 x$
- Interpretation: $\%\Delta y = \beta_1 \Delta x \times 100$

Log-Log (an elasticity):

- $log(y) = \beta_0 + \beta_1 log(x)$
- Interpretation: $\%\Delta y = \beta_1 \%\Delta x$

# New?: Non-standard standard errors

# Non-standard standard errors

A standard error estimates the uncertainty around an estimated parameter.

Formally we have

$$se = \sqrt{\widehat{Var(\hat{\beta})}}.$$

Just like calculating $\hat{\beta}$, it is incredibly important to get your standard errors right.

You have to know what you don't know!

- Robust standard errors
- Clustered standard errors

# Robust standard errors

Using the diamonds data set from `ggplot2`: regress price on carats.

```
reg1<-felm(price~carat, diamonds)

summary(reg1)
```

```
##
## Call:
##    felm(formula = price ~ carat, data = diamonds)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -18585.3   -804.8    -18.9    537.4  12731.7
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -2256.36      13.06  -172.8   <2e-16 ***
## carat        7756.43      14.07   551.4   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1549 on 53938 degrees of freedom
## Multiple R-squared(full model): 0.8493   Adjusted R-squared: 0.8493
## Multiple R-squared(proj model): 0.8493   Adjusted R-squared: 0.8493
## F-statistic(full model):3.041e+05 on 1 and 53938 DF, p-value: < 2.2e-16
## F-statistic(proj model): 3.041e+05 on 1 and 53938 DF, p-value: < 2.2e-16
```

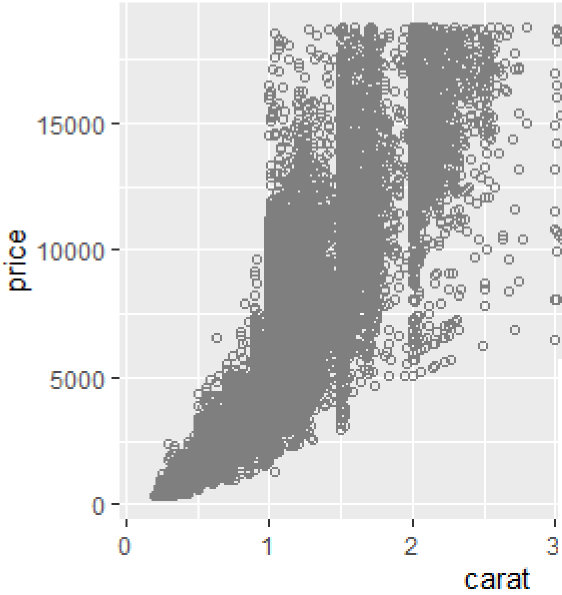
# Robust standard errors

Cool.

Plot the data to check OLS assumptions:

```
myPlot <- ggplot(data = diamonds, aes(y = price, x = carat)) +
geom_point(color = "gray50", shape = 21)
```

# Robust standard errors




# Robust standard errors

You should have the econometric heebie jeebies.

Homoskedastic assumption needed for OLS is not valid!

- The higher the carat, the greater the variance in price.
- $\Rightarrow$ OLS standard errors are likely to be wrong.

Thankfully all is not lost!

# Eicker, Huber and White to the rescue!

Econometricians Eicker, Huber and White figured out a way to do calculate “robust”, or “heteroskedasticity-robust” standard errors.

Robust standard errors are larger than regular standard errors, and thus more conservative (which is the right thing to be... you want to know what you don’t know).

# Robust standard errors

## How can we find these in R?

```
reg1<-felm(price~carat, diamonds)

summary(reg1, robust=TRUE)
```

```
##
## Call:
##    felm(formula = price ~ carat, data = diamonds)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -18585.3   -804.8    -18.9    537.4  12731.7
##
## Coefficients:
##             Estimate Robust s.e t value Pr(>|t|)
## (Intercept) -2256.36      16.13  -139.9   <2e-16 ***
## carat        7756.43      25.40   305.4   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1549 on 53938 degrees of freedom
## Multiple R-squared(full model): 0.8493   Adjusted R-squared: 0.8493
## Multiple R-squared(proj model): 0.8493   Adjusted R-squared: 0.8493
## F-statistic(full model, *iid*):3.041e+05 on 1 and 53938 DF, p-value: < 2.2e-16
## F-statistic(proj model): 9.326e+04 on 1 and 53938 DF, p-value: < 2.2e-16
```

# Robust standard errors

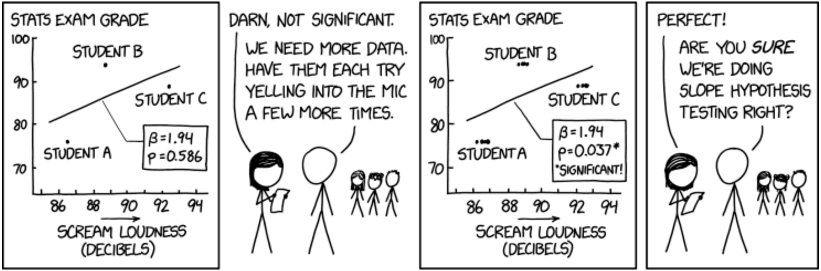
Or if you want to put them in a stargazer table:

```
stargazer(reg1, type = "latex" , se =  list(reg1$rse), header=FALSE)
```

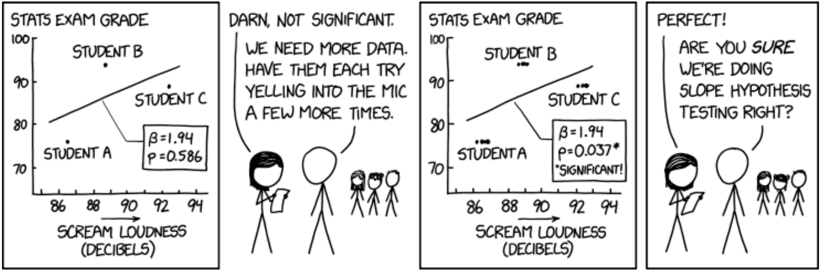
Table 1:

| | *Dependent variable:* |
|---|---|
| | price |
| carat | 7,756.426$^{***}$ |
| | (25.399) |
| Constant | $-$2,256.361$^{***}$ |
| | (16.128) |
| Observations | 53,940 |
| $R^2$ | 0.849 |
| Adjusted $R^2$ | 0.849 |
| Residual Std. Error | 1,548.562 (df = 53938) |
| *Note:* | $^{*}$p$<$0.1; $^{**}$p$<$0.05; $^{***}$p$<$0.01 |

# Clustered standard errors



# Clustered standard errors




**Econometricians Haiku**

T-stats looks too good

Try cluster standard errors

significance gone.

from Angrist and Pischke 2008

# Clustered standard errors

Suppose that every observation belongs to (only) one of G groups.

The assumption we make when we cluster:

- there is no correlation across groups
- we allow for arbitrary within-group correlation.

Need to have a fairly large number of clusters (40+) for the estimate to be credible.

# Clustered standard errors

Example: consider individuals within a village.

It may be reasonable to think that individuals' error terms are:

- correlated within a village
- aren't correlated across villages

# Clustered standard errors

When should I cluster?

Where does the variation in your explanatory variable come from?

- Variation between individual observations?
- Variation between groups of observations?

If the variation comes from group level variation, cluster by groups.

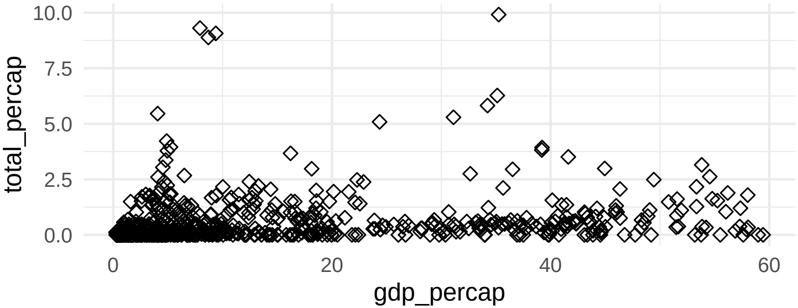
# Clustered standard errors

Recall our olympics data

```
olympics <- read.csv("olympics_data.csv")
olympics2<-olympics %>% select(country, year, type, gold, silver, bronze, population, gdp)%>%
  filter(type == "summer" & !is.na(population) & !is.na(gdp)) %>%
  mutate(total = gold + silver + bronze)

olympics2<-olympics2%>%mutate(gdp_percap=gdp/population, total_percap=total/population)%>%
  filter(total_percap<12,  gdp_percap<60)

my_plot1<-ggplot(data = olympics2, aes(x = gdp_percap, y = total_percap))+
  geom_point(size=2, shape=23)+
  theme_minimal()
my_plot1
```



# Clustered standard errors

```
reg1<-felm(total_percap~gdp_percap, olympics2)
reg2<-felm(total_percap~gdp_percap|0|0|country, olympics2)

stargazer(reg1, reg2, type = "latex", header=FALSE)
```

Table 2:

| | *Dependent variable:* | |
|---|---|---|
| | total_percap | |
| | (1) | (2) |
| gdp_percap | 0.014*** | 0.014*** |
| | (0.003) | (0.005) |
| Constant | 0.347*** | 0.347*** |
| | (0.055) | (0.085) |
| Observations | 680 | 680 |
| $R^2$ | 0.042 | 0.042 |
| Adjusted $R^2$ | 0.040 | 0.040 |
| Residual Std. Error (df = 678) | 1.049 | 1.049 |
| *Note:* | *p<0.1; **p<0.05; ***p<0.01 | |