Note

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1 Signal and background

Signal events are 3 Higgs events in BSM. I use MadGraph to generate by following commands:

```
import model cxSM_VLF_EFT
generate g g > h h h
```

then pass the events through the Pythia and Delphes. In Pythia, I set the branching ratio of Higgs so that it only decays to b quarks.

Background events are 6b events in SM. I use MadGraph to generate by following commands:

```
generate p p > b b b b~ b~ b~
```

then pass the events through the Pythia and Delphes.

2 Transverse momentum and pseudorapidity distribution

In each event there are 6 b quarks. I order these b quarks by transverse momentum p_{T}, then plot p_{T} and pseudorapidity η distributions.

Figure 1 is the p_{T} and η distributions of b quarks for background.

Figure 2 is the p_{T} and η distributions of b quarks for signal.

3 Delta R and transverse momentum distribution

For signal, I plot the ΔR(b,b) distribution of two b decays from the same Higgs. Then I plot ΔR(b,b) against p_{T}(b,b). The result is in Figure 3. As can be seen from the plot, ΔR get smaller at higher p_{T}(b,b).

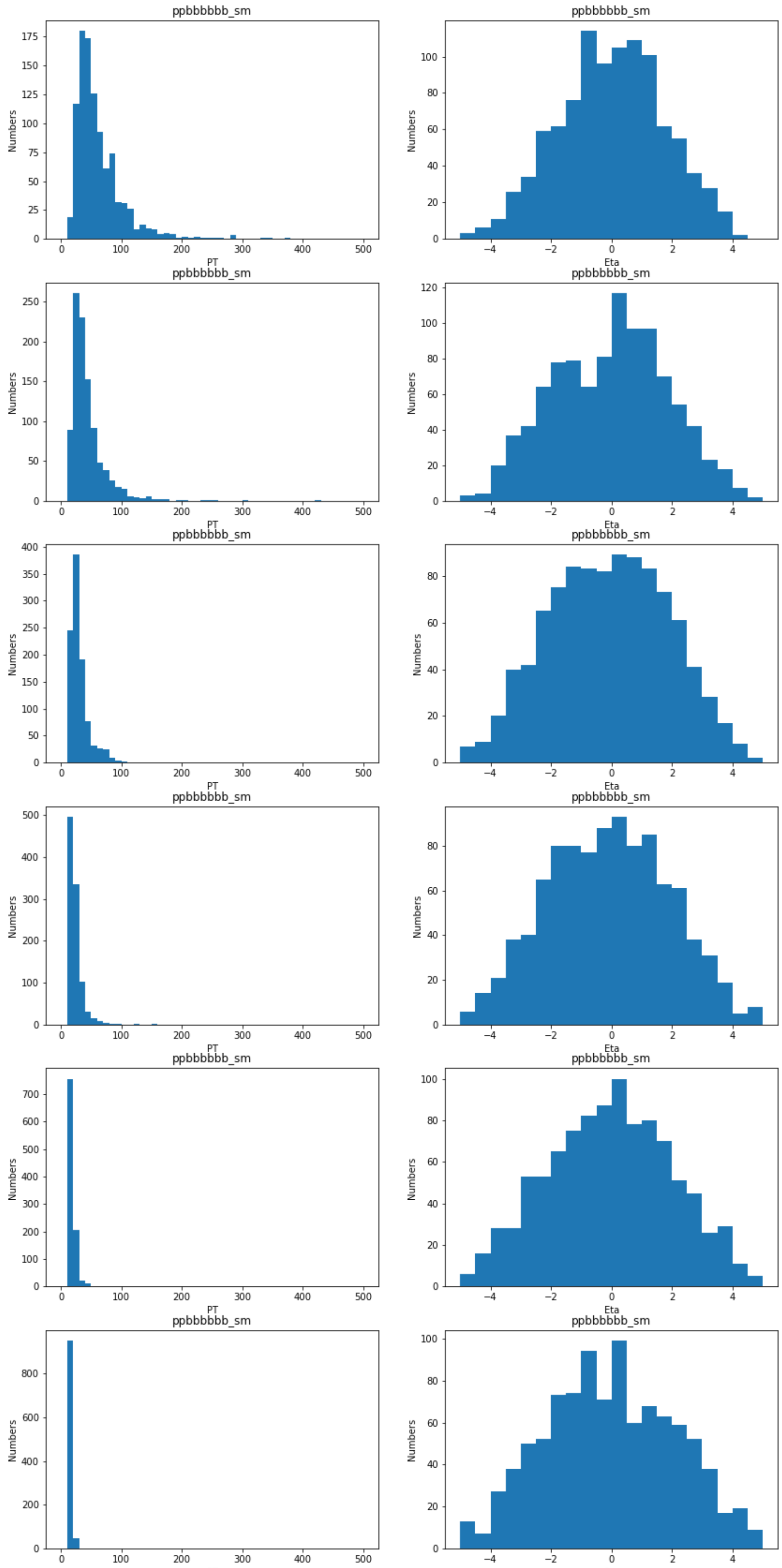

Figure 1: p_{T} and η distributions of b partons for background events. They are ordered by p_{T}.

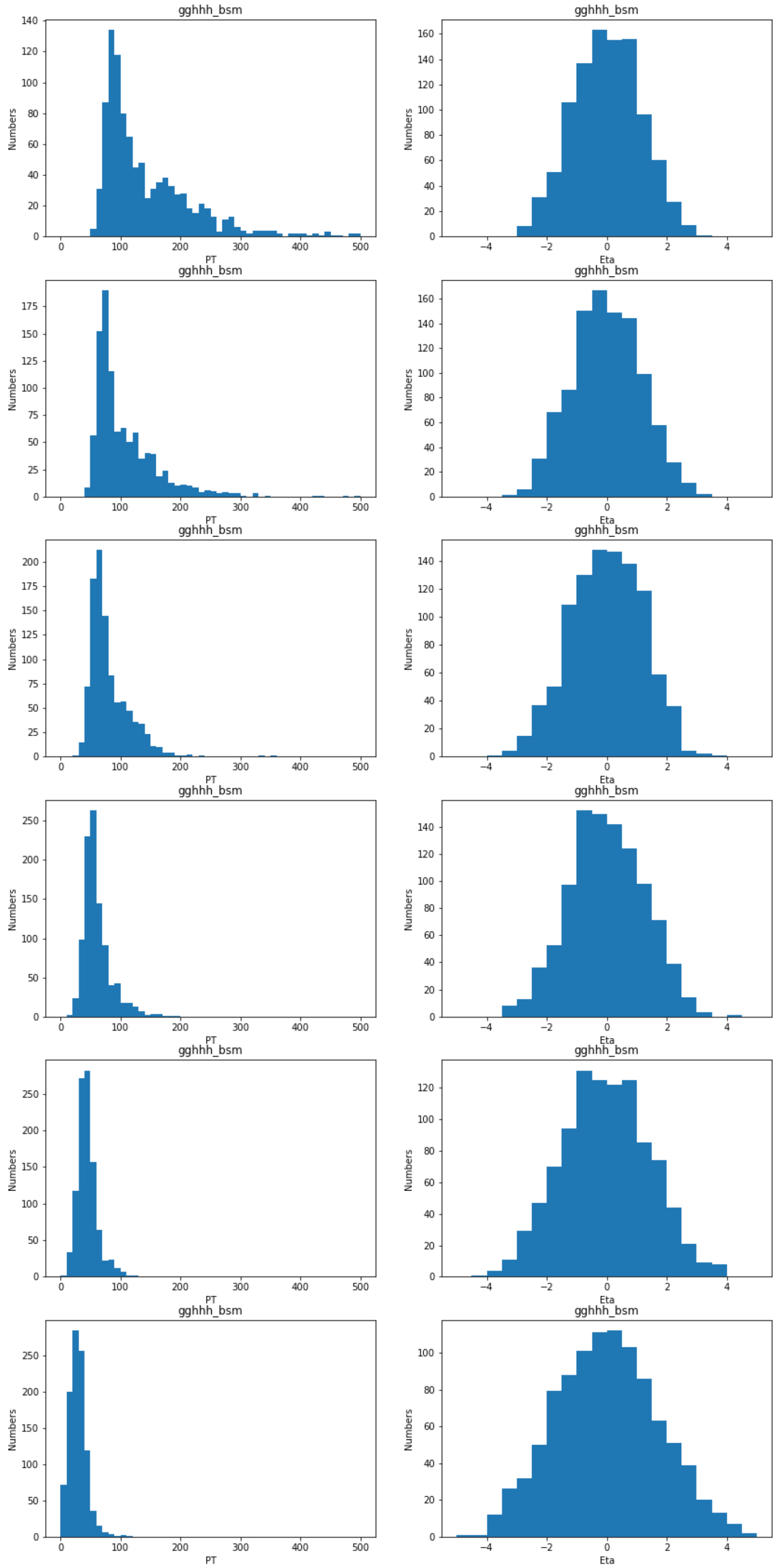

Figure 2: p_{T} and η distributions of b partons for signal events. They are ordered by p_{T}.

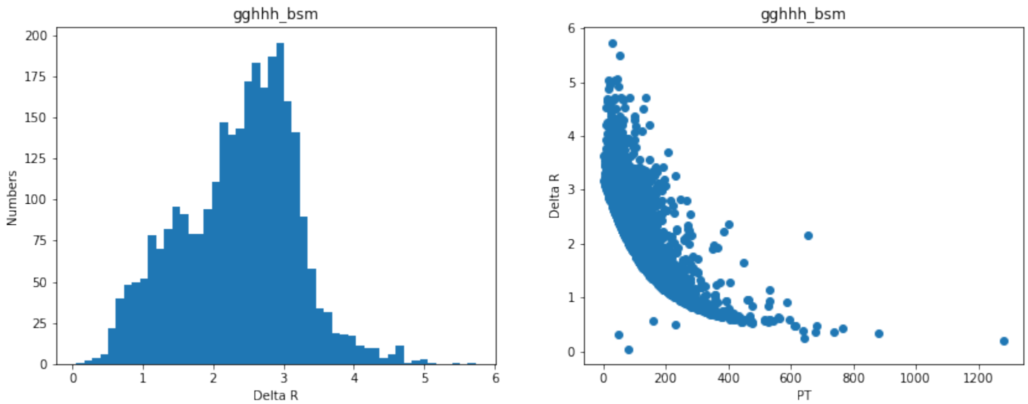

Figure 3: ΔR(b,b) distribution and ΔR(b,b) against p_{T}(b,b) plot

4 Cutflow table

I apply the η and p_{T} cuts sequentially and count how many event can pass the cuts. The results are in Table 1 and Table 2.

Table 1: 1000 background events. Those background events is generated in $\sqrt{s} = 13$ TeV with the cut b $p_{\mathrm{T}} > 10$ GeV in MadGraph.

Cut	number of events pass
6 b-partons $\lvert\eta\rvert < 2.5$	330
6 b-partons $p_{\mathrm{T}} < 25$ GeV	11
4 b-partons $p_{\mathrm{T}} < 40$ GeV	8

Table 2: 1000 signal events.

Cut	number of events pass
6 b-partons $\lvert\eta\rvert < 2.5$	732
6 b-partons $p_{\mathrm{T}} < 25$ GeV	534
4 b-partons $p_{\mathrm{T}} < 40$ GeV	498

5 Construction of bb-pairs

To construct the bb-pairs from the same Higgs, the typical strategy is to try all combinatoric and find the minimal mass difference between all pairs, i.e. minimize this

$$\chi^2 = [M(\mathrm{b}_1\mathrm{b}_2) - M(\mathrm{b}_3\mathrm{b}_4)]^2 + [M(\mathrm{b}_1\mathrm{b}_2) - M(\mathrm{b}_5\mathrm{b}_6)]^2 + [M(\mathrm{b}_3\mathrm{b}_4) - M(\mathrm{b}_5\mathrm{b}_6)]^2$$

where $M(\mathrm{b}_i\mathrm{b}_j)$ means the invariant mass of b-parton i and b-parton j.

The other strategy is to minimize mass difference to the SM Higgs mass

$$\chi^2 = [M(\mathrm{b}_1\mathrm{b}_2) - M_\mathrm{H}]^2 + [M(\mathrm{b}_3\mathrm{b}_4) - M_\mathrm{H}]^2 + [M(\mathrm{b}_5\mathrm{b}_6) - M_\mathrm{H}]^2$$

I implement these two methods to construct bb-pairs and test on the 10,000 signal events in parton level. The result is as follows:

- Method 1 (mass difference between all pairs): accuracy = 0.863
- Method 2 (mass difference to the SM Higgs mass): accuracy = 0.875

Method 2 is better for identifying the true Higgs pair.

6 Absolute cross section

Absolute cross section is defined as follow

$$\sigma_{\mathrm{abs}} = \sigma \times \frac{\text{number of events pass the cut}}{\text{number of events}}$$

I have generated background events with different cuts in MadGraph, the detailed information of cuts is in Table 3. I calculate their absolute cross section and check if it is the same or not. The result is in Table 4.

From the Table 4, if the number of event is large enough the absolute cross section will be similar.

7 The problem for generating pp $\to$ 6b events

Failed to generate the requested number of events

Table 3: The cuts applied on the different run. Where p_{T} means the minimum transverse momentum of b and $|\eta|$ means the range of b (-1 means no restriction).

No.	p_{T} (GeV)	$\|\eta\|$
1	0	-1
2	10	5
3	10	3
4	20	3
5	0	-1
6	10	-1

Table 4: Absolute cross section. Where the selection cut is $p_{\mathrm{T}} > 20$ GeV, $|\eta| < 3$.

No.	total event	cross section	events pass selection	absolute cross section
1	339	2731.9362	0	0.0
2	4761	71.316651	190	2.846
3	9054	35.225852	694	2.700
4	4207	2.7698473	4183	2.754
5	1000	2531.9304	0	0.0
6	1000	70.158431	27	1.894

8 pp $\to$ 4b

I use MadGraph to generate 4b events by following commands:

```
generate p p > b b b~ b~
```

In MadGraph, I apply the cuts: $p_{\mathrm{T}} > 25$ GeV, $|\eta| < 2.5$ for b. Pass those events through Pythia, then check how many events there are 6 b-hadrons final state.

In 100,000 events, there are 6,916 events having greater than or equal to 6 b-hadrons. The distribution of number of b-hadrons is in Figure 4.

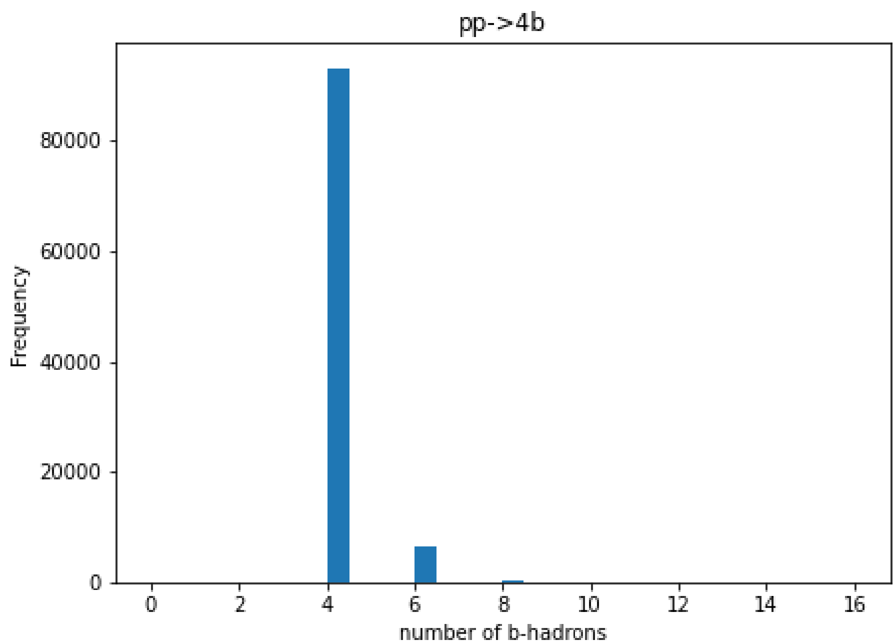

Figure 4: Number of b-hadrons in Pythia final state.

Pass events through Delphes. In Delphes the b-tagging efficiency is set to 1.

Figure 5 is the number of jets and number of b-jets distributions. In 100,000 events, there are 173 events having greater than or equal to 6 b-jets.

9 Comparison for pp $\to$ 4b and pp $\to$ 6b event

To compare pp->4b and pp->6b events, I plot the p_{T}, η and total invariant mass of 6 b-jets for both.

The number of events of 6 b-jets for pp->4b is 2051 and for pp->6b is 1408. I scaled the number of events for pp->4b to be the same as pp->6b.

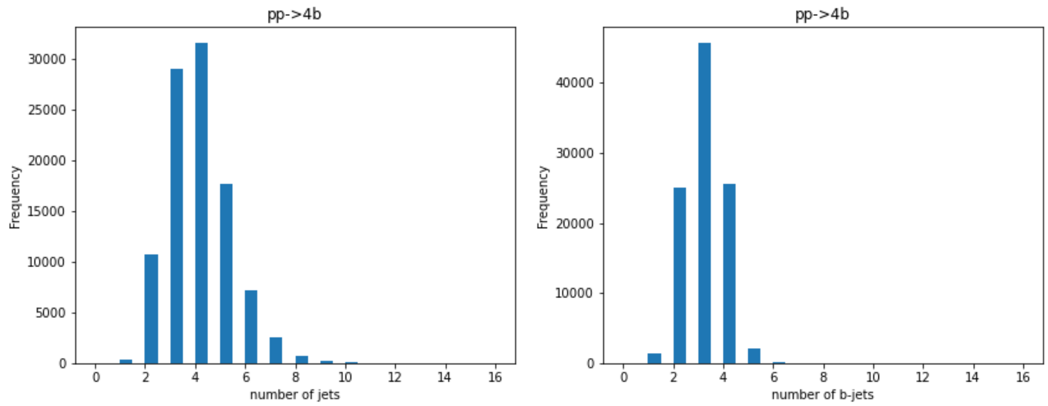

Figure 5: Number of jets and number of b-jets.

Figure 6 is p_{T} and η distributions. Figure 7 is total invariant mass of 6 b-jets distributions. Their distributions look similar.

10 Cutflow table for b-jets

I apply following cut sequentially and count how many events can pass these cuts. Table 5 is the result for pp->4b events. Table 6 is the result for pp->6b events. Table 7 is the result for signal events.

- Cut 1: There are greater than or equal to 6 b-jets.
- Cut 2: There are greater than or equal to 6 b-jets satisfy $|\eta| < 2.5$.
- Cut 3: There are greater than or equal to 6 b-jets satisfy $p_{\mathrm{T}} > 25$ GeV.
- Cut 4: There are greater than or equal to 4 b-jets satisfy $p_{\mathrm{T}} > 40$ GeV.

Table 5: 1,000,000 pp->4b events. Those events are generated in $\sqrt{s} = 14$ TeV with the cuts: $p_{\mathrm{T}} > 25$ GeV, $|\eta| < 2.5$ for b in MadGraph.

Cut	number of event pass
1	1783
2	1565
3	1059
4	735

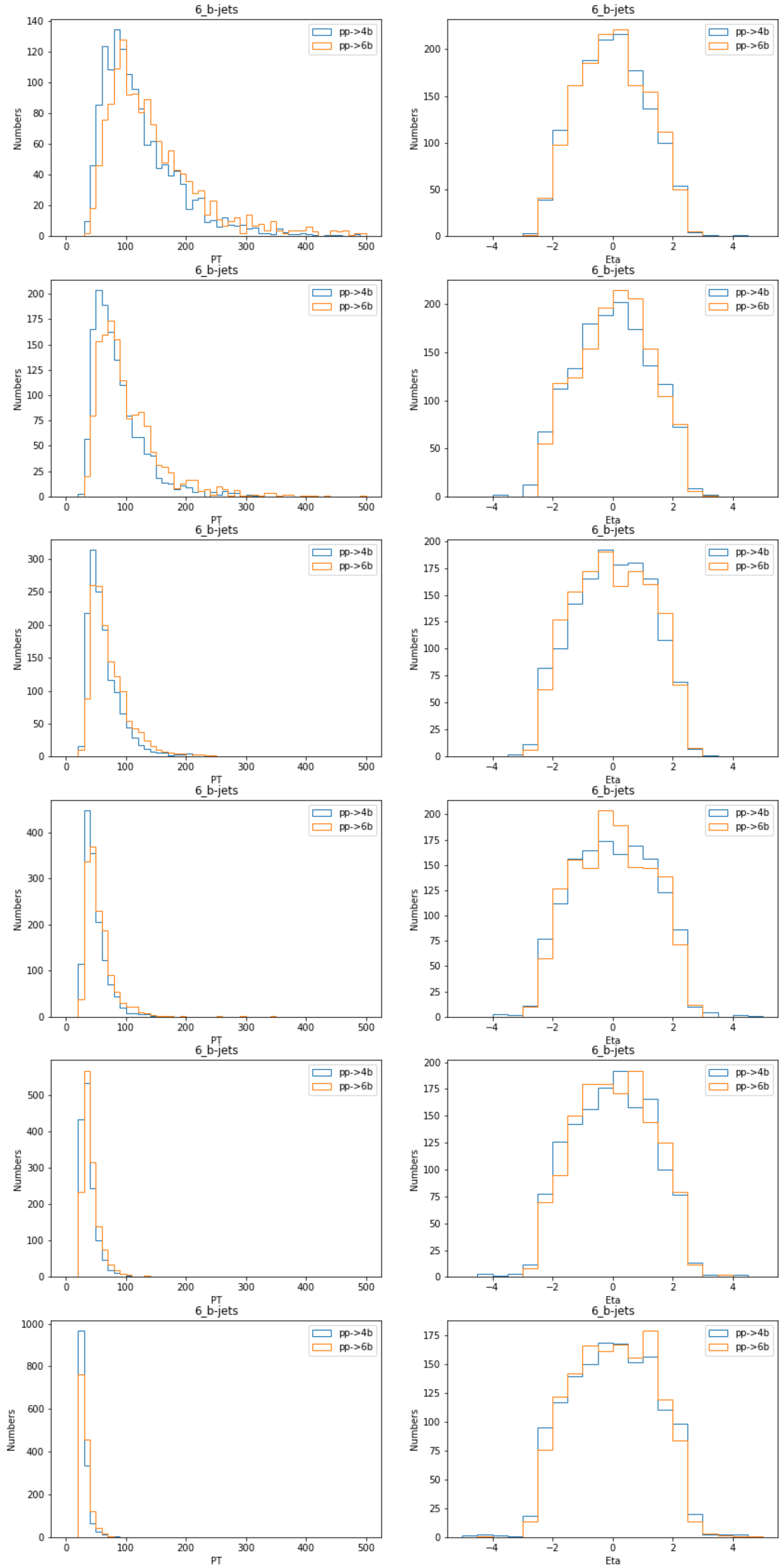

Figure 6: p_{T} and η distributions of b-jets for pp->4b and pp->6b events. They are ordered by p_{T}.

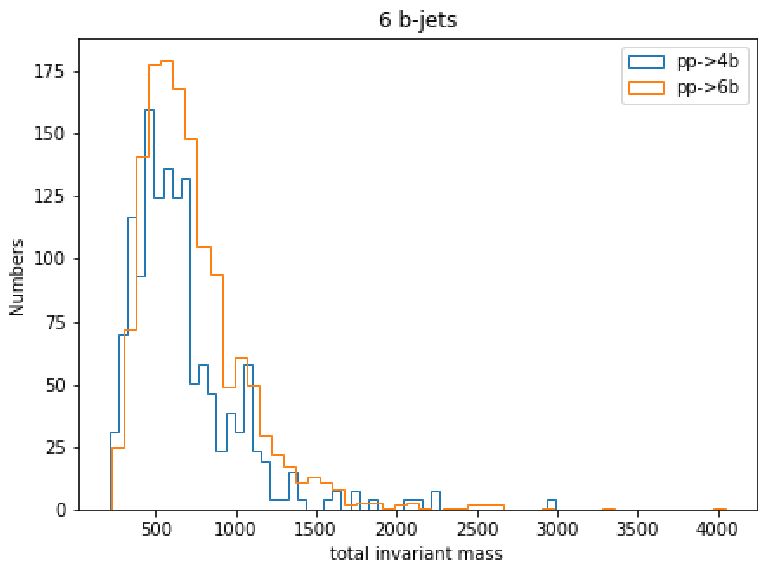

Figure 7: The distribution of total invariant mass of 6 b-jets for pp->4b and pp->6b events.

Table 6: 10,000 pp->6b events. Those events are generated in $\sqrt{s} = 14$ TeV with the cuts: $p_{\mathrm{T}} > 25$ GeV, $|\eta| < 2.5$ for b in MadGraph.

Cut	number of event pass
1	1408
2	1322
3	1052
4	822

Table 7: 100,000 signal events. Those events are generated in $\sqrt{s} = 14$ TeV.

Cut	number of event pass
1	21,814
2	21,254
3	17,130
4	14,142

11 Signal

Generate resonant channel gg->h3, h3->h2h, h2->hh in MadGraph by following commands:

```
import model cxSM_VLF_EFT
generate g g > h3, (h3 > h2 h, h2 > h h)
```

then check this channel is dominated in signal (gg->3h) or not.

I generate this channel and signal in $\sqrt{s} = 14$ TeV, the cross sections are 2.094 pb and 4.067 pb, respectively.

Figure 8 is the total invariant mass of 6 b-jets for signal events. Figure 9 is the total invariant mass of 6 b-jets for this resonant channel. From the results, there is a peak around 400 GeV, because the mass of h_3 is $m_3 = 420$ GeV.

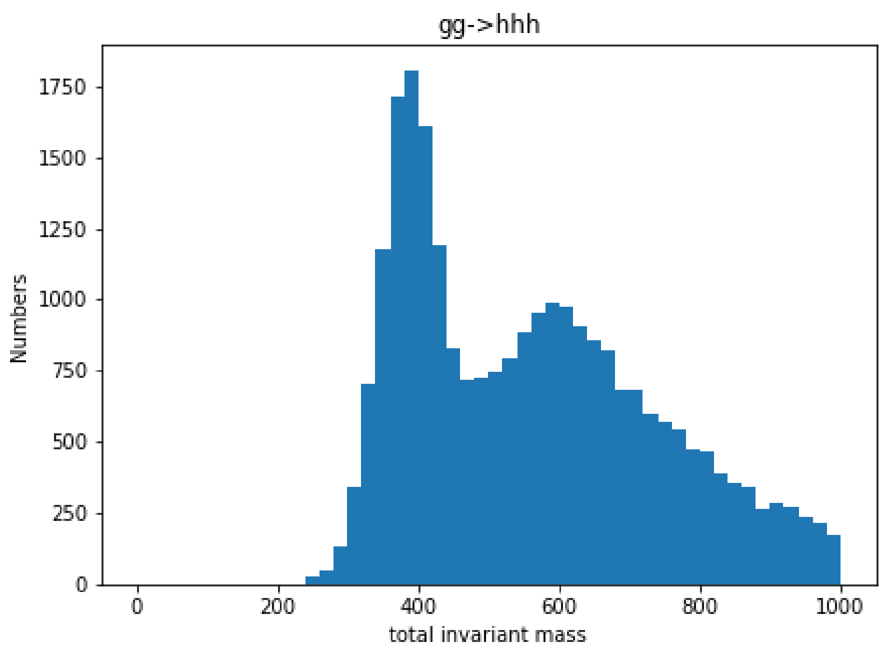

Figure 8: Total invariant mass of 6 b-jets for signal events.

A bump around 600 GeV in Figure 8? In run card, some parameters are not set correctly. After setting all parameters correctly and regenerating these event, the results are in Figure 10. There is no bump around 600 GeV.

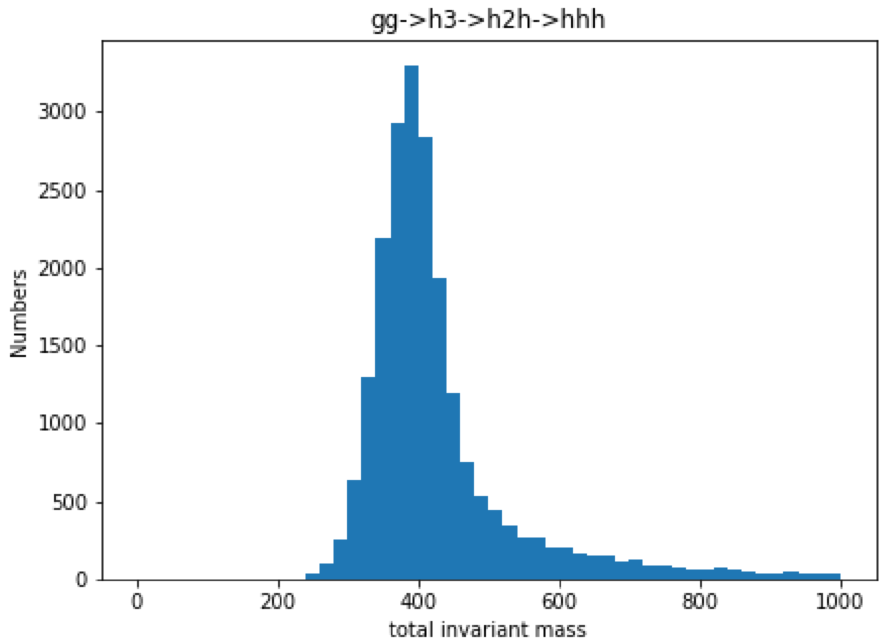

Figure 9: Total invariant mass of 6 b-jets for resonant channel.

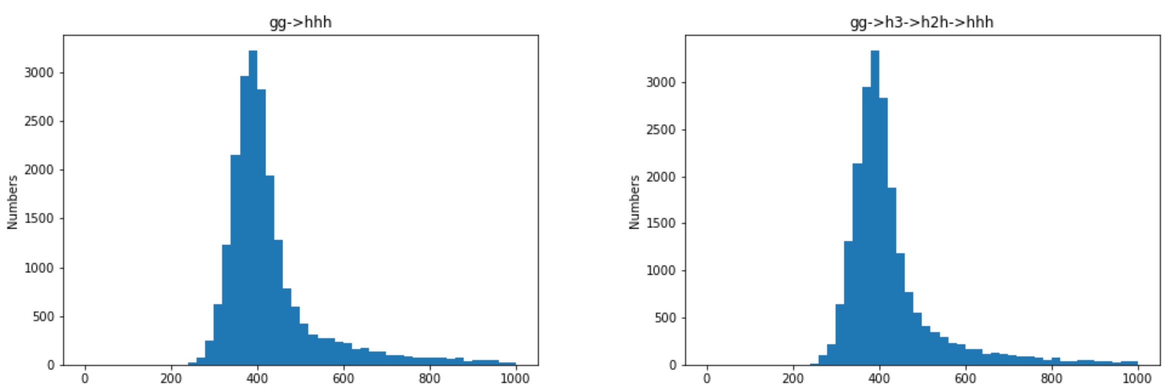

Figure 10: Total invariant mass of 6 b-jets for signal events and resonant channel (correct parameters).

12 The cross section in MadGraph

Table 8 is the cross sections calculated by MadGraph and in paper. They are different because in MadGraph we only consider LO. But in the paper, the numbers are quoted from the LHC Higgs Cross Section Working Group, and they have considered up to NNLO. The "k factor" is around 2.5, which accounts for the values in gg>h3.

Table 8: Cross section from MadGraph and paper

Process	σ MadGraph (fb)	σ in paper (fb)
g g > h3	21	55
gg > h3 > h h h	4.0	38.2

The problem of cross section: If we use the default run card to generate the decay process, the value of cross section will be problematic.

Solution:

In madevent, use command `compute_widths` to compute decay widths of h_2, h_3, then replace the `run_card.dat` by `run_card_default.dat`.

Regenerate the signal and resonant event, the cross sections are 11.1 fb and 10.39 fb, respectively. This channel is indeed dominated in the signal.

13 Comparision for pp $\to$ hhh and gg $\to$ hhh

Generate pp->hhh in MadGraph by following commands:

```
import model cxSM_VLF_EFT
define p = p b b~
generate p p > h h h QCD<=8
```

Generate gg->hhh in MadGraph by following commands:

```
import model cxSM_VLF_EFT
generate g g > h h h
```

These events are generated in 14 TeV and the sample size are 100k. The cross section for pp->hhh and gg->hhh are 11.22 fb and 11.11 fb, respectively. They only differ by 1%.

14 Comparision for gg $\to$ hhh and resonant channel

To compare gg $\to$ hhh and resonant channel, I plot the p_{T}, η, and total invariant mass of 6 b-jets for both.

These events are generated in 14 TeV and the sample size are 100k. The cross section for gg->hhh and resonant channel are 11.11 fb and 10.38 fb, respectively.

The number of events of 6 b-jets for gg->hhh is 21,814 and for the resonant channel is 21,475. The numbers are very close. I scaled the number of events for gg->3h to be the same as the resonant channel.

Figure 11 is p_{T} and η distributions. Figure 12 is total invariant mass of 6 b-jets distributions. Their distributions look similar.

I apply the cuts same in Sec.10 and count how many events can pass these cuts. Table 9 is the result. Those results for both events are similar.

Table 9: Number of events pass the selection cuts. Total number of events for gg->hhh and resonant channel both are 100,000.

Cut	gg->3h	resonant channel
1	21,814	21,475
2	21,254	20,898
3	17,130	16,828
4	14,142	13,730

15 Branching ratios and decay widths

The branching ratios and decay widths are calculated by MadGraph and Mathematica notebook. Mathematica notebook does not include uu dd ss ee decay modes. Since the contribution of these modes is very small.

Table 10 is calculated by MadGraph. Table 11 is calculated by Mathematica notebook.

The exact values from both are slightly different because MG can not correctly expand the parameter ϵ in the model.

Set the ϵ to 0.001 and calculate again. The results are in Table 12 and Table 13.

This time, the decay widths and branching ratios calculated by MG and Mathematica notebook become close.

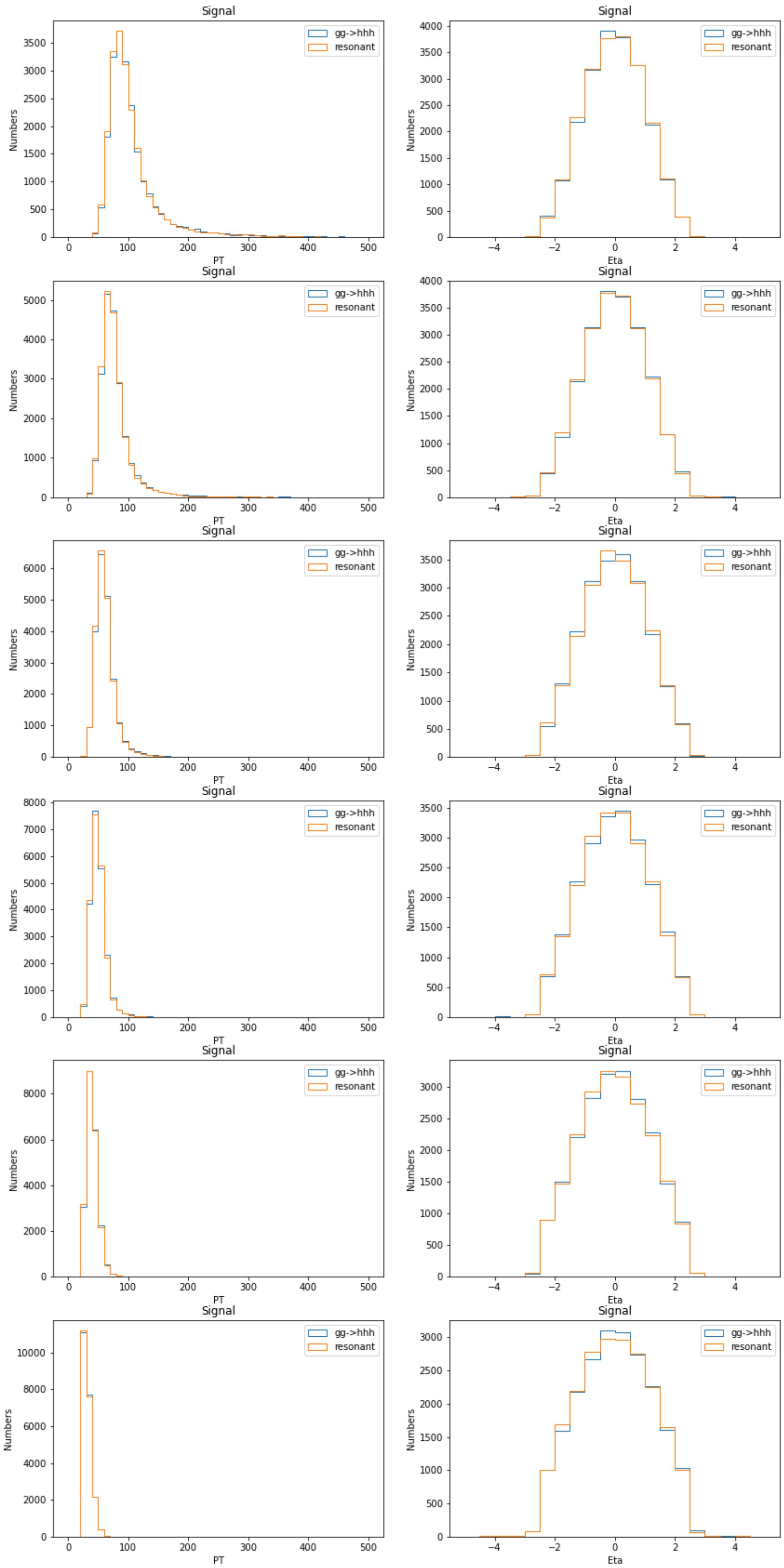

Figure 11: p_{T} and η distributions of b-jets for gg->3h and resonant channel. They are ordered by p_{T}.

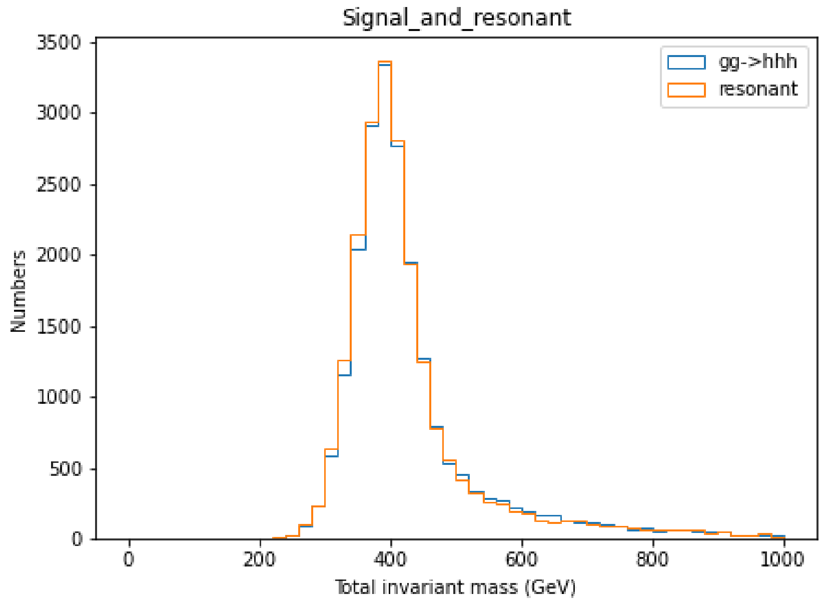

Figure 12: The distribution of total invariant mass of 6 b-jets for gg->3h and resonant channel.

Table 10: Decay widths and branching ratios calulated by MadGraph at BP1.

	BR	Width (GeV)		BR	Width (GeV)
$h_2 \to h_1h_1$	0.7506	0.085616	$h_3 \to h_1h_2$	0.6795	0.8414
$h_2 \to WW$	0.1734	0.019782	$h_3 \to h_1h_1$	0.1711	0.21183
$h_2 \to ZZ$	0.07548	0.0086097	$h_3 \to WW$	0.08756	0.10842
$h_2 \to bb$	0.0004724	5.3887e-05	$h_3 \to ZZ$	0.04102	0.050792
$h_2 \to cc$	3.455e-05	3.9407e-06	$h_3 \to tt$	0.02065	0.025569
$h_2 \to \tau\tau$	2.254e-05	2.5714e-06	$h_3 \to gg$	8.662e-05	0.00010725
$h_2 \to ss$	2.185e-07	2.4927e-08	$h_3 \to bb$	8.158e-05	0.00010102
$h_2 \to gg$	1.561e-07	1.7811e-08	$h_3 \to cc$	5.961e-06	7.3813e-06
$h_2 \to \mu\mu$	7.972e-08	9.0933e-09	$h_3 \to \tau\tau$	3.89e-06	4.8167e-06
$h_2 \to \gamma\gamma$	6.495e-09	7.409e-10	$h_3 \to \gamma\gamma$	3.852e-07	4.7697e-07
$h_2 \to \gamma Z$	9.447e-10	1.0776e-10	$h_3 \to \gamma Z$	4.937e-08	6.1131e-08
$h_2 \to dd$	5.441e-10	6.207e-11	$h_3 \to ss$	3.77e-08	4.6686e-08
$h_2 \to uu$	1.393e-10	1.5889e-11	$h_3 \to \mu\mu$	1.375e-08	1.7031e-08
$h_2 \to ee$	1.865e-12	2.1269e-13	$h_3 \to dd$	9.389e-11	1.1625e-10
			$h_3 \to uu$	2.403e-11	2.976e-11
			$h_3 \to ee$	3.217e-13	3.9835e-13

Table 11: Decay widths and branching ratios calulated by Mathematica at BP1.

	BR	Width (GeV)		BR	Width (GeV)
$h_2 \to h_1h_1$	0.8321	0.1407	$h_3 \to h_1h_2$	0.8219	2.158
$h_2 \to WW$	0.1166	0.019721	$h_3 \to h_1h_1$	0.1049	0.27543
$h_2 \to ZZ$	0.05108	0.0086386	$h_3 \to WW$	0.04131	0.10846
$h_2 \to gg$	0.0001204	2.0359e-05	$h_3 \to ZZ$	0.01941	0.050962
$h_2 \to bb$	9.91e-05	1.6758e-05	$h_3 \to tt$	0.01242	0.032613
$h_2 \to \tau\tau$	1.526e-05	2.5802e-06	$h_3 \to gg$	6.364e-05	0.00016711
$h_2 \to cc$	4.59e-06	7.7613e-07	$h_3 \to bb$	1.195e-05	3.1386e-05
$h_2 \to \gamma\gamma$	3.161e-06	5.346e-07	$h_3 \to \tau\tau$	1.841e-06	4.8326e-06
$h_2 \to \gamma Z$	4.822e-07	8.154e-08	$h_3 \to cc$	5.536e-07	1.4536e-06
$h_2 \to \mu\mu$	5.43e-08	9.1826e-09	$h_3 \to \gamma\gamma$	1.823e-07	4.7857e-07
$h_2 \to tt$	4.355e-19	7.3649e-20	$h_3 \to \gamma Z$	2.334e-08	6.1277e-08
			$h_3 \to \mu\mu$	6.55e-09	1.7198e-08

Table 12: Decay widths and branching ratios calulated by MadGraph at BP1 with $\epsilon = 0.001$.

	BR	Width (GeV)		BR	Width (GeV)
$h_2 \to h_1h_1$	0.8311	1.4041e-05	$h_3 \to h_1h_2$	1.0	2.1493
$h_2 \to WW$	0.1174	1.9835e-06	$h_3 \to h_1h_1$	1.279e-05	2.748e-05
$h_2 \to ZZ$	0.0511	8.6329e-07	$h_3 \to WW$	5.058e-06	1.0871e-05
$h_2 \to bb$	0.0003198	5.4032e-09	$h_3 \to ZZ$	2.369e-06	5.0929e-06
$h_2 \to cc$	2.339e-05	3.9513e-10	$h_3 \to tt$	1.193e-06	2.5638e-06
$h_2 \to \tau\tau$	1.526e-05	2.5783e-10	$h_3 \to gg$	5.007e-09	1.0761e-08
$h_2 \to ss$	1.479e-07	2.4994e-12	$h_3 \to bb$	4.713e-09	1.0129e-08
$h_2 \to \mu\mu$	5.397e-08	9.1178e-13	$h_3 \to cc$	3.443e-10	7.4011e-10
$h_2 \to dd$	3.684e-10	6.2237e-15	$h_3 \to \tau\tau$	2.247e-10	4.8297e-10
$h_2 \to uu$	9.431e-11	1.5932e-15	$h_3 \to \gamma\gamma$	2.227e-11	4.7857e-11
$h_2 \to gg$	1.056e-11	1.7841e-16	$h_3 \to \gamma Z$	2.854e-12	6.1335e-12
$h_2 \to ee$	1.262e-12	2.1326e-17	$h_3 \to ss$	2.178e-12	4.6812e-12
$h_2 \to \gamma\gamma$	4.393e-13	7.4213e-18	$h_3 \to \mu\mu$	7.945e-13	1.7077e-12
$h_2 \to \gamma Z$	6.389e-14	1.0794e-18	$h_3 \to dd$	5.423e-15	1.1657e-14
			$h_3 \to uu$	1.388e-15	2.984e-15
			$h_3 \to ee$	1.858e-17	3.9942e-17

Table 13: Decay widths and branching ratios calulated by Mathematica at BP1 with $\epsilon = 0.001$.

	BR	Width (GeV)		BR	Width (GeV)
$h_2 \to h_1h_1$	0.8321	1.407e-05	$h_3 \to h_1h_2$	1.0	2.158
$h_2 \to WW$	0.1166	1.9721e-06	$h_3 \to h_1h_1$	1.276e-05	2.7543e-05
$h_2 \to ZZ$	0.05108	8.6386e-07	$h_3 \to WW$	5.026e-06	1.0846e-05
$h_2 \to gg$	0.0001204	2.0359e-09	$h_3 \to ZZ$	2.361e-06	5.0962e-06
$h_2 \to bb$	9.91e-05	1.6758e-09	$h_3 \to tt$	1.511e-06	3.2613e-06
$h_2 \to \tau\tau$	1.526e-05	2.5802e-10	$h_3 \to gg$	7.743e-09	1.6711e-08
$h_2 \to cc$	4.59e-06	7.7613e-11	$h_3 \to bb$	1.454e-09	3.1386e-09
$h_2 \to \gamma\gamma$	3.161e-06	5.346e-11	$h_3 \to \tau\tau$	2.239e-10	4.8326e-10
$h_2 \to \gamma Z$	4.822e-07	8.154e-12	$h_3 \to cc$	6.736e-11	1.4536e-10
$h_2 \to \mu\mu$	5.43e-08	9.1826e-13	$h_3 \to \gamma\gamma$	2.218e-11	4.7857e-11
$h_2 \to tt$	4.355e-15	7.3649e-20	$h_3 \to \gamma Z$	2.839e-12	6.1277e-12
			$h_3 \to \mu\mu$	7.969e-13	1.7198e-12

For color channels, they are still doesn't look the same. The reason is that in MadGraph it only calculates to LO, but the numbers in Mathematica notebook are calculated to NLO.

16 SPANet

Code: Symmetry Preserving Attention Networks

Symmetry Preserving Attention NETworks (Spa-Net) is used to do the jet assignment task. The jet assignment task is the identification of the original particle which leads to a reconstructed jet.

16.1 Prepare training data

1. Defining the event topology in `.ini` file. The structure of the `.ini` file follows this format:

```
[SOURCE]
FEATURE_1 = FEATURE_OPTION
FEATURE_2 = FEATURE_OPTION
FEATURE_3 = FEATURE_OPTION
...
```

```
[EVENT]
particles = (PARTICLE_1, PARTICLE_2, ...)
permutations = EVENT_SYMMETRY_GROUP

[PARTICLE_1]
jets = (JET_1, JET_2, ...)
permutations = JET_SYMMETRY_GROUP

[PARTICLE_2]
jets = (JET_1, JET_2, ...)
permutations = JET_SYMMETRY_GROUP

...
```

2. Create training dataset in HDF5 format.

3. Write option-file in JSON format.

16.2 Training

Training:

```
python train.py -of <OPTIONS_FILE> --log_dir <LOG_DIR>  --name <NAME> --gpus 1
```

<OPTIONS_FILE>: JSON file with option overloads. <LOG_DIR>: output directory. <NAME>: subdirectory for this run.

Evaluation:

```
python test.py <log_directory> --gpu
```

<log_directory>: directory containing the checkpoint and options file.

```
python test.py <log_directory> -tf <TEST_FILE> --gpu
```

<TEST_FILE> will replace the test file in option file.

17 SPANet for ttbar event

The training and testing datasets from here: Link

For full ttbar event:

- Training sample:
  - Total sample size: 10,009,520
  - 2t sample size: 2,967,955
- Testing sample size:
  - Total sample size: 358,946
  - 2t sample size: 116,342

Figure 13 is the training results for full ttbar events. The results are the same as the numbers given in the SPANet paper.

18 SPANet for di-Higgs event

Generate di-Higgs events in MadGraph by following commands:

```
import model 2HDMtII_NLO
generate p p > h2 [QCD] QED<=99 QCD<=99
```

then use the MadSpin let h2 decay to h1h1 and h1 decay to $b\bar{b}$. The mass of h2 is 1000 GeV.

The `.ini` file for di-Higgs event (h > b $\bar{\text{b}}$)

```
[SOURCE]
mass = log_normalize
pt = log_normalize
eta = normalize
phi = normalize

[EVENT]
particles = (h1, h2)
permutations = [(h1, h2)]

[h1]
```

Event Type: *t

Jet Limit	Event Proportion	Jet Proportion	Event Purity	T Purity
== 6	0.812	0.308	0.640	0.694
== 7	0.851	0.341	0.597	0.665
>= 8	0.881	0.351	0.523	0.610
Full	0.850	1.000	0.583	0.652

Event Type: 0t

Jet Limit	Event Proportion	Jet Proportion	Event Purity	T Purity
== 6	0.188	0.308	1.000	N/A
== 7	0.149	0.341	1.000	N/A
>= 8	0.119	0.351	1.000	N/A
Full	0.150	1.000	1.000	N/A

Event Type: 1t

Jet Limit	Event Proportion	Jet Proportion	Event Purity	T Purity
== 6	0.566	0.308	0.567	0.567
== 7	0.531	0.341	0.556	0.556
>= 8	0.484	0.351	0.523	0.523
Full	0.525	1.000	0.549	0.549

Event Type: 2t

Jet Limit	Event Proportion	Jet Proportion	Event Purity	T Purity
== 6	0.246	0.308	0.807	0.841
== 7	0.320	0.341	0.666	0.755
>= 8	0.397	0.351	0.523	0.663
Full	0.324	1.000	0.637	0.735

Figure 13: The training result for full ttbar events.

```
jets = (b1, b2)
permutations = [(b1, b2)]

[h2]
jets = (b1, b2)
permutations = [(b1, b2)]
```

There are three types of events, 0h, 1h, 2h. The number means how many identifiable Higgs in a event.

Definition of some parameters:

- Event proportion:

$$\text{Event proportion} \equiv \frac{\text{number of some type events with } i \text{ jets}}{\text{number of events with } i \text{ jets}} \tag{1}$$

- Jet proportion:

$$\text{Jet proportion} \equiv \frac{\text{number of events with } i \text{ jets}}{\text{total number of events}} \tag{2}$$

- Event purity:

$$\epsilon^{\text{event}} \equiv \frac{\text{number of some type events with and all Higgs are correctly identified}}{\text{number of some type events}} \tag{3}$$

- H purity:

$$\epsilon^{\text{h}} \equiv \frac{\text{number of correctly identified Higgs in some type events}}{\text{number of identifiable Higgs in some type events}} \tag{4}$$

18.1 Training sample for di-Higgs events

Di-Higgs events pp->h2, h2->hh, h->bb was generated at $\sqrt{s} = 14$ TeV using MadGraph. Then pass these events to Pythia8 for showering and hadronization. Then pass to Delphes for detector simulation.

Jets are reconstructed by the anti-kT algorithm with radius $R = 0.5$ and are required to have $p_{\mathrm{T}} \geq 20$ GeV.

Preselection requirements: ≥ 4 jets in $|\eta| < 2.5$.

Define the correct jet assignments by matching them to the simulated truth quarks within an angular distance of $\Delta R = \sqrt{\Delta\eta^2 + \Delta\phi^2} < 0.4$.

18.2 Training result for di-Higgs event

For 100k di-Higgs event with $m_{h_2} = 1000$ GeV:

- Training sample:
  - Total sample size: 78,785
  - 2h sample size: 36,765
  - 5% used on validation
- Testing sample:
  - Total sample size: 8,753
  - 2h sample size: 4,130

Event Type: *h

Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
== 4	0.887	0.291	0.945	0.960
== 5	0.925	0.331	0.908	0.940
>= 6	0.947	0.378	0.792	0.865
Full	0.922	1.000	0.874	0.914

Event Type: 0h

Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
== 4	0.113	0.291	1.000	N/A
== 5	0.075	0.331	1.000	N/A
>= 6	0.053	0.378	1.000	N/A
Full	0.078	1.000	1.000	N/A

Event Type: 1h

Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
== 4	0.545	0.291	0.911	0.911
== 5	0.442	0.331	0.888	0.888
>= 6	0.385	0.378	0.827	0.827
Full	0.450	1.000	0.876	0.876

Event Type: 2h

Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
== 4	0.342	0.291	1.000	1.000
== 5	0.483	0.331	0.926	0.963
>= 6	0.562	0.378	0.769	0.878
Full	0.472	1.000	0.871	0.933

Figure 14: The training result for 100k di-Higgs events.

Figure 14 is the training results for 100k di-Higgs events. For 2h events, $\epsilon^{\text{event}} = 0.871, \epsilon^{\text{h}} = 0.933$.

For 1M di-Higgs event with $m_{\text{h}_2} = 1000$ GeV:

- Training sample:
  - Total sample size: 788,160
  - 2h sample size: 364,773
  - 5% used on validation
- Testing sample:
  - Total sample size: 87,702
  - 2h sample size: 40,695

Figure 15 is the training results for 1M di-Higgs events. For 2h events, $\epsilon^{\text{event}} = 0.914, \epsilon^{\text{h}} = 0.955$.

For di-Higgs event with $m_{\text{h}_2} = 500$ GeV:

- Training sample:
  - Total sample size: 764,676
  - 2h sample size: 396,588
  - 5% used on validation
- Testing sample:
  - Total sample size: 84,687
  - 2h sample size: 43,818

Figure 16 is the training results for 500 GeV di-Higgs events. For 2h events, $\epsilon^{\text{event}} = 0.779, \epsilon^{\text{h}} = 0.871$.

18.3 Apply the pre-trained model on different $m_{\mathbf{h}_2}$ sample

The SPANet models in Sec.18.2 are trained on $m_{\text{h}_2} = 500$ GeV and $m_{\text{h}_2} = 1000$ GeV. Test these models on samples with $m_{\text{h}_2} = 500$ GeV, 750 GeV, 1000 GeV.

The results are summarized in Table 14.

Event Type: *h

Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
== 4	0.877	0.301	0.950	0.964
== 5	0.920	0.326	0.919	0.946
>= 6	0.944	0.372	0.831	0.891
Full	0.916	1.000	0.894	0.928

Event Type: 0h

Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
== 4	0.123	0.301	1.000	N/A
== 5	0.080	0.326	1.000	N/A
>= 6	0.056	0.372	1.000	N/A
Full	0.084	1.000	1.000	N/A

Event Type: 1h

Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
== 4	0.533	0.301	0.917	0.917
== 5	0.445	0.326	0.885	0.885
>= 6	0.394	0.372	0.813	0.813
Full	0.452	1.000	0.873	0.873

Event Type: 2h

Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
== 4	0.345	0.301	1.000	1.000
== 5	0.475	0.326	0.950	0.975
>= 6	0.551	0.372	0.844	0.918
Full	0.464	1.000	0.914	0.955

Figure 15: The training result for 1M di-Higgs events.

Table 14: SPANet models test on different m_{h_2} samples.

Training m_{h_2} (GeV)	Testing m_{h_2} (GeV)	Event purity	H purity
500	500	0.779	0.871
500	750	0.408	0.672
500	1000	0.390	0.665
1000	500	0.383	0.638
1000	750	0.731	0.860
1000	1000	0.914	0.955

Event Type: *h

Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
== 4	0.872	0.319	0.879	0.914
== 5	0.923	0.325	0.802	0.868
>= 6	0.944	0.356	0.633	0.753
Full	0.914	1.000	0.763	0.836

Event Type: 0h

Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
== 4	0.128	0.319	1.000	N/A
== 5	0.077	0.325	1.000	N/A
>= 6	0.056	0.356	1.000	N/A
Full	0.086	1.000	1.000	N/A

Event Type: 1h

Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
== 4	0.497	0.319	0.793	0.793
== 5	0.391	0.325	0.748	0.748
>= 6	0.312	0.356	0.666	0.666
Full	0.397	1.000	0.743	0.743

Event Type: 2h

Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
== 4	0.375	0.319	0.994	0.994
== 5	0.532	0.325	0.841	0.912
>= 6	0.632	0.356	0.617	0.775
Full	0.517	1.000	0.779	0.871

Figure 16: The training result for di-Higgs events with $m_{\mathrm{h}_2} = 500$ GeV.

19 SPANet for tri-Higgs event

The tri-Higgs events are signal events (gg->hhh) generated in previous.

19.1 Training sample for tri-Higgs event

TriHiggs events gg->hhh was generated at $\sqrt{s} = 14$ TeV using MadGraph. Then pass these events to Pythia8 for showering and hadronization. In Pythia8, the Higgs are forced to decay to $\mathrm{b\bar{b}}$. Then pass to Delphes for detector simulation.

Jets are reconstructed by the anti-kT algorithm with radius $R = 0.5$ and are required to have $p_{\mathrm{T}} \geq 20$ GeV.

Preselection requirements: ≥ 6 jets in $|\eta| < 2.5$.

Define the correct jet assignments by matching them to the simulated truth quarks within an angular distance of $\Delta R = \sqrt{\Delta\eta^2 + \Delta\phi^2} < 0.4$.

19.2 Training result for tri-Higgs event

For 1M tri-Higgs event:

- Training sample:
  - Total sample size: 542,681
  - 3h sample size: 104,467
  - 5% used on validation
- Testing sample:
  - Total sample size: 60,173
  - 3h sample size: 11,598

Figure 17 is the training results for 1M tri-Higgs events. For 3h events, $\epsilon^{\text{event}} = 0.344, \epsilon^{\text{h}} = 0.538$.

Event Type: 0h

Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
== 6	0.186	0.439	1.000	N/A
== 7	0.147	0.313	1.000	N/A
>= 8	0.119	0.248	1.000	N/A
Full	0.157	1.000	1.000	N/A

Event Type: 1h

Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
== 6	0.386	0.439	0.452	0.452
== 7	0.337	0.313	0.405	0.405
>= 8	0.312	0.248	0.334	0.334
Full	0.353	1.000	0.412	0.412

Event Type: 2h

Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
== 6	0.303	0.439	0.293	0.489
== 7	0.298	0.313	0.228	0.429
>= 8	0.285	0.248	0.152	0.343
Full	0.297	1.000	0.239	0.435

Event Type: 3h

Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
== 6	0.124	0.439	0.603	0.705
== 7	0.218	0.313	0.336	0.546
>= 8	0.283	0.248	0.152	0.400
Full	0.193	1.000	0.344	0.538

Figure 17: The training result for 1M tri-Higgs events.

20 χ^2 method

20.1 Di-Higgs

Try all possible combination of final jets and find the minimal mass difference between SM Higgs mass, i.e. minimize this

$$\chi^2 = [M(j_1 j_2) - M_{\mathrm{H}}]^2 + [M(j_3 j_4) - M_{\mathrm{H}}]^2 \tag{5}$$

where $M(j_i j_j)$ means the invariant mass of jet i and jet j.

The results is presented in Table 15.

Table 15: Performance comparison for di-Higgs events.

	Event	SPANet Efficiency		χ^2 Efficiency	
N_{Jet}	Fraction	Event	Higgs	Event	Higgs
$= 4$	0.224	1.000	1.000	1.000	1.000
$= 5$	0.334	0.950	0.975	0.675	0.737
≥ 6	0.442	0.844	0.918	0.313	0.441
Total	1.000	0.914	0.955	0.582	0.665