Note

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1   Generate di-Higgs samples in SM

Generate the double Higgs events in the standard model by MadGraph with `loop_sm` model. Following are the MadGraph scripts for generating di-Higgs samples:

```
import model loop_sm
generate p p > h h [QCD] QED^2<=99 QCD^2<=99
output /home/r10222035/CPVDM/Di-Higgs-SM/di-Higgs-sm


launch /home/r10222035/CPVDM/Di-Higgs-SM/di-Higgs-sm


shower=OFF
detector=OFF
analysis=OFF
done


set run_card nevents 10000
set run_card ebeam1 6500.0
set run_card ebeam2 6500.0


done
```

1.1   Variation with κ_λ

Reference: [How to change the trilinear Higgs coupling in Madgraph?](#)
The definition of κ_λ

$$\kappa_\lambda \equiv \frac{\lambda_{HHH}}{\lambda_{HHH}^{\mathrm{SM}}} \tag{1}$$

Following the below steps, we can add a parameter κ_λ in the model

1. Go to the MadGraph model file directory. Copy `loop_sm` to `my_loop_sm`.

2. Go to `my_loop_sm` directory.

3. In `parameters.py`, add a new parameter for κ_λ by

```
khhh = Parameter(name = 'khhh',
  nature = 'external',
  type = 'real',
  value = 1,
  texname = '\\text{khhh}',
  lhablock = 'SMINPUTS',
  lhacode = [ 10 ])
```

4. In `vertices.py`, we can find the coupling for three Higgs vertex in the form `GC_XX`.

5. In `couplings.py`, multiply the value for `GC_XX` found in step 4 by `khhh`.

6. In `restrict_default.dat`, add

```
10 2.000000e+00 # khhh
```

in Block SMINPUTS.

Finish the above setting we can use the following scripts to generate di-Higgs samples:

```
import model my_loop_sm
generate p p > h h [QCD] QED^2<=99 QCD^2<=99
output /home/r10222035/CPVDM/Di-Higgs-SM/di-Higgs-sm-kappa

launch /home/r10222035/CPVDM/Di-Higgs-SM/di-Higgs-sm-kappa

shower=OFF
detector=OFF
analysis=OFF
done

set param_card khhh 1

set run_card nevents 10000
```

```
set run_card ebeam1 6500.0
set run_card ebeam2 6500.0

done
```

1.2 Results

The cross sections of various κ_λ are shown in Table 1.

Table 1: The cross sections of various κ_λ. My data is the results from MadGraph. The reference data is from here.

| | 13 TeV | | | 14 TeV | | | | |
| | Cross section (fb) | | | Cross section (fb) | | | | |
κ_λ	Ref.	My data	Ref./My	Ref.	My data	Ref./My	Ref. K	Ref.K/My K
-1	116.71	74.62	1.564	136.91	87.93	1.56	1.86	1.19
0	62.51	41.96	1.490	73.64	49.45	1.49	1.79	1.20
1	27.84	20.27	1.373	32.88	24.05	1.37	1.66	1.21
2	12.42	9.56	1.299	14.75	11.34	1.30	1.56	1.20
2.4	11.65	8.33	1.399	13.79	9.90	1.39	1.65	1.18
3	16.28	9.81	1.660	19.07	11.55	1.65	1.90	1.15
5	81.74	43.55	1.877	95.22	50.68	1.88	2.14	1.14

The m_{HH} distribution with various κ_λ is presented in Figure 1. In the left plot, the data is the parton level data from MadGraph. The right plot comes from the ATLAS reference. Here, the $\sqrt{s} = 13$ TeV

Figure 2 and 3 are generated at $\sqrt{s} = 14$ TeV.

2 Non-resonant di-Higgs event selection

2.1 Sample

Non-resonant Higgs pair process is generated by MadGraph. Then pass to Pythia for showering and hadronization. Then pass to Delphes for detector simulation.

Jets are reconstructed using the anti-k_t algorithm with radius parameter $R = 0.4$.

The b-tagging part in the Delphes card is changed such that same as the DL1r b-tagger at 77% WP. The b-jet efficiency is set to 0.77. The c-jet missing rate is set to 0.204. The light jet missing rate is set to 0.0077.
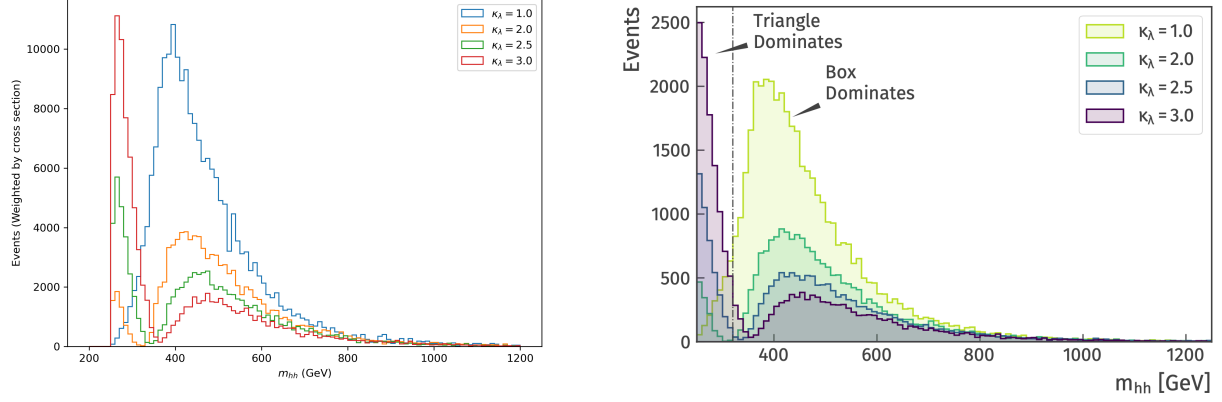
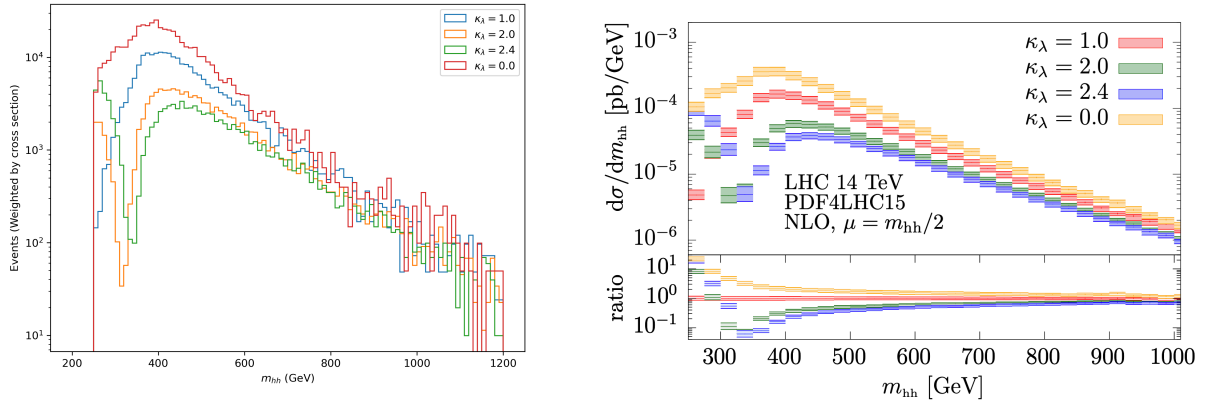Figure 1: The m_{hh} distribution with various κ_λ. The bin height is weighted by the cross section.

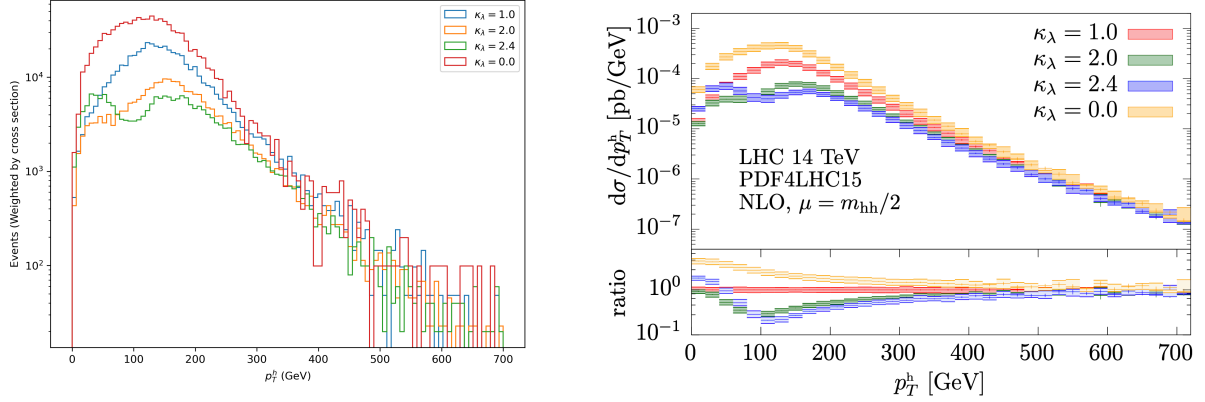Figure 2: The m_{hh} distribution with various κ_λ. The bin height is weighted by the cross section.

Figure 3: The p_{T}^{h} distribution with various κ_λ. The bin height is weighted by the cross section.

2.2   Event selection

Reference: ATLAS CONF Note CONF-HDBS-2022-35

The selection steps:

- Four tag: The event contains at least 4 b-tagged anti-k_t $R = 0.4$ jets with $p_{\mathrm{T}} > 40$ GeV and $|\eta| < 2.5$.

- The four jets with the highest p_{T} are paired to construct two Higgs boson candidates.

- min-ΔR pairing method: Choose the pairing in which the higher-p_{T} jet pair has the smallest ΔR separation.

- Higgs Eta:

$$|\Delta \eta_{HH}| < 1.5$$

- Top veto: Every possible pair of jets with $p_{\mathrm{T}} > 40$ GeV and $|\eta| < 2.5$, including those that were not selected for the H candidates, to form "W candidates". "Top quark candidates" are built by pairing W candidates with each remaining jet that was selected for H candidates. The quantity X_{Wt} is defined as

$$X_{Wt} = \sqrt{\left(\frac{m_W - 80.4 \text{ GeV}}{0.1 m_W}\right)^2 + \left(\frac{m_t - 172.5 \text{ GeV}}{0.1 m_t}\right)^2}$$

Events with the smallest $X_{Wt} < 1.5$ are vetoed.

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- Signal region:

$$X_{HH} = \sqrt{\left(\frac{m_{H_1} - 124 \text{ GeV}}{0.1 m_{H_1}}\right)^2 + \left(\frac{m_{H_2} - 117 \text{ GeV}}{0.1 m_{H_2}}\right)^2} < 1.6$$

Table 2: The selection passing rate and efficiency at each stage. The b-tagging part is the same as the DL1r 77% WP.

	ATLAS		My sample	
Cut	pass rate	efficiency	pass rate	efficiency
Four tag	0.0649	0.0649	0.0852	0.0852
Higgs Eta	0.0543	0.8360	0.0688	0.8074
Top veto	0.0456	0.8401	0.0553	0.8044
Signal region	0.0220	0.4818	0.0181	0.3283

Correct selection: Consider the events in which four jets can be matched one-to-one (within $\Delta R < 0.3$) to the four b-quarks decayed from the Higgs bosons. For the highest p_T there are 89% of simulated signal events reaching this selection.

Correct pairing: Consider the correct selection events, for min-ΔR pairing method there 85% of events are correctly paired.

Figure 4 shows the Higgs mass distribution. There is a deviation between the mass distribution peak and the signal region's center.
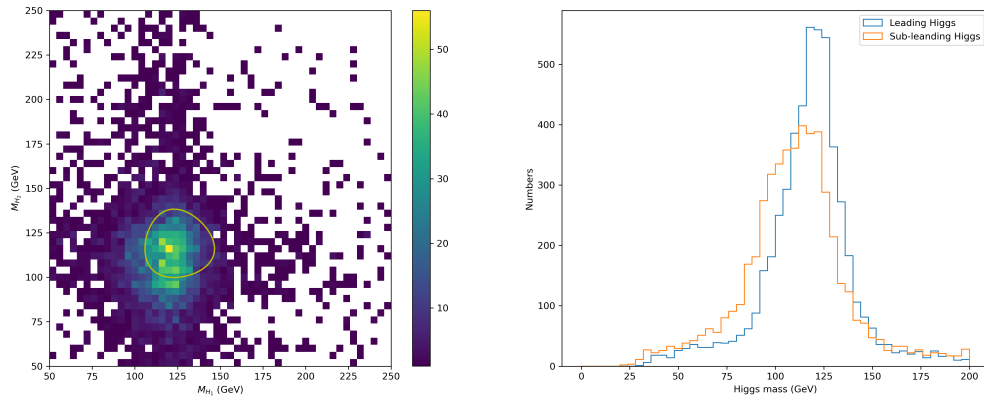

Figure 4: The mass plane and distribution for Higgs candidate.

2.2.1 Old method

Reference: Search for pair production of Higgs bosons in the $b\bar{b}b\bar{b}$ final state using proton-proton collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector

The selection steps:

- Four tag: The event contains at least 4 b-tagged anti-kt small-R ($R = 0.4$) jets with $p_{\mathrm{T}} > 40$ GeV and $|\eta| < 2.5$. The four jets with the highest b-tagging score are paired to construct two Higgs boson candidates.

- The four jets with the highest p_{T} are paired to construct two Higgs boson candidates in my samples.

- Delta R: Pairing jets to Higgs boson candidate need to satisfy the following requirements:

$$\left.\begin{array}{c} \dfrac{360 \text{ GeV}}{m_{4\mathrm{j}}} - 0.5 < \Delta R_{jj,\mathrm{lead}} < \dfrac{653 \text{ GeV}}{m_{4\mathrm{j}}} + 0.475 \\ \dfrac{235 \text{ GeV}}{m_{4\mathrm{j}}} < \Delta R_{jj,\mathrm{subl}} < \dfrac{875 \text{ GeV}}{m_{4\mathrm{j}}} + 0.35 \end{array}\right\} \text{ if } m_{4\mathrm{j}} < 1250 \text{ GeV}$$

$$\left.\begin{array}{c} 0 < \Delta R_{jj,\mathrm{lead}} < 1 \\ 0 < \Delta R_{jj,\mathrm{subl}} < 1 \end{array}\right\} \text{ if } m_{4\mathrm{j}} > 1250 \text{ GeV}$$

- If there are more than 2 pairings satisfies the Delta R requirement. Calculate D_{HH}

$$D_{HH} = \frac{\left| m_{2\mathrm{j}}^{\mathrm{lead}} - \frac{120}{110} m_{2\mathrm{j}}^{\mathrm{subl}} \right|}{\sqrt{1 + \left(\frac{120}{110} \right)^2}}$$

the pairing with the smallest value of D_{HH} is chosen.

- Higgs PT:

$$p_{\mathrm{T}}^{\mathrm{lead}} > m_{4\mathrm{j}} \times 0.5 - 103 \text{ GeV}$$

$$p_{\mathrm{T}}^{\mathrm{subl}} > m_{4\mathrm{j}} \times 0.33 - 73 \text{ GeV}$$

- Higgs Eta:

$$|\Delta \eta_{HH}| < 1.5$$

- Signal region:

$$X_{HH} = \sqrt{\left(\frac{m_{2\mathrm{j}}^{\mathrm{lead}} - 120 \text{ GeV}}{0.1 m_{2\mathrm{j}}^{\mathrm{lead}}} \right)^2 + \left(\frac{m_{2\mathrm{j}}^{\mathrm{subl}} - 110 \text{ GeV}}{0.1 m_{2\mathrm{j}}^{\mathrm{subl}}} \right)^2} < 1.6$$

- Top veto: Every possible pair of jets with $p_{\mathrm{T}} > 40$ GeV and $|\eta| < 2.5$, including those that were not selected for the H candidates, to form "W candidates". "Top quark candidates" are built by pairing W candidates with each remaining jet that was selected for H candidates

$$X_{Wt} = \sqrt{\left(\frac{m_W - 80 \text{ GeV}}{0.1 m_W}\right)^2 + \left(\frac{m_t - 173 \text{ GeV}}{0.1 m_t}\right)^2}$$

Events with the smallest $X_{Wt} < 1.5$ are vetoed.

The results are in Table 3.

Table 3: The selection passing rate and efficiency at each stage.

Cut	ATLAS		My sample	
	pass rate	efficiency	pass rate	efficiency
Four tag	0.0490	0.0490	0.0563	0.0563
Delta R	0.0448	0.9143	0.0471	0.8370
Higgs PT	0.0422	0.9420	0.0446	0.9480
Higgs Eta	0.0380	0.9005	0.0398	0.8911
Signal region	0.0193	0.5079	0.0170	0.4280
Top veto	0.0179	0.9275	0.0145	0.8537

2.3 Background event selection

Apply the same selection step to the background samples. The cutflow table is in Table 4 and the mass distribution is in Figure 5.

Table 4: The selection passing rate and efficiency at each stage for "min-ΔR" pairing method.

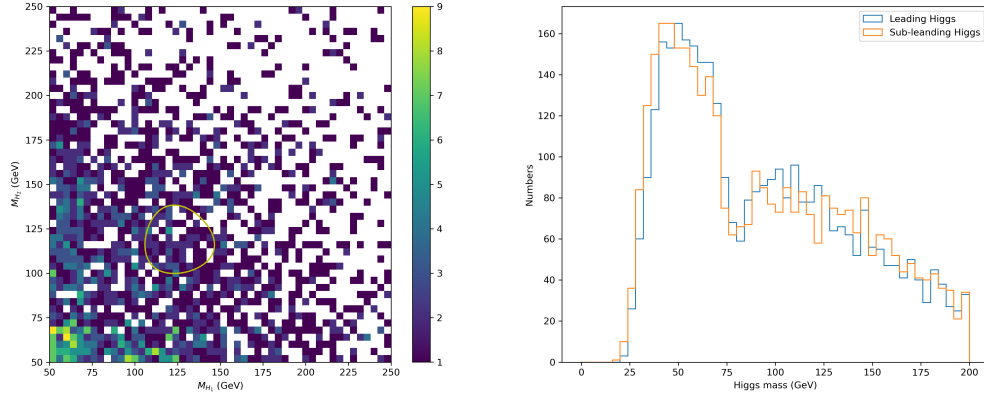| Cut	pp4b min-ΔR	
	pass rate	efficiency
Four tag	0.0096	0.0096
Higgs Eta	0.0056	0.5835
Top veto	0.0042	0.7423
Signal region	0.0001	0.0232

Figure 5: The mass plane and distribution for background events Higgs candidate.

3 Apply SPANET on non-resonant di-Higgs event

3.1 Training SPANET

The training samples are required to pass the "Four tag cut", i.e., there are at least four b-tagged jets with $p_\mathrm{T} > 40$ GeV and $|\eta| < 2.5$. The b-tagging efficiency is the same as the DL1r b-tagger at 77% WP.

- Training sample:

  - Total sample size: 76,131

  - 1h sample size: 14,527

  - 2h sample size: 60,122

  - 5% used on validation

- Testing sample:

  - Total sample size: 8,460

  - 1h sample size: 1,577

  - 2h sample size: 6,744

The training results are presented in Table 5.

Table 5: SPA-NET training results on the di-Higgs samples with "Four tag cut".

N_{Jet}	Event Fraction	Event Efficiency	Higgs Efficiency
$= 4$	0.280	0.961	0.961
$= 5$	0.287	0.878	0.913
≥ 6	0.229	0.740	0.819
Total	0.797	0.868	0.903

Table 6: The selection passing rate and efficiency at each stage for "min-ΔR" and SPA-NET pairing.

	min-ΔR				SPA-NET	
	ATLAS		My sample		My sample	
Cut	pass rate	efficiency	pass rate	efficiency	pass rate	efficiency
Four tag	0.0649	0.0649	0.0852	0.0852	0.0852	0.0852
Higgs Eta	0.0543	0.8360	0.0688	0.8074	0.0635	0.7454
Top veto	0.0456	0.8401	0.0553	0.8044	0.0508	0.8006
Signal region	0.0220	0.4818	0.0181	0.3283	0.0027	0.0541

Table 7: The selection passing rate and efficiency at each stage for "min-ΔR" and SPA-NET pairing.

	pp4b			
	min-ΔR		SPA-NET	
Cut	pass rate	efficiency	pass rate	efficiency
Four tag	0.0096	0.0096	0.0096	0.0096
Higgs Eta	0.0056	0.5835	0.0055	0.5733
Top veto	0.0042	0.7423	0.0042	0.7607
Signal region	0.0001	0.0232	0.0001	0.0181

3.2 Use SPANET for event selection

The "min-ΔR" pairing is replaced by SPA-NET pairing. Other cuts remained unchanged. The cutflow tables for signal and background are in Table 6, 7.

Figure 6 shows the Higgs mass distribution for SPA-NET pairing.
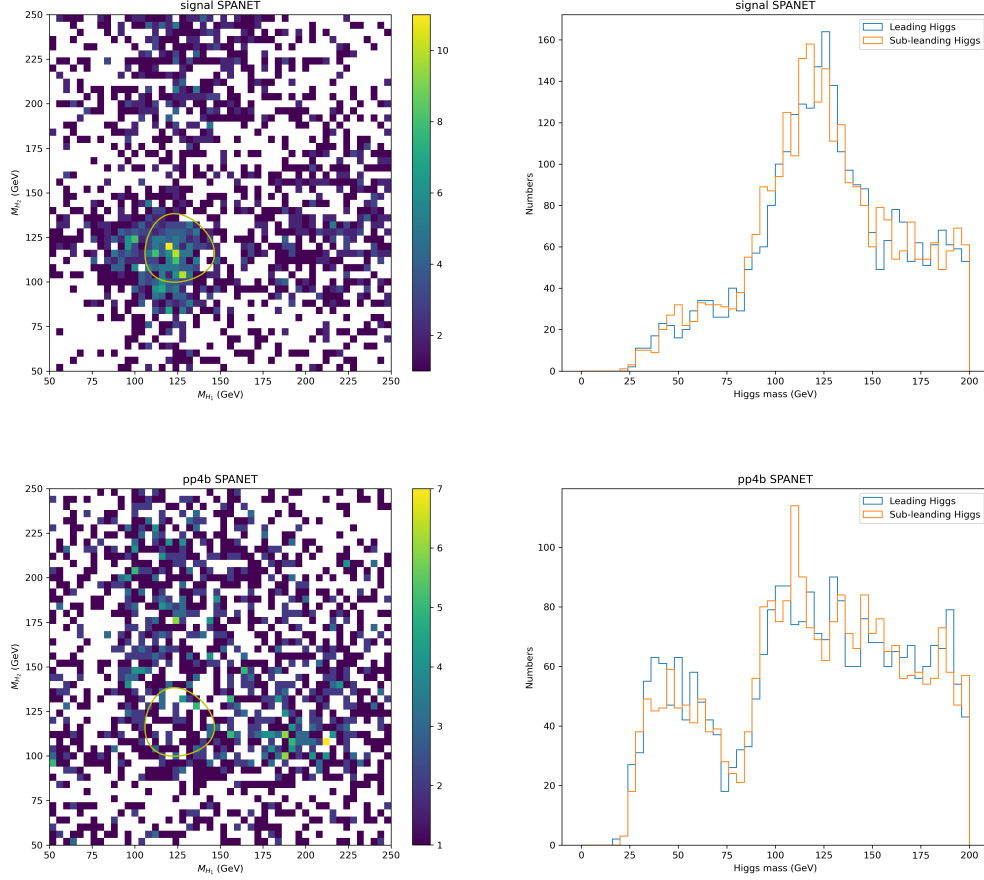

Figure 6: The mass plane and distribution for Higgs candidate for SPA-NET pairing method. The above figure is for the signal sample and the below one is for the background sample.

4 Pairing performace

The truth pairing is defined by matching the jets to the simulated quarks within the angular distance of $\Delta R = \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} < 0.4$.

The "min-ΔR" pairing method contain two-step: first select four b-jets with the highest p_{T}, and second, choose min-ΔR configuration. For the first step, the correct selection rate is 89%. For the second step, the correct pairing rate is 91%. The pairing efficiency is 81%.

The "SPA-NET" pairing method: The pairing efficiency is 86.8%.

The mass distribution for truth pairing is in Figure 7. The results are similar to the "min-ΔR" ones, but much different to the SPA-NET's. There are indeed some bugs in the SPA-NET pairing code.
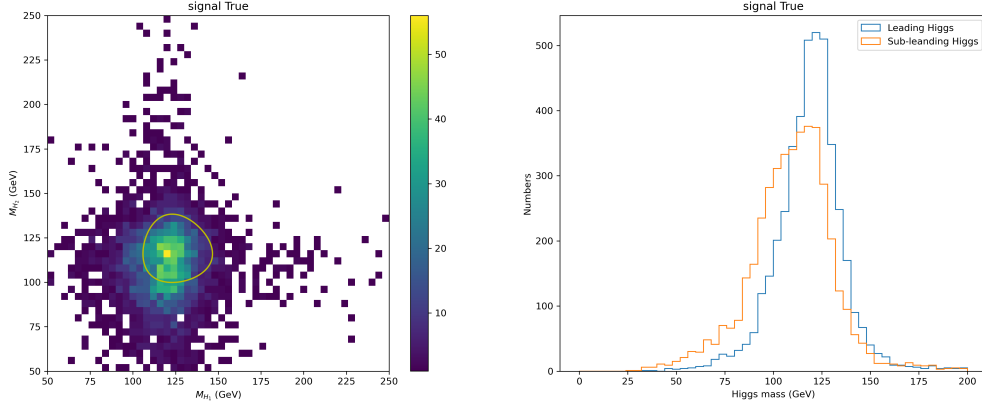

Figure 7: The mass plane and distribution for truth pairing.

5 Apply SPANET to non-resonant di-Higgs event again

The bugs for Sec.3: Some variables are misdefined twice, so they will get an incorrect number when calculating some physical quantities.

Fix this bug, then the cutflow tables for signal and background are in Table 8, 9.

Table 8: The selection passing rate and efficiency at each stage for "min-ΔR" and SPA-NET pairing.

| | min-ΔR | | | | SPA-NET | |
| | ATLAS | | My sample | | My sample | |
Cut	pass rate	efficiency	pass rate	efficiency	pass rate	efficiency
Four tag	0.0649	0.0649	0.0852	0.0852	0.0852	0.0852
Higgs Eta	0.0543	0.8360	0.0688	0.8074	0.0676	0.7942
Top veto	0.0456	0.8401	0.0553	0.8044	0.0544	0.8051
Signal region	0.0220	0.4818	0.0181	0.3283	0.0194	0.3567

Figure 8 shows the Higgs mass distribution for SPA-NET pairing.

The "min-ΔR" result is in Table 10. The SPANet result is in Table 11.

Table 9: The selection passing rate and efficiency at each stage for "min-ΔR" and SPA-NET pairing.

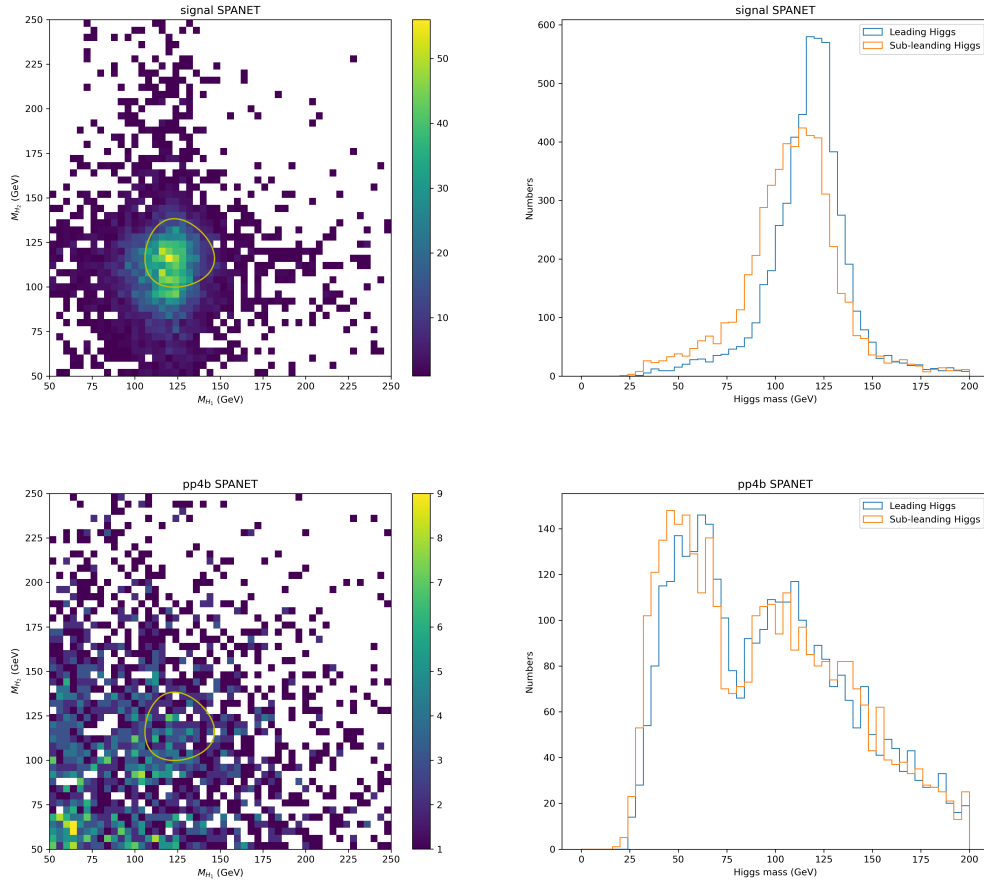| | pp4b | | | |
	min-ΔR		SPA-NET	
Cut	pass rate	efficiency	pass rate	efficiency
Four tag	0.0096	0.0096	0.0096	0.0096
Higgs Eta	0.0056	0.5835	0.0050	0.5202
Top veto	0.0042	0.7423	0.0037	0.7341
Signal region	0.0001	0.0232	0.0002	0.0443

Figure 8: The mass plane and distribution for Higgs candidate for SPA-NET pairing method. The above figure is for the signal sample and the below one is for the background sample.

The SPA-NET method will let more signal events pass the Higgs signal cut, but also true for the background process. Therefore, the SPA-NET pairing method can not improve the $S/\sqrt{B}$.

Table 10: The cross sections for the di-Higgs signal and background processes at different cuts. The pairing method is "min-ΔR".

| | Cross section (fb) | | | $\mathcal{L} = 139$ fb^{-1} | $\mathcal{L} = 300$ fb^{-1} | $\mathcal{L} = 3000$ fb^{-1} |
	Signal	Background	S/B	$S/\sqrt{B}$	$S/\sqrt{B}$	$S/\sqrt{B}$
No cut	6.774	6.29e+05	1.08e-05	0.101	0.148	0.468
Four tag	0.577	6.06e+03	9.52e-05	0.0874	0.128	0.406
Higgs Eta	0.466	3.54e+03	0.000132	0.0923	0.136	0.429
Top veto	0.375	2.62e+03	0.000143	0.0862	0.127	0.401
Higgs signal	0.123	61.0	0.00202	0.186	0.273	0.862

Table 11: The cross sections for the di-Higgs signal and background processes at different cuts. The pairing method is SPA-NET.

| | Cross section (fb) | | | $\mathcal{L} = 139$ fb^{-1} | $\mathcal{L} = 300$ fb^{-1} | $\mathcal{L} = 3000$ fb^{-1} |
	Signal	Background	S/B	$S/\sqrt{B}$	$S/\sqrt{B}$	$S/\sqrt{B}$
No cut	6.774	6.29e+05	1.08e-05	0.101	0.148	0.468
Four tag	0.577	6.06e+03	9.52e-05	0.0874	0.128	0.406
Higgs Eta	0.458	3.15e+03	0.000145	0.0962	0.141	0.447
Top veto	0.369	2.31e+03	0.000159	0.0904	0.133	0.420
Higgs signal	0.132	1.02e+02	0.00128	0.153	0.225	0.712