

Note

Feng-Yang Hsieh

1 Signal and background

Signal events are 3 Higgs events in BSM. I use MadGraph to generate by following commands:

```
import model cxSM_VLF_EFT
generate g g > h h h
```

then pass the events through the Pythia and Delphes. In Pythia, I set the branching ratio of Higgs so that it only decays to b quarks.

Background events are 6b events in SM. I use MadGraph to generate by following commands:

```
generate p p > b b b b~ b~ b~
```

then pass the events through the Pythia and Delphes.

2 Transverse momentum and pseudorapidity distribution

In each event there are 6 b quarks. I order these b quarks by transverse momentum p_T , then plot p_T and pseudorapidity η distributions.

Figure 1 is the p_T and η distributions of b quarks for background.

Figure 2 is the p_T and η distributions of b quarks for signal.

3 Delta R and transverse momentum distribution

For signal, I plot the $\Delta R(b,b)$ distribution of two b decays from the same Higgs. Then I plot $\Delta R(b,b)$ against $p_T(b,b)$. The result is in Figure 3. As can be seen from the plot, ΔR get smaller at higher $p_T(b,b)$.



Figure 1: p_T and η distributions of b partons for background events. They are ordered by p_T .



Figure 2: p_T and η distributions of b partons for signal events. They are ordered by p_T .



Figure 3: $\Delta R(b,b)$ distribution and $\Delta R(b,b)$ against $p_T(b,b)$ plot

4 Cutflow table

I apply the η and p_T cuts sequentially and count how many event can pass the cuts. The results are in Table 1 and Table 2.

Table 1: 1000 background events. Those background events is generated in $\sqrt{s} = 13$ TeV with the cut b $p_T > 10$ GeV in MadGraph.

Cut	number of events pass
6 b-partons $ \eta < 2.5$	330
6 b-partons $p_T < 25$ GeV	11
4 b-partons $p_T < 40$ GeV	8

Table 2: 1000 signal events.

Cut	number of events pass
6 b-partons $ \eta < 2.5$	732
6 b-partons $p_T < 25$ GeV	534
4 b-partons $p_T < 40$ GeV	498

5 Construction of bb-pairs

To construct the bb-pairs from the same Higgs, the typical strategy is to try all combinatoric and find the minimal mass difference between all pairs, i.e. minimize this

$$\chi^2 = [M(b_1b_2) - M(b_3b_4)]^2 + [M(b_1b_2) - M(b_5b_6)]^2 + [M(b_3b_4) - M(b_5b_6)]^2$$

where $M(b_i b_j)$ means the invariant mass of b-parton i and b-parton j .

The other strategy is to minimize mass difference to the SM Higgs mass

$$\chi^2 = [M(b_1b_2) - M_H]^2 + [M(b_3b_4) - M_H]^2 + [M(b_5b_6) - M_H]^2$$

I implement these two methods to construct bb-pairs and test on the 10,000 signal events in parton level. The result is as follows:

- Method 1 (mass difference between all pairs): accuracy = 0.863
- Method 2 (mass difference to the SM Higgs mass): accuracy = 0.875

Method 2 is better for identifying the true Higgs pair.

6 Absolute cross section

Absolute cross section is defined as follow

$$\sigma_{\text{abs}} = \sigma \times \frac{\text{number of events pass the cut}}{\text{number of events}}$$

I have generated background events with different cuts in MadGraph, the detailed information of cuts is in Table 3. I calculate their absolute cross section and check if it is the same or not. The result is in Table 4.

From the Table 4, if the number of event is large enough the absolute cross section will be similar.

7 The problem for generating $pp \rightarrow 6b$ events

Failed to generate the requested number of events

Table 3: The cuts applied on the different run. Where p_T means the minimum transverse momentum of b and $|\eta|$ means the range of b (-1 means no restriction).

No.	p_T (GeV)	$ \eta $
1	0	-1
2	10	5
3	10	3
4	20	3
5	0	-1
6	10	-1

Table 4: Absolute cross section. Where the selection cut is $p_T > 20$ GeV, $|\eta| < 3$.

No.	total event	cross section	events pass selection	absolute cross section
1	339	2731.9362	0	0.0
2	4761	71.316651	190	2.846
3	9054	35.225852	694	2.700
4	4207	2.7698473	4183	2.754
5	1000	2531.9304	0	0.0
6	1000	70.158431	27	1.894

8 $pp \rightarrow 4b$

I use MadGraph to generate 4b events by following commands:

```
generate p p > b b b~ b~
```

In MadGraph, I apply the cuts: $p_T > 25$ GeV, $|\eta| < 2.5$ for b. Pass those events through Pythia, then check how many events there are 6 b-hadrons final state.

In 100,000 events, there are 6,916 events having greater than or equal to 6 b-hadrons. The distribution of number of b-hadrons is in Figure 4.



Figure 4: Number of b-hadrons in Pythia final state.

Pass events through Delphes. In Delphes the b-tagging efficiency is set to 1.

Figure 5 is the number of jets and number of b-jets distributions. In 100,000 events, there are 173 events having greater than or equal to 6 b-jets.

9 Comparison for $pp \rightarrow 4b$ and $pp \rightarrow 6b$ event

To compare $pp \rightarrow 4b$ and $pp \rightarrow 6b$ events, I plot the p_T , η and total invariant mass of 6 b-jets for both.

The number of events of 6 b-jets for $pp \rightarrow 4b$ is 2051 and for $pp \rightarrow 6b$ is 1408. I scaled the number of events for $pp \rightarrow 4b$ to be the same as $pp \rightarrow 6b$.



Figure 5: Number of jets and number of b-jets.

Figure 6 is p_T and η distributions. Figure 7 is total invariant mass of 6 b-jets distributions. Their distributions look similar.

10 Cutflow table for b-jets

I apply following cut sequentially and count how many events can pass these cuts. Table 5 is the result for pp->4b events. Table 6 is the result for pp->6b events. Table 7 is the result for signal events.

- Cut 1: There are greater than or equal to 6 b-jets.
- Cut 2: There are greater than or equal to 6 b-jets satisfy $|\eta| < 2.5$.
- Cut 3: There are greater than or equal to 6 b-jets satisfy $p_T > 25$ GeV.
- Cut 4: There are greater than or equal to 4 b-jets satisfy $p_T > 40$ GeV.

Table 5: 1,000,000 pp->4b events. Those events are generated in $\sqrt{s} = 14$ TeV with the cuts: $p_T > 25$ GeV, $|\eta| < 2.5$ for b in MadGraph.

Cut	number of event pass
1	1783
2	1565
3	1059
4	735



Figure 6: p_T and η distributions of b-jets for $pp \rightarrow 4b$ and $pp \rightarrow 6b$ events. They are ordered by p_T .

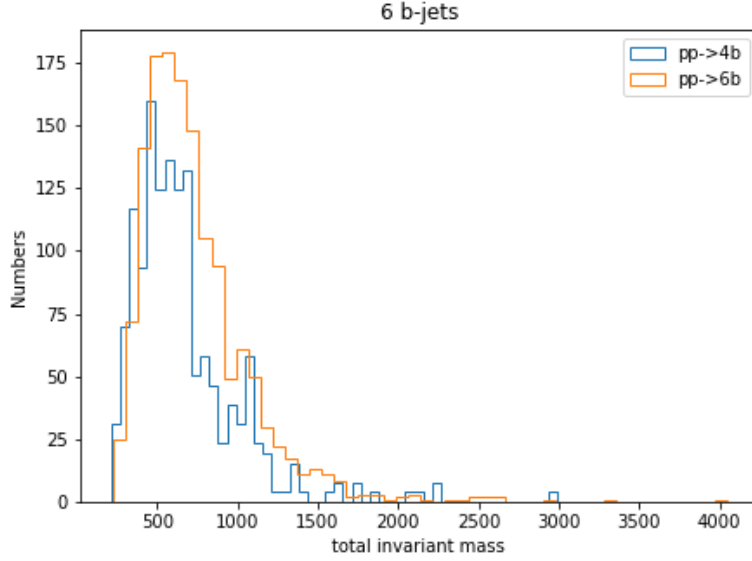


Figure 7: The distribution of total invariant mass of 6 b-jets for pp->4b and pp->6b events.

Table 6: 10,000 pp->6b events. Those events are generated in $\sqrt{s} = 14$ TeV with the cuts: $p_T > 25$ GeV, $|\eta| < 2.5$ for b in MadGraph.

Cut	number of event pass
1	1408
2	1322
3	1052
4	822

Table 7: 100,000 signal events. Those events are generated in $\sqrt{s} = 14$ TeV.

Cut	number of event pass
1	21,814
2	21,254
3	17,130
4	14,142

11 Signal

Generate resonant channel $gg \rightarrow h_3$, $h_3 \rightarrow h_2 h$, $h_2 \rightarrow hh$ in MadGraph by following commands:

```
import model cxSM_VLF_EFT
generate g g > h3, (h3 > h2 h, h2 > h h)
```

then check this channel is dominated in signal ($gg \rightarrow 3h$) or not.

I generate this channel and signal in $\sqrt{s} = 14$ TeV, the cross sections are 2.094 pb and 4.067 pb, respectively.

Figure 8 is the total invariant mass of 6 b-jets for signal events. Figure 9 is the total invariant mass of 6 b-jets for this resonant channel. From the results, there is a peak around 400 GeV, because the mass of h_3 is $m_3 = 420$ GeV.

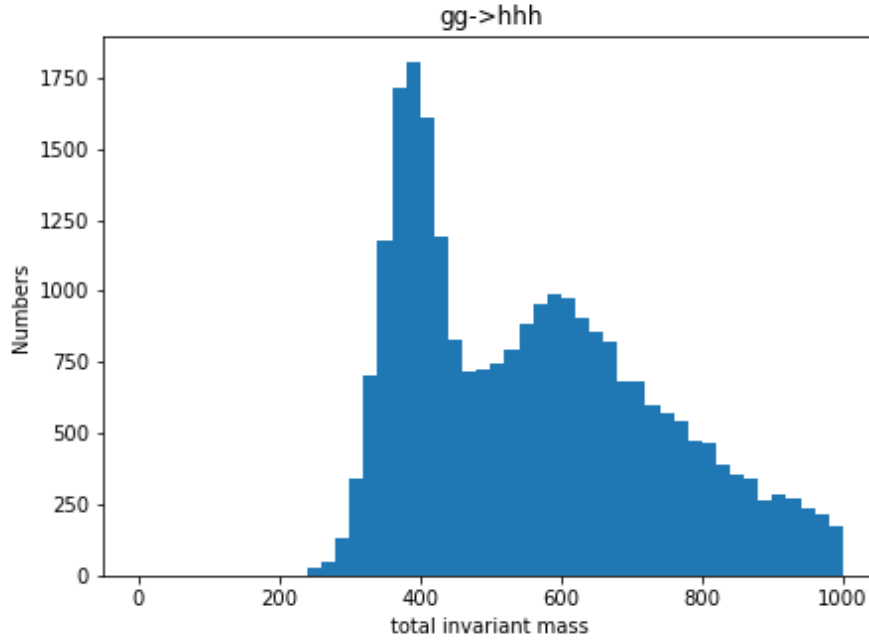


Figure 8: Total invariant mass of 6 b-jets for signal events.

A bump around 600 GeV in Figure 8? In run card, some parameters are not set correctly. After setting all parameters correctly and regenerating these event, the results are in Figure 10. There is no bump around 600 GeV.



Figure 9: Total invariant mass of 6 b-jets for resonant channel.



Figure 10: Total invariant mass of 6 b-jets for signal events and resonant channel (correct parameters).

12 The cross section in MadGraph

Table 8 is the cross sections calculated by MadGraph and in paper. They are different because in MadGraph we only consider LO. But in the paper, the numbers are quoted from the LHC Higgs Cross Section Working Group, and they have considered up to NNLO. The “k factor” is around 2.5, which accounts for the values in $gg \rightarrow h3$.

Table 8: Cross section from MadGraph and paper

Process	σ MadGraph (fb)	σ in paper (fb)
$g g \rightarrow h3$	21	55
$gg \rightarrow h3 \rightarrow h h h$	4.0	38.2

The problem of cross section: If we use the default run card to generate the decay process, the value of cross section will be problematic.

Solution:

In madevent, use command `compute_widths` to compute decay widths of h_2, h_3 , then replace the `run_card.dat` by `run_card_default.dat`.

Regenerate the signal and resonant event, the cross sections are 11.1 fb and 10.39 fb, respectively. This channel is indeed dominated in the signal.

13 Comparision for $pp \rightarrow hhh$ and $gg \rightarrow hhh$

Generate $pp \rightarrow hhh$ in MadGraph by following commands:

```
import model cxSM_VLF_EFT
define p = p b b~
generate p p > h h h QCD<=8
```

Generate $gg \rightarrow hhh$ in MadGraph by following commands:

```
import model cxSM_VLF_EFT
generate g g > h h h
```

These events are generated in 14 TeV and the sample size are 100k. The cross section for $pp \rightarrow hhh$ and $gg \rightarrow hhh$ are 11.22 fb and 11.11 fb, respectively. They only differ by 1%.

14 Comparision for $gg \rightarrow hhh$ and resonant channel

To compare $gg \rightarrow hhh$ and resonant channel, I plot the p_T , η , and total invariant mass of 6 b-jets for both.

These events are generated in 14 TeV and the sample size are 100k. The cross section for $gg \rightarrow hhh$ and resonant channel are 11.11 fb and 10.38 fb, respectively.

The number of events of 6 b-jets for $gg \rightarrow hhh$ is 21,814 and for the resonant channel is 21,475. The numbers are very close. I scaled the number of events for $gg \rightarrow 3h$ to be the same as the resonant channel.

Figure 11 is p_T and η distributions. Figure 12 is total invariant mass of 6 b-jets distributions. Their distributions look similar.

I apply the cuts same in Sec.10 and count how many events can pass these cuts. Table 9 is the result. Those results for both events are similar.

Table 9: Number of events pass the selection cuts. Total number of events for $gg \rightarrow hhh$ and resonant channel both are 100,000.

Cut	$gg \rightarrow 3h$	resonant channel
1	21,814	21,475
2	21,254	20,898
3	17,130	16,828
4	14,142	13,730

15 Branching ratios and decay widths

The branching ratios and decay widths are calculated by MadGraph and Mathematica notebook. Mathematica notebook does not include uu dd ss ee decay modes. Since the contribution of these modes is very small.

Table 10 is calculated by MadGraph. Table 11 is calculated by Mathematica notebook.

The exact values from both are slightly different because MG can not correctly expand the parameter ϵ in the model.

Set the ϵ to 0.001 and calculate again. The results are in Table 12 and Table 13.

This time, the decay widths and branching ratios calculated by MG and Mathematica notebook become close.



Figure 11: p_T and η distributions of b-jets for $gg \rightarrow 3h$ and resonant channel. They are ordered by p_T .

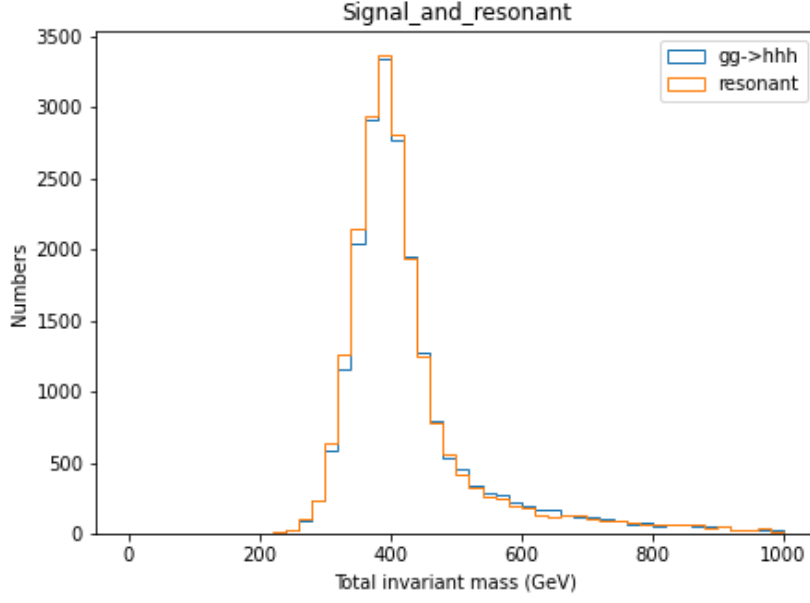


Figure 12: The distribution of total invariant mass of 6 b-jets for $gg \rightarrow 3h$ and resonant channel.

Table 10: Decay widths and branching ratios calculated by MadGraph at BP1.

	BR	Width (GeV)		BR	Width (GeV)
$h_2 \rightarrow h_1 h_1$	0.7506	0.085616	$h_3 \rightarrow h_1 h_2$	0.6795	0.8414
$h_2 \rightarrow WW$	0.1734	0.019782	$h_3 \rightarrow h_1 h_1$	0.1711	0.21183
$h_2 \rightarrow ZZ$	0.07548	0.0086097	$h_3 \rightarrow WW$	0.08756	0.10842
$h_2 \rightarrow bb$	0.0004724	5.3887e-05	$h_3 \rightarrow ZZ$	0.04102	0.050792
$h_2 \rightarrow cc$	3.455e-05	3.9407e-06	$h_3 \rightarrow tt$	0.02065	0.025569
$h_2 \rightarrow \tau\tau$	2.254e-05	2.5714e-06	$h_3 \rightarrow gg$	8.662e-05	0.00010725
$h_2 \rightarrow ss$	2.185e-07	2.4927e-08	$h_3 \rightarrow bb$	8.158e-05	0.00010102
$h_2 \rightarrow gg$	1.561e-07	1.7811e-08	$h_3 \rightarrow cc$	5.961e-06	7.3813e-06
$h_2 \rightarrow \mu\mu$	7.972e-08	9.0933e-09	$h_3 \rightarrow \tau\tau$	3.89e-06	4.8167e-06
$h_2 \rightarrow \gamma\gamma$	6.495e-09	7.409e-10	$h_3 \rightarrow \gamma\gamma$	3.852e-07	4.7697e-07
$h_2 \rightarrow \gamma Z$	9.447e-10	1.0776e-10	$h_3 \rightarrow \gamma Z$	4.937e-08	6.1131e-08
$h_2 \rightarrow dd$	5.441e-10	6.207e-11	$h_3 \rightarrow ss$	3.77e-08	4.6686e-08
$h_2 \rightarrow uu$	1.393e-10	1.5889e-11	$h_3 \rightarrow \mu\mu$	1.375e-08	1.7031e-08
$h_2 \rightarrow ee$	1.865e-12	2.1269e-13	$h_3 \rightarrow dd$	9.389e-11	1.1625e-10
			$h_3 \rightarrow uu$	2.403e-11	2.976e-11
			$h_3 \rightarrow ee$	3.217e-13	3.9835e-13

Table 11: Decay widths and branching ratios calulated by Mathematica at BP1.

	BR	Width (GeV)		BR	Width (GeV)
$h_2 \rightarrow h_1 h_1$	0.8321	0.1407	$h_3 \rightarrow h_1 h_2$	0.8219	2.158
$h_2 \rightarrow WW$	0.1166	0.019721	$h_3 \rightarrow h_1 h_1$	0.1049	0.27543
$h_2 \rightarrow ZZ$	0.05108	0.0086386	$h_3 \rightarrow WW$	0.04131	0.10846
$h_2 \rightarrow gg$	0.0001204	2.0359e-05	$h_3 \rightarrow ZZ$	0.01941	0.050962
$h_2 \rightarrow bb$	9.91e-05	1.6758e-05	$h_3 \rightarrow tt$	0.01242	0.032613
$h_2 \rightarrow \tau\tau$	1.526e-05	2.5802e-06	$h_3 \rightarrow gg$	6.364e-05	0.00016711
$h_2 \rightarrow cc$	4.59e-06	7.7613e-07	$h_3 \rightarrow bb$	1.195e-05	3.1386e-05
$h_2 \rightarrow \gamma\gamma$	3.161e-06	5.346e-07	$h_3 \rightarrow \tau\tau$	1.841e-06	4.8326e-06
$h_2 \rightarrow \gamma Z$	4.822e-07	8.154e-08	$h_3 \rightarrow cc$	5.536e-07	1.4536e-06
$h_2 \rightarrow \mu\mu$	5.43e-08	9.1826e-09	$h_3 \rightarrow \gamma\gamma$	1.823e-07	4.7857e-07
$h_2 \rightarrow tt$	4.355e-19	7.3649e-20	$h_3 \rightarrow \gamma Z$	2.334e-08	6.1277e-08
			$h_3 \rightarrow \mu\mu$	6.55e-09	1.7198e-08

Table 12: Decay widths and branching ratios calulated by MadGraph at BP1 with $\epsilon = 0.001$.

	BR	Width (GeV)		BR	Width (GeV)
$h_2 \rightarrow h_1 h_1$	0.8311	1.4041e-05	$h_3 \rightarrow h_1 h_2$	1.0	2.1493
$h_2 \rightarrow WW$	0.1174	1.9835e-06	$h_3 \rightarrow h_1 h_1$	1.279e-05	2.748e-05
$h_2 \rightarrow ZZ$	0.0511	8.6329e-07	$h_3 \rightarrow WW$	5.058e-06	1.0871e-05
$h_2 \rightarrow bb$	0.0003198	5.4032e-09	$h_3 \rightarrow ZZ$	2.369e-06	5.0929e-06
$h_2 \rightarrow cc$	2.339e-05	3.9513e-10	$h_3 \rightarrow tt$	1.193e-06	2.5638e-06
$h_2 \rightarrow \tau\tau$	1.526e-05	2.5783e-10	$h_3 \rightarrow gg$	5.007e-09	1.0761e-08
$h_2 \rightarrow ss$	1.479e-07	2.4994e-12	$h_3 \rightarrow bb$	4.713e-09	1.0129e-08
$h_2 \rightarrow \mu\mu$	5.397e-08	9.1178e-13	$h_3 \rightarrow cc$	3.443e-10	7.4011e-10
$h_2 \rightarrow dd$	3.684e-10	6.2237e-15	$h_3 \rightarrow \tau\tau$	2.247e-10	4.8297e-10
$h_2 \rightarrow uu$	9.431e-11	1.5932e-15	$h_3 \rightarrow \gamma\gamma$	2.227e-11	4.7857e-11
$h_2 \rightarrow gg$	1.056e-11	1.7841e-16	$h_3 \rightarrow \gamma Z$	2.854e-12	6.1335e-12
$h_2 \rightarrow ee$	1.262e-12	2.1326e-17	$h_3 \rightarrow ss$	2.178e-12	4.6812e-12
$h_2 \rightarrow \gamma\gamma$	4.393e-13	7.4213e-18	$h_3 \rightarrow \mu\mu$	7.945e-13	1.7077e-12
$h_2 \rightarrow \gamma Z$	6.389e-14	1.0794e-18	$h_3 \rightarrow dd$	5.423e-15	1.1657e-14
			$h_3 \rightarrow uu$	1.388e-15	2.984e-15
			$h_3 \rightarrow ee$	1.858e-17	3.9942e-17

Table 13: Decay widths and branching ratios calculated by Mathematica at BP1 with $\epsilon = 0.001$.

	BR	Width (GeV)		BR	Width (GeV)
$h_2 \rightarrow h_1 h_1$	0.8321	1.407e-05	$h_3 \rightarrow h_1 h_2$	1.0	2.158
$h_2 \rightarrow WW$	0.1166	1.9721e-06	$h_3 \rightarrow h_1 h_1$	1.276e-05	2.7543e-05
$h_2 \rightarrow ZZ$	0.05108	8.6386e-07	$h_3 \rightarrow WW$	5.026e-06	1.0846e-05
$h_2 \rightarrow gg$	0.0001204	2.0359e-09	$h_3 \rightarrow ZZ$	2.361e-06	5.0962e-06
$h_2 \rightarrow bb$	9.91e-05	1.6758e-09	$h_3 \rightarrow tt$	1.511e-06	3.2613e-06
$h_2 \rightarrow \tau\tau$	1.526e-05	2.5802e-10	$h_3 \rightarrow gg$	7.743e-09	1.6711e-08
$h_2 \rightarrow cc$	4.59e-06	7.7613e-11	$h_3 \rightarrow bb$	1.454e-09	3.1386e-09
$h_2 \rightarrow \gamma\gamma$	3.161e-06	5.346e-11	$h_3 \rightarrow \tau\tau$	2.239e-10	4.8326e-10
$h_2 \rightarrow \gamma Z$	4.822e-07	8.154e-12	$h_3 \rightarrow cc$	6.736e-11	1.4536e-10
$h_2 \rightarrow \mu\mu$	5.43e-08	9.1826e-13	$h_3 \rightarrow \gamma\gamma$	2.218e-11	4.7857e-11
$h_2 \rightarrow tt$	4.355e-15	7.3649e-20	$h_3 \rightarrow \gamma Z$	2.839e-12	6.1277e-12
			$h_3 \rightarrow \mu\mu$	7.969e-13	1.7198e-12

For color channels, they still don't look the same. The reason is that in MadGraph only calculates to LO, but the numbers in the Mathematica notebook are calculated to NLO.

16 SPANet

Code: [Symmetry Preserving Attention Networks](#)

Symmetry Preserving Attention NETWORKs (Spa-Net) is used to do the jet assignment task. The jet assignment task is the identification of the original particle which leads to a reconstructed jet.

16.1 Prepare training data

1. Defining the event topology in .ini file. The structure of the .ini file follows this format:

```
[SOURCE]
FEATURE_1 = FEATURE_OPTION
FEATURE_2 = FEATURE_OPTION
FEATURE_3 = FEATURE_OPTION
...
```

```
[EVENT]
particles = (PARTICLE_1, PARTICLE_2, ...)
permutations = EVENT_SYMMETRY_GROUP
```

```
[PARTICLE_1]
jets = (JET_1, JET_2, ...)
permutations = JET_SYMMETRY_GROUP
```

```
[PARTICLE_2]
jets = (JET_1, JET_2, ...)
permutations = JET_SYMMETRY_GROUP
```

```
...
```

2. Create training dataset in HDF5 format.
3. Write option-file in JSON format.

16.2 Training

Training:

```
python train.py -of <OPTIONS_FILE> --log_dir <LOG_DIR> --name <NAME> --gpus 1
```

<OPTIONS_FILE>: JSON file with option overloads. <LOG_DIR>: output directory. <NAME>: subdirectory for this run.

Evaluation:

```
python test.py <log_directory> --gpu
```

<log_directory>: directory containing the checkpoint and options file.

```
python test.py <log_directory> -tf <TEST_FILE> --gpu
```

<TEST_FILE> will replace the test file in option file.

17 SPANet for ttbar event

The training and testing datasets from here: [Link](#)

For full ttbar event:

- Training sample:
 - Total sample size: 10,009,520
 - 2t sample size: 2,967,955
- Testing sample size:
 - Total sample size: 358,946
 - 2t sample size: 116,342

Figure 13 is the training results for full ttbar events. The results are the same as the numbers given in the SPANet paper.

18 SPANet for di-Higgs event

Generate di-Higgs events in MadGraph by following commands:

```
import model 2HDMtII_NLO
generate p p > h2 [QCD] QED<=99 QCD<=99
```

then use the MadSpin let h2 decay to h1h1 and h1 decay to b \bar{b} . The mass of h2 is 1000 GeV.

The .ini file for di-Higgs event (h > b \bar{b})

```
[SOURCE]
mass = log_normalize
pt = log_normalize
eta = normalize
phi = normalize

[EVENT]
particles = (h1, h2)
permutations = [(h1, h2)]

[h1]
```

Event Type: *t				
Jet Limit	Event Proportion	Jet Proportion	Event Purity	T Purity
= 6	0.812	0.308	0.640	0.694
= 7	0.851	0.341	0.597	0.665
>= 8	0.881	0.351	0.523	0.610
Full	0.850	1.000	0.583	0.652
Event Type: 0t				
Jet Limit	Event Proportion	Jet Proportion	Event Purity	T Purity
= 6	0.188	0.308	1.000	N/A
= 7	0.149	0.341	1.000	N/A
>= 8	0.119	0.351	1.000	N/A
Full	0.150	1.000	1.000	N/A
Event Type: 1t				
Jet Limit	Event Proportion	Jet Proportion	Event Purity	T Purity
= 6	0.566	0.308	0.567	0.567
= 7	0.531	0.341	0.556	0.556
>= 8	0.484	0.351	0.523	0.523
Full	0.525	1.000	0.549	0.549
Event Type: 2t				
Jet Limit	Event Proportion	Jet Proportion	Event Purity	T Purity
= 6	0.246	0.308	0.807	0.841
= 7	0.320	0.341	0.666	0.755
>= 8	0.397	0.351	0.523	0.663
Full	0.324	1.000	0.637	0.735

Figure 13: The training result for full $t\bar{t}$ events.

```
jets = (b1, b2)
permutations = [(b1, b2)]
```

```
[h2]
jets = (b1, b2)
permutations = [(b1, b2)]
```

There are three types of events, 0h, 1h, and 2h. The number means how many identifiable Higgs is in an event.

Definition of some parameters:

- Event proportion:

$$\text{Event proportion} \equiv \frac{\text{number of some type events with } i \text{ jets}}{\text{number of events with } i \text{ jets}} \quad (1)$$

- Jet proportion:

$$\text{Jet proportion} \equiv \frac{\text{number of events with } i \text{ jets}}{\text{total number of events}} \quad (2)$$

- Event purity:

$$\epsilon^{\text{event}} \equiv \frac{\text{number of some type events with and all Higgs are correctly identified}}{\text{number of some type events}} \quad (3)$$

- H purity:

$$\epsilon^{\text{h}} \equiv \frac{\text{number of correctly identified Higgs in some type events}}{\text{number of identifiable Higgs in some type events}} \quad (4)$$

18.1 Training sample for di-Higgs events

Di-Higgs events $pp \rightarrow h2$, $h2 \rightarrow hh$, $h \rightarrow bb$ was generated at $\sqrt{s} = 14$ TeV using MadGraph. Then pass these events to Pythia8 for showering and hadronization. Then pass to Delphes for detector simulation.

Jets are reconstructed by the anti-kT algorithm with radius $R = 0.5$ and are required to have $p_T \geq 20$ GeV.

Preselection requirements: ≥ 4 jets in $|\eta| < 2.5$.

Define the correct jet assignments by matching them to the simulated truth quarks within an angular distance of $\Delta R = \sqrt{\Delta\eta^2 + \Delta\phi^2} < 0.4$.

18.2 Training result for di-Higgs event

For 100k di-Higgs event with $m_{h_2} = 1000$ GeV:

- Training sample:
 - Total sample size: 78,785
 - 2h sample size: 36,765
 - 5% used on validation
- Testing sample:
 - Total sample size: 8,753
 - 2h sample size: 4,130

Event Type: *h				
Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
= 4	0.887	0.291	0.945	0.960
= 5	0.925	0.331	0.908	0.940
>= 6	0.947	0.378	0.792	0.865
Full	0.922	1.000	0.874	0.914
Event Type: 0h				
Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
= 4	0.113	0.291	1.000	N/A
= 5	0.075	0.331	1.000	N/A
>= 6	0.053	0.378	1.000	N/A
Full	0.078	1.000	1.000	N/A
Event Type: 1h				
Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
= 4	0.545	0.291	0.911	0.911
= 5	0.442	0.331	0.888	0.888
>= 6	0.385	0.378	0.827	0.827
Full	0.450	1.000	0.876	0.876
Event Type: 2h				
Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
= 4	0.342	0.291	1.000	1.000
= 5	0.483	0.331	0.926	0.963
>= 6	0.562	0.378	0.769	0.878
Full	0.472	1.000	0.871	0.933

Figure 14: The training result for 100k di-Higgs events.

Figure 14 is the training results for 100k di-Higgs events. For 2h events, $\epsilon^{\text{event}} = 0.871$, $\epsilon^{\text{h}} = 0.933$.

For 1M di-Higgs event with $m_{\text{h}_2} = 1000$ GeV:

- Training sample:
 - Total sample size: 788,160
 - 2h sample size: 364,773
 - 5% used on validation
- Testing sample:
 - Total sample size: 87,702
 - 2h sample size: 40,695

Figure 15 is the training results for 1M di-Higgs events. For 2h events, $\epsilon^{\text{event}} = 0.914$, $\epsilon^{\text{h}} = 0.955$.

For di-Higgs event with $m_{\text{h}_2} = 500$ GeV:

- Training sample:
 - Total sample size: 764,676
 - 2h sample size: 396,588
 - 5% used on validation
- Testing sample:
 - Total sample size: 84,687
 - 2h sample size: 43,818

Figure 16 is the training results for 500 GeV di-Higgs events. For 2h events, $\epsilon^{\text{event}} = 0.779$, $\epsilon^{\text{h}} = 0.871$.

18.3 Apply the pre-trained model on different m_{h_2} sample

The SPANet models in Sec.18.2 are trained on $m_{\text{h}_2} = 500$ GeV and $m_{\text{h}_2} = 1000$ GeV. Test these models on samples with $m_{\text{h}_2} = 500$ GeV, 750 GeV, 1000 GeV.

The results are summarized in Table 14.

Event Type: *h				
Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
= 4	0.877	0.301	0.950	0.964
= 5	0.920	0.326	0.919	0.946
>= 6	0.944	0.372	0.831	0.891
Full	0.916	1.000	0.894	0.928
Event Type: 0h				
Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
= 4	0.123	0.301	1.000	N/A
= 5	0.080	0.326	1.000	N/A
>= 6	0.056	0.372	1.000	N/A
Full	0.084	1.000	1.000	N/A
Event Type: 1h				
Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
= 4	0.533	0.301	0.917	0.917
= 5	0.445	0.326	0.885	0.885
>= 6	0.394	0.372	0.813	0.813
Full	0.452	1.000	0.873	0.873
Event Type: 2h				
Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
= 4	0.345	0.301	1.000	1.000
= 5	0.475	0.326	0.950	0.975
>= 6	0.551	0.372	0.844	0.918
Full	0.464	1.000	0.914	0.955

Figure 15: The training result for 1M di-Higgs events.

Table 14: SPANet models test on different m_{h_2} samples.

Training m_{h_2} (GeV)	Testing m_{h_2} (GeV)	Event purity	H purity
500	500	0.779	0.871
500	750	0.408	0.672
500	1000	0.390	0.665
1000	500	0.383	0.638
1000	750	0.731	0.860
1000	1000	0.914	0.955

Event Type: *h				
Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
== 4	0.872	0.319	0.879	0.914
== 5	0.923	0.325	0.802	0.868
>= 6	0.944	0.356	0.633	0.753
Full	0.914	1.000	0.763	0.836
Event Type: 0h				
Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
== 4	0.128	0.319	1.000	N/A
== 5	0.077	0.325	1.000	N/A
>= 6	0.056	0.356	1.000	N/A
Full	0.086	1.000	1.000	N/A
Event Type: 1h				
Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
== 4	0.497	0.319	0.793	0.793
== 5	0.391	0.325	0.748	0.748
>= 6	0.312	0.356	0.666	0.666
Full	0.397	1.000	0.743	0.743
Event Type: 2h				
Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
== 4	0.375	0.319	0.994	0.994
== 5	0.532	0.325	0.841	0.912
>= 6	0.632	0.356	0.617	0.775
Full	0.517	1.000	0.779	0.871

Figure 16: The training result for di-Higgs events with $m_{h_2} = 500$ GeV.

19 SPANet for tri-Higgs event

The tri-Higgs events are signal events ($gg \rightarrow hhh$) generated in the previous.

19.1 Training sample for tri-Higgs event

Tri-Higgs events $gg \rightarrow hhh$ was generated at $\sqrt{s} = 14$ TeV using MadGraph. Then pass these events to Pythia8 for showering and hadronization. In Pythia8, the Higgs is forced to decay to $b\bar{b}$. Then pass to Delphes for detector simulation.

Jets are reconstructed by the anti-kT algorithm with radius $R = 0.5$ and are required to have $p_T \geq 20$ GeV.

Preselection requirements: ≥ 6 jets in $|\eta| < 2.5$.

Define the correct jet assignments by matching them to the simulated truth quarks within an angular distance of $\Delta R = \sqrt{\Delta\eta^2 + \Delta\phi^2} < 0.4$.

19.2 Training result for tri-Higgs event

For 1M tri-Higgs event:

- Training sample:
 - Total sample size: 542,681
 - 3h sample size: 104,467
 - 5% used on validation
- Testing sample:
 - Total sample size: 60,173
 - 3h sample size: 11,598

Figure 17 is the training results for 1M tri-Higgs events. For 3h events, $\epsilon^{\text{event}} = 0.344$, $\epsilon^h = 0.538$.

Event Type: 0h				
Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
= 6	0.186	0.439	1.000	N/A
= 7	0.147	0.313	1.000	N/A
>= 8	0.119	0.248	1.000	N/A
Full	0.157	1.000	1.000	N/A
Event Type: 1h				
Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
= 6	0.386	0.439	0.452	0.452
= 7	0.337	0.313	0.405	0.405
>= 8	0.312	0.248	0.334	0.334
Full	0.353	1.000	0.412	0.412
Event Type: 2h				
Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
= 6	0.303	0.439	0.293	0.489
= 7	0.298	0.313	0.228	0.429
>= 8	0.285	0.248	0.152	0.343
Full	0.297	1.000	0.239	0.435
Event Type: 3h				
Jet Limit	Event Proportion	Jet Proportion	Event Purity	H Purity
= 6	0.124	0.439	0.603	0.705
= 7	0.218	0.313	0.336	0.546
>= 8	0.283	0.248	0.152	0.400
Full	0.193	1.000	0.344	0.538

Figure 17: The training result for 1M tri-Higgs events.

20 χ^2 method

20.1 Di-Higgs

Try all possible combination of final jets and find the minimal mass difference between SM Higgs mass, i.e. minimize this

$$\chi^2 = [M(j_1 j_2) - M_H]^2 + [M(j_3 j_4) - M_H]^2 \quad (5)$$

where $M(j_i j_j)$ means the invariant mass of jet i and jet j .

The results are presented in Table 15.

Table 15: Performance comparison for di-Higgs events.

N_{Jet}	Event Fraction	SPANet Efficiency		χ^2 Efficiency	
		Event	Higgs	Event	Higgs
= 4	0.224	1.000	1.000	1.000	1.000
= 5	0.334	0.950	0.975	0.675	0.737
≥ 6	0.442	0.844	0.918	0.313	0.441
Total	1.000	0.914	0.955	0.582	0.665

20.2 Di-Higgs with b-tag

Similarly, try to minimize (5), but only try all possible permutation of b-tagged jets. Here, we only consider the events with ≥ 4 b-jets.

The results are presented in Table 16 and Table 17.

Table 16: Performance comparison for di-Higgs events with ≥ 4 b-jets (b-tagging efficiency = 100%).

N_{Jet}	Event Fraction	SPANet Efficiency		χ^2 Efficiency	
		Event	Higgs	Event	Higgs
= 4	0.224	1.000	1.000	1.000	1.000
= 5	0.334	0.980	0.990	0.974	0.979
≥ 6	0.442	0.946	0.972	0.916	0.933
Total	1.000	0.969	0.984	0.954	0.963

Table 17: Performance comparison for di-Higgs events with ≥ 4 b-jets (b-tagging efficiency = default).

N_{Jet}	Event Fraction	SPANet Efficiency		χ^2 Efficiency	
		Event	Higgs	Event	Higgs
= 4	0.200	1.000	1.000	1.000	1.000
= 5	0.318	0.950	0.975	0.892	0.914
≥ 6	0.482	0.865	0.930	0.725	0.776
Total	1.000	0.919	0.958	0.833	0.865

20.3 Tri-Higgs

Try all possible combination of final jets and find the minimal mass difference between SM Higgs mass, i.e. minimize this

$$\chi^2 = [M(j_1 j_2) - M_{\text{H}}]^2 + [M(j_3 j_4) - M_{\text{H}}]^2 + [M(j_5 j_6) - M_{\text{H}}]^2 \quad (6)$$

where $M(j_i j_j)$ means the invariant mass of jet i and jet j .

Testing sample are required with ≥ 0 b-jets and b-tagging efficiency: default. The results are presented in Table 18.

Table 18: Performance comparison for tri-Higgs events. The results are for complete events (3 Higgs).

N_{Jet}	Event Fraction	SPANet Efficiency		χ^2 Efficiency	
		Event	Higgs	Event	Higgs
= 6	0.062	0.580	0.689	0.515	0.559
= 7	0.076	0.382	0.596	0.178	0.228
≥ 8	0.081	0.238	0.465	0.043	0.133
Total	0.291	0.384	0.564	0.223	0.303

Below, only consider the permutation of b-tagged jets. Testing events are required with ≥ 6 b-jets. The results are presented in Table 19 and Table 20.

21 SPANet for di-Higgs events with b-tag

In this section, SPANet will take the b-tagging information.

The `.ini` file for di-Higgs event

Table 19: Performance comparison for tri-Higgs events with ≥ 6 b-jets (b-tagging efficiency = 100%). The results are for complete events (3 Higgs).

N_{Jet}	Event Fraction	SPANet Efficiency		χ^2 Efficiency	
		Event	Higgs	Event	Higgs
= 6	0.150	0.633	0.729	0.517	0.560
= 7	0.183	0.550	0.675	0.459	0.512
≥ 8	0.196	0.443	0.597	0.363	0.425
Total	0.529	0.534	0.662	0.440	0.494

Table 20: Performance comparison for tri-Higgs events with ≥ 6 b-jets (b-tagging efficiency = default). The results are for complete events (3 Higgs).

N_{Jet}	Event Fraction	SPANet Efficiency		χ^2 Efficiency	
		Event	Higgs	Event	Higgs
= 6	0.100	0.488	0.613	0.484	0.535
= 7	0.175	0.329	0.502	0.347	0.413
≥ 8	0.257	0.185	0.394	0.173	0.254
Total	0.532	0.290	0.417	0.289	0.359

[SOURCE]

```

mass = log_normalize
pt = log_normalize
eta = normalize
phi = normalize
btag = none

```

[EVENT]

```

particles = (h1, h2)
permutations = [(h1, h2)]

```

[h1]

```

jets = (b1, b2)
permutations = [(b1, b2)]

```

[h2]

```

jets = (b1, b2)

```

permutations = [(b1, b2)]

For training data preparation is almost same in Sec.18.1. 2 b-tagged jets or 4 b-tagged jets will be required in the following training data. The default b-tagging efficiency in Delphes is

$$\text{eff.} = 0.85 \tanh(0.0025 p_T) \frac{25.0}{1 + 0.063 p_T} \quad (7)$$

21.1 Training result for di-Higgs event

Case 1: ≥ 0 b-tagged jets, b-tagging efficiency: 100%

- Training sample:
 - Total sample size: 788,108
 - 1h sample size: 357,082
 - 2h sample size: 364,717
 - 5% used on validation
- Testing sample:
 - Total sample size: 87,748
 - 1h sample size: 39,816
 - 2h sample size: 40,635

The training results for Case 1 is presented in Table 21.

Table 21: SPA-NET results on di-Higgs with at least 0 b-tagged jets (b-tagging efficiency: 100%).

N_{Jet}	Event Fraction	Event Efficiency	Higgs Efficiency
= 4	0.224	1.000	1.000
= 5	0.333	0.981	0.990
≥ 6	0.443	0.948	0.973
Total	1.000	0.970	0.985

Case 2: ≥ 2 b-tagged jets, b-tagging efficiency: 100%

- Training sample:
 - Total sample size: 787,097
 - 1h sample size: 357,078
 - 2h sample size: 364,717
 - 5% used on validation
- Testing sample:
 - Total sample size: 87,641
 - 1h sample size: 39,816
 - 2h sample size: 40,635

The training results for Case 2 is presented in Table 22.

Table 22: SPA-NET results on di-Higgs with at least 2 b-tagged jets (b-tagging efficiency: 100%).

N_{Jet}	Event Fraction	Event Efficiency	Higgs Efficiency
$= 4$	0.224	1.000	1.000
$= 5$	0.333	0.984	0.992
≥ 6	0.443	0.953	0.976
Total	1.000	0.974	0.987

Case 3: ≥ 4 b-tagged jets, b-tagging efficiency: 100%

- Training sample:
 - Total sample size: 454,768
 - 1h sample size: 85,074
 - 2h sample size: 363,904
 - 5% used on validation
- Testing sample:
 - Total sample size: 50,735

- 1h sample size: 9,547
- 2h sample size: 40,541

The training results for Case 3 is presented in Table 23.

Table 23: SPA-NET results on di-Higgs with at least 4 b-tagged jets (b-tagging efficiency: 100%).

N_{Jet}	Event Fraction	Event Efficiency	Higgs Efficiency
$= 4$	0.224	1.000	1.000
$= 5$	0.333	0.980	0.990
≥ 6	0.443	0.946	0.972
Total	1.000	0.969	0.984

Case 4: ≥ 0 b-tagged jets, b-tagging efficiency: default

- Training sample:
 - Total sample size: 788,160
 - 1h sample size: 357,137
 - 2h sample size: 364,773
 - 5% used on validation
- Testing sample:
 - Total sample size: 87,702
 - 1h sample size: 39,659
 - 2h sample size: 40,695

The training results for Case 4 is presented in Table 24.

Case 5: ≥ 2 b-tagged jets, b-tagging efficiency: default

- Training sample:
 - Total sample size: 633,726
 - 1h sample size: 270,325
 - 2h sample size: 328,427

Table 24: SPA-NET results on di-Higgs with at least 0 b-tagged jets (b-tagging efficiency: default)

N_{Jet}	Event Fraction	Event Efficiency	Higgs Efficiency
= 4	0.224	1.000	1.000
= 5	0.334	0.961	0.981
≥ 6	0.442	0.887	0.941
Total	1.000	0.937	0.968

– 5% used on validation

- Testing sample:
 - Total sample size: 70,436
 - 1h sample size: 29,847
 - 2h sample size: 36,742

The training results for Case 5 is presented in Table 25.

Table 25: SPA-NET results on di-Higgs with at least 2 b-tagged jets (b-tagging efficiency: default)

N_{Jet}	Event Fraction	Event Efficiency	Higgs Efficiency
= 4	0.223	1.000	1.000
= 5	0.332	0.964	0.982
≥ 6	0.445	0.891	0.943
Total	1.000	0.939	0.969

Case 6: ≥ 4 b-tagged jets, b-tagging efficiency: default

- Training sample:
 - Total sample size: 108,266
 - 1h sample size: 19,808
 - 2h sample size: 87,196
 - 5% used on validation
- Testing sample:

- Total sample size: 11,992
- 1h sample size: 2,205
- 2h sample size: 9,643

The training results for Case 6 is presented in Table 26.

Table 26: SPA-NET results on di-Higgs with at least 4 b-tagged jets (b-tagging efficiency: default)

N_{Jet}	Event Fraction	Event Efficiency	Higgs Efficiency
= 4	0.200	1.000	1.000
= 5	0.318	0.950	0.975
≥ 6	0.482	0.865	0.930
Total	1.000	0.919	0.958

Table 27 is the summary for b-tag training results.

Table 27: Summary for b-tag training results. 2b means at least 2 b-tagged jets. 100% and default are the b-tagging efficiency in Delphes.

Case	Event Efficiency	Higgs Efficiency	Testing sample
no b-tag	0.914 ± 0.0014	0.955 ± 0.0007	40,695
≥ 0 b 100%	0.970 ± 0.0008	0.985 ± 0.0004	40,635
≥ 2 b 100%	0.974 ± 0.0008	0.987 ± 0.0004	40,635
≥ 4 b 100%	0.969 ± 0.0009	0.984 ± 0.0004	40,541
≥ 0 b default	0.937 ± 0.0012	0.968 ± 0.0006	40,695
≥ 2 b default	0.939 ± 0.0012	0.969 ± 0.0006	36,742
≥ 4 b default	0.919 ± 0.0028	0.958 ± 0.0014	9,643

22 SPANet for tri-Higgs events with b-tag

The .ini file for tri-Higgs event

```
[SOURCE]
mass = log_normalize
pt = log_normalize
eta = normalize
```

```

phi = normalize
btag = none

[EVENT]
particles = (h1, h2, h3)
permutations = [(h1, h2, h3)]

[h1]
jets = (b1, b2)
permutations = [(b1, b2)]

[h2]
jets = (b1, b2)
permutations = [(b1, b2)]

[h3]
jets = (b1, b2)
permutations = [(b1, b2)]

```

22.1 Training sample preparation for tri-Higgs events

Tri-Higgs events $gg \rightarrow hhh$ was generated at $\sqrt{s} = 14$ TeV using MadGraph. Then pass these events to Pythia8 for showering and hadronization. In Pythia8, the Higgs is forced to decay to $b\bar{b}$. Then pass to Delphes for detector simulation.

Jets are reconstructed by the anti-kT algorithm with radius $R = 0.4$ and are required to have $p_T \geq 20$ GeV.

Preselection requirements: ≥ 6 jets in $|\eta| < 2.5$.

Define the correct jet assignments by matching them to the simulated truth quarks within an angular distance of $\Delta R = \sqrt{\Delta\eta^2 + \Delta\phi^2} < 0.4$.

22.2 Trainging result for tri-Higgs events

All cases partial event training are set true.

Case 1: ≥ 0 b-tagged jets, b-tagging efficiency: 100%

- Training sample:

- Total sample size: 519,372
- 1h sample size: 168,385
- 2h sample size: 175,616
- 3h sample size: 112,770
- 5% used on validation
- Testing sample:
 - Total sample size: 57,652
 - 1h sample size: 18,639
 - 2h sample size: 19,456
 - 3h sample size: 12,679

The training results for Case 1 is presented in Table 28.

Table 28: SPA-NET results on tri-Higgs with at least 0 b-tagged jets (b-tagging efficiency: 100%).

N_{Jet}	Event Fraction	Event Efficiency	Higgs Efficiency
= 6	0.062	0.608	0.711
= 7	0.076	0.519	0.652
≥ 8	0.081	0.426	0.584
Total	0.220	0.510	0.644

Case 2: ≥ 2 b-tagged jets, b-tagging efficiency: 100%

- Training sample:
 - Total sample size: 519,318
 - 1h sample size: 168,385
 - 2h sample size: 175,616
 - 3h sample size: 112,770
 - 5% used on validation
- Testing sample:

- Total sample size: 57,646
- 1h sample size: 18,639
- 2h sample size: 19,456
- 3h sample size: 12,679

The training results for Case 2 is presented in Table 29.

Table 29: SPA-NET results on tri-Higgs with at least 2 b-tagged jets (b-tagging efficiency: 100%).

N_{Jet}	Event Fraction	Event Efficiency	Higgs Efficiency
$= 6$	0.063	0.637	0.732
$= 7$	0.076	0.556	0.679
≥ 8	0.081	0.442	0.601
Total	0.220	0.537	0.665

Case 3: ≥ 4 b-tagged jets, b-tagging efficiency: 100%

- Training sample:
 - Total sample size: 503,404
 - 1h sample size: 164,104
 - 2h sample size: 175,572
 - 3h sample size: 112,770
 - 5% used on validation
- Testing sample:
 - Total sample size: 55,989
 - 1h sample size: 18,197
 - 2h sample size: 19,454
 - 3h sample size: 12,679

The training results for Case 3 is presented in Table 30.

Table 30: SPA-NET results on tri-Higgs with at least 4 b-tagged jets (b-tagging efficiency: 100%).

N_{Jet}	Event Fraction	Event Efficiency	Higgs Efficiency
$= 6$	0.064	0.635	0.732
$= 7$	0.079	0.563	0.684
≥ 8	0.084	0.443	0.602
Total	0.226	0.539	0.668

Case 4: ≥ 6 b-tagged jets, b-tagging efficiency: 100%

- Training sample:
 - Total sample size: 212,885
 - 1h sample size: 5,349
 - 2h sample size: 33,675
 - 3h sample size: 112,330
 - 5% used on validation
- Testing sample:
 - Total sample size: 23,886
 - 1h sample size: 3,853
 - 2h sample size: 6,804
 - 3h sample size: 12,639

The training results for Case 4 is presented in Table 31.

Table 31: SPA-NET results on tri-Higgs with at least 6 b-tagged jets (b-tagging efficiency: 100%).

N_{Jet}	Event Fraction	Event Efficiency	Higgs Efficiency
$= 6$	0.150	0.633	0.729
$= 7$	0.183	0.550	0.675
≥ 8	0.196	0.443	0.597
Total	0.529	0.534	0.662

Case 5: ≥ 0 b-tagged jets, b-tagging efficiency: default

- Training sample:
 - Total sample size: 519,404
 - 1h sample size: 168,781
 - 2h sample size: 175,235
 - 3h sample size: 112,740
 - 5% used on validation
- Testing sample:
 - Total sample size: 57,744
 - 1h sample size: 18,447
 - 2h sample size: 19,572
 - 3h sample size: 12,655

The training results for Case 5 is presented in Table 32.

Table 32: SPA-NET results on tri-Higgs with at least 0 b-tagged jets (b-tagging efficiency: default).

N_{Jet}	Event Fraction	Event Efficiency	Higgs Efficiency
$= 6$	0.062	0.580	0.689
$= 7$	0.076	0.382	0.569
≥ 8	0.081	0.238	0.465
Total	0.219	0.384	0.564

Case 6: ≥ 2 b-tagged jets, b-tagging efficiency: default

- Training sample:
 - Total sample size: 486,815
 - 1h sample size: 155,932
 - 2h sample size: 166,234
 - 3h sample size: 109,947

- 5% used on validation
- Testing sample:
 - Total sample size: 54,132
 - 1h sample size: 17,102
 - 2h sample size: 18,522
 - 3h sample size: 12,321

The training results for Case 6 is presented in Table 33.

Table 33: SPA-NET results on tri-Higgs with at least 2 b-tagged jets (b-tagging efficiency: default).

N_{Jet}	Event Fraction	Event Efficiency	Higgs Efficiency
= 6	0.064	0.575	0.683
= 7	0.079	0.386	0.573
≥ 8	0.085	0.236	0.460
Total	0.228	0.383	0.562

Case 7: ≥ 4 b-tagged jets, b-tagging efficiency: default

- Training sample:
 - Total sample size: 238,155
 - 1h sample size: 16,272
 - 2h sample size: 65,770
 - 3h sample size: 72,441
 - 5% used on validation
- Testing sample:
 - Total sample size: 26,561
 - 1h sample size: 7,261
 - 2h sample size: 9,300
 - 3h sample size: 8,124

The training results for Case 7 is presented in Table 34.

Table 34: SPA-NET results on tri-Higgs with at least 4 b-tagged jets (b-tagging efficiency: default).

N_{Jet}	Event Fraction	Event Efficiency	Higgs Efficiency
= 6	0.081	0.553	0.668
= 7	0.105	0.373	0.559
≥ 8	0.120	0.236	0.460
Total	0.306	0.367	0.549

Case 8: ≥ 6 b-tagged jets, b-tagging efficiency: default

- Training sample:
 - Total sample size: 20,335
 - 1h sample size: 3,300
 - 2h sample size: 5,639
 - 3h sample size: 10,840
 - 5% used on validation
- Testing sample:
 - Total sample size: 2,166
 - 1h sample size: 357
 - 2h sample size: 592
 - 3h sample size: 1,153

The training results for Case 8 is presented in Table 35.

Table 35: SPA-NET results on tri-Higgs with at least 6 b-tagged jets (b-tagging efficiency: default).

N_{Jet}	Event Fraction	Event Efficiency	Higgs Efficiency
= 6	0.100	0.488	0.613
= 7	0.175	0.329	0.502
≥ 8	0.257	0.185	0.394
Total	0.532	0.290	0.417

Table 36 is the summary for tri-Higgs training results.

Table 36: Summary for tri-Higgs training results. 100% and default are the b-tagging efficiency in Delphes.

Case	Event Efficiency	Higgs Efficiency	Testing sample
≥ 0 b 100%	0.510 ± 0.0044	0.644 ± 0.0025	12,679
≥ 2 b 100%	0.537 ± 0.0044	0.665 ± 0.0024	12,679
≥ 4 b 100%	0.539 ± 0.0044	0.668 ± 0.0024	12,679
≥ 6 b 100%	0.534 ± 0.0044	0.662 ± 0.0024	12,639
≥ 0 b default	0.384 ± 0.0043	0.564 ± 0.0025	12,655
≥ 2 b default	0.383 ± 0.0044	0.562 ± 0.0026	12,321
≥ 4 b default	0.367 ± 0.0053	0.549 ± 0.0032	8,124
≥ 6 b default	0.290 ± 0.0034	0.417 ± 0.0084	1,153