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# Recommender Systems



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# Session-based Recommendations with Recurrent Neural Networks



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- instead of long user histories (as in the case of Netflix)
- short session-based data (e.g. a small sportswear website)
- Factor models (sparse user-item interaction matrix)
- Neighborhood methods (co-occurrences of items in sessions)

## Gated Recurrent Unit (GRU)

$$h_t = (1 - z_t)h_{t-1} + z_t \tilde{h}_t$$

$$z_t = \sigma(W_z x_t + U_z h_{t-1}) \rightarrow \text{update gate}$$

$$\tilde{h}_t = \tanh(Wx_t + U(r_t \odot h_{t-1}))$$

$$r_t = \sigma(W_r x_t + U_r h_{t-1}) \rightarrow \text{reset gate}$$

**Input:** actual state of the session

**Output:** item of the next event in the session

## State of the session:

- item of the actual event (1-of- $N$  encoding)
- events in the session so far (weighted sum)

$N \rightarrow$  number of items

**Output:** predicted preference of the items

(i.e. the likelihood of being the next  
in the session for each item)

## Session-parallel mini-batches

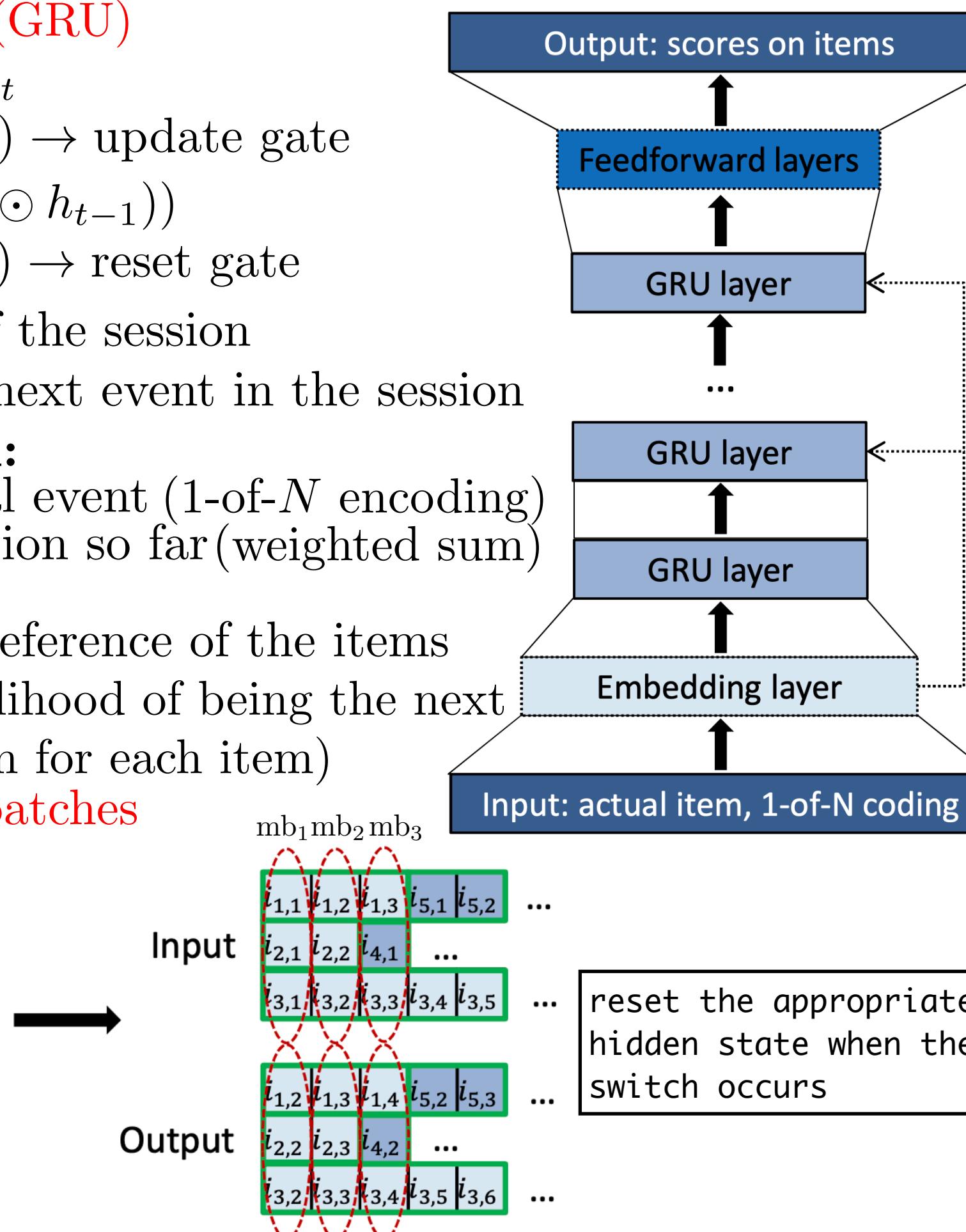
Session1  $i_{1,1} | i_{1,2} | i_{1,3} | i_{1,4}$

Session2  $i_{2,1} | i_{2,2} | i_{2,3}$

Session3  $i_{3,1} | i_{3,2} | i_{3,3} | i_{3,4} | i_{3,5} | i_{3,6}$

Session4  $i_{4,1} | i_{4,2}$

Session5  $i_{5,1} | i_{5,2} | i_{5,3}$



## Ranking Loss

BPR: Bayesian Personalized Ranking

user prefers item  $i$  over item  $j$

$i \rightarrow$  desired item (next item in the session)

$j \rightarrow$  negative samples

$\hat{r}_s \rightarrow$  scores at a given point of the session  $s$

$$L_s = -\frac{1}{N_S} \sum_{j=1}^{N_S} \log(\sigma(\hat{r}_{s,i} - \hat{r}_{s,j}))$$

loss at a given point of the session  $s$

$N_S \rightarrow$  sample size

TOP1 loss

$$\frac{1}{N} \sum_{j=1}^N I\{\hat{r}_{s,j} > \hat{r}_{s,i}\}$$

relative rank of the relevant item

– regularized approximation

$$L_s = \frac{1}{N_S} \cdot \sum_{j=1}^{N_S} \sigma(\hat{r}_{s,j} - \hat{r}_{s,i}) + \sigma(\hat{r}_{s,j}^2)$$

## Evaluation

**Recall@20:** proportion of cases having the desired item amongst the top-20 items in all test cases

**MRR@20:** Mean Reciprocal Rank (average of the reciprocal ranks of the desired items)

Loss / #Units	RSC15	
	Recall@20	MRR@20
TOP1 100	0.5853 (+15.55%)	0.2305 (+12.58%)
BPR 100	0.6069 (+19.82%)	0.2407 (+17.54%)
Cross-entropy 100	0.6074 (+19.91%)	0.2430 (+18.65%)
TOP1 1000	0.6206 (+22.53%)	<b>0.2693 (+31.49%)</b>
BPR 1000	<b>0.6322 (+24.82%)</b>	0.2467 (+20.47%)
Cross-entropy 1000	0.5777 (+14.06%)	0.2153 (+5.16%)

Compared to the best baseline (item-KNN)	VIDEO	
	Recall@20	MRR@20
0.6141 (+11.50%)	0.3511 (+3.84%)	
0.5999 (+8.92%)	0.3260 (-3.56%)	
0.6372 (+15.69%)	0.3720 (+10.04%)	
<b>0.6624 (+20.27%)</b>	<b>0.3891 (+15.08%)</b>	
0.6311 (+14.58%)	0.3136 (-7.23%)	

Baseline	RSC15		VIDEO	
	Recall@20	MRR@20	Recall@20	MRR@20
POP	0.0050	0.0012	0.0499	0.0117
S-POP	0.2672	0.1775	0.1301	0.0863
Item-KNN	0.5065	0.2048	0.5508	0.3381
BPR-MF	0.2574	0.0618	0.0692	0.0374

Best parametrizations for datasets/loss functions

Dataset	Loss	Mini-batch	Dropout
RSC15	TOP1	50	0.5
RSC15	BPR	50	0.2
RSC15	Cross-entropy	500	0
VIDEO	TOP1	50	0.4
VIDEO	BPR	50	0.3
VIDEO	Cross-entropy	200	0.1

	Learning rate	Momentum
VIDEO:	0.01	0
YouTube-like	0.05	0.2
OTT video service	0.01	0
platform	0.05	0

RSC15: RecSys Chalenge 2015



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# AutoRec: Autoencoders Meet Collaborative Filtering



[YouTube Video](#)

Collaborative filtering (CF): exploit information about users' preferences for items (e.g. star ratings) to provide personalised recommendations.

- matrix factorisation
- neighbourhood models
- rating-based collaborative filtering

$m \rightarrow$  number of users

$n \rightarrow$  number of items

$R \in \mathbb{R}^{m \times n} \rightarrow$  user-item rating matrix

$u \in U = \{1, 2, \dots, m\} \rightarrow$  user

$i \in I = \{1, 2, \dots, n\} \rightarrow$  item

$r^u = (R_{u1}, \dots, R_{un}) \in \mathbb{R}^n$

( partially observed vector  
(representing user  $u$ ))

$r^i = (R_{1i}, \dots, R_{mi}) \in \mathbb{R}^m$

( partially observed vector  
(representing item  $i$ ))

## Item-based AutoRec

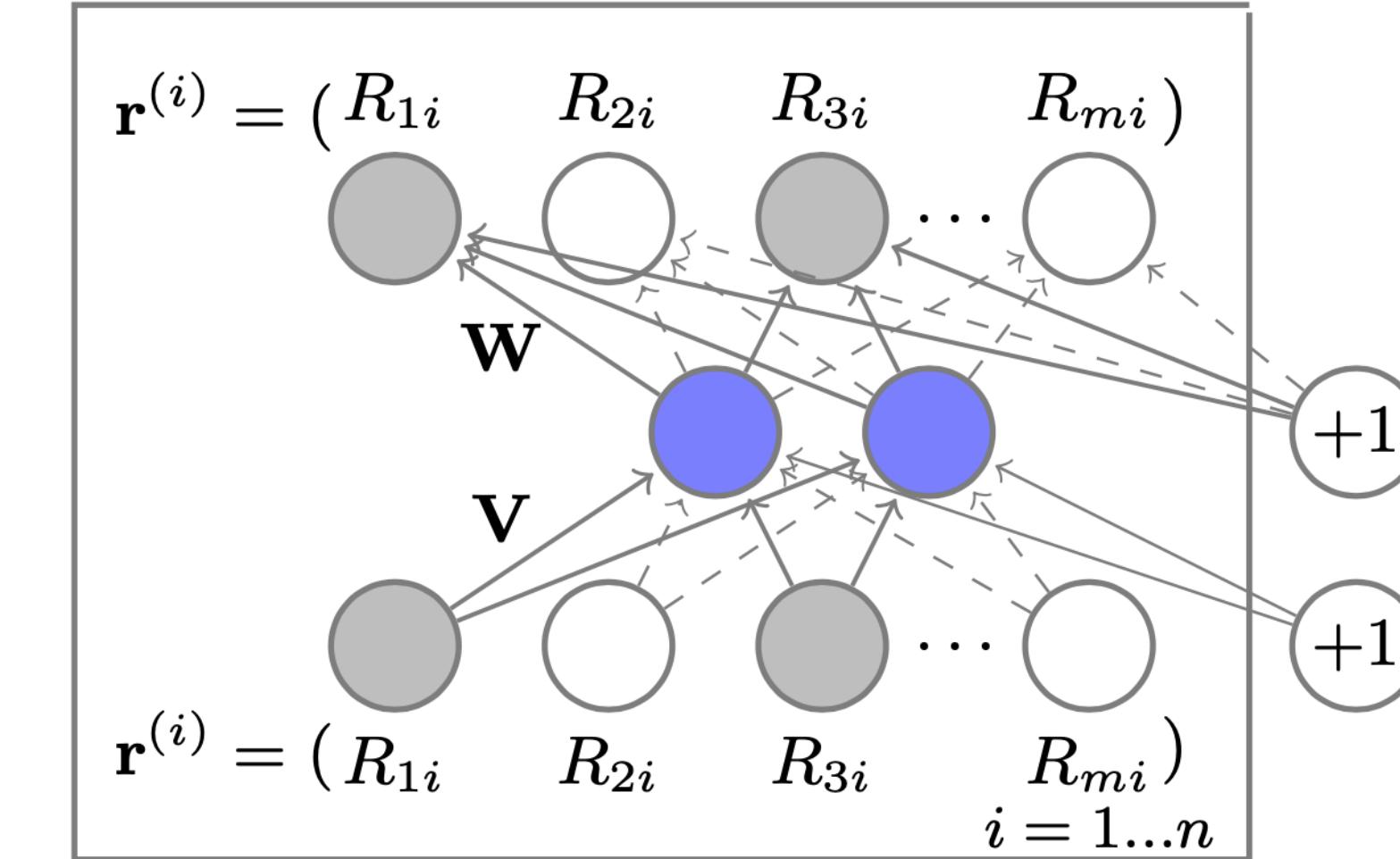
$$\min_{\theta} \sum_{i=1}^n \|r^i - h(r^i; \theta)\|_{\mathcal{O}}^2 + \frac{\lambda}{2} (\|W\|_F^2 + \|V\|_F^2)$$

$$h(r; \theta) = f(Wg(Vr + \mu) + b)$$

$\|\cdot\|_{\mathcal{O}} \rightarrow$  consider only the contribution  
of observed ratings

$$\hat{R}_{ui} = h(r^i; \theta)_u$$

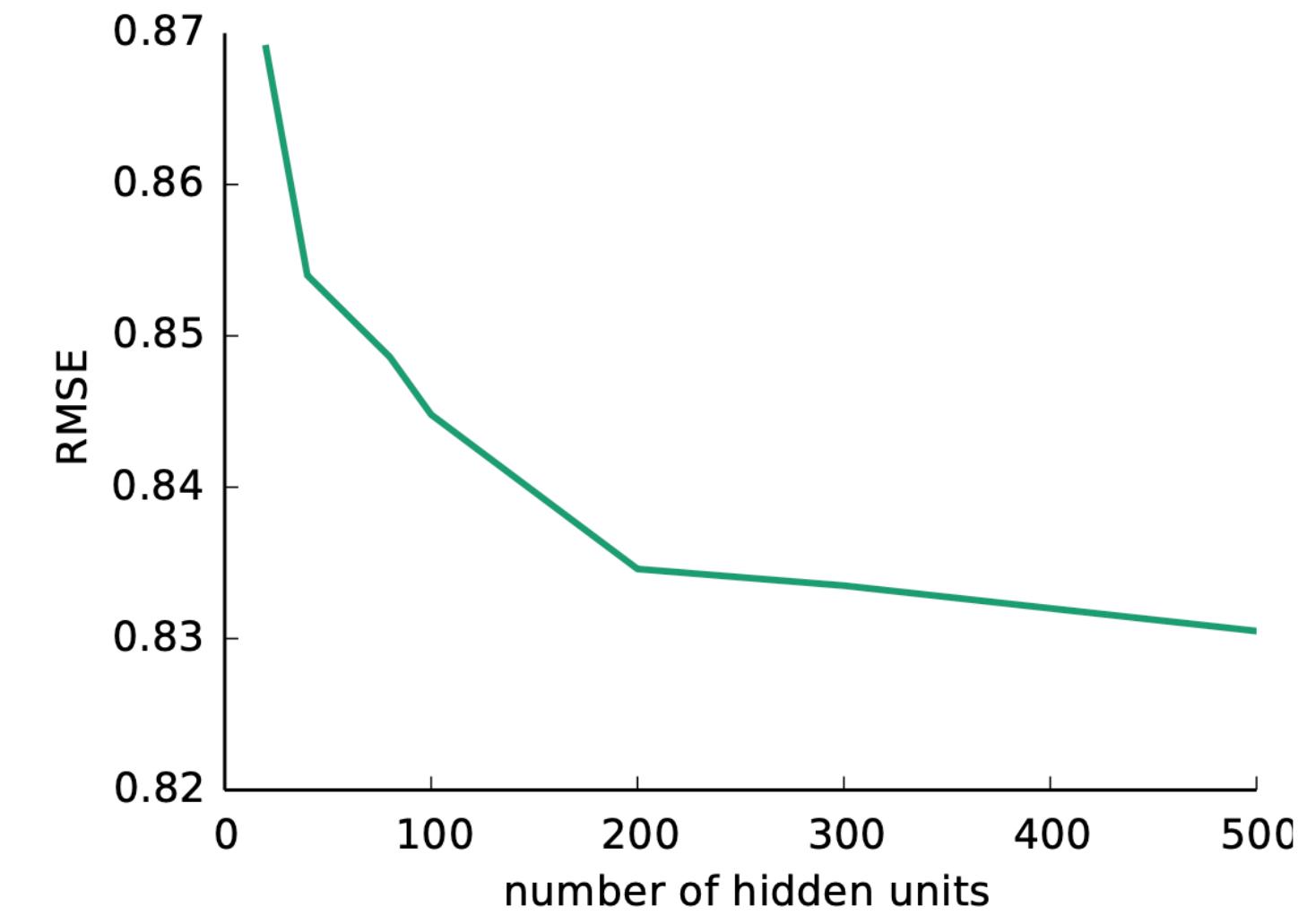
User-based AutoRec  $\rightarrow$  similar



RMSE	Movielens 1M & 10M	
	ML-1M	ML-10M
U-AutoRec	0.874	0.867
I-AutoRec	<b>0.831</b>	<b>0.782</b>

$f(\cdot)$	$g(\cdot)$	RMSE
Identity	Identity	0.872
Sigmoid	Identity	0.852
Identity	Sigmoid	<b>0.831</b>
Sigmoid	Sigmoid	0.836

	ML-1M	ML-10M	Netflix
BiasedMF	0.845	0.803	0.844
I-RBM	0.854	0.825	-
U-RBM	0.881	0.823	0.845
LLORMA	0.833	<b>0.782</b>	0.834
<b>I-AutoRec</b>	<b>0.831</b>	<b>0.782</b>	<b>0.823</b>





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# Wide & Deep Learning for Recommender Systems

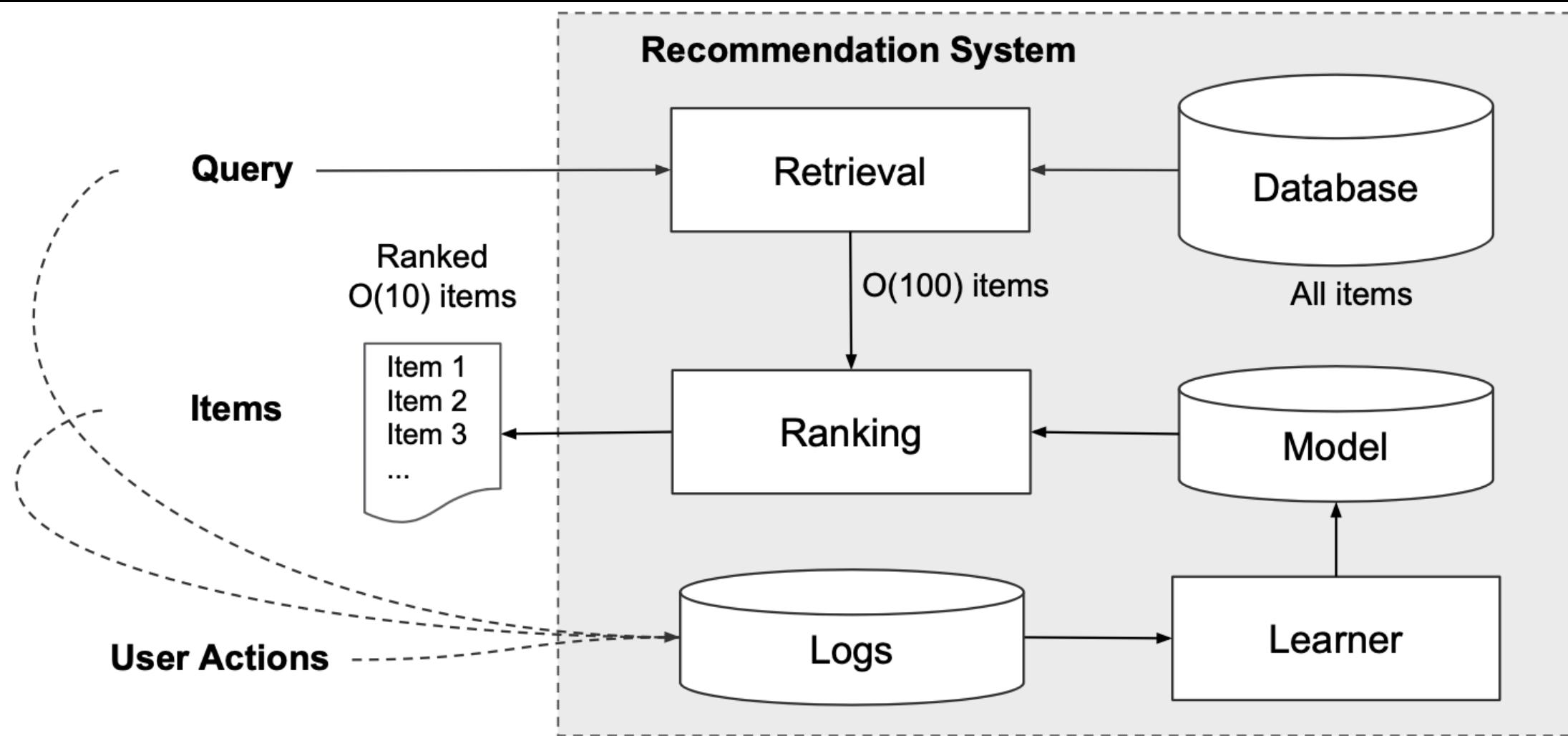


[YouTube Video](#)

Google Play: one billion active users & one million apps

Impressions: list of apps

"A recommender system can be viewed as a search ranking system, where the input query is a set of user and contextual information, and the output is a ranked list of items"



- user features (e.g., country, language, demographics)
- contextual features (e.g., device, hour of the day, day of the week)
- impression features (e.g., app age, historical statistics of an app)

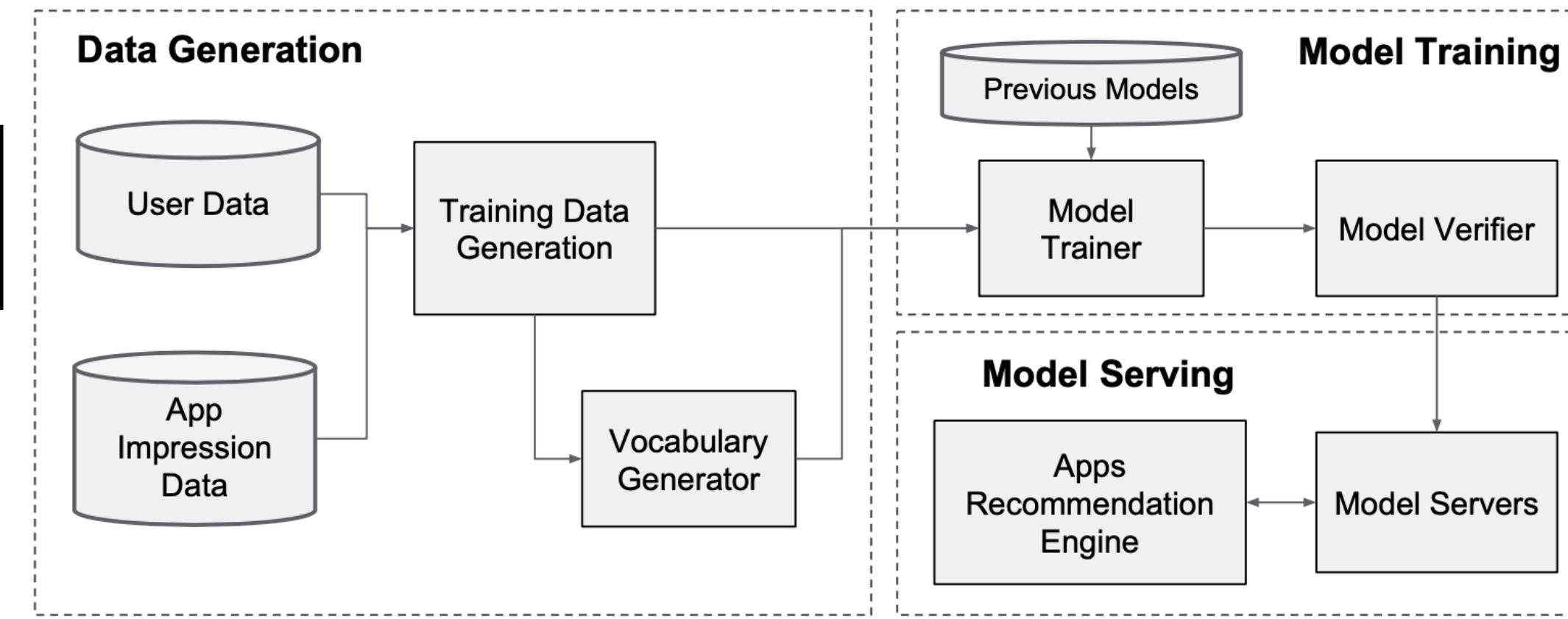
The label is app acquisition: 1 if the impressed app was installed, and 0 otherwise

$$p(y=1|x) = \sigma(w_{\text{wide}}^T[x, \phi(x)] + w_{\text{deep}}^T a^L + b)$$

$x = [x_1, x_2, \dots, x_d]$  is a vector of  $d$  features

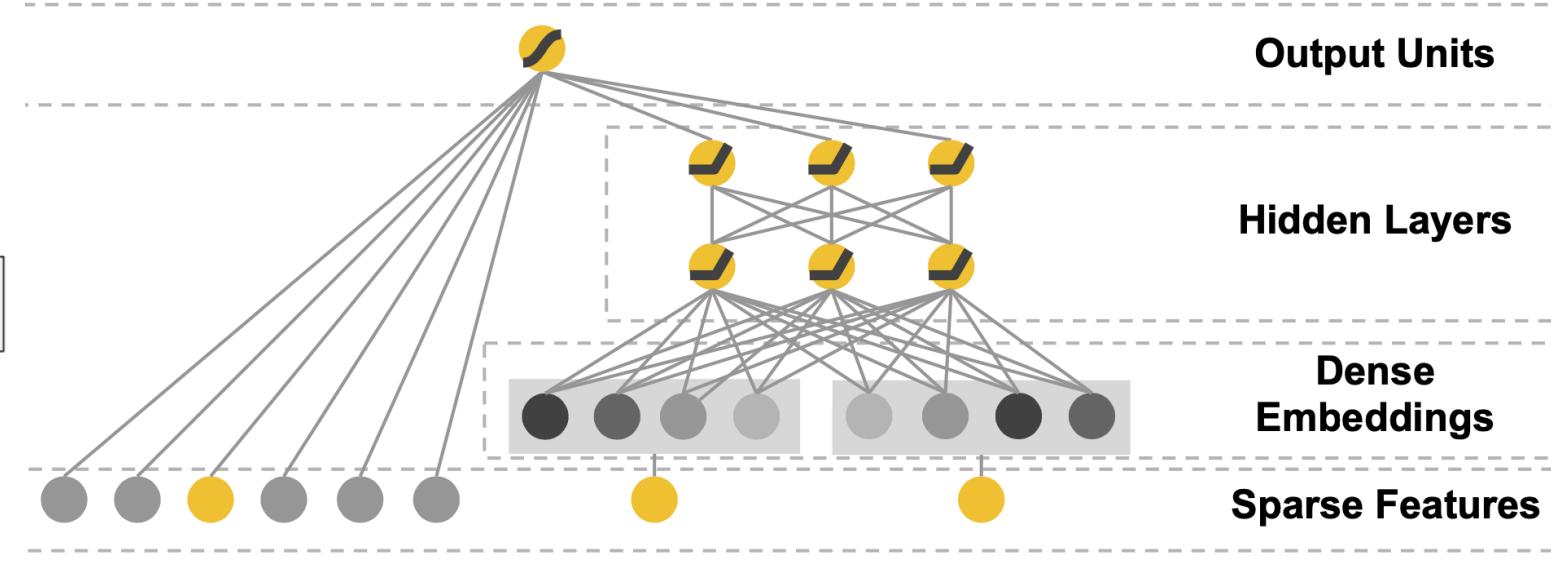
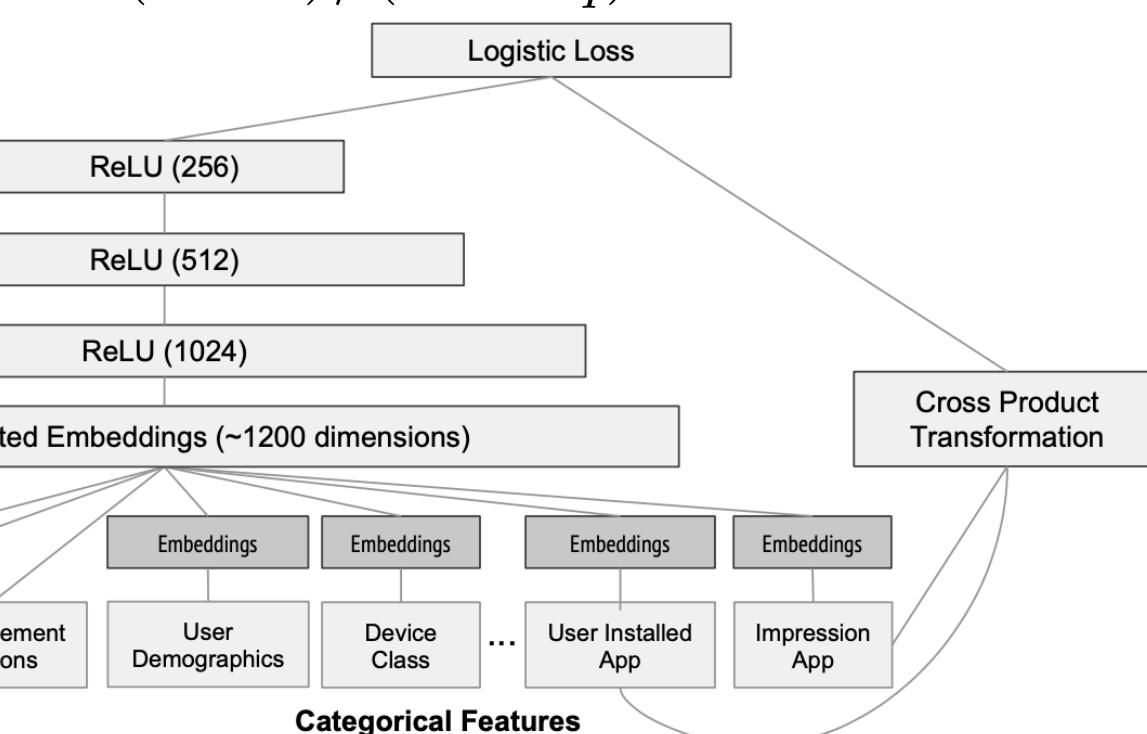
$$\phi_k(x) = \prod_{i=1}^d x_i^{c_{ki}}, c_{ki} \in \{0, 1\}$$

$$a^{l+1} = f(W^l a^l + b^l)$$



For binary features, a cross-product transformation (e.g., "AND(gender=female, language=en)") is 1 if and only if the constituent features ("gender=female" and "language=en") are all 1, and 0 otherwise.

Continuous real-valued features are normalized to  $[0, 1]$  by mapping a feature value  $x$  to its cumulative distribution function  $P(X \leq x)$ , divided into  $n_q$  quantiles. The normalized value is  $(i - 1)/(1 - n_q)$  for values in the  $i$ -th quantiles.



- memorization
- generalization

Model	Wide (control)
Deep	0.726
Wide & Deep	0.722
Offline AUC	0.728
Online Acquisition Gain	0%
	+2.9%
	+3.9%



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[YouTube Video](#)

# Neural Collaborative Filtering

Collaborative Filtering: modeling users' preference on items based on their past interactions (e.g., ratings and clicks)

**Learning from implicit data**

$M \rightarrow$  number of users

$N \rightarrow$  number of items

$Y \in \mathbb{R}^{M \times N} \rightarrow$  user-item interaction matrix

$y_{ui} = 1$  if interaction (user  $u$ , item  $i$ ) is observed

doesn't mean that user  $u$  likes item  $i$

$y_{ui} = 0$  otherwise

doesn't mean that user  $u$  didn't like item  $i$

$\hat{y}_{ui} = f(u, i; \theta)$

interaction function

predicted score of interaction  $y_{ui}$

pointwise loss  $\rightarrow$  squared loss btw  $\hat{y}_{ui}$  &  $y_{ui}$

pairwise loss  $\rightarrow$  maximizes the margin btw  $\hat{y}_{ui}$  &  $y_{uj}$

**Matrix Factorization (MF)**

$p_u \rightarrow$  latent vector for user  $u$

$q_i \rightarrow$  latent vector for item  $i$

$\hat{y}_{ui} = f(u, i; p_u, q_i) = p_u^T q_i = \sum_{k=1}^K p_{uk} q_{ik}$

$K \rightarrow$  dimension of the latent space

**Neural Collaborative Filtering**

$v_u^U \rightarrow$  one-hot encoding feature vector for user  $u$

$v_i^I \rightarrow$  one-hot encoding feature vector for item  $i$

$\hat{y}_{ui} = f(P^T v_u^U, Q^T v_i^I; P, Q, \theta_f)$

$$L_{sqr} = \sum_{(u,i) \in \mathcal{Y} \cup \mathcal{Y}^-} w_{ui} (y_{ui} - \hat{y}_{ui})^2$$

$\mathcal{Y} \rightarrow$  set of observed interactions in  $Y$

$\mathcal{Y}^- \rightarrow$  set of negative instances

all (or sampled from) unobserved interactions

$w_{ui} \rightarrow$  weight of training instance

**Likelihood**

$$p(\mathcal{Y}, \mathcal{Y}^-; P, Q, \theta_f) = \prod_{(u,i) \in \mathcal{Y}} \hat{y}_{ui} \prod_{(u,j) \in \mathcal{Y}^-} (1 - \hat{y}_{uj})$$

$\hat{y}_{ui} \rightarrow$  how likely  $i$  is relevant to  $u$

$$L = - \sum_{(u,i) \in \mathcal{Y}} \log \hat{y}_{ui} - \sum_{(u,j) \in \mathcal{Y}^-} \log(1 - \hat{y}_{uj})$$

$$= - \sum_{(u,i) \in \mathcal{Y} \cup \mathcal{Y}^-} y_{ui} \log \hat{y}_{ui} + (1 - y_{ui}) \log(1 - \hat{y}_{ui})$$

**Generalized Matrix Factorization (GMF)**

$$p_u = P^T v_u^U \quad \hat{y}_{ui} = a_{\text{out}}(h^T(p_u \odot q_i))$$

$$q_i = Q^T v_i^I$$

If  $h = 1$  and  $a_{\text{out}} \equiv \text{id} \implies \text{GMF} \equiv \text{MF}$

**Fusion of GMF and MLP (NeuMF)**

$$\phi^{GMF} = p_u^G \odot q_i^G,$$

$$\phi^{MLP} = a_L(\mathbf{W}_L^T(a_{L-1}(\dots a_2(\mathbf{W}_2^T \begin{bmatrix} p_u^M \\ q_i^M \end{bmatrix} + \mathbf{b}_2) \dots)) + \mathbf{b}_L)$$

$$\hat{y}_{ui} = \sigma(\mathbf{h}^T \begin{bmatrix} \phi^{GMF} \\ \phi^{MLP} \end{bmatrix}),$$

**Multi-Layer Perceptron (MLP)**

$$\mathbf{z}_1 = \phi_1(\mathbf{p}_u, \mathbf{q}_i) = \begin{bmatrix} \mathbf{p}_u \\ \mathbf{q}_i \end{bmatrix},$$

$$\phi_2(\mathbf{z}_1) = a_2(\mathbf{W}_2^T \mathbf{z}_1 + \mathbf{b}_2),$$

.....

$$\phi_L(\mathbf{z}_{L-1}) = a_L(\mathbf{W}_L^T \mathbf{z}_{L-1} + \mathbf{b}_L),$$

$$\hat{y}_{ui} = \sigma(\mathbf{h}^T \phi_L(\mathbf{z}_{L-1})),$$

**Evaluation**

$$\text{Hit}@\ell = \frac{1}{m} \sum_{u \in \mathcal{U}} \mathbb{1}\{\text{rank}_{u,g_u} \leq \ell\}$$

$\text{rank}_{u,g_u} \rightarrow$  ranking of the ground-truth item  $g_u$

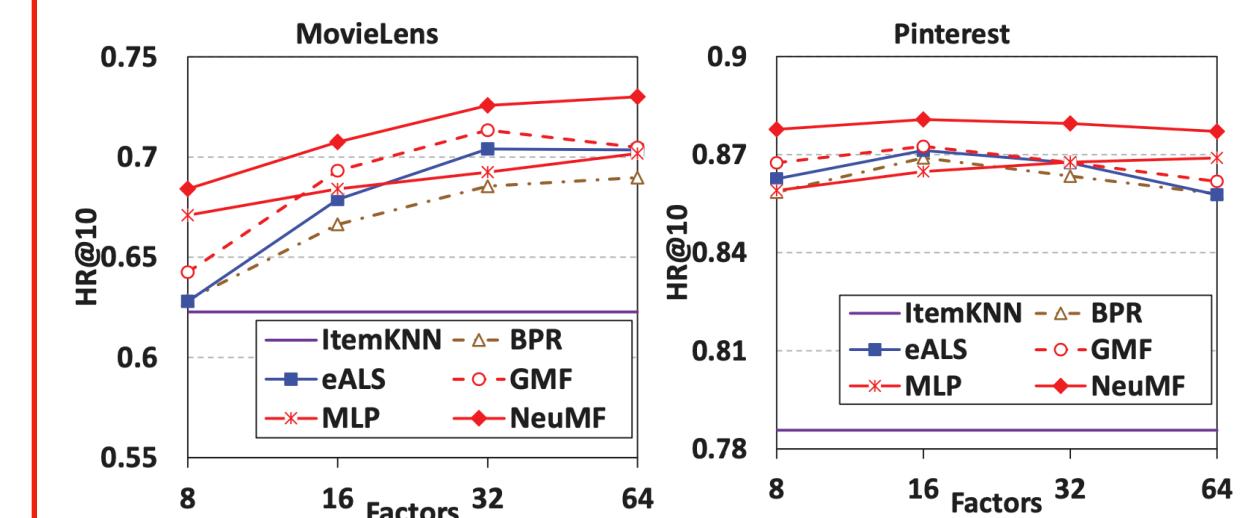
$\mathcal{U} \rightarrow$  user set

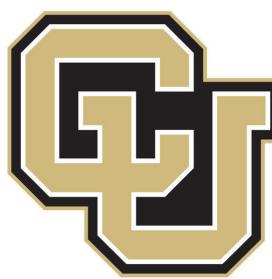
$m \rightarrow$  number of users

$$\text{AUC} = \frac{1}{m} \sum_{u \in \mathcal{U}} \frac{1}{|I \setminus S_u|} \sum_{j \in I \setminus S_u} \mathbb{1}\{\text{rank}_{u,g_u} < \text{rank}_{u,g_j}\}$$

$I \rightarrow$  item set

$S_u \rightarrow$  candidate items of user  $u$





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# Neural Factorization Machines for Sparse Predictive Analytics



[YouTube Video](#)

- recommendation systems – targeted advertising
- search ranking – visual analysis – event detection

Online advertising: predict how likely a particular occupation will click on user IDs & demographics (e.g., genders & occupations) one-hot encoding

Interaction btw features (cross features)

occupation = {banker, doctor}

gender = {M, F}

occupation\_gender = {banker\_M, banker\_F, doctor\_M, doctor\_F}

Factorization Machines (FMs)

$x \in \mathbb{R}^n \rightarrow$  sparse real valued feature vector

$$\hat{y}_{\text{FM}} = w_0 + \sum_{i=1}^n w_i x_i + \sum_{i=1}^n \sum_{j=i+1}^n v_i^T v_j x_i x_j$$

$v_i \in \mathbb{R}^k \rightarrow$  embedding vector for feature  $i$

$k \rightarrow$  number of latent factors

Matrix Factorization: Models the relation of two entities only!

Neural Factorization Machines (NFMs)

$x_i = 0$  means  $i$ -th feature does not exist in the instance

$$\hat{y}_{\text{NFM}}(x) = w_0 + \sum_{i=1}^n w_i x_i + f(x)$$

Embedding Layer

$v_i \in \mathbb{R}^k \rightarrow$  embedding vector for the  $i$ -th feature  
 $\mathcal{V}_x = \{x_1 v_1, \dots, x_n v_n\} \rightarrow$  set of embedding vectors (representing  $x$ )

Bi-Interaction Layer

$$f_{\text{BI}}(\mathcal{V}_x) = \sum_{i=1}^n \sum_{j=i+1}^n x_i v_i \odot x_j v_j$$

Hidden Layers

$$\mathbf{z}_1 = \sigma_1(\mathbf{W}_1 f_{\text{BI}}(\mathcal{V}_x) + \mathbf{b}_1),$$

$$\mathbf{z}_2 = \sigma_2(\mathbf{W}_2 \mathbf{z}_1 + \mathbf{b}_2),$$

.....

$$\mathbf{z}_L = \sigma_L(\mathbf{W}_L \mathbf{z}_{L-1} + \mathbf{b}_L),$$

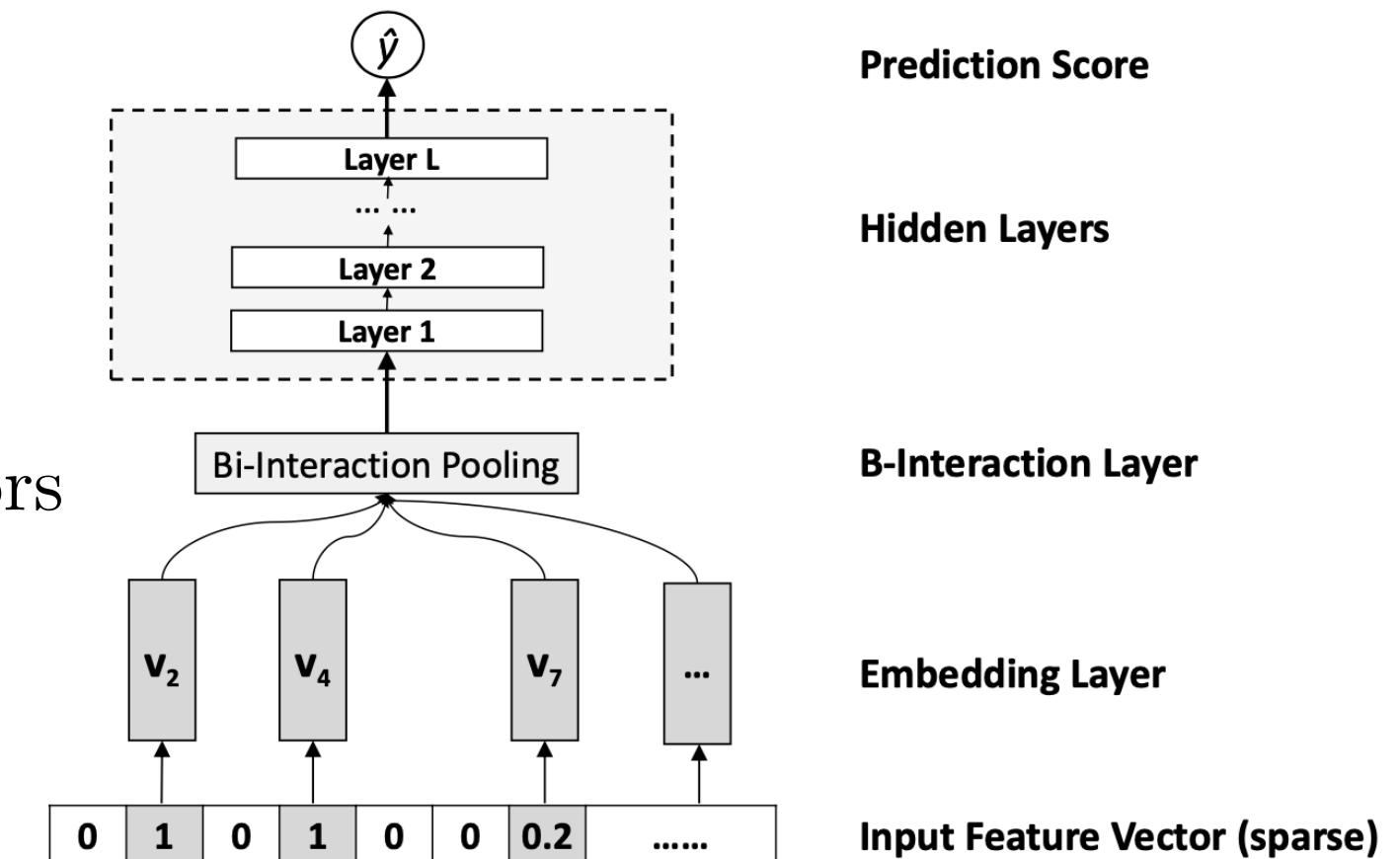
Prediction Layer

$$f(x) = h^T z_L$$

NFM generalizes FM

$$\begin{aligned} \hat{y}_{\text{NFM}-0} &= w_0 + \sum_{i=1}^n w_i x_i + \mathbf{h}^T \sum_{i=1}^n \sum_{j=i+1}^n x_i \mathbf{v}_i \odot x_j \mathbf{v}_j \\ &= w_0 + \sum_{i=1}^n w_i x_i + \sum_{i=1}^n \sum_{j=i+1}^n \sum_{f=1}^k h_f v_{if} v_{jf} \cdot x_i x_j \end{aligned}$$

$$h = (1, 1, \dots, 1) \implies \text{FM}$$



Learning

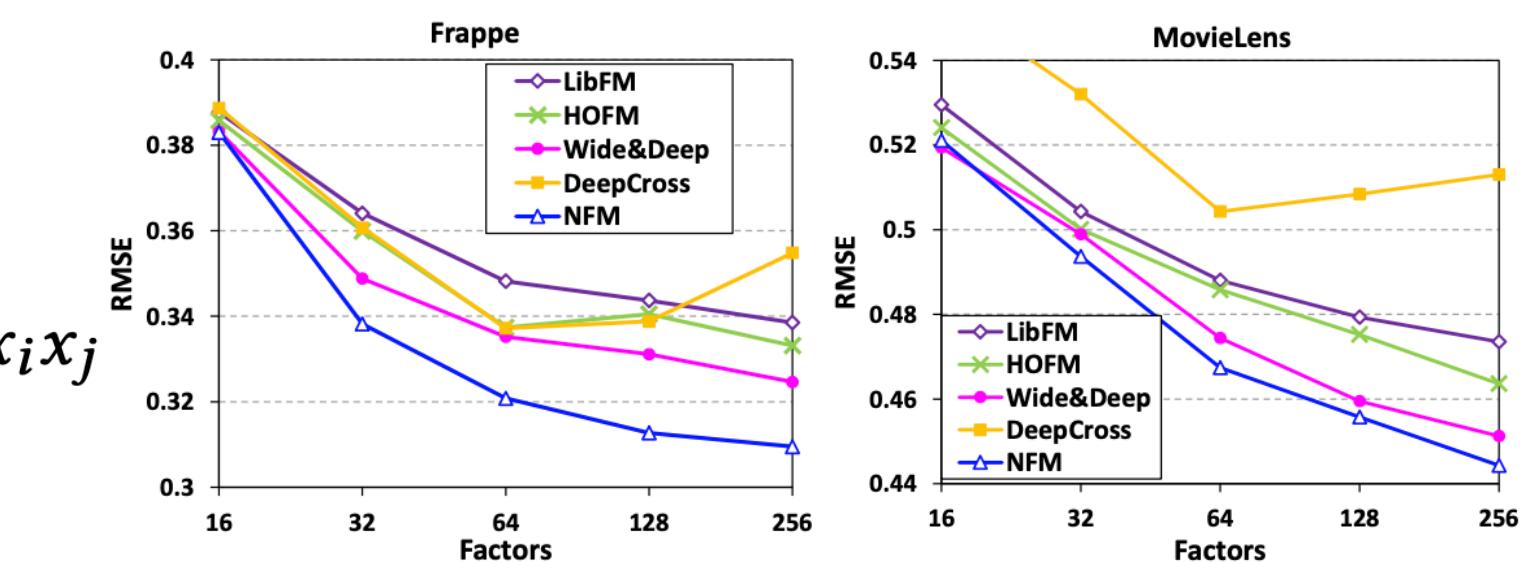
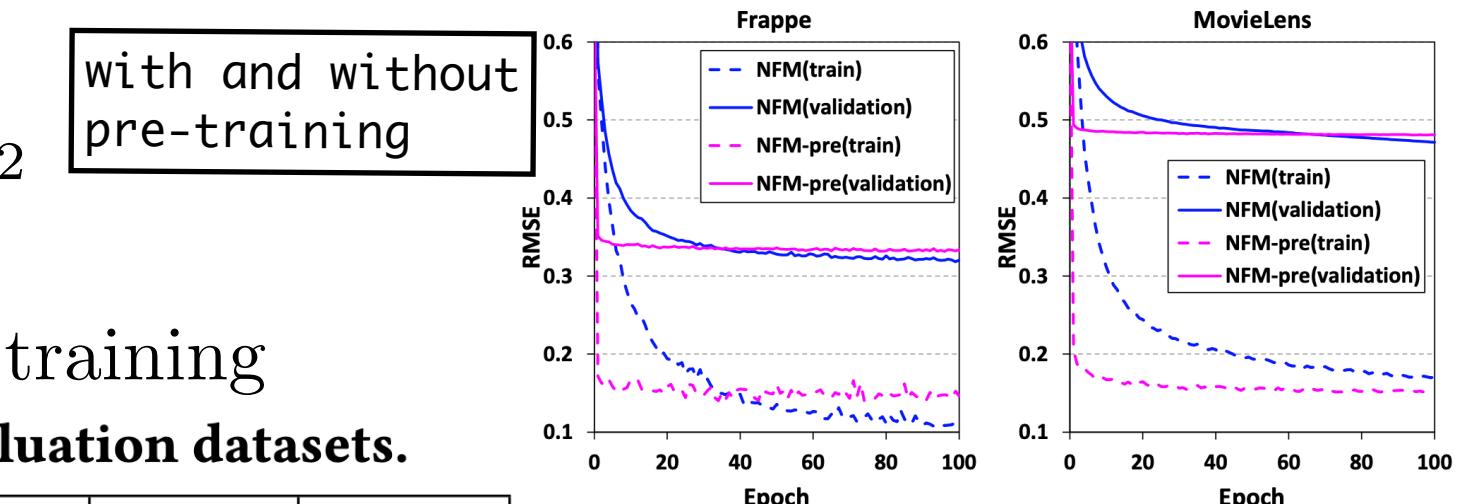
$$L_{\text{reg}} = \sum_{x \in \mathcal{X}} (\hat{y}(x) - y(x))^2$$

$\mathcal{X} \rightarrow$  set of instances for training

Statistics of the evaluation datasets.

Dataset	Instance#	Feature#	User#	Item#
Frappe	288,609	5,382	957	4,082
MovieLens	2,006,859	90,445	17,045	23,743

→ context-aware app prediction  
→ personalized tag recommendation





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# DeepFM: A Factorization-Machine based Neural Network for CTR Prediction



[YouTube Video](#)

CTR: Click Through Rate

Estimate the probability a user will click on a recommended item

$CTR \times bid \rightarrow$  online advertising

$bid \rightarrow$  benefit the system receives if the item is clicked by a user

$n \rightarrow$  number of instances

$(\mathcal{X}, y) \rightarrow$  an instance

$\mathcal{X} \rightarrow m$ -fields data record (usually involving a pair of user and item)

$y \in \{0, 1\} \rightarrow$  associated label indicating user behaviors

$y = 1 \rightarrow$  user clicked the item

categorical fields (e.g., gender, location)  $\rightarrow$  one-hot encoding

continuous fields (e.g., age)

$x = [x_{field_1}, \dots, x_{field_j}, \dots, x_{field_m}] \rightarrow$  sparse & high-dimensional

$x_{field_j} \rightarrow$  vector representation of the  $j$ -th field of  $\mathcal{X}$

$\hat{y} = \text{CTR\_model}(x) \rightarrow$  prediction model

(estimate the probability of a user clicking  
a specific app in a given context)

## DeepFM

learn both low- and high-order interactions

$\hat{y} = \text{sigmoid}(y_{FM} + y_{DNN})$

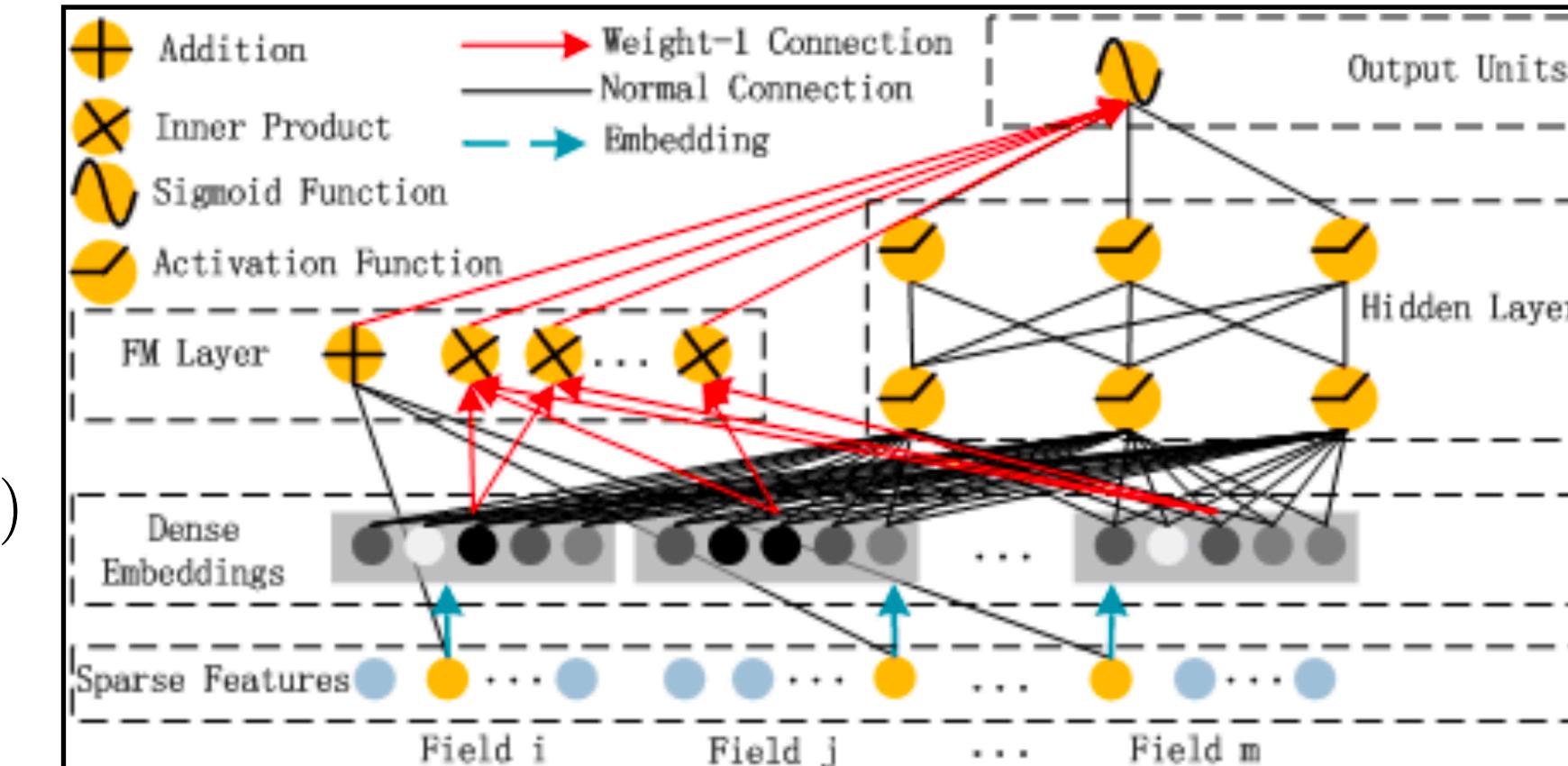
$$y_{FM} = \langle w, x \rangle + \sum_{j_1=1}^d \sum_{j_2=j_1+1}^d \langle V_i, V_j \rangle x_{j_1} \cdot x_{j_2}$$

$w \in \mathbb{R}^d, V_i \in \mathbb{R}^k$  ( $k$  is given)

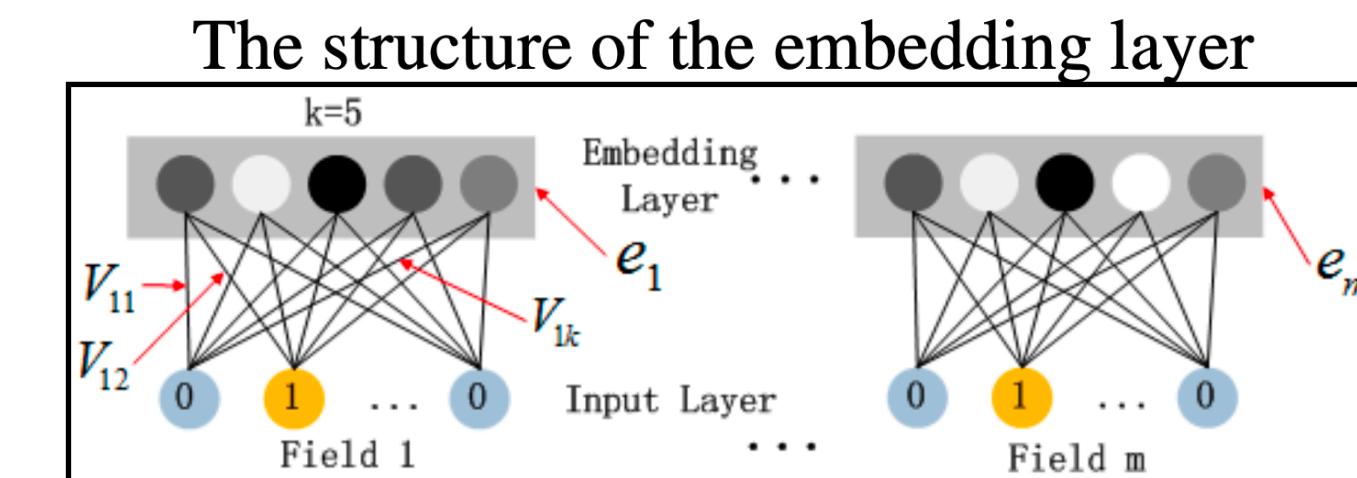
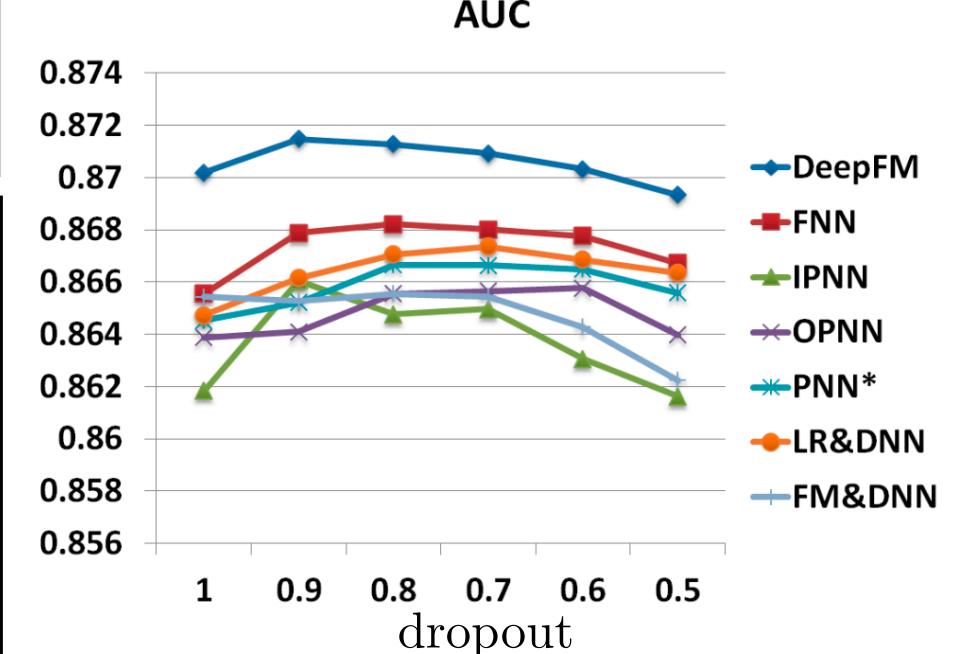
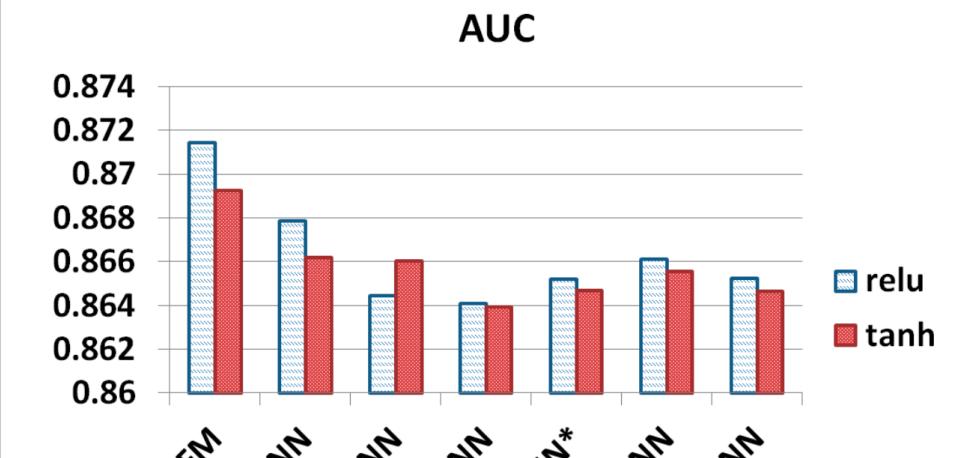
$a^{(0)} = [e_1, e_2, \dots, e_m] \rightarrow$  output of the embedding layer

$m \rightarrow$  number of fields

$e_i \rightarrow$  embedding of the  $i$ -th field



"The FM component and deep component share the same feature embedding, which brings two important benefits: 1) it learns both low- and high-order feature interactions from raw features; 2) there is no need for expertise feature engineering of the input, as required in Wide & Deep."



$$a^{(\ell+1)} = \sigma(W^{(\ell)} a^{(\ell)} + b^{(\ell)})$$

$$y_{DNN} = \sigma(W^{|H|+1} a^{|H|+1} + b^{|H|+1})$$

$|H| \rightarrow$  number of hidden layers

	Company*		Criteo	
	AUC	LogLoss	AUC	LogLoss
LR	0.8640	0.02648	0.7686	0.47762
FM	0.8678	0.02633	0.7892	0.46077
FNN	0.8683	0.02629	0.7963	0.45738
IPNN	0.8664	0.02637	0.7972	0.45323
OPNN	0.8658	0.02641	0.7982	0.45256
PNN*	0.8672	0.02636	0.7987	0.45214
LR & DNN	0.8673	0.02634	0.7981	0.46772
FM & DNN	0.8661	0.02640	0.7850	0.45382
DeepFM	<b>0.8715</b>	<b>0.02618</b>	<b>0.8007</b>	<b>0.45083</b>

# Personalizing Session-based Recommendations with Hierarchical Recurrent Neural Networks


[YouTube Video](#)

HRNN: Hierarchical RNN

$$S_m = \{i_{m,1}, i_{m,2}, \dots, i_{m,N_m}\} \rightarrow \text{session } m$$

$$s_{m,n} = \text{GRU}_{\text{ses}}(i_{m,n}, s_{m,n-1}), n = 1, 2, \dots, N_m - 1$$

$s_{m,n}$  → hidden state at step  $n$

$$s_{m,0} = 0$$

$i_{m,n}$  → one-hot vector of the current item ID

$\hat{r}_{m,n}$  → output of the RNN  
 (a score for every item in the catalog)  
 (the likelihood of being the next  
 item in the session)

$$\hat{r}_{m,n} = g(s_{m,n}), n = 1, \dots, N_m - 1$$

$g(\cdot)$  is a non-linear function like softmax or tanh  
 ranking loss functions: cross-entropy, BPR, TOP1  
 Bayesian Personalized Ranking

**TOP1 Loss**

$$\frac{1}{N} \sum_{j=1}^N I\{\hat{r}_{s,j} > \hat{r}_{s,i}\} - \text{relative rank of the relevant item}$$

score of a sampled irrelevant item

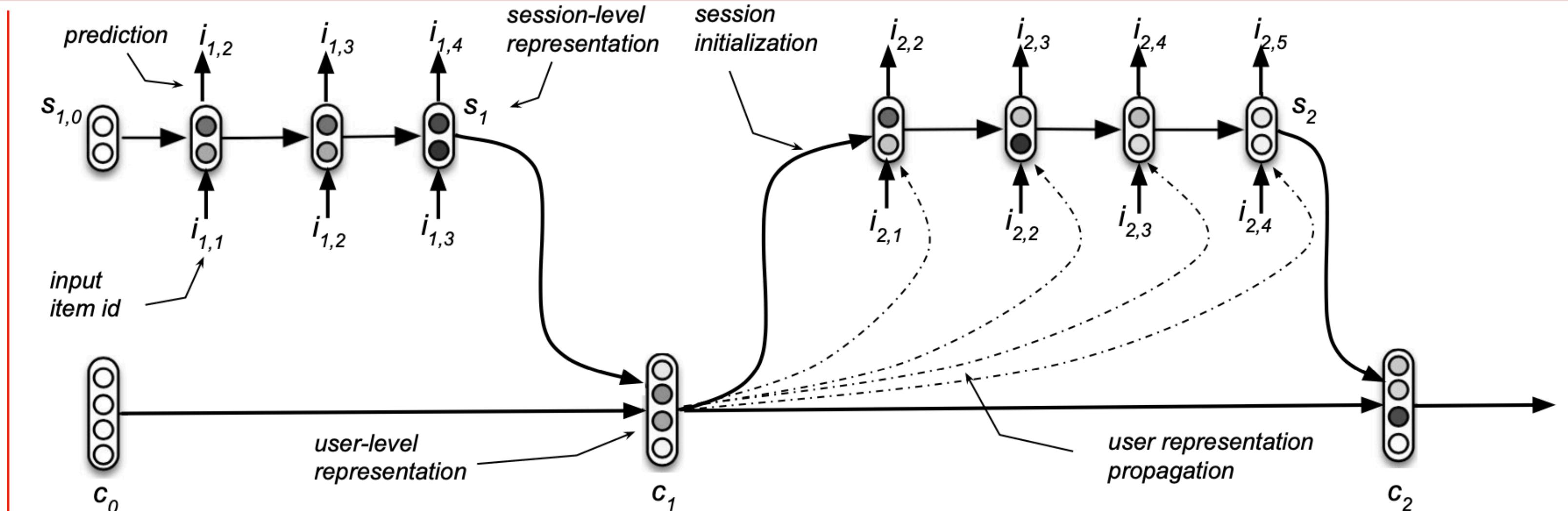
$$- \text{regularized approximation}$$

$$L_s = \frac{1}{N_S} \cdot \sum_{j=1}^{N_S} \sigma(\hat{r}_{s,j} - \hat{r}_{s,i}) + \sigma(\hat{r}_{s,j}^2)$$

**Model user activity across user sessions**

$\text{GRU}_{\text{user}}$  → user-level GRU

$$C^u = \{S_1^u, S_2^u, \dots, S_{M_u}^u\} \rightarrow \text{sessions of user } u$$



$$s_1^u, s_2^u, \dots, s_{M_u}^u \rightarrow \text{session-level representation}$$

$$s_m^u = s_{m,N_m-1}^u \rightarrow \text{last hidden state of } \text{GRU}_{\text{ses}}$$

$$c_m^u \rightarrow \text{user-level representation}$$

$$c_m = \text{GRU}_{\text{usr}}(s_m, c_{m-1}), m = 1, \dots, M_u$$

$$c_0 = 0$$

$$s_{m+1,0} = \tanh(W_{\text{init}} c_m + b_{\text{init}})$$

$$s_{m+1,n} = \text{GRU}_{\text{ses}}(i_{m+1,n}, s_{m+1,n-1}, [c_m])$$

$$n = 1, \dots, N_{m+1} - 1$$

**User-parallel mini-batches**

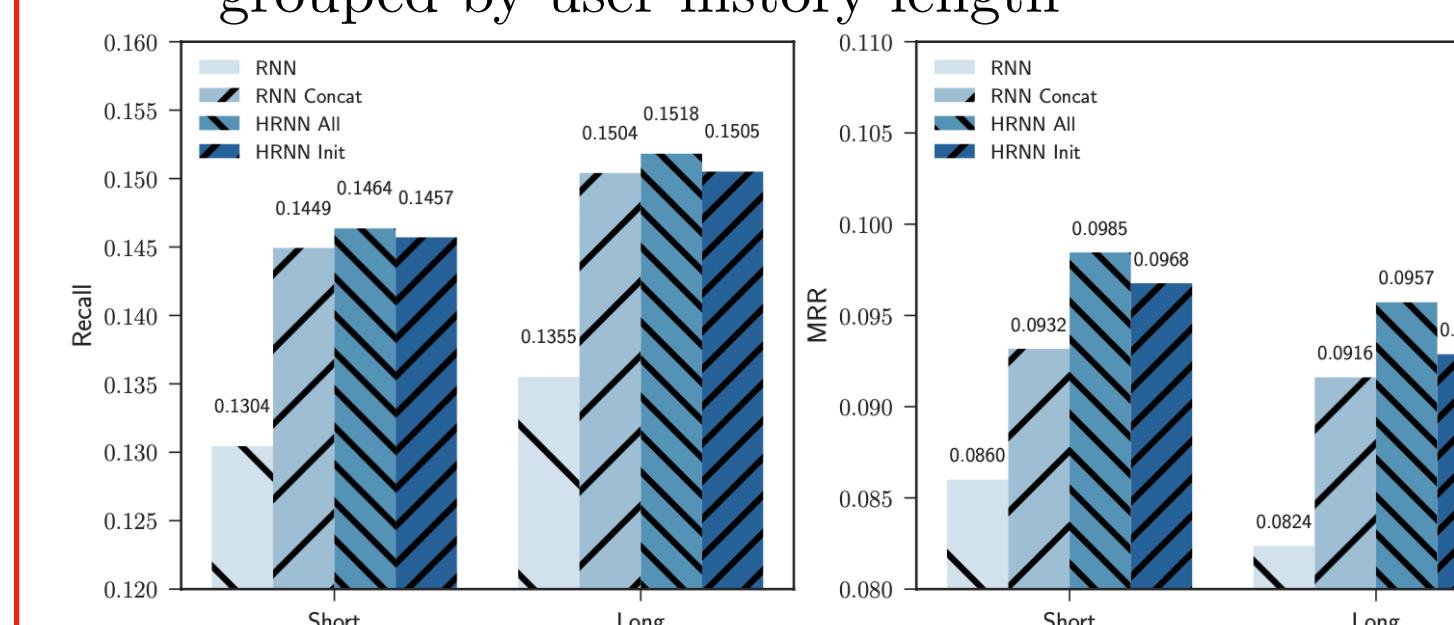
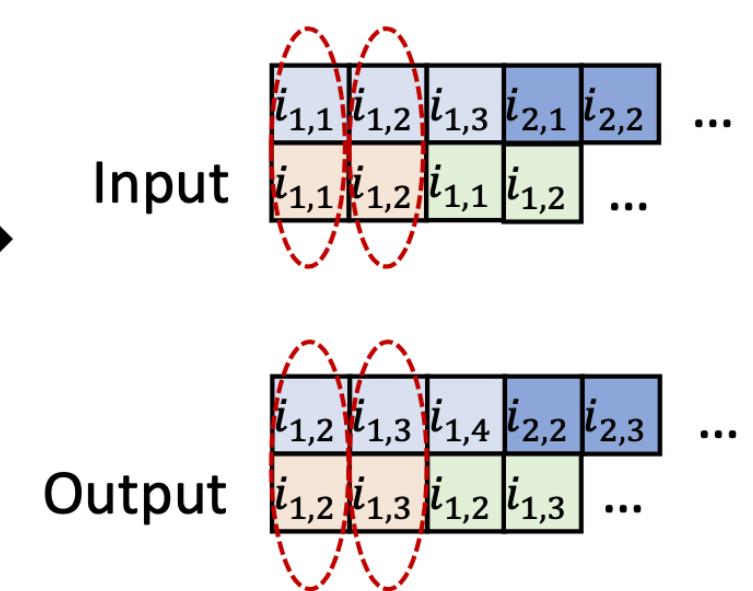
User 1      Session1  $i_{1,1} | i_{1,2} | i_{1,3} | i_{1,4}$

Session2  $i_{2,1} | i_{2,2} | i_{2,3}$

User 2      Session1  $i_{1,1} | i_{1,2} | i_{1,3}$

Session1  $i_{1,1} | i_{1,2} | i_{1,3} | i_{1,4}$

Session2  $i_{2,1} | i_{2,2}$





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# Variational Autoencoders for Collaborative Filtering



YouTube Video

 $u \in \{1, 2, \dots, U\} \rightarrow \text{users}$  $i \in \{1, 2, \dots, I\} \rightarrow \text{items}$  $X \in \mathbb{R}^{U \times I} \rightarrow \text{user-by-item interaction matrix}$  $x_u = [x_{u1}, x_{u2}, \dots, x_{uI}]^T \rightarrow \text{click history}$  $x_{ui} \rightarrow \text{number of clicks for item } i \text{ from user } u$ 

For simplicity, let us binarize the click matrix.

 $z_u \sim \mathcal{N}(0, I_K)$  $z_u \in \mathbb{R}^K \rightarrow \text{latent representation}$  $x_u \sim \text{Mult}(N_u, \pi(z_u))$  $\pi(z_u) \propto \exp(f_\theta(z_u))$ probability distribution over  $I$  items $N_u = \sum_{i=1}^I x_{ui} \rightarrow \text{total number of clicks from user } u$  $\log p_\theta(x_u | z_u) \stackrel{c}{=} \sum_i x_{ui} \log \pi_i(z_u) \rightarrow \text{multinomial likelihood}$  $\log p_\theta(x_u | z_u) \stackrel{c}{=} -\sum_i \frac{c_{ui}}{2} (x_{ui} - f_{ui})^2 \rightarrow \text{Gaussian likelihood}$  $f_\theta(z_u) = [f_{u1}, \dots, f_{uI}]^T$  $c_{ui} = c_{x_{ui}} \rightarrow \text{"confidence weight } (c_1 > c_0)}$  $\log p_\theta(x_u | z_u) \stackrel{c}{=} -\sum_i x_{ui} \log \sigma(f_{ui}) + (1 - x_{ui}) \log(1 - \sigma(f_{ui}))$   
logistic likelihood $p(z_u | x_u) \rightarrow \text{posterior distribution (intractable)}$  $q(z_u) = \mathcal{N}(\mu_u, \text{diag}(\sigma_u^2)) \rightarrow \text{variational distribution}$ 

$$\text{KL}(q(z_u) \| p(z_u | x_u))$$

$$g_\phi(x_u) = [\mu_\phi(x_u), \sigma_\phi(x_u)]$$

$$q_\phi(z_u | x_u) = \mathcal{N}(\mu_\phi(x_u), \text{diag}(\sigma_\phi^2(x_u)))$$

$$\log p(x_u; \theta) \geq \mathbb{E}_{q_\phi(z_u | x_u)} [\log p_\theta(x_u | z_u)] - \text{KL}[q_\phi(z_u | x_u) \| p(z_u)] := \mathcal{L}(x_u; \theta, \phi)$$

 $z_u = \mu_\phi(x_u) + \epsilon \odot \sigma_\phi(x_u) \rightarrow \text{re-parametrization trick}$ 

$$\epsilon \sim \mathcal{N}(0, I_K)$$

$$\mathcal{L}(x_u; \theta, \phi) = \mathbb{E}_{q_\phi(z_u | x_u)} [\log p_\theta(x_u | z_u)] - \beta \text{KL}[q_\phi(z_u | x_u) \| p(z_u)]$$

$$\theta^{\text{AE}}, \phi^{\text{AE}} = \arg \max_{\theta, \phi} \sum_u \mathbb{E}_{\delta(z_u - g_\phi(x_u))} [\log p_\theta(x_u | z_u)]$$

$$= \arg \max_{\theta, \phi} \sum_u \log p_\theta(x_u | g_\phi(x_u))$$

→ Regular Autoencoder

Prediction

 $x \rightarrow \text{user history}$  $z = \mu_\phi(x)$  and rank based on  $f_\theta(z)$ 

Metrics

 $\text{Recall}@R(u, \omega) := \frac{\sum_{r=1}^R \mathbb{I}[\omega(r) \in I_u]}{\min(R, |I_u|)} \rightarrow$  considers all items ranked within the first  $R$  to be equally important $w(r) \rightarrow \text{item at rank } r$  $I_u \rightarrow \text{set of held-out items that user } u \text{ clicked on}$  $\text{DCG}@R(u, \omega) := \sum_{r=1}^R \frac{2^{\mathbb{I}[\omega(r) \in I_u]} - 1}{\log(r+1)} \rightarrow$  uses a monotonically increasing discount to emphasize the importance of higher ranks versus lower ones

Truncated discounted cumulative gain

NDCG@R is the DCG@R linearly normalized to  $[0, 1]$  after dividing by the best possible DCG@R, where all the held-out items are ranked at the top.

- MovieLens-20M (ML-20M)
- Netflix Prize (Netflix)
- Million Song Dataset (MSD)

	ML-20M	Netflix	MSD
# of users	136,677	463,435	571,355
# of items	20,108	17,769	41,140
# of interactions	10.0M	56.9M	33.6M
% of interactions	0.36%	0.69%	0.14%
# of held-out users	10,000	40,000	50,000
	Recall@20	Recall@50	NDCG@100
Mult-VAE <sup>PR</sup>	<b>0.395</b>	<b>0.537</b>	<b>0.426</b>
Gaussian-VAE <sup>PR</sup>	0.383	0.523	0.415
Logistic-VAE <sup>PR</sup>	0.388	0.523	0.419
Mult-DAE	<b>0.387</b>	<b>0.524</b>	<b>0.419</b>
Gaussian-DAE	0.376	0.515	0.409
Logistic-DAE	0.381	0.516	0.414



# Personalized Top-N Sequential Recommendation via Convolutional Sequence Embedding


[YouTube Video](#)

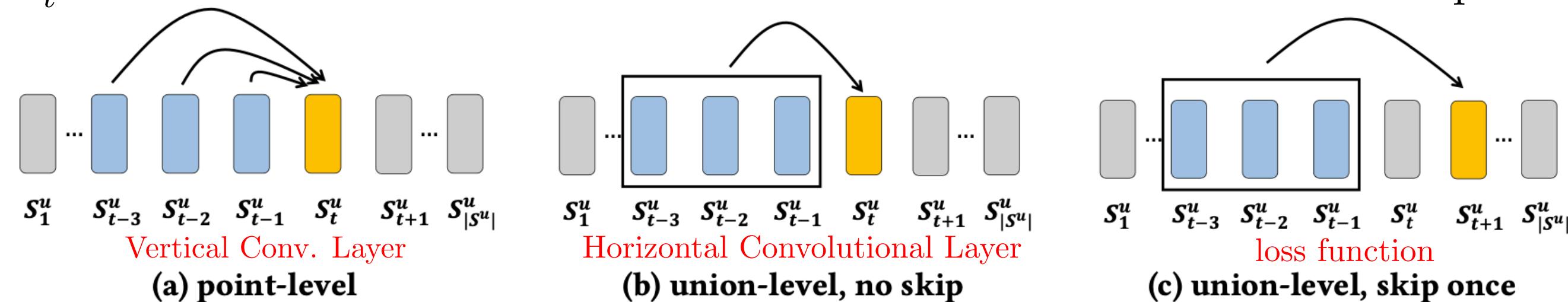
Convolutional Sequence Embedding Recommendation Model (Caser)

$\mathcal{U} = \{u_1, u_2, \dots, u_{|\mathcal{U}|}\} \rightarrow$  set of users

$\mathcal{I} = \{i_1, i_2, \dots, i_{|\mathcal{I}|}\} \rightarrow$  universe of items

$\mathcal{S}^u = (\mathcal{S}_1^u, \dots, \mathcal{S}_{|\mathcal{S}^u|}^u) \rightarrow$  sequence of items from  $\mathcal{I}$  associated with user  $u$

$\mathcal{S}_t^u \in \mathcal{I} \rightarrow$  index  $t$  denotes the order in which an action occurs in the sequence



input:  $L$  successive items from the user's sequence  $\mathcal{S}^u$

targets: next  $T$  items

$L + T \rightarrow$  size of the sliding window over the user's sequence

$(u, \text{previous } L \text{ items}, \text{next } T \text{ items}) \rightarrow$  training instance triplet

**Embedding Look-up**

$Q_i \in \mathbb{R}^d \rightarrow$  embedding for item  $i$        $d \rightarrow$  number of latent dimensions

$E^{(u,t)} = [Q_{\mathcal{S}_{t-L}^u}, \dots, Q_{\mathcal{S}_{t-1}^u}]^T \in \mathbb{R}^{L \times d} \rightarrow$  embedding matrix

$P_u \in \mathbb{R}^d \rightarrow$  user embedding

Regard  $E \in \mathbb{R}^{L \times d}$  as an image!

**Horizontal Convolutional Layer**

$F^k \in \mathbb{R}^{h \times d}, k = 1, \dots, n \rightarrow n$  horizontal filters

$h \in \{1, 2, \dots, L\} \rightarrow$  height of the filter

$c_i^k = \phi_c(E_{i:i+h-1} \odot F^k) \rightarrow$   $i$ -th convolution value  
activation func. inner product

$$c^k = [c_1^k, \dots, c_{L-h+1}^k]$$

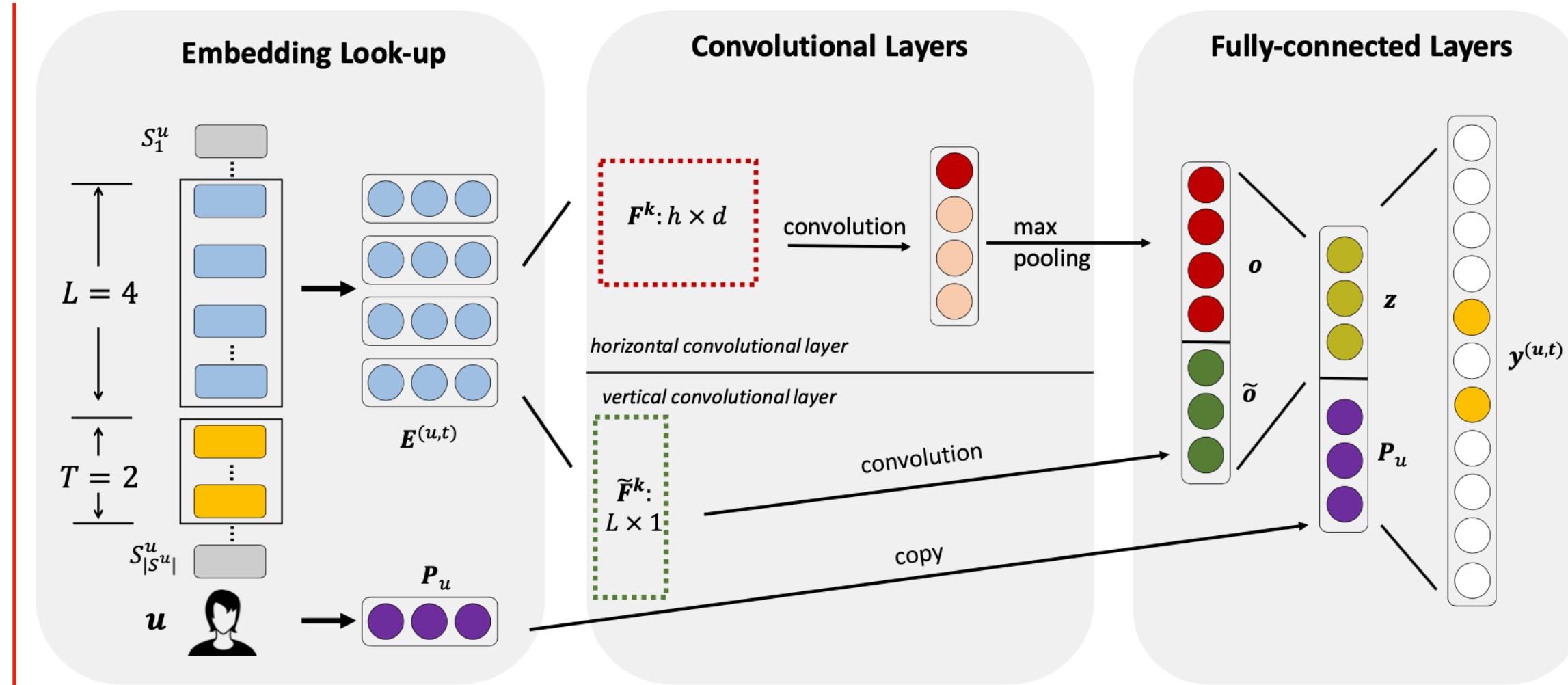
$$o = [\max(c^1), \dots, \max(c^n)]$$

**Vertical Conv. Layer**

$$\tilde{F}^k \in \mathbb{R}^{L \times 1}, k = 1, \dots, \tilde{n}$$

$$\tilde{c}^k = [\tilde{c}_1^k, \dots, \tilde{c}_d^k]$$

$$\tilde{o} = [\tilde{c}^1, \dots, \tilde{c}^n] \in \mathbb{R}^{d\tilde{n}}$$



**Network Training**

$$p(\mathcal{S}_t^u | \mathcal{S}_{t-1}^u, \dots, \mathcal{S}_{t-L}^u) = \sigma(y_{\mathcal{S}_t^u}^{(u,t)}) \rightarrow \text{sigmoid}$$

$$\mathcal{C}^u \rightarrow \text{collection of time-steps for which we would like to make predictions for user } u$$

$$p(\mathcal{S} | \theta) = \prod_u \prod_{t \in \mathcal{C}^u} \sigma(y_{\mathcal{S}_t^u}^{(u,t)}) \prod_{j \neq \mathcal{S}_t^u} (1 - \sigma(y_j^{(u,t)}))$$

likelihood of all sequences in the dataset

To capture the skip behavior, replace  $\mathcal{S}_t^u$  with  $\mathcal{D}_t^u$ .

$$\ell = \sum_u \sum_{t \in \mathcal{C}^u} \sum_{i \in \mathcal{D}_t^u} -\log(\sigma(y_i^{(u,t)})) + \sum_{j \neq i} -\log(1 - \sigma(y_j^{(u,t)}))$$

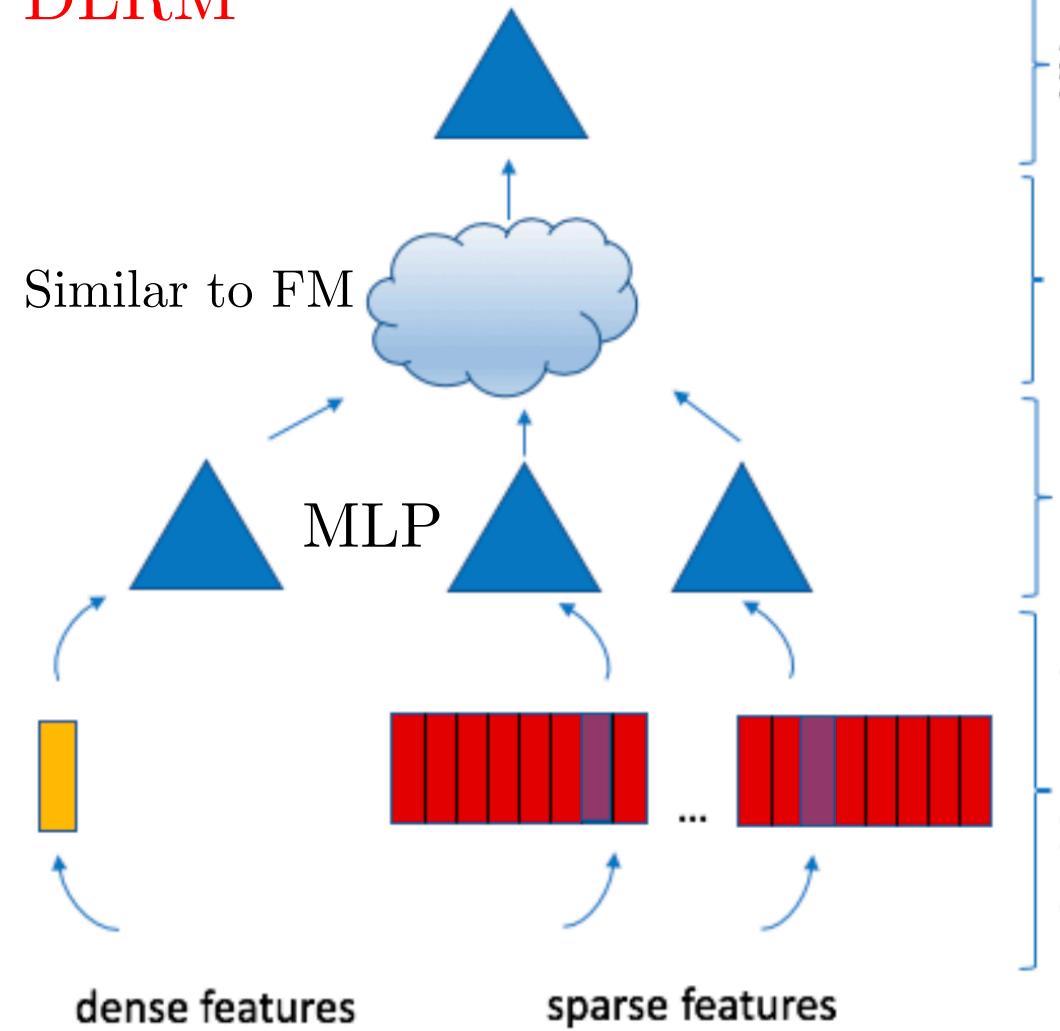
binary cross-entropy

**Evaluation Metrics**

$$\text{Prec}@N = \frac{|R \cap \hat{R}_{1:N}|}{N} \quad \text{Recall}@N = \frac{|R \cap \hat{R}_{1:N}|}{|R|} \quad \text{AP} = \frac{\sum_{N=1}^{|\hat{R}|} \text{Prec}@N \times \text{rel}(N)}{|\hat{R}|}$$

$\hat{R}_{1:N} \rightarrow$  list of top  $N$  predicted items for a user       $\text{rel}(N) = 1$  if the  $N$ -th item in  $\hat{R}$  is in  $R$   
 $R \rightarrow$  last 20% of actions in a user's sequence

# Deep Learning Recommendation Model for Personalization and Recommendation Systems


[YouTube Video](#)
**DLMR**

**Embeddings**

$e_i \in \{0, 1\}^m \rightarrow$  one-hot vector with the  $i$ -th position being 1 while others are 0, where index  $i$  corresponds to  $i$ -th category

$W \in \mathbb{R}^{m \times d} \rightarrow$  embedding table

$w_i^T = e_i^T W \rightarrow$   $i$ -th row of  $W$

$a^T = [0, \dots, a_{i_1}, \dots, a_{i_k}, \dots, 0]$   
multi-hot vector (weighted combination of multiple items)

$S = A^T W \rightarrow$  mini-batch of  $t$  embedding lookups

$A = [a_1, \dots, a_t]$

**Matrix Factorization (MF)**
 $\mathcal{S} \rightarrow$  set of users that have rated some products

 $w_i \in \mathbb{R}^d, i = 1, \dots, m \rightarrow$  representation of the  $i$ -th product

 $v_j \in \mathbb{R}^d, j = 1, \dots, n \rightarrow$  representation of the  $j$ -th user

 $\mathcal{S} = \{(i, j) : \text{when the } i\text{-th product has been rated by user } j\}$ 

$$\min \sum_{(i,j) \in \mathcal{S}} r_{ij} - w_i^T v_j$$

rating of the  $i$ -th product by the  $j$ -th user

$$R = [r_{ij}]$$

$$W^T = [w_1, \dots, w_m] \implies R \approx WV^T$$

$$V^T = [v_1, \dots, v_n]$$

**Factorization Machine (FM)**

$$\phi : \mathbb{R}^n \rightarrow T$$

prediction function

 $x \in \mathbb{R}^n \rightarrow$  input data point

 $y \in T \rightarrow$  target label

click-through rate (CTR):  $T = \{+1, -1\}$ 

$$\hat{y} = b + w^T x + x^T \text{upper}(VV^T)x$$

$$V \in \mathbb{R}^{n \times d}$$

strictly upper triangular part

**Multilayer Perceptrons (MLP)**

$$\hat{y} = W_k \sigma(W_{k-1} \sigma(\dots \sigma(W_1 x + b_1) \dots) + b_{k-1}) + b_k$$

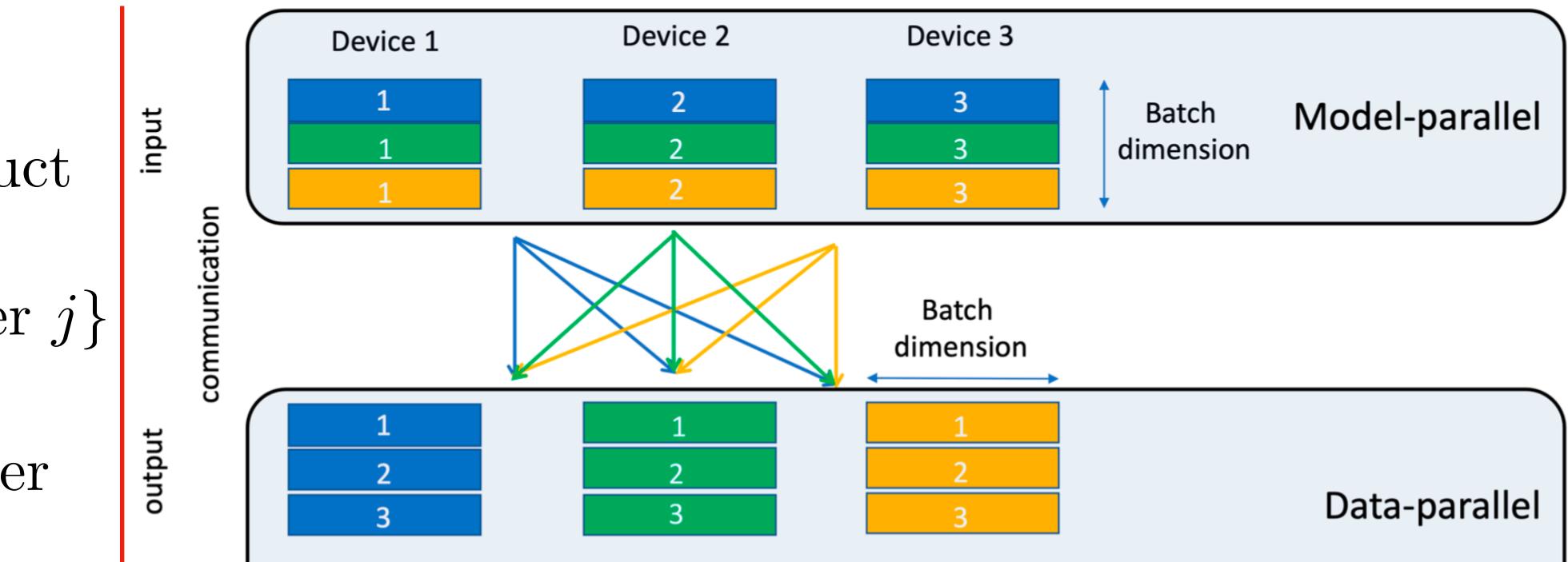
**Parallelism**

 Embeddings  $\rightarrow$  Memory Bound (Model Parallel)

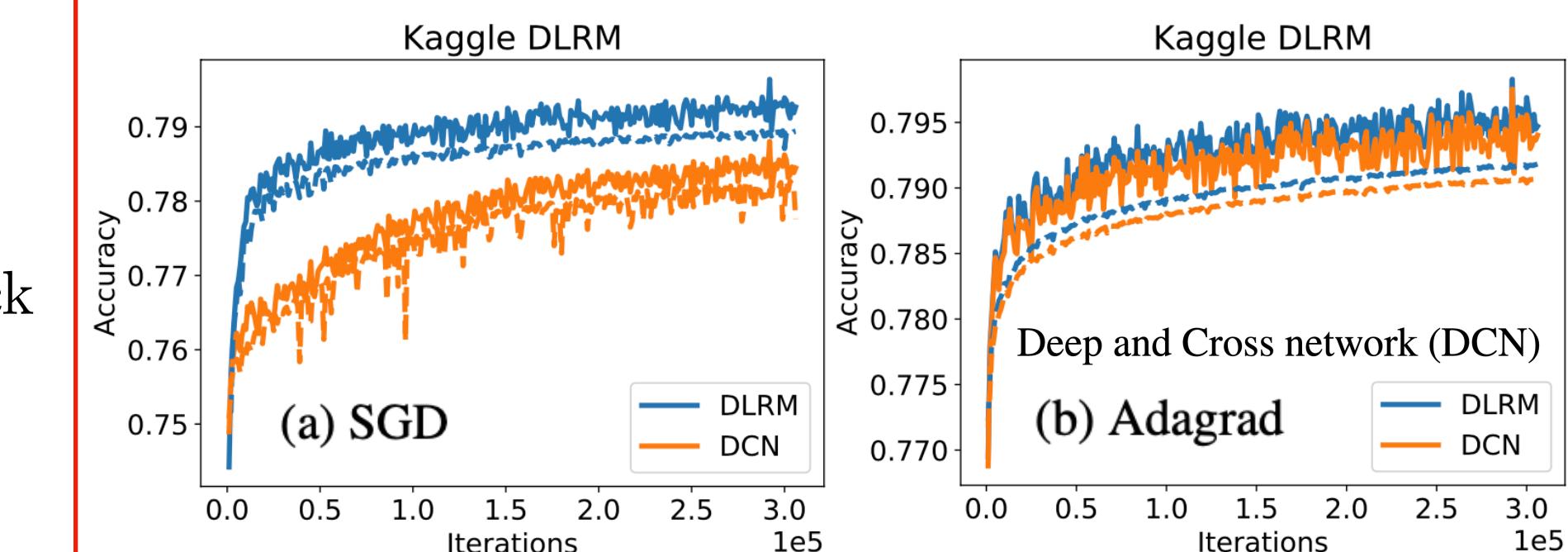
 MLP  $\rightarrow$  Compute Bound (Data Parallel)

**Data**

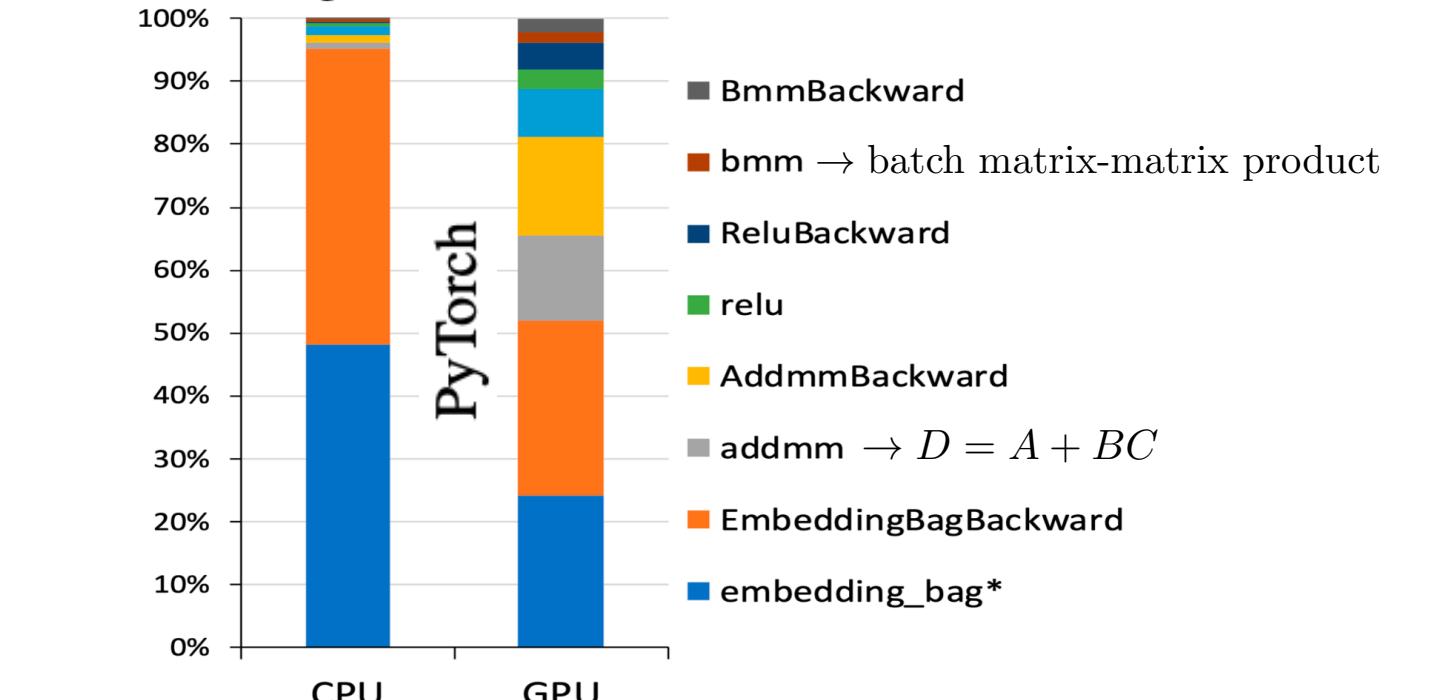
Criteo AI Labs Ad Kaggle and Terabyte data sets for ad CTR prediction



Butterfly shuffle for the all-to-all (personalized) communication



Comparison of training (solid) and validation (dashed) accuracies of DLRM and DCN





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# Questions?

[YouTube Playlist](#)

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