Integration

Indefinite integral

$$\int f(x)dx$$

Definite integral

$$\int_a^b f(x)dx$$

Remark: if the fun. f(x) is continuous on the interval [a, b], then the fun. is integrable on [a, b].

Integration Rules

1- $\int a\ dx = ax + c, a$ is continuous and c is integral constant.
2- $\int x^n\ dx = \frac{x^{n+1}}{n+1} + c, n\epsilon R, n \neq -1.$
3- $\int [f(x)]^n\ f'(x)\ dx = \frac{[f(x)\}^{n+1}}{n+1} + c, n\epsilon R, n \neq -1.$

Properties of integral

1- The linearity of integral
   I- $\int c\ f(x)dx = c \int f(x)dx$
   II- $\int [f(x) \pm g(x)]\ dx = \int f(x)\ dx \pm \int g(x)\ dx$

   Ex1: find 1- $\int -2x\ dx$

   Sol: $\int -2x\ dx = -2 \int x\ dx = -2\left(\frac{x^2}{2}\right) + c = -x + c$

   Ex2: find $\int [(x^2+1)^2 x + 1]\ dx$

   Sol: $\int (x^2+1)^2 x\ dx + \int dx = \frac{2}{2}\int (x^2+1)^2 x\ dx + \int dx$

   $\frac{1}{2}\int (x^2+1)^2 (2x)\ dx + \int dx = \frac{1}{2}\frac{(x^2+1)^3}{3} + x + c$

   $$= \frac{(x^2+1)^3}{6} + x + c$$

2- Additivity of the integral on adjacent intervals

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx\,, a < c < b.$$

Ex3: if $f(x) = \begin{cases} 3x^2, & x \geq 2 \\ 6x, & x < 2 \end{cases}$, find $\int_0^5 f(x)dx$

Sol: $\int_0^5 f(x)dx = \int_0^2 6x\, dx + \int_2^5 3x^2\, dx$

$$= 6\left[\frac{x^2}{2}\right]_0^2 + 3\left[\frac{x^3}{3}\right]_2^5$$

$$6\left[\frac{4}{2} - 0\right] + 3\left[\frac{125}{3} - \frac{8}{3}\right]$$

$$= 12 + 117 = 129$$

Ex4: find $\int_{-2}^{2} |x - 1|\, dx$

Sol: since $\begin{cases} x - 1, & x \geq 1 \\ -(x + 1), & x < 1 \end{cases}$

$\therefore\ \int_{-2}^{2} |x - 1|\, dx = \int_{-2}^{1} (-x + 1)\, dx + \int_1^2 (x - 1)\, dx$

$$= -\frac{x^2}{2} + x \Bigg]\!\!\Bigg]_{-2}^{1} + \frac{x^2}{2} - x \Bigg]\!\!\Bigg]_{1}^{2}$$

$$= \frac{9}{2} + \frac{1}{2} = 5$$

3- $\int_a^b f(x)dx = -\int_b^a f(x)dx$

4- $\int_a^a f(x)\, dx = 0$

5- $\frac{d}{dx}\int_a^b f(x)\, dx = 0$

6- $if\ f(x) \geq 0$ on [a, b], then $\int_a^b f(x)dx\ \geq 0$

7- $if\ f(x) \geq g(x)$ on [a, b], then $\int_a^b f(x)dx\ \geq\ \int_a^b g(x)dx$

Examples 5: H.W

1- $\int_0^1 (4x^2 - 3x + 1)\, dx$	2- $\int_1^5 x\sqrt{x^2+1}\, dx$
3- $\int \sqrt{x^2 + 4x^2}\, dx$	4- $\int \frac{x^2-6x+5}{x-1}$
5- $\int \frac{\sqrt{x}+x}{\sqrt{x}}$	6- $\int_{-2}^1 \sqrt{t^2+2t-1}\, dt$
7- $\int_1^1 (y^2+1)\, dy$	8- $\int_1^n nx\, dx$
9- $\int_{-1}^1 dx$	10- $\int 0\, dx$

Ex6: if $\int_2^5 f(x)\, dx = 7$ and $\int_{-2}^5 f(x)\, dx = 4$, find $\int_{-2}^2 f(x)\, dx$.

Sol: since $\int_{-2}^5 f(x)\, dx = \int_{-2}^2 f(x)\, dx + \int_2^5 f(x)\, dx$

$$4 = \int_{-2}^{2} f(x)\, dx + 7$$

$$\int_{-2}^{2} f(x)\, dx = 4 - 7 = -3$$

Ex 7: if $\int_3^4 f(x)\, dx = -9$, show that $\int_4^3 f(x)\, dx = 27$ H.W

Ex 8: find $\int_2^2 \sqrt{x+1}\, dx$ H.W

Ex 9: find $\frac{d}{dx}\int_2^3 f(x)\, dx$ H.W

Ex 10: find $\int (1-u)(1+u+u^2)\, du$ H.W

Ex 11: find $\frac{d}{dx}\int_{2x}^{x^2} (1+2t)\, dt = \frac{d}{dx}\left[(t+t^2)\right]_{2x}^{x^2}$

$$= \frac{d}{dx}\left[x^2 + x^4 - 2x - 4x^2\right] = -6x + 4x^3 - 2$$