The function

Def: A function is a rule of correspondence between two non-empty sets of elements called the domain and the rang of the function such that to each element of the domain corresponds one and only one element of the range and each element of the range is the correspondence of the least one element of the domain and is denoted by $y = (x)$.

Def: The domain is the set of real values taken by the independent variables

Def: The range is the set of real values taken by the dependent variables

1) If function is polynomial

$$D_f = R$$

Ex1: find the domain and range of $f(x) = X^2$

Sol: $D_f = R$

$$R_f = R$$

2) If function is roots $\sqrt[n]{f(x)}$, then

(a) if n is even, then $D_f = solution\ f(x) \geq 0$.

(b) if n is odd, then $D_f = R$

Ex2: find the domain and range of $f(x) = \sqrt{1 - x^2}$

Sol: $1 - x^2 \geq 0 \rightarrow (1 - x)(1 + x) \geq 0$

$$D_f = \{x: -1 \leq x \leq 1\} = [-1,1]$$

$$R_f = \{y: 0 \leq y \leq 1\}$$

3) If function is quotient $\frac{f(x)}{g(x)}, g(x) \neq 0$, then

$$D_f = R - \{g(x) = 0\}$$

Ex3: find the domain and range of $y = \frac{5x+3}{2x-1}$

Sol: $2x - 1 = 0 \rightarrow x = \frac{1}{2}$

$D_y = R - \left\{\frac{1}{2}\right\}$

To find the range

$y = \frac{5x+3}{2x-1} \rightarrow 2xy - y = 5x + 3$

$\rightarrow 2xy - 5x = 3 + y \ \rightarrow x(2y - 5) = 3 + y$

$\rightarrow x = \frac{3+y}{2y-5}$

$\rightarrow 2y - 5 = 0 \ \rightarrow y = \frac{5}{2}$

$R_y = R - \left\{\frac{5}{2}\right\}$

Operations on function

If f and g are function with domain Df and Dg then

1) $(f \mp g)(x) = f(x) \mp g(x)$

2) $(f * g)(x) = f(x) * g(x)$

3) $(\ \ g\ )\ (x) = f(x)\ g(x)$, $g(x) \neq 0$

Ex: let $f(x) = \sqrt{x+1}$ and $(x) = \sqrt{x}$, find:

$f - g, f + g, f . g, \frac{f}{g}, \frac{g}{f}$

Sol: H.W

Composite

Def: Let f and g be two functions, then the composite function is

$(f \circ g)(x) = f(g(x))$

Ex: if $f(x) = x^2 - 1$, and $g(x) = \sqrt{x+2}$ find

$(f \circ g)(x), (g \circ f)(x), D_{f \circ g}, D_{g \circ f}$

Sol:

To find $(f \circ g)(x)$,

$(f \circ g)(x) = f\big(g(x)\big) = f\big(\sqrt{x+2}\big) = \big(\sqrt{x+2}\big)^2 - 1$

$\rightarrow x + 2 - 1$

$\rightarrow x + 1$

$\therefore (f \circ g)(x) = x + 1$

To find $D_{f \circ g}$

Since $f \circ g$ is polynomial function then $D_{f \circ g} = R$

Inverse function

If F is a function such that each $y \in R_f$, there corresponds exactly one number $x \in D_f$, then the function f^{-1} is defined by

$x = f^{-1}(y)$ if and only if $y = f(x)$

f^{-1} is called the inverse of f

Ex: if $f(x) = \sqrt{2x+5}$ find $D_f, R_f, f^{-1}, D_{f^{-1}}, R_{f^{-1}}$

Sol: $D_f = \left\{x : x \geq \frac{-5}{2}\right\}$

$R_f = \{y : y \geq 0\}$

$$y = \sqrt{2x+5} \rightarrow y^2 = 2x + 5 \rightarrow x = \frac{y^2 - 5}{2} = f^{-1}(y)$$

$D_{f^{-1}} = \{y: y \geq 0\} = R_f$

$R_{f^{-1}} = \left\{x: x \geq \frac{-5}{2}\right\} = D_f$

Ex: if $f(x) = x^2 + 3x + 1$ find D_f, R_f, f^{-1}

Sol: H.W

Special function

1- Constant function: is defined by

$f(x) = k,\ k \in R$

$Df = R,\ Rf = \{k\}$

Ex: $(x) = 2$

$Df = R,\ Rf = \{2\}$

2- Identity function: is defined by $(x) = x$

$Df = R,\ Rf = R$

3-Rational function

$$f(x) = \frac{a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_n}{b_0x^m + b_1x^{m-1} + b_2x^{m-2} + \cdots + b_m}$$

Provided that $b_0x^m + b_1x^{m-1} + b_2x^{m-2} + \cdots + b_m \neq 0$

4-Square root function: $y = f(x) = \sqrt{x}$

$Df = [0, \infty),\ f = [0, \infty)$

5- The absolute value function: is defined by $(x) = |x|$

$Df = R,\ Rf = R+$

6-Polynomial function: is defined by $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_n$ Where $a_0, a_1, \dots, a_n,\ a_0 \neq 0$

n is the degree of the polynomial

If $n = 0 \Rightarrow f(x) = a_0$ $constant\ function$

If $n = 1 \Rightarrow f(x) = a_0x^1 + a_1$

If $n = 2 \Rightarrow f(x) = a_0x^2 + a_1x^1 + a_2$

Graph of function

Def: the point(x,y) whose coordinate satisfy the function y=f(x) make up the graph of the function in the xy-plane.

Ex: Graph the following function $y = x$

x	Y
-2	-2
-1	-1
0	0
1	1
2	2
10	10