The differential equation

1- $f(x) = \int f'(x)dx$
2- $f'(x) = \int f''(x)dx$
3- $s = \int v(t)dt$, s is distance, v is velocity>
4- $v = \int a(t)dt$

Examples:

1- Find the solution of the differential equation
$y' = x \; if \; y(1) = 2$

Sol: $\int y'dx = \int x\,dx$

$y = \frac{x^2}{2} + c \Longrightarrow 2 = \frac{1}{2} + c \Rightarrow c = \frac{3}{2}$

2- Find the solution of the differential equation
$y'' = x \; if \; y(1) = \frac{1}{2} \; and \; y'(1) = -\frac{1}{2}$

Sol: $\int y''dx = \int x\,dx$

$y' = \frac{x^2}{2} + c_1 \Rightarrow -\frac{1}{2} = \frac{1}{2} + c_1 \Rightarrow c_1 = -1$

$\therefore y' = \frac{x^2}{2} - 1$

$\int y'dx = \int (\frac{x^2}{2} - 1)\,dx \Rightarrow y = \frac{x^3}{6} - x + c_2 \Rightarrow \frac{1}{2} = \frac{1}{6} - 1 + c_2 \Rightarrow$
$c_2 = \frac{4}{3}$

$\therefore y = \frac{x^3}{6} - x + \frac{4}{3}$

3- An object moves along a coordinate line with velocity $v(t) = 2 - 3t + t^2$ and it's initial position at (t=0) is 2 unit to the right of the origin, find
a- The position of the object 4 second later?
b- The total distance travelled by the object during those the first 4 second?

Solution:

a- $x(4) = x(0) + \int_0^4 v(t)dt = 2 + \int_0^4 (2 - 3t + t^2)dt$
=2+$[2t - \frac{3}{2}t^2 + \frac{t^3}{3}]_0^4 = 2 + \frac{16}{3} = \frac{22}{3}$

b- $s = \int_0^4 |v(t)|dt = \int_0^4 |2 - 3t + t^2|dt$

$\because 2 - 3t + t^2 = 0 \Longrightarrow (2 - t)(1 - t) = 0 \Longrightarrow t = 2 \; or \; t = 1$

$$\therefore s = \left|\int_0^1 v(t)dt\right| + \left|\int_1^2 v(t)dt\right| + \left|\int_2^4 v(t)dt\right|$$

$$= \left|[2t - \frac{3}{2}t^2 + \frac{t^3}{3}]_0^1\right| + \left|[2t - \frac{3}{2}t^2 + \frac{t^3}{3}]_1^2\right| + \left|[2t - \frac{3}{2}t^2 + \frac{t^3}{3}]_2^4\right| = \frac{52}{3}$$

4- Find the equation of the curve if it's slope $\frac{3x^2}{2y}$ and pass through the point (1, 3).

Sol: $\frac{dy}{dx} = \frac{3x^2}{2y} \Rightarrow 2y\,dy = 3x^2\,dx \;\Rightarrow 2\int y\,dy = 3\int x^2\,dx$

$y^2 = x^3 + c \Longrightarrow (3)^2 = (1)^3 + c \Rightarrow c = 8$

$y^2 = x^3 + 8$

5- If (0, 1) is a critical point for $f(x)$, where $f''(x) = 3x + 1$. Find f(x). H.W.

Integral of trigonometric functions:

A-

$\int \sin x\,dx = -\cos x + C$ $\qquad$ $\int \cos x\,dx = \sin x + C$

$\int \sec^2 x\,dx = \tan x + C$ $\qquad$ $\int \csc^2 x\,dx = -\cot x + C$

$\int \sec x \tan x\,dx = \sec x + C$ $\qquad$ $\int \csc x \cot x\,dx = -\csc x + C$

Ex1: find $\int \cos 2x\,dx$

Sol: $\frac{1}{2}\int 2\cos 2x\,dx \;= \frac{1}{2}\sin 2x + c$

Ex2: find $\int (10x^4 - 2sec^2 x)\,dx$

Sol: $10\int x^4 dx \;- 2\int sec^2 x\,dx$

$= 10\frac{x^5}{5} - 2\tan x + c$

Ex3: $\int \frac{cos\theta}{\sin^2\theta}\ d\theta$

Sol: $\int \frac{cos\theta}{\sin^2\theta}\ d\theta = \int \left(\frac{1}{\sin\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right) d\theta$

$= \int \csc\theta \cot\theta\ d\theta = -\csc\theta + c$

Ex4: $\int \sin x \cos x\ dx$, H.W

B-

$\int \sin^2 x\ dx = \int \frac{1}{2}(1 - \cos 2x)\ dx$ H.W

$\int \cos^2 x\ dx = \int \frac{1}{2}(\cos 2x + 1)\ dx$ H.W

Ex1: $\int \cos^2 x\ dx = \int \frac{1}{2}(\cos 2x + 1)\ dx$

$\int \frac{1}{2}(\cos 2x + 1)\ dx = \frac{1}{2}\int \cos 2x\ \ dx + \frac{1}{2}\int dx$

$= -\frac{1}{4}\sin 2x + \frac{1}{2}x + c$

Ex2: $\int \sin^3 x\ dx = \int \sin x \sin^2 x\ dx = \int \sin x\,(1 - \cos^2 x)\ dx$

$$= \int \sin x\ dx - \int \sin x\ \cos^2 x\ \ dx$$

$$= -\cos x + \frac{1}{3}\cos^3 x + c$$

Ex3: $\int \cos^3 x\ dx$ H.W

Ex4: $\int \sin^4 x\ dx$ H.W

C-

$\int \tan^2 x\ dx = \int (\sec^2 x - 1)\ dx$ H.W

$\int \cos^2 x\ dx = \int (\csc^2 x - 1)\ dx$ H.W

Ex1: $\int \tan^2 x\ dx = \int (\sec^2 x - 1)\ dx = \tan x - x + c$

Ex2: $\int \frac{\tan x}{\cos x} dx = \int \frac{\sin x/\cos x}{\cos x} dx = \int \frac{\sin x}{\cos^2 x} dx$

$= \int \sin x \ \cos^{-2} x \, dx = \frac{1}{\cos x} + c$

Find : $\int \cos\frac{x}{2} \, dx$, $\int_1^x (\cos^2 t + \sin^2 t) \, dt$

The natural logarithm functions

F(x)	${dy}/{dx}$	Integral
$y = \ln x$	$\frac{dy}{dx} = \frac{1}{x}$	$\int \frac{1}{x} \, dx = \ln x + \text{c}$
$y = e^x$	$\frac{dy}{dx} = e^x$	$\int e^x \, dx = e^x + c$
$y = a^u$	$\frac{dy}{du} = a^u . \frac{du}{dx} . \ln a$	$\int a^u \, du = \frac{1}{\ln a} a^u + c$

Ex1: $\int \frac{2x}{x^2+5} dx = \ln|x^2 + 5| + c$

Ex2: $\int \tan x \ dx \ = \int \frac{\sin x}{\cos x} dx = -\ln|\cos x| + c$

Ex3: $\int \cot x \ dx$ H.W

Ex4: $\int \sec x \ dx \ = \int \sec x \frac{\sec x + \tan x}{\sec x + \tan x} \ dx$

$= \int \frac{\sec^2 x + \sec x \ \tan x}{\sec x + \tan x} \ dx = \ln|\sec x + \ \tan x| + c$

Ex5: $\int \csc x \ dx$ H.W

Ex6: $\int \frac{6t+1}{5-t}\ dt = \int \left(-6 + \frac{31}{5-t}\right) dt$

$= -6\int dt\ - 31\int \frac{-1}{5-t} dt = -6t - 31\ln|5-t| + c$

Ex7: $\int e^{\sin x} \cos x\ dx = e^{\sin x} + c$

Ex8: $\int \frac{(e^x-1)^2}{e^x}\ dx = \int \frac{e^{2x}-2e^x+1}{e^x}\ dx$

$= \int (\frac{e^{2x}}{e^x} - 2\frac{e^x}{e^x} + \frac{1}{e^x})\ dx = \int (e^x - 2 + e^{-x})\ dx = e^x - 2x - e^{-x} + c$

Ex8: $\int e^{3x+1}\ dx$ H.W

Ex9: $\int 5^{2x}\ dx = \frac{1}{2\ \ln 5}\int 2\ln 5\ 5^{2x}\ dx = \frac{5^{2x}}{2\ \ln 5} + c$