Derivative of trigonometric

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

EXAMPLE 2 Differentiate $f(x) = \dfrac{\sec x}{1 + \tan x}$. For what values of x does the graph of f have a horizontal tangent?

SOLUTION The Quotient Rule gives

$$f'(x) = \frac{(1 + \tan x)\dfrac{d}{dx}(\sec x) - \sec x \dfrac{d}{dx}(1 + \tan x)}{(1 + \tan x)^2}$$

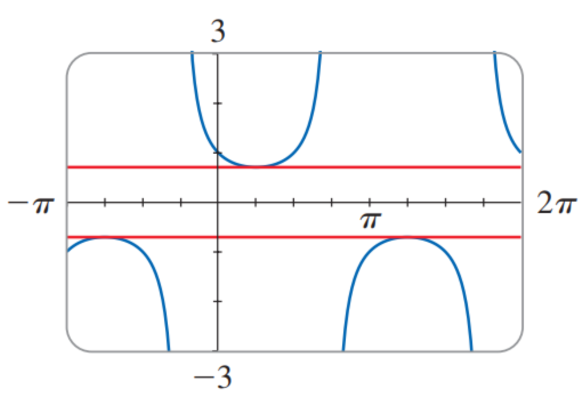
$$= \frac{(1 + \tan x)\sec x \tan x - \sec x \cdot \sec^2 x}{(1 + \tan x)^2}$$

$$= \frac{\sec x\,(\tan x + \tan^2 x - \sec^2 x)}{(1 + \tan x)^2}$$

$$= \frac{\sec x\,(\tan x - 1)}{(1 + \tan x)^2} \qquad (\sec^2 x = \tan^2 x + 1)$$

Because $\sec x$ is never 0, we see that $f'(x) = 0$, when $\tan x = 1$,

And this occurs when $x = \frac{\pi}{4} + n\pi$, where $n \in Z$, see fig. below



Ex: if $f(x) = 3\sin x - 3\cos x$ find f'

Sol: $f' = 3\cos x - 3(-\sin x)$

$= 3\cos x + 3\sin x$

Ex: if $y = \sec\theta \tan\theta$ find $\frac{dy}{d\theta}$

Sol: $y' = \sec\theta\,(sec^2\theta) + \tan\theta\,(\sec\theta\tan\theta)$

$= \sec\theta\,(sec^2\theta + tan^2\theta)$

$= \sec\theta\,(2sec^2\theta + 1)$

H.W

1 $h(\theta) = \theta^2 \sin\theta$

2 $y = \sec\theta\ \tan\theta$

3 $f(\theta) = (\theta - \cos\theta)\sin\theta$

4 $H(t) = \cos^2 t$

Exponential Functions

The function $f(x) = 2^x$ is called an *exponential function* because the variable, x, is the exponent. It should not be confused with the power function $g(x) = x^2$, in which the variable is the base.

An exponential function is a function of the form

$$f(x) = b^x$$

Where b is a positive constant.

- If $x = n$, a positive integer, then $b^n = b.b \dots \dots . b$
- If $x = 0$, then $b^0 = 1$, and if $x = -n$, where n is a positive integer, then

$$b^{-n} = \frac{1}{b^n}$$

- If $x$is a rational number, $x = {}^{p}/_{q}$, where p and q are integers and $q > 0$, then

$$b^x = b^{p/q} = \sqrt[q]{b^p} = (\sqrt[q]{b})^p$$

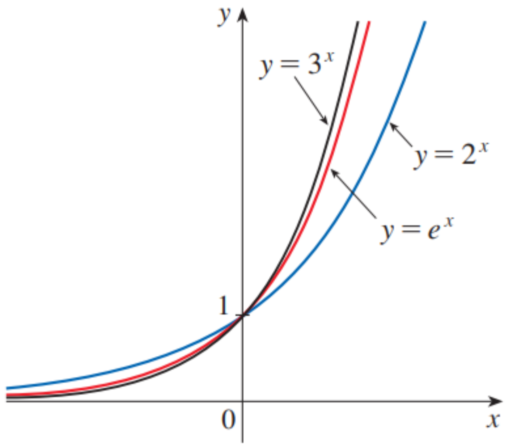
Laws of Exponents If a and b are positive numbers and x and y are any real numbers, then

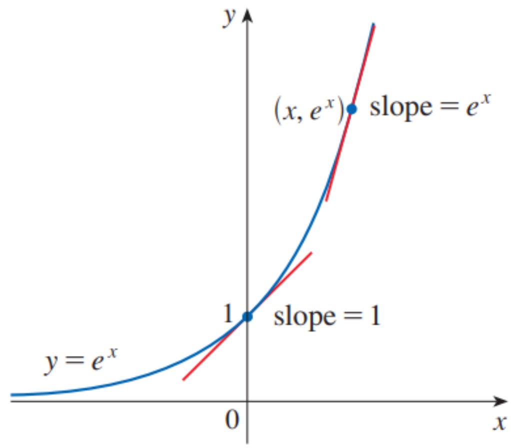
1. $b^{x+y} = b^x b^y$ **2.** $b^{x-y} = \dfrac{b^x}{b^y}$ **3.** $(b^x)^y = b^{xy}$ **4.** $(ab)^x = a^x b^x$

The number *e*

Def: e is the number such that $\lim_{h\to 0} \frac{e^h - 1}{h} = 1$.

Geometrically, this means that of all possible exponential functions $y = b^x$, the function $f(x) = e^x$ is the one whose tangent line at (0, 1) has a slope $f'(0) = 1$. See fig. below





Derivative of natural exponential number function

$$\frac{d}{dx}(e^x) = e^x$$

If $f(x) = e^x - x$, find f' and f''. Compare the graphs of f and f'.

Sol:

Using the Difference Rule, we have

$$f'(x) = \frac{d}{dx}(e^x - x) = \frac{d}{dx}(e^x) - \frac{d}{dx}(x) = e^x - 1$$

the second derivative as the derivative of f', so

$$f''(x) = \frac{d}{dx}(e^x - 1) = \frac{d}{dx}(e^x) - \frac{d}{dx}(1) = e^x$$