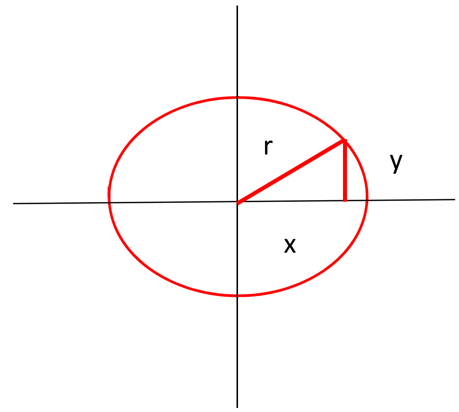
Transcendental function

A function that is not algebra is called transcendental function:

1- trigonometric function:




r: Hypotnuse (Hyp.)

y: opposite (Opp.)

x: Adjcent (Adj.)

There are six basics trigonometric functions:

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x} = \frac{sin\theta}{cos\theta} \qquad \cot\theta = \frac{x}{y} = \frac{1}{tan\theta} = \frac{cos\theta}{sin\theta}$$

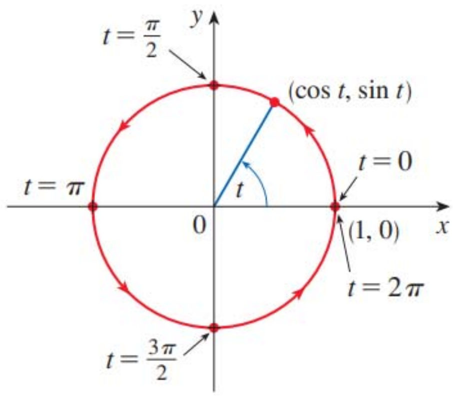
$$\csc\theta = \frac{r}{y} = \frac{1}{sin\theta} \qquad \sec\theta = \frac{r}{x} = \frac{1}{cos\theta}$$

Special angle

| $\theta$ | radians | $\sin\theta$ | $\cos\theta$ | $\tan\theta$ |
|---|---|---|---|---|
| $0^\circ$ | 0 | 0 | 1 | 0 |
| $30^\circ$ | $\pi/6$ | $1/2$ | $\sqrt{3}/2$ | $\sqrt{3}/3$ |
| $45^\circ$ | $\pi/4$ | $\sqrt{2}/2$ | $\sqrt{2}/2$ | 1 |
| $60^\circ$ | $\pi/3$ | $\sqrt{3}/2$ | $1/2$ | $\sqrt{3}$ |
| $90^\circ$ | $\pi/2$ | 1 | 0 | — |

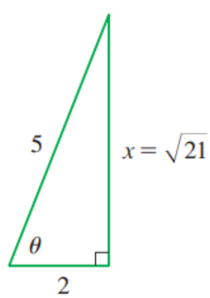
Unit circle:

$(x, y) = (cos\theta, sin\theta)$




EX: If $cos\theta = \frac{2}{5}$ and $0 < \theta < 2\pi$, find the other five trigonometric functions of $\theta$.

Sol: since $cos\theta = \frac{2}{5}$, this mean that the hypotenuse = 5 and the adjacent = 2. By using Pythagorean theorem gives $y^2 + 4 = 25$, then the opposite is $\sqrt{21}$.




Trigonometric identities:

$$sin^2\theta + cos^2\theta = 1 \dots\dots\dots\dots\dots (*)$$

- Dividing eq.(*) by $cos^2\theta$ then

$$tan^2\theta + 1 = sec^2\theta$$

- Dividing eq.(*) by $sin^2\theta$ then

$$1 + cot^2\theta = cos^2\theta$$

Note: the sine is an odd function and cosine is an even function, we have

$$\sin(-\theta) = -\sin\theta \ \& \ \cos(-\theta) = \cos\theta$$

Note: Since the angles $\theta$ and $\theta + 2\pi$ have the same terminal side, we have

$$\sin(\theta + 2\pi) = \sin\theta \ \& \ \cos(\theta + 2\pi) = \cos\theta$$

These show that sine and cosine are periodic with period $2\pi$

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

And

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

By dividing the two equations above, we have

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

If we put $y = x$ in the equation

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

We get the double-angle-formulas

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

By using $sin^2\theta + cos^2\theta = 1$, we have

$$\cos 2x = 2\cos^2 x - 1$$

$$\cos 2x = 1 - 2\sin^2 x$$

Now, we solve these equations:

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

Finally, we state the product identities

$$\sin x \cos y = \tfrac{1}{2}[\sin(x + y) + \sin(x - y)]$$

$$\cos x \cos y = \tfrac{1}{2}[\cos(x + y) + \cos(x - y)]$$

$$\sin x \sin y = \tfrac{1}{2}[\cos(x - y) - \cos(x + y)]$$

Find all values of $x$ in the interval $[0, 2\pi]$ such that $sinx = sin2x$.

Sol: by using the double-angle-formula

$$\sin x = 2\sin x \cos x \qquad \text{or} \qquad \sin x(1 - 2\cos x) = 0$$

Therefore there are two possibilities:

$$\sin x = 0 \qquad \text{or} \qquad 1 - 2\cos x = 0$$

$$x = 0, \pi, 2\pi \qquad\qquad \cos x = \tfrac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

The given equation has five solutions: 0, $\pi/3$, $\pi$, $5\pi/3$, and $2\pi$.