Differentiation Rules

Rule 1: Derivative of a constant

If C is constant, then $\frac{d}{dx}(c) = 0$

Ex: $\frac{d}{dx}(5) = 0$

Rule 2: The Power Rule If n is a positive integer, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Ex: if $f(x) = x^6$, then $f'(x) = 6x^5$

Rule 3: The Constant Multiple Rule. If c is a constant and f is a differentiable function, then

$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$

Ex: if $f(x) = 3x^4$, then $f'(x) = 12x^3$

Rule 4: The Sum and Difference Rules. If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

Ex: if $f(x) = x^8 - 3x^4 + 3x^2 + 10$, then $f'(x) = 8x^7 - 12x^3 + 6x$

Rule 5: The Product Rule, If f and g are both differentiable, then

$$\frac{d}{dx}[f(x).g(x)] = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$$

Ex: if $f(x) = x^2(3x^4 + 10)$, then $f'(x) = x^2(12x^3) + (3x^4 + 10)(2x) = 12x^5 + 6x^5 + 20x$

Rule 6: The Quotient Rule. If f and g are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

Ex: if $f(x) = \frac{x^2+x-2}{x^3+6}$, then

$$f'(x) = \frac{x^3+6(2x+1)-(x^2+x-2)(3x^2)}{(x^3+6)^2}$$

$$= \frac{(2x^4+x^3+12x+6)-(3x^4+3x^3-6x^2)}{(x^3+6)^2}$$

$$= \frac{-x^4 - 2x^3 + 6x^2 + 12x + 6}{(x^3 + 6)^2}$$

Second and Higher Order Derivatives

The derivative $y' = \frac{dy}{dx}$ is the first derivative of y with respect to x. If so, its derivative $y'' = (\frac{dy}{dx})^2 = \frac{dy}{dx}\frac{dy}{dx} = \frac{d^2y}{dx^2}$ is called the second derivative of y with respect to x. Likewise for $y''' = (\frac{dy}{dx})^3 = \frac{d}{dx}(\frac{d^2y}{dx^2}) = \frac{d^3y}{dx^3}$ is called the third derivative of y with respect to x etc. Finally, $y^n = \frac{d}{dx}(y)^{n-1}$ is called the n-th derivative of y with respect to x.

Ex: if $f(x) = 4x^4 + 3x^3 + 2x^2 + x + 10$ find $y', y'', y''', y^{(4)}, y^{(5)}$

The chain Rule

The Chain Rule If g is differentiable at x and f is differentiable at $g(x)$, then the composite function $F = f \circ g$ defined by $F = f(g(x))$ is differentiable at x and F' is given by the product

$$F'(x) = f'\big(g(x)\big).\, g'(x)$$

Or In Leibniz notation, if $y = f(u)$ and $u = g(x)$ are both differentiable functions, then

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Ex: Find$F'(x)$ if $F(x) = \sqrt{x^2+1}$.

Sol: let $u = x^2 + 1$ and $y = \sqrt{u}$ then

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{1}{2\sqrt{u}}(2x) = \frac{x}{\sqrt{x^2+1}}$$

The Power Rule Combined with the Chain Rule.

If n is any real number and $u = g(x)$is differentiable, then

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$$

Alternatively, $\frac{d}{dx}[g(x)]^n = n[[g(x)]^{n-1} \cdot g'(x)$

Ex: Find the derivative of the function $g(t) = (\frac{t-2}{2t+1})^9$

Sol:

$$g'(t) = 9\left(\frac{t-2}{2t+1}\right)^8\left(\frac{(2t+1)\cdot 1 - 2(t-2)}{(2t+1)^2}\right)$$

$$\frac{45(t-2)^8}{(2t+1)^{10}}$$