



## LECTURE 4

# Types of FSM

### Introduction

Every DFA is an NFA, so for every NFA we can construct an equivalent DFA, one which accepts the same language.

## 1. Deterministic (DFA) and Nondeterministic (NFA) FSM:

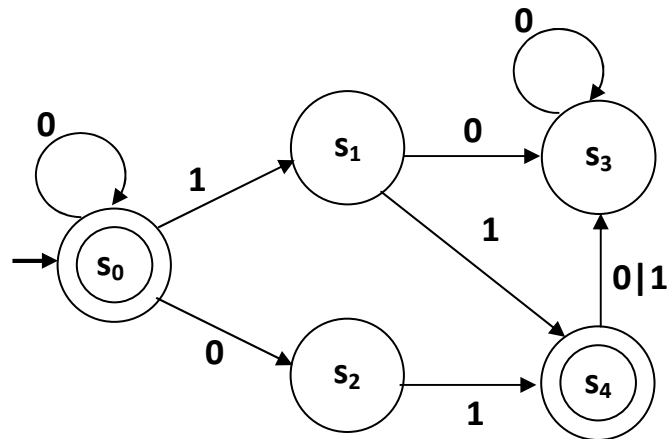
The FSM discussed so far are **deterministic**, because for each pair of state and input value there is a unique next state given by the transition function.

There is other important type of FSM in which there may be several possible next states for each pair of input value and state. Such machines are called **nondeterministic** FSM. The FSMs below describe the difference between these two types:

Type	FSMs Example	Transition Function
<b>Deterministic Finite-state Automaton (DFA)</b>	<pre> graph LR     start(( )) --&gt; s0(((s0)))     s0 -- 0 --&gt; s1((s1))     s0 -- 1 --&gt; s2(((s2)))     s1 -- 1 --&gt; s2     style start fill:none,stroke:none     </pre>	$(s_0, 0) = s_1$ $(s_0, 1) = s_2$ $(s_1, 1) = s_2$
<b>Nondeterministic Finite-state Automaton (NFA)</b>	<pre> graph LR     start(( )) --&gt; s0(((s0)))     s0 -- 0 --&gt; s1((s1))     s0 -- 0 --&gt; s2(((s2)))     s1 -- 1 --&gt; s2     style start fill:none,stroke:none     </pre>	$(s_0, 0) = s_1$ $(s_0, 0) = s_2$ $(s_1, 1) = s_2$

### Example 1:

Find the next-state table for the nondeterministic FSM with the state diagram shown below and find the language recognized by this FSM



**Solution:**

		F	
		0	1
<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">→</div> <div style="border: 1px solid black; border-radius: 50%; padding: 2px;">⊙</div> </div>	Input symbol \ State		
	s <sub>0</sub>	s <sub>0</sub> , s <sub>2</sub>	s <sub>1</sub>
	s <sub>1</sub>	s <sub>3</sub>	s <sub>4</sub>
	s <sub>2</sub>	---	s <sub>4</sub>
	s <sub>3</sub>	s <sub>3</sub>	---
⊙	s <sub>4</sub>	s <sub>3</sub>	s <sub>3</sub>

the language recognized by the machine is  $\{0^n, 0^n01, 0^n11 \mid n \geq 0\}$ .

## 2. Converting an NFA to a DFA:

The idea of conversion is that such a set of NFA states will become a single DFA state. The conversion from NFA to DFA can be achieved by following steps:

The states of DFA are subsets of state sets of the NFA.

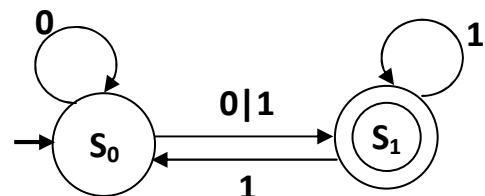
The next state of a subset, under an input symbol, is the subset containing the next states in the NFA of all elements in this subset.

The start state of DFA is the start state of NFA

The final states of DFA is the final states of NFA and any other subsets that include any final state.

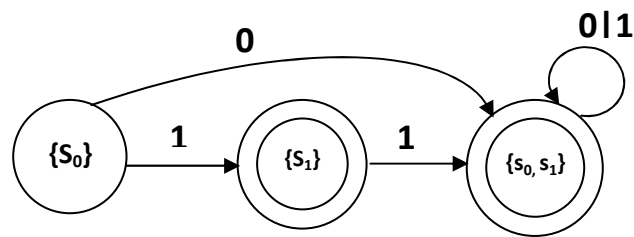
### Example 2:

Find a deterministic finite-state automaton (DFA) that equivalent to the following nondeterministic finite-state automaton (NFA).



**Solution:**

		$f$	
		0	1
→	State \ Input symbol		
	$\{s_0\}$	$\{s_0, s_1\}$	$\{s_1\}$
	$\{s_1\}$		$\{s_0, s_1\}$
	$\{s_0, s_1\}$	$\{s_0, s_1\}$	$\{s_0, s_1\}$

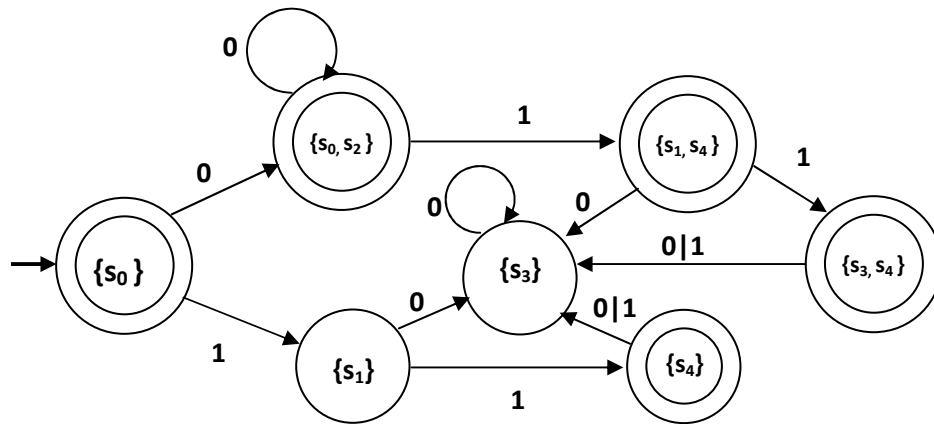


**Example 3:**

Find a DFA that recognizes the same language as the NFA in Example 1

**Solution:**

		$F$	
		0	1
→	State \ Input symbol		
	$\{s_0\}$	$\{s_0, s_2\}$	$\{s_1\}$
	$\{s_1\}$	$\{s_3\}$	$\{s_4\}$
	$\{s_2\}$	---	$\{s_4\}$
	$\{s_3\}$	$\{s_3\}$	---
	$\{s_4\}$	$\{s_3\}$	$\{s_3\}$
	$\{s_0, s_2\}$	$\{s_0, s_2\}$	$\{s_1, s_4\}$
	$\{s_1, s_4\}$	$\{s_3\}$	$\{s_4, s_3\}$
	$\{s_4, s_3\}$	$\{s_3\}$	$\{s_3\}$



### 3. NFA with $\epsilon$ - Transition:

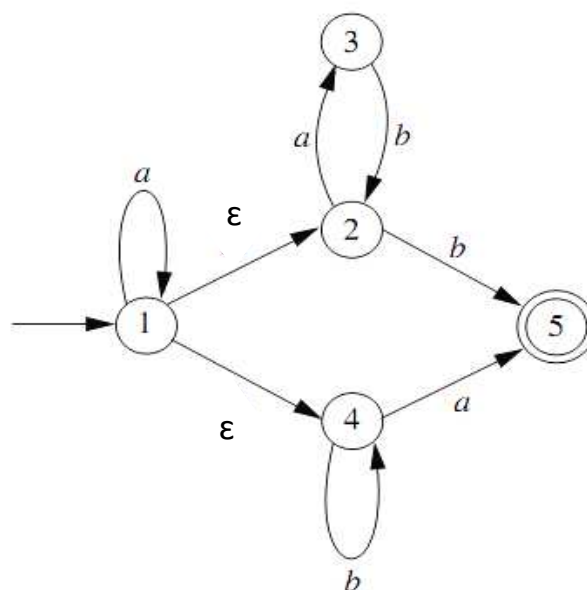
For any given input symbol, a next-state can be found by either following a transition with that symbol or by first doing any number of  $\epsilon$ -transitions and then a transition with that input symbol.

The set of all states that can be reached from states  $S$  using any number of  $\epsilon$ -transitions is called  $\epsilon$ -closure( $S$ ).

To convert an NFA with  $\epsilon$ -transitions to a DFA, we first doing any number of  $\epsilon$ -transitions and then following transitions with input symbols.

#### Example 3:

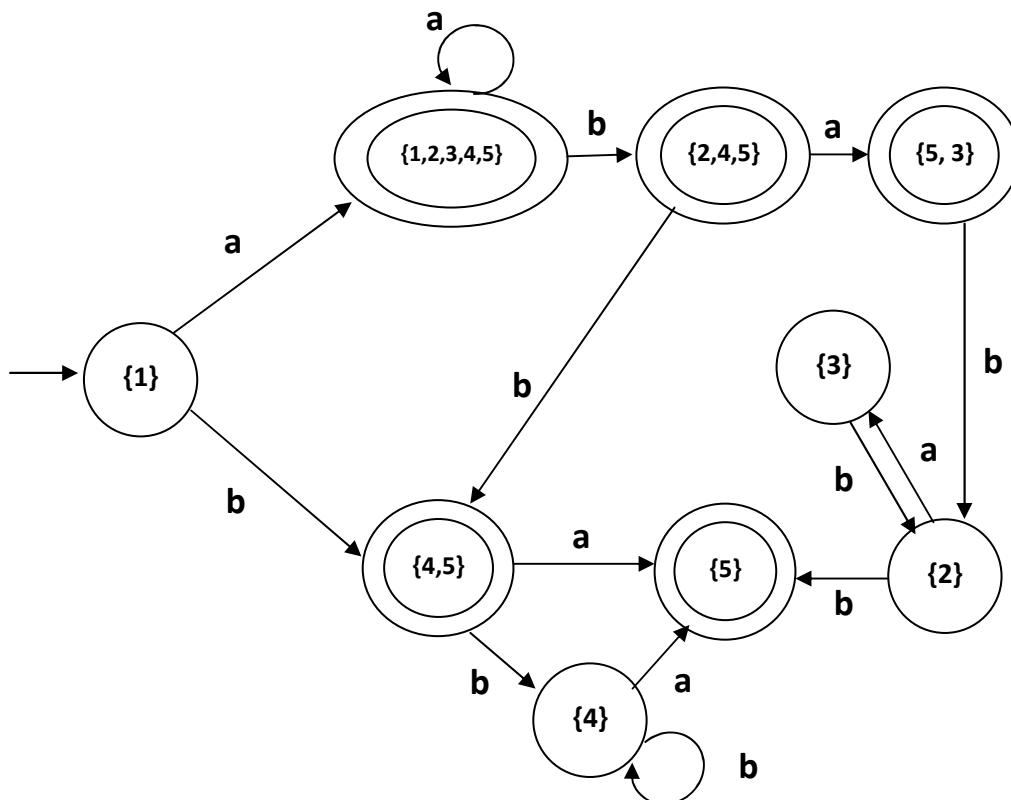
Find a DFA that recognizes the same language as the following NFA.





**Solution:**

		$F$	
		A	b
→	{1}	{1, 2, 3, 4, 5}	{4, 5}
	{2}	{3}	{5}
	{3}	---	{2}
	{4}	{5}	{4}
⊙	{5}	---	---
⊙	{1, 2, 3, 4, 5}	{1, 2, 3, 4, 5}	{4, 5, 2}
⊙	{4, 5}	{5}	{4}
⊙	{4, 5, 2}	{5, 3}	{4, 5}
⊙	{5, 3}	---	{2}





## Note:

The  $\epsilon$ -closure of the DFA states in example 3 are as follows:

$$\epsilon\text{-closure}(1) = \{1, 2, 4\}$$

$$\epsilon\text{-closure}(2) = \{2\}$$

$$\epsilon\text{-closure}(3) = \{3\}$$

$$\epsilon\text{-closure}(4) = \{4\}$$

$$\epsilon\text{-closure}(5) = \{5\}$$

## 4. Homework:

### HW 1:

Find the DFA that recognizes the same language as the following NFA.

