1 Dp

1.1 01_knapsack

```cpp
for(int i = 0; i < 100005; i++) dp[i] = dp1[i] = 205;
dp[0] = dp1[0] = 0;
for(int i = 1; i <= n; i++) {
    for(int j = 1e5+1; j >= a[i]; j--) { // 到著做回
        dp[j] = min(dp[j-a[i]] + 1, dp[j]);
    }
}
```

1.2 Josephus

```cpp
int josephus(int n, int k) { // n people, kth is killed
    if (n == 1) return 1;
    else return (josephus(n - 1, k) + k - 1) % n + 1;
        /* The position returned by josephus(n - 1, k)
        is adjusted because the recursive call
        josephus(n - 1, k) considers the
        original position k % n + 1 as position 1 */
}
```

1.3 Bitmask

```cpp
// n個城市，m個單向邊，求從1出發走到n的所有 徑數
// 遞迴版本，存反向圖
ll alln;
ll tbl[20][1<<20]; // 建表
ll dp(int i, ll vs) {
    if(tbl[i][vs]) return tbl[i][vs];
    if(vs == alln && i == 0) return 1;
    if(vs == alln || i == 0) return 0;
    ll r = 0;
    For(j, n) {
        if(!g[i][j]) continue;
        if(vs&(1<<j)) continue;
        r += dp(j, vs|(1<<j))*g[i][j];
        r %= mod;
    }
    return tbl[i][vs] = r % mod;
}

alln = (1<<n)-1;
ans = dp(n-1, 1<<(n-1))%mod; //從最後一點遞迴回去，bitmask n
    -1位為1，其餘為0

// TLE版本，迴圈版很難壓常，存正向圖
N = (1<<n)-1; // 可表示n個bit的bitmask
dp[0][1] = 1;
for(int mask = 1; mask <= N; mask++) {
    for(int i = 0; i < n; i++) {
        if(!(1 & mask>>i)) continue;
        int mask2 = mask - (1<<i);
        for(int j = 0; j < n; j++) {
            if(!(1 & mask2>>j) || g[j][i] == 0) continue;
            dp[i][mask] += dp[j][mask2]*g[j][i]; // 非簡單
                圖，可能有重複單向邊，g[i][j]存邊數
            dp[i][mask] %= mod;
        }
    }
}
cout << dp[n-1][N] % mod << '\n';
```

1.4 InfinitKnapsack

```cpp
// 找零問題
int main() {       //O(n^2)
    dp[0] = 1;
    for(ll i = 1; i <= n; i++) {
        for(ll j = a[i]; j < 30001; j++) { // 順著做過去
            dp[j] += dp[j-a[i]];
            if(dp[j-a[i]]) coin[j] = i; // 此   額當前拿力哪一
                個錢幣
        }
    }
    ll ans = dp[sum]; // sum = 所求  額
    while (sum) {
        ans.push_back(coin[sum]);
        sum -= a[coin[sum]];     // 遞迴找用過哪些錢幣
    }
}
```

2 Data Structure

2.1 DSU

```cpp
class DSU{
public:
    DSU(int n ){
        this->n = n;
        reset();
    }
    int n;
    vector<int> boss;
    vector<int> rank;
    vector<int> size;
    void reset(){
        this->boss.resize(n);
        this->rank.resize(n,0);
        this->size.resize(n,0);
        for(int i =0;i<n;i++){
            boss[i] = i;
        }
    }
    int find(int x){
        if(boss[x]!= x){
            boss[x] = find(boss[x]);
        }
        return boss[x];
    }
    int get_size(int x){
        return size[find(x)];
    }
    void merge(int x, int y){
        int a = find(x);
        int b = find(y);
        if(a!=b){
            if(rank[a]<rank[b]){
                boss[a] = b;
                size[b] += size[a];
            }else if (rank[a]<rank[b]){
                boss[b] = a;
                size[a] += size[b];
            }else{
                boss[a] = b;
                size[b] += size[a];
                rank[b]++;
            }
        }
    }
    bool aresame(int a,int b){
        return find(a)==find(b);
    }
};
```

2.2 Monotonic Queue

```cpp
class Monotonic_queue{
private:
    deque<int> qu;
public:
    void push(int n){
        while(!qu.empty()&&qu.back()<n){
            qu.pop_back();
        }
        qu.push_back(n);
    }
    int max(){
        return qu.front();
    }
    int min(){
        return qu.back();
    }
    int  size(){
        return qu.size();
    }
    void pop(){
        qu.pop_front();
    }
};
```

2.3 BIT

```cpp
class BIT{
public:
    vector<int> bit;
    int N;
    BIT(int n){
        this->N = n;
        this->bit.resize(n);
    }
    void update(int x,int d){
```

```
10        while(x<=N){
11            bit[x] +=d;
12            x +=x&(-x);// lowest bit in x;
13        }
14    }
15    int query(int x){
16        int res = 0;
17        while(x){
18            res+= bit[x];
19            x -= x& -x;
20        }
21        return res;
22    }
23 };
```

2.4 Treap

```
1 // 區間加值、反轉、rotate、刪除、插入元素、求區間
2 // srand(time(0))
3 class Treap {
4   private:
5     struct Node {
6         int pri = rand(), size = 1;
7         ll val, mn, inc = 0; bool rev = 0;
8         Node *lc = 0, *rc = 0;
9         Node(ll v) { val = mn = v; }
10    };
11    Node* root = 0;
12    void rev(Node* t) {
13        if (!t) return;
14        swap(t->lc, t->rc), t->rev ^= 1;
15    }
16    void update(Node* t, ll v) {
17        if (!t) return;
18        t->val += v, t->inc += v, t->mn += v;
19    }
20    void push(Node* t) {
21        if (t->rev) rev(t->lc), rev(t->rc), t->rev = 0;
22        update(t->lc, t->inc), update(t->rc, t->inc);
23        t->inc = 0;
24    }
25    void pull(Node* t) {
26        t->size = 1 + size(t->lc) + size(t->rc);
27        t->mn = t->val;
28        if (t->lc) t->mn = min(t->mn, t->lc->mn);
29        if (t->rc) t->mn = min(t->mn, t->rc->mn);
30    }
31    void discard(Node* t) { // 看要不要釋放記憶體
32        if (!t) return;
33        discard(t->lc), discard(t->rc);
34        delete t;
35    }
36    void split(Node* t, Node*& a, Node*& b, int k) {
37        if (!t) return a = b = 0, void();
38        push(t);
39        if (size(t->lc) < k) {
40            a = t;
41            split(t->rc, a->rc, b, k - size(t->lc) - 1);
42            pull(a);
43        } else {
44            b = t;
45            split(t->lc, a, b->lc, k);
46            pull(b);
```

```
47        }
48    }
49    Node* merge(Node* a, Node* b) {
50        if (!a || !b) return a ? a : b;
51        if (a->pri > b->pri) {
52            push(a);
53            a->rc = merge(a->rc, b);
54            pull(a);
55            return a;
56        } else {
57            push(b);
58            b->lc = merge(a, b->lc);
59            pull(b);
60            return b;
61        }
62    }
63    inline int size(Node* t) { return t ? t->size : 0; }
64  public:
65    int size() { return size(root); }
66    void add(int l, int r, ll val) {
67        Node *a, *b, *c, *d;
68        split(root, a, b, r);
69        split(a, c, d, l - 1);
70        update(d, val);
71        root = merge(merge(c, d), b);
72    }
73    // 反轉區間 [l, r]
74    void reverse(int l, int r) {
75        Node *a, *b, *c, *d;
76        split(root, a, b, r);
77        split(a, c, d, l - 1);
78        swap(d->lc, d->rc);
79        d->rev ^= 1;
80        root = merge(merge(c, d), b);
81    }
82    // 區間 [l, r] 向右 rotate k 次，k < 0 表向左 rotate
83    void rotate(int l, int r, int k) {
84        int len = r - l + 1;
85        Node *a, *b, *c, *d, *e, *f;
86        split(root, a, b, r);
87        split(a, c, d, l - 1);
88        k = (k + len) % len;
89        split(d, e, f, len - k);
90        root = merge(merge(c, merge(f, e)), b);
91    }
92    // 插入一個元素 val 使其 index = i <= size
93    void insert(int i, ll val) {
94        if (i == size() + 1) {
95            push_back(val); return;
96        }
97        assert(i <= size());
98        Node *a, *b;
99        split(root, a, b, i - 1);
100       root = merge(merge(a, new Node(val)), b);
101   }
102   void push_back(ll val) {
103       root = merge(root, new Node(val));
104   }
105   void remove(int l, int r) {
106       int len = r - l + 1;
107       Node *a, *b, *c, *d;
108       split(root, a, b, l - 1);
109       split(b, c, d, len);
110       discard(c); // 看你要不要釋放記憶體
111       root = merge(a, d);
```

```
112       }
113   ll minn(int l, int r) {
114       Node *a, *b, *c, *d;
115       split(root, a, b, r);
116       split(a, c, d, l - 1);
117       int ans = d->mn;
118       root = merge(merge(c, d), b);
119       return ans;
120   }
121 };
```

2.5 Segment Tree

```
1 class SegmentTree{
2 private:
3     const int n;
4     const vl arr;
5     // vl st;
6     vl summ;
7     vl minn;
8     vl maxx;
9     vl tag;
10    void pull(int l,int r,int v){
11        if(r-l==1)
12            return;
13        // st[v]=st[2*v+1]+st[2*v+2];
14        int mid=(l+r)/2;
15        push(l,mid,2*v+1);
16        push(mid,r,2*v+2);
17        summ[v]=summ[2*v+1]+summ[2*v+2];
18        // minn[v]=min(minn[2*v+1],minn[2*v+2]);
19        // maxx[v]=max(maxx[2*v+1],minn[2*v+2]);
20    }
21    void push(int l,int r,int v){
22        summ[v]+=tag[v]*(r-l);
23        if(r-l==1)
24            return tag[v]=0,void();
25        tag[2*v+1]+=tag[v];
26        tag[2*v+2]+=tag[v];
27        tag[v]=0;
28    }
29    void build(int l,int r,int v=0){
30        if(r-l==1){
31            summ[v]=arr[l];
32            // summ[v]=minn[v]=maxx[v]=arr[l];
33            return;
34        }
35        int mid=(l+r)/2;
36        build(l,mid,2*v+1);
37        build(mid,r,2*v+2);
38        pull(l,r,v);
39    }
40
41 public:
42    SegmentTree(vl&_arr,int _n):arr(_arr),n(_n){
43        assert(arr.size()==n);
44        summ.assign(4*n,0);
45        // minn.assign(4*n,1e9);
46        // maxx.assign(4*n,-1e9);
47        tag.assign(4*n,0);
48        build(0,arr.size());
49    }
50    void modify(int x,int val,int l,int r,int v=0){
```

```
51        }
52        }
53        // query sum
54        loli query(int L,int R,int l,int r,int v=0){
55            // dbn(L,R,l,r,v)
56            push(l,r,v);
57            if(l==L && R==r){
58                return summ[v];
59                return minn[v];
60                return maxx[v];
61            }
62            int mid=(l+r)/2;
63            if(R<=mid)
64                return query(L,R,l,mid,2*v+1);
65            else if(mid<=L)
66                return query(L,R,mid,r,2*v+2);
67            else
68                return query(L,mid,l,mid,2*v+1)+query(mid,R,mid,r
                    ,2*v+2);
69        }
70        // plus `val` to every element in [L,R)
71        void update(int L,int R,loli val,int l,int r,int v=0){
72            // dbn(L,R,l,r,v)
73            push(l,r,v);
74            if(l==L && R==r){
75                tag[v]+=val;
76                push(l,r,v);
77                return;
78            }
79            int mid=(l+r)/2;
80            if(R<=mid)
81                update(L,R,val,l,mid,2*v+1);
82            else if(mid<=L)
83                update(L,R,val,mid,r,2*v+2);
84            else
85                update(L,mid,val,l,mid,2*v+1),update(mid,R,val,
                    mid,r,2*v+2);
86            pull(l,r,v);
87        }
88    };
89
90    void solve(){
91        int n,q;
92        cin>>n>>q;
93        vl arr(n);
94        for(auto&x:arr)
95            cin>>x;
96        SegmentTree st(arr,n);
97        while(q--){
98            int op=0;
99            // str op;
100           cin>>op;
101           if(op&1){
102               loli l,r,val;
103               cin>>l>>r>>val;
104               assert(r>=l);
105               st.update(l-1,r,val,0,n);
106               // loli k,u;
107               // cin>>k>>u;
108               // st.update(k-1,k,u-arr[k-1],0,n);
109               // arr[k-1]=u;
110           }else{
111               int x,y;
112               cin>>x>>y;
113               assert(y>=x);
114               cout<<st.query(x-1,y,0,n)<<endl;
```

```
115           }
116       }
117   }
```

2.6 Sparse Table

```
1   int a[N], sp[__lg(N) + 1][N]{};
2   void init(int n) { //0-based
3       for (int i = 0; i < n; ++i) {
4           sp[0][i] = a[i];
5       }
6       for (int i = 0; i < __lg(n); ++i) {
7           for (int j = 0; j+(1<<i) < n; ++j) {
8               sp[i + 1][j] = max(sp[i][j], sp[i][j+(1<<i)]);
9           }
10      }
11  }
12  int query(int l, int r) { //[l, r]
13      int p = __lg(r - l + 1);
14      return max(sp[p][l], sp[p][r-(1<<p)+1]);
15  }
```

2.7 Monotonic Stack

```
1   vector<int> monotonic_stack(vector<int> nums){
2       int n = nums.size();
3       vector<int> res(n);
4       stack<int>  st;
5       for(int i = n-1;i>=0;i--){
6           while(!st.empty() && st.top()<=nums[i]){
7               st.pop();
8           }
9           if(st.empty())res[i] = -1;
10          else res[i] = st.top();
11          st.push(nums[i]);
12      }
13      return res;
14  }
```

3 Flow

3.1 Maximum Simple Graph Matching

```
1   struct GenMatch { // 1-base
2       int V, pr[N];
3       bool el[N][N], inq[N], inp[N], inb[N];
4       int st, ed, nb, bk[N], djs[N], ans;
5       void init(int _V) {
6           V = _V;
7           for (int i = 0; i <= V; ++i) {
8               for (int j = 0; j <= V; ++j) el[i][j] = 0;
9               pr[i] = bk[i] = djs[i] = 0;
10              inq[i] = inp[i] = inb[i] = 0;
11          }
```

```
12      }
13      void add_edge(int u, int v) {
14          el[u][v] = el[v][u] = 1;
15      }
16      int lca(int u, int v) {
17          fill_n(inp, V + 1, 0);
18          while (1)
19              if (u = djs[u], inp[u] = true, u == st) break;
20              else u = bk[pr[u]];
21          while (1)
22              if (v = djs[v], inp[v]) return v;
23              else v = bk[pr[v]];
24          return v;
25      }
26      void upd(int u) {
27          for (int v; djs[u] != nb;) {
28              v = pr[u], inb[djs[u]] = inb[djs[v]] = true;
29              u = bk[v];
30              if (djs[u] != nb) bk[u] = v;
31          }
32      }
33      void blo(int u, int v, queue<int> &qe) {
34          nb = lca(u, v), fill_n(inb, V + 1, 0);
35          upd(u), upd(v);
36          if (djs[u] != nb) bk[u] = v;
37          if (djs[v] != nb) bk[v] = u;
38          for (int tu = 1; tu <= V; ++tu)
39              if (inb[djs[tu]])
40                  if (djs[tu] = nb, !inq[tu])
41                      qe.push(tu), inq[tu] = 1;
42      }
43      void flow() {
44          fill_n(inq + 1, V, 0), fill_n(bk + 1, V, 0);
45          iota(djs + 1, djs + V + 1, 1);
46          queue<int> qe;
47          qe.push(st), inq[st] = 1, ed = 0;
48          while (!qe.empty()) {
49              int u = qe.front();
50              qe.pop();
51              for (int v = 1; v <= V; ++v)
52                  if (el[u][v] && djs[u] != djs[v] &&
53                      pr[u] != v) {
54                      if ((v == st) ||
55                          (pr[v] > 0 && bk[pr[v]] > 0))
56                          blo(u, v, qe);
57                      else if (!bk[v]) {
58                          if (bk[v] = u, pr[v] > 0) {
59                              if (!inq[pr[v]]) qe.push(pr[v]);
60                          } else
61                              return ed = v, void();
62                      }
63                  }
64          }
65      }
66      void aug() {
67          for (int u = ed, v, w; u > 0;)
68              v = bk[u], w = pr[v], pr[v] = u, pr[u] = v,
69              u = w;
70      }
71      int solve() {
72          fill_n(pr, V + 1, 0), ans = 0;
73          for (int u = 1; u <= V; ++u)
74              if (!pr[u])
75                  if (st = u, flow(), ed > 0) aug(), ++ans;
76          return ans;
77      }
```

```
78 };
```

3.2 Dinic

```
1  #define maxn 2005
2  #define INF 0x3f3f3f3f
3  struct MaxFlow{
4      struct edge{
5          int  to, cap, flow,rev;
6          edge( int v, int c, int f,int r) :  to(v), cap(c),
                  flow(f),rev(r) {}
7      };
8      vector<edge> G[maxn];
9      int s,t,dis[maxn],cur[maxn],vis[maxn];
10     void add_edge(int from,int to,int cap){
11         G[from].push_back(edge(to,cap,0,G[to].size()));
12         G[to].push_back(edge(from,0,0,G[from].size()-1));
13     }
14     bool bfs(){
15         memset(dis, -1, sizeof(dis));
16         queue<int> qu;
17         qu.push(s);
18         dis[s] = 0;
19         while (!qu.empty()) {
20             int from = qu.front();
21             qu.pop();
22             for (auto &e: G[from]) {
23                 if (dis[e.to]==-1 && e.cap != e.flow) {
24                     dis[e.to] = dis[from] + 1;
25                     qu.push(e.to);
26                 }
27             }
28         }
29         return dis[t]!=-1;
30     }
31     int dfs(int from,int cap){
32         if(from==t ||cap==0)return cap;
33         for(int &i = cur[from];i<G[from].size();i++){
34             edge &e = G[from][i];
35             if(dis[e.to]==dis[from]+1 && e.flow!=e.cap){
36                 int df = dfs(e.to,min(e.cap-e.flow,cap));
37                 if(df){
38                     e.flow+=df;
39                     G[e.to][e.rev].flow-=df;
40                     return df;
41                 }
42             }
43         }
44         dis[from] = -1;
45         return 0;
46     }
47     int Maxflow(int s,int t){
48         this->s = s,this->t =t;
49         int flow = 0;
50         int df;
51         while(bfs()){
52             memset(cur,0,sizeof(cur));
53             while(df = dfs(s,INF)){
54                 flow +=df;
55             }
56         }
57         return flow;
58     }
```

```
59 };
60 int main(){
61     int n = 4,m = 6;
62     MaxFlow maxflow;
63     for(int i =0;i<m;i++){
64         int a,b,cap;
65         cin >>a>>b>>cap;
66         maxflow.add_edge(a,b,cap);
67     }
68     cout << maxflow.Maxflow(1,3)<<endl;;
69 }
```

4 Formula

4.1 formula

4.1.1 Pick 公式

給定頂點坐標均是整點的簡單多邊形，面積 = 內部格點數 + 邊上格點數/2-1

4.1.2 圖論

1. 對於平面圖，$F = E - V + C + 1$，C 是連通分 數
2. 對於平面圖，$E \le 3V - 6$
3. 對於連通圖 G，最大獨立點集的大小設為 I(G)，最大匹配大小設為 M(G)，最小點覆蓋設為 Cv(G)，最小邊覆蓋設為 Ce(G)。對於任意連通圖：

   (a) $I(G) + Cv(G) = |V|$
   (b) $M(G) + Ce(G) = |V|$

4. 對於連通二分圖：

   (a) $I(G) = Cv(G)$
   (b) $M(G) = Ce(G)$

5. 最大權閉合圖：

   (a) $C(u,v) = \infty, (u,v) \in E$
   (b) $C(S,v) = W_v, W_v > 0$
   (c) $C(v,T) = -W_v, W_v < 0$
   (d) ans=$\sum_{W_v > 0} W_v - flow(S,T)$

6. 最大密度子圖：

   (a) 求 $max\left(\frac{W_e + W_v}{|V'|}\right), e \in E', v \in V'$
   (b) $U = \sum_{v \in V} 2W_v + \sum_{e \in E} W_e$
   (c) $C(u,v) = W_{(u,v)}, (u,v) \in E$，雙向邊
   (d) $C(S,v) = U, v \in V$
   (e) $D_u = \sum_{(u,v) \in E} W_{(u,v)}$
   (f) $C(v,T) = U + 2g - D_v - 2W_v, v \in V$
   (g) 二分搜 g：
       $l = 0, r = U, eps = 1/n^2$
       if$((U \times |V| - flow(S,T))/2 > 0)$ $l = mid$
       else $r = mid$
   (h) ans=$min\_cut(S,T)$
   (i) $|E| = 0$ 要特殊判斷

7. 弦圖：

   (a) 點數大於 3 的環都要有一條弦

   (b) 完美消除序 從後往前依次給每個點染色，給每個點染上可以染的最小顏色
   (c) 最大團天小 = 色數
   (d) 最大獨立集：完美消除序 從前往後能選就選
   (e) 最小團覆蓋：最大獨立集的點和他延伸的邊構成
   (f) 區間圖是弦圖
   (g) 區間圖的完美消除序 ：將區間按造又端點由小到大排序
   (h) 區間圖染色：用線段樹做

4.1.3 dinic 特殊圖複雜度

1. 單位流：$O\left(min\left(V^{3/2}, E^{1/2}\right)E\right)$
2. 二分圖：$O\left(V^{1/2}E\right)$

4.1.4 0-1 分數規劃

$x_i = \{0,1\}$，x_i 可能會有其他限制，求 $max\left(\frac{\sum B_i x_i}{\sum C_i x_i}\right)$

1. $D(i,g) = B_i - g \times C_i$
2. $f(g) = \sum D(i,g)x_i$
3. $f(g) = 0$ 時 g 為最佳解，$f(g) < 0$ 沒有意義
4. 因為 $f(g)$ 單調可以二分搜 g
5. 或用 Dinkelbach 通常比較快

```
1  binary_search(){
2      while(r-l>eps){
3          g=(l+r)/2;
4          for(i:所有元素)D[i]=B[i]-g*C[i];//D(i,g)
5          找出一組合法x[i]使f(g)最大;
6          if(f(g)>0) l=g;
7          else r=g;
8      }
9      Ans = r;
10 }
11 Dinkelbach(){
12     g=任意 態(通常設為0);
13     do{
14         Ans=g;
15         for(i:所有元素)D[i]=B[i]-g*C[i];//D(i,g)
16         找出一組合法x[i]使f(g)最大;
17         p=0,q=0;
18         for(i:所有元素)
19             if(x[i])p+=B[i],q+=C[i];
20         g=p/q;// 新解，注意q=0的情況
21     }while(abs(Ans-g)>EPS);
22     return Ans;
23 }
```

4.1.5 學長公式

1. $\sum_{d|n} \phi(n) = n$
2. $g(n) = \sum_{d|n} f(d) => f(n) = \sum_{d|n} \mu(d) \times g(n/d)$
3. Harmonic series $H_n = \ln(n) + \gamma + 1/(2n) - 1/(12n^2) + 1/(120n^4)$
4. $\gamma = 0.5772156649015328606065120900824024310421 5$
5. 格雷碼 $= n \oplus (n >> 1)$
6. $SG(A + B) = SG(A) \oplus SG(B)$
7. 旋轉矩陣 $M(\theta) = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$

4.1.6 基本數論

1. $\sum_{d|n} \mu(n) = [n == 1]$
2. $g(m) = \sum_{d|m} f(d) \Leftrightarrow f(m) = \sum_{d|m} \mu(d) \times g(m/d)$
3. $\sum_{i=1}^{n} \sum_{j=1}^{m}$ 互質數 $= \sum \mu(d) \lfloor \frac{n}{d} \rfloor \lfloor \frac{m}{d} \rfloor$
4. $\sum_{i=1}^{n} \sum_{j=1}^{n} lcm(i,j) = n \sum_{d|n} d \times \phi(d)$

4.1.7 排組公式

1. k 卡特 $\frac{C_n^{kn}}{n(k-1)+1}$, $C_m^n = \frac{n!}{m!(n-m)!}$
2. $H(n,m) \cong x_1 + x_2 \ldots + x_n = k, num = C_k^{n+k-1}$
3. Stirling number of 2nd, n 人分 k 組方法數目

   (a) $S(0,0) = S(n,n) = 1$
   (b) $S(n,0) = 0$
   (c) $S(n,k) = kS(n-1,k) + S(n-1,k-1)$

4. Bell number, n 人分任意多組方法數目

   (a) $B_0 = 1$
   (b) $B_n = \sum_{i=0}^{n} S(n,i)$
   (c) $B_{n+1} = \sum_{k=0}^{n} C_k^n B_k$
   (d) $B_{p+n} \equiv B_n + B_{n+1} mod p$, p is prime
   (e) $B_{p^m+n} \equiv mB_n + B_{n+1} mod p$, p is prime
   (f) From $B_0 : 1, 1, 2, 5, 15, 52,$
       203, 877, 4140, 21147, 115975

5. Derangement, 錯排, 沒有人在自己位置上

   (a) $D_n = n!(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} \ldots + (-1)^n \frac{1}{n!})$
   (b) $D_n = (n-1)(D_{n-1} + D_{n-2}), D_0 = 1, D_1 = 0$
   (c) From $D_0 : 1, 0, 1, 2, 9, 44,$
       265, 1854, 14833, 133496

6. Binomial Equality

   (a) $\sum_k \binom{r}{m+k}\binom{s}{n-k} = \binom{r+s}{m+n}$
   (b) $\sum_k \binom{l}{m+k}\binom{s}{n+k} = \binom{l+s}{l-m+n}$
   (c) $\sum_k \binom{l}{m+k}\binom{s+k}{n}(-1)^k = (-1)^{l+m}\binom{s-m}{n-l}$
   (d) $\sum_{k \le l} \binom{l-k}{m}\binom{s}{k-n}(-1)^k = (-1)^{l+m}\binom{s-m-1}{l-n-m}$
   (e) $\sum_{0 \le k \le l} \binom{l-k}{m}\binom{q+k}{n} = \binom{l+q+1}{m+n+1}$
   (f) $\binom{r}{k} = (-1)^k \binom{k-r-1}{k}$
   (g) $\binom{r}{m}\binom{m}{k} = \binom{r}{k}\binom{r-k}{m-k}$
   (h) $\sum_{k \le n} \binom{r+k}{k} = \binom{r+n+1}{n}$
   (i) $\sum_{0 \le k \le n} \binom{k}{m} = \binom{n+1}{n}$
   (j) $\sum_{k \le m} \binom{m+r}{k} x^k y^k = \sum_{k \le m} \binom{-r}{k}(-x)^k(x+y)^{m-k}$

4.1.8 冪次, 冪次和

1. $a^b \% P = a^{b\%\varphi(p)+\varphi(p)}, b \ge \varphi(p)$
2. $1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$
3. $1^4 + 2^4 + 3^4 + \ldots + n^4 = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30}$
4. $1^5 + 2^5 + 3^5 + \ldots + n^5 = \frac{n^6}{6} + \frac{n^5}{2} + \frac{5n^4}{12} - \frac{n^2}{12}$
5. $0^k + 1^k + 2^k + \ldots + n^k = P(k), P(k) = \frac{(n+1)^{k+1} - \sum_{i=0}^{k-1} C_i^{k+1} P(i)}{k+1}, P(0) = n+1$
6. $\sum_{k=0}^{m-1} k^n = \frac{1}{n+1} \sum_{k=0}^{n} C_k^{n+1} B_k m^{n+1-k}$

7. $\sum_{j=0}^{m} C_j^{m+1} B_j = 0, B_0 = 1$
8. 除 $B_1 = -1/2$, 剩下的奇數項都是 0
9. $B_2 = 1/6, B_4 = -1/30, B_6 = 1/42, B_8 = -1/30, B_{10} = 5/66, B_{12} = -691/2730, B_{14} = 7/6, B_{16} = -3617/510, B_{18} = 43867/798, B_{20} = -174611/330,$

4.1.9 Burnside's lemma

1. $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$
2. $X^g = t^{c(g)}$
3. G 表示有幾種轉法, X^g 表示在那種轉法下, 有幾種是會保持對稱的, t 是顏色數, $c(g)$ 是循環節不動的面數。
4. 正立方體塗三顏色, 轉 0 有 3^6 個元素不變, 轉 90 有 6 種, 每種有 3^3 不變, 180 有 3×3^4, 120 (角) 有 8×3^2, 180 (邊) 有 6×3^3, 全部 $\frac{1}{24}(3^6 + 6 \times 3^3 + 3 \times 3^4 + 8 \times 3^2 + 6 \times 3^3) = 57$

4.1.10 Count on a tree

1. Rooted tree: $s_{n+1} = \frac{1}{n} \sum_{i=1}^{n}(i \times a_i \times \sum_{j=1}^{\lfloor n/i \rfloor} a_{n+1-i \times j})$
2. Unrooted tree:

   (a) Odd: $a_n - \sum_{i=1}^{n/2} a_i a_{n-i}$
   (b) Even: $Odd + \frac{1}{2} a_{n/2}(a_{n/2} + 1)$

3. Spanning Tree

   (a) 完全圖 n^{n-2}
   (b) 一般圖 (Kirchhoff's theorem) $M[i][i] = degree(V_i), M[i][j] = -1$, if have $E(i,j)$, 0 if no edge. delete any one row and col in A, $ans = det(A)$

5 Geometry

5.1 Sort by Angle

```
bool cmp(pii a, pii b) {
#define is_neg(k) (k.y < 0 || (k.y == 0 && k.x < 0));
  int A = is_neg(a), B = is_neg(b);
  if (A != B)
    return A < B;
  if (cross(a, b) == 0)
    return (a.x*a.x + a.y*a.y) < (b.x*b.x + b.y*b.y);
  return cross(a, b) > 0;
}
```

5.2 Geometry

```
const double PI=atan2(0.0,-1.0);
template<typename T>
struct point{
  T x,y;
  point(){}
  point(const T&x,const T&y):x(x),y(y){}
  point operator+(const point &b)const{
    return point(x+b.x,y+b.y); }
  point operator-(const point &b)const{
    return point(x-b.x,y-b.y); }
  point operator*(const T &b)const{
    return point(x*b,y*b); }
  point operator/(const T &b)const{
    return point(x/b,y/b); }
  bool operator==(const point &b)const{
    return x==b.x&&y==b.y; }
  T dot(const point &b)const{
    return x*b.x+y*b.y; }
  T cross(const point &b)const{
    return x*b.y-y*b.x; }
  point normal()const{//求法向
    return point(-y,x); }
  T abs2()const{//向  長度的平方
    return dot(*this); }
  T rad(const point &b)const{//  向  的弧度
return fabs(atan2(fabs(cross(b)),dot(b))); }
  T getA()const{//對x軸的弧度
    T A=atan2(y,x);//超過180度會變負的
    if(A<=-PI/2)A+=PI*2;
    return A;
  }
};
template<typename T>
struct line{
  line(){}
  point<T> p1,p2;
  T a,b,c;//ax+by+c=0
  line(const point<T>&x,const point<T>&y):p1(x),p2(y){}
  void pton(){//轉成一般式
    a=p1.y-p2.y;
    b=p2.x-p1.x;
    c=-a*p1.x-b*p1.y;
  }
  T ori(const point<T> &p)const{//點和有向直線的關係，>0左
      邊、=0在線上<0右邊
    return (p2-p1).cross(p-p1);
  }
  T btw(const point<T> &p)const{//點投影  在線段上<=0
    return (p1-p).dot(p2-p);
  }
  bool point_on_segment(const point<T>&p)const{//點是否在線段
      上
    return ori(p)==0&&btw(p)<=0;
  }
  T dis2(const point<T> &p,bool is_segment=0)const{//點跟直線
      /線段的距離平方
    point<T> v=p2-p1,v1=p-p1;
    if(is_segment){
      point<T> v2=p-p2;
      if(v.dot(v1)<=0)return v1.abs2();
      if(v.dot(v2)>=0)return v2.abs2();
    }
    T tmp=v.cross(v1);
    return tmp*tmp/v.abs2();
  }
  T seg_dis2(const line<T> &l)const{//  線段距離平方
    return min({dis2(l.p1,1),dis2(l.p2,1),l.dis2(p1,1),l.dis2
        (p2,1)});
  }
```

```cpp
 66   point<T> projection(const point<T> &p)const{//點對直線的投
           影
 67     point<T> n=(p2-p1).normal();
 68     return p-n*(p-p1).dot(n)/n.abs2();
 69   }
 70   point<T> mirror(const point<T> &p)const{
 71     //點對直線的鏡射，要先呼叫pton轉成一般式
 72     point<T> R;
 73     T d=a*a+b*b;
 74     R.x=(b*b*p.x-a*a*p.x-2*a*b*p.y-2*a*c)/d;
 75     R.y=(a*a*p.y-b*b*p.y-2*a*b*p.x-2*b*c)/d;
 76     return R;
 77   }
 78   bool equal(const line &l)const{//直線相等
 79     return ori(l.p1)==0&&ori(l.p2)==0;
 80   }
 81   bool parallel(const line &l)const{
 82     return (p1-p2).cross(l.p1-l.p2)==0;
 83   }
 84   bool cross_seg(const line &l)const{
 85     return (p2-p1).cross(l.p1-p1)*(p2-p1).cross(l.p2-p1)<=0;
           //直線是否交線段
 86   }
 87   int line_intersect(const line &l)const{//直線相交情況，-1無
           限多點、1交於一點、0不相交
 88     return parallel(l)?(ori(l.p1)==0?-1:0):1;
 89   }
 90   int seg_intersect(const line &l)const{
 91     T c1=ori(l.p1), c2=ori(l.p2);
 92     T c3=l.ori(p1), c4=l.ori(p2);
 93     if(c1==0&&c2==0){//共線
 94       bool b1=btw(l.p1)>=0,b2=btw(l.p2)>=0;
 95       T a3=l.btw(p1),a4=l.btw(p2);
 96       if(b1&&b2&&a3==0&&a4>=0) return 2;
 97       if(b1&&b2&&a3>=0&&a4==0) return 3;
 98       if(b1&&b2&&a3>=0&&a4>=0) return 0;
 99       return -1;//無限交點
100     }else if(c1*c2<=0&&c3*c4<=0)return 1;
101     return 0;//不相交
102   }
103   point<T> line_intersection(const line &l)const{/*直線交點*/
104     point<T> a=p2-p1,b=l.p2-l.p1,s=l.p1-p1;
105     //if(a.cross(b)==0)return INF;
106     return p1+a*(s.cross(b)/a.cross(b));
107   }
108   point<T> seg_intersection(const line &l)const{//線段交點
109     int res=seg_intersect(l);
110     if(res<=0) assert(0);
111     if(res==2) return p1;
112     if(res==3) return p2;
113     return line_intersection(l);
114   }
115 };
116 template<typename T>
117 struct polygon{
118   polygon(){}
119   vector<point<T> > p;//逆時針順序
120   T area()const{//面積
121     T ans=0;
122     for(int i=p.size()-1,j=0;j<(int)p.size();i=j++)
123       ans+=p[i].cross(p[j]);
124     return ans/2;
125   }
```

```cpp
126   point<T> center_of_mass()const{//重心
127     T cx=0,cy=0,w=0;
128     for(int i=p.size()-1,j=0;j<(int)p.size();i=j++){
129       T a=p[i].cross(p[j]);
130       cx+=(p[i].x+p[j].x)*a;
131       cy+=(p[i].y+p[j].y)*a;
132       w+=a;
133     }
134     return point<T>(cx/3/w,cy/3/w);
135   }
136   char ahas(const point<T>& t)const{//點是否在簡單多邊形內，
           是的話回傳1、在邊上回傳-1、否則回傳0
137     bool c=0;
138     for(int i=0,j=p.size()-1;i<p.size();j=i++){
139       if(line<T>(p[i],p[j]).point_on_segment(t))return -1;
140       else if((p[i].y>t.y)!=(p[j].y>t.y)&&
141       t.x<(p[j].x-p[i].x)*(t.y-p[i].y)/(p[j].y-p[i].y)+p[i].x
             )
142         c=!c;
143     return c;
144   }
145   char point_in_convex(const point<T>&x)const{
146     int l=1,r=(int)p.size()-2;
147     while(l<=r){//點是否在凸多邊形內，是的話回傳1、在邊上回傳
             -1、否則回傳0
148       int mid=(l+r)/2;
149       T a1=(p[mid]-p[0]).cross(x-p[0]);
150       T a2=(p[mid+1]-p[0]).cross(x-p[0]);
151       if(a1>=0&&a2<=0){
152         T res=(p[mid+1]-p[mid]).cross(x-p[mid]);
153         return res>0?1:(res>=0?-1:0);
154       }else if(a1<0)r=mid-1;
155       else l=mid+1;
156     }
157     return 0;
158   }
159   vector<T> getA()const{//凸包邊對x軸的夾角
160     vector<T>res;//一定是遞增的
161     for(size_t i=0;i<p.size();++i)
162       res.push_back((p[(i+1)%p.size()]-p[i]).getA());
163     return res;
164   }
165   bool line_intersect(const vector<T>&A,const line<T> &l)
           const{//O(logN)
166     int f1=upper_bound(A.begin(),A.end(),(l.p1-l.p2).getA())-
             A.begin();
167     int f2=upper_bound(A.begin(),A.end(),(l.p2-l.p1).getA())-
             A.begin();
168     return l.cross_seg(line<T>(p[f1],p[f2]));
169   }
170   polygon cut(const line<T> &l)const{//凸包對直線切割，得到直
           線l左側的凸包
171     polygon ans;
172     for(int n=p.size(),i=n-1,j=0;j<n;i=j++){
173       if(l.ori(p[i])>=0){
174         ans.p.push_back(p[i]);
175         if(l.ori(p[j])<0)
176           ans.p.push_back(l.line_intersection(line<T>(p[i],p[
                 j])));
177       }else if(l.ori(p[j])>0)
178         ans.p.push_back(l.line_intersection(line<T>(p[i],p[j
                 ])));
179     }
180     return ans;
```

```cpp
181   }
182   static bool graham_cmp(const point<T>& a,const point<T>& b)
           {//凸包排序函數
183     return (a.x<b.x)||(a.x==b.x&&a.y<b.y);
184   }
185   void graham(vector<point<T> > &s){//凸包
186     sort(s.begin(),s.end(),graham_cmp);
187     p.resize(s.size()+1);
188     int m=0;
189     for(size_t i=0;i<s.size();++i){
190       while(m>=2&&(p[m-1]-p[m-2]).cross(s[i]-p[m-2])<=0)--m;
191       p[m++]=s[i];
192     }
193     for(int i=s.size()-2,t=m+1;i>=0;--i){
194       while(m>=t&&(p[m-1]-p[m-2]).cross(s[i]-p[m-2])<=0)--m;
195       p[m++]=s[i];
196     }
197     if(s.size()>1)--m;
198     p.resize(m);
199   }
200   T diam(){//直徑
201     int n=p.size(),t=1;
202     T ans=0;p.push_back(p[0]);
203     for(int i=0;i<n;i++){
204       point<T> now=p[i+1]-p[i];
205       while(now.cross(p[t+1]-p[i])>now.cross(p[t]-p[i]))t=(t
             +1)%n;
206       ans=max(ans,(p[i]-p[t]).abs2());
207     }
208     return p.pop_back(),ans;
209   }
210   T min_cover_rectangle(){//最小覆蓋矩形
211     int n=p.size(),t=1,r=1,l;
212     if(n<3)return 0;//也可以做最小周長矩形
213     T ans=1e99;p.push_back(p[0]);
214     for(int i=0;i<n;i++){
215       point<T> now=p[i+1]-p[i];
216       while(now.cross(p[t+1]-p[i])>now.cross(p[t]-p[i]))t=(t
             +1)%n;
217       while(now.dot(p[r+1]-p[i])>now.dot(p[r]-p[i]))r=(r+1)%n
             ;
218       if(!i)l=r;
219       while(now.dot(p[l+1]-p[i])<=now.dot(p[l]-p[i]))l=(l+1)%
             n;
220       T d=now.abs2();
221       T tmp=now.cross(p[t]-p[i])*(now.dot(p[r]-p[i])-now.dot(
             p[l]-p[i]))/d;
222       ans=min(ans,tmp);
223     }
224     return p.pop_back(),ans;
225   }
226   T dis2(polygon &p1){//凸包最近距離平方
227     vector<point<T> > &P=p,&Q=p1.p;
228     int n=P.size(),m=Q.size(),l=0,r=0;
229     for(int i=0;i<n;++i)if(P[i].y<P[l].y)l=i;
230     for(int i=0;i<m;++i)if(Q[i].y<Q[r].y)r=i;
231     P.push_back(P[0]),Q.push_back(Q[0]);
232     T ans=1e99;
233     for(int i=0;i<n;++i){
234       while((P[l]-P[l+1]).cross(Q[r+1]-Q[r])<0)r=(r+1)%m;
235       ans=min(ans,line<T>(P[l],P[l+1]).seg_dis2(line<T>(Q[r],
             Q[r+1])));
236       l=(l+1)%n;
237     }
238     return P.pop_back(),Q.pop_back(),ans;
```

```cpp
239      }
240      static char sign(const point<T>&t){
241          return (t.y==0?t.x:t.y)<0;
242      }
243      static bool angle_cmp(const line<T>& A,const line<T>& B){
244          point<T> a=A.p2-A.p1,b=B.p2-B.p1;
245          return sign(a)<sign(b)||(sign(a)==sign(b)&&a.cross(b)>0);
246      }
247      int halfplane_intersection(vector<line<T> > &s){//半平面交
248          sort(s.begin(),s.end(),angle_cmp);//線段左側為該線段半平
                 面
249          int L,R,n=s.size();
250          vector<point<T> > px(n);
251          vector<line<T> > q(n);
252          q[L=R=0]=s[0];
253          for(int i=1;i<n;++i){
254              while(L<R&&s[i].ori(px[R-1])<=0)--R;
255              while(L<R&&s[i].ori(px[L])<=0)++L;
256              q[++R]=s[i];
257              if(q[R].parallel(q[R-1])){
258                  --R;
259                  if(q[R].ori(s[i].p1)>0)q[R]=s[i];
260              }
261              if(L<R)px[R-1]=q[R-1].line_intersection(q[R]);
262          }
263          while(L<R&&q[L].ori(px[R-1])<=0)--R;
264          p.clear();
265          if(R-L<=1)return 0;
266          px[R]=q[R].line_intersection(q[L]);
267          for(int i=L;i<=R;++i)p.push_back(px[i]);
268          return R-L+1;
269      }
270  };
271  template<typename T>
272  struct triangle{
273      point<T> a,b,c;
274      triangle(){}
275      triangle(const point<T> &a,const point<T> &b,const point<T>
             &c):a(a),b(b),c(c){}
276      T area()const{
277          T t=(b-a).cross(c-a)/2;
278          return t>0?t:-t;
279      }
280      point<T> barycenter()const{//重心
281          return (a+b+c)/3;
282      }
283      point<T> circumcenter()const{//外心
284          static line<T> u,v;
285          u.p1=(a+b)/2;
286          u.p2=point<T>(u.p1.x-a.y+b.y,u.p1.y+a.x-b.x);
287          v.p1=(a+c)/2;
288          v.p2=point<T>(v.p1.x-a.y+c.y,v.p1.y+a.x-c.x);
289          return u.line_intersection(v);
290      }
291      point<T> incenter()const{//內心
292          T A=sqrt((b-c).abs2()),B=sqrt((a-c).abs2()),C=sqrt((a-b).
                 abs2());
293          return point<T>(A*a.x+B*b.x+C*c.x,A*a.y+B*b.y+C*c.y)/(A+B
                 +C);
294      }
295      point<T> perpencenter()const{//垂心
296          return barycenter()*3-circumcenter()*2;
297      }
298  };
299  template<typename T>
```

```cpp
300  struct point3D{
301      T x,y,z;
302      point3D(){}
303      point3D(const T&x,const T&y,const T&z):x(x),y(y),z(z){}
304      point3D operator+(const point3D &b)const{
305          return point3D(x+b.x,y+b.y,z+b.z);}
306      point3D operator-(const point3D &b)const{
307          return point3D(x-b.x,y-b.y,z-b.z);}
308      point3D operator*(const T &b)const{
309          return point3D(x*b,y*b,z*b);}
310      point3D operator/(const T &b)const{
311          return point3D(x/b,y/b,z/b);}
312      bool operator==(const point3D &b)const{
313          return x==b.x&&y==b.y&&z==b.z;}
314      T dot(const point3D &b)const{
315          return x*b.x+y*b.y+z*b.z;}
316      point3D cross(const point3D &b)const{
317          return point3D(y*b.z-z*b.y,z*b.x-x*b.z,x*b.y-y*b.x);}
318      T abs2()const{//向量長度的平方
319          return dot(*this);}
320      T area2(const point3D &b)const{//和b、原點圍成面積的平方
321          return cross(b).abs2()/4;}
322  };
323  template<typename T>
324  struct line3D{
325      point3D<T> p1,p2;
326      line3D(){}
327      line3D(const point3D<T> &p1,const point3D<T> &p2):p1(p1),p2
             (p2){}
328      T dis2(const point3D<T> &p,bool is_segment=0)const{//點跟直
                 線/線段的距離平方
329          point3D<T> v=p2-p1,v1=p-p1;
330          if(is_segment){
331              point3D<T> v2=p-p2;
332              if(v.dot(v1)<=0)return v1.abs2();
333              if(v.dot(v2)>=0)return v2.abs2();
334          }
335          point3D<T> tmp=v.cross(v1);
336          return tmp.abs2()/v.abs2();
337      }
338      pair<point3D<T>,point3D<T> > closest_pair(const line3D<T> &
             l)const{
339          point3D<T> v1=(p1-p2),v2=(l.p1-l.p2);
340          point3D<T> N=v1.cross(v2),ab(p1-l.p1);
341          //if(N.abs2()==0)return NULL;平行或重合
342          T tmp=N.dot(ab),ans=tmp*tmp/N.abs2();//最近點對距離
343          point3D<T> d1=p2-p1,d2=l.p2-l.p1,D=d1.cross(d2),G=l.p1-p1
                 ;
344          T t1=(G.cross(d2)).dot(D)/D.abs2();
345          T t2=(G.cross(d1)).dot(D)/D.abs2();
346          return make_pair(p1+d1*t1,l.p1+d2*t2);
347      }
348      bool same_side(const point3D<T> &a,const point3D<T> &b)
             const{
349          return (p2-p1).cross(a-p1).dot((p2-p1).cross(b-p1))>0;
350      }
351  };
352  template<typename T>
353  struct plane{
354      point3D<T> p0,n;//平面上的點和法向
355      plane(){}
356      plane(const point3D<T> &p0,const point3D<T> &n):p0(p0),n(n)
             {}
357      T dis2(const point3D<T> &p)const{//點到平面距離的平方
```

```cpp
358          T tmp=(p-p0).dot(n);
359          return tmp*tmp/n.abs2();
360      }
361      point3D<T> projection(const point3D<T> &p)const{
362          return p-n*(p-p0).dot(n)/n.abs2();
363      }
364      point3D<T> line_intersection(const line3D<T> &l)const{
365          T tmp=n.dot(l.p2-l.p1);//等於0表示平行或重合該平面
366          return l.p1+(l.p2-l.p1)*(n.dot(p0-l.p1)/tmp);
367      }
368      line3D<T> plane_intersection(const plane &p1)const{
369          point3D<T> e=n.cross(p1.n),v=n.cross(e);
370          T tmp=p1.n.dot(v);//等於0表示平行或重合該平面
371          point3D<T> q=p0+(v*(p1.n.dot(p1.p0-p0))/tmp);
372          return line3D<T>(q,q+e);
373      }
374  };
375  template<typename T>
376  struct triangle3D{
377      point3D<T> a,b,c;
378      triangle3D(){}
379      triangle3D(const point3D<T> &a,const point3D<T> &b,const
             point3D<T> &c):a(a),b(b),c(c){}
380      bool point_in(const point3D<T> &p)const{//點在該平面上的投
                 影在三角形中
381          return line3D<T>(b,c).same_side(p,a)&&line3D<T>(a,c).
                 same_side(p,b)&&line3D<T>(a,b).same_side(p,c);
382      }
383  };
384  template<typename T>
385  struct tetrahedron{//四面體
386      point3D<T> a,b,c,d;
387      tetrahedron(){}
388      tetrahedron(const point3D<T> &a,const point3D<T> &b,const
             point3D<T> &c,const point3D<T> &d):a(a),b(b),c(c),d(d)
             {}
389      T volume6()const{//體積的六倍
390          return (d-a).dot((b-a).cross(c-a));
391      }
392      point3D<T> centroid()const{
393          return (a+b+c+d)/4;
394      }
395      bool point_in(const point3D<T> &p)const{
396          return triangle3D<T>(a,b,c).point_in(p)&&triangle3D<T>(c,
                 d,a).point_in(p);
397      }
398  };
399  template<typename T>
400  struct convexhull3D{
401      static const int MAXN=1005;
402      struct face{
403          int a,b,c;
404          face(int a,int b,int c):a(a),b(b),c(c){}
405      };
406      vector<point3D<T>> pt;
407      vector<face> ans;
408      int fid[MAXN][MAXN];
409      void build(){
410          int n=pt.size();
411          ans.clear();
412          memset(fid,0,sizeof(fid));
413          ans.emplace_back(0,1,2);//注意不能共線
414          ans.emplace_back(2,1,0);
415          int ftop = 0;
```

```
416    for(int i=3, ftop=1; i<n; ++i,++ftop){
417      vector<face> next;
418      for(auto &f:ans){
419        T d=(pt[i]-pt[f.a]).dot((pt[f.b]-pt[f.a]).cross(pt[f.
             c]-pt[f.a]));
420        if(d<=0) next.push_back(f);
421        int ff=0;
422        if(d>0) ff=ftop;
423        else if(d<0) ff=-ftop;
424        fid[f.a][f.b]=fid[f.b][f.c]=fid[f.c][f.a]=ff;
425      }
426      for(auto &f:ans){
427        if(fid[f.a][f.b]>0 && fid[f.a][f.b]!=fid[f.b][f.a])
428          next.emplace_back(f.a,f.b,i);
429        if(fid[f.b][f.c]>0 && fid[f.b][f.c]!=fid[f.c][f.b])
430          next.emplace_back(f.b,f.c,i);
431        if(fid[f.c][f.a]>0 && fid[f.c][f.a]!=fid[f.a][f.c])
432          next.emplace_back(f.c,f.a,i);
433      }
434      ans=next;
435    }
436  }
437  point3D<T> centroid()const{
438    point3D<T> res(0,0,0);
439    T vol=0;
440    for(auto &f:ans){
441      T tmp=pt[f.a].dot(pt[f.b].cross(pt[f.c]));
442      res=res+(pt[f.a]+pt[f.b]+pt[f.c])*tmp;
443      vol+=tmp;
444    }
445    return res/(vol*4);
446  }
447 };
```

5.3 Convex Hull

```
1  using pdd = pair<double, double>;
2  #define F first
3  #define S second
4  pdd operator-(pdd a, pdd b) {
5    return {a.F - b.F, a.S - b.S};
6  }
7  double cross(pdd a, pdd b) {
8    return a.F * b.S - a.S * b.F;
9  }
10 void solve() {
11   int n;
12   cin >> n;
13   vector<pdd> pnts;
14   for (int i = 0; i < n; ++i) {
15     double x, y;
16     cin >> x >> y;
17     pnts.push_back(x, y);
18   }
19   sort(iter(pnts));
20   vector<pdd> hull;
21   for (int i = 0; i < 2; ++i) {
22     int t = hull.size();
23     for (pdd j: pnts) {
24       while(hull.size() - t >= 2 && cross(j - hull[hull.size
               () - 2], hull.back() - hull[hull.size() - 2]) >=
               0)
25         hull.pop_back();
```

```
26       hull.push_back(j);
27     }
28     hull.pop_back();
29     reverse(iter(pnts));
30   }
31   double area = 0;
32   for (int i=0; i < hull.size(); ++i){
33     area += cross(hull[i], hull[(i + 1) % hull.size()]);
34   }
35   area /= 2.0;
36 }
```

5.4 Min Covering Circle

```
1  double dis(pdd a, pdd b) {
2    double dx = a.x - b.x, dy = a.y - b.y;
3    return sqrt(dx*dx + dy*dy);
4  }
5  double sq(double x) {
6    return x * x;
7  }
8  pdd excenter(pdd p1, pdd p2, pdd p3) {
9    double a1 = p1.x - p2.x, a2 = p1.x - p3.x;
10   double b1 = p1.y - p2.y, b2 = p1.y - p3.y;
11   double c1 = (sq(p1.x) - sq(p2.x) + sq(p1.y) - sq(p2.y)) /
           2;
12   double c2 = (sq(p1.x) - sq(p3.x) + sq(p1.y) - sq(p3.y)) /
           2;
13   double dd = a1*b2 - a2*b1;
14   return {(c1*b2 - c2*b1) / dd, (a1*c2 - a2*c1) / dd};
15 }
16 void solve(pdd a[], int n) {
17   shuffle(a, a + n, rng);
18   pdd center = a[0];
19   double r = 0;
20   for (int i = 1; i < n; ++i) {
21     if (dis(center, a[i]) <= r) continue;
22     center = a[i], r = 0;
23     for (int j = 0; j < i; ++j) {
24       if (dis(center, a[j]) <= r) continue;
25       center.x = (a[i].x + a[j].x) / 2;
26       center.y = (a[i].y + a[j].y) / 2;
27       r = dis(center, a[i]);
28       for (int k = 0; k < j; ++k) {
29         if (dis(center, a[k]) <= r) continue;
30         center = excenter(a[i], a[j], a[k]);
31         r = dis(center, a[i]);
32       }
33     }
34   }
35   cout << fixed << setprecision(10) << r << '\n';
36   cout << center.x << ' ' << center.y << '\n';
37 }
```

5.5 Point in Polygon

```
1  const ll inf = 2000000000;
2  struct Point {
3    ll x, y;
4    Point(ll x = 0, ll y = 0):x(x), y(y){}
```

```
5    Point operator+(const Point p) const {
6      return Point(x + p.x, y + p.y); }
7    Point operator-(const Point p) const {
8      return Point(x - p.x, y - p.y); }
9    ll operator*(const Point p) const { //dot
10     return x * p.x + y * p.y; }
11   ll operator^(const Point p) const { //cross
12     return x * p.y - y * p.x; }
13 };
14 bool onseg(Point a, Point b, Point o) {
15   return ((a - o) ^ (b - o)) == 0 && ((a - o) * (b - o)) <=
           0;
16 }
17 int ori(Point a, Point b, Point o) {
18   ll w = (a - o) ^ (b - o);
19   return (w ? (w > 0 ? 1 : -1) : 0);
20 }
21 bool inters(Point a, Point b, Point c, Point d) {
22   if (onseg(a, b, c) || onseg(a, b, d)) return 1;
23   if (onseg(c, d, a) || onseg(c, d, b)) return 1;
24   if (ori(a, b, c) * ori(a, b, d) < 0 && ori(c, d, a) * ori(c
           , d, b) < 0) return 1;
25   return 0;
26 }
27 Point poly[maxn];
28 void solve(int n, Point p) {
29   poly[n] = poly[0];
30   int cnt = 0;
31   for (int i = 0; i < n; ++i) {
32     if (onseg(poly[i], poly[i + 1], p)) {
33       cnt = -1;
34       break;
35     }
36     if (inters(poly[i], poly[i + 1], p, Point(inf, p.y))) {
37       ++cnt;
38     }
39     Point hi = (poly[i].y > poly[i + 1].y ? poly[i] : poly[i
             + 1]);
40     if (hi.y == p.y && hi.x > p.x) {
41       --cnt;
42     }
43   }
44   if (cnt < 0)
45     cout << "BOUNDARY\n";
46   else if (cnt % 2)
47     cout << "INSIDE\n";
48   else
49     cout << "OUTSIDE\n";
50 }
```

6 Graph

6.1 Bipartite Matching

```
1  const int MAXN = 100;
2
3  struct Bipartite_matching{
4      int mx[MAXN], my[MAXN], vy[MAXN]; //matchX, matchY,
             visitY
5      vector<int> edge[MAXN]; //adjcent list;
6      int x_cnt;
```

```cpp
 7      bool dfs(int x){
 8          for(auto y: edge[x]){ //對 x 可以碰到的邊進 檢查
 9              if(vy[y] == 1) continue; //避免遞迴 error
10
11              vy[y] = 1;
12              if(my[y] == -1 || dfs(my[y])){ //分析 3
13                  mx[x] = y;
14                  my[y] = x;
15                  return true;
16              }
17          }
18          return false; //分析 4
19      }
20
21      int bipartite_matching(){
22          memset(mx, -1, sizeof(mx)); //分析 1,2
23          memset(my, -1, sizeof(my));
24          int ans = 0;
25          for(int i = 0; i < x_cnt; i++){  //對每一個 x 節點進
                     DFS(最大匹配)
26              memset(vy, 0, sizeof(vy));
27              if(dfs(i)) ans++;
28          }
29          return ans;
30      }
31      vector<vector<int>> get_match(){
32          vector<vector<int>> res;
33          for(int i =0 ;i<x_cnt;i++){
34              if(mx[i]!=-1){
35                  res.push_back({i,mx[i]});
36              }
37          }
38          return res;
39      }
40      void add_edge(int i,int j){
41          edge[i].push_back(j);
42      }
43      void init(int x){
44          x_cnt = x;
45      }
46 };
47 int main(){
48      int n,m;
49      Bipartite_matching bm;
50      for(int i = 0;i<m;i++){
51          int a , b;cin >>a>>b;
52          bm.add_edge(a,b);
53      }
54      bm.init(n);
55      cout << bm.bipartite_matching()<<endl;
56      auto match = bm.get_match();
57      for(auto t: match){
58          cout << t[0]<<" "<<t[1]<<endl;
59      }
60
61 }
```

6.2 Tarjan SCC

```cpp
 1 const int n = 16;
 2 vector<vector<int>> graph;
 3 int visit[n], low[n],t = 0;
 4 int st[n], top =0;
 5 bool instack[n];
 6 int contract[n]; // 每個點收縮到的點
 7 vector<vector<int>> block;
 8 void dfs(int x,int parent){
 9     // cout <<x<<endl;
10     visit[x] = low[x] = ++t;
11   st[top++] = x;
12   instack[x] = true;
13      for(auto to: graph[x]){
14          if(!visit[to])
15              dfs(to,x);
16
17          if(instack[to])
18              low[x] = min(low[x],low[to]);
19      }
20      if(visit[x]==low[x]){ //scc 裡最早拜訪的
21          int j;
22          block.push_back({});
23          do{
24              j = st[--top];
25              instack[j] = false;
26              block[block.size()-1].push_back(j);
27              contract[j] =x;
28          }while(j!=x);
29      }
30 }
31 int main(){
32      for(int i =0;i<n;i++){
33          if (!visit[i])
34          dfs(i, i);
35      }
36      for(auto t: block){
37          for(auto x:t){
38              cout << x <<" ";
39          }cout <<endl;
40      }
41 }
```

6.3 Bridge

```cpp
 1 const int n = 9;
 2 vector<vector<int>> graph;
 3 vector<int> visit(n, 0);
 4 vector<int> trace(n, 0);
 5 vector<vector<int>> bridge;
 6 int t = 0;
 7 void dfs(int x, int parent){
 8     visit[x] = ++t;
 9     trace[x] = x; // 最高祖先預設為自己
10      for (auto to : graph[x]){
11          if (visit[to]){ // back edge
12              if (to != parent){
13                  trace[x] = to;
14              }
15          }
16          else{ // treeedge
17              dfs(to, x);
18              if (visit[trace[to]] < visit[trace[x]])
19                  trace[x] = trace[to];
20
21              // 子樹回不到祖先暨自身。
22              if (visit[trace[to]] > visit[x])
23                  bridge.push_back({x, to});
24          }
25      }
26 }//call for()dfs(i,-1)
27 int main(){
28      for(int i =0;i<9;i++){
29          if(!visit[i])
30              dfs(i,-1);
31      }
32      for(auto x: bridge){
33          cout << x[0]<<" "<< x[1]<<endl;
34      }
35 }
```

6.4 2 SAT

```cpp
 1 class TwoSAT{
 2 public:
 3     TwoSAT(int n) : n(n), graph(2 * n), visited(2 * n, false)
            {}
 4     void addClause(int a, int b) {// 0-base;
 5         a *=2;
 6         b *=2;
 7         // Add implications (~a => b) and (~b => a)
 8         graph[a ^ 1].push_back(b);
 9         graph[b ^ 1].push_back(a);
10     }
11     bool solve() {// Find SCCs and check for contradictions
12         for (int i = 0; i < 2 * n; ++i) {
13             if (!visited[i]) {
14                 dfs1(i);
15             }
16         }
17         reverse(processingOrder.begin(), processingOrder.end
                ());//topological sort
18         for (int i = 0; i < 2 * n; ++i) {
19             visited[i] = false;
20         }
21         for (int node : processingOrder) {
22             if (!visited[node]) {
23                 scc.clear();
24                 dfs2(node);
25                 if (!checkSCCConsistency()) {
26                     return false;
27                 }
28             }
29         }
30
31         return true;
32     }
33
34 private:
35     int n;
36     vector<vector<int>> graph;
37     vector<bool> visited;
38     vector<int> processingOrder;
39     vector<int> scc;
40
41     void dfs1(int node) {
42         visited[node] = true;
43         for (int neighbor : graph[node]) {
44             if (!visited[neighbor]) {
```

```cpp
45              dfs1(neighbor);
46          }
47      }
48      processingOrder.push_back(node);
49  }
50
51  void dfs2(int node) {
52      visited[node] = true;
53      scc.push_back(node);
54      for (int neighbor : graph[node]) {
55          if (!visited[neighbor]) {
56              dfs2(neighbor);
57          }
58      }
59  }
60
61  bool checkSCCConsistency() {
62      for (int node : scc) {
63          if (find(scc.begin(), scc.end(), node ^ 1) != scc
                .end()) {
64              return false; // Contradiction found in the
                    same SCC
65          }
66      }
67      return true;
68  }
69 };
70 int main() {
71     int n, m;// Number of variables and clauses
72     TwoSAT twoSat(n);
73     for (int i = 0; i < m; ++i) {
74         int a, b;
75         twoSat.addClause(a, b);
76     }
77     if (twoSat.solve()) {
78         cout << "Satisfiable" << endl;
79     } else {
80         cout << "Unsatisfiable" << endl;
81     }
82 }
```

6.5 Kosaraju 2DFS

```cpp
1  const int n = 16;
2  vector<vector<int>> graph;
3  vector<vector<int>> reverse_graph;
4  int visit[n];
5  int contract[n]; // 每個點收縮到的點
6  vector<vector<int>> block;
7  vector<int> finish;//fake topological sort
8  // need graph and reverse praph
9  void dfs1(int x){
10     visit[x] = true;
11     for(auto to:graph[x]){
12         if(!visit[to]){
13             dfs1(to);
14         }
15     }
16     finish.push_back(x);
17 }
18 void dfs2(int x,int c){
19     contract[x] = c;
20     block[c].push_back(x);
```

```cpp
21     visit[x] = true;
22     for(auto to:reverse_graph[x]){
23         if(!visit[to]){
24             dfs2(to,c);
25         }
26     }
27 }
28 int main(){
29     graph = {};
30     reverse_graph = {};
31
32     for(int i =0;i<n;i++){
33         if (!visit[i])
34       dfs1(i);
35     }
36     int c =0;
37     memset(visit,0,sizeof(visit));
38     for(int i =  n-1;i>=0;i--){
39         if(!visit[finish[i]]){
40             block.push_back({});
41             dfs2(finish[i],c++);
42         }
43     }
44     for(auto t: block){
45         for(auto x:t){
46             cout << x <<" ";
47         }cout <<endl;
48     }
49 }
```

6.6 Minimum Steiner Tree

```cpp
1  // Minimum Steiner Tree
2  // O(V 3^T + V^2 2^T)
3  struct SteinerTree { // 0-base
4    static const int T = 10, N = 105, INF = 1e9;
5    int n, dst[N][N], dp[1 << T][N], tdst[N];
6    int vcost[N]; // the cost of vertexs
7    void init(int _n) {
8      n = _n;
9      for (int i = 0; i < n; ++i) {
10       for (int j = 0; j < n; ++j) dst[i][j] = INF;
11       dst[i][i] = vcost[i] = 0;
12     }
13   }
14   void add_edge(int ui, int vi, int wi) {
15     dst[ui][vi] = min(dst[ui][vi], wi);
16   }
17   void shortest_path() {
18     for (int k = 0; k < n; ++k)
19       for (int i = 0; i < n; ++i)
20         for (int j = 0; j < n; ++j)
21           dst[i][j] =
22             min(dst[i][j], dst[i][k] + dst[k][j]);
23   }
24   int solve(const vector<int> &ter) {
25     shortest_path();
26     int t = SZ(ter);
27     for (int i = 0; i < (1 << t); ++i)
28       for (int j = 0; j < n; ++j) dp[i][j] = INF;
29     for (int i = 0; i < n; ++i) dp[0][i] = vcost[i];
30     for (int msk = 1; msk < (1 << t); ++msk) {
31       if (!(msk & (msk - 1))) {
```

```cpp
32         int who = __lg(msk);
33         for (int i = 0; i < n; ++i)
34           dp[msk][to] =
35             vcost[ter[who]] + dst[ter[who]][i];
36       }
37       for (int i = 0; i < n; ++i)
38         for (int submsk = (msk - 1) & msk; submsk;
39               submsk = (submsk - 1) & msk)
40           dp[msk][i] = min(dp[msk][i],
41             dp[submsk][i] + dp[msk ^ submsk][i] -
42               vcost[i]);
43       for (int i = 0; i < n; ++i) {
44         tdst[i] = INF;
45         for (int j = 0; j < n; ++j)
46           tdst[i] =
47             min(tdst[i], dp[msk][j] + dst[j][i]);
48       }
49       for (int i = 0; i < n; ++i) dp[msk][i] = tdst[i];
50     }
51     int ans = INF;
52     for (int i = 0; i < n; ++i)
53       ans = min(ans, dp[(1 << t) - 1][i]);
54     return ans;
55   }
56 };
```

6.7 Dijkstra

```cpp
1  #define maxn 200005
2  vector<int> dis(maxn,-1);
3  vector<int> parent(maxn,-1);
4  vector<bool> vis(maxn,false);
5  vector<vector<pair<int,int>>> graph;
6  void dijsktra(int source){
7      dis[source] =0 ;
8
9      priority_queue<pair<int,int>,vector<pair<int,int>>,
          greater<pair<int,int>>> pq;
10     pq.push({0,source});
11     while(!pq.empty()){
12         int from  = pq.top().second;
13         pq.pop();
14         // cout <<vis[from]<<endl;
15         if(vis[from])continue;
16         vis[from] = true;
17         for(auto next : graph[from]){
18             int to = next.second;
19             int weight = next.first;
20             // cout <<from<<' ' <<to<<' ' <<weight;
21             if(dis[from]+weight< dis[to] || dis[to]==-1){
22                 dis[to] = dis[from]+weight;
23                 parent[to] = from;
24                 pq.push({dis[from]+weight,to});
25             }
26         }
27     }
28 }
29 int main(){
30     int startpoint;
31     dijsktra(startpoint);
32     //dis and parent
33 }
```

6.8 Maximum Clique Dyn

```cpp
const int N = 150;
struct MaxClique { // Maximum Clique
  bitset<N> a[N], cs[N];
  int ans, sol[N], q, cur[N], d[N], n;
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; i++) a[i].reset();
  }
  void addEdge(int u, int v) { a[u][v] = a[v][u] = 1; }
  void csort(vector<int> &r, vector<int> &c) {
    int mx = 1, km = max(ans - q + 1, 1), t = 0,
        m = r.size();
    cs[1].reset(), cs[2].reset();
    for (int i = 0; i < m; i++) {
      int p = r[i], k = 1;
      while ((cs[k] & a[p]).count()) k++;
      if (k > mx) mx++, cs[mx + 1].reset();
      cs[k][p] = 1;
      if (k < km) r[t++] = p;
    }
    c.resize(m);
    if (t) c[t - 1] = 0;
    for (int k = km; k <= mx; k++)
      for (int p = cs[k]._Find_first(); p < N;
           p = cs[k]._Find_next(p))
        r[t] = p, c[t] = k, t++;
  }
  void dfs(vector<int> &r, vector<int> &c, int l,
    bitset<N> mask) {
    while (!r.empty()) {
      int p = r.back();
      r.pop_back(), mask[p] = 0;
      if (q + c.back() <= ans) return;
      cur[q++] = p;
      vector<int> nr, nc;
      bitset<N> nmask = mask & a[p];
      for (int i : r)
        if (a[p][i]) nr.push_back(i);
      if (!nr.empty()) {
        if (l < 4) {
          for (int i : nr)
            d[i] = (a[i] & nmask).count();
          sort(nr.begin(), nr.end(),
            [&](int x, int y) { return d[x] > d[y]; });
        }
        csort(nr, nc), dfs(nr, nc, l + 1, nmask);
      } else if (q > ans) ans = q, copy_n(cur, q, sol);
      c.pop_back(), q--;
    }
  }
  int solve(bitset<N> mask = bitset<N>(
              string(N, '1'))) { // vertex mask
    vector<int> r, c;
    ans = q = 0;
    for (int i = 0; i < n; i++)
      if (mask[i]) r.push_back(i);
    for (int i = 0; i < n; i++)
      d[i] = (a[i] & mask).count();
    sort(r.begin(), r.end(),
      [&](int i, int j) { return d[i] > d[j]; });
    csort(r, c), dfs(r, c, 1, mask);
    return ans; // sol[0 ~ ans-1]
  }
```

```cpp
} graph;
```

6.9 Minimum Clique Cover

```cpp
struct Clique_Cover { // 0-base, O(n2^n)
  int co[1 << N], n, E[N];
  int dp[1 << N];
  void init(int _n) {
    n = _n, fill_n(dp, 1 << n, 0);
    fill_n(E, n, 0), fill_n(co, 1 << n, 0);
  }
  void add_edge(int u, int v) {
    E[u] |= 1 << v, E[v] |= 1 << u;
  }
  int solve() {
    for (int i = 0; i < n; ++i)
      co[1 << i] = E[i] | (1 << i);
    co[0] = (1 << n) - 1;
    dp[0] = (n & 1) * 2 - 1;
    for (int i = 1; i < (1 << n); ++i) {
      int t = i & -i;
      dp[i] = -dp[i ^ t];
      co[i] = co[i ^ t] & co[t];
    }
    for (int i = 0; i < (1 << n); ++i)
      co[i] = (co[i] & i) == i;
    fwt(co, 1 << n);
    for (int ans = 1; ans < n; ++ans) {
      int sum = 0;
      for (int i = 0; i < (1 << n); ++i)
        sum += (dp[i] *= co[i]);
      if (sum) return ans;
    }
    return n;
  }
};
```

6.10 Floyd Warshall

```cpp
#define maxn 2005
vector<vector<int>> dis(maxn,vector<int>(maxn,9999999));
vector<vector<int>> mid(maxn,vector<int>(maxn,-1));
vector<vector<pair<int,int>>> graph;

void floyd_warshall(int n ){ // n is n nodes
  for(int i =0;i<n;i++){
      for(auto path:graph[i]){
          dis[i][path.second] = path.first;
      }
  }
  for (int i=0; i<n; i++)
    dis[i][i] = 0;
  for (int k=0; k<n; k++){
    for (int i=0; i<n; i++){
      for (int j=0; j<n; j++){
        if (dis[i][k] + dis[k][j] < dis[i][j] || dis[i][j
            ]==-1){
          dis[i][j] = dis[i][k] + dis[k][j];
          mid[i][j] = k;  // 由 i 點走到 j 點經過   k 點
```

```cpp
      }
    }
  }
}
void find_path(int s, int t){   // 印出最短   徑
  if (mid[s][t] == -1) return;  // 沒有中繼點就結束
  find_path(s, mid[s][t]);      // 前半段最短   徑
  cout << mid[s][t];            // 中繼點
  find_path(mid[s][t], t);      // 後半段最短   徑
}
int main(){
    int n;
    floyd_warshall(n);
    for(int i =0;i<4;i++){
        for(int j = 0 ; j <4;j++)
            cout << dis[i][j]<<" ";
        cout << endl;
    }
    find_path(0,2);
}
```

6.11 Articulation Vertex

```cpp
const int n = 9;
int t = 0;
vector<int> disc(n, -1);       // Discovery time
vector<int> low(n, -1);        // Low time
vector<int> parent_array(n, -1); // Parent in DFS tree
vector<bool> visited(n, false);
vector<bool> is_articulation(n, false);
vector<vector<int>> graph;
void dfs_articulation(int node, int parent){
    visited[node] = true;
    disc[node] = t;
    low[node] = t;
    t++;
    int children = 0;

    for (int neighbor : graph[node])
    {
        if (!visited[neighbor])
        {
            children++;
            parent_array[neighbor] = node;
            dfs_articulation(neighbor, node);
            low[node] = min(low[node], low[neighbor]);

            if (low[neighbor] >= disc[node] && parent != -1)
            {
                is_articulation[node] = true;
            }
        }
        else if (neighbor != parent)
        {
            low[node] = min(low[node], disc[neighbor]);
        }
    }

    if (parent == -1 && children > 1)
    {
        is_articulation[node] = true;
```

```
39        }
40 }//call for() dfs(i,-1)
41 int main(){
42     for (int i = 0; i < n; ++i) {
43         if (!visited[i]) {
44             dfs_articulation(i, -1);
45         }
46     }
47     cout << "Articulation Points: ";
48     for (int i = 0; i < n; ++i) {
49         if (is_articulation[i]) {
50             cout << i << " ";
51         }
52     }cout << endl;
53 }
```

6.12 Number of Maximal Clique

```
 1 struct BronKerbosch { // 1-base
 2   int n, a[N], g[N][N];
 3   int S, all[N][N], some[N][N], none[N][N];
 4   void init(int _n) {
 5     n = _n;
 6     for (int i = 1; i <= n; ++i)
 7       for (int j = 1; j <= n; ++j) g[i][j] = 0;
 8   }
 9   void add_edge(int u, int v) {
10     g[u][v] = g[v][u] = 1;
11   }
12   void dfs(int d, int an, int sn, int nn) {
13     if (S > 1000) return; // pruning
14     if (sn == 0 && nn == 0) ++S;
15     int u = some[d][0];
16     for (int i = 0; i < sn; ++i) {
17       int v = some[d][i];
18       if (g[u][v]) continue;
19       int tsn = 0, tnn = 0;
20       copy_n(all[d], an, all[d + 1]);
21       all[d + 1][an] = v;
22       for (int j = 0; j < sn; ++j)
23         if (g[v][some[d][j]])
24           some[d + 1][tsn++] = some[d][j];
25       for (int j = 0; j < nn; ++j)
26         if (g[v][none[d][j]])
27           none[d + 1][tnn++] = none[d][j];
28       dfs(d + 1, an + 1, tsn, tnn);
29       some[d][i] = 0, none[d][nn++] = v;
30     }
31   }
32   int solve() {
33     iota(some[0], some[0] + n, 1);
34     S = 0, dfs(0, 0, n, 0);
35     return S;
36   }
37 };
```

6.13 DominatorTree

```
 1 struct dominator_tree { // 1-base
 2   vector<int> G[N], rG[N];
```

```
 3   int n, pa[N], dfn[N], id[N], Time;
 4   int semi[N], idom[N], best[N];
 5   vector<int> tree[N]; // dominator_tree
 6   void init(int _n) {
 7     n = _n;
 8     for (int i = 1; i <= n; ++i)
 9       G[i].clear(), rG[i].clear();
10   }
11   void add_edge(int u, int v) {
12     G[u].pb(v), rG[v].pb(u);
13   }
14   void dfs(int u) {
15     id[dfn[u] = ++Time] = u;
16     for (auto v : G[u])
17       if (!dfn[v]) dfs(v), pa[dfn[v]] = dfn[u];
18   }
19   int find(int y, int x) {
20     if (y <= x) return y;
21     int tmp = find(pa[y], x);
22     if (semi[best[y]] > semi[best[pa[y]]])
23       best[y] = best[pa[y]];
24     return pa[y] = tmp;
25   }
26   void tarjan(int root) {
27     Time = 0;
28     for (int i = 1; i <= n; ++i) {
29       dfn[i] = idom[i] = 0;
30       tree[i].clear();
31       best[i] = semi[i] = i;
32     }
33     dfs(root);
34     for (int i = Time; i > 1; --i) {
35       int u = id[i];
36       for (auto v : rG[u])
37         if (v = dfn[v]) {
38           find(v, i);
39           semi[i] = min(semi[i], semi[best[v]]);
40         }
41       tree[semi[i]].pb(i);
42       for (auto v : tree[pa[i]]) {
43         find(v, pa[i]);
44         idom[v] =
45           semi[best[v]] == pa[i] ? pa[i] : best[v];
46       }
47       tree[pa[i]].clear();
48     }
49     for (int i = 2; i <= Time; ++i) {
50       if (idom[i] != semi[i]) idom[i] = idom[idom[i]];
51       tree[id[idom[i]]].pb(id[i]);
52     }
53   }
54 };
```

6.14 Topological Sort

```
 1 vector<vector<int>> graph;
 2 vector<int> visit(10,0);
 3 vector<int> order;
 4 int n;
 5 bool cycle;    // 記錄DFS的過程中是否偵測到環
 6 void DFS(int i){ //reverse(order) is topo
 7   if (visit[i] == 1) {cycle = true; return;}
 8   if (visit[i] == 2) return;
```

```
 9   visit[i] = 1;
10   for(auto to :graph[i])
11       DFS(to);
12   visit[i] = 2;
13   order.push_back(i);
14 }//for() if(!vis[i])DFS(i)
15 int main() {
16   for (int i=0; i<n; ++i){
17     if (!visit[i])
18       DFS(i);
19   }
20   if (cycle)
21     cout << "圖上有環";
22   else
23     for (int i=n-1; i>=0; --i)
24       cout << order[i];
25 }
```

6.15 Closest Pair

```
 1 template<typename _IT=point<T>* >
 2 T closest_pair(_IT L, _IT R){
 3   if(R-L <= 1) return INF;
 4   _IT mid = L+(R-L)/2;
 5   T x = mid->x;
 6   T d = min(closest_pair(L,mid),closest_pair(mid,R));
 7   inplace_merge(L, mid, R, ycmp);
 8   static vector<point> b; b.clear();
 9   for(auto u=L;u<R;++u){
10     if((u->x-x)*(u->x-x)>=d) continue;
11     for(auto v=b.rbegin();v!=b.rend();++v){
12       T dx=u->x-v->x, dy=u->y-v->y;
13       if(dy*dy>=d) break;
14       d=min(d,dx*dx+dy*dy);
15     }
16     b.push_back(*u);
17   }
18   return d;
19 }
20 T closest_pair(vector<point<T>> &v){
21   sort(v.begin(),v.end(),xcmp);
22   return closest_pair(v.begin(),v.end());
23 }
```

6.16 Minimum Mean Cycle

```
 1 ll road[N][N]; // input here
 2 struct MinimumMeanCycle {
 3   ll dp[N + 5][N], n;
 4   pll solve() {
 5     ll a = -1, b = -1, L = n + 1;
 6     for (int i = 2; i <= L; ++i)
 7       for (int k = 0; k < n; ++k)
 8         for (int j = 0; j < n; ++j)
 9           dp[i][j] =
10             min(dp[i - 1][k] + road[k][j], dp[i][j]);
11     for (int i = 0; i < n; ++i) {
12       if (dp[L][i] >= INF) continue;
13       ll ta = 0, tb = 1;
```

```
14        for (int j = 1; j < n; ++j)
15          if (dp[j][i] < INF &&
16              ta * (L - j) < (dp[L][i] - dp[j][i]) * tb)
17            ta = dp[L][i] - dp[j][i], tb = L - j;
18        if (ta == 0) continue;
19        if (a == -1 || a * tb > ta * b) a = ta, b = tb;
20      }
21      if (a != -1) {
22        ll g = __gcd(a, b);
23        return pll(a / g, b / g);
24      }
25      return pll(-1LL, -1LL);
26    }
27    void init(int _n) {
28      n = _n;
29      for (int i = 0; i < n; ++i)
30        for (int j = 0; j < n; ++j) dp[i + 2][j] = INF;
31    }
32  };
```

6.17 Planar

```
1  class Graph {
2  public:
3    int V;
4    vector<vector<int>> adj;
5    Graph(int vertices) : V(vertices), adj(vertices) {}
6    void addEdge(int u, int v) {
7      adj[u].push_back(v);
8      adj[v].push_back(u);
9    }
10 };
11
12 bool containsSubgraph(const Graph& graph, const vector<int>&
     subgraph) {
13   unordered_set<int> subgraphVertices(subgraph.begin(),
        subgraph.end());
14   for (int vertex : subgraphVertices) {
15     for (int neighbor : graph.adj[vertex]) {
16       if (subgraphVertices.count(neighbor) == 0) {
17         bool found = true;
18         for (int v : subgraph) {
19           if (v != vertex && v != neighbor) {
20             if (graph.adj[v].size() < 3) {
21               found = false;
22               break;
23             }
24           }
25         }
26         if (found)
27           return true;
28       }
29     }
30   }
31   return false;
32 }
33
34 bool isPlanar(const Graph& graph) {
35   // Subgraphs isomorphic to K  and K ,
36   vector<int> k5 = {0, 1, 2, 3, 4};      // Vertices of K
37   vector<int> k33a = {0, 1, 2};          // Vertices of K
        ,   (part A)
```

```
38     vector<int> k33b = {3, 4, 5};          // Vertices of K
         ,   (part B)
39
40     if (containsSubgraph(graph, k5) || containsSubgraph(graph
        , k33a) || containsSubgraph(graph, k33b)) {
41       return false; // The graph is non-planar
42     }
43     return true; // The graph is planar
44 }
45 int main() {
46     int vertices, edges;
47     Graph graph(vertices);
48     for (int i = 0; i < edges; ++i) {
49         int u, v;cin >> u >> v;
50         graph.addEdge(u, v);
51     }
52     if (isPlanar(graph)) {
53         cout << "The graph is planar." << endl;
54     } else {
55         cout << "The graph is non-planar." << endl;
56     }
57 }
```

6.18 Heavy Light Decomposition

```
1  int dep[N], pa[N], sz[N], nxt[N];
2  int id[N], rt[N];
3  int dfs(int u, int lst, int d = 0) {
4    dep[u] = d;
5    pa[u] = lst;
6    sz[u] = 1;
7    nxt[u] = -1;
8    for (int v: g[u]) {
9      if (v == lst) continue;
10     sz[u] += dfs(v, u, d + 1);
11     if (nxt[u] == -1 || sz[v] > sz[nxt[u]]) {
12       nxt[u] = v;
13     }
14   }
15   return sz[u];
16 }
17 int tn = 0;
18 void mapId(int u, int lst, int root) {
19   id[u] = ++tn;
20   rt[u] = root;
21   if (~nxt[u]) mapId(nxt[u], u, root);
22   for (int v: g[u]) {
23     if (v == lst || v == nxt[u]) continue;
24     mapId(v, u, v);
25   }
26 }
27 void solve() {
28   while (rt[a] != rt[b]) {
29     if (dep[rt[a]] > dep[rt[b]]) swap(a, b);
30     //...
31     b = pa[rt[b]];
32   }
33   if (a != b) {
34     if (id[a] > id[b]) swap(a, b);
35     //...
36   } else {
37     //...
38   }
```

6.19 Centroid Decomposition

```
1  int sz[maxn]{};
2  bool ok[maxn]{};
3  int get_subtree_size(int u, int lst) {
4    sz[u] = 1;
5    for (int v: g[u]) {
6      if (v == lst || ok[v]) continue;
7      sz[u] += get_subtree_size(v, u);
8    }
9    return sz[u];
10 }
11 int get_centroid(int u, int lst, int tree_size) {
12   for (int v: g[u]) {
13     if (v == lst || ok[v]) continue;
14     if (2 * sz[v] >= tree_size) {
15       return get_centroid(v, u, tree_size);
16     }
17   }
18   return u;
19 }
20 void centroid_decomp(int u = 1) { //1-based
21   int centroid = get_centroid(u, u, get_subtree_size(u, u));
22   //...
23   ok[centroid] = 1;
24   for (int v: g[centroid]) if (!ok[v]) {
25     centroid_decomp(v);
26   }
27 }
```

6.20 KM_O

```
1  // 二分圖最大權完美匹配
2  #define MAXN 100
3  #define INF INT_MAX
4  int g[MAXN][MAXN],lx[MAXN],ly[MAXN],slack_y[MAXN];
5  int px[MAXN],py[MAXN],match_y[MAXN],par[MAXN];
6  int n;
7  void adjust(int y){//把增廣 上所有邊反轉
8    match_y[y]=py[y];
9    if(px[match_y[y]]!=-2)
10     adjust(px[match_y[y]]);
11 }
12 bool dfs(int x){//DFS找增廣
13   for(int y=0;y<n;++y){
14     if(py[y]!=-1)continue;
15     int t=lx[x]+ly[y]-g[x][y];
16     if(t==0){
17       py[y]=x;
18       if(match_y[y]==-1){
19         adjust(y);
20         return 1;
21       }
22       if(px[match_y[y]]!=-1)continue;
23       px[match_y[y]]=y;
24       if(dfs(match_y[y]))return 1;
25     }else if(slack_y[y]>t){
```

```
26        slack_y[y]=t;
27        par[y]=x;
28      }
29    }
30    return 0;
31  }
32  inline int km(){
33    memset(ly,0,sizeof(int)*n);
34    memset(match_y,-1,sizeof(int)*n);
35    for(int x=0;x<n;++x){
36      lx[x]=-INF;
37      for(int y=0;y<n;++y){
38        lx[x]=max(lx[x],g[x][y]);
39      }
40    }
41    for(int x=0;x<n;++x){
42      for(int y=0;y<n;++y)slack_y[y]=INF;
43      memset(px,-1,sizeof(int)*n);
44      memset(py,-1,sizeof(int)*n);
45      px[x]=-2;
46      if(dfs(x))continue;
47      bool flag=1;
48      while(flag){
49        int cut=INF;
50        for(int y=0;y<n;++y)
51          if(py[y]==-1&&cut>slack_y[y])cut=slack_y[y];
52        for(int j=0;j<n;++j){
53          if(px[j]!=-1)lx[j]-=cut;
54          if(py[j]!=-1)ly[j]+=cut;
55          else slack_y[j]-=cut;
56        }
57        for(int y=0;y<n;++y){
58          if(py[y]==-1&&slack_y[y]==0){
59            py[y]=par[y];
60            if(match_y[y]==-1){
61              adjust(y);
62              flag=0;
63              break;
64            }
65            px[match_y[y]]=y;
66            if(dfs(match_y[y])){
67              flag=0;
68              break;
69            }
70          }
71        }
72      }
73    }
74    int ans=0;
75    for(int y=0;y<n;++y) if(g[match_y[y]][y]!=-INF) ans += g[
         match_y[y]][y];
76    return ans;
77  }
```

6.21 Minimum Arborescence

```
1  struct zhu_liu { // O(VE)
2    struct edge {
3      int u, v;
4      ll w;
5    };
6    vector<edge> E; // 0-base
7    int pe[N], id[N], vis[N];
```

```
8   ll in[N];
9   void init() { E.clear(); }
10  void add_edge(int u, int v, ll w) {
11    if (u != v) E.pb(edge{u, v, w});
12  }
13  ll build(int root, int n) {
14    ll ans = 0;
15    for (;;) {
16      fill_n(in, n, INF);
17      for (int i = 0; i < SZ(E); ++i)
18        if (E[i].u != E[i].v && E[i].w < in[E[i].v])
19          pe[E[i].v] = i, in[E[i].v] = E[i].w;
20      for (int u = 0; u < n; ++u) // no solution
21        if (u != root && in[u] == INF) return -INF;
22      int cntnode = 0;
23      fill_n(id, n, -1), fill_n(vis, n, -1);
24      for (int u = 0; u < n; ++u) {
25        if (u != root) ans += in[u];
26        int v = u;
27        while (vis[v] != u && !~id[v] && v != root)
28          vis[v] = u, v = E[pe[v]].u;
29        if (v != root && !~id[v]) {
30          for (int x = E[pe[v]].u; x != v;
31               x = E[pe[x]].u)
32            id[x] = cntnode;
33          id[v] = cntnode++;
34        }
35      }
36      if (!cntnode) break; // no cycle
37      for (int u = 0; u < n; ++u)
38        if (!~id[u]) id[u] = cntnode++;
39      for (int i = 0; i < SZ(E); ++i) {
40        int v = E[i].v;
41        E[i].u = id[E[i].u], E[i].v = id[E[i].v];
42        if (E[i].u != E[i].v) E[i].w -= in[v];
43      }
44      n = cntnode, root = id[root];
45    }
46    return ans;
47  }
48  };
```

6.22 Maximum Clique

```
1  struct Maximum_Clique {
2    typedef bitset<MAXN> bst;
3    bst N[MAXN], empty;
4    int p[MAXN], n, ans;
5    void BronKerbosch2(bst R, bst P, bst X) {
6      if (P == empty && X == empty)
7        return ans = max(ans, (int)R.count()), void();
8      bst tmp = P | X;
9      int u;
10     if ((R | P | X).count() <= ans) return;
11     for (int uu = 0; uu < n; ++uu) {
12       u = p[uu];
13       if (tmp[u] == 1) break;
14     }
15     // if (double(clock())/CLOCKS_PER_SEC > .999)
16     // return;
17     bst now2 = P & ~N[u];
18     for (int vv = 0; vv < n; ++vv) {
19       int v = p[vv];
```

```
20       if (now2[v] == 1) {
21         R[v] = 1;
22         BronKerbosch2(R, P & N[v], X & N[v]);
23         R[v] = 0, P[v] = 0, X[v] = 1;
24       }
25     }
26   }
27   void init(int _n) {
28     n = _n;
29     for (int i = 0; i < n; ++i) N[i].reset();
30   }
31   void add_edge(int u, int v) {
32     N[u][v] = N[v][u] = 1;
33   }
34   int solve() { // remember srand
35     bst R, P, X;
36     ans = 0, P.flip();
37     for (int i = 0; i < n; ++i) p[i] = i;
38     random_shuffle(p, p + n), BronKerbosch2(R, P, X);
39     return ans;
40   }
41  };
```

7 Math

7.1 Miller Robin

```
1  // n < 4,759,123,141      3 : 2, 7, 61
2  // n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
3  // n < 3,474,749,660,383  6 : pirmes <= 13
4  // n < 2^64               7 : 2, 325, 9375, 28178, 450775,
       9780504, 1795265022
5
6  //From jacky860226
7  typedef long long LL;
8  inline LL mul(LL a,LL b,LL m){//a*b%m
9      return (a%m)*(b%m)%m;
10  }
11  /*LL mul(LL a,LL b,LL m){//a*b%m
12      a %= m, b %= m;
13      LL y = (LL)((double)a*b/m+0.5); //fast for m < 2^58
14      LL r = (a*b-y*m)%m;
15      return r<0 ? r+m : r;
16  }*/
17  template<typename T> T
18  pow(T a,T b,T mod) { //a^b%mod
19      T ans = 1;
20      while(b) {
21          if(b&1) ans = mul(ans,a,mod);
22          a = mul(a,a,mod);
23          b >>= 1;
24      } return ans;
25  }
26  template<typename T>
27  bool isprime(T n, int num) { //num = 3,7
28      int sprp[3] = {2,7,61}; //int範圍可解
29      //int llsprp[7] =
             {2,325,9375,28178,450775,9780504,1795265022}; //至少
             unsigned long long範圍
30      if(n==2) return true;
```

```
31    if(n<2 || n%2==0) return false;
32    //n-1 = u * 2^t
33    int t = 0;  T u = n-1;
34    while(u%2==0) u >>= 1, t++;
35    for(int i=0; i<num; i++) {
36        T a = sprp[i]%n;
37        if(a==0 || a==1 || a==n-1) continue;
38        T x = pow(a,u,n);
39        if(x==1 || x==n-1) continue;
40        for(int j=1; j<t; j++) {
41            x = mul(x,x,n);
42            if(x==1) return false;
43            if(x==n-1) break;
44        }
45        if(x!=n-1) return false;
46    } return true;
47 }
```

7.2 fpow

```
1 ll fpow(ll b, ll p, ll mod) {
2   ll res = 1;
3   while (p) {
4     if (p & 1) res = res * b % mod;
5     b = b * b % mod, p >>= 1;
6   }
7   return res;
8 }
```

7.3 Big Number

```
1 template<typename T>
2 inline string to_string(const T& x){
3   stringstream ss;
4   return ss<<x,ss.str();
5 }
6 struct bigN:vector<ll>{
7   const static int base=1000000000,width=log10(base);
8   bool negative;
9   bigN(const_iterator a,const_iterator b):vector<ll>(a,b){}
10  bigN(string s){
11    if(s.empty())return;
12    if(s[0]=='-')negative=1,s=s.substr(1);
13    else negative=0;
14    for(int i=int(s.size())-1;i>=0;i-=width){
15      ll t=0;
16      for(int j=max(0,i-width+1);j<=i;++j)
17        t=t*10+s[j]-'0';
18      push_back(t);
19    }
20    trim();
21  }
22  template<typename T>
23  bigN(const T &x):bigN(to_string(x)){}
24  bigN():negative(0){}
25  void trim(){
26    while(size()&&!back())pop_back();
27    if(empty())negative=0;
28  }
29  void carry(int _base=base){
```

```
30    for(size_t i=0;i<size();++i){
31      if(at(i)>=0&&at(i)<_base)continue;
32      if(i+1u==size())push_back(0);
33      int r=at(i)%_base;
34      if(r<0)r+=_base;
35      at(i+1)+=(at(i)-r)/_base,at(i)=r;
36    }
37  }
38  int abscmp(const bigN &b)const{
39    if(size()>b.size())return 1;
40    if(size()<b.size())return -1;
41    for(int i=int(size())-1;i>=0;--i){
42      if(at(i)>b[i])return 1;
43      if(at(i)<b[i])return -1;
44    }
45    return 0;
46  }
47  int cmp(const bigN &b)const{
48    if(negative!=b.negative)return negative?-1:1;
49    return negative?-abscmp(b):abscmp(b);
50  }
51  bool operator<(const bigN&b)const{return cmp(b)<0;}
52  bool operator>(const bigN&b)const{return cmp(b)>0;}
53  bool operator<=(const bigN&b)const{return cmp(b)<=0;}
54  bool operator>=(const bigN&b)const{return cmp(b)>=0;}
55  bool operator==(const bigN&b)const{return !cmp(b);}
56  bool operator!=(const bigN&b)const{return cmp(b)!=0;}
57  bigN abs()const{
58    bigN res=*this;
59    return res.negative=0, res;
60  }
61  bigN operator-()const{
62    bigN res=*this;
63    return res.negative=!negative,res.trim(),res;
64  }
65  bigN operator+(const bigN &b)const{
66    if(negative)return -(-(*this)+(-b));
67    if(b.negative)return *this-(-b);
68    bigN res=*this;
69    if(b.size()>size())res.resize(b.size());
70    for(size_t i=0;i<b.size();++i)res[i]+=b[i];
71    return res.carry(),res.trim(),res;
72  }
73  bigN operator-(const bigN &b)const{
74    if(negative)return -(-(*this)-(-b));
75    if(b.negative)return *this+(-b);
76    if(abscmp(b)<0)return -(b-(*this));
77    bigN res=*this;
78    if(b.size()>size())res.resize(b.size());
79    for(size_t i=0;i<b.size();++i)res[i]-=b[i];
80    return res.carry(),res.trim(),res;
81  }
82  bigN operator*(const bigN &b)const{
83    bigN res;
84    res.negative=negative!=b.negative;
85    res.resize(size()+b.size());
86    for(size_t i=0;i<size();++i)
87      for(size_t j=0;j<b.size();++j)
88        if((res[i+j]+=at(i)*b[j])>=base){
89          res[i+j+1]+=res[i+j]/base;
90          res[i+j]%=base;
91        }
92    return res.trim(),res;
93  }
94  bigN operator/(const bigN &b)const{
95    int norm=base/(b.back()+1);
```

```
96    bigN x=abs()*norm;
97    bigN y=b.abs()*norm;
98    bigN q,r;
99    q.resize(x.size());
100   for(int i=int(x.size())-1;i>=0;--i){
101     r=r*base+x[i];
102     int s1=r.size()<=y.size()?0:r[y.size()];
103     int s2=r.size()<y.size()?0:r[y.size()-1];
104     int d=(ll(base)*s1+s2)/y.back();
105     r=r-y*d;
106     while(r.negative)r=r+y,--d;
107     q[i]=d;
108   }
109   q.negative=negative!=b.negative;
110   return q.trim(),q;
111  }
112  bigN operator%(const bigN &b)const{
113    return *this-(*this/b)*b;
114  }
115  friend istream& operator>>(istream &ss,bigN &b){
116    string s;
117    return ss>>s, b=s, ss;
118  }
119  friend ostream& operator<<(ostream &ss,const bigN &b){
120    if(b.negative)ss<<'-';
121    ss<<(b.empty()?0:b.back());
122    for(int i=int(b.size())-2;i>=0;--i)
123      ss<<setw(width)<<setfill('0')<<b[i];
124    return ss;
125  }
126  template<typename T>
127  operator T(){
128    stringstream ss;
129    ss<<*this;
130    T res;
131    return ss>>res,res;
132  }
133 };
```

7.4 modinv

```
1 // 解 (ax == 1) mod p。p 必須是質數，a 是正整數。
2 ll modinv(ll a, ll p) {
3   if (p == 1) return 0;
4   ll pp = p, y = 0, x = 1;
5   while (a > 1) {
6     ll q = a / p, t = p;
7     p = a % p, a = t, t = y, y = x - q * y, x = t;
8   }
9   if (x < 0) x += pp;
10  return x;
11 }
12 // 解 (ax == b) mod p。p 必須是質數，a 和 b 是正整數。
13 ll modinv(ll a, ll b, ll p) {
14   ll ret = modinv(a, p);
15   return ret * b % p;
16 }
```

7.5 Matrix

- 旋轉矩陣

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

· 縮放矩陣

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

· 反射矩陣

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2u_x^2 - 1 & 2u_x u_y \\ 2u_x u_y & 2u_y^2 - 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

· 正投影

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} u_x^2 & u_x u_y \\ u_x u_y & u_y^2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

7.6 PollardRho

```
// does not work when n is prime
ll f(ll x,ll mod){ return add(mul(x,x,mod),1,mod); }
ll pollard_rho(ll n){
  if(!(n&1)) return 2;
  while(1){
    ll y=2,x=rand()%(n-1)+1,res=1;
    for(int sz=2;res==1;y=x,sz*=2)
      for(int i=0;i<sz&&res<=1;++i)
        x=f(x,n),res=__gcd(abs(x-y),n);
    if(res!=0&&res!=n) return res;
  }
}
```

7.7 Euler Totient Function

```
ll phi[maxn];
for (int i = 0; i < maxn; ++i) {
  phi[i] = i;
}
for (int i = 2; i < maxn; ++i) if (phi[i] == i) {
  phi[i] = i - 1; //prime
  for (int j = 2; i*j < maxn; ++j) { //overflow
    phi[i*j] = (phi[i*j] / i) * (i - 1);
  }
}
```

7.8 extGCD

```
(1)  int extgcd(int a,int b,int &x,int &y){//a*x +b*y = 1
         if(b==0){
             x = 1;
             y = 0;
             return a;   //到達遞歸邊界開始向上一層返回
         }
         int r = extgcd(b,a%b,x,y);
(2)      int temp=y;     //把x y變成上一層的
         y = x - (a / b) * y;
         x = temp;
         return r;       //得到a b的最大公因數
     }
     int main(){
         int a = 55,b = 80;
         int x,y;//a*x+b*y = 1;
(3)      int GCD = extgcd(a,b,x,y);
     }
```

7.9 random

```
(4)  inline int ran(){
         static int x = 20167122;
         return x = (x * 0xdefaced + 1) & INT_MAX;
     }
```

7.10 FFT

```
//OI Wiki
#include <complex>
using cd = complex<double>;
const double PI = acos(-1);
void change(vector<cd> &y) {
  vector<int> rev(y.size());
  for (int i = 0; i < y.size(); ++i) {
    rev[i] = rev[i >> 1] >> 1;
    if (i & 1) {
      rev[i] |= y.size() >> 1;
    }
  }
  for (int i = 0; i < y.size(); ++i) {
    if (i < rev[i]) {
      swap(y[i], y[rev[i]]);
    }
  }
}
void fft(vector<cd> &y, bool inv) {
  change(y);
  for (int h = 2; h <= y.size(); h <<= 1) {
    cd wn(cos(2 * PI / h), sin(2 * PI / h));
    for (int j = 0; j < y.size(); j += h) {
      cd w(1, 0);
      for (int k = j; k < j + h / 2; ++k) {
        cd u = y[k];
        cd t = w * y[k + h / 2];
        y[k] = u + t;
        y[k + h / 2] = u - t;
        w = w * wn;
      }
    }
```

```
  }
  if (inv) {
    reverse(begin(y) + 1, end(y));
    for (int i = 0; i < y.size(); ++i) {
      y[i] /= y.size();
    }
  }
}
void solve() {
  int n;
  int m = 1 << (__lg(n) + 1); //power of 2
  vector<cd> a(m), b(m);
  //...
  fft(a, 0);
  fft(b, 0);
  vector<cd> c(m);
  for (int i = 0; i < m; ++i) {
    c[i] = a[i] * b[i];
  }
  fft(c, 1);
  for (auto p: c) {
    int ans = int(p.real() + 0.25);
  }
}
```

7.11 mu

```
int mu[MAXN];
bool isnp[MAXN];
vector<int> primes;
void init(int n)
{
    mu[1] = 1;
    for (int i = 2; i <= n; i++)
    {
        if (!isnp[i])
            primes.push_back(i), mu[i] = -1; // 质数为-1
        for (int p : primes)
        {
            if (p * i > n)
                break;
            isnp[p * i] = 1;
            if (i % p == 0)
            {
                mu[p * i] = 0; // 有平方因数为0
                break;
            }
            else
                mu[p * i] = mu[p] * mu[i]; // 互质，用积性
                                            函数性质
        }
    }
}
```

7.12 Chinese Remainder

```cpp
// Chinese remainder theorem (special case): find z such that
// z % x = a, z % y = b.  Here, z is unique modulo M = lcm(x,
    y).
// Return (z,M).  On failure, M = -1.
PII chinese_remainder_theorem(int x, int a, int y, int b) {
  int s, t;
  int d = extended_euclid(x, y, s, t);
  if (a%d != b%d) return make_pair(0, -1);
  return make_pair(mod(s*b*x+t*a*y,x*y)/d, x*y/d);
}

// Chinese remainder theorem: find z such that
// z % x[i] = a[i] for all i.  Note that the solution is
// unique modulo M = lcm_i (x[i]).  Return (z,M).  On
// failure, M = -1.  Note that we do not require the a[i]'s
// to be relatively prime.
PII chinese_remainder_theorem(const VI &x, const VI &a) {
  PII ret = make_pair(a[0], x[0]);
  for (int i = 1; i < x.size(); i++) {
    ret = chinese_remainder_theorem(ret.second, ret.first, x[
        i], a[i]);
    if (ret.second == -1) break;
  }
  return ret;
}

// computes x and y such that ax + by = c; on failure, x = y
    =-1
void linear_diophantine(int a, int b, int c, int &x, int &y)
    {
  int d = gcd(a,b);
  if (c%d) {
    x = y = -1;
  } else {
    x = c/d * mod_inverse(a/d, b/d);
    y = (c-a*x)/b;
  }
}
```

8 Misc

8.1 Mo's Algorithm

```cpp
struct Query {
  int L, R;
  //...
};
vector<Query> query;
void solve() { //K = n / sqrt(q)
  sort(iter(query), [&](Query &a, Query &b) {
    if (a.L / K != b.L / K) return a.L < b.L;
    return a.L / K % 2 ? a.R < b.R : a.R > b.R;
  });
  int L = 0, R = 0;
  for (auto x: query) {
    while (R < x.R) add(arr[++R]);
    while (L > x.L) add(arr[--L]);
    while (R > x.R) sub(arr[R--]);
    while (L < x.L) sub(arr[L++]);
    //...
  }
}
```

8.2 pbds

```cpp
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

template<typename T>
using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;

int32_t main() {
  ordered_set<int64_t> rbt;
  // .insert(x); .erase(x)
  // .lower_bound(x); .upper_bound(x): iter
  // .find_by_order(k): find k-th small value(iter)
  // .order_of_key(x): return x is k-th big
  // .join(rbt2): merge with no mutiple same element
  // .split(key, rbt2): rbt keeps value <= key, others to
      rbt2
}
```

8.3 Misc

```cpp
mt19937 rng(chrono::steady_clock::now().time_since_epoch().
    count());
int randint(int lb, int ub) {
  return uniform_int_distribution<int>(lb, ub)(rng);
} //static unsigned x = 19; ++(x *= 0xdefaced);

#define SECS ((double)clock() / CLOCKS_PER_SEC)

struct KeyHasher {
  size_t operator()(const Key& k) const {
    return k.first + k.second * 100000;
} };
typedef unordered_map<Key,int,KeyHasher> map_t;

__lg
__gcd

int __builtin_ffs(unsigned int x)
int __builtin_ffsl(unsigned long)
int __builtin_ffsll(unsigned long long)
```
返回右起第一個1的位置
Returns one plus the index of the least significant 1-bit of
    x, or if x is zero, returns zero.

```cpp
int __builtin_clz(unsigned int x)
int __builtin_clzl(unsigned long)
int __builtin_clzll(unsigned long long)
```
返回左起第一個1之前0的個數
Returns the number of leading 0-bits in x, starting at the
    most significant bit position. If x is 0, the result is
    undefined.

```cpp
int __builtin_ctz(unsigned int x)
int __builtin_ctzl(unsigned long)
int __builtin_ctzll(unsigned long long)
```
返回右起第一個1之後的0的個數
Returns the number of trailing 0-bits in x, starting at the
    least significant bit position. If x is 0, the result is
    undefined.

```cpp
int __builtin_popcount(unsigned int x)
int __builtin_popcountl(unsigned long)
int __builtin_popcountll(unsigned long long)
```
返回1的個數
Returns the number of 1-bits in x.

```cpp
int __builtin_parity(unsigned int x)
int __builtin_parityl(unsigned long)
int __builtin_parityll(unsigned long long)
```
返回1的個數的奇偶性(1的個數 mod 2的值)
Returns the parity of x, i.e. the number of 1-bits in x
    modulo 2.

9 String

9.1 Hashing

```cpp
const ll P = 401, M = 998244353;

ll hashes[10005], modp[10005];
ll hashp(string s, bool saveval) {
  ll val = 0;
  int index = 0;
  for (char c: s) {
    val = ((val * P) % M + c) % M;
    if (saveval) hashes[index++] = val;
  }
  return val;
}
void init(int base, int exp) {
  ll b = 1;
  modp[0] = 1;
  for (int i = 0; i < exp; i++) {
    b = (b * base) % M;
    modp[i + 1] = b;
  }
}
ll subseq(int l, int r) { //[l, r]
  if (l == 0) return hashes[r];
  return ((hashes[r] - hashes[l-1] * modp[r-l+1]) % M + M) %
      M;
}
```

9.2 Trie

```cpp
struct node {
  int ch[26]{};
  int cnt = 0;
```

```
 4 };
 5 struct Trie {
 6   vector<node> t;
 7   void init() {
 8     t.clear();
 9     t.emplace_back(node());
10   }
11   void insert(string s) {
12     int ptr = 0;
13     for (char i: s) {
14       if (!t[ptr].ch[i - 'a']) {
15         t[ptr].ch[i - 'a'] = (int)t.size();
16         t.emplace_back(node());
17       }
18       ptr = t[ptr].ch[i - 'a'];
19     }
20     t[ptr].cnt++;
21   }
22 } trie;
```

9.3 Zvalue

```
 1 vector<int> Zvalue(string &s) { //t + # + s
 2   vector<int> Z(s.size());
 3   int x = 0, y = 0;
 4   for (int i=0; i<s.size(); ++i) {
 5     Z[i] = max(0, min(y - i + 1, Z[i - x]));
 6     while (i + Z[i] < s.size() && s[Z[i]] == s[i + Z[i]])
 7       x = i, y = i + Z[i], ++Z[i];
 8   }
 9   return Z;
10 }
```

9.4 KMP

```
 1 int F[maxn]{};
 2 vector<int> match(string& s, string& t) {
 3   int p = F[0] = -1;
 4   for (int i = 1; i < t.size(); ++i) {
 5     while (p != -1 && t[p + 1] != t[i]) p = F[p];
 6     if (t[p + 1] == t[i]) ++p;
 7     F[i] = p;
 8   }
 9   p = -1;
10   vector<int> v;
11   for (int i = 0; i < s.size(); ++i) {
12     while (p != -1 && t[p + 1] != s[i]) p = F[p];
13     if (t[p + 1] == s[i]) ++p;
14     if (p == t.size() - 1) v.push_back(i - p), p = F[p];
15   }
16   return v; //0-based
17 }
```

9.5 Manacher

```
 1 int z[maxn * 2]{};
 2 int manacher(string& s) {
 3   string t = "#";
 4   for (char c: s) t += c, t += '#';
 5   int l = 0, r = 0, ans = 0; //l: mid, r: right
 6   for (int i = 1; i < t.size(); ++i) {
 7     z[i] = (r > i ? min(z[2 * l - i], r - i) : 1);
 8     while (i - z[i] >= 0 && i + z[i] < t.size()) {
 9       if (t[i - z[i]] == t[i + z[i]])
10         ++z[i];
11       else
12         break;
13     }
14     if (i + z[i] > r) r = i + z[i], l = i;
15   }
16   for (int i = 1; i < t.size(); ++i) ans = max(ans, z[i] - 1)
       ;
17   string res;
18   for (int i = 1; i < t.size(); ++i) if (ans == z[i] - 1) {
19     for (int j = i - ans + 1; j < i + ans; ++j) if (t[j] != '
          #') {
20       res += t[j];
21     }
22     break;
23   }
24   return ans;
25 }
```

10 Tree

10.1 LCA

```
 1 int n, logn,t=0;
 2 vector<vector<int>> graph;
 3 vector<vector<int>> ancestor;
 4 vector<int> tin, tout;
 5 void dfs(int x){
 6   tin[x] = t++;
 7   for(auto y:graph[x]){
 8       if(y!= ancestor[x][0]){
 9           ancestor[y][0] = x;
10           dfs(y);
11       }
12   }
13   tout[x] = t++;
14 }
15 bool is_ancestor(int x, int y){
16   return tin[x] <= tin[y] && tout[x] >= tout[y];
17 }
18 void table(){
19   for (int i=1; i<logn; i++)// 上二輩祖先、上四輩祖先、上八輩
          祖先、……
20     for (int x=0; x<n; ++x)
21       ancestor[x][i] = ancestor[ancestor[x][i-1]][i-1];
22 }
23
24 int kth_ancestor(int x, int k){
25   for (int i=0; i<logn; i++)// k拆解成二進位位數，找到第k祖
          先。不斷上升逼近之。
26     if (k & (1<<i))
```

```
27       x = ancestor[x][i];
28   return x;
29 }
30
31 void rooted_tree(int root){// build the tree with root at "
       root"
32   ancestor[root][0] = root;
33   dfs(root);
34   table();
35 }
36 int LCA(int x,int y){
37     if (is_ancestor(x, y)) return x;
38   if (is_ancestor(y, x)) return y;
39     for (int i=logn-1; i>=0; i--)
40     if (!is_ancestor(ancestor[x][i], y))
41       x = ancestor[x][i];
42   return ancestor[x][0];
43 }
44 int main(){
45     graph = {
46         {1,2},
47         {3},
48         {5,6},
49         {7},
50         {},
51         {},
52         {},
53         {8},
54         {4},
55     };
56     n = 9;
57     logn = ceil(log2(n));
58     ancestor.resize(n,vector<int>(logn));
59     tin.resize(n);
60     tout.resize(n);
61
62     rooted_tree(0);
63     while(true){
64         int a,b;
65         cin >>a>>b;
66         cout <<LCA(a,b)<<endl;;
67     }
68 }
69 int main(){
70     n = 9;
71     logn = ceil(log2(n));
72     ancestor.resize(n,vector<int>(logn));
73     tin.resize(n);
74     tout.resize(n);
75     rooted_tree(0);
76     while(true){
77         int a,b;
78         cin >>a>>b;
79         cout <<LCA(a,b)<<endl;;
80     }
81 }
```

10.2 Diameter

```
 1 vector<vector<int>> graph;
 2 int diameter = 0;
 3 int dfs(int start, int parent){
 4     int h1 = 0, h2 = 0;
```

```
 5        for (auto child : graph[start]){
 6            if (child != parent){
 7                int h = dfs(child, start) + 1;
 8                if (h > h1){
 9                    h2 = h1;
10                    h1 = h;
11                }
12                else if (h > h2){
13                    h2 = h;
14                }
15            }
16        }
17        diameter = max(diameter, h1 + h2);
18        return h1;
19    }
20    // call diameter
21    int main(){
22        dfs(0,-1);
23        cout << diameter<<endl;
24    }
```

10.3   Radius

```
 1    // Perform DFS to find the farthest node and its distance
         from the given node
 2    pair<int, int> dfs(int node, int distance, vector<bool> &
         visited, const vector<vector<int>> &adj_list){
 3        visited[node] = true;
 4        int max_distance = distance;
 5        int farthest_node = node;
 6
 7        for (int neighbor : adj_list[node]){
 8            if (!visited[neighbor]){
 9                auto result = dfs(neighbor, distance + 1, visited
                     , adj_list);
10                if (result.first > max_distance){
11                    max_distance = result.first;
12                    farthest_node = result.second;
13                }
14            }
15        }
16
17        return make_pair(max_distance, farthest_node);
18    }
19
20    // Calculate the radius of the tree using DFS
21    int tree_radius(const vector<vector<int>> &adj_list){
22        int num_nodes = adj_list.size();
23        vector<bool> visited(num_nodes, false);
24
25        // Find the farthest node from the root (node 0)
26        auto farthest_result = dfs(0, 0, visited, adj_list);
27
28        // Reset visited array
29        fill(visited.begin(), visited.end(), false);
30
31        // Calculate the distance from the farthest node
32        int radius = dfs(farthest_result.second, 0, visited,
             adj_list).first;
33
34        return radius;
35    }
36    int main() {
```

```
37        vector<vector<int>> adj_list;
38        int radius = tree_radius(adj_list);
39        cout << "Tree radius: " << radius << endl;
40        return 0;
41    }
```

10.4   Spanning Tree

```
 1    const int V = 100, E = 1000;
 2    struct Edge {int a, b, c;} e[E];   // edge list
 3    bool operator<(Edge e1, Edge e2) {return e1.c < e2.c;}
 4
 5    int p[V];
 6    void init() {for (int i=0; i<V; ++i) p[i] = i;}
 7    int find(int x) {return x == p[x] ? x : (p[x] = find(p[x]));}
 8    void merge(int x, int y) {p[find(x)] = find(y);}
 9
10    void Kruskal(){
11        init();
12        sort(e, e+E);
13        int i, j;
14        for (i = 0, j = 0; i < V-1 && j < E; ++i){
15            while (find(e[j].a) == find(e[j].b)) j++;
16            merge(e[j].a, e[j].b);
17            cout << "起點：" << e[j].a<< "終點：" << e[j].b<< "權重：
                 " << e[j].c;
18            j++;
19        }
20        if (i == V-1) cout << "得到最小生成樹";
21        else          cout << "得到最小生成森  ";
22    }
```

NYCU_Segmentree Codebook

Contents