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Numerical methods Assignment 4

1.

```
clear;
clc;
n = 6;
x = [0.15 0.21 0.23 0.27 0.32 0.35];
f = @(x) 1+log(x)/log(10);
m = zeros(n,n+2);
for i = 1:n
    m(i,1) = i-1;
end
m(:,2) = x;
for i = 1:n
    m(i,3) = f(m(i,2));
end

for i = 1:n-1
    for j = 1:n-i
        m(j,i+3) = (m(j+1,i+2)-m(j,i+2))/(m(j+i,2)-m(j,2));
    end
end

g = @(x0,i) m(i,3)+m(i,4).*(x0-x(i))+m(i,5).*(x0-x(i)).*(x0-x(i+1));
error = @(x0,i) m(i,6).*(x0-x(i)).*(x0-x(i+1)).*(x0-x(i+2));
error(0.268,1);%0.0040
error(0.268,2);%-3.8567e-05
error(0.268,3);%2.3570e-05
g_diff = @(x0,i) m(i,4)+m(i,5).*((x0-x(i))+(x0-x(i+1)));
g_diff(0.268,3)
```

Command Window

```
ans =

    1.6348
```

We first use the for loop to generate the divided differences table

```
m =

         0    0.1500    0.1761    2.4355   -5.7505   15.3474  -38.8116   94.5795
    1.0000    0.2100    0.3222    1.9754   -3.9088    8.7494  -19.8957         0
    2.0000    0.2300    0.3617    1.7409   -2.9464    5.9640         0         0
    3.0000    0.2700    0.4314    1.4757   -2.2307         0         0         0
    4.0000    0.3200    0.5051    1.2973         0         0         0         0
    5.0000    0.3500    0.5441         0         0         0         0         0
```

i	x_i	f_i	$f[x_i, x_{i+1}]$	$f[x_i, \dots, x_{i+2}]$	$f[x_i, \dots, x_{i+3}]$	$f[x_i, \dots, x_{i+4}]$	F[xi…xi+5]
0	0.15	0.1761	2.4355	-5.7505	15.3474	-38.8116	94.5795
1	0.21	0.3222	1.9754	-3.9088	8.7494	-19.8957	
2	0.23	0.3617	1.7409	-2.9464	5.9640		
3	0.27	0.4314	1.4757	-2.2307			
4	0.32	0.5051	1.2973				
5	0.35	0.5441					

And calculate the error of quadratic interpolating polynomial, since the error of start from x(3) have min error, we calculate the f'(0.268)from x3 and use three point(0.23, 0.3617) (0.27,0.4314) (0.32,0.5051)

f'(0.268) = 1.6348

```
ans =

    1.6348
```

2.

```
clear;
clc;
n = 7;
x = [0.3 0.5 0.7 0.9 1.1 1.3 1.5];
f = @(x) x+sin(x)/3;
f_diff = @(x) 1+cos(x)/3;
m = zeros(n,n+1);
m(:,1) = x;
for i = 1:n
    m(i,2) = f(m(i,1));
end
for i = 1:n-1
    for j = 1:n-i
        m(j,i+2) = (m(j+1,i+1)-m(j,i+1));
    end
end
m
```

We first use the for loop to generate the divided differences table

```
m =

    0.3000    0.3985    0.2613   -0.0064   -0.0022    0.0003    0.0001   -0.0000
    0.5000    0.6598    0.2549   -0.0086   -0.0018    0.0004    0.0001         0
    0.7000    0.9147    0.2464   -0.0104   -0.0014    0.0005         0         0
    0.9000    1.1611    0.2360   -0.0118   -0.0010         0         0         0
    1.1000    1.3971    0.2241   -0.0128         0         0         0         0
    1.3000    1.6212    0.2113         0         0         0         0         0
    1.5000    1.8325         0         0         0         0         0         0
```

Xi	fi	△fi	△2fi	△3fi	△4fi	△5fi	△6fi
0.3	0.3985	0.2613	-0.0064	-0.0022	0.0003	0.0001	0
0.5	0.6598	0.2549	-0.0086	-0.0018	0.0004	0.0001	

0.7	0.9147	0.2464	-0.0104	-0.0014	0.0005		
0.9	1.1611	0.2360	-0.0118	-0.0010			
1.1	1.3971	0.2241	-0.0128				
1.3	1.6212	0.2113					
1.5	1.8325						

(a)

```
target = 0.72;
i = 3;% i from 1~4
s = (target-x(i))/0.2;
g_3 =@(s,i) m(i,2)+m(i,3)*s+m(i,4)*s*(s-1)/2+m(i,5)*s*(s-1)*(s-2)/6;
g_3_diff = @(s,i) ( m(i,3) + m(i,4)*(s+(s-1))/2 + m(i,5)*(s*(s-1)+(s-1)*(s-2)+s*(s-2))/6 )/0.2;
error = abs(g_3_diff(s,i)-f_diff(x(i)))
g_3_diff(s,i)
```

We choose the starting point 0.7

the f'(0.72) =1.2510

```
error =

    0.0040


ans =

    1.2510
```

(b)

```
target = 1.33;
i = 5;% i from 1~5
s = (target-x(i))/0.2;
g_2 =@(s,i) m(i,2)+m(i,3)*s+m(i,4)*s*(s-1)/2;
g_2_diff = @(s,i) (m(i,3) + m(i,4)*(s+(s-1))/2)/0.2;
abs(g_2_diff(s,i)-f_diff(x(i)))
g_2_diff(s,i)
```

We choose the starting point 1.3

the f'(1.33) =1.0790

```
error =

    0.0722


ans =

    1.0790
```

(c)

```
target = 0.50;
i = 1;% i from 1~3
s = (target-x(i))/0.2;
g_4 =@(s,i) m(i,2) + m(i,3)*s + m(i,4)*s*(s-1)/2 + m(i,5)*s*(s-1)*(s-2)/6 + m(i,6)*s*(s-1)*(s-2)*(s-3)/24;
g_4_diff = @(s,i) (m(i,3) + m(i,4)*(s+(s-1))/2 + m(i,5)*(s*(s-1)+(s-1)*(s-2)+s*(s-2))/6 + m(i,6)*(s*(s-1)*(s-2)+s*(s-1)*(s-3)+s*(s
error = abs(g_4_diff(s,i)-f_diff(x(i)))
g_4_diff(s,i)
```

We choose the starting point 0.3

the f'(0.5) =1.2925

```
error =

    0.0259


ans =

    1.2925
```

3.

$$f''(x) = C_{-2}f_{-2} + C_{-1}f_{-1} + C_0f_0 + C_1f_1 + C_2f_2$$

$P(u)=1 \Rightarrow f_{-2}=f_{-1}=f_0=f_1=f_2=1$

$f''(x) = C_{-2}+C_{-1}+C_0+C_1+C_2 = P''(0)=0$

$P(u)=u \Rightarrow P(-2h)=-2h=f_{-2}$ $P(-h)=-h=f_{-1}$ $P(0)=0=f_0$

$P(2h)=2h=f_2$ $P(h)=h=f_1$

$\Rightarrow f''(x) = (-2h)C_{-2} + (-h)C_{-1} + 0C_0 + hC_1 + 2hC_2 = P''(0)=0$

$P(u)=u^2 \Rightarrow P(-2h)=4h^2=f_{-2}$ $P(-h)=h^2=f_{-1}$

$P(2h)=4h^2=f_2$ $P(h)=h^2=f_1$ $P(0)=0=f_0$

$\Rightarrow f''(x) = 4h^2C_{-2} + h^2C_{-1} + 0C_0 + h^2C_1 + 4h^2C_2 = P''(0)=2$

$P(u)=u^3 \Rightarrow P(-2h)=-8h^3=f_{-2}$ $P(-h)=-h^3=f_{-1}$

$P(2h)=8h^3=f_2$ $P(h)=h^3=f_1$ $P(0)=0=f_0$

$\Rightarrow f''(x) = (-8h^3)C_{-2} + (-h^3)C_{-1} + 0C_0 + h^3C_1 + 8h^3C_2 = P''(0)=0$

$P(u)=u^4$ $P(-2h)=16h^4=f_{-2}$ $P(-h)=h^4=f_{-1}$

$P(2h)=16h^4=f_2$ $P(h)=h^4=f_1$ $P(0)=0=f_0$

$\Rightarrow f''(x) = 16h^4C_{-2} + h^4C_{-1} + 0C_0 + h^4C_1 + 16h^4C_2 = P''(0)=0$

for $f''(x)$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -2 & 1 & 0 & 1 & 2 \\ 4h^2 & h^2 & 0 & h^2 & 4h^2 \\ -8 & -1 & 0 & 1 & -8 \\ 16 & 1 & 0 & 1 & 16 \end{bmatrix} \begin{bmatrix} C_{-2} \\ C_{-1} \\ C_0 \\ C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} C_{-2} \\ C_{-1} \\ C_0 \\ C_1 \\ C_2 \end{bmatrix} = \frac{1}{12h^2} \begin{bmatrix} -1 \\ 16 \\ -30 \\ 16 \\ -1 \end{bmatrix}$$

$$\Rightarrow f''(x_0) = \frac{-f_{-2} + 16f_{-1} - 30f_0 + 16f_1 - f_2}{12h^2}$$

note that $x_2 - x_1 = x_1 - x_0 = x_0 - x_{-1} = x_{-1} - x_{-2} = h$

$f(x_0 + nh) = f_n \Rightarrow f(x_0)=f_0$ $f(x_1)=f_1$ $f(x_2)=f_2$

$f(x_{-1})=f_{-1}$ $f(x_{-2})=f_{-2}$

$\Rightarrow$ for $f'''(x)$ a little different: $P(u)=u^3$ $P'''(0)=6$.

$P(u)=u^2$ $P''(0)=0$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -2h & -h & 0 & h & 2h \\ 4h^2 & h^2 & 0 & h^2 & 4h^2 \\ -8h^3 & -h^3 & 0 & h^3 & 8h^3 \\ 16h^4 & h^4 & 0 & h^4 & 16h^4 \end{bmatrix} \begin{bmatrix} C_{-2} \\ C_{-1} \\ C_0 \\ C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 6 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} C_{-2} \\ C_{-1} \\ C_0 \\ C_1 \\ C_2 \end{bmatrix} = \frac{1}{2h^3} \begin{bmatrix} -1 \\ 2 \\ 0 \\ -2 \\ 1 \end{bmatrix}$$

$$\Rightarrow f'''(x_0) = \frac{-f_{-2} + 2f_{-1} - 2f_1 + f_2}{2h^3}$$

4.

```
clear;
clc;
x = [1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8];
y = [1.543 1.669 1.811 1.971 2.151 2.352 2.577 2.828 3.107];
h = 0.1;
n = 9;
trap = 0;
for i = 1:n
    trap =trap+ y(i);
end
for i = 2:n-1
    trap = trap+y(i);
end
trap  = trap*h/2
%simp 1/3 rule
simp_2222 = 0;
simp_2222 = simp_2222+(y(1)+4*y(2)+y(3));
simp_2222 = simp_2222+(y(3)+4*y(4)+y(5));
simp_2222 = simp_2222+(y(5)+4*y(6)+y(7));
simp_2222 = simp_2222+(y(7)+4*y(8)+y(9));
simp_2222 = simp_2222*h/3
```

we first initial the points and use it to calculate the integral by Simpson's 1/3 rule and Trapezoidal Rule.

Trapezoidal Rule: 1.7684

Simpson's 1/3 Rule:1.7669

For 9 points (8 panels), we can use two 3/8 rule and one 1/3 rule (3+3+2 = 8) to calculate the integral

There are three possible:

1.3/8 rule on points(1~4),(4~7) 1/3 rule on point (7~9)

```
% 3 3 2
simp_332 = 0;
simp_332 = simp_332+(y(1)+3*y(2)+3*y(3)+y(4))*3*h/8;
simp_332 = simp_332+(y(4)+3*y(5)+3*y(6)+y(7))*3*h/8;
simp_332 = simp_332+(y(7)+4*y(8)+y(9))*h/3;
simp_332 % 1.7669
abs(simp_2222-simp_332) % 1.2500e-05
```

Ans = 1.7669

Error = 1.25e-05

2.3/8 rule on points(1~4),(6~9) 1/3 rule on point(4~6)

```
% 3 2 3
simp_323 = 0;
simp_323 = simp_323+(y(1)+3*y(2)+3*y(3)+y(4))*3*h/8;
simp_323 = simp_323+(y(4)+4*y(5)+y(6))*h/3;
simp_323 = simp_323+(y(6)+3*y(7)+3*y(8)+y(9))*3*h/8;
simp_323 % 1.7670
abs(simp_2222-simp_323) % 1.6667e-05
```

Ans = 1.7670

Error = 1.6667e-05

3.3/8 rule on points(3~6),(6~9) 1/3 rule on point (1~3)

```
% 2 3 3
simp_233 = 0;
simp_233 = simp_233+(y(1)+4*y(2)+y(3))*h/3;
simp_233 = simp_233+(y(3)+3*y(4)+3*y(5)+y(6))*3*h/8;
simp_233 = simp_233+(y(6)+3*y(7)+3*y(8)+y(9))*3*h/8;
simp_233 % 1.7669
abs(simp_2222-simp_233) % 1.2500e-05
```

Ans = 1.7669

Error = 1.25e-05

The minimum error is 1.25e-05 (error is compare with Simpson 1/3 rule)

Ans = 1.7669

With 3/8 rule on points(1~4),(4~7) 1/3 rule on point (7~9)

5.

```
clear;
clc;
f = @(x) 1/x.^2;
prev = 0;
now =0;
iteration = 1;
a = 0.2;
b = 1.0;
while(1)
    prev = now;
    points = 2^iteration+1;
    h = (b-a)/(points-1);
    p = linspace(a,b,points);
    now = 0;
    for i = 1:points
        now = now + f(p(i));
    end
    for i = 2:points-1
        now = now + f(p(i));
    end
    now = now *h/2;

    fprintf("%d iteration: ",iteration);
    fprintf("value = %f\n",now);
    fprintf("with %d points, h = %f \n",points,h);
    if(abs(now-prev)< 0.02 )
        break;
    end
    iteration = iteration+1;
end
fprintf("\nans = %f\n",now);
fprintf("at %d iteration with %d points, h = %f \n",iteration,points,h);
```

```
Command Window
ans = 4.003227
at 6 iteration with 65 points, h = 0.012500
fx >>
```

First we initial the interval a, b and the function f. we use the while loop and use if(abs(now-prev)<0.02)to determine how many iteration to run. In while loop we first calculate the points and h and use these value with f(x) to calculate the value of Integrate $f(x) = 1/x$ 2 over the interval [0.2, 1].

Ans = 4.003227

at 6th iteration with 65 points, h = 0.012500

6.

$$\int_{-0.2}^{1.4}\int_{0.4}^{2.6} e^{x}\sin(2y)\,dy\,dx = \int_{-0.2}^{1.4} e^{x}dx \int_{0.4}^{2.6}\sin(2y)\,dy$$

$$\int_{-0.2}^{1.4} e^{x}dx = \int_{-0.2}^{-0.1} e^{x}dx + \int_{-0.1}^{0} e^{x}dx + \dots + \int_{1.3}^{1.4} e^{x}dx$$

$$\int_{1.4}^{2.6}\sin(2y)dy = \int_{0.4}^{0.5}\sin(2y)dy + \int_{0.5}^{0.6}\sin(2y)dy + \dots + \int_{2.5}^{2.6}\sin(2y)dy$$

$$\int_{a}^{b} f(x)\,dx = \frac{b-a}{2}\int_{-1}^{1} f\left(\frac{(b-a)t+a+b}{2}\right)dt \quad \left(x = \frac{(b-a)t+a+b}{2}\right)$$

$$= \frac{b-a}{2}\left[\frac{5}{9}\times f\left(\frac{(b-a)(-\frac{\sqrt{3}}{\sqrt{5}})+a+b}{2}\right) + \frac{8}{9}\times f\left(\frac{a+b}{2}\right) + \frac{5}{9}\times f\left(\frac{(b-a)(\frac{\sqrt{3}}{\sqrt{5}})+a+b}{2}\right)\right]$$

$$\int_{-0.2}^{1.4} e^{x}dx = 3.23646921$$

$$\int_{0.4}^{2.6}\sin(2y)dy = 0.11409502.$$

$$\Rightarrow \int_{-0.2}^{1.4} e^{x}dx \int_{0.4}^{2.6}\sin(2y)dy$$

$$= \int_{-0.2}^{1.4}\int_{0.4}^{2.6} e^{x}\sin(2y)\,dy\,dx$$

$$= 3.23646921 \times 0.11409502.$$

$$= 0.36926502.$$

```
clear;
clc;
in_a= 0.4;
in_b = 2.6;
out_a = -0.2;
out_b = 1.4;
h = 0.1;
in_n = (in_b-in_a)/h+1
out_n = (out_b-out_a)/h+1
x_points = linspace(out_a,out_b,out_n)
y_points = linspace(in_a,in_b,in_n)
sum_x = 0;
sum_y = 0;
w = [5/9 8/9 5/9];
t = [-sqrt(3/5) 0 sqrt(3/5)]

f = @(x) exp(x);
for i = 1:out_n-1;
    a = x_points(i);
    b = a+0.1;
    temp = 0;
    for j = 1:3
        input = ((b-a)*t(j)+a+b)/2;
        temp = temp+w(j)*f(input);
    end
    temp = temp*(b-a)/2;
    sum_x =sum_x+temp;
end
fprintf("sum_x = %.8f\n",sum_x);

g = @(y) sin(2*y);
for i = 1:in_n-1;
    a = y_points(i);
    b = a+0.1;
    temp = 0;
    for j = 1:3
        input = ((b-a)*t(j)+a+b)/2;
        temp = temp+w(j)*g(input);
    end
    temp = temp*(b-a)/2;
    sum_y =sum_y+temp;
end
fprintf("sum_y = %.8f\n",sum_y);
ans = sum_y*sum_x;
```

```
sum_x = 3.23646921
sum_y = 0.11409502
ans= 0.36926502>>
```

We use two for loop to calculate the single integral outer for loop is two calculate different a,b

And the inner for loop is to calculate the integral by three term Gaussian quadrature.

we get the integral of e^x = 3.23646921, integral of sin(2y) =0.11409502 ans = 0.36926502