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Numerical methods Assignment 3

1.(a)

```
clear;
clc;
f = @(x) 0.79168 + (x-0.12)*(-0.01834)/0.12+(x-0.12)*(x-0.24)*(-0.01129)/2/0.12/0.12;
temp = f(0.231);
fprintf("%.8f",temp);
```

We form the function by differences method and put the value into the function (only use to degree 2).

And we can get the value of f(0.231) = 0.77510712

```
Command Window
fx 0.77510712>>
```

(b)

```
clear;
clc;
f = @(x) 0.79168 + (x-0.12)*(-0.01834)/0.12+(x-0.12)*(x-0.24)*(-0.01129)/0.12/0.12/2+(x-0.12)*(x-0.24)*(x-0.36)*0.00134/0.12/0.12/0.12/6;
temp = f(0.231);
fprintf("%.8f",temp);

```

We form the function by differences method and put the value into the function (use to degree 3).

And we can get the value of f(0.231) = 0.77512378

```
Command Window
fx 0.77512378>>
```

(c)

```
clear;
clc;

error_f1 = @(x) (x-0.12)*(x-0.24)*(x-0.36)*0.00134/0.12/0.12/0.12/6;
e_f1 = error_f1(0.231);

error_f2 = @(x) (x-0.12)*(x-0.24)*(x-0.36)*(x-0.48)*0.00038/0.12/0.12/0.12/0.12/24;
e_f2 = error_f2(0.231);

fprintf("%.8f\n",e_f1);
fprintf("%.8f\n",e_f2);
```

The error will be the next degree polynomial value, ex: the error of degree 2 function, the error will be kx^3 in the next iteration
And the error in (a) and (b) is

0.00001666

-0.00000245

```
Command Window
0.00001666
-0.00000245
fx >>
```

(d)

Like part 1(c) I calculate the error
start from X0 = 0.24: -0.0001075
start from X0 = 0.36: 0.000125
the error start from X0 = 0.24 is smaller
➔ It is better to choose X0 start from 0.24

```
clear;
clc;

error_f1 = @(x) (x-0.24)*(x-0.36)*(x-0.48)*0.00172/0.12/0.12/0.12/6;
e_f1 = error_f1(0.42);

error_f2 = @(x) (x-0.36)*(x-0.48)*(x-0.60)*0.00200/0.12/0.12/0.12/6;
e_f2 = error_f2(0.42);

fprintf("%.8f\n",e_f1);
fprintf("%.8f\n",e_f2);
```

```
-0.00010750
0.00012500
fx >>
```

2.

$f(-1)=0 \quad f(-0.5)=0 \quad f(0)=1 \quad f(0.5)=0 \quad f(1)=0$

$g_i(x)=a_i(x-x_i)^3+b_i(x-x_i)^2+c_i(x-x_i)+d_i$

$g_i'(x)=3a_i(x-x_i)^2+2b_i(x-x_i)+c_i$

$g_i''(x)=6a_i(x-x_i)^2+2b_i$

$g_i(x_i)=y_i=d_i$

$g_i(x_{i+1})=g_{i+1}(x_{i+1})=y_{i+1}=d_{i+1}=a_ih^3+b_ih^2+c_ih+d_i=\frac{1}{8}a_i+\frac{1}{4}b_i+\frac{1}{2}c_i+d_i$

$g_i'(x_{i+1})=g_{i+1}'(x_{i+1})=c_{i+1}=3a_ih^2+2b_ih+c_i=\frac{3}{4}a_i+b_i+c_i$

$g_i''(x_{i+1})=g_{i+1}''(x_{i+1})=b_{i+1}=6a_ih^2+2b_i=\frac{3}{2}a_i+b_i$

$\frac{3}{4}a_3$

$$\Rightarrow\begin{cases} d_0=0 \quad d_1=0 \quad d_2=1 \quad d_3=0 \quad c_0=0 \\ d_1=\frac{1}{8}a_0+\frac{1}{4}b_0+\frac{1}{2}c_0+d_0 \\ d_2=\frac{1}{8}a_1+\frac{1}{4}b_1+\frac{1}{2}c_1+d_1 \\ d_3=\frac{1}{8}a_2+\frac{1}{4}b_2+\frac{1}{2}c_2+d_2 \\ c_1=\frac{3}{4}a_0+b_0+c_0 \\ c_2=\frac{3}{4}a_1+b_1+c_1 \\ c_3=\frac{3}{4}a_2+b_2+c_2 \\ b_1=\frac{3}{2}a_0+b_0 \\ b_2=\frac{3}{2}a_1+b_1 \\ b_3=\frac{3}{2}a_2+b_2 \\ \frac{1}{8}a_3+\frac{1}{4}b_3+\frac{1}{2}c_3+d_3=0 \\ \frac{3}{4}a_3+b_3+c_3=0 \end{cases}$$

$\Rightarrow$

$a_0=6$	$a_1=-10$	$a_2=10$	$a_3=-6$
$b_0=-3$	$b_1=6$	$b_2=-9$	$b_3=6$
$c_0=0$	$c_1=3/2$	$c_2=0$	$c_3=-3/2$
$d_0=0$	$d_1=0$	$d_2=1$	$d_3=0$

$\Rightarrow g_0=6(x+1)^3-3(x+1)^2$

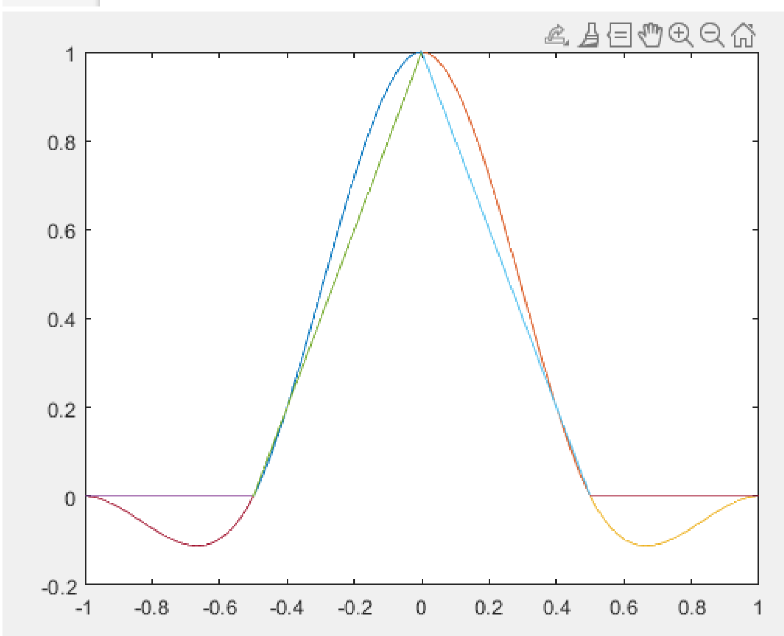
$g_1=-10(x+0.5)^3+6(x+0.5)^2+\frac{3}{2}(x+0.5)$

$g_2=10x^3-9x^2+1$

$g_3=-6(x-0.5)^3+6(x-0.5)^2-\frac{3}{2}(x-0.5)$

Write the function in matlab and plot the curve we calculate

```matlab
clear;
clc;
g0 = @(x) 6*(x+1).^3-3*(x+1).^2;
g1 = @(x) -10*(x+0.5).^3 +6*(x+0.5).^2 +3/2*(x+0.5);
g2 = @(x) 10*x.^3 - 9*x.^2 + 1;
g3 = @(x) -6*(x-0.5).^3 +6*(x-0.5).^2 -3/2*(x-0.5);
x0 = linspace(-1,-0.5);
x1 = linspace(-0.5,0);
x2 = linspace(0,0.5);
x3 = linspace(0.5,1);
plot(x0,g0(x0));
hold on;
plot(x1,g1(x1));
hold on;
plot(x2,g2(x2));
hold on;
plot(x3,g3(x3));
hold on;
g1(-0.5)
g1(0)
p0 = @(x) 0*x;
p1 = @(x) 1+2*x;
p2 = @(x) 1-2*x;
p3 = @(x) 0*x;

plot(x0,p0(x0));
plot(x1,p1(x1));
plot(x2,p2(x2));
plot(x3,p3(x3));
```



3.(a)

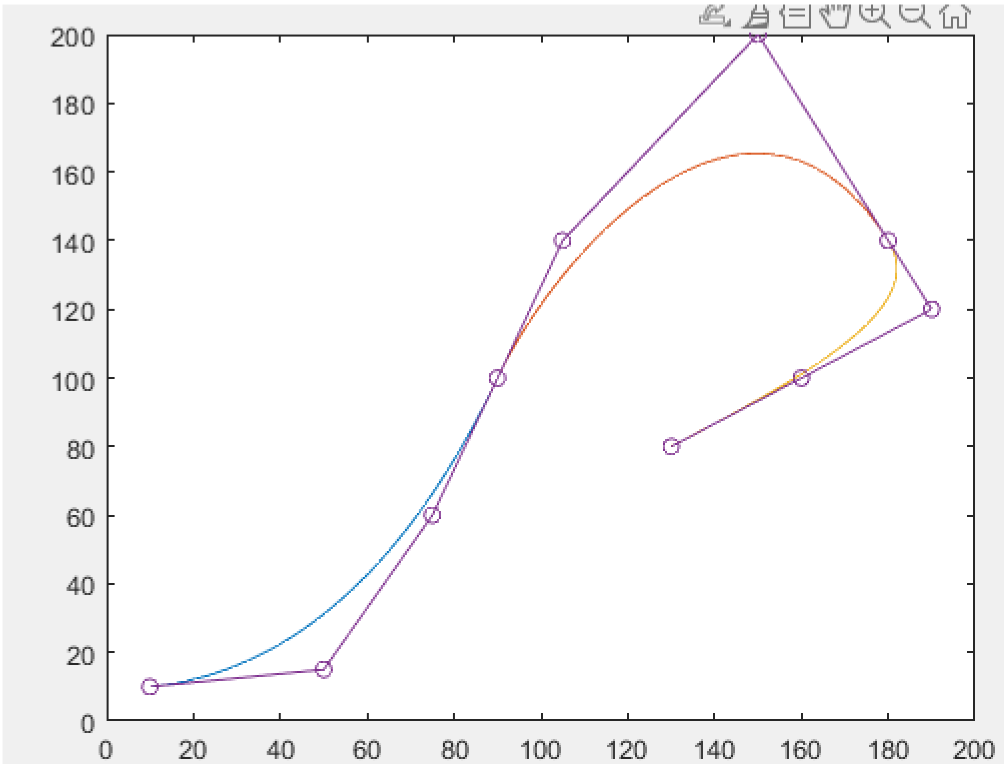
```
clear;
clc;
xy=[10 10;
    50 15;
    75 60;
    90 100;
    105 140;
    150 200;
    180 140;
    190 120;
    160 100;
    130 80;];
x = xy(:,1);
y = xy(:,2);

%g = @(u) [(1-u).^3 3.*u.*((1-u).^2) 3.*u.^2.*(1-u) u.^3;];
%g1 = @(u) g(u)*xy([1:4],:);
%g2 = @(u) g(u)*xy([4:7],:);
%g3 = @(u) g(u)*xy([7:10],:);
g1_x = @(u) x(1).*(1-u).^3 + x(2).*3.*u.*((1-u).^2)+x(3).*3.*u.^2.*(1-u)+x(4).*u.^3;
g1_y = @(u) y(1).*(1-u).^3 + y(2).*3.*u.*((1-u).^2)+y(3).*3.*u.^2.*(1-u)+y(4).*u.^3;

g2_x = @(u) x(4).*(1-u).^3 + x(5).*3.*u.*((1-u).^2)+x(6).*3.*u.^2.*(1-u)+x(7).*u.^3;
g2_y = @(u) y(4).*(1-u).^3 + y(5).*3.*u.*((1-u).^2)+y(6).*3.*u.^2.*(1-u)+y(7).*u.^3;

g3_x = @(u) x(7).*(1-u).^3 + x(8).*3.*u.*((1-u).^2)+x(9).*3.*u.^2.*(1-u)+x(10).*u.^3;
g3_y = @(u) y(7).*(1-u).^3 + y(8).*3.*u.*((1-u).^2)+y(9).*3.*u.^2.*(1-u)+y(10).*u.^3;
u = linspace(0,1);

plot(g1_x(u),g1_y(u));
hold on;
plot(g2_x(u),g2_y(u));
hold on;
plot(g3_x(u),g3_y(u));
hold on;
plot(x,y,'-O');
```

We first write down the value of x y

And from the Bezier curve matrix we can get the formula of x and y

From u = [0,1] we can get every x y point

And plot it on the 2d graph.



(b)

Since the point 2 3 4 are collinear so the graph are smoothy connected at point 6

Since the point 5 6 7 are collinear so the graph are smoothy connected at point 3

(c)

3.(c) $P(u)=au^3+bu^2+cu+d \quad P'(u)=3au^2+2bu+c$

$P(1)=P_3=a+b+c+d$

$P(2)=P_6=8a+4b+2c+d$

$P'(1)=3(P_4-P_3)=3a+2b+c$

$P'(2)=3(P_6-P_5)=12a+4b+c$

$$\Rightarrow \begin{bmatrix} 1&1&1&1\\ 8&4&2&1\\ 3&2&1&0\\ 12&4&1&0 \end{bmatrix}\begin{bmatrix} a\\ b\\ c\\ d \end{bmatrix}=\begin{bmatrix} 1&0&0&0\\ 0&0&0&1\\ -3&3&0&0\\ 0&0&-3&3 \end{bmatrix}\begin{bmatrix} P_3\\ P_4\\ P_5\\ P_6 \end{bmatrix}$$

$$\Rightarrow P(u)=[u^3\ u^2\ u\ 1]\begin{bmatrix} a\\ b\\ c\\ d \end{bmatrix}=[u^3\ u^2\ u\ 1]\begin{bmatrix} 2&-2&1&1\\ -9&9&-5&-4\\ 12&-12&8&5\\ -4&5&-4&-2 \end{bmatrix}\begin{bmatrix} 1&0&0&0\\ 0&0&0&1\\ -3&3&0&0\\ 0&0&-3&3 \end{bmatrix}\begin{bmatrix} P_3\\ P_4\\ P_5\\ P_6 \end{bmatrix}$$

$$=[u^3\ u^2\ u\ 1]\begin{bmatrix} -1&3&-3&1\\ 6&-15&12&-2\\ -12&24&-15&3\\ 8&-12&6&-1 \end{bmatrix}\begin{bmatrix} P_3\\ P_4\\ P_5\\ P_6 \end{bmatrix}$$

$P(2)=P_6=8a+4b+2c+d$

$P(3)=P_9=27a+9b+3c+d$

$P'(2)=3(P_7-P_6)=12a+4b+c$

$P'(3)=3(P_9-P_8)=27a+6b+c$

$$\Rightarrow \begin{bmatrix} 8&4&2&1\\ 27&9&3&1\\ 12&4&1&0\\ 27&6&1&0 \end{bmatrix}\begin{bmatrix} a\\ b\\ c\\ d \end{bmatrix}=\begin{bmatrix} 1&0&0&0\\ 0&0&0&1\\ -3&3&0&0\\ 0&0&-3&3 \end{bmatrix}\begin{bmatrix} P_6\\ P_7\\ P_8\\ P_9 \end{bmatrix}$$

$$\Rightarrow P(u)=[u^3\ u^2\ u\ 1]\begin{bmatrix} a\\ b\\ c\\ d \end{bmatrix}=[u^3\ u^2\ u\ 1]\begin{bmatrix} 2&-2&1&1\\ -15&15&-8&-7\\ 36&-36&21&16\\ -27&28&-18&-12 \end{bmatrix}\begin{bmatrix} 1&0&0&0\\ 0&0&0&1\\ -3&3&0&0\\ 0&0&-3&3 \end{bmatrix}\begin{bmatrix} P_6\\ P_7\\ P_8\\ P_9 \end{bmatrix}$$

$$=[u^3\ u^2\ u\ 1]\begin{bmatrix} -1&3&-3&1\\ 9&-24&21&-6\\ -27&63&-48&12\\ 27&-54&36&-8 \end{bmatrix}\begin{bmatrix} P_6\\ P_7\\ P_8\\ P_9 \end{bmatrix}$$

4.

(a)

```
clear;
clc;
xy=[10 10;
    50 15;
    75 60;
    90 100;
    105 140;
    150 200;
    180 140;
    190 120;
    160 100;
    130 80;];
x = xy(:,1);
y = xy(:,2);

%g = @(u) [(1-u).^3/6 u.^3/2-u.^2+2/3 -u.^3/2+u.^2/2+u/2+1/6 u.^3/6];
%g1 = @(u) g(u)*xy([1:4],:);
%g2 = @(u) g(u)*xy([2:5],:);
%g3 = @(u) g(u)*xy([3:6],:);
%g4 = @(u) g(u)*xy([4:7],:);
%g5 = @(u) g(u)*xy([5:8],:);
%g6 = @(u) g(u)*xy([6:9],:);
%g7 = @(u) g(u)*xy([7:10],:);
```


```
%g = @(u) [(1-u).^3/6 u.^3/2-u.^2+2/3 -u.^3/2+u.^2/2+u/2+1/6 u.^3/6];
%g1 = @(u) g(u)*xy([1:4],:);
%g2 = @(u) g(u)*xy([2:5],:);
%g3 = @(u) g(u)*xy([3:6],:);
%g4 = @(u) g(u)*xy([4:7],:);
%g5 = @(u) g(u)*xy([5:8],:);
%g6 = @(u) g(u)*xy([6:9],:);
%g7 = @(u) g(u)*xy([7:10],:);
```


```


g1_x = @(u) x(1).*((1-u).^3/6) + x(2).*(u.^3/2-u.^2+2/3)+x(3).*(-u.^3/2+u.^2/2+u/2+1/6)+x(4).*(u.^3/6);
g1_y = @(u) y(1).*((1-u).^3/6) + y(2).*(u.^3/2-u.^2+2/3)+y(3).*(-u.^3/2+u.^2/2+u/2+1/6)+y(4).*(u.^3/6);
g2_x = @(u) x(2).*((1-u).^3/6) + x(3).*(u.^3/2-u.^2+2/3)+x(4).*(-u.^3/2+u.^2/2+u/2+1/6)+x(5).*(u.^3/6);
g2_y = @(u) y(2).*((1-u).^3/6) + y(3).*(u.^3/2-u.^2+2/3)+y(4).*(-u.^3/2+u.^2/2+u/2+1/6)+y(5).*(u.^3/6);
g3_x = @(u) x(3).*((1-u).^3/6) + x(4).*(u.^3/2-u.^2+2/3)+x(5).*(-u.^3/2+u.^2/2+u/2+1/6)+x(6).*(u.^3/6);
g3_y = @(u) y(3).*((1-u).^3/6) + y(4).*(u.^3/2-u.^2+2/3)+y(5).*(-u.^3/2+u.^2/2+u/2+1/6)+y(6).*(u.^3/6);
g4_x = @(u) x(4).*((1-u).^3/6) + x(5).*(u.^3/2-u.^2+2/3)+x(6).*(-u.^3/2+u.^2/2+u/2+1/6)+x(7).*(u.^3/6);
g4_y = @(u) y(4).*((1-u).^3/6) + y(5).*(u.^3/2-u.^2+2/3)+y(6).*(-u.^3/2+u.^2/2+u/2+1/6)+y(7).*(u.^3/6);
g5_x = @(u) x(5).*((1-u).^3/6) + x(6).*(u.^3/2-u.^2+2/3)+x(7).*(-u.^3/2+u.^2/2+u/2+1/6)+x(8).*(u.^3/6);
g5_y = @(u) y(5).*((1-u).^3/6) + y(6).*(u.^3/2-u.^2+2/3)+y(7).*(-u.^3/2+u.^2/2+u/2+1/6)+y(8).*(u.^3/6);
g6_x = @(u) x(6).*((1-u).^3/6) + x(7).*(u.^3/2-u.^2+2/3)+x(8).*(-u.^3/2+u.^2/2+u/2+1/6)+x(9).*(u.^3/6);
g6_y = @(u) y(6).*((1-u).^3/6) + y(7).*(u.^3/2-u.^2+2/3)+y(8).*(-u.^3/2+u.^2/2+u/2+1/6)+y(9).*(u.^3/6);
g7_x = @(u) x(7).*((1-u).^3/6) + x(8).*(u.^3/2-u.^2+2/3)+x(9).*(-u.^3/2+u.^2/2+u/2+1/6)+x(10).*(u.^3/6);
g7_y = @(u) y(7).*((1-u).^3/6) + y(8).*(u.^3/2-u.^2+2/3)+y(9).*(-u.^3/2+u.^2/2+u/2+1/6)+y(10).*(u.^3/6);
```

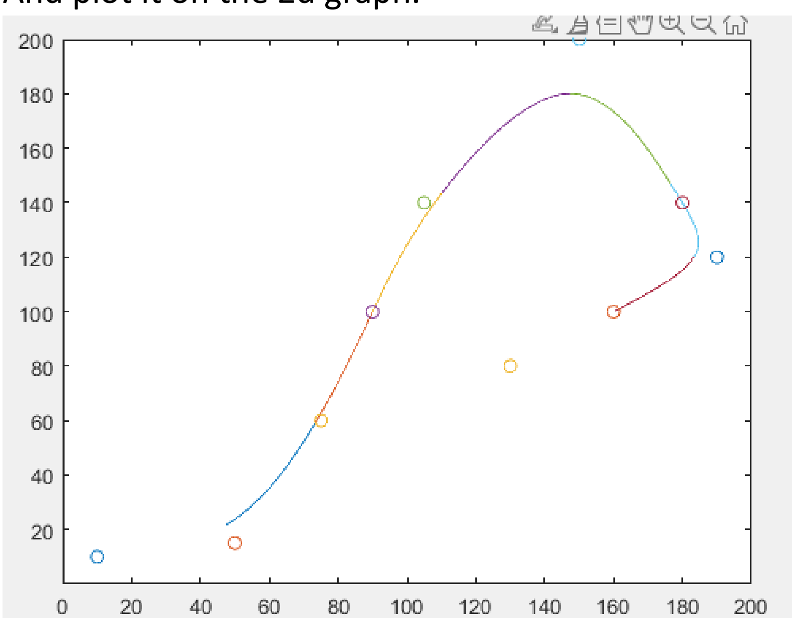
```
g1_x = @(u) x(1).*((1-u).^3/6) + x(2).*(u.^3/2-u.^2+2/3)+x(3).*(-u.^3/2+u.^2/2+u/2+1/6)+x(4).*(u.^3/6);
g1_y = @(u) y(1).*((1-u).^3/6) + y(2).*(u.^3/2-u.^2+2/3)+y(3).*(-u.^3/2+u.^2/2+u/2+1/6)+y(4).*(u.^3/6);
g2_x = @(u) x(2).*((1-u).^3/6) + x(3).*(u.^3/2-u.^2+2/3)+x(4).*(-u.^3/2+u.^2/2+u/2+1/6)+x(5).*(u.^3/6);
g2_y = @(u) y(2).*((1-u).^3/6) + y(3).*(u.^3/2-u.^2+2/3)+y(4).*(-u.^3/2+u.^2/2+u/2+1/6)+y(5).*(u.^3/6);
g3_x = @(u) x(3).*((1-u).^3/6) + x(4).*(u.^3/2-u.^2+2/3)+x(5).*(-u.^3/2+u.^2/2+u/2+1/6)+x(6).*(u.^3/6);
g3_y = @(u) y(3).*((1-u).^3/6) + y(4).*(u.^3/2-u.^2+2/3)+y(5).*(-u.^3/2+u.^2/2+u/2+1/6)+y(6).*(u.^3/6);
g4_x = @(u) x(4).*((1-u).^3/6) + x(5).*(u.^3/2-u.^2+2/3)+x(6).*(-u.^3/2+u.^2/2+u/2+1/6)+x(7).*(u.^3/6);
g4_y = @(u) y(4).*((1-u).^3/6) + y(5).*(u.^3/2-u.^2+2/3)+y(6).*(-u.^3/2+u.^2/2+u/2+1/6)+y(7).*(u.^3/6);
g5_x = @(u) x(5).*((1-u).^3/6) + x(6).*(u.^3/2-u.^2+2/3)+x(7).*(-u.^3/2+u.^2/2+u/2+1/6)+x(8).*(u.^3/6);
g5_y = @(u) y(5).*((1-u).^3/6) + y(6).*(u.^3/2-u.^2+2/3)+y(7).*(-u.^3/2+u.^2/2+u/2+1/6)+y(8).*(u.^3/6);
g6_x = @(u) x(6).*((1-u).^3/6) + x(7).*(u.^3/2-u.^2+2/3)+x(8).*(-u.^3/2+u.^2/2+u/2+1/6)+x(9).*(u.^3/6);
g6_y = @(u) y(6).*((1-u).^3/6) + y(7).*(u.^3/2-u.^2+2/3)+y(8).*(-u.^3/2+u.^2/2+u/2+1/6)+y(9).*(u.^3/6);
g7_x = @(u) x(7).*((1-u).^3/6) + x(8).*(u.^3/2-u.^2+2/3)+x(9).*(-u.^3/2+u.^2/2+u/2+1/6)+x(10).*(u.^3/6);
g7_y = @(u) y(7).*((1-u).^3/6) + y(8).*(u.^3/2-u.^2+2/3)+y(9).*(-u.^3/2+u.^2/2+u/2+1/6)+y(10).*(u.^3/6);

u = linspace(0,1);

plot(g1_x(u),g1_y(u));
hold on;
plot(g2_x(u),g2_y(u));
hold on;
plot(g3_x(u),g3_y(u));
hold on;
plot(g4_x(u),g4_y(u));
hold on;
plot(g5_x(u),g5_y(u));
hold on;
plot(g6_x(u),g6_y(u));
hold on;
plot(g7_x(u),g7_y(u));
hold on;
for i =1:10
    plot(x(i),y(i),'o');
end
```

We first write down the value of x y

And from the b-spline matrix we can get the formula of x and y

From u = [0,1] we can get every x y point

And plot it on the 2d graph.



(b)b-spline are C2 continuous so at every point will smoothly connected

(c)

4(c)

$P(1) = (P_{i-1} + 4P_i + P_{i+1})/6 = a+b+c+d$

$P(2) = (P_i + 4P_{i+1} + P_{i+2})/6 = 8a+4b+2c+d$

$P'(1) = (-P_{i-1} + P_{i+1})/2 = 3a+2b+c$

$P'(2) = (-P_i + P_{i+2})/2 = 12a+4b+c$

$$\Rightarrow P(u) = [u^3\ u^2\ u\ 1]\begin{bmatrix} 2 & -2 & 1 & 1 \\ -9 & 7 & -5 & -4 \\ 12 & -12 & 8 & 5 \\ -4 & 5 & -4 & -2 \end{bmatrix} \frac{\begin{bmatrix} 1 & 4 & 1 & 0 \\ 0 & 1 & 4 & 1 \\ -3 & 0 & 3 & 0 \\ 0 & -3 & 0 & 3 \end{bmatrix}}{6} \begin{bmatrix} P_{i-1} \\ P_i \\ P_{i+1} \\ P_{i+2} \end{bmatrix}$$

$$= [u^3\ u^2\ u\ 1]\begin{bmatrix} -1 & 3 & -3 & 1 \\ 6 & -15 & 12 & -3 \\ -12 & 21 & -12 & 3 \\ 8 & -5 & 4 & -1 \end{bmatrix}\begin{bmatrix} P_{i-1} \\ P_i \\ P_{i+1} \\ P_{i+2} \end{bmatrix}.$$

$P(2)\ (P_{i-1} + 4P_i + P_{i+1})/6 = 8a+4b+2c+d$

$P(3)\ (P_i + 4P_{i+1} + P_{i+2})/6 = 27a+9b+3c+d$

$P'(2)\ (-P_{i-1} + P_{i+1})/2 = 12a+4b+c$

$P'(3)\ (-P_i + P_{i+2})/2 = 27a+6b+c.$

$$\Rightarrow P(u) = [u^3\ u^2\ u\ 1]\begin{bmatrix} 2 & -2 & 1 & 1 \\ -15 & 15 & -8 & -7 \\ 36 & -36 & 21 & 16 \\ -27 & 28 & -18 & -12 \end{bmatrix}\begin{bmatrix} 1 & 4 & 1 & 0 \\ 0 & 1 & 4 & 1 \\ -3 & 0 & 3 & 0 \\ 0 & -3 & 0 & 3 \end{bmatrix}\begin{bmatrix} P_{i-1} \\ P_i \\ P_{i+1} \\ P_{i+2} \end{bmatrix}$$

$$= [u^3\ u^2\ u\ 1]\begin{bmatrix} -1 & 3 & -3 & 1 \\ 9 & -24 & 21 & -6 \\ -27 & 60 & -45 & 12 \\ 27 & -44 & 31 & -8 \end{bmatrix}\begin{bmatrix} P_{i-1} \\ P_i \\ P_{i+1} \\ P_{i+2} \end{bmatrix}$$

5.

(a)

5.(a)

5. $Aa = b.$

$$\Rightarrow \begin{bmatrix} 0.40 & 0.70 & 1 \\ 1.2 & 2.1 & 1 \\ 3.4 & 4.0 & 1 \\ 4.1 & 4.9 & 1 \\ 5.7 & 6.3 & 1 \\ 7.2 & 8.1 & 1 \\ 9.3 & 8.9 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \cong \begin{bmatrix} 0.031 \\ 0.933 \\ 3.058 \\ 3.349 \\ 4.870 \\ 5.757 \\ 8.921 \end{bmatrix} = Aa = b$$

$$A^T A a = A^T b.$$

$$\begin{bmatrix} 0.4 & 1.2 & 3.4 & 4.1 & 5.7 & 7.2 & 9.3 \\ 0.7 & 2.1 & 4.0 & 4.9 & 6.3 & 8.1 & 8.9 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0.4 & 0.7 & 1 \\ 1.2 & 2.1 & 1 \\ 3.4 & 4.0 & 1 \\ 4.1 & 4.9 & 1 \\ 5.7 & 6.3 & 1 \\ 7.2 & 8.1 & 1 \\ 9.3 & 8.9 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0.4 & 1.2 & 3.4 & 4.1 & 5.7 & 7.2 & 9.3 \\ 0.7 & 2.1 & 4.0 & 4.9 & 6.3 & 8.1 & 8.9 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0.031 \\ 0.933 \\ 3.058 \\ 3.349 \\ 4.870 \\ 5.757 \\ 8.921 \end{bmatrix}.$$

(b)

```
clear;
clc;
A = [0.4 0.7 1;
     1.2 2.1 1;
     3.4 4.0 1;
     4.1 4.9 1;
     5.7 6.3 1;
     7.2 8.1 1;
     9.3 8.9 1;]
b = [0.031;
     0.933;
     3.058;
     3.349;
     4.870;
     5.757;
     8.921;
    ];
AT = A.' % A transpose
% (AT)Aa = (AT)b -> Ka = C
K = AT*A;
C = AT*b;
%Ka =C -> a = K_1 C
K_1 = inv(K);
a = K_1*C
```

```
a =

    1.5961
   -0.7024
    0.2207
```

From the normal equation we can get the ((AT)A)a = (AT)b

We multiply the matrix make it become (K)a = C

And a = (K)^-1C

We can get the a =

[1.5961

 -0.7024;

  0.2207;]

(c)

```
clear;
clc;
A = [0.4 0.7 1;
     1.2 2.1 1;
     3.4 4.0 1;
     4.1 4.9 1;
     5.7 6.3 1;
     7.2 8.1 1;
     9.3 8.9 1;]
a = [ 1.5961;
     -0.7024;
      0.2207;];
b = [0.031;
     0.933;
     3.058;
     3.349;
     4.870;
     5.757;
     8.921;
    ];
x = A*a
r = A*a-b
E = r.*r
sum(E)
```

```
ans =

    0.3194
```

From the a we got, we can get the Aa;

And calculate the error r = A*a-b;

And use the r to calculate the th sum of squares of the deviations = 0.3194

6.

6.

$$\cos^2 x = \frac{1}{2}(1+\cos(2x)) = \frac{1}{2}\left(1+\sum_{n=0}^{\infty}(-1)^n\frac{(2x)^{2n}}{(2n)!}\right) = \frac{1}{2}\left(2+\sum_{n=1}^{\infty}(-1)^n\frac{(2x)^{2n}}{(2n)!}\right)$$

$$\approx 1-\frac{(2x)^2}{2!\times 2}+\frac{(2x)^4}{4!\times 2}-\frac{(2x)^6}{6!\times 2} = 1-x^2+\frac{1}{3}x^4-\frac{2}{45}x^6$$

$$1-x^2+\frac{1}{3}x^4-\frac{2}{45}x^6 = \frac{a_0+a_1x+a_2x^2+a_3x^3}{1+b_1x+b_2x^2+b_3x^3}$$

$$\begin{aligned}
a_0 &= 1 & a_0 &= 1 \\
a_1 &= b_1 & a_1 &= 0 \\
a_2 &= -1+b_2 & a_2 &= -2/3 \\
a_3 &= -b_1+b_3 \quad\Rightarrow & a_3 &= 0 \\
0 &= \tfrac{1}{3}-b_2 & b_1 &= 0 \\
0 &= 1/3\, b_1-b_3 & b_2 &= 1/3 \\
0 &= -2/45+1/3\, b_2 & b_3 &= 0
\end{aligned}
\quad\Rightarrow\quad \cos^2(x) \approx \frac{1-2/3\,x^2}{1+1/3\,x^2}.$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\Rightarrow \sin(x^4 - x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x^4-x)^{2n+1}}{(2n+1)!}$$

$$\approx \frac{1 \times (x^4 - x)}{1} + \frac{-1(x^4-x)^3}{3!} + \frac{1 \times (x^4-x)^5}{5!} \approx (x^4 - x) + \frac{-1(x^{12} - 3x^9 + 3x^6 - x^3)}{6} + \frac{-x^5}{120}$$

$$\approx 0 - x + 0 + \tfrac{1}{6}x^3 + x^4 - \tfrac{1}{120}x^5 - \tfrac{1}{2}x^6 \approx \frac{a_0 + a_1 x + a_2 x^2 + a_3 x^3}{1 + b_1 x + b_2 x^2 + b_3 x^3}$$

$a_0 = 0$

$a_1 = -1$ $\qquad 0 = 1 + \tfrac{1}{6} b_1 - b_3$

$a_2 = -b_1$ $\qquad 0 = -\tfrac{1}{120} + b_1 + \tfrac{1}{6} b_2$

$a_3 = \tfrac{1}{6} - b_2$ $\qquad 0 = -\tfrac{1}{2} - \tfrac{1}{120} b_1 + b_2 + \tfrac{1}{6} b_3$

$$\Rightarrow \begin{bmatrix} \frac{1}{6} & 0 & -1 \\ 1 & \frac{1}{6} & 0 \\ -\frac{1}{120} & 1 & \frac{1}{6} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} -1 \\ \frac{1}{120} \\ \frac{1}{2} \end{bmatrix} \Rightarrow \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} -102/2153 \\ 14393/43060 \\ 2136/2153 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 102/2153 \\ -21649/129180 \end{bmatrix}$$

$$\Rightarrow \sin(x^4 + x) = \frac{-x + \frac{102}{2153}x^2 - \frac{21649}{129180}x^3}{1 - \frac{102}{2153}x + \frac{14393}{43060} + \frac{2136}{2153}}$$

$$xe^x \approx x + \cancel{x^2} + \frac{x^3}{2} + \frac{x^4}{6} + \frac{x^5}{24} + \frac{x^6}{120} = \frac{a_0 + a_1 x + a_2 x^2 + a_3 x^3}{1 + b_1 x + b_2 x^2 + b_3 x^3}$$

$a_0 = 0$ $\qquad 0 = \frac{1}{6} + \frac{1}{2} b_1 + b_2 + b_3$

$a_1 = 1$

$a_2 = 1 + b_1$ $\qquad 0 = \frac{1}{24} + \frac{1}{6} b_1 + \frac{1}{2} b_2 + b_3$

$a_3 = \frac{1}{2} + b_1 + b_2$ $\qquad 0 = \frac{1}{120} + \frac{1}{24} b_1 + \frac{1}{6} b_2 + \frac{1}{2} b_3$

$$\Rightarrow \begin{bmatrix} \frac{1}{2} & 1 & 1 \\ \frac{1}{6} & \frac{1}{2} & 1 \\ \frac{1}{24} & \frac{1}{6} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} -\frac{1}{6} \\ -\frac{1}{24} \\ -\frac{1}{120} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} -3/5 \\ 3/20 \\ -1/60 \end{bmatrix} \Rightarrow \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2/5 \\ 1/20 \end{bmatrix}$$

$$\Rightarrow xe^x = \frac{x + \frac{2}{5}x^2 + \frac{1}{20}x^3}{1 - \frac{3}{5}x + \frac{3}{20}x^2 - \frac{1}{60}x^3}$$

7.

7.

$f(x) = xe^{-x}$ $\qquad f(1) = 1/e \quad f(2) = 2/e^2 \quad f(3) = 3/e^3$

$f'(x) = e^{-x} - xe^{-x} = (1-x)e^{-x} \Rightarrow f'(1) = 0 \quad f'(2) = -1/e^2 \quad f'(3) = -2/e^3$

$$P_1(u) = [u^3\ u^2\ u\ 1]\begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1/e \\ 2/e^2 \\ 0 \\ -1/e^2 \end{bmatrix} = \left(2/e - 5/e^2\right)u^3 + \left(-3/e + 7/e^2\right)u^2 + 1/e.$$

$$P_2(u) = [u^3\ u^2\ u\ 1]\begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 2/e^2 \\ 3/e^3 \\ -1/e^2 \\ -2/e^3 \end{bmatrix} = \left(3/e^2 - 8/e^3\right)u^3 + \left(-4/e^2 + 11/e^3\right)u^2 - 1/e^2\,u + 2/e^2.$$

(b)

$f\left(\frac{3}{2}\right) = P_1\left(\frac{1}{2}\right) = 0.3362.$