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Numerical methods Assignment 5

1.

We first initial the table at start point, and we use the for loop to iterate and complete the table to get the answer at t= 2;

```
clear;
clc;
dydt = @(y,t) y^2+ t^2;
h = 0.1;
start = 1;
end_ = 2;
n = (end_-start)/h+1;

x = linspace(start,end_,n);
x = x.';
table = zeros(n,4);
table(:,1) = x;
table(1,2) = 0;
table(1,3) = dydt(table(1,2),table(1,1));
table(1,4) = table(1,3)*h;
for i = 2:n
    table(i,2) = table(i-1,2)+table(i-1,4);
    table(i,3) = dydt(table(i,2),table(i,1));
    table(i,4) = table(i,3)*h;
end
table
```

```
clear;
clc;
dydt = @(y,t) y^2+ t^2;
h = 0.05;
start = 1;
end_ = 2;
n = (end_-start)/h+1;

x = linspace(start,end_,n);
x = x.';
table = zeros(n,4);
table(:,1) = x;
table(1,2) = 0;
table(1,3) = dydt(table(1,2),table(1,1));
table(1,4) = table(1,3)*h;
for i = 2:n
    table(i,2) = table(i-1,2)+table(i-1,4);
    table(i,3) = dydt(table(i,2),table(i,1));
    table(i,4) = table(i,3)*h;
end
table
```

Result for h = 0.1

t	y	y'	hy'
1	0	1	0.1
1.1	0.1	1.22	0.122
1.2	0.222	1.489284	0.148928
1.3	0.370928	1.827588	0.182759
1.4	0.553687	2.26657	0.226657
1.5	0.780344	2.858937	0.285894
1.6	1.066238	3.696863	0.369686
1.7	1.435924	4.951878	0.495188
1.8	1.931112	6.969193	0.696919
1.9	2.628031	10.51655	1.051655
2	3.679686	17.54009	1.754009

Result for h = 0.05

t	y	y'	hy'
1	0	1	0.05
1.05	0.05	1.105	0.05525
1.1	0.10525	1.221078	0.061054
1.15	0.166304	1.350157	0.067508
1.2	0.233812	1.494668	0.074733
1.25	0.308545	1.6577	0.082885
1.3	0.39143	1.843218	0.092161
1.35	0.483591	2.05636	0.102818
1.4	0.586409	2.303876	0.115194
1.45	0.701603	2.594746	0.129737
1.5	0.83134	2.941126	0.147056
1.55	0.978396	3.35976	0.167988
1.6	1.146384	3.874197	0.19371
1.65	1.340094	4.518353	0.225918
1.7	1.566012	5.342393	0.26712
1.75	1.833132	6.422871	0.321144
1.8	2.154275	7.880901	0.394045
1.85	2.54832	9.916436	0.495822
1.9	3.044142	12.8768	0.64384
1.95	3.687982	17.40371	0.870186
2	4.558168	24.77689	1.238845

Result for h = 0.1 is 3.679686

Result for h = 0.05 is 4.558168

We use the answer we got before and use will loop for iteration until the difference is less than 1e-6. We can get the accuracy of using h=0.05 is 93.11%.

```
clear;
clc;
n = 2;
h_01 = 3.679686;
h_005= 4.558168;
pre = 3.679686;
cur = 4.558168;
while abs(cur - pre) > 1e-6
    next = cur + (cur - pre)/(2^n - 1);
    pre = cur;
    cur = next;
    n = n + 1;
end
1-(abs(cur-h_005)/cur)
```

```
ans =

    0.9311
```

2.

2.

$$\int_{t_n}^{t_{n+1}} f(t)\,dt = X_{n+1} - X_n = C_0 f_{n+1} + C_1 f_n + C_2 f_{n-1} + C_3 f_{n-3}$$

if $f(t) = 1$

$$\Rightarrow \int_0^h 1\,dt = h = C_0 + C_1 + C_2 + C_3$$

$$\int_0^h h\,dt = \frac{1}{2}h^2 = C_0 \times h + C_1 \times 0 + C_2 \times (-h) + C_3 \times (-2h)$$

$$\int_0^h h^2\,dt = \frac{1}{3}h^3 = C_0 h^2 + C_1 \times 0 + C_2 \times (-h)^2 + C_3 \times (-2h)^2.$$

$$\int_0^h h^3\,dt = \frac{1}{4}h^4 = C_0 h^3 + C_1 \times 0 + C_2 \times (-h)^3 + C_3 \times (-2h)^3$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ h & 0 & -h & -2h \\ h^2 & 0 & h^2 & 4h^2 \\ h^3 & 0 & -h^3 & -8h^3 \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} h \\ \frac{1}{2}h^2 \\ \frac{1}{3}h^3 \\ \frac{1}{4}h^4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & -2 \\ 1 & 0 & 1 & 4 \\ 1 & 0 & -1 & -8 \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \end{bmatrix} = h \begin{bmatrix} 1 \\ 1/2 \\ 1/3 \\ 1/4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \end{bmatrix} = h \times \begin{bmatrix} 0 & 1/3 & 1/2 & 1/6 \\ 1 & 1/2 & -1 & -1/2 \\ 0 & -1 & 1/2 & 1/2 \\ 0 & 1/6 & 0 & -1/6 \end{bmatrix} \begin{bmatrix} 1 \\ 1/2 \\ 1/3 \\ 1/4 \end{bmatrix} \Rightarrow \begin{bmatrix} 9/24 \\ 19/24 \\ -5/24 \\ 1/24 \end{bmatrix}$$

$$\Rightarrow \tilde{X}_{n+1} = X_n + \frac{h}{24}\left[9f_{n+1} + 19f_n - 5f_{n-1} + f_{n-2}\right] - \frac{19}{720} h^5 x^{(5)}(\xi_n)$$

To get the error term

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ h & 0 & -h & -2h & -3h \\ h^2 & 0 & h^2 & 4h^2 & 9h^2 \\ h^3 & 0 & -h^3 & -8h^3 & -27h^3 \\ h^4 & 0 & h^4 & 16h^4 & 81h^3 \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} h \\ h^2/2 \\ h^3/3 \\ h^4/4 \\ h^5/5 \end{bmatrix} \Rightarrow \begin{bmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \frac{h}{720} \begin{bmatrix} -19 \\ 106 \\ -264 \\ 646 \\ 251 \end{bmatrix}$$

$$\Rightarrow \text{error} = \frac{-19}{720} h.$$

3.

We first initial the function

```
function dydt = f(t, y)
    dydt = [y(2); y(3); t-t*y(2)+2*y(1)];
end

```

And use ode 45 to predict y

From T = 0 0.2 ... 1.0

```
%a
T = linspace(0.2,1,5);

for i = 1:5

    [t, y] = ode45(@f, [0 T(i)], [0; 1; 0]);
    predict_y(i+1) = y(length(y),1);
    dy(i+1) = y(length(y),2);
end
predict_y
```

```
predict_y =

         0    0.2001    0.4021    0.6108    0.8340    1.0825
```

(b)

Having y(0) y(0.2) y(0.4) y(0.6) we can use Adams-Moulton method to get y(5) and y(6) to get the solution y(1) = 1.1577

```
%b
for i = 4:5
    predict_y(i+1) = predict_y(i) + 0.2/24*(55*dy(i)-59*dy(i-1)+37*dy(i-2)-9*dy(i-3));
end

corrector = predict_y(5) + 0.2/24*(9*dy(6)+19*dy(5)-5*dy(4)+dy(3))
```

```
ans =

    1.1577
```

(C) we use corrector to get the preciser ans and use it to compare with origin y(1) to calculate the error rate = 99.992445%

```
%c
corrector = predict_y(5) + 0.2/24*(9*dy(6)+19*dy(5)-5*dy(4)+dy(3))
error_rate = abs((corrector-predict_y(6))/corrector)
accuracy = 1-error_rate;
fprintf("accuracy=%.8f",accuracy)
```

```
corrector =

    1.1576


error_rate =

   7.5553e-05

accuracy=0.99992445>>
```

4.

(a)

4. (a)

$y'' = -y/4. \quad y(0)=0 \quad y(\pi)=2$

$y'(\theta_i) = \frac{y(\theta_{i+1}) - y(\theta_{i-1})}{2h} \quad y''(\theta_i) = \frac{y(\theta_{i+1}) - 2y(\theta_i) + y(\theta_{i-1})}{h^2}$

$\Rightarrow \frac{\pi - 0}{\pi/4} = 4 \Rightarrow y_0 = 0 \quad y_1 = \frac{\pi}{4} \ldots \quad y_4 = 1$

$y_0 = 0$

from $y'' = -\cancel{y/4} = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$

$\Rightarrow \frac{-\pi^2}{64} y_i = y_{i+1} - 2y_i + y_{i-1}$

$\Rightarrow y_0 + \left(\frac{\pi^2}{64} - 2\right) y_1 + y_2 = 0$

$y_1 + \left(\frac{\pi^2}{64} - 2\right) y_2 + y_3 = 0$

$y_2 + \left(\frac{\pi^3}{64} - 2\right) y_3 + y_4 = 0$

$y_4 = 2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & \frac{\pi^2}{64}-2 & 1 & & \\ & 1 & \frac{\pi^2}{64}-2 & 1 & 0 \\ & & 1 & \frac{\pi^2}{64} & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.7702 \\ 1.4215 \\ 1.8537 \\ 2.0000 \end{bmatrix}$$

since $y_i' = \frac{y_{i+1} - y_{i-1}}{2h} \Rightarrow \begin{bmatrix} y_1' \\ y_2' \\ y_3' \end{bmatrix} = \begin{bmatrix} 1.4215/2h \\ 1.0835/2h \\ 0.5785/2h \end{bmatrix} = \begin{bmatrix} 0.9050 \\ 0.6898 \\ 0.3683 \end{bmatrix}$

$y = 2\sin\left(\frac{\theta}{2}\right) \Rightarrow y' = \cos\left(\frac{\theta}{2}\right) \Rightarrow \begin{bmatrix} y'(\frac{\pi}{4}) \\ y'(\frac{\pi}{2}) \\ y'(\frac{3\pi}{4}) \end{bmatrix} = \begin{bmatrix} 0.9239 \\ 0.7071 \\ 0.3827 \end{bmatrix} \Rightarrow \text{error} = \begin{bmatrix} 0.0189 \\ 0.0173 \\ 0.0144 \end{bmatrix}$

(b)

5.

```
clear;
clc;
n = 15;
h = pi/n;
A = [1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0;
     1 h^2/4-2 1 0 0 0 0 0 0 0 0 0 0 0 0 0;
     0 1 h^2/4-2 1 0 0 0 0 0 0 0 0 0 0 0 0;
     0 0 1 h^2/4-2 1 0 0 0 0 0 0 0 0 0 0 0;
     0 0 0 1 h^2/4-2 1 0 0 0 0 0 0 0 0 0 0;
     0 0 0 0 1 h^2/4-2 1 0 0 0 0 0 0 0 0 0;
     0 0 0 0 0 1 h^2/4-2 1 0 0 0 0 0 0 0 0;
     0 0 0 0 0 0 1 h^2/4-2 1 0 0 0 0 0 0 0;
     0 0 0 0 0 0 0 1 h^2/4-2 1 0 0 0 0 0 0;
     0 0 0 0 0 0 0 0 1 h^2/4-2 1 0 0 0 0 0;
     0 0 0 0 0 0 0 0 0 1 h^2/4-2 1 0 0 0 0;
     0 0 0 0 0 0 0 0 0 0 1 h^2/4-2 1 0 0 0;
     0 0 0 0 0 0 0 0 0 0 0 1 h^2/4-2 1 0 0;
     0 0 0 0 0 0 0 0 0 0 0 0 1 h^2/4-2 1 0;
     0 0 0 0 0 0 0 0 0 0 0 0 0 1 h^2/4-2 1;
     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1];
b = [0; 0; 0; 0; 0; 0; 0; 0; 0; 0;0;0;0;0;0; 2];
angle = linspace(0,pi,n+1);
y = A\b;
dy = [];
for i = 2:n-1
    dy(i-1) = (y(i+1) - y(i-1))/(2*pi/n);
end
true = [];
for i = 2:n-1
    true(i-1) = cos(angle(i)/2);
end
dy
true
error = abs(dy-true)
error_rate = error./true
max_error_rate = max(error_rate)
```

Command Window

```
max_error_rate =

    0.0043
```

We use the matlab to calculate that we need h = pi/15 to have max error rate=0.0043< 0.005=0.5%

5.

5. $x''-tx'+t^2x=t^3 \Rightarrow x''=t^3-t^2x+tx'=f(t,x,x')$

$$\begin{cases} x(0)+x'(0)-x(1)+x'(1)=3 \\ x(0)-x'(0)+x(1)+x'(1)=2 \end{cases} \Rightarrow x_0=5/2 \quad , h=1/4$$

$$x_i'=\frac{x_{i+1}-x_{i-1}}{2h} \quad x_i''=\frac{x_{i+1}-2x_i+x_{i-1}}{h^2}$$

$$\Rightarrow (x_{i+1}-2x_i+x_{i-1})-h^2\times f\left(t_i,x_i,\frac{x_{i+1}-x_{i-1}}{2h}\right)=0$$

let $t_i=ih$

$$\Rightarrow x_{i+1}-2x_i+x_{i-1}-h^2\times t_i^3+h^2\times t_i^2x_i-h^2t_i\frac{x_{i+1}-x_{i-1}}{2h}=0$$

$$\Rightarrow (1-\tfrac{1}{2}ht_i)x_{i+1}+(-2+h^2t_i^2)x_i+(1+\tfrac{1}{2}ht_i)x_{i-1}=h^2t_i^3$$

$$\Rightarrow (1-\tfrac{1}{2}ih^2)x_{i+1}+(-2+i^2h^4)x_i+(1+\tfrac{1}{2}ih^2)x_{i-1}=i^3h^5$$

$$\Rightarrow x_0=5/2$$

$i=1$: $\frac{31}{32}x_2+\frac{-511}{256}x_1+\frac{33}{32}x_0=\frac{1}{1024}$

$i=2$: $\frac{30}{32}x_3+\frac{-508}{256}x_2+\frac{34}{32}x_1=\frac{8}{1024}$

$i=3$: $\frac{29}{32}x_4+\frac{-503}{256}x_3+\frac{35}{32}x_2=\frac{27}{1024}$

$$x(0)'-x(1)+x(1)'=1/2$$

$$x_0'-x_4+x_4'=1/2$$

$$\Rightarrow x_0'-x_4+\frac{x_4-x_3}{h}=1/2$$

$$\Rightarrow \frac{x_1-x_0}{h}-x_4+\frac{x_4-x_3}{h}=1/2$$

$$\Rightarrow x_0-x_1-x_3+x_4-hx_4=\tfrac{1}{2}h$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{33}{32} & \frac{-511}{256} & \frac{31}{32} & 0 & 0 \\ 0 & \frac{34}{32} & \frac{-508}{256} & \frac{30}{32} & 0 \\ 0 & 0 & \frac{35}{32} & \frac{-503}{256} & \frac{27}{32} \\ 1 & -1 & 0 & -1 & \frac{3}{4} \end{bmatrix}\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}=\begin{bmatrix} 5/2 \\ 1/1024 \\ 8/1024 \\ 27/1024 \\ 1/8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}=\begin{bmatrix} x(0) \\ x(0.25) \\ x(0.5) \\ x(0.75) \\ x(1) \end{bmatrix}=\begin{bmatrix} 2.5 \\ 1.741 \\ 0.727 \\ -0.782 \\ -2.154 \end{bmatrix}$$