DSP Additionals.

(1) $$x[n] = \begin{cases} 1+\frac{n}{3} & ; -3 \le n \le -1 \\ 1 & ; 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

(a) determine its values and sketch the signal $x(n)$.

$x(-3) = 1 + \frac{-3}{3} = 0$

$x(-2) = 1 - \frac{2}{3} = 0.33$

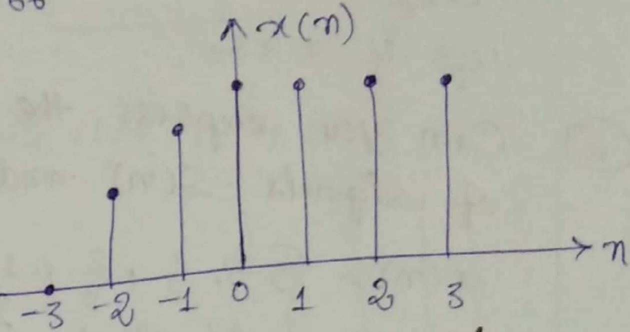
$x(-1) = 1 - \frac{1}{3} = 0.66$

$x(0) = 1$

$x(1) = 1$

$x(2) = 1$

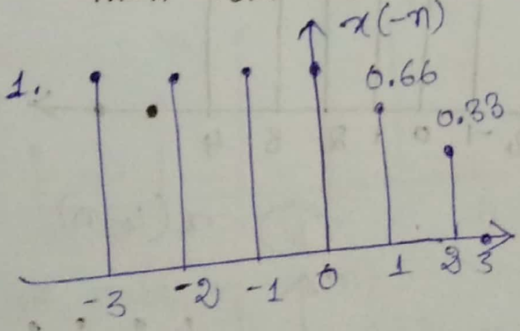
$x(3) = 1$

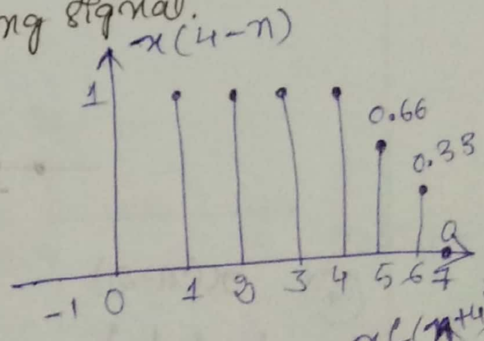


(b) Sketch the sigs that result if we:

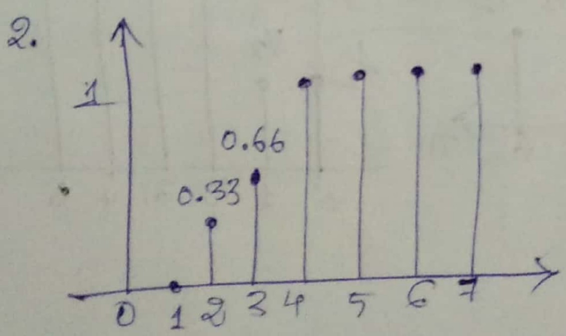
1. First fold $x(n)$ and then delay the resulting signal by four samples.
2. First delay $x(n)$ by four samples and then fold the resulting signal.

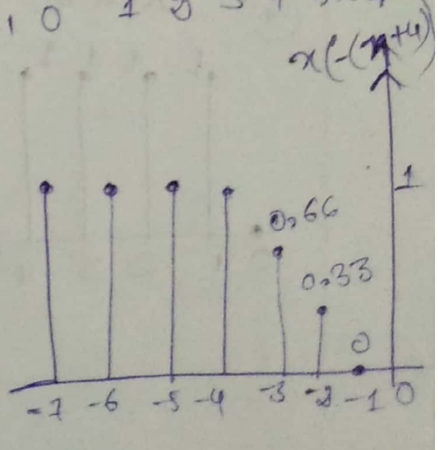
1.





2.





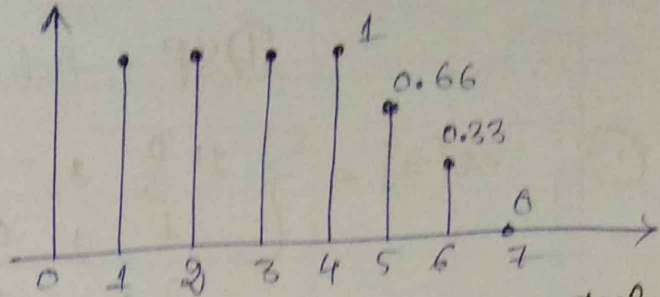
(c) Sketch the signal $x(-n+4)$

$-8 < -n+4 < 8$

$-7 < -n < -1$

$7 > n > 1$

$1 < n < 7$



(d) Compare the results in parts (b) & (c) and derive a rule for obtaining the signal $x(-n+k)$ from $x(n)$.

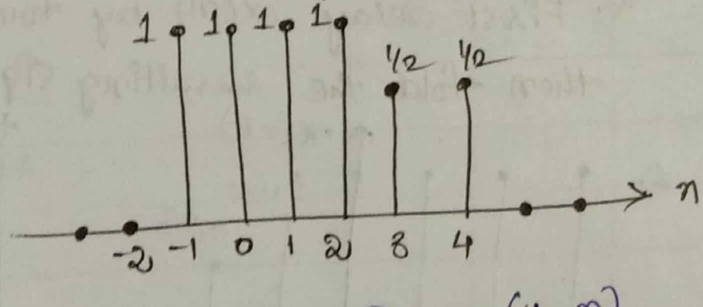
To obtain $x(-n+k)$, first fold $x(-n)$ which gives $x(-n)$. Then we have to shift by k samples to the right if $k>0$, or k samples left if $k<0$.

(e) Can you express the signal $x(n)$ in terms of signals $\delta(n)$ and $u(n)$?

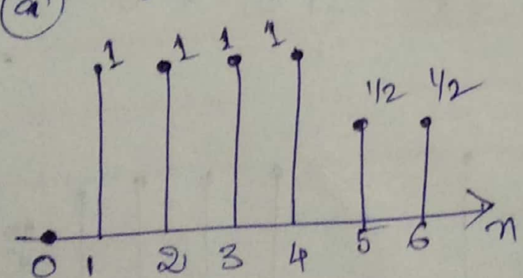
$$x(n) = \left\{ \begin{matrix} 0, & \frac{1}{3}, & \frac{2}{3}, & 1, & 1, & 1, & 1 \\ -3 & -2 & -1 & \underset{\uparrow}{0} & & & \end{matrix} \right\}$$

$$x(n) = \frac{1}{3}\,\delta(n-2) + \frac{2}{3}\,\delta(n-1) + \delta(n) + u(n) - u(n-4)$$

(2) A discrete-time signal $x(n)$ is shown in fig. Sketch the label carefully each of the following sig.



(a) $x(n-2)$

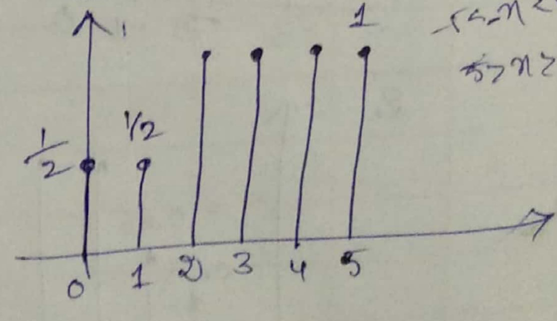


(b) $x(4-n)$

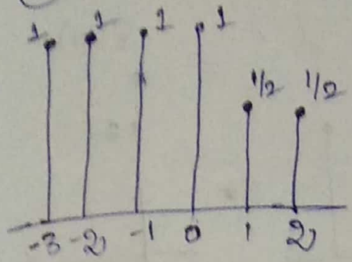
$-1 < 4-n < 4$

$-5 < -n < 0$

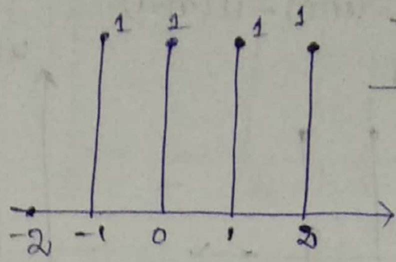
$5 > n > 0$



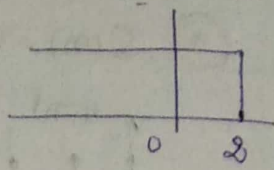
(c) $x(n+2)$



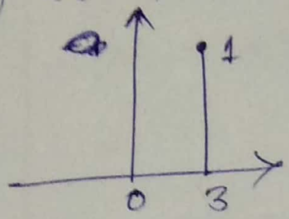
(d) $x(n)\,u(2-n)$



$u(2-n)$



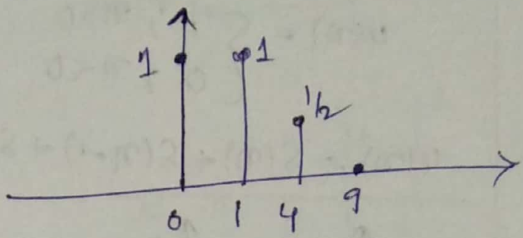
(e) $x(n-1)\,\delta(n-3)$



(f) $x(n^2)$

$x(9)=0 \quad x(4)=1/2 \quad x(1)=1 \quad x(0)=1$

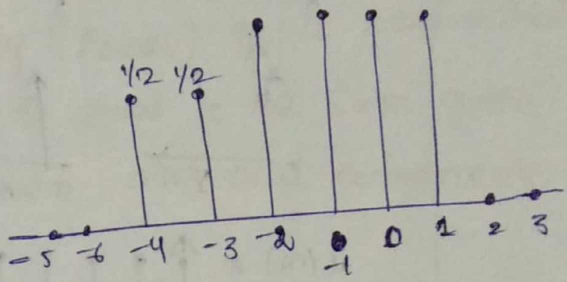
$x(1)=1 \quad x(4)=1/2 \quad x(9)=0$



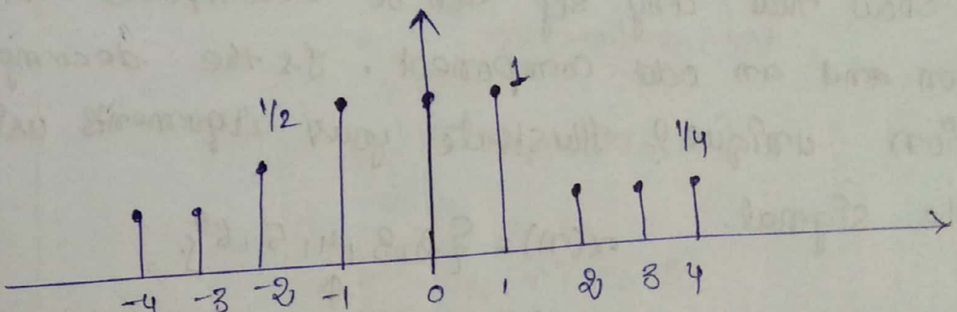
$\{0, 1/2, 1, 1, 1, 1/2, 0\}$

(g) even part of $x(n)$

$$x_e(n) = \frac{x(n)+x(n)}{2}$$



$$x_e(n) = \left\{ \frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 1, \underset{\uparrow}{1}, 1, \frac{1}{4}, \frac{1}{4}, \frac{1}{4} \right\}$$

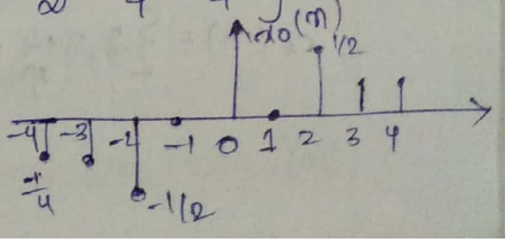


$x_e(n)$

(h) Odd part of $x(n)$

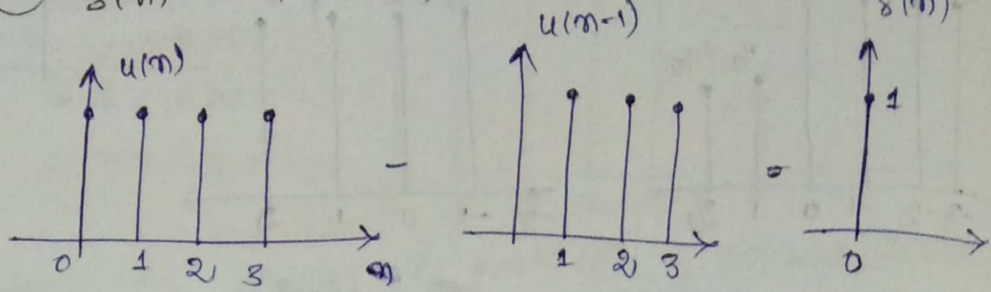
$$x_o(n) = \frac{x(n)-x(-n)}{2}$$

$$x_o(n) = \left\{ \frac{-1}{4}, \frac{-1}{4}, \frac{-1}{2}, 0, \underset{\uparrow}{0}, 0, \frac{1}{2}, \frac{1}{4}, \frac{1}{4} \right\}$$



(3) Show that

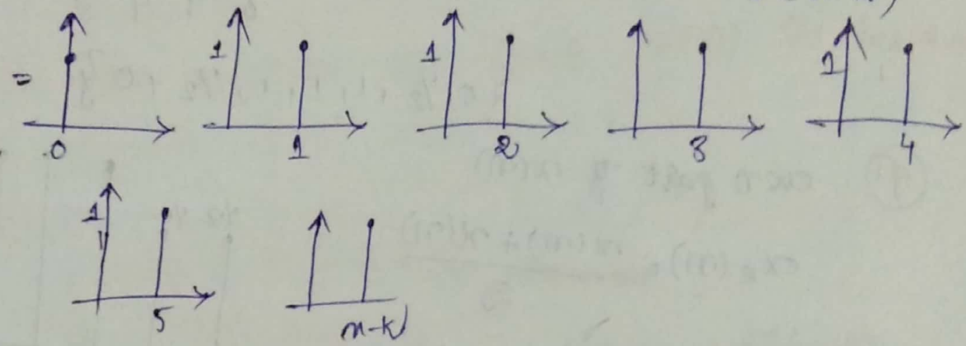
(a) $\delta(n) = u(n) - u(n-1)$

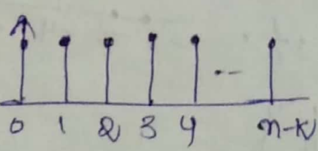


(b) $u(n) = \sum_{k=-\infty}^{n} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k)$

$$u(n) = \begin{cases} 1 \; ; & n > 0 \\ 0 \; ; & n < 0 \end{cases}$$

$u(n) = \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3) + \delta(n-4) + \delta(n-5)$ ---- $\delta(n-k)$



$u(n) =$  $= \sum_{k=0}^{\infty} \delta(n-k)$

(4) Show that any sig can be decomposed into an even and an odd component. Is the decomposition unique? Illustrate your arguments using the signal.

$$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$$

Ans

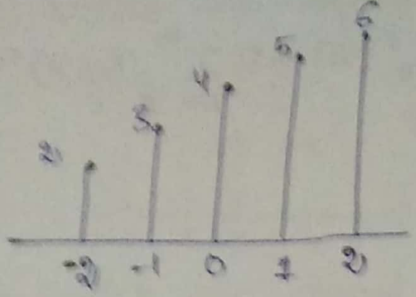
$$x_e(n) = \frac{x(n) + x(-n)}{2} \qquad x_e(n) = x_e(-n)$$

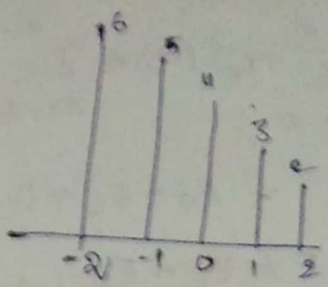
$$x_o(n) = \frac{x(n) - x(-n)}{2} \qquad x_o(n) = -x_o(n)$$

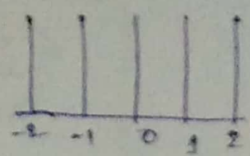
$$\Rightarrow x(n) = x_e(n) + x_o(n)$$

$$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$$

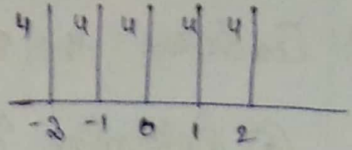
$x(-n)$





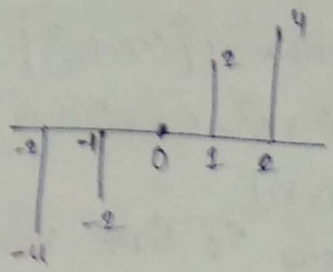


$$x_e(n) = \frac{x_e(n) + x(-n)}{2} \Rightarrow$$

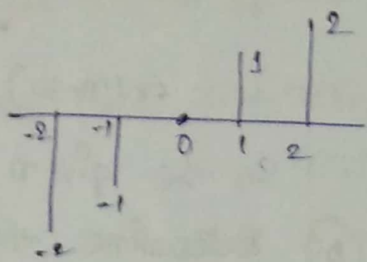


$x(n) - x(-n)$

$\Rightarrow$



$$x_o(n) = \frac{x(n) - x(-n)}{2}$$



(5) Show that the energy (Power) of a real-valued energy (Power) signal is equal to the sum of the energies (Power) of its even and odd components.

Sol First prove

$$\sum_{n=-\infty}^{\infty} x_e(n) + x_o(n) = 0 \Rightarrow \sum_{n=-\infty}^{\infty} x_e(n) + x_o(n)$$

$$= \sum_{m=-\infty}^{\infty} x_e(-m)\, x_o(-m)$$

$$= -\sum_{m=-\infty}^{\infty} x_e(m)\, x_o(m)$$

$$= -\sum_{m=-\infty}^{\infty} x_e(n)\, x_o(n)$$

$$= \sum_{n=-\infty}^{\infty} x_e(n)\, x_o(n) \underline{\underline{=0}}$$

energies (power)

$$\sum_{n=-\infty}^{\infty} x^2(n) = \sum_{n=-\infty}^{\infty} \left(x_e(n) + x_o(n)\right)^2$$

$$= \sum_{n=-\infty}^{\infty} x_e^2(n) + x_o^2(n) + 2x_e(n)\, x_o(n)$$

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$$= \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n) + 2\sum_{n=-\infty}^{\infty} x_e(n)\,x_o(n)$$

$$= E_e + E_o + 0$$

$$E = E_e + E_o$$

(6) Consider the s/m $y(n) = T[x(n)] = x(n^2)$

(a) Determine if the s/m is time invariant

Given $y(n) = T[x(n)] = x(n^2)$

$$x(n-k) \rightarrow y_1(n) = x[(n-k)^2]$$

$$= x(n^2 + k^2 - 2nk)$$

$$x(n-k) \neq y(n-k)$$

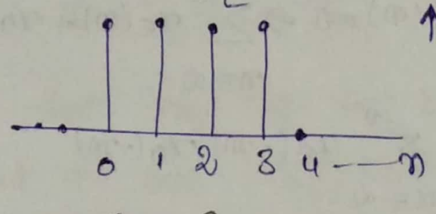
So the given system is time variant

(b) Determine the result in part (a) assume that the signal $x(n) = \begin{cases} 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$

is applied into the s/m.

(1) Sketch the signal $x(n)$

$$x(n) = \{ \ldots 0, \underset{\uparrow}{1}, 1, 1, 1, 0, 0, \ldots \}$$

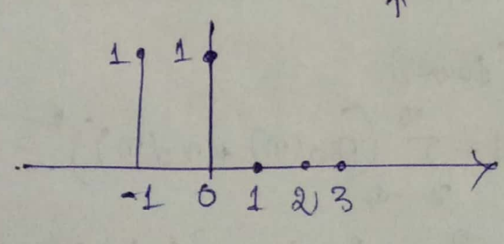


(2) Determine & sketch the signal $y(n) = T[x(n)]$

$$y(n) = T[x(n)] = x(n^2) \Rightarrow \{x(0), x(1), x(2^2), x(3^2), \ldots\}$$

$$\Rightarrow \{x(0), x(1), x(4), x(9), x(16) \ldots\}$$

$$y(n) = x(n^2) = \{ \ldots 1, \underset{\uparrow}{1}, 0, 0, 0, \ldots \}$$



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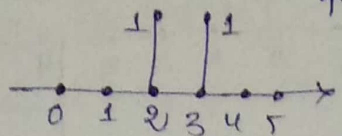
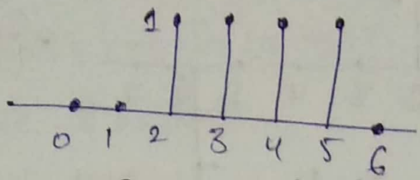
(3) Sketch the signal $y_2'(n) = y(n-2)$.

$y(n-2) = \{ \cdots 0, 0, 1, 1, 0, 0, 0 \cdots \}$
$\uparrow$



(4) Sketch the signal $x_2(n) = x(n-2)$.

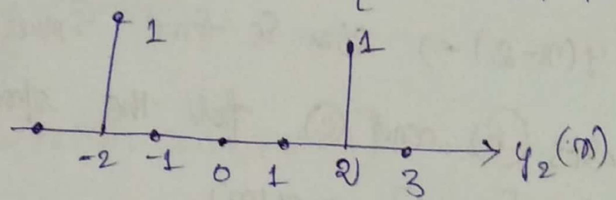
$x(n-2) = \{ \cdots 0, 0, 1, 1, 1, 1, 0, \cdots \}$
$\uparrow$



(5) Determine & sketch the signal $y_2(n) = T[x_2(n)]$

$y_2(n) = T[x_2(n-2)] = \{ x(0), x(1), x(2), x(3), x(4), x(5), x(6) \}$

$= \{ \cdots 0, 1, 0, 0, 0, 1, 0 \cdots \}$
$\uparrow$



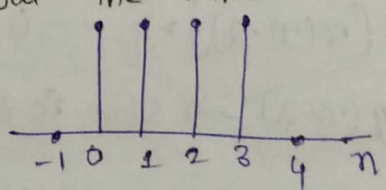
(6) Compare the signal $y_2(n)$ and $y(n-2)$.
What is conclusion?

$y_2(n) \neq y(n-2) \rightarrow$ system is time variant

(c) Repeat part (b) for the s/m.

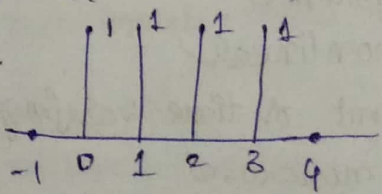
$y(n) = x(n) - x(n-1)$

Can you use this result to make any statement about the time invariance of the s/m.

(1) $x(n) =$  $= \{ 1, 1, 1, 1 \}$
$\uparrow$

(2) $y(n) = x(n) - x(n-1)$

$x(n) -$

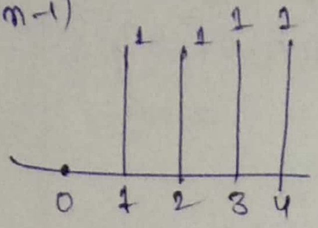


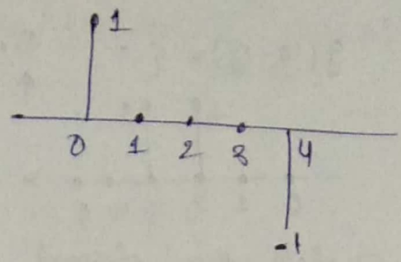
y(n).

$y(n) = x(n) - x(n-1)$

$y(n) \Rightarrow x(n) - x(n-1)$

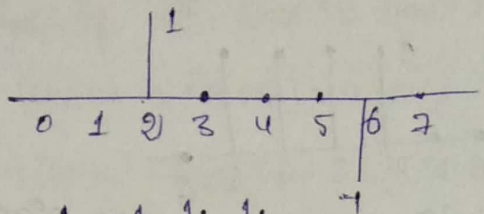
$x(n-1)$





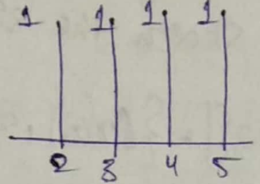
$\rightarrow \{0, 1, 0, 0, 0, -1\}$
$\uparrow$

(3) $y(n-2) \Rightarrow$



$= \{0, 0, 1, 0, 0, 0, -1\}$

(4) $x(n-2) \Rightarrow$



$\Rightarrow \{0, 0, 1, 1, 1, 1\}$

(5) $y_2(n) = \{0, 0, 1, 0, 0, 0, -1\}$

(6) $y_2(n) = y(n-2) \rightarrow$ s/m is time invariant.

(d) Repeat parts (b) and (c) for the s/m:
$y(n) = T[x(n)] = n\,x(n)$
$n \rightarrow$ integer value from 0 –

1) $y(n) = n\,x(n)$
$x(n) = \{ \ldots 0, 1, 1, 1, 1, 1, 0, \ldots \}$

(2) $y(n) = \{ \ldots 0, 1, 2, 3, 4, \ldots \}$
$\uparrow$

(3) $y(n-2) = \{ \ldots 0, 0, 0, 1, 2, 3, 4 \ldots \}$
$\uparrow$

(4) $x(n-2) = \{ \ldots 0, 0, 0, 1, 1, 1, 1 \ldots \}$
$\uparrow$

(5) $y_2(n) = T[x(n-2)] = \{ \ldots 0, 0, 2, 3, 4, 5 \ldots \}$

(6) $y_2(n) \neq y(n-2) \Rightarrow$ S/m is time variant

(7) A discrete-time system can be
1. Static or dynamic
2. Linear or nonlinear
3. Time invariant or time varying
4. Causal or noncausal
5. Stable or unstable

Examine the following systems with respect to the properties above.

(a) $y(n) = \cos[x(n)]$

i] static (only present i/p)

iv) only present i/p → causal

v) stable

ii) $y_1(n) = \cos[x_1(n)]$

$y_2(n) = \cos[x_2(n)]$

$y(n) = \cos[x_1(n)] + \cos[x_2(n)]$

$y'(n) = \cos[x_1(n) + x_2(n)]$

⇒ Non-linear

iii] $y(n) = \cos[x(n-n_0)]$

$y(n) = \cos[x(n-n_0)]$

⇒ Time variant

(b) $y(n) = \sum_{k=-\infty}^{n+1} x(k)$

Dynamic (depends on future values)

linear, time invariant, non causal (also depends on future values).

unstable.

(c) $y(n) = x(n) \cos[w_0(n)]$

Static, linear, Time varient, Causal, stable

$y(n) = x(n-n_0) \cos[w_0(n-n_0)]$

$y'(n) = x(n-n_0) \cos w_0 n$

(d) $y(n) = x(-n+2)$

Dynamic

at $n=0 \Rightarrow y(0) = x(2)$

future value

$y_1(n) = x_1(-n+2) + x_2(-n+2)$

$y_2(n) = (x + x_2(-n+2))$

$\Rightarrow x_1(-n+2) + x_2(-n+2)$

→ linear

Non Causal, stable, Time invariant

(e) $y(n) = \text{Trun}[x(n)]$

Static, non-linear, time invariant, causal, stable

(f) $y(n) = \text{Round}[x(n)]$

Static, non-linear, time invariant, Causal, stable.

g) $y(n) = |x(n)|$

Static, non-linear, time invariant, causal, stable

(h) $y(n) = x(n)\,u(n)$

Static, linear, time invariant, causal, stable.

(i) $y(n) = x(n) + n\,x(n+1)$

Dynamic, linear, time variant, Non-causal, Stable

(j) $y(n) = x(2n)$

Dynamic, linear, Time variant, Non-causal, Stable

(k)
$$y(n) = \begin{cases} x(n), & \text{if } x(n) \geq 0 \\ 0, & \text{if } x(n) < 0 \end{cases}$$

Static, linear, time invariant, Non-Causal, Stable

(l) $y(n) = x(-n)$

Dynamic, linear, Time variant, Causal, stable.

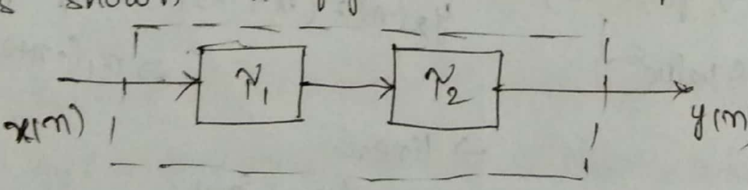
(m) $y(n) = \mathrm{Sign}[x(n)]$

static, non linear, Time invariant, causal, Stable

(n) $x(n) = x_a(nT) \qquad -\infty < n < \infty$

Static, linear, time variant, non-causal, Stable.

(8) Two discrete-time systems $\mathcal{T}_1$ and $\mathcal{T}_2$ are connected in cascade to form a new system $\mathcal{T}$ as shown in fig. Prove or disprove the following



$\mathcal{T} = \mathcal{T}_1 \mathcal{T}_2$

(a) If $\mathcal{T}_1$ and $\mathcal{T}_2$ are linear, then $\mathcal{T}$ is linear (i.e. the cascade connection of two linear systems is linear).

True if $v_1(n) = \mathcal{T}_1[x_1(n)]$ and $v_2(n) = \mathcal{T}_1[x_2(n)]$

then

$\alpha_1 x_1(n) + \alpha_2 x_2(n)$ yields
$\alpha_1 v_1(n) + \alpha_2 v_2(n)$

By the linearity property of τ_1 similarly, if
$y_1(n) = \tau_2 [v_1(n)]$ and
$y_2(n) = \tau_2 [v_2(n)]$

then $\beta_1 v_1(n) + \beta_2 v_2(n) \rightarrow y(n) = \beta_1 y_1(n) + \beta_2 y_2(n)$

by the linearity property of τ_2, since
$v_1(n) = \tau_1 [x_1(n)]$ and
$v_2(n) = \tau_2 [x_2(n)]$

it follows that, $A_1 x_1(n) + A_2 x_2(n)$
yields the output
$A_1 \tau [x_1(n)] + A_2 \tau [x_2(n)]$
where $\tau = \tau_1 \tau_2$ Hence τ is linear

(b) If τ_1 and τ_2 are time invariant, then τ is time invariant.

True for τ_1, if $x(n) \rightarrow v(n)$ &
$x(n-k) \rightarrow v(n-k)$

for τ_2, if
$v(n) \rightarrow y(n)$
and $v(n-k) \rightarrow y(n-k)$

Hence, for $\tau_1 \tau_2$ if $x(n) \rightarrow y(n)$
$x(n-k) \rightarrow y(n-k)$

$\tau = \tau_1 \tau_2$ is time invariant.

(c) If τ_1 and τ_2 are causal, then τ is causal.

True, τ_1 is causal $\Rightarrow v(n)$ depends only on $x(k)$ for $k \leq n$. τ_2 is causal $\Rightarrow y(n)$ depends only on $v(k)$ for $k \leq n$. Therefore, $y(n)$ depends only on $x(k)$ for $k \leq n$. Hence τ is causal.

(d) If τ_1 and τ_2 are linear and time invariant the same holds for τ.

True. Combine (a) and (b) then we will get to understand that τ_1 and τ_2 are linear and time invariant that holds for γ.

(e) If γ_1 and γ_2 are linear and time invariant then interchanging their order doesn't change the s/m γ.

True this follows from $h_1(n) * h_2(n) = h_2(n) * h_1(n)$

(f) As in part (e) except that γ_1, γ_2 are now time varying.

false. Ex:- $\gamma_1 : y(n) = n x(n)$

$\gamma_2 : y(n) = n x(n+1)$

$\gamma_2 \cdot [\gamma_1 [\delta(n)]] = \gamma_2 (0) = 0$

$\gamma_1 \cdot [\gamma_2 [\delta(n)]] = \gamma_1 [\delta(n+1)]$

$= -\delta(n+1)$

$\neq 0$

(g) If γ_1 and γ_2 are non linear, then γ is nonlinear.

false. Ex:- $\gamma_1 : y(n) = x(n) + b$ and

$\gamma_2 : y(n) = x(n) - b$, where $b \neq 0$

$\gamma[x(n)] = \gamma_2 [\gamma_1 [x(n)]] = \gamma_2 [x(n) + b] = x(n)$

Hence γ is linear

(h) If γ_1 and γ_2 are stable, then γ is stable

True. γ_1 is stable $\Rightarrow v(n)$ is bounded if $x(n)$ is bounded

γ_2 is stable $\Rightarrow y(n)$ is bounded if $v(n)$ is bounded.

Hence, $y(n)$ is bounded if $x(n)$ is bounded $\Rightarrow$

$\gamma = \gamma_1 \gamma_2$ is stable.

(i) Show by an example that the inverse of parts (c) and (h) do not hold in general.

Inverse of (c). γ_1 and for γ_2 are noncausal

$\Rightarrow \gamma$ is noncausal ex:- $\gamma_1 : y(n) = x(n+1)$. &

$\gamma_2 : y(n) = x(n-2)$

$\gamma : y(n) = x(n-1)$

which is causal, Hence the inverse of (c) is false.

Inverse of (h); γ_1 and / or γ_2 is unstable, implies γ is unstable. Ex: $\gamma_1: y(n) = e^{x(n)}$, Stable and ~~Another~~ than $\gamma_2: y(n) = \ln[x(n)]$ which is unstable.

But $\gamma: y(n) = x(n)$, which is stable. Hence the inverse of (h) is false.

(9) Let γ be an LTI, relaxed and BIBO stable s/m with i/p $x(n)$ and o/p $y(n)$. Show that

(a) If $x(n)$ is periodic with period N [i.e. $x(n) = x(n+N)$ for all $n \geq 0$] the o/p $y(n)$ tends to a periodic sig with the same period.

$$x(n) = x(n+N) \quad \forall n \geq 0$$

$$y(n) = \sum_{K=-\infty}^{\infty} h(K)\, x(h-K)$$

$$y(n+N) = \sum_{K=-\infty}^{n+N} h(K)\, x(n+N-k)$$

$$y(n+N) = \sum_{k=n+1}^{n+N} h(K)\, x(n-K) + \sum_{K=-\infty}^{n} h(K)\, x(n-K)$$

$$y(n+N) = y(n) + \sum_{K=n+1}^{n+N} h(k)\, x(n-k)$$

for BIBO s/m $\lim_{n\to\infty} |h(n)| = 0$

$$\lim_{n\to\infty} \sum_{k=n+1}^{n+N} h(k)\, x(n-k) = 0$$

$$\lim_{n\to\infty} y(n+N) = y(N)$$

$$\therefore y(N) = y(n+N)$$

(b) If $x(n)$ is bounded and tends to a constant, the o/p will also tend to a constant.

$x(n) = x_0(n) + a\,u(n)$ $\quad x_0(n) \to$ bounded with $\lim_{n\to\infty} x_0(n) = 0$

$$\sum_n x_0^2(n) < \infty \Rightarrow \sum_n y_0^2(n) < \infty.$$

hence $\lim_{n\to\infty} |y_0(n)| = 0 = a \sum_{k=0}^{n} h(K) = \text{constant}$

(c) If $x(n)$ is an energy signal, the output $y(n)$ will also be an energy signal.

$$y(n) = \sum_{k} h(k)\, x(n-k)$$

$$\sum_{-\infty}^{\infty} y^2(n) = \sum_{-\infty}^{\infty} \left[\sum_{k} h(k)\, x(n-k) \right]^2$$

$$= \sum_{k} \sum_{l} h(k)\, h(l) \sum_{n} x(n-k)\, x(n-l)$$

but $\sum_{n} x(n-k)\, x(n-l) \leq \sum_{n} x^2(n)\, |h(l)|$

for BIBO stable s/m $\sum_{k} |h(k)| < M$

Hence $E_y \leq M^2 E_x$, so that $E_y < 0$ if $E_x <$

(10) The following i/p - o/p pairs have been observ[ed] during the operation of a time invariant s/m

$$x_1(n) = \{1, \underset{\uparrow}{0}, 2\} \xleftrightarrow{T} y_1(n) = \{0, \underset{\uparrow}{1}, 2\}$$

$$x_2(n) = \{\underset{\uparrow}{0}, 0, 3\} \xleftrightarrow{T} y_2(n) = \{0, 1, \underset{\uparrow}{0}, 2\}$$

$$x_3(n) = \{\underset{\uparrow}{0}, 0, 0, 1\} \xleftrightarrow{T} y_3(n) = \{1, 2, 1\}$$

Can you draw any conclusions regarding the linearity of the s/m. What is the impulse response of the s/m?

Sol As this is a time-invariant s/m.

$y_2(n)$ should have only 3 elements and $y_3(n)$ should have 4 elements.

So, it is non linear

(11) The following input-output pairs have bee[n] observed during the operation of a linear s/m.

$$x_1(n) = \{-1, 2, 1\} \xrightarrow{T} y_1(n) = \{1, \underset{\uparrow}{2}, -1, 0, 1\}$$

$$x_2(n) = \{1, -1, -1\} \xrightarrow{T} y_2(n) = \{-1, \underset{\uparrow}{1}, 0, 2\}$$

$$x_3(n) = \{0, 1\} \xrightarrow{T} y_3(n) = \{\underset{\uparrow}{1}, 2, 1\}$$

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Can you draw any conclusions about the time invariance of this system?

$x_1(n) + x_2(n) = \delta(n)$

& system is linear the impulse response of the s/m is $y_1(n) + y_2(n) = \{0, 3, -1, 2, 1\}$
$\uparrow$

if s/m were time invariant the response of $x_3(n)$ would be $\{3, 2, 1, 3, 1\}$
$\uparrow$

(12) The only available information about the s/m consists of N input-output pairs, of s/gs

$y_i(n) = \tau[x_i(n)]$, $i = 1, 2, \ldots N$.

(a) what is the class of input s/gs for which we can determine the o/p, using the information above, if the s/m is known to be linear?

linear combination of signal in the form of $x_i(n)$ & $i = 1, 2, \ldots N$

bcoz if we take $i = 1, 3$

$y_1(n) = x_1(n)$

$y_3(n) = x_3(n)$

$\Rightarrow y(n) = y_1(n) + y_3(n) = x_1(n) + x_3(n)$

linear

(b) Same repeat, if the s/m is known to be time invariant

Any $x_i(n-k)$ where k is any integer, $i = 1, 2, - N$

1st replace $n = n - n_0 \Rightarrow x_i(n - n_0 - k)$

$x(n)$ by $x(n - n_0) \Rightarrow x_i(n - k - n_0)$

(time invariant)

(13) Show that the necessary and sufficient condition for a relaxed LTI system to be BIBO stable is $\sum_{n=-\infty}^{\infty} |h(n)| \leq M_h < \infty$

for some constant M_n,

→ A s/m to be BIBO stable only when bounded output should produce bounded input

$$y(n) = \sum_{k} h(k) \cdot x(n-k)$$

$$|y(n)| = \sum_{k} |h(k)| \, |x(n-k)|$$

$$= \sum_{k} |x(n-k)| \leq M_n \quad \{\text{some cons}$$

So $|y(n)| = M_n \sum_{k} |h(k)|$

$|y(n)| < \infty$ for all n, if and only if

$\sum_{k} |h(k)| < \infty$ So $\sum_{n=-\infty}^{\infty} |y(n)|$

→ A s/m to be BIBO stable only when bounded input produce bounded output.

$$y(n) = \sum_{k=-\infty}^{\infty} h(k) \, x(n-k) \; ; \; n \leq n-k, \; k \geq 0$$

$$|y(n)| = \sum_{k=-\infty}^{\infty} |h(k)| \cdot |x(n-k)|$$

as $\sum_{k=-\infty}^{\infty} |x(n-k)| \leq M_n$ for some constant

$$|y(n)| = M_n \sum_{k=-\infty}^{\infty} |h(k)| \; ; \; n \leq n-k, \; k \geq 0$$

$|y(n)|$ is $< \infty$, if & only if $\sum_{k=-\infty}^{\infty} |h(k)| < \infty$

So $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$

14. Show that

(a) A relaxed linear s/m is causal if & only if for any input $x(n)$ such that

$x(n) = 0$ for $n < n_0 \Rightarrow y(n) = 0$ for $n < n_0$

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If a system is causal output depends only on the present and past input as $x(n)=0$ for $n<n_0$. the $y(n)$ also becomes zero for $n<n_0$

(b) A relaxed LTI s/m is causal if & only if $h(n)=0$ for $n<0$

$$y(n)=\sum_{k=-\infty}^{\infty} h(k)\,x(n-k)$$

for finite impulse response

$h(n)=0,\ n<0$ and $n\geq m$

so $y(n)$ reduces to $y(n)=\sum_{k^{n}}^{n-1} h(k)\,a(n-k)$

If it is infinite impulse response then $y(n)=\sum_{k=0}^{\infty} h(k)\,x(n-k)$

(15) (a) Show that for any real or complex constant a, and any finite integer numbers M and N, we have

$$\sum_{n=M}^{N} a^n = \begin{cases} \dfrac{a^M-a^{N+1}}{1-a}, & \text{if } a\neq 1 \\ N-M+1, & \text{if } a=1 \end{cases}$$

for $a=1$, $\sum_{n=M}^{N} a^n = N-M+1$

for $a\neq 1$, $\sum_{n=m}^{N} a^n = \dfrac{a^m-a^{N+1}}{1-a}$

$$(1-a)\sum_{n=M}^{N} a^n = a^m + a^{m+1} - a^{m+1} + \dots - a^N - a^N - a^{N+1}$$

$$= a^M - a^{N+1}$$

(b) Show that if $|a|<1$, then

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

For $M=0$, $|a|<1$ and $N \to \infty$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, \quad |a|<1$$

(16) (a) If $y(n) = x(n) * h(n)$, show that $\Sigma_y = \Sigma_x \Sigma_h$

where $\Sigma_x = \sum_{n=-\infty}^{\infty} x(n)$.

$$y(n) = \sum_{k} h(k)\, x(n-k)$$

$$\sum_{n} y(n) = \sum_{n} \sum_{k} h(k)\, x(n-k)$$

$$\sum_{n} y(n) = \sum_{k} h(k) \cdot \sum_{n=-\infty}^{\infty} x(n-k)$$

$$\sum_{n} y(n) = \left(\sum_{k} h(k)\right)\left(\sum_{n} x(n)\right)$$

(b) Compute the convolution $y(n) = x(n) * h(n)$ of the following signals and check the correctness of the results by using the test (a).

(i) $x(n) = \{1, 2, 4\}$, $h(n) = \{1, 1, 1, 1, 1\}$

$$y(n) = \{1, 3, 7, 7, 7, 6, 4\}$$

$\sum_{n} y(n) = 35$; $\sum_{n} x(n) = 7$, $\sum_{n} y(n) = 5$

$$\sum_{n} y(n) = \sum_{n} x(n) \sum_{n} h(n)$$

$35 = 7 \times 5$

$35 = 35$

Tabular Method

$h(n)$ \ $x(n)$	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4

(2) $x(n) = \{1,2,-1\}$, $h(n) = x(n)$

$x(n) = \{1,2,-1\}$, $h(n) = \{1,2,-1\}$

$h(n)$ \ $x(n)$	1	2	-1
1	1	2	-1
2	2	4	-2
-1	-1	-2	1

$y(n) = x(n) * h(n)$

$y(n) = \{1,4,2,-4,1\}$

$\sum_n y(n) = 4$; $\sum_n x(n) = 2$, $\sum_n h(n) = 2$

$\sum_n y(n) = \sum_n (x) * \sum_n h(n)$

$4 = 4$

(3) $x(n) = \{0,1,-2,3,-4\}$, $h(n) = \{\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}\}$

$y(n) = \{0, \frac{1}{2}, \frac{-1}{2}, \frac{3}{2}, -2, 0, \frac{-5}{2}, 2\}$

$h(n)$ \ $x(n)$	0	1	-2	3	-4
$\frac{1}{2}$	0	1/2	-1	3/2	-2
$\frac{1}{2}$	0	1/2	-1	3/2	-2
1	0	1	-2	3	-4
$\frac{1}{2}$	0	1/2	-1	3/2	-2

$\sum_n y(n) = -5$, $\sum_n x(n) = -2$,

$\sum_n h(n) = 5/2$

$\sum_n y(n) = \sum_n x(n) \, h(n)$

$-5 = 5$

(4) $x(n) = \{1,2,3,4,5\}$, $h(n) = \{1\}$

$y(n) = \{1,2,3,4,5\}$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$15 = 15$

(5) $x(n) = \{1,2,3\}$, $h(n) = \{0,0,1,1,1,1\}$

$y(n) = \{0,0,1,-1,2,2,1,3\}$

$h(n)$ \ $x(n)$	1	-2	3
0	0	0	0
0	0	0	0
1	1	-2	3
1	1	-2	3
1	1	-2	3
1	1	-2	3

$\sum_n y(n) = 8$; $\sum_n x(n) = 2$;

$\sum_n h(n) = 4$

$\sum_n y(n) = \sum_n x(n) \cdot \sum_n h(n)$ $\quad 8 = 8$

(6) $x(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1\}$, $h(n) = \{1, \underset{\uparrow}{-2}, 3\}$

$y(n) = \{\underset{\uparrow}{0}, 0, 1, -1, 2, 2, 1, 3\}$

$h(n) \backslash x(n)$	0	0	1	1	1	1
1	0	0	1	1	1	1
-2	0	0	-2	-2	-2	-2
3	0	0	3	3	3	3

$\sum_n y(n) = 8$, $\sum_n x(n) = 4$, $\sum_n h(n) = 2$

$\sum_n y(n) = \sum_n x(n) \sum h(n)$

$8 = 8$

(7) $x(n) = \{\underset{\uparrow}{0}, 1, 4, -3\}$,

$h(n) = \{\underset{\uparrow}{1}, 0, -1, -1\}$.

$y(n) = \{\underset{\uparrow}{0}, 1, 4, -4, -5, -1, 3\}$

$h(n) \backslash x(n)$	0	1	4	-3
1	0	1	4	-3
0	0	0	0	0
-1	0	-1	-4	3
-1	0	-1	-4	3

$\sum_n y(n) = -2$; $\sum_n x(n) = -2$;

$\sum_n h(n) = 1$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

(8) $x(n) = \{\underset{\uparrow}{1}, 1, 2\}$, $h(n) = u(n)$

$y(n) = \{\underset{\uparrow}{1}, 2, 4, 3, 2\}$

$h(n) \backslash x(n)$	1	1	2
1	1	1	2
1	1	1	2
1	1	1	2

$\sum_n y(n) = 12$; $\sum_n x(n) = 4$;

$\sum_n h(n) = 3$.

$\sum_n y(n) = \sum_n x(n) \sum h(n)$

$12 = 4 \times 3$

(9) $x(n) = \{1, 1, 0, 1, 1\}$, $h(n) = \{1, -2, -3, \underset{\uparrow}{4}\}$

$y(n) = \{1, -1, -5, 2, 3, -5, 1, 4\}$

$h(n) \backslash x(n)$	1	1	0	1	1
1	1	1	0	1	1
-2	-2	-2	0	-2	-2
-3	-3	-3	0	-3	-3
-4	-4	-4	0	-4	-4

$\sum_n y(n) = 0$ $\sum_n x(n) = 4$

$\sum_n h(n) = 0$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$0 = 0$

(10) $x(n) = \{1, 2, \underset{\uparrow}{0}, 2, 1\}$. $h(n) = x(n)$

$y(n) = \{1, 4, 4, 4, \underset{\uparrow}{10}, 4, 4, 4, 1\}$

$h(n) \backslash x(n)$	1	2	0	2	1
1	1	2	0	2	1
2	2	4	0	4	2
0	0	0	0	0	0
2	2	4	0	4	2
1	1	2	0	2	1

$\sum_n y(n) = 36 \quad \sum_n x(n) = 6 \quad \sum_n h(n) = 6$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$36 = 36$

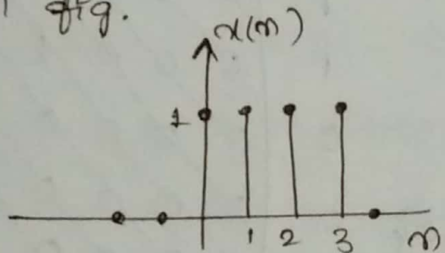
(11) $x(n) = \left(\frac{1}{2}\right)^n u(n), \; h(n) = \left(\frac{1}{4}\right)^n u(n)$

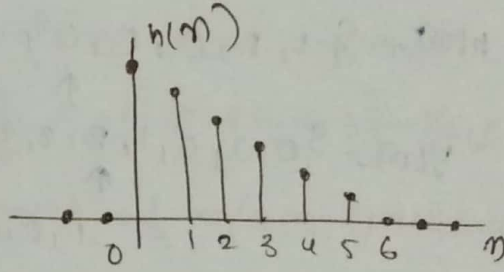
$y(n) = \left(2\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n u(n)\right)$

$\sum_n y(n) = \frac{8}{3} \; ; \; \sum_n h(n) = \frac{4}{3} \; ; \; \sum_n x(n) = 2.$

(17) Compute and plot convolutions $x(n) * h(n)$ and $h(n) * x(n)$ for the pairs of signals shown in fig.

(a)





$x(n) = \{\underset{\uparrow}{1}, 1, 1, 1\}$

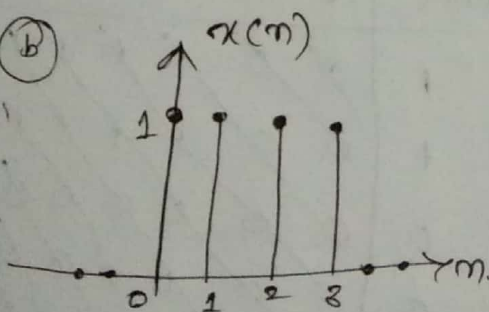
$h(n) = \{\underset{\uparrow}{6}, 5, 4, 3, 2, 1, 0\}$

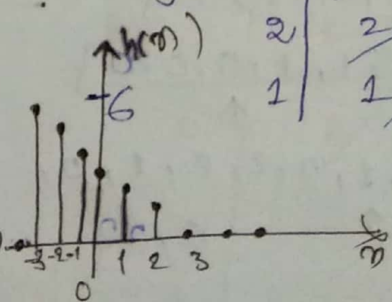
$y(n) = x(n) * h(n)$

$y(n) = \{\underset{\uparrow}{6}, 11, 15, 18, 14, 10, 6, 3, 1\}$

$h(n) \backslash x(n)$	1	1	1
6	6	6	6
5	5	5	5
4	4	4	4
3	3	3	3
2	2	2	2
1	1	1	1

(b)



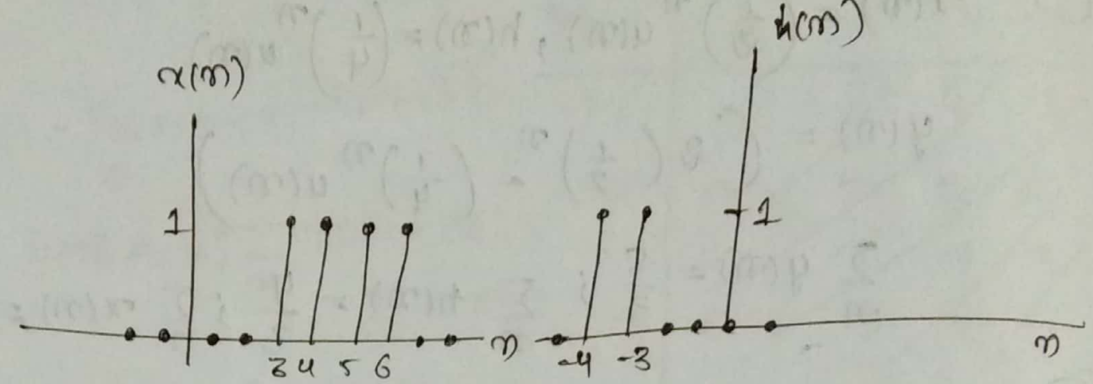


$h(n) = \{6, 5, 4, \underset{\uparrow}{3}, 2, 1, 0\}$

$y(n) = \{6, 11, 15, \underset{\uparrow}{18}, 14, 10, 6, 3, 1\}$

6	6	6	6
5	5	5	5
4	4	4.	4
3	3	3	3
2	2	2	2
1	1	1	1

(c)



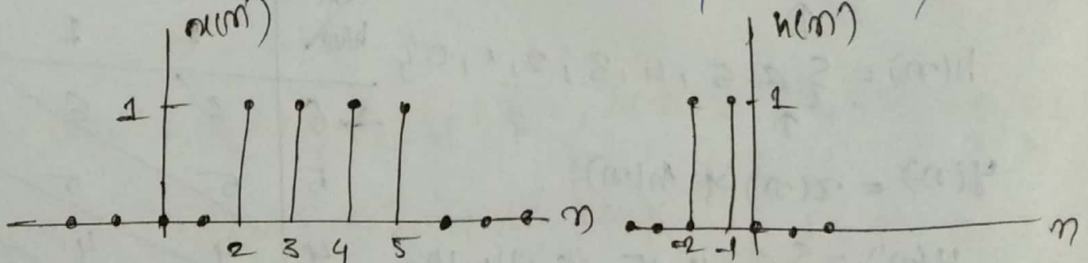
$x(n) = \{\underset{\uparrow}{0}, 0, 0, 1, 1, 1, 1\}$

$h(n) = \{1, 1, 0, 0, \underset{\uparrow}{0}\}$

$y(n) = \{0, 0, 0, 1, \underset{\uparrow}{2}, 2, 2, 1, 0, 0, 0\}$

h(n) \ x(n)	0	0	0	1	1	1	1
1	0	0	0	1	1	1	1
1	0	0	0	1	1	1	1
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

(d)



$x(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1\}$

$h(n) = \{1, 1, \underset{\uparrow}{0}, 0, 0\}$

$y(n) = \{0, 0, \underset{\uparrow}{1}, 2, 2, 2, 1, 0, 0, 0, 0\}$

h(n) \ x(n)	0	0	1	1	1	1
1	0	0	1	1	1	1
1	0	0	1	1	1	1
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

(18) Determine and sketch the convolution $y(n)$ of the signals

$$x(n) = \begin{cases} \frac{1}{3}n, & 0 \le n \le 6 \\ 0, & \text{elsewhere} \end{cases} \qquad h(n) = \begin{cases} 1, & -2 \le n \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Graphically (b) Analytically.

(a) $x(n) = \{0, \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, 2\}$

$h(n) = \{1, 1, 1, 1, 1\}$

$y(n) = \{0, \frac{1}{3}, \underset{\uparrow}{1}, 2, \frac{10}{3}, 5, \frac{20}{3}, 6, 5, \frac{11}{3}, 2\}$

$h(n) \backslash x(n)$	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	2/3	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	2/3	1	4/3	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	2/3	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	2/3	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	2/3	1	$\frac{4}{3}$	$\frac{5}{3}$	2

(b) $x(n) = \frac{1}{3}n\,[u(n) - u(n-7)]$

$h(n) = u(n+2) - u(n-3)$

$y(n) = \frac{1}{3}n\,[u(n) - u(n-7)] * u(n+2) - u(n-3)$

$= \frac{1}{3}n[u(n) * u(n+2)] - \frac{1}{3}n[u(n) * u(n-3)] - \frac{1}{3}n\,[u(n-7) * u(n+2)] + \frac{1}{3}n\,(u(n-7) * u(n-3))$

(20) Consider the following three operations

(a) Multiply the integer numbers ; 131 and 122

$131 \times 122 = 15982$

(b) Compute the convolution of signals : $\{1, 3, 1\} * \{1, 2, 2\}$.

$y(n) = \{1, 5, 9, 8, 2\}$

$h(n) \backslash x(n)$	1	2	2
1	1	2	2
3	3	6	6
1	1	2	2

(c) Multiply the polynomials: $1+3z+z^2$ and $1+2z+2z^2$.

$(z^2+3z+1)\times(2z^2+2z+1) \Rightarrow 2z^4+6z^3+2z^2+2z^3 + 6z^2+2z+z^2+3z+1$

$\Rightarrow 2z^4+8z^3+9z^2+5z+1$

(d) Repeat part (a) for the numbers 1.31 & 12.2

$1.31 \times 12.2 = 15.982$

(e) Comment on your results.

These are different ways to perform convolu

(2) Compute the convolution $y(n) = x(n) * h(n)$ of the following pairs of signals.

(a) $x(n) = a^n u(n)$, $h(n) = b^n u(n)$ when $a \neq b$ & when $a = b$.

$y(n) = x(n) * h(n)$

$= a^n u(n) * b^n u(n)$

$= [a^n * b^n] u(n)$

$y(n) = \sum_{k=0}^{n} a^k u(k)\, b^{n-k} u(n-k)$

$= b^n \sum_{k=0}^{n} a^k u(k)\, b^{-k}$

$= b^n \cdot \sum_{k=0}^{n} (ab)^{-k}$

if $a \neq b$ then $y(n) = \dfrac{b^{n+1} - a^{n+1}}{b-a} u(n)$

if $a = b \Rightarrow b^n (n+1) u(n)$

(b) $x(n) = \begin{cases} 1; & n = -2, 0, 1 \\ 2; & n = -1 \\ 0; & \text{elsewhere} \end{cases}$

$h(n) = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-5)$

$x(n) = \{1, 2, 1, 1\}$

$h(n) = \{\underset{\uparrow}{1}, -1, 0, 0, 1, 1\}$

$y(n) = \{1, 1, -1, \underset{\uparrow}{0}, 0, 3, 3, 2, 1\}$

(c) $x(n) = u(n+1) - u(n-4) - \delta(n-5)$;

$h(n) = [u(n+2) - u(n-3)] \cdot (3 - |n|)$

$x(n) = \{1, \underset{\uparrow}{1}, 1, 1, 1, 0, -1\}$

$h(n) = \{1, 2, \underset{\uparrow}{3}, 2, 1\}$

$y(n) = \{1, 3, 6, 8, 9, 8, 5, 1, -2, -2, 1\}$

(b)

$h(n)$ \ $x(n)$	1	2	1	1
1	1	2	1	1
-1	-1	-2	-1	-1
0	0	0	0	0
0	0	0	0	0
1	1	2	1	1
1	1	2	1	1

(d) $x(n) = u(n) - u(n-5)$,

$h(n) = u(n-2) - u(n-8) + u(n-11) - u(n-17)$

(c)

$h(n)$ \ $x(n)$	1	1	1	1	1	0	-1
1	1	1	1	1	1	0	-1
2	2	2	2	2	2	0	-2
3	3	3	3	3	3	0	-3
2	2	2	2	2	2	0	-2
1	1	1	1	1	1	0	-1

(d)

$x(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$

$h'(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1, 1, 1\}$

$h(n) = h'(n) + h'(n-9)$

$y(n) = y'(n) + y'(n-9)$, where

$y'(n) = \{0, 0, 1, 2, 3, 4, 5, 5, 4, 3, 2, 1\}$

$h(n)$ \ $x(n)$	1	1	1	1	1
0	0	0	0	0	0
0	0	0	0	0	0
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1

(22) Let $x(n)$ be the i/p signal to a discrete-time filter with impulse response $h_i(n)$ and let $y_i(n)$ be the corresponding output

(a) Compute and sketch $x(n)$ and $y_i(n)$ in the following cases, using the same scale in all figures. $x(n) = \{1, 4, 2, 3, 5, 3, 3, 4, 5, 7, 6, 9\}$

$h_1(n) = \{1, 1\}$ $\quad h_4(n) = \{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\}$

$h_2(n) = \{1, 2, 1\}$ $\quad h_5(n) = \{\frac{1}{4}, -\frac{1}{2}, \frac{1}{4}\}$

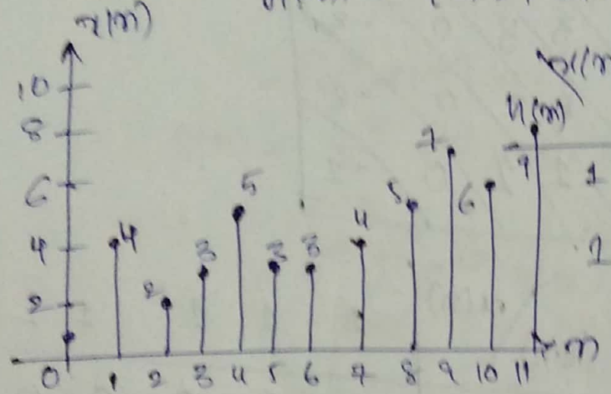
$h_3(n) = \{\frac{1}{2}, \frac{1}{2}\}$

Sketch $x(n)$, $y_1(n)$, $y_2(n)$ on one graph and $x(n)$, $y_3(n)$, $y_4(n)$, $y_5(n)$ on another graph.

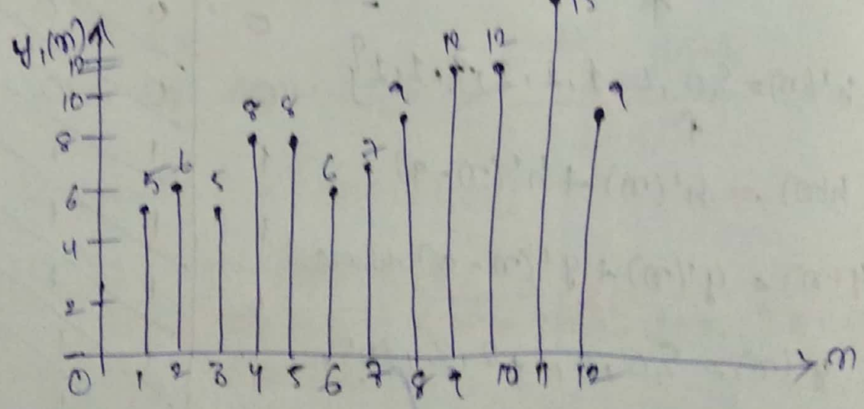
Sol

$y_1(n) = x(n) * h_1(n)$

$y_1(n) = \{1, 5, 6, 5, 8, 8, 6, 7, 9, 12, 12, 15, 9\}$

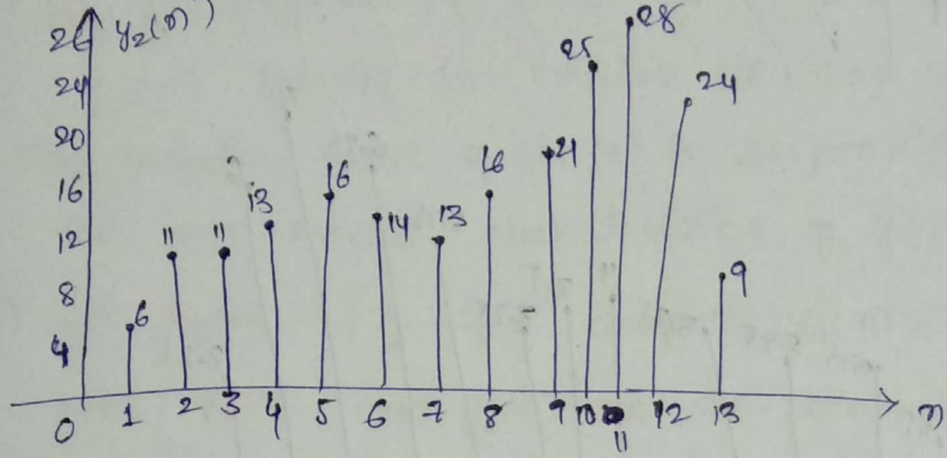


$h(n)$ \ $x(n)$	1	4	2	3	5	3	3	4	5	7	6
1	1	4	2	3	5	3	3	4	5	7	6
1	1	4	2	3	5	3	3	4	5	7	6



$y_2(n) = x(n) * h_2(n) \Rightarrow \{1, 6, 11, 11, 13, 16, 14, 13, 16, 21, 25, 28, 24, 9\}$

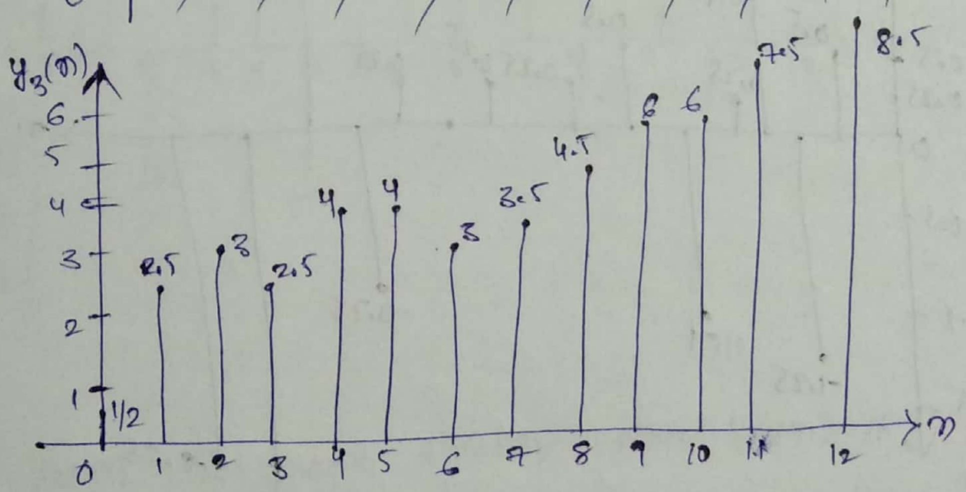
$h(n)$ \ $x(n)$	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9
2	2	8	4	6	10	6	6	8	10	14	12	18
1	1	4	2	3	5	3	3	4	5	7	6	9



$y_3(n) = x(n) * h_3(n) \Rightarrow \{\frac{1}{2}, 2.5, 3, 2.5, 4, 4, 3, 3.5,$

$4.5, 6, 6, 7.5, \frac{9}{2}\}$

$h(n)$ \ $x(n)$	1	4	2	3	5	3	3	4	5	7	6	9
$\frac{1}{2}$	$\frac{1}{2}$	2	1	3/2	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	2	$\frac{5}{2}$	$\frac{7}{2}$	3	$\frac{9}{2}$
$\frac{1}{2}$	$\frac{1}{2}$	2	1	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	2	$\frac{5}{2}$	$\frac{7}{2}$	3	$\frac{9}{2}$



$\Rightarrow y_4(n) = x(n) * h_4(n) \Rightarrow \{0.25, 1.5, 2.75, 2.75, 3.25,$

$4, 3.5, 3.25, 5.25, 6.25, 7, 6, 2.25$

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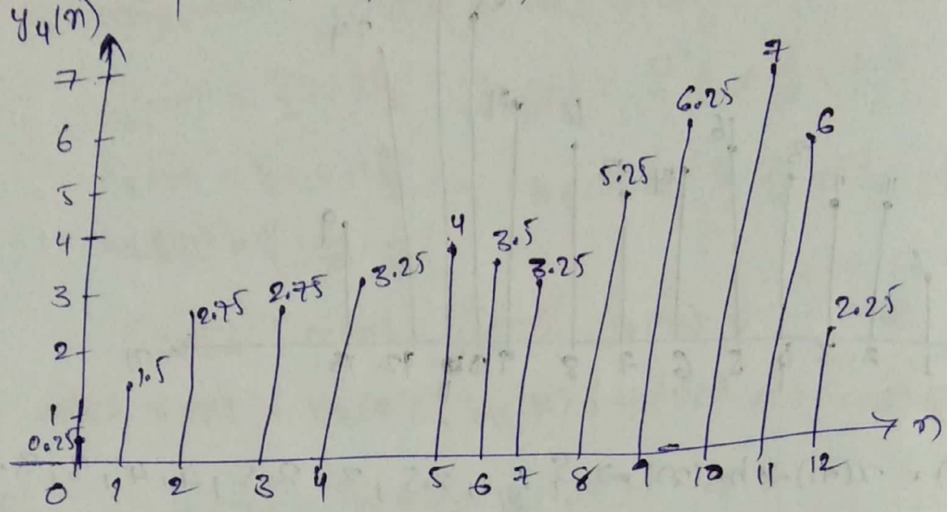
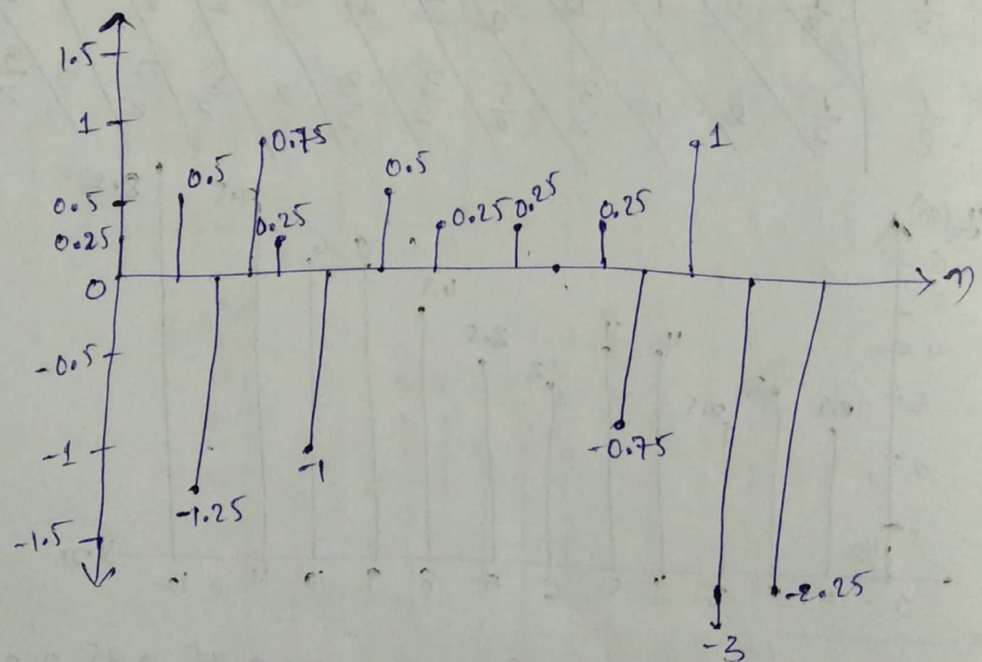
$h(n) \backslash x(n)$	1	4	2	3	5	3	3	4	5	7	6	9
$\frac{1}{4}$	$\frac{1}{4}$	1	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	1	$\frac{5}{4}$	$\frac{7}{4}$	$\frac{3}{2}$	$\frac{9}{4}$
$\frac{1}{2}$	$\frac{1}{2}$	2	1	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	2	$\frac{5}{2}$	$\frac{7}{2}$	3	$\frac{9}{2}$
$\frac{1}{4}$	$\frac{1}{4}$	1	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	1	$\frac{5}{4}$	$\frac{7}{4}$	$\frac{3}{2}$	$\frac{9}{4}$



$\Rightarrow y_5(n) = x(n) * h_5(n) \Rightarrow \{0.25, 0.5, -1.25, 0.75, 0.25, -1, 0.5, 0.25, 0.25, 0, 0.25, -0.75, 1, -3, -2.25\}$



(b) What is the difference b/w $y_1(n)$ & $y_2(n)$ and b/w $y_3(n)$ & $y_4(n)$?

$y_3(n) = \frac{1}{2} y_1(n)$; $h_3(n) = \frac{1}{2} h_1(n)$

$y_4(n) = \frac{1}{4} y_2(n)$; $h_4(n) = \frac{1}{4} h_2(n)$

(c) Comment on the smoothness of $y_2(n)$ and $y_4(n)$. which factors affect the smoothness?

$y_2(n)$ and $y_4(n)$ are smoother than $y_1(n)$ becoz of smaller Scalar factor.

(d) Compare $y_4(n)$ with $y_5(n)$. what is the difference? can you explain it?

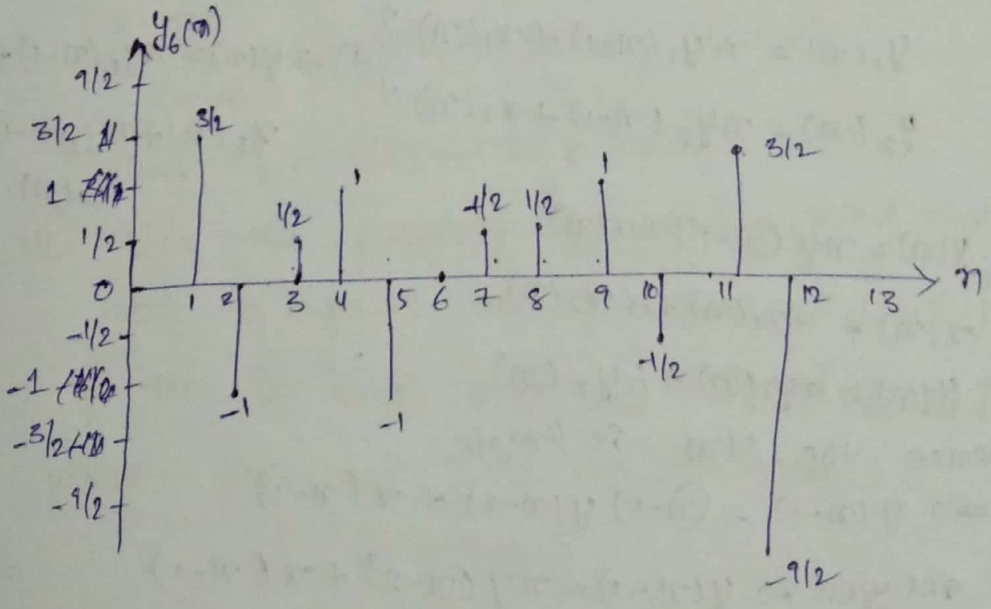
$y_4(n)$ results in smaller o/p, than $y_5(n)$.

The negative value of $h_5(0)$ is responsible for the non-smooth characteristics of $y_5(n)$.

(e) Let $h_6(n) = \{\frac{1}{2}, \frac{-1}{2}\}$ compute $y_6(n)$. Sketch $x(n)$, $y_2(n)$ and $y_6(n)$ on the same figure and comment on the results.

$$y_6(n) = x(n) * h_6(n)$$

$$y_6(n) = \{\frac{1}{2}, \frac{3}{2}, -1, \frac{1}{2}, 1, -1, 0, \frac{1}{2}, \frac{1}{2}, 1, \frac{-1}{2}, \frac{3}{2}, \frac{-9}{2}\}$$



$y_2(n)$ is more smaller than $y_6(n)$

(23) Express the o/p $y(n)$ of a linear time-invariant s/m with impulse response $h(n)$ in terms of its step response $s(n) = h(n) * u(n)$ and the input $x(n)$.

Sol:- $\delta(n) = u(n) - u(n-1)$

$h(n) = h(n) * \delta(n)$

$= h(n) * [(u(n) - u(n-1)]$

$= h(n) * u(n) - h(n) * u(n-1)$

$= s(n) - s(n-1)$

then $y(n) = h(n) * x(n)$

$= [s(n) - s(n-1)] * x(n)$

$= s(n) * x(n) - s(n-1) * x(n)$

Q4) The discrete-time system

$$y(n) = ny(n-1) + x(n), \quad n \geq 0$$

is at rest [i.e., $y(-1)=0$]. Check if the s/m is linear time invariant and BIBO stable.

Ans

$y(n) = ny(n-1) + x(n), \quad n \geq 0$

$\left.\begin{array}{l} y_1(n) = n y_1(n-1) + x_1(n) \\ y_2(n) = n y_2(n-1) + x_2(n) \end{array}\right\} \oplus \Rightarrow y(n) = n y_1(n-1) + x_1(n) + n y_2(n-1) + x_2(n).$

$y(n) = ny(n-1) + x(n)$

$x(n) = a x_1(n) + b x_2(n)$

$y(n) = a y_1(n) + b y_2(n)$

Hence the s/m is linear.

$\Rightarrow y(n-1) = (n-1) y(n-2) + x(n-1)$

delayed $\Rightarrow y(n-1) = ny(n-2) + x(n-1)$

So the s/m is TI.

$\Rightarrow$ If $x(n) = u(n)$, then $|x(n)| \leq 1$, for this bounded i/p, o/p is $y(0) = 0$, $y(1) = 2$, $y(2) = 5$, ---, unbounded. So s/m is unstable

(25) Consider the signal $\gamma(n) = a^n u(n)$, $0 < a < 1$

(a) Show that any sequence $x(n)$ can be decomposed as $x(n) = \sum_{n=-\infty}^{\infty} c_k \cdot \gamma(n-k)$ and express c_k in terms of $x(n)$.

Sol

$$\delta(n) = \gamma(n) - a\gamma(n-1)$$

$$\delta(n-k) = \gamma(n-k) - a\gamma(n-k-1)$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot \delta(n-k)$$

$$= \sum_{k=-\infty}^{+\infty} x(k) \cdot [\gamma(n-k) - a\gamma(n-k-1)]$$

$$x(n) = \sum_{k=-\infty}^{+\infty} x(k)\gamma(n-k) - a\sum_{k=-\infty}^{+\infty} x(k)\gamma(n-k-1)$$

$$x(n) = \sum_{k=-\infty}^{+\infty} x(k)\gamma(n-k) - a\sum_{k=-\infty}^{+\infty} x(k-1)\gamma(n-k)$$

$$= \sum_{k=-\infty}^{\infty} [x(k) - a x(k-1)]\, \gamma(n-k)$$

Thus $c_k = x(k) - a x(k-1)$

(b) Use the property of linearity and time invariance to express the o/p $y(n) = T[x(n)]$ in terms of the i/p $x(n)$ and the signal $g(n) = \mathcal{T}[\gamma(n)]$, where $\mathcal{T}[\cdot]$ is an LTI sym.

Sol

$$y(n) = \mathcal{T}[x(n)]$$

$$= \mathcal{T}\left[\sum_{k=-\infty}^{+\infty} c_k\, \gamma(n-k)\right]$$

$$= \sum_{k=-\infty}^{+\infty} c_k\, \mathcal{T}[\gamma(n-k)]$$

$$= \sum_{k=-\infty}^{+\infty} c_k\, g(n-k)$$

(c) Express the impulse response $h(n) = \gamma[\delta(n)]$ in terms of $g(n)$.

$$h(n) = \gamma[\delta(n)]$$

$$h(n) = \gamma[\gamma(n) - a\gamma(n-1)]$$

$$= g(n) - a\,g(n-1)$$

(26) Determine the zero-input response of the sim described by the second order difference equation

$$x(n) - 3y(n-1) - 4y(n-2) = 0$$

Ans

with $x(n) = 0$

$$-3y(n-1) - 4y(n-2) = 0 \qquad \left(\tfrac{1}{6}(-3)\right)$$

$$y(n-1) + \frac{4}{3}y(n-2) = 0$$

at $n=0$; $y(-1) = -\frac{4}{3}y(-2)$

at $n=1$; $y(0) = \left(\frac{-4}{3}\right)y(-1) = \left(\frac{-4}{3}\right)^2 y(-2)$

$$y(1) = \left(\frac{-4}{3}\right)^3 y(-2)$$

$\vdots$

$$y(k) = \left(\frac{-4}{3}\right)^{k+2} y(-2)$$

$\hookrightarrow$ zero-input response

(27) Determine the particular solution of the difference eqn

$y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n)$ when the forcing fun. is $x(n) = 2^n u(n)$.

$$y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n)$$

$$x(n) = y(n) - \frac{5}{6}y(n-1) + \frac{1}{6}y(n-2)$$

Characteristic. eqn is,

$$\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = 0; \quad \lambda = 1/2, 1/3$$

So $y_h(n) = c_1\left(\frac{1}{2}\right)^n + c_2\left(\frac{1}{3}\right)^n$

$x(n) = 2^n u(n)$

$y_p(n) = k(2^n)u(n)$

So, $k(2^n)u(n) - k\left(\frac{5}{6}\right)(2^{n-1})u(n-1) + k\left(\frac{1}{6}\right)(2^{n-2})u(n-2) = 2^n u(n)$

for $n=2$,

$$4k - \frac{5k}{3} + \frac{k}{6} = 4$$

$$k = 8/5$$

Total solution is

$$y_p(n) + y_h(n) = y(n).$$

$$y(n) = \frac{8}{5}(2^n)u(n) + c_1\left(\frac{1}{2}\right)^n u(n) + c_2\left(\frac{1}{3}\right)^n u(n)$$

Assume. $y(-2) = y(-1) = 0$ So $y(0) = 1$

then $y(1) = \frac{5}{6}y(0) + 2 = \frac{17}{6}$

$$\frac{8}{5} + c_1 + c_2 = 1$$

$$c_1 + c_2 = 3/5 \quad \text{—— (1)}$$

$$\frac{16}{5} + \frac{1}{2}c_1 + \frac{1}{3}c_2 = \frac{17}{6}$$

$$3c_1 + 2c_2 = -\frac{11}{5} \quad \text{—— (2)}$$

By solving . Eq 2

$$c_1 = -1, \quad c_2 = 2/5$$

So the total solution is

$$y(n) = \left[\frac{8}{5}(2^n) - \left(\frac{1}{2}\right)^n + \frac{2}{5}\left(\frac{1}{3}\right)^n\right]u(n)$$

(28)

at $y(-1) = 1$

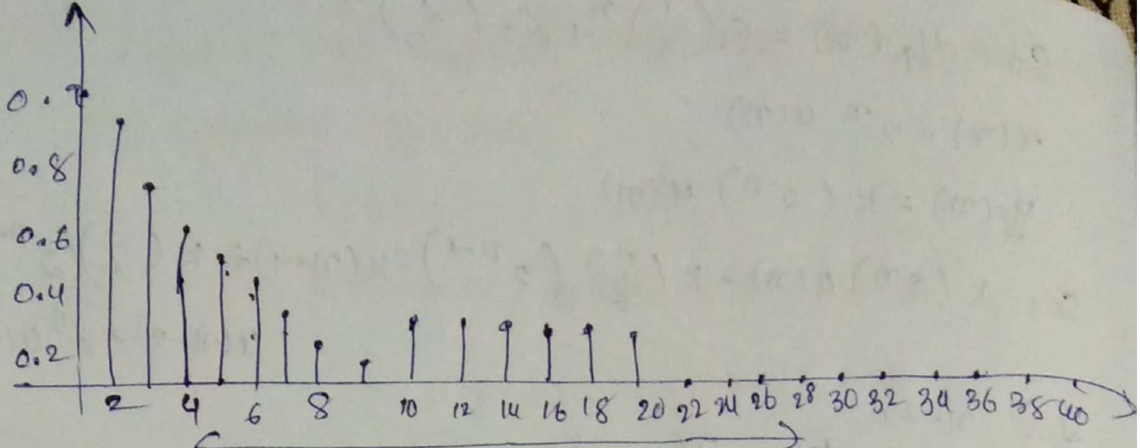
The given eq^n is $y(n) = (-a)^{n+1} + \frac{(1-(-a)^{n+1})}{1+a}$

$$y(n) = y_{zi}(n) + y_{zs}(n)$$

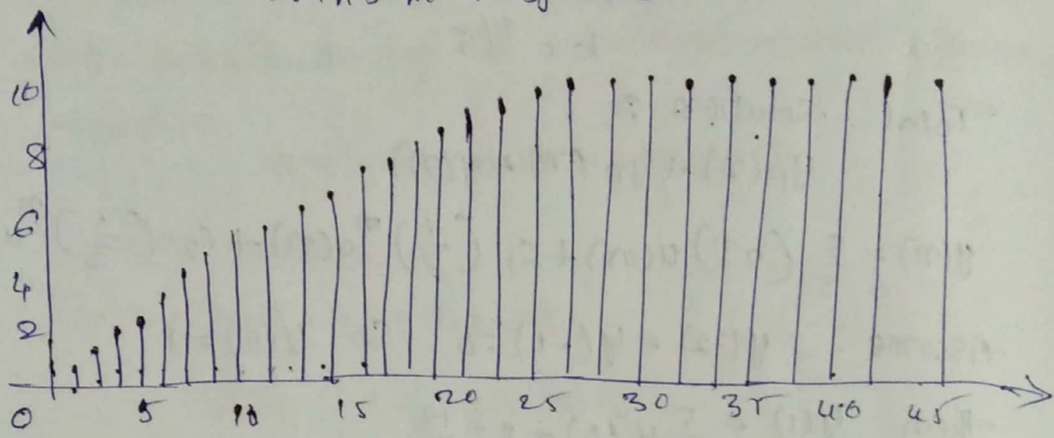
$$= \text{Transient} + \text{steady state}$$

$y(0) = 0.9$

$y(1) = 0.87$ $\quad y(2) = 0.72$



Transient Response



⟵ Steady state Response ⟶

(29) $h_1(n) = a^n[u(n) - u(n-N)]$ & $h_2(n) = [u(n) - u(n-M)]$

$$h(n) = h_1(n) * h_2(n)$$

$$= \sum_{k=-\infty}^{\infty} a^k [u(k)\, u(k-N)][u(n-k) - u(n-k-M)]$$

$$= \sum_{k=-\infty}^{\infty} a^k u(k)\, u(n-k) - \sum_{k=-\infty}^{\infty} a^k u(k)\, u(n-k-M) -$$

$$\sum_{k=-\infty}^{\infty} a^k u(k-N)\, u(n-k) + \sum_{k=-\infty}^{\infty} a^k u(k-N)\, u(n-k-m)$$

$$= \left(\sum_{k=0}^{n} a^k - \sum_{k=0}^{n-m} a^k\right) - \left(\sum_{k=N}^{n} a^k - \sum_{k=N}^{n-m} a^k\right)$$

$h(n) = 0$

(30) $y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$ to i/p

i/p $x(n) = 4^n u(n)$.

Ans $y(n)-3y(n-1)-4y(n-2)=x(n)+2x(n-1)$

characteristic equation is

$\lambda^2-3\lambda-4=0$

$\lambda=4,-1$

So $y_h(n)=c_1 4^n+c_2(-1)^n$

$x(n)=4^n u(n)$

$y_p(n)=kn4^n u(n)$

$kn4^n u(n)-3k(n-1)4^{n-1}u(n-1)-4k(n-2)4^{n-2}u(n-2)$

$=4^n u(n)+2(4)^{n-1}u(n-1)$

for $n=2$, $k(32-12)=4^2+8=24 \rightarrow k=6/5$

The total solution is,

$y(n)=y_p(n)+y_h(n)$

$=\left[\frac{6}{5}n4^n+c_1 4^n+c_2(-1)^n\right]u(n)$

to find c_1 & c_2, Let $y(-2)$ & $y(-1)=0$ then

$y(0)=1$ $\quad y(1)=3y(0)+4+2=9$

$c_2+c_1=1$ —— ①

$\frac{24}{5}+4c_1-c_2=9 \Rightarrow 4c_1-c_2=\frac{21}{5}$ —— ②

from ① & ②,

$c_1=\frac{26}{25}$ & $c_2=\frac{-1}{25}$

So $y(n)=\left[\frac{6}{5}n4^n+\frac{26}{25}4^n-\frac{1}{25}(-1)^n\right]u(n)$.

31 $y(n)-3y(n-1)-4y(n-2)=x(n)+2x(n-1)$

Sol Characteristic eq :- $\lambda^2-3\lambda-4=0$

$\lambda=-4,1$

$y_h(n)=c_1 4^n+c_2(-1)^n$

$x(n)=\delta(n)$

$y(0)=1$ and $y(1)-3y(0)=2$

$y(1)=5$

So $c_1 + c_2 = 1$ — (1)

$4c_1 - c_2 = 5$ — (2)

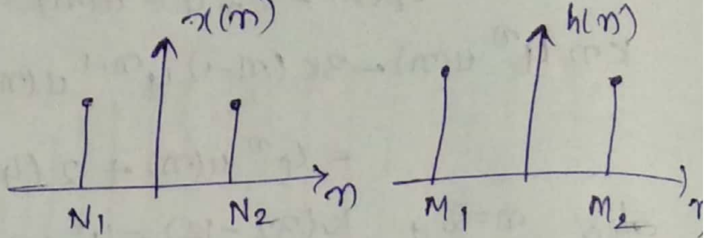
from (1) & (2) $c_1 = \frac{6}{5}$ & $c_2 = \frac{-1}{5}$

$\therefore h(n) = \left[\frac{6}{5} 4^n - \frac{1}{3}(-1)^n\right] u(n)$

(22) (a) Determine the range $L_1 \le n \le L_2$ of their convolution in terms of N_1, N_2, M_1 & M_2.

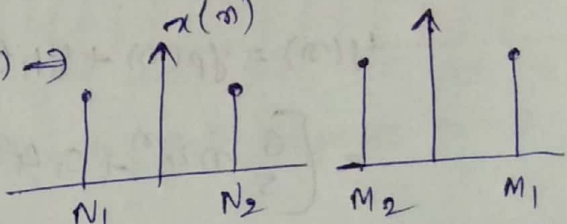
$L_1 = N_1 + M_1$

$L_2 = N_2 + M_2$



(b) Partial overlap from left

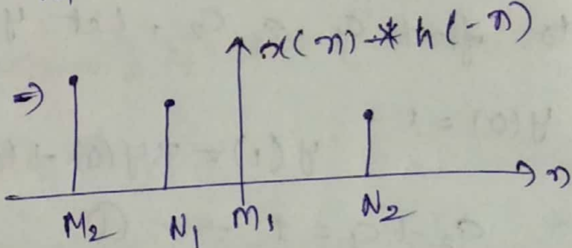
$x(n) * h(n) \Rightarrow$



low $N_1 + M_1$ &

high $M_2 + N_1 - 1$

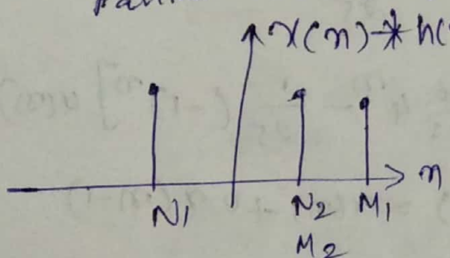
$\Rightarrow$



If fully overlap then

$N_1 + M_2$ (low) & high $N_2 + M_1$

Partial overlap from right

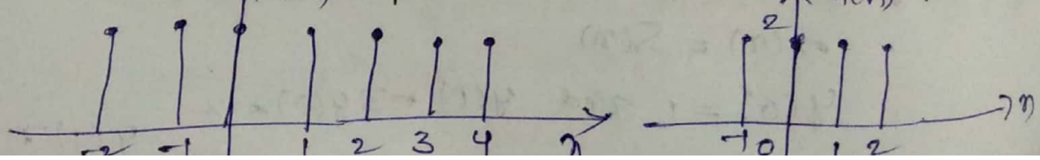


low $\Rightarrow N_2 + M_1 + 1$

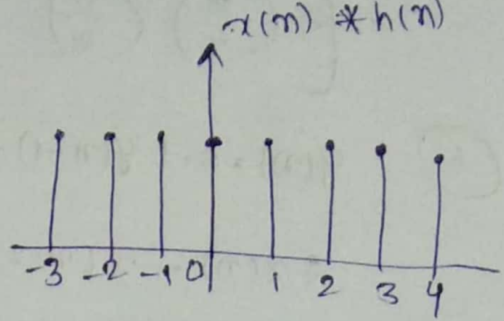
high $\Rightarrow N_2 + M_2$

If fully overlaped high $N_2 + M_1$; Low $= N_1 + M_2$

(c) $x(n) = \{1, 1, 1, 1, 1, 1, 1\}$ $h(n) = \{2, 2, 2, 2\}$



$N_1 = -2 \quad N_2 = 4 \quad M_1 = -1, \ M_2 = 2$



Partial overlap from left

low $N_1 + M_1 = -3$

high $M_2 + N_1 - 1 = 2 - 2 - 1 = -1$

full overlap $n = 0, \ n = 3$

partial right ; $n = 4, \ n = 6, \ L_2 = 6.$

(23) (a) $y(n) = 0.6\,y(n-1) - 0.08\,y(n-2) + x(n)$

Sol $x(n) = y(n) - 0.6\,y(n-1) - 0.08\,y(n-2)$

characteristic equation

$$\lambda^2 - 0.6\lambda + 0.08 = 0$$

$$\lambda = \frac{1}{2}, \frac{2}{5}$$

$$y_h(n) = c_1 \left(\frac{1}{5}\right)^n + c_2 \left(\frac{2}{5}\right)^n$$

Impulse response $x(n) = \delta(n)$ with $y(0) = 1$

$$y(1) = -0.6\,y(0) = 0 \Rightarrow y(1) = 0.6$$

So $c_1 + c_2 = 1$ — (1)

$\frac{1}{5} c_1 + \frac{2}{5} c_2 = 0.6$ — (2)

from (1) & (2) . $c_1 = -1, \ c_2 = 3$

$$h(n) = \left[\left[-\left(\frac{1}{5}\right)^n + 2\left(\frac{2}{5}\right)^n\right] u(n)\right.$$

step response $x(n) = u(n)$

$$s(n) = \sum_{k=0}^{n} y\,h(n-k), \quad n \geq 0$$

$$= \sum_{k=0}^{n} \left[2\left(\frac{2}{5}\right)^{n-k} - \left(\frac{1}{5}\right)^{n-k}\right]$$

$$= 2\left(\frac{2}{5}\right)^n \sum_{k=0}^{n} \left(\frac{5}{2}\right)^k - \left(\frac{1}{5}\right)^n \sum_{k=0}^{n} (5)^k$$

$$= \left[2\left(\frac{2}{5}\right)^n \left(\frac{5}{2}\right)^{n+1} - 1\right] - \left[\left(\frac{1}{5}\right)^k \cdot \left(5^{n+1} - 1\right)\right] u(n)$$

(b) $y(n) = 0.7\,y(n-1) - 0.1\,y(n-2) + 2x(n) - x(n-2)$

$2x(n) - x(n-2) = y(n) - 0.7\,y(n-1) + 0.1\,y(n-2)$

characteristic equation

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$$\lambda = 1/2,\ 1/5$$

$$y_h(n) = c_1\left(\frac{1}{2}\right)^n + c_2\left(\frac{1}{5}\right)^n$$

Impulse Response $x(n) = \delta(n)$, $y(0) = 2$

$$y(1) - 0.7\,y(0) = 0 \Rightarrow y(1) = 1.4$$

$$c_1 + c_2 = 2$$

$$\frac{1}{2}c_1 + \frac{1}{5}c_2 = \frac{7}{5} \quad \text{— (1)}$$

$$c_1 + \frac{2}{5}c_2 = \frac{14}{5} \quad \text{— (2)}$$

Solving (1) & (2)

$$c_1 = \frac{10}{3},\quad c_2 = \frac{-4}{3}$$

So $h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right] u(n)$

Step Response $S(n) = \sum_{k=0}^{n} h(n-k)$

$$= \frac{10}{3}\sum_{k=0}^{n}\left(\frac{1}{2}\right)^{n-k} - \frac{4}{3}\sum_{k=0}^{n}\left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k - \frac{4}{3}\cdot\left(\frac{1}{5}\right)^n \sum_{k=0}^{n} 5^k$$

$$= \frac{10}{3}\left[\frac{1}{2}^n\left(2^{n+1} - 1\right) u(n)\right] - \frac{4}{3}\left[\frac{1}{5}^n\left(5^{n+1} - 1\right)\right] u($$

(34) $h(n) = \begin{cases} \left(\frac{1}{2}\right)^n, & 0 \le n \le 4 \\ 0, & \text{elsewhere} \end{cases}$

$$y(n) = \{\underset{\uparrow}{1}, 2, 2.5, 3, 3, 3, 2, 1, 0\}$$

$h(n) = \{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, --- \}$

$y(n) = \{1, 2, 2.5, 3, 3, 3, 2, 1, 0, --- \}$

$y(0) = x(0)\, h(0)$

$y(0) = x(0) \cdot 1 \Rightarrow x(0) = 1$

$y(1) = x(1) + h(1)\, x(0)$

$2 = x(1) + \frac{1}{2}(1) \Rightarrow x(1) = \frac{3}{2}$

$y(2) = x(2) + h(2)\, x(1) + h(1)\, x(0)$

$2.5 = x(2) + \frac{1}{4}\left(\frac{3}{2}\right) + \frac{1}{2}(1)$

so $x(n) = \{1, \frac{3}{2}, \frac{3}{2}, \frac{7}{4}, \frac{3}{2}, --- \}$

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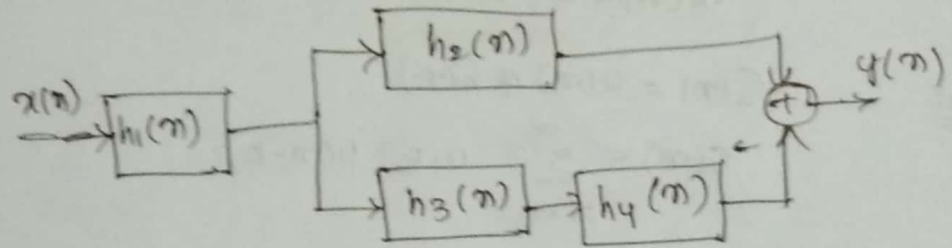
(a)



(a) Express the overall impulse response in terms of $h_1(n)$, $h_2(n)$, $h_3(n)$, and $h_4(n)$.

Sol $h(n) = h_1(n) * [h_2(n) - \{h_3(n) * h_4(n)\}]$

(b) Determine $h(n)$ when $h_1(n) = \{\frac{1}{2}, \frac{1}{4}, \frac{1}{2}\}$

$h_2(n) = h_3(n) = (n+1)\, u(n)$

$h_4(n) = \delta(n-2)$

Sol:- $h_3(n) * h_4(n) = (n+1)\, u(n) * \delta(n-2)$

$= (n+1)\, u(n-2) = (n+1)\, u(n-2)$

$h_2(n) - [h_3(n) * h_4(n)] = (n+1)\, u(n) - (n+1)\, u(n-2)$

$= 2u(n) - \delta(n)$

$h_1(n) = \frac{1}{2}\delta(n) + \frac{1}{4}\cdot\delta(n-1) + \frac{1}{2}\cdot\delta(n-2)$

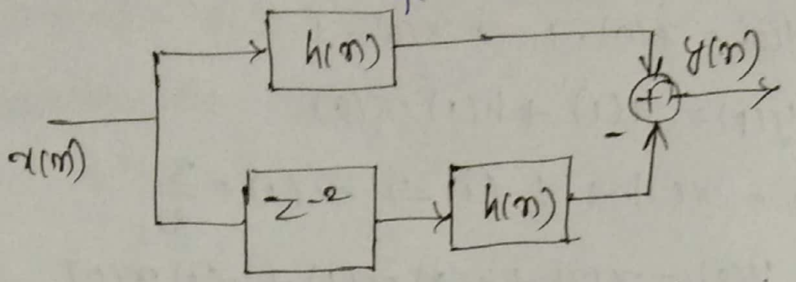
$h(n) = [\frac{1}{2}\cdot\delta(n) + \frac{1}{4}\cdot\delta(n-1) + \frac{1}{2}\cdot\delta(n-2)] * [2u(n) - \delta(n)]$

$= \frac{1}{2}\delta(n) + \frac{5}{4}\delta(n-1) + 2\delta(n-2) + \frac{5}{2}u(n-3)$

(c) Determine the response of the s/m in part (b) if $x(n) = \delta(n+2) + 3\delta(n-1) - 4\delta(n-3)$

Ans $x(n) = \{1, 0, 0, \underset{\uparrow}{3}, 0, -4\}$

(36)



$h(n) = a^n u(n)$, $-1 < a < 1$, Determine the response $y(n)$ of the s/m to the excitation

$$x(n) = u(n+5) - u(n-10)$$

Sol

$$S(n) = u(n) * h(n)$$

$$S(n) = \sum_{k=0}^{\infty} u(k)\, h(n-k)$$

$$= \sum_{k=0}^{\infty} h(n-k)$$

$$= \sum_{k=0}^{\infty} a^{n-k} = \frac{a^{n+k} - 1}{a-1} \quad ; n \geq 0$$

for $x(n) = u(n+5) - u(n-10)$ then

$$S(n+5) - S(n-10) = \frac{a^{n+6} - 1}{a-1} u(n+5) - \frac{a^{n-9} - 1}{a-1} u(n-10)$$

from given figure

$$y(n) = x(n) * h(n) - x(n) * h(n-2)$$

$$y(n) = \frac{a^{n+6} - 1}{a-1} u(n+5) - \frac{a^{n-9} - 1}{a-1} u(n-10) - \frac{a^{n+4} - 1}{a-1} u(n+3) + \frac{a^{n+1} - 1}{a-1} u(n-12)$$

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37) Compute & Sketch the step response of the sm

$$y(n) = \frac{1}{M} \sum_{k=0}^{M-1} x(n-k)$$

Sol

$$h(n) = \left[\frac{u(n) - u(n-m)}{m} \right]$$

$$S(n) = \sum_{k=-\infty}^{\infty} u(k)\, h(n-k)$$

$$= \sum_{k=0}^{n} h(n-k) = \begin{cases} \frac{n+1}{m}, & n < m \\ 1, & n \geq m \end{cases}$$

38) $h(n) = \begin{cases} a^n, & n \geq 0,\ n \text{ even} \\ 0, & \text{otherwise} \end{cases}$

Sol

$$\sum_{k=-\infty}^{\infty} |h(n)| = \sum_{\substack{n=0 \\ n=\text{even}}}^{\infty} |a|^n$$

$$= \sum_{n=0}^{\infty} |a|^{2n} = \frac{1}{1-|a|^2}$$

Stable if $|a| < 1$

39) $h(n) = a^n u(n)$ to i/p signal $x(n) = u(n) - u(n-10)$

Sol

$$h(n) = a^n u(n)$$

$$y_1(n) = \sum_{k=0}^{\infty} u(k)\, h(n-k)$$

$$= \sum_{k=0}^{n} a^{n-k} = a^n \sum_{k=0}^{n} a^{-k}$$

$$= \frac{1 - a^{n+1}}{1-a}\, u(n)$$

$$y(n) = y_1(n) - y_1(n-10)$$

$$= \frac{1}{1-a} \left[(1-a^{n+1})\, u(n) - (1-a^{n-9})\, u(n-10) \right]$$

$h(n) = \left(\frac{1}{2}\right)^n u(n)$

(40) $x(n) = \begin{cases} 1, & 0 \le n \le 10 \\ 0, & \text{otherwise} \end{cases}$

from '36' problem with $a = 1/2$

$$y(n) = 2\left[1 - \left(\frac{1}{2}\right)^{n+1}\right]u(n) - 2\left[1\left(\frac{1}{2}\right)^{n-9}\right]u(n-10)$$

(41) $h(n) = \left(\frac{1}{2}\right)^n u(n)$

Determine respone for given i/p signals.

(a) $x(n) = 2^n u(n)$

Sol $x(n) = 2^n u(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k 2^{n-k} = 2^n \sum_{k=0}^{n} \left(\frac{1}{4}\right)^k$$

$$= 2^n \left[1 - \left(\frac{1}{4}\right)^{n+1}\right]\left(\frac{4}{3}\right) = \frac{2}{3}\left[2^{n+1} - \left(\frac{1}{2}\right)^{n+1}\right]$$

(b) $x(n) = u(-n)$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=0}^{\infty} h(k) = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k = 2, \quad n < 0$$

$$y(n) = \sum_{k=n}^{\infty} h(k) \Rightarrow \sum_{k=n}^{\infty} \left(\frac{1}{2}\right)^k$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k - \sum_{k=0}^{n-1} \left(\frac{1}{2}\right)^k$$

$$= 2 - \left(\frac{1 - \left(\frac{1}{2}\right)^n}{\frac{1}{2}}\right)$$

$$= 2\left(\frac{1}{2}\right)^n, \quad n \ge 0$$

(42) Three s/m with impulse response $h_1(n) = \delta(n) - \delta(n-1)$, $h_2(n) = h(n)$ and $h_3(n) = u(n)$ connected in cascade.

(a) What is the $h_c(n)$, (impulse response of overall s/m)?

$$h_c(n) = h_1(n) * h_2(n) * h_3(n)$$

$$= [\delta(n) - \delta(n-1)] * u(n) * h(n)$$

$$= [u(n) - u(n-1)] * h(n)$$

$$= \delta(n) * h(n) = h(n)$$

(b) Does the order of interconnection affect the overall s/m?

Sol NO.

(43) (a) Prove & explain graphically the difference b/w the relations

$x(n)\,\delta(n-n_0) = x(n_0)\,\delta(n-n_0)$ and

$x(n) * \delta(n-n_0) = x(n-n_0)$

Sol → $x(n)\,\delta(n-n_0) = x(n_0)$ Thus only the value of $x(n)$ at $n=n_0$ is of interest

→ $x(n) * \delta(n-n_0) = x(n-n_0)$. Thus, we obtained shifted version of $x(n)$ sequence.

(b) $$y(n) = \sum_{k=-\infty}^{\infty} h(k)\,x(n-k)$$

$$= h(n) * x(n)$$

Linearity:- $x_1(n) \longrightarrow y_1(n) = h(n) * x_1(n)$

$x_2(n) \longrightarrow y_2(n) = h(n) * x_2(n)$

$= \alpha\, h(n) * x_1(n) + \beta\, h(n) * x_2(n)$

$= \alpha\, y_1(n) + \beta\, y_2(n)$.

Time invariance: $x(n) \rightarrow y_1(n) = h(n) * x(n)$

$x(n-n_0) \rightarrow y_1(n) = h(n) * x(n-n_0)$

$= \sum_k h(k)\, x(n-n_0-k)$

$= y(n-n_0)$

(c) $h(n) = \delta(n-n_0)$

(45) Compute zero state response for sm

$y(n) + \frac{1}{2} y(n-1) = x(n) + 2x(n-2)$

to the input $x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 2, 1\}$

Sol

$y(n) = \frac{-1}{2} y(n-1) + x(n) + 2x(n-2)$

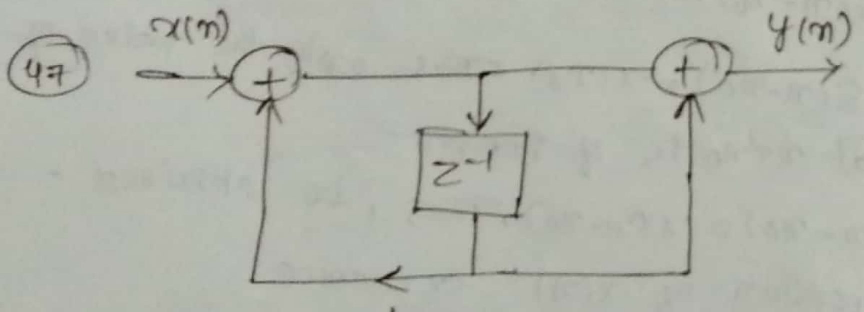
at $y(-2) = \frac{-1}{2} y(-3) + x(-2) + 2x(-4) = 1$

$y(-1) = \frac{-1}{2} y(-2) + x(-1) + 2x(-3) = 3/2$

$y(0) = \frac{-1}{2} y(-1) + 2x(-2) + x(0) = \frac{17}{4}$

$y(1) = \frac{-1}{2} y(0) + x(1) + 2x(-1) = \frac{47}{8}$

(47)



(a) Compute the first samples of its impulse response.

Sol $x(n) = \{\underset{\uparrow}{1}, 0, 0, \ldots\}$

$y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$

$y(0) = x(0) = 1$

$y(1) = \frac{1}{2} y(0) + x(1) + x(0) = 3/2$

$y(2) = 1/2\, y(1) + x(2) + x(1) = 3/4$ Thus we obtain

$y(n) = \{1, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \frac{3}{32}, ---\}$

(b) find the input-output relation.

Sol $y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$

(c) for i/p $x(n) = \{\underset{\uparrow}{1}, 1, 1 ---\}$ compute first 10 samples.

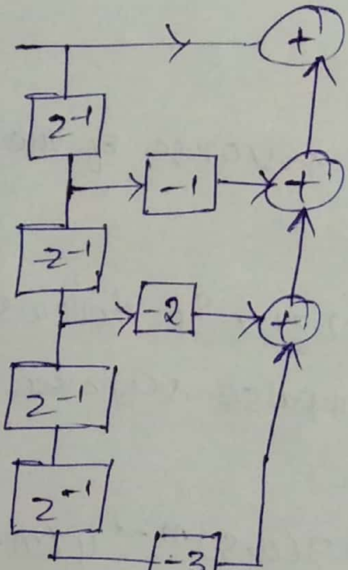
as in part (a) we obtain

$$y(n) = \{1, \frac{5}{2}, \frac{13}{4}, \frac{29}{8}, \frac{61}{16}, ---\}$$

(d) Compute 10 samples for i/p given (c), by using convolution.

$$y(n) = u(n) * h(n)$$

$$= \sum_{k} u(k)\, h(n-k)$$



$$= \sum_{k=0}^{n} h(n-k)$$

$y(0) = h(0) = 1$

$y(1) = h(0) + h(1) = 5/2$

$y(2) = h(0) + h(1) + h(2) = \frac{13}{4}$ etc.

(e) from part (a), $h(n) = 0$ for $n<0 \Rightarrow$ The s/m is Causal.

$$\sum_{n=0}^{\infty} |h(n)| = 1 + \frac{3}{2}\left(1 + \frac{1}{2} + \frac{1}{4} + --\right) = 4$$

$\Rightarrow$ s/m is stable.

(48) $y(n) = a y(n-1) + b x(n)$

(a) Determine b in terms of 'a' so that

$$\sum_{n=-\infty}^{\infty} h(n) = 1$$

Sol $y(n) = ay(n-1) + bx(n)$

$h(n) = ba^n u(n)$

$$\sum_{n=0}^{\infty} h(n) = \frac{b}{1-a} = 1$$

$b = 1-a$

(b) $s(n) = \sum_{k=0}^{n} h(n-k)$

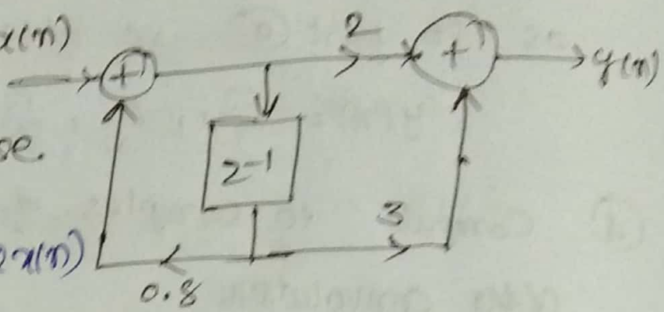
$$= b\left[\frac{1-a^{(n+1)}}{1-a}\right] u(n)$$

$$s(\infty) = \frac{b}{1-a} = 1$$

$b = 1-a$

(c) $b = 1-a$ in both cases.

(49) (a) Determine the impulse Response.



Ans $y(n) = 0.8y(n-1) + 2x(n) + 3x(n-1)$

$y(n) - 0.8y(n-1) = 2x(n) + 3x(n-1)$

The characteristic equation is

$r - 0.8 = 0$

$r = 0.8$

$y_h(n) = c(0.8)^n$

Let's first consider the response of the sm

$y(n) - 0.8y(n-1) = x(n)$

to $x(n) = \delta(n)$, Since $y(0) = 1$, it follows that $c = 1$. Then, the impulse response of the original slm is

$h(n) = 2(0.8)^n u(n) + 3(0.8)^{n-1} u(n-1)$

$= 2\delta(n) + 4.6(0.8)^{n-1} u(n-1)$

(b) The inverse slm is characterized by the difference eq

$x(n) = -1.5x(n-1) + \frac{1}{2}y(n) - 0.4y(n-1)$

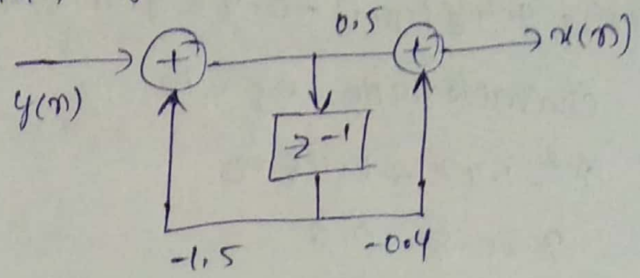
(50) (a) $y(n) = 0.9y(n-1) + x(n) + 2x(n-1) + 3x(n-2)$

$y(n) - 0.9y(n-1) = x(n) + 2x(n-1) + 3x(n-2)$

for $x(n) = \delta(n)$, we have

$y(0)=1,\quad y(1)=2.9,\quad y(2)=5.61\ ;\ y(3)=5.049$

$y(4)=4.544,\quad y(5)=4.090$



(b) $S(0)=y(0)=1$

$S(1)=y(0)+y(1)=3.91$

$S(2)=y(0)+y(1)+y(2)=9.51$

$S(3)=y(0)+y(1)+y(2)+y(3)=14.56$

$S(4)=\sum_{0}^{4} y(n)=19.10$

$S(5)=\sum_{0}^{5} y(n)=23.19.$

(c) $h(n)=(0.9)^n u(n)+2(0.9)^{n-1}u(n-1)+3(0.9)^{n-2}u(n-2)$

$=\delta(n)+2.9\,\delta(n-1)+5.61(0.9)^{n-2}u(n-2).$

5)

(a) $y(n)=\frac{1}{3}x(n)+\frac{1}{3}x(n-3)+y(n-1)$

for $x(n)=\delta(n)$, we have

$h(n)=\left\{\frac{1}{3},\frac{1}{3},\frac{1}{3},\frac{2}{3},\frac{2}{3},\frac{2}{3},\frac{2}{3},\dots\right\}$

(b) $y(n)=\frac{1}{2}y(n-1)+\frac{1}{8}y(n-2)+\frac{1}{2}x(n-2)$

with $x(n)=\delta(n)$ and

$y(-1)=y(-2)=0$, we obtain

$h(n)=\left\{0,0,\frac{1}{2},\frac{1}{4},\frac{3}{16},\frac{1}{8},\frac{11}{128},\frac{15}{256},\frac{41}{1024},\dots\right\}$

(c) $y(n)=1.4\,y(n-1)-0.48\,y(n-2)+x(n)$

with $x(n)=\delta(n)$ and

$y(-1)=y(-2)=0$, we obtain

$h(n)=\{1,1.4,1.48,1.4,1.2496,1.0774,$

$0.9086,\dots\}$

(d) All three systems are IIR.

(e) $y(n) = 1.4y(n-1) - 0.48y(n-2) + x(n)$

The characteristic eq is

$r^2 - 1.4r + 0.48 = 0$

$r = 0.8, 0.6$

$y_h(n) = c_1(0.8)^n + c_2(0.6)^n$ for $x(n) = \delta(n)$.

$c_1 + c_2 = 1$ and

$0.8c_1 + 0.6c_2 = 1.4$

$c_1 = 4, \quad c_2 = -3$

$h(n) = [4(0.8)^n - 3(0.6)^n]u(n).$

(52) (a) $h_1(n) = c_0\,\delta(n) + c_1\,\delta(n-1) + c_2\,\delta(n-2)$

$h_2(n) = b_2\,\delta(n) + b_1\,\delta(n-1) + b_0\,\delta(n-2).$

$h_3(n) = a_0\,\delta(n) + (a_1 + a_0 + a_2)\,\delta(n-1) + a_1 a_2\,\delta(n-2).$

(b) The only question is whether

$h_3(n) \overset{?}{=} h_2(n) = h_1(n)$

let $a_0 = c_0$

$a_1 + a_2 c_0 = c_1, \Rightarrow a_1 + a_2 c_0 - c_1 = 0$

$a_2 a_1 = c_2, \Rightarrow \frac{c_2}{a_2} = a_1$

$\Rightarrow \frac{c_2}{a_2} + a_2 c_0 - c_1 = 0$

$\Rightarrow c_0 a_2^2 - c_1 a_2 + c_2 = 0$

for $c_0 \neq 0$, the quadratic has a real solution if & only if $c_1^2 - 4c_0c_2 \geq 0$.

(53) (a) $y(n) = \frac{1}{2}y(n) + x(n) + x(n-1)$

for $y(n) - \frac{1}{2}y(n-1) = x(n) + x(n-1)$ $\quad x(n) = \delta(n)$

$h(n) = \left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^{n-1} u(n-1)$

(b) $h_1(n) * [\delta(n) + \delta(n-1)] = \left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^{n-1} u(n-1).$

(54) (a) $x_1(n) = \{1, \underset{\uparrow}{2}, 4\}$ $h(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1\}$

Convolution: $y_1(n) = \{1, 3, \underset{\uparrow}{7}, 7, 7, 6, 4\}$

Correlation: $r_1(n) = \{1, 3, 7, 7, \underset{\uparrow}{7}, 6, 4\}$

(b) $x_2(n) = \{0, \underset{\uparrow}{1}, -2, 3, -4\}$ $h_2(n) = \{\frac{1}{2}, 1, \underset{\uparrow}{2}, 1, \frac{1}{2}\}$

Convolution: $y_2(n) = \{\frac{1}{2}, 0, \underset{\uparrow}{\frac{3}{2}}, -2, \frac{1}{2}, -6, \frac{-5}{2}, -2\}$

Correlation: $r_2(n) = \{\frac{1}{2}, \underset{\uparrow}{0}, \frac{3}{2}, -2, \frac{1}{2}, -6, \frac{-5}{2}, -2\}$

Note $y_2(n) = r_2(n)$, $\because h_2(-n) = h_2(n)$ (c)

(c) $x_3(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$ $h_3(n) = \{\underset{\uparrow}{4}, 3, 2, 1\}$

Convolution: $y_3(n) = \{\underset{\uparrow}{4}, 11, 20, 30, 20, 11, 4\}$

Correlation: $r_3(n) = \{1, 4, 10, \underset{\uparrow}{20}, 25, 24, 16\}$

(d) $x_4(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$ $h_4(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

Convolution: $y_4(n) = \{\underset{\uparrow}{1}, 4, 10, 20, 25, 24, 16\}$

Correlation: $r_4(n) = \{4, 11, 20, \underset{\uparrow}{30}, 20, 11, 4\}$

Note that $h_3(-n) = h_4(n+3)$

$r_3(n) = y_4(n+3)$

$h_4(-n) = h_3(n+3)$

$r_4(n) = y_3(n+3)$

(55) Ans length of $h(n) = 2$

$h(n) = \{h_0, h_1\}$

$h_0 = 1$

$3h_0 + h_1 = 4$ $\boxed{h_0 = 1, \; h_1 = 1}$

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(56) (2.5.6) $y(n) = -\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k)$

(2.5.9) $w(n) = -\sum_{k=1}^{N} a_k\, w(n-k) + x(n)$

(2.5.10) $y(n) = \sum_{k=0}^{M} b_k\, w(n-k)$

From 2.5.9 we obtain ~~x(n) + w(n) y(n)~~

$x(n) = w(n) + \sum_{k=1}^{N} a_k\, w(n-k)$ by substituting

(2.5.10) for $y(n)$ and (A) into (2.5.6)

we obtain L.H.S = R.H.S.

(57) Sol $y(-1) = y(-2) = 0$

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

The characteristic equation is

$\lambda^2 - 4\lambda + 4 = 0$

$\lambda = 2, 2.$ Hence

$y_h(n) = c_1 2^n + c_2 n 2^n$

The partial solution is

$y_p(n) = k(-1)^n u(n).$

Substituting this solution is into the difference eq, we obtain

$k(-1)^n u(n) - 4k(-1)^{n-1} u(n-1) + 4k(-1)^{n-2} u(n-2)$

$= (-1)^n u(n) - (-1)^{n-1} u(n).$

For $n=2$, $k(1+4+4) = 2 \Rightarrow k = \frac{2}{9}$. The total solution is $y(n) = \left[c_1 2^n + c_2 n 2^n + \frac{2}{9}(-1)^n\right] u(n)$

from the initial conditions, we obtain

$y(0) = 1,\ y(1) = 2.$

$$c_1 + \frac{2}{9} = 1$$

$$\Rightarrow c_1 = \frac{7}{9}$$

$$2c_1 + 2c_2 - \frac{2}{9} = 2$$

$$c_2 = 1/3$$

(58)

Sol

From problem (57)

$$h(n) = \left[c_1 2^n + c_1 n 2^n\right] u(n)$$

with $y(0) = 1$, $y(1) = 3$ we have

$c_1 = 1$; $2c_1 + 3c_2 = 1$

$$c_2 = 1/2$$

Thus $h(n) = \left[2^n + \frac{1}{2} n 2^n\right] u(n)$.

(59)

Sol

$$u(n-k) = \begin{cases} 1, & n \geq k \\ 0, & \text{otherwise} \end{cases}$$

$$x(n) = x(n) * \delta(n)$$

$$= x(n) * \left[u(n) - u(n-1)\right]$$

$$= \left[x(n) - x(n-1)\right] * u(n)$$

$$x(n) = \sum_{k=-\infty}^{\infty} \left[x(k) - x(k-1)\right] u(n-k)$$

(60)

Sol:-

$$\cancel{u(n-k)} = \begin{cases} \cancel{1}, & \cancel{n \geq k} \\ \cancel{0}, & \cancel{\text{otherwise}} \end{cases}$$

$$\cancel{x(n) = x(n) * \delta(n)}$$

$$\cancel{= x(n) * \left[u(n) - u(n-1)\right]}$$

$$\cancel{= \left[x(n) - x(n-1)\right] * u(n)}$$

Let $h(n)$ be the impulse response of the s/m.

$$S(k) = \sum_{m=-\infty}^{k} h(m)$$

$$h(k) = S(k) - S(k-1)$$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} \left[S(k) - S(k-1)\right] x(n-k)$$

(61)

Sol

$$x(n)=\begin{cases}1, & n_0-N\le n\le n_0+N\\ 0, & \text{otherwise}\end{cases}$$

$$y(n)=\begin{cases}1, & -N\le n\le N\\ 0, & \text{otherwise}\end{cases}$$

$$r_{xx}(l)=\sum_{n=-\infty}^{\infty} x(n)\,x(n-l)$$

The range of Non-zero values of $r_{xx}(l)$ is determined by $n_0-N\le n\le n_0+N$

$n_0-N\le n-l\le n_0+N$

which implies

$-2N\le l\le 2N$

for a given shift l, the no. of terms in the summation for which both $x(n)$ and the value each non-zero is $2N+1-|l|$ and the term is 1. Hence,

$$r_{xx}(l)=\begin{cases}2N+1-|l|, & -2N\le l\le 2N\\ 0, & \text{otherwise}\end{cases}$$

for $r_{xy}(l)$ we have

$$r_{xy}(l)=\begin{cases}2N+1-|l-n_0|, & n_0-2N\le l\le n_0+2N\\ 0, & \text{otherwise}\end{cases}$$

(62)

(a) $$r_{xx}(l)=\sum_{n=-\infty}^{\infty} x(n)\,x(n-l)$$

$r_{xx}(-3)=x(0)\,x(3)=1$

$r_{xx}(-2)=x(0)\,x(2)+x(1)+x(3)=3$

$r_{xx}(-1)=x(0)\,x(1)+x(1)\,x(2)+x(2)\,x(3)=5$

$$r_{xx}(0)=\sum_{n=0}^{3} x^2(n)=7$$

also $r_{xx}(-1)=r_{xx}(l)$

$\therefore \; r_{xx}(l) = \{1, 3, 5, \underset{\uparrow}{7}, 5, 3, 1\}$

(b) $$r_{yy}(l) = \sum_{n=-\infty}^{\infty} y(n)\, y(n-l)$$

we obtain $r_{yy}(l) = \{1, 3, 5, 7, 5, 3, 1\}$

we obtain $y(n) = x(-n+3)$. which is equivalent to reverse the sequence $x(n)$. This has not changed the value.

(62)

Sol $$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$$

$$= \begin{cases} 2N+1-|l| & , -2N \le l \le 2N \\ 0 & , \text{otherwise} \end{cases}$$

$r_{xx}(0) = 2N+1$

$\therefore$ The normalized auto correlation is

$$\rho_{xx}(l) = \begin{cases} \frac{1}{2N+1}(2N+1-|l|) & , -2N \le l \le 2N \\ 0 & , \text{otherwise} \end{cases}$$

(64)

(a) $$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$$

$$= \sum_{n=-\infty}^{\infty} [s(n) + r_1 s(n-k_1) + r_2 s(n-k_2)] * [s(n-l) + r_1 s(n-l-k_1) + r_2 s(n-l-k_2)]$$

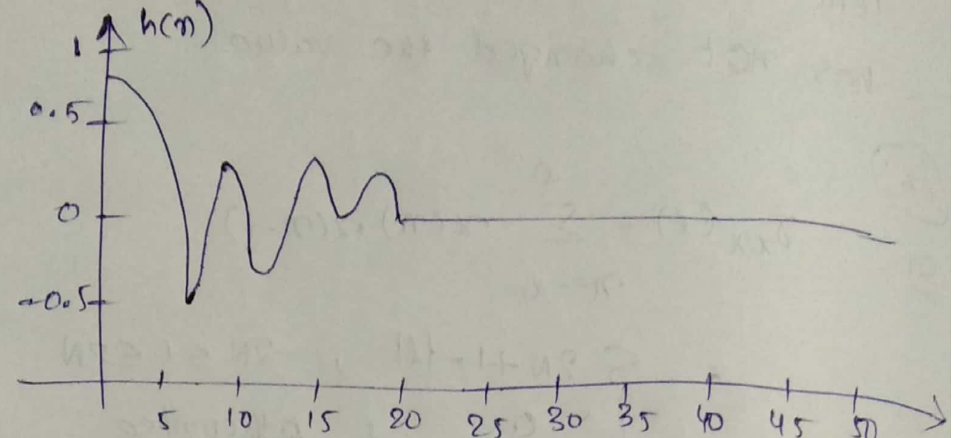
$$= (1 + r_1^2 + r_2^2)\, r_{ss}(l) + r_1 [r_{ss}(l+k_1) + r_{ss}(l-k_1)] + r_2 [r_{ss}(l+k_2) + r_{ss}(l-k_2)] + r_1 r_2 [r_{ss}(l+k_1-k_2) + r_{ss}(l+k_2-k_1)]$$

(b) $r_{ss}(l)$ has peaks at $l = 0, \pm k_1, \pm k_2$ and $\pm(k_1+k_2)$ suppose that $k_1 < k_2$. Then, we can determine γ_1 and k_1. The problem is to determine γ_2 and k_2 from the other peaks.

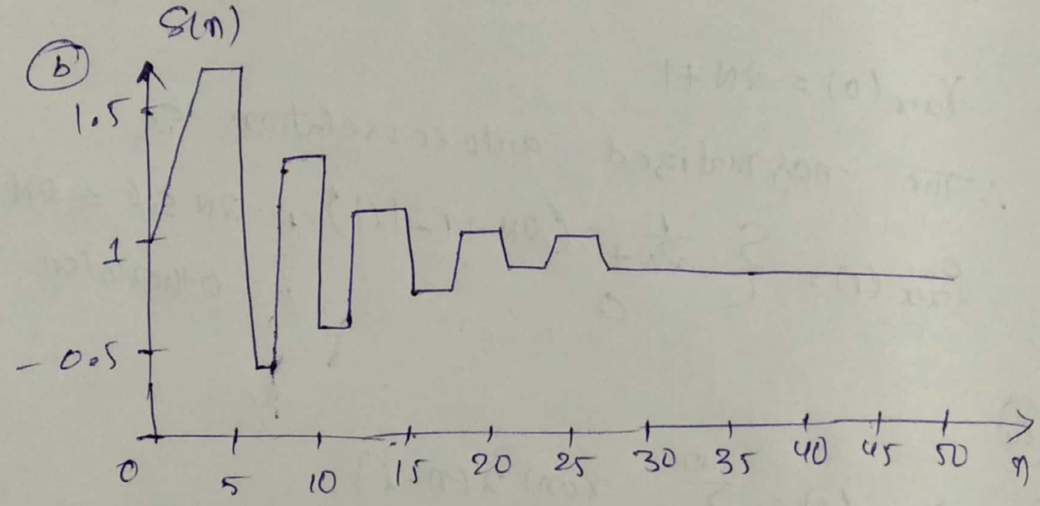
(c) If $\gamma_2 = 0$, the peaks occur at $l = 0$, and $l = \pm k_1$. That is easy to obtain γ_1 and k_1

(6B)

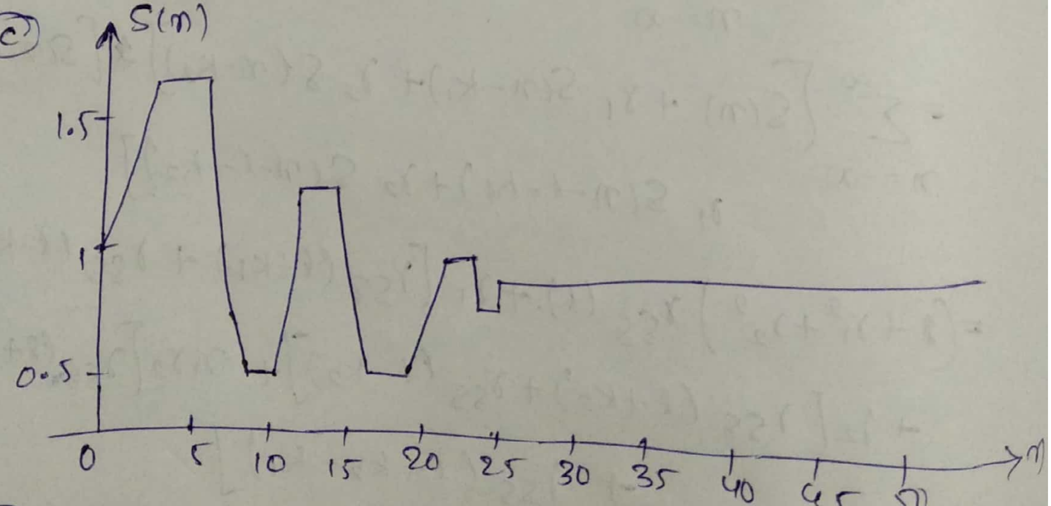
(a)



(b)



(c)

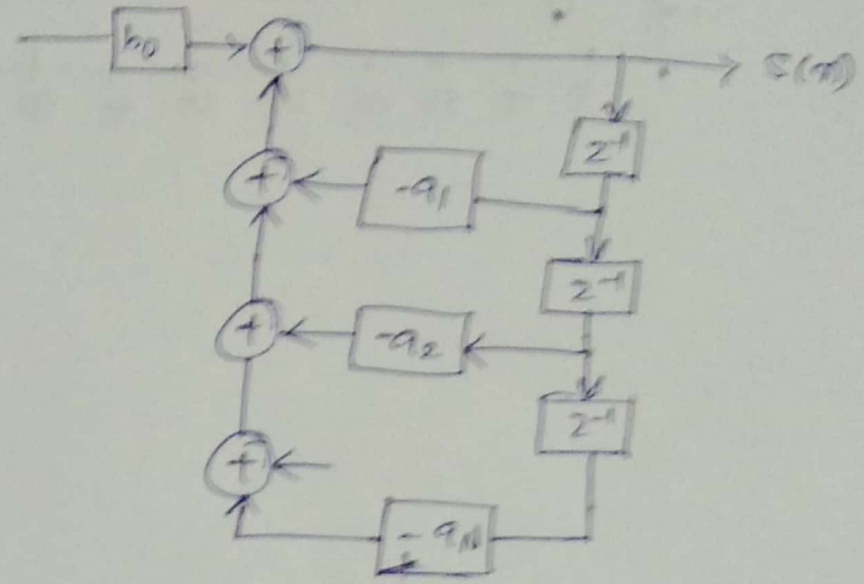


(d) C & b are similar except c have steady state after $n = 20$, where b have nearly at $n = 30$.

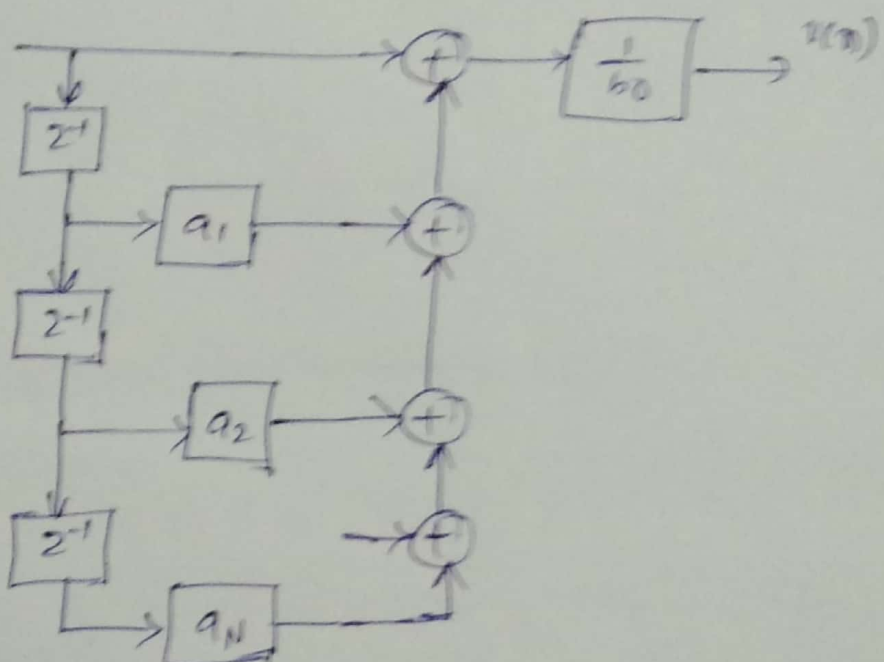
(44) (a) $s(n) = -a_1 s(n-1) - a_2 s(n-2) \cdots - a_N s(n-N) + b_0 v(n)$

(b) $v(n) = \frac{1}{b_0}\left[s(n) + a_1 s(n-1) + a_2 s(n-2) + \cdots + a_N s(n-N)\right]$

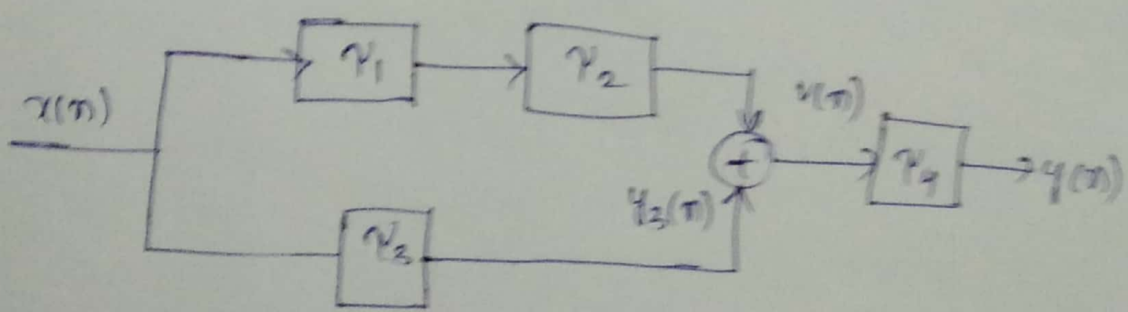
a)



(b)



(67) Plot $h(n)$ for $0 \le n \le 99$.



Sol:-

o/p