DSP problems 2 - Unit

1) A discrete-time signal $x(n)$ is defined as

$$x(n) = \begin{cases} 1+\frac{n}{3}, & -3 \le n \le -1 \\ 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

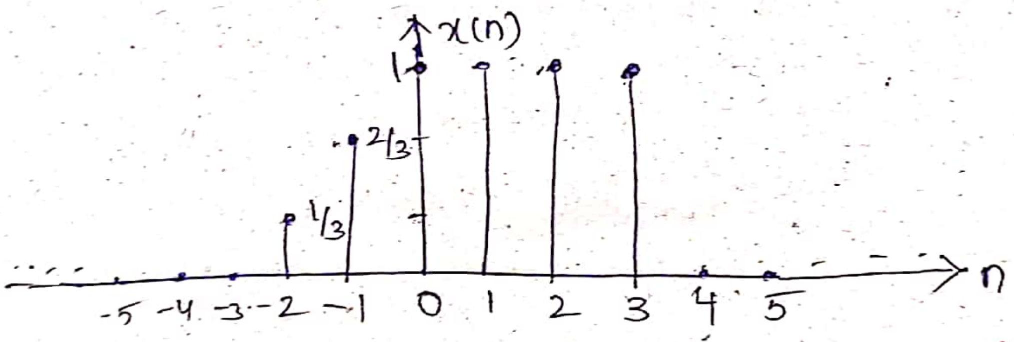
a) Determine its values and sketch the signal $x(n)$.

In the interval $-3 \le n \le -1$ $x(n) = 1+\frac{n}{3}$

for $n=-3$, $1-3/3 = 0$

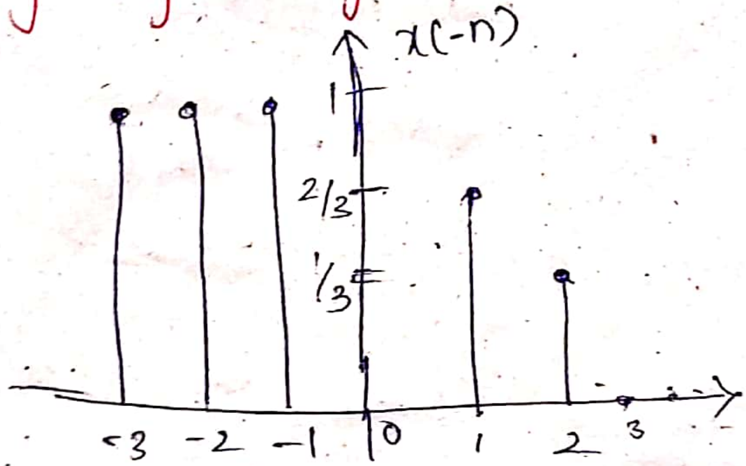
$n=-2$, $1-2/3 = 1/3$

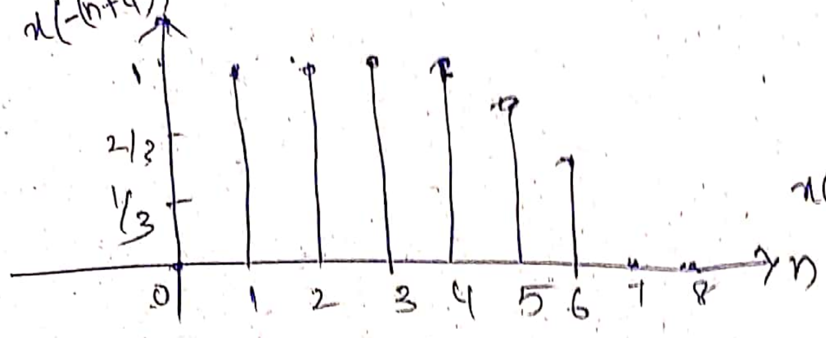
$n=-1$, $1-1/3 = 2/3$.



b) Sketch the signals that result if we:

1) First fold $x(n)$ and then delay the resulting signal by samples.





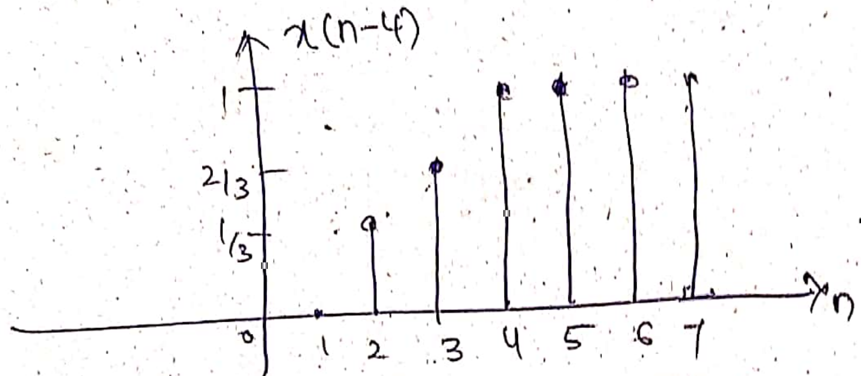
$x(-n-4) = x(-(n+4))$

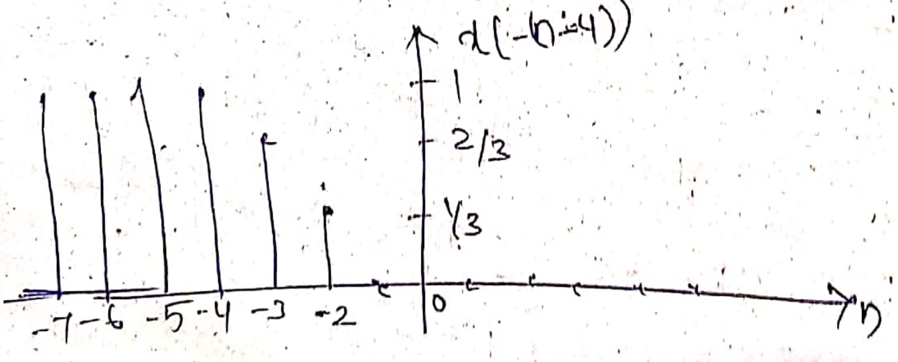
2) First delay x(n) by four samples and then fold the resulting signal.

$n-4=3 \Rightarrow n=7$

$n-4=-2 \Rightarrow n=2$

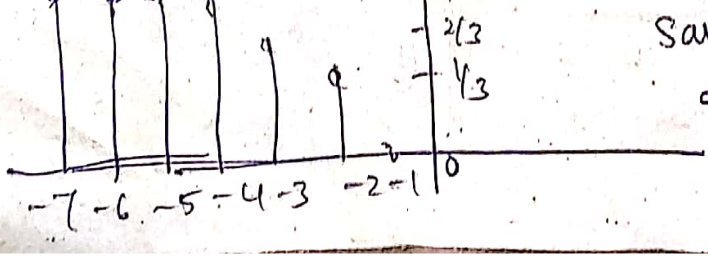
limits $2 \le n \le 7$





c) Sketch the signal x(-n+4)

x(-n+4) can be written as x(-(n-4))



Same as above diagram in 2

d) compare the results in parts (b) and (c) and derive a rule for obtaining the signal $x(-n+k)$ from $x(n)$.

$x(-n+k)$ can be obtained as $x(-(n-k))$

First shifting the signal by four samples and then reversing

$x(-n+4)$

$-n+4=3$

$n=4-3=1$

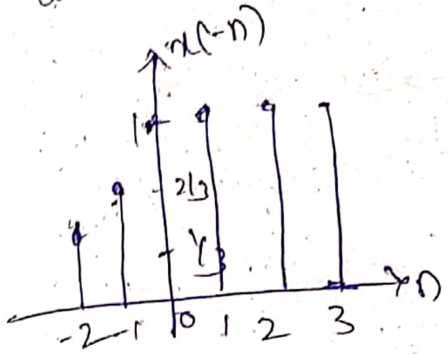
$n=1$

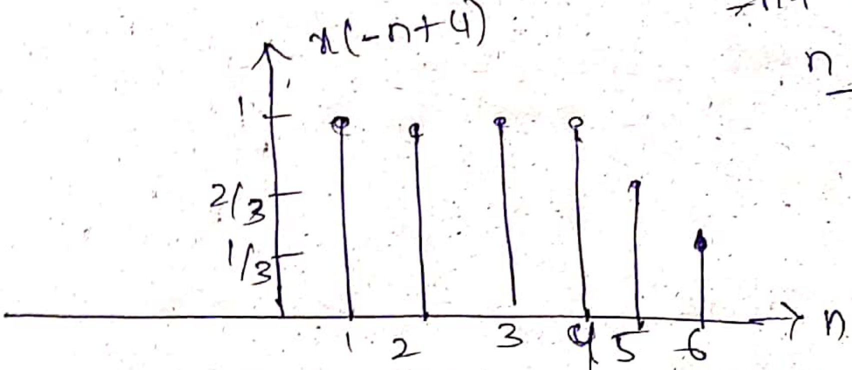
$-n+4=1$

$n=3$

$-n+4=-2$

$n=6$

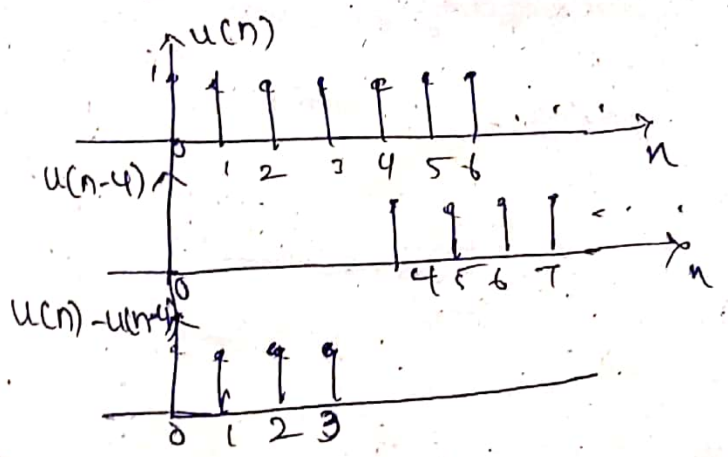




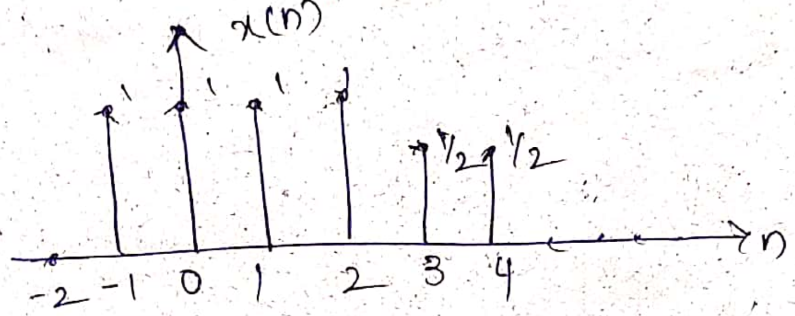
e) Can you express the signal $x(n)$ in terms of signals $\delta(n)$ and $u(n)$?

Sol:- $x(n)$ in terms of $\delta(n)$

$$x(n) = \frac{1}{3}\delta[n+2] + 2/3\,\delta(n+1) + u(n) - u(n-4)$$



2.2) A discrete time signal x(n) is shown in fig P 2.2 sketch and label carefully each of the following signals.



a) $x(n-2)$

$n-2=-2$

$\underline{n=0}$

$n-2=4$

$\underline{n=6}$

$0 \leq n \leq 6$

b) $x(4-n)$

$4-n=-2$

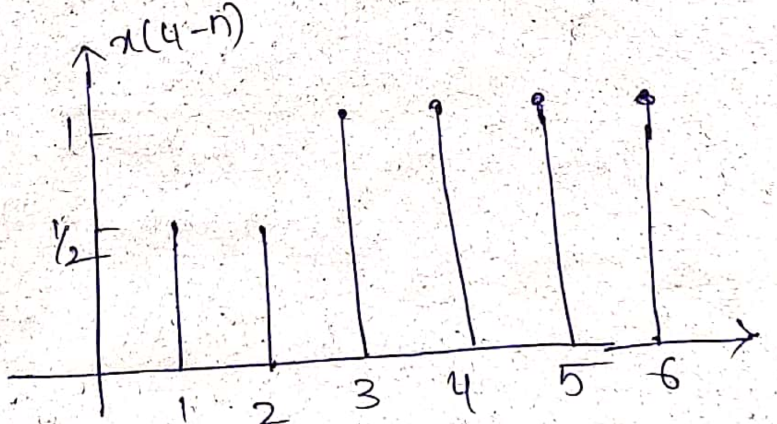
$4+2=n$

$\underline{n=6}$

$4-n=4$

$4-4=n$

$\underline{n=0}$



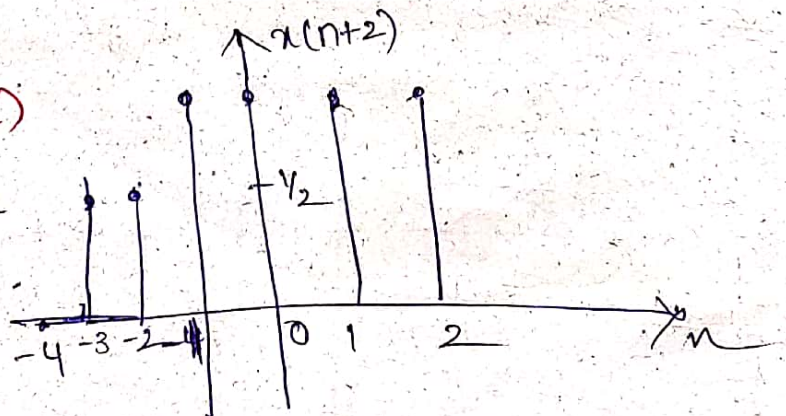
c) $x(n+2)$

$n+2=-2$

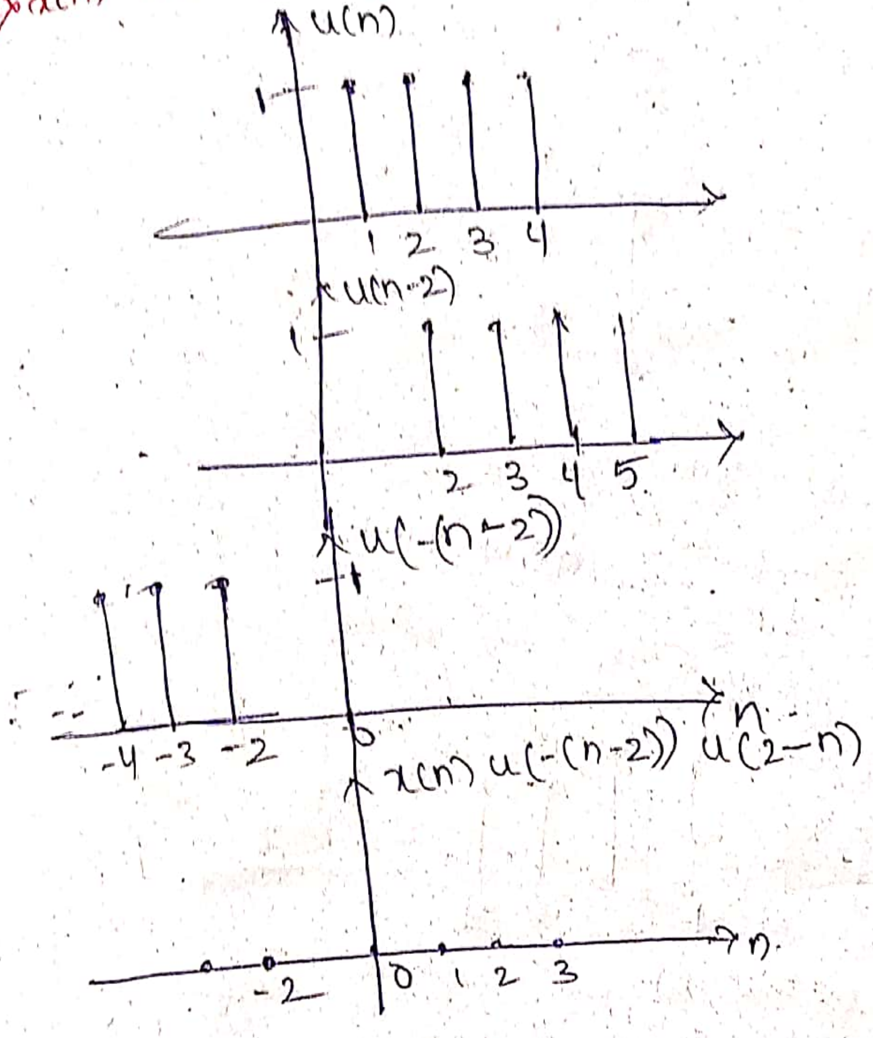
$\underline{n=-4}$

$n+2=4$

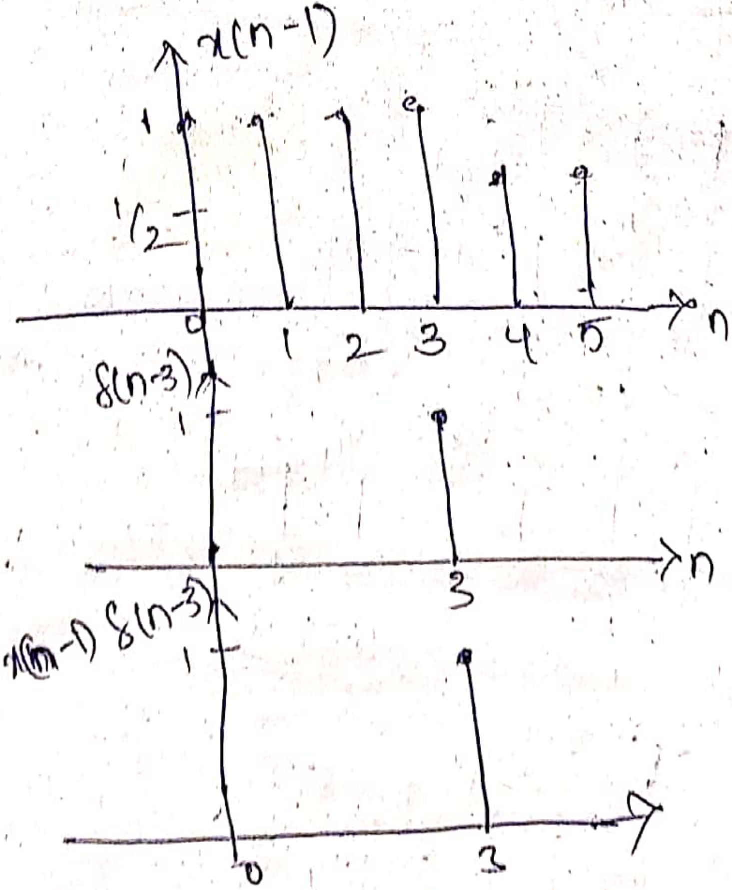
$\underline{n=2}$



d) $x(n)\ u(2-n)\ u(-(n-2))$



e) $x(n-1)\ \delta(n-3)$



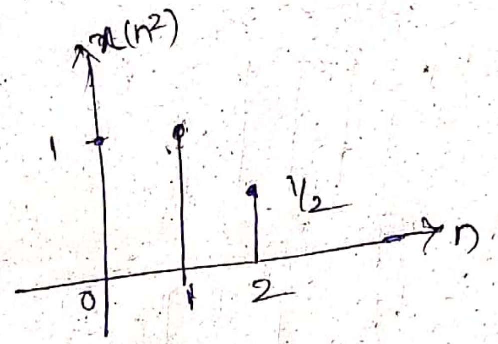
f) $x(n^2)$

$n^2 = 4 \Rightarrow n = 2$

$n^2 = 3 \Rightarrow n = \pm\sqrt{3}$

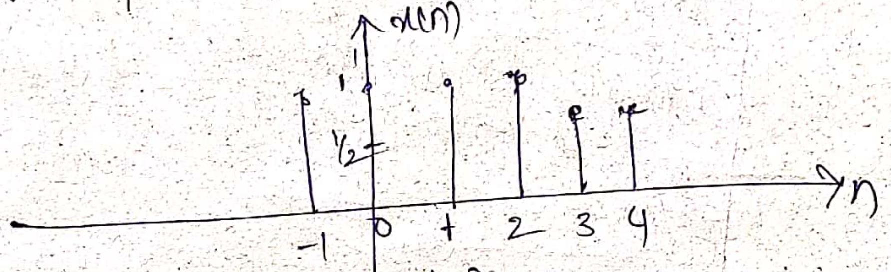
$n^2 = 1 \Rightarrow n = \pm 1$

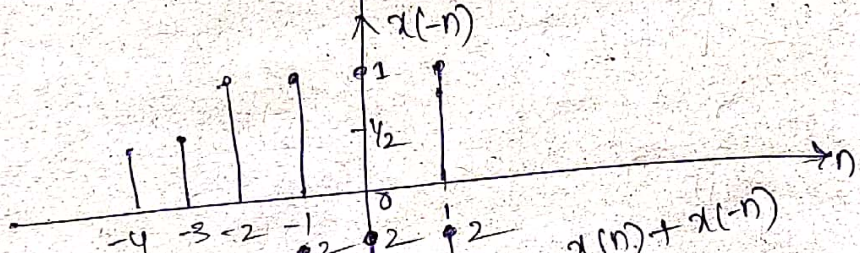
$n^2 = 0 \Rightarrow n = 0$

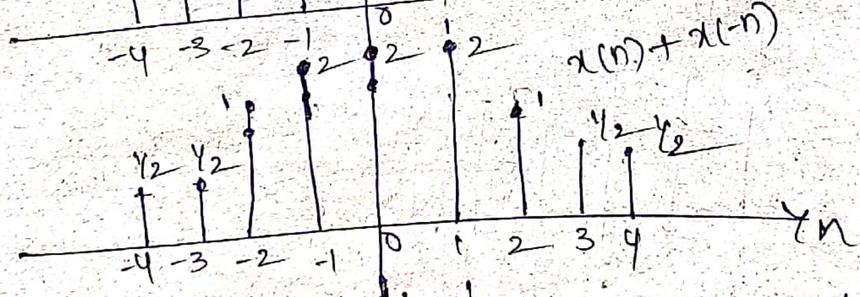


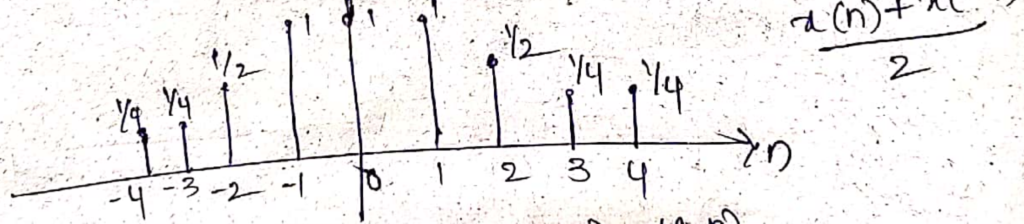
g) even part of $x(n)$

$$\text{even part of } x(n) = \frac{x(n) + x(-n)}{2}$$

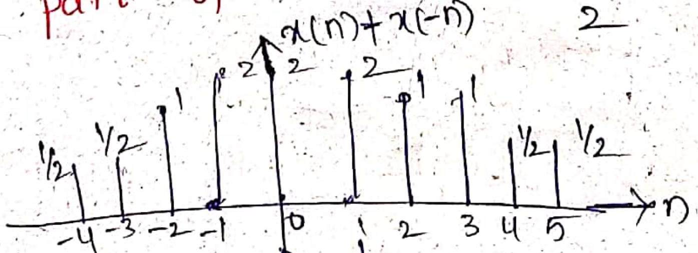


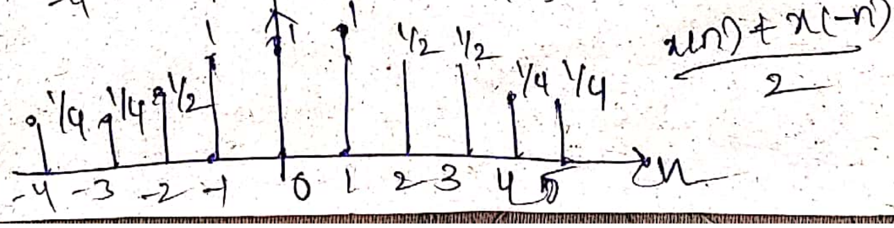


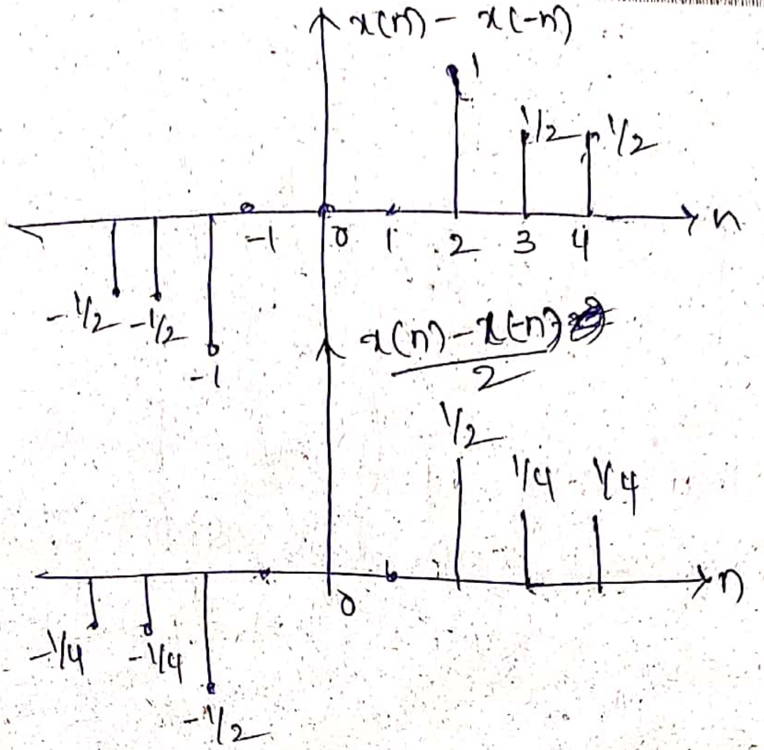




h) odd part of $x(n) = \frac{x(n) - x(-n)}{2}$

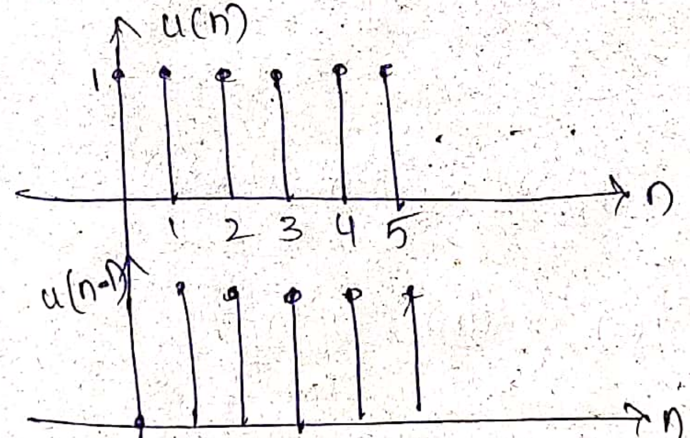


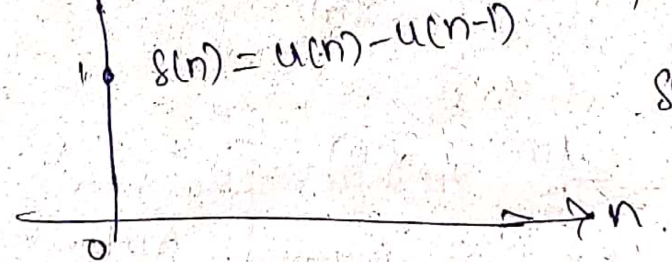




2.3) Show that

a) $\delta(n) = u(n) - u(n-1)$



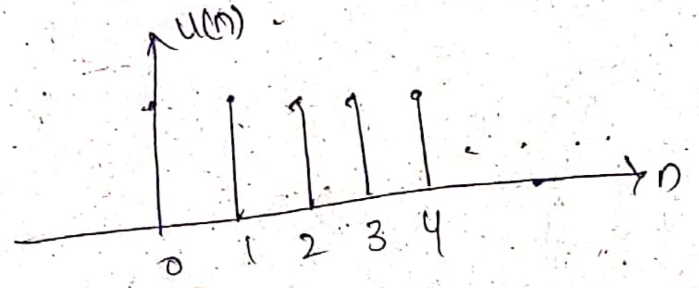


$\delta(n) = 1$ at $n=0$

b) $u(n) = \sum_{k=-\infty}^{n} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k)$

$$\sum_{k=-\infty}^{n} \delta(k) = \sum_{k=-\infty}^{0} \delta(k) + \sum_{k=0}^{n} \delta(k)$$

$$\sum_{k=0}^{\infty} \delta(n-k) = \delta(n) + \delta(n-1) + \delta(n-2) + \cdots$$



2.4> Show that any signal can be decomposed into an even and an odd component. Is the decomposition unique. Illustrate your arguments using the signal.

$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$

Sol:-

even component of the signal

$$= \frac{x(n) + x(-n)}{2}$$

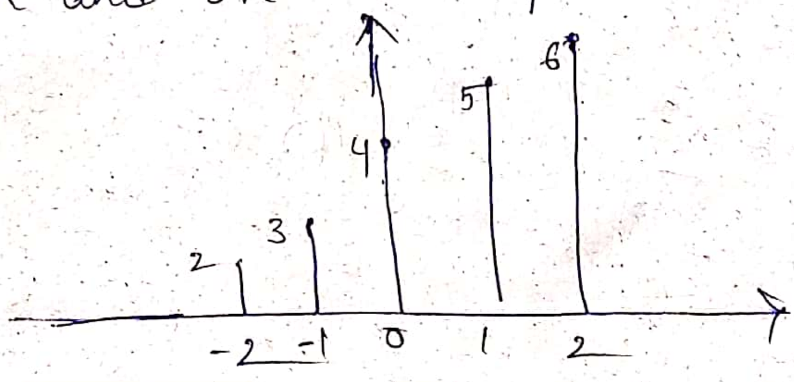
If $x(n) = x(-n)$ it is a even signal

$$\frac{x(n) + x(n)}{2} = \frac{2x(n)}{2} = x(n)$$

for odd signal $x(n) = -x(-n)$

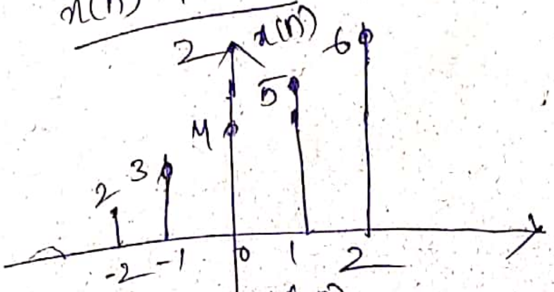
$$\frac{x(n) + x(n)}{2} = \frac{2x(n)}{2} = x(n)$$

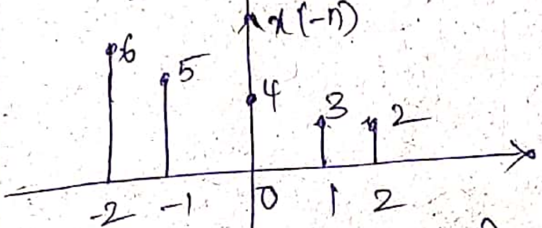
any signal can be decomposed into an even and an odd component.

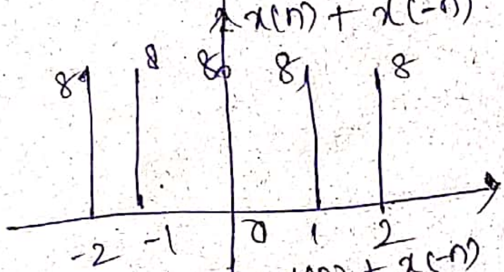


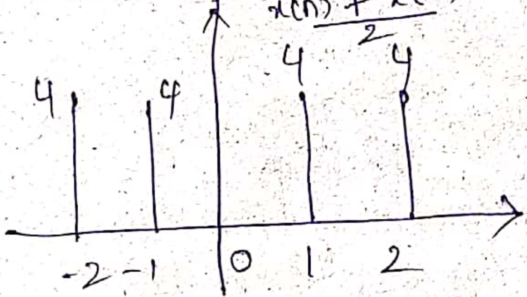
even component

$$\frac{x(n) + x(-n)}{2}$$



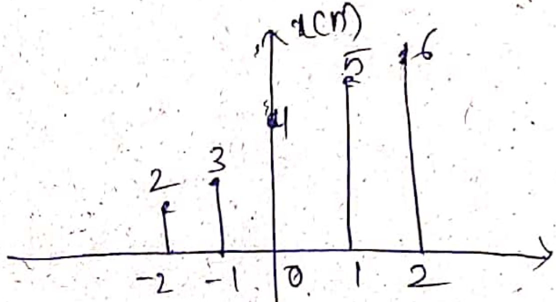


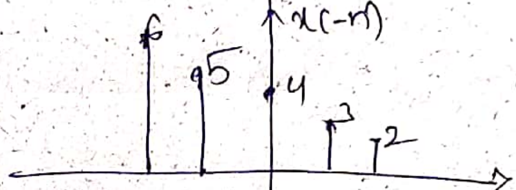


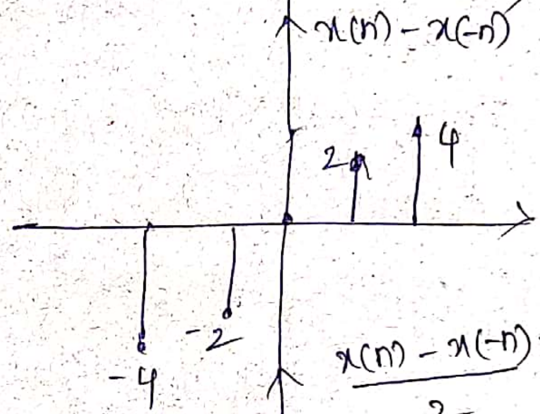


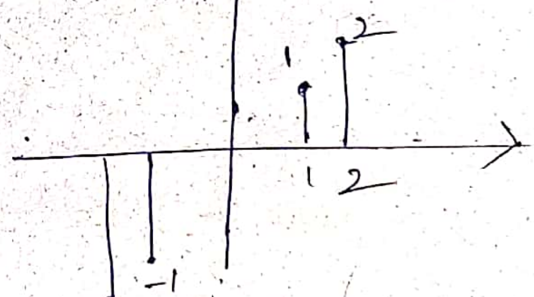
odd component

$$\frac{x(n) - x(-n)}{2}$$

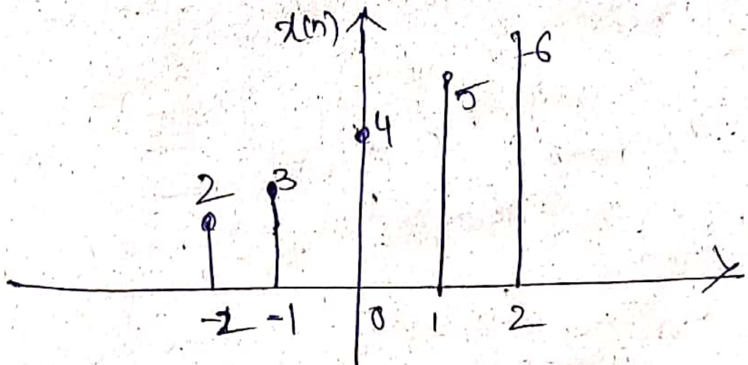








$$x(n) = \frac{x(n) + x(-n)}{2} + \frac{x(n) - x(-n)}{2}$$



we get the original signals by adding odd and even components

2.6 Consider the system

$$y(n) = T[x(n)] = x(n^2)$$

a) Determine if the system is time invariant

$$y(n) = x(n^2)$$

delaying the input

$$T[x(n-n_0)] = x((n-n_0)^2)$$

$$= x(n^2 + n_0^2 - 2nn_0)$$

$$x(n) \longrightarrow \boxed{} \longrightarrow y(n) = x(n^2)$$

$$x(n-n_0) \qquad x((n-n_0)^2) = y(n)$$

$$y(n) = x((n-n_0)^2) \neq y(n)$$

delaying the o/p

$$y(n-n_0) = x(n^2-n_0)$$

$$y(n) = y(n-n_0)$$

The system is time variant system

b) To clarify the result in part (a) assume that the signal

$$x(n) = \begin{cases} 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

is applied into the system.

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1) sketch the signal $x(n)$

2) Determine and sketch the signal $y(n)$

$$y(n) = T[x(n)]$$

$y(n) = x(n^2)$

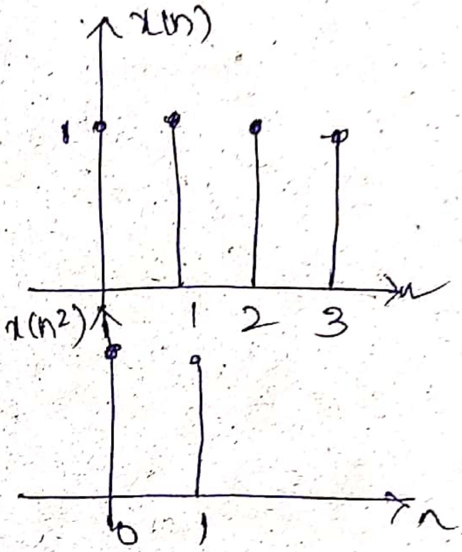
$n^2 = 0$ $\quad$ $n^2 = 1$

$n = 0$ $\quad$ $n = \pm 1$

$n^2 = 2$

$n^2 = 3$ $\quad$ $n = \pm\sqrt{2}$

$n = \pm\sqrt{3}$



3) sketch the signal $y_2'(n) = y(n-2)$

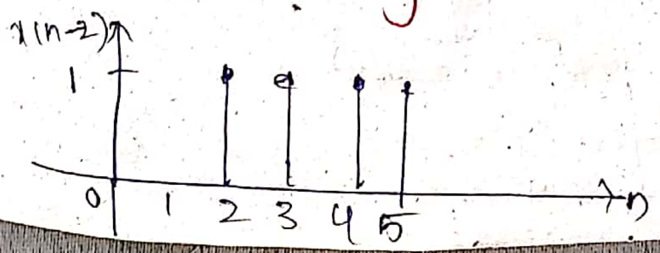
$y(n-2) = x((n-2)^2)$

$y(n) = x(n^2)$

so, $y(n)$ is shifted right by 2 units

4) Determine and sketch the signal $x_2(n) = x(n-2)$

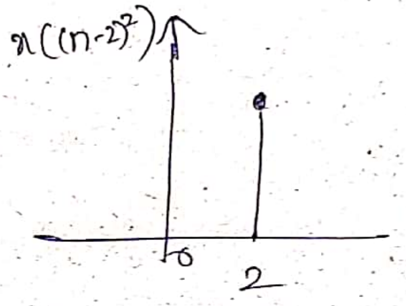
$x(n)$ is shifted right side by 2 units



b) Determine and sketch the signal

$y_2(n) = t\,[x_2(n)]$

$y_2(n) = T[x_2(n)] = x((n-2)^2)$



$(n-2)^2 = 0$

$n-2 = 0$

$n = 2$

b) compare the signal $y_2(n)$ and $y(n-2)$.

what is your conclusion

sol:- $y_2(n) \neq y(n-2)$

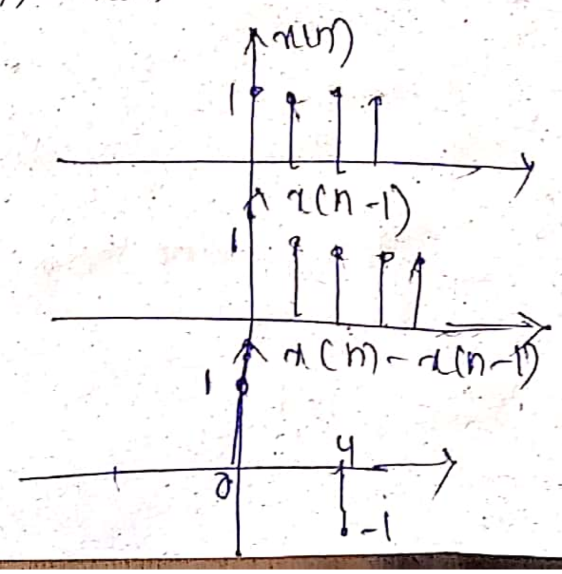
because the given system is time variant

c) Report part (b) for the system.

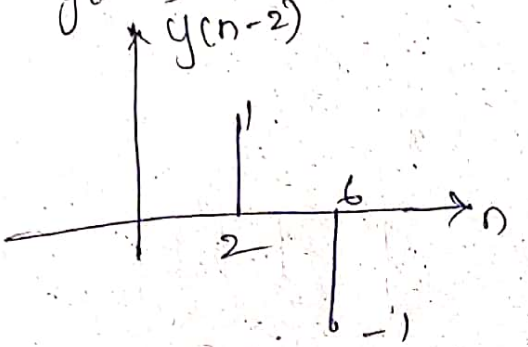
$y(n) = x(n) - x(n-1)$

Can you use this result to make any statement about the time invariance of this system? why?

sol:- $y(n) = x(n) - x(n-1)$

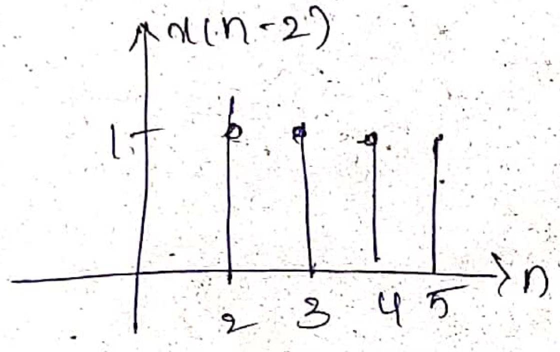


$y(n-2)$



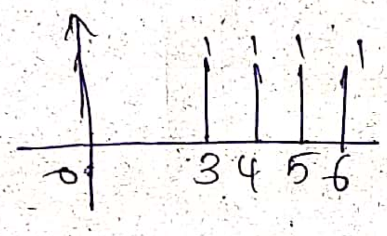
$n-2=4$ $\quad$ $n-2=0$

$n=6$ $\quad$ $n=2$



$y_2(n) = T[x_2(n)]$

$x(n-2) - x(n-3)$

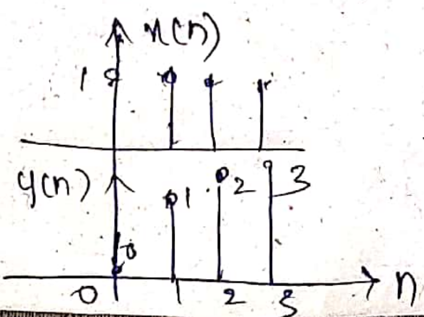


$y_2'(n) = y_2(n)$

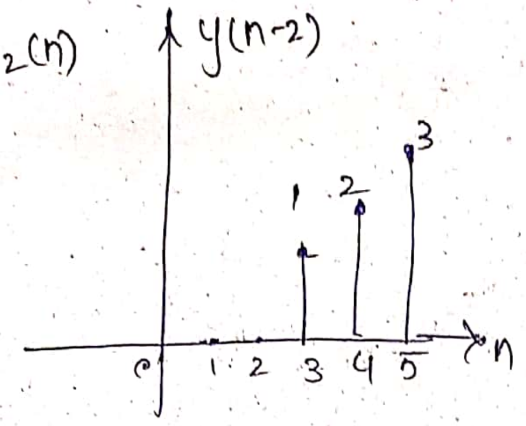
means that the given system is a time invariant. If we the o/p the result is equal to delayed i/p and o/p

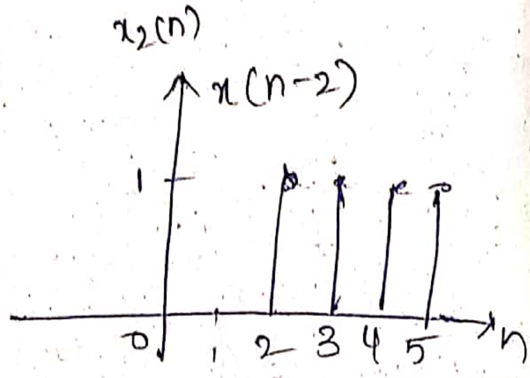
d) Report parts (b) and (c) for the system.

$y(n) = T[x(n)] = n x(n)$



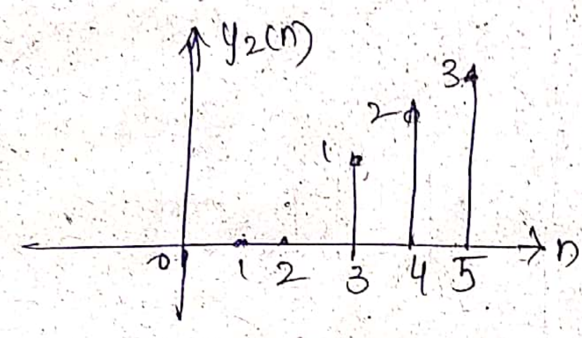
$y_2(n)$





$y_2(n) = T(x_2(n))$

$= (n-2)\ x(n-2)$



$y_2'(n) = y_2(n)$ so, it is time invariant S/m.

2.7) A discrete-time system can be

1) Static or dynamic

2) Linear or non-linear

3) Time invariant or time varying

4) causal or noncausal

5) Stable or unstable

Examine the following systems with respect to the properties

$$y(n) = \cos[x(n)]$$

1) static or dynamic

for $n=0$

$y(0) = \cos[x(0)]$

present o/p is depends on present i/p not on past i/p so, it is a static s/m.

2) linear or non-linear

$a x_1[n] + b x_2[n] = a y_1(n) + b y_2[n)$

$y_1(n) = \cos(x_1(n)] \quad y_2(n) = \cos(x_2(n))$

$a \cos(x_1(n)) + b \cos(x_2(n))$

$\cos[a x_1(n) + b x_2(n)) \neq a \cos(x_1(n)) + b \cos(x_2(n))$

so; the s/m is non linear.

3) Time invariant or time varying

$y(n-n_0) = \cos[x(n-n_0)] = y'(n)$

$x(n-n_0)$

$y(n) = \cos(x(n-n_0))$

$y'(n) \neq y(n)$

So, the given system is time invariant.

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4) causal or non-causal

Every static system is a causal system but every causal is not static s/m.

$y(n)$ is a causal system.

5) Stable or unstable

Stable.

6) $y(n) = \sum_{K=-\infty}^{n+1} x(K)$

1) $y(0) = \sum_{K=-\infty}^{n+1} x(K)$ static or dynamic

$= x(-\infty) + \cdots x(0) + x(1) + \cdots + x(n) + x(n+1)$

consider $n=0$

$y(0) = x(0) + x(0+1)$

the given s/m depends on past i/p's and future i/p

so, the given s/m is a dynamic system.

2) linear or non-linear

$x(K) = a\, x_1(K) + b x_2(K)$

$a\, y_1(n) + b\, y_2(n)$.

$a\, y_1(n) = a \sum_{K=-\infty}^{n+1} x_1(K)$

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$by_2(n) = b \sum_{k=-\infty}^{n+1} x_1(k)$

$ay_1(n) + by_2(n) = a \sum_{k=-\infty}^{n+1} x_1(k) + b \sum_{k=-\infty}^{n+1} x_1(k)$

$y(n) = \sum_{k=-\infty}^{n+1} a x_1(k) + b x_2(k) = a \sum_{k=-\infty}^{n+1} a x_1(k) + b \sum_{k=-\infty}^{n+1} x_2(k)$

$\therefore$ the s/m is linear

3) Time invariant s/m

4) noncausal s/m

5) unstable.

c) $y(n) = x(n) \cos(\omega_0 n)$

1) $y(0) = x(0) \cos(0)$

Static system

2) $x(n) = a x_1(n) + b x_2(n)$

$y(n) = a y_1(n) + b y_2(n)$ $\quad by_2(n) = b x_2(n) \cos(\omega_0 n)$

$a y_1(n) = a x_1(n) \cos(\omega_0 n)$

$y'(n) = a x_1(n) \cos(\omega_0 n) + b x_2(n) \cos(\omega_0 n)$

$y(n) = a x_1(n) \cos(\omega_0 n) + b x_2(n) \cos(\omega_0 n)$

$\therefore$ it is a linear s/m.

3) $y(n - n_0) = x(n - n_0) \cos(\omega_0 (n - n_0))$

$y(n) = x(n - n_0) \cos(\omega_0 n)$ $\quad \therefore$ Time varying s/m

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4) causal s/m

5) unstable

d) $y(n) = x(-n+2)$

1) $y(0) = x(0+2)$ $\quad y(1) = x(-1+2) = x(1)$

$y(-1) = x(1+2) = x(3)$

$\therefore$ dynamic system.

2) linear system

3) $y(n) = x(-n+2)$

$y(n-n_0) = x(-(n-n_0)+2)$

$y'(n) = x(-n+n_0+2)$

$y(n) = x(-n+2+n_0)$

$\therefore$ Time invarying.

4) causal or noncausal

$\therefore$ noncausal s/m

5) stable.

e) $y(n) = T_{run}[x(n)]$, where $T_{run}(x(n))$ denotes the integer part of $x(n)$, obtained by truncation

1) static

$a\, y_1(n) + b\, y_2(n) \neq \text{Round}\,[a x_1(n) + b x_2(n)]$

2> non linear

3> $y(n) = y(n-n_0) = \text{Round}\,[x(n-n_0)]$

$x(n-n_0)$ $\quad y(n) = \text{Round}\,[x(n-n_0)]$ Time invariant.

4> causal s/m

5> stable

f> $y(n) = \text{Round}\,[x(n)]$, where $\text{Round}\,[x(n)]$ denotes the integer part of $x(n)$ obtained by truncation.

g> $y(n) = |x(n)|$

1> $y(0) = |x(0)|$ Static s/m

2> $a_1 |y_1(n)| + b y_2(n) = |a_1 x_1(n) + b\, x_2(n)|$

$a\, |x_1(n)| + b|x_2(n)| = a_1\, |x_1(n)| + b\,(x_2(n))$

non linear

3> Time invariant

4> Causal

h> $y(n) = x(n)\, u(n)$

1> static s/m

2> $a x_1(n)\, u(n) + x_2\, u(n) = a x_1(n)\, u(n) + b\, x_2(n)\, u(n)$

linear s/m.

3> $y(n) = y(n-n_0) = x(n-n_0)\, u(n-n_0)$

$x(n-n_0)\, u(n-n_0) = y(n)$

Time invariant

4> causal s/m

5> stable

8> $y(n) = x(n) + n\, x(n+1)$

1> $y(0) = x(0) + 0\, x(0+1) \Rightarrow x(1) + 1\, x(2) = y(1)$

dynamic s/m

2> linear s/m

3> Time variant

4> no-causal s/m

5> stable.

9> $y(n) = x(2n)$

1> $y(1) = x(2)$ dynamic s/m

2> $a\, y_1(n) + b\, y_2(n) = [a x_1(2n) + b x_2(n)]$

linear s/m

3> $y(n-n_0) = x(2(n-n_0))$

$y(n) = x(2n-n_0)$

time variant s/m.

4) non- causal

5) stable.

k) $y(n) \begin{cases} x(n) ; & \text{if } x(n) \geq 0 \\ 0 ; & \text{if } x(n) < 0 \end{cases}$

1) $y(0) = x(0)$ Static s/m

2) non- linear s/m

3) $y(n-n_0) = x(n-n_0)$ time invariant s/m

4) causal s/m

5) stable.

l) $y(n) = x(-n)$

1) $y(0) = x(-0)$

$y(1) = x(-1)$

dynamic s/m

2) linear s/m

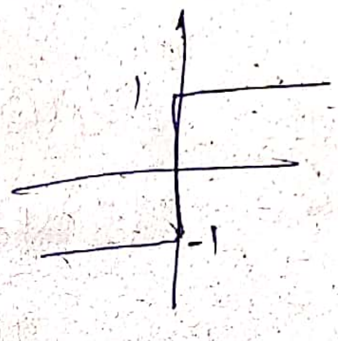
3) $y(n-n_0) = x(-(n-n_0))$

4) causal s/m

5) Stable.

m> $y(n) = \text{sign}[x(n)]$

1> $y(0) = \text{sign}(x(0))$



Static s/m

2> non-linear

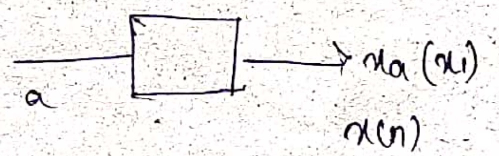
3> time invariant

4> causal s/m

5> Stable

n> The ideal sampling system with i/p $x_a(t)$ and o/p $x(n) = x_a(nT)$ $-\infty < n < \infty$

$x(n) = x_a(nT)$



1> $x(n) = x_a(nT)$

2> linear

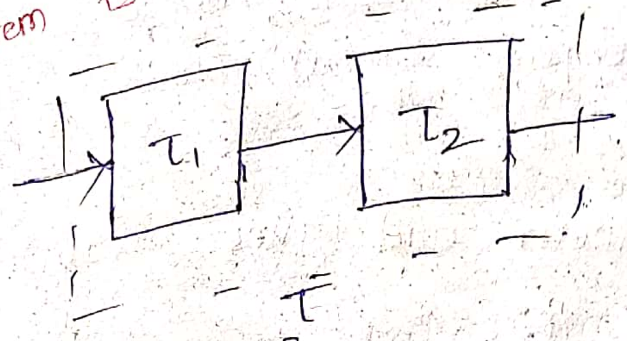
3> Time invariant

4> causal

5> stable.

2.8> Two discrete-time systems T_1 and T_2 are connected in cascade to form a new system T as shown in fig P2.8. prove or disprove the following statements.

a> If T_1 and T_2 are linear, then T is linear (i.e.) The cascade connection of two linear system is linear)



$v_1(n) = T_1[x_1(n)]$

$v_2(n) = T_1[x_2(n)]$

Then $a x_1(n) + b x_2(n)$ gives

$a v_1(n) + b v_2(n)$

$y_1(n) = T_2[v_1(n)]$ and $y_2(n) = T_2[v_2(n)]$,

$y(n) = a y_1(n) + b y_2(n)$

$= a v_1(n) + b v_2(n)$

$a T[x_1(n)] + b T[x_2(n)]$ $\quad T = T_1 T_2$

The system is linear

b) If T_1 and T_2 are time invariant, then T is time invariant this statement is true.

$x(n) \longrightarrow v(n)$

$x(n-k) \longrightarrow v(n-k)$ for T_1

$v(n) \longrightarrow y(n)$ $\quad v(n-k) \longrightarrow y(n\cancel{+k})$

for T_2

$x(n) \longrightarrow y(n)$

$x(n-k) \longrightarrow y(n-k)$

$T = T_1 T_2$ is time invariant.

c) If T_1 and T_2 are causal, then T is causal.

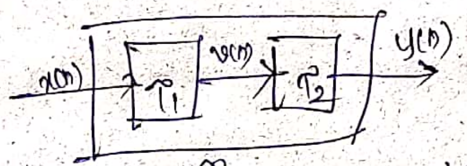
If T_1 is causal $\rightarrow v(n)$ depends on $x(k)$ for $k \le n$

T_2 is causal $\therefore$ $y(n)$ depends on $x(k)$ for $k \le n$

Hence T is causal.

d) If T_1 and T_2 are linear and time invariant, the same holds for T.

Yes, T holds the same as T_1 and T_2



the above proof from 'b' and 'c' suits this answer.

e) If T_1 and T_2 are linear and time invariant, then interchanging their order does not change the system.

True

$h_1(n) * h_2(n) = h_2(n) * h_1(n)$

f) As in part (e) except that T_1, T_2 are now time varying (Hint: use an example)

$T_1 : y(n) = n\,x(n)$

$T_2 : y(n) = n\,x(n+1)$

here, two S/m's are not time varying

$T_2[T_1[\delta(n)]] = T_2(0) = 0$

$T_1[T_2[\delta(n)]] = T_1[\delta(n+1)]$

$= \delta(n+1)$

$\neq 0$

b) If T_1 and T_2 are stable, then T is stable.

$r(n)$ is bounded i/p. $u(n)$ is bounded o/p

$u(n)$ is i/p to the next S/m. o/p will be bounded.

$x(n)$ is bounded i/p $y(n)$ is bounded o/p

∴ overall S/m is a stable system.

Q. Show by an example that the inverses of parts (c) and (b) do not hold in general.

Inverse of part (c) is

If c is causal then only T_1 and T_2 are caused.

T is stable then only T_1 and T_2 are stable.

2.9) Let T be an LTI, relaxed and BIBO stable S/m with input $x(n)$ and o/p $y(n)$.

Show that:

a) If $x(n)$ is periodic with period N (i.e. $x(n) = x(n+N)$ for all $n \geq 0$) the o/p $y(n)$ tends to a periodic signal with the same period

Sol:-

$$y(n) = \sum_{K=-\infty}^{n} h(K)\, x(n-K), \quad x(n)=0 \quad n<0$$

$$y(n+N) = \sum_{K=-\infty}^{n+N} h(K)\, x(n+N-K) =$$

$$= \sum_{K=-\infty}^{n+N} h(K)\, x(n-K)$$

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$$= \sum_{k=-\infty}^{n} h(k)\, x(n-k) + \sum_{k=n+1}^{n+N} h(k)\, x(n-k)$$

$$= y(n) + \sum_{k=n+1}^{n+N} h(k)\, h(n-k)$$

For a BIBO s/m, $\lim_{n\to\infty} |h(n)| = 0$

$$= \lim_{n\to\infty} \sum_{k=n+1}^{n+N} h(k)\, x(n-k) = 0$$

$$\Rightarrow \lim_{n\to\infty} y(n+N) = y(N)$$

b) If $x(n)$ is bounded and tends to a constant, the o/p will also tend to a constant

Let, $x(n) = x_0(n) + a\,u(n)$, where a is a constant and $x_0(n)$ is a bounded signal with $\lim_{n\to\infty} x_0(n) = 0$

$$y(n) = a \sum_{k=0}^{\infty} h(k)\, u(n-k) + \sum_{k=0}^{\infty} h(k)\, x_0(n-k)$$

$$= a \sum_{k=0}^{n} h(k) + y_0(n)$$

$$\sum_{n} x_0^2(n) < \infty \Rightarrow \sum_{n} y_0^2(n) < \infty$$

Hence $\lim_{n\to\infty} |y_0(n)| = 0$

$$\lim_{n\to\infty} y(n) = a \sum_{k=0}^{n} h(k) = \text{Constant.}$$

9) If $x(n)$ is an energy signal, the o/p $y(n)$ will also be an energy signal

$$y(n) = \sum_{k} h(k)\, x(n-k)$$

$$\sum_{-\infty}^{\infty} y^2(n) = \sum_{-\infty}^{\infty} \left[\sum_{k} h(k)\, x(n-k)\right]^2$$

$$= \sum_{k} \sum_{q} h(k)\, h(q) \sum_{n} x(n-k)\, x(n-q)$$

$$\sum_{n} x(n-k)\, x(n-q) \leq \sum_{n} x^2(n) = E_q$$

$$\sum_{n} y^2(n) \leq E_x \sum_{k} |h(k)| \sum_{L} |h(L)|$$

For a BIBO stable s/m

$$\sum_{k} |h(k)| < M$$

$E_y \leq M^2 E_x$ so that

$E_y < 0$ if $E_x < 0$

10) The following input-output pairs have been observed during the operation of a time-invariant s/m:-

$$x_1(n) = \{\underset{\uparrow}{1}, 0, 2\} \xleftrightarrow{T} y_1(n) = \{\underset{\uparrow}{0}, 1, 2\}$$

$$x_2(n) = \{\underset{\uparrow}{0}, 0, 3\} \xleftrightarrow{T} y_2(n) = \{\underset{\uparrow}{0}, 1, 0, 2\}$$

$$x_3(n) = \{\underset{\uparrow}{0}, 0, 0, 1\} \xleftrightarrow{T} y_3(n) = \{1, \underset{\uparrow}{2}, 1\}$$

Can you draw any conclusions regarding the linearity of the s/m. what is the impulse response of the s/m?

The S/m is non linear. This is evident from observation of the pairs

$x_3(n) \longrightarrow y_3(n)$ and $x_2(n) \longrightarrow y_2(n)$

If the s/m were linear, $y_2(n)$ would be of the form

$$y_2(n) = \{3, 6, 3\}$$

The s/m is time invariant s/m.

11) The following i/p o/p pairs have been observed during the operation of a linear s/m:-

$$x_1(n) = \{-1, \underset{\uparrow}{2}, 1\} \xleftrightarrow{T} y_1(n) = \{1, \underset{\uparrow}{2}, -1, 0, 1\}$$

$$x_2(n) = \{1, \underset{\uparrow}{-1}, -1\} \xleftrightarrow{T} y_2(n) = \{-1, \underset{\uparrow}{1}, 0, 2\}$$

$$x_3(n) = \{0, \underset{\uparrow}{1}, 1\} \xleftrightarrow{T} y_3(n) = \{\underset{\uparrow}{1}, 2, 1\}$$

Can you draw any conclusions about the time invariance of this s/m?

$$x_1(n) + x_2(n) = \delta(n)$$

and system is linear, the impulse response of the s/m is

$$y_1(n) + y_2(n) = \{0, \underset{\uparrow}{3}, -1, 2, 1\}$$

If the s/m were time invariant, the response to $x_3(n)$ would be $\{3, \underset{\uparrow}{2}, 1, 3, 1\}$.

2.12) The only available information about a system consists of N input – output pairs of signals $y(n) = T[x_i(n)]$, $i = 1, 2, \cdots N$.

a) what is the class of input signals for which we consider the output, using the information above, if the system is known to be linear?

$x_i(n) = 1, 2, 3, \cdots, N$ (Any weighted linear combination of signals).

b) The same as above, if the s/m is known to be time invariant.

$x_i(n-k)$, where k is any integer and $i = 1, 2, 3, \cdots N$.

2.13) Show that the necessary and sufficient condition for a relaxed LTI system to the BIBO stable is $\sum_{n=-\infty}^{\infty} |h(n)| \leq M_h < \infty$ for some constant M_h.

A. System is BIBO stable if and only if a bounded input produces a bounded output.

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$$y(n) = \sum_{k} h(k)\, x(n-k)$$

$$|y(n)| = \sum_{k} |h(k)|\, |x(n-k)|$$

$$= M_x \sum_{k} |h(k)|$$

$|x(n-k)| \leq M_x$; $|y(n)| < \infty$ for all n,

iff and only iff $\sum_{k} h(k) < \infty$

2.14) Show that for any real or complex constant a, and any finite integer numbers m and N, we have

a)
$$\sum_{n=M}^{N} a^n = \begin{cases} \dfrac{a^M - a^{N+1}}{1-a} ; & a \neq 1 \\ N-M+1 ; & a = 1 \end{cases}$$

for $a=1$, $\sum_{n=M}^{N} a^n = N-M+1$

for $a \neq 1$, $\sum_{n=M}^{N} a^n = a^M + a^{M+1} + \cdots + a^N$

$$(1-a)\sum_{n=M}^{N} a^n = a^M + a^{M+1} + (-a^{M+1}) + \cdots + a^N - a^N - a^{N+1}$$

$$(1-a)\sum_{n=M}^{N} a^n = a^M - a^{N+1}$$

b) Show that if $|a| < 1$, then

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

for $M=0$, $|a|<1$, and $N \to \infty$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, \quad |a|<1$$

2.15) Show that

a) A relaxed linear system is causal if and only if for any input $(x(n))$ such that $x(n)=0$ for $n<n_0 \Rightarrow y(n)=0$ for $n<n_0$

A system is causal $\Leftrightarrow$ the o/p becomes non zero after the input becomes non-zero

$x(n)=0$ for $n<n_0 \Rightarrow y(n)=0$ for $n<n_0$

b) A relaxed LTI system is causal if and only if $h(n)=0$ for $n<0$

$$y(n) = \sum_{-\infty}^{n} h(k)\, x(n-k), \quad (x(n)=0, n<0)$$

if $h(k)=0$ for $k=0$, then

$$y(n) = \sum_{0}^{n} h(k)\, x(n-k); \quad (y(n)=0 \text{ for } n<0)$$

if $y(n)=0$ for $n<0$, then

$$\sum_{k=-\infty}^{n} h(k)\, x(n-k) \Rightarrow h(k)=0, \; k<0$$

2.16) a) If $y(n) = x(n) * h(n)$; show that

$$\sum_y = \sum_x \sum_h, \text{ where } \sum_x = \sum_{n=-\infty}^{\infty} x(n)$$

$$y(n) = \sum_k h(k)\, x(n-k)$$

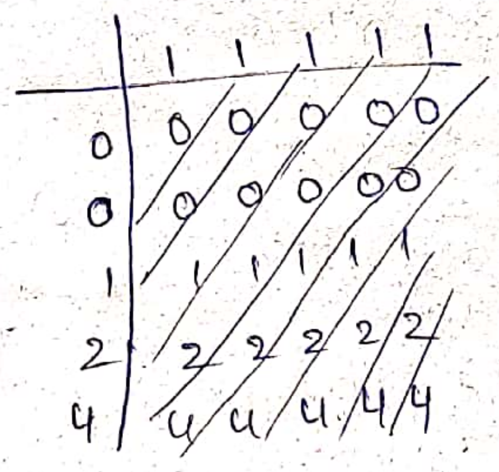
$$\sum_n y(n) = \sum_n \sum_k h(k)\, x(n-k) = \sum_k h(k) \sum_{n=-\infty}^{\infty} x(n-k)$$

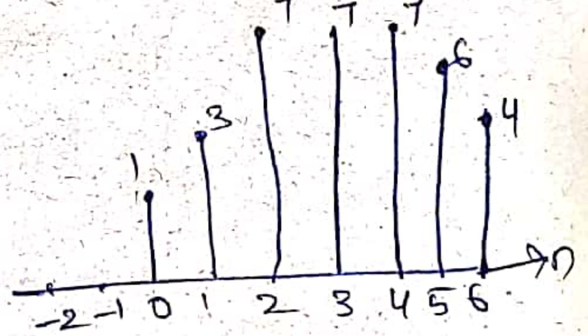
$$\sum_n (y(n)) = \left(\sum_k h(k)\right)\left(\sum_n x(n)\right)$$

b) compute the convolution $y(n) = x(n) * h(n)$ of the following signals and check the correctness of the results by using the test in (a)

1) $x(n) = \{1, 2, 4\}$, $h(n) = \{1, 1, 1, 1, 1\}$

$y(n) = x(n) * h(n)$





$y(n) = \{0, 0, \underset{\uparrow}{1}, 3, 7, 7, 7, 6, 4\}$

$$\sum_{n=0}^{6} y(n) = y(0) + y(1) + y(2) + y(3) + y(4) + y(5) + y(6)$$

$$= 1+3+7+7+7+6+4$$

$$= 35$$

$$\sum_{n=0}^{N} y(n) = \left(\sum_{K} h(K)\right)\left(\sum_{K} x(K)\right)$$

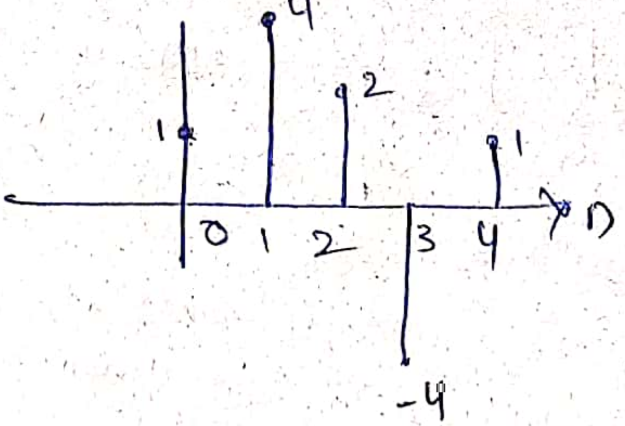
$$\sum_{K} h(K) = 5, \quad \sum_{K} x(K) = 7.$$

2) $x(n) = \{1, 2, -1\}$, $h(n) = x(n)$

$$y(n) = x(n) * h(n) = x(n) * x(n)$$

	1	2	-1
1	1	2	-1
2	2	4	-2
-1	-1	-2	1

$$y(n) = \{1, 4, 2, -4, 1\}$$



$$\sum_{n=0}^{4} y(n) = y(0) + y(1) + y(2) + y(3) + y(4)$$

$$= 1+4+2-4+2 = 2+2 = 4.$$

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$$\sum_{n=0} y(n) = \left(\sum_K h(K)\right)\left(\sum_K x(K)\right)$$

$$= (2)\,2 = 4$$

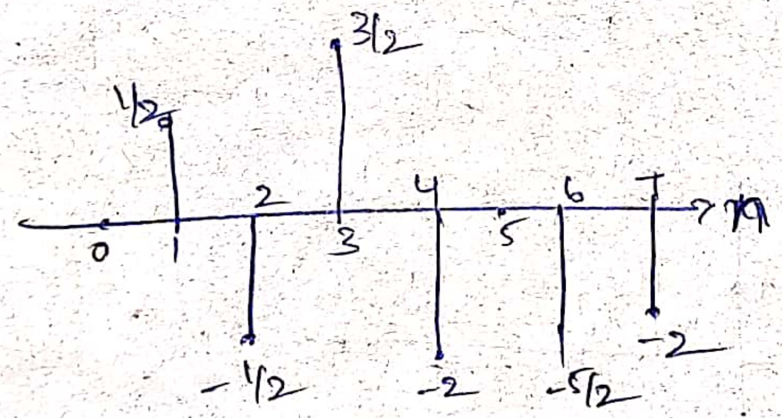
$$\sum_K h(K) = 1+2-1 = 2$$

$$\sum_K x(K) = 1+-1 = 2$$

3) $x(n) = \{0, 1, -2, 3, -4\}$, $h(n) = \{\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}\}$

$x(n)$ \ $h(n)$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$
0	0	0	0	0
1	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$
-2	-1	-1	-2	-1
3	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{3}$	$\frac{3}{2}$
-4	-2	-2	-4	-2

$y(n) = \{0, \frac{1}{2}, -\frac{1}{2}, \frac{3}{2}, -2, 0, -\frac{5}{2}, -2\}$



$$\sum_{n=0}^{7} y(n) = y(0) + y(1) + y(2) + y(3) + y(4) + y(5) + y(6) + y(7).$$

$= 0 + \frac{1}{2} + \frac{3}{2} + \left(-\frac{1}{2}\right) - 2 + 0 - \frac{5}{2} - 2$

$= -5$.

$$\sum_{n=0}^{7} y(n) = \left(\sum_k h(k)\right)\left(\sum_k x(k)\right)$$

$= (2.5)\ (-2)$

$= -5$

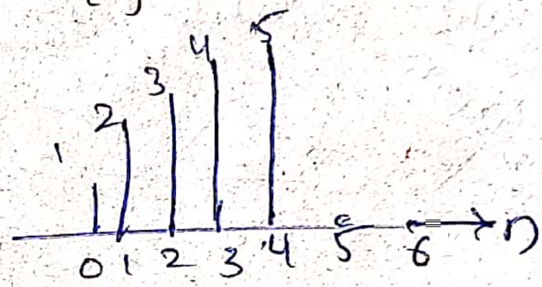
$\sum_k h(k) = 2.5$

$\sum_k x(k) = -2$

4) $x(n) = \{1, 2, 3, 4, 5\}$, $h(n) = \{1\}$

$y(n) = x(n) * h(n)$

$y(n) = \{1, 2, 3, 4, 5\}$

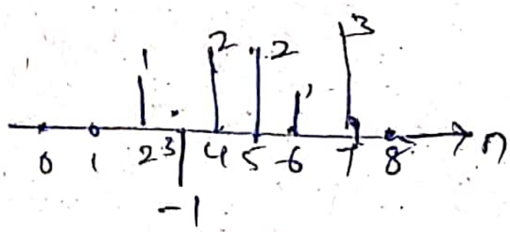


$\sum_k h(k) = 1$, $\sum_k x(k) = 1+2+3+4+5 = 15$.

5) $x(n) = \{\underset{\uparrow}{1}, -2, 3\}$, $h(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1\}$

	0	0	1	1	1	1
1	0	0	1	1	1	1
-2	0	0	-2	-2	-2	-2
3	0	0	3	3	3	3
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$y(n) = \{0, 0, 1, -1, 2, 1, 3, 0, 0\}$.



$$\sum_{n=0}^{7} y(n) = y(0) + y(1) + y(2) + y(3) + y(4) + y(5) + y(6)$$

$$= 0+0+1+(-1)+2+2+1+3+0+0+0 = 8.$$

$$\sum_{n=0}^{n} y(n) = \left(\sum_{k} x(k)\right)\left(\sum_{k} h(k)\right)$$

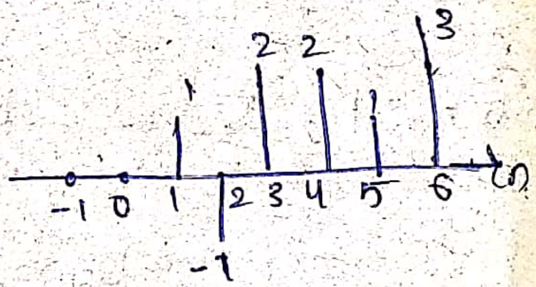
$$= (1-2+3)(0+0+1+1+1+1)$$

$$= 2\times 4 = 8$$

6) $x(n) = \{0,0,1,1,1,1\}$, $h(n) = \{1,-2,3\}$

x(n) \ h(n)	1	-2	3
0	0	0	0
0	0	0	0
1	1	-2	3
1	1	-2	3
1	1	-2	3
1	1	-2	3

$y(n) = \{0,0,1,-1,2,2,1,1\}$



$$\sum_{n=0}^{6} y(n) = y(0) + y(1) + y(2) + y(3) + y(4) + y(5) + y(6)$$

$$\sum_{n=0}^{6} y(n) = \left(\sum_{n} x(n)\right)\left(\sum_{k} h(k)\right)$$

$$= 4\times 2$$

$$= 8$$

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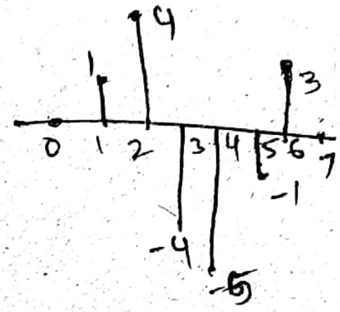
7) $x(n) = \{\underset{\uparrow}{0}, 1, 4, -3\}$, $h(n) = \{\underset{\uparrow}{1}, 0, -1, -1\}$

$x(n)$ \ $h(n)$	1	0	-1	-1
0	0	0	0	0
1	1	0	-1	-1
4	4	0	-4	-4
-3	-3	0	3	3

$y(n) = \{0, 1, 4, -4, -5, -1, 3\}$



$$\sum_{n=0}^{6} y(n) = y(0) + y(1) + y(2) + y(3) + y(4) + y(5) + y(6)$$
$$= 0 + 1 + 4 - 4 - 5 - 1 + 3 = -2$$

$$\sum_{n=0}^{n} y(n) = \left(\sum_{k} x(k)\right)\left(\sum_{k} h(k)\right) = 2 \times -1 = -2$$

8) $x(n) = \{1, 1, 2\}$, $h(n) = u(n)$

$y(n) = u(n) + u(n-1) + 2u(n-2)$

$\Rightarrow x(n) = \delta(n) + \delta(n-1) + 2\delta(n-2)$ $\Rightarrow h(n) = \delta(n)$

$\Rightarrow y(n) = x(n) * h(n)$

$\Rightarrow y(n) = \delta(n) * u(n) + \delta(n-1) * u(n) + 2\delta(n-2) * u(n)$

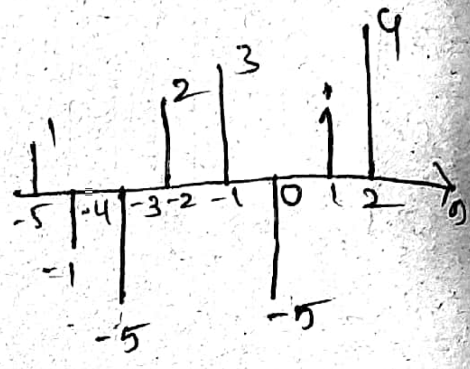
$\Rightarrow y(n) = u(n) = u(n) + u(n-1) + 2u(n-2)$

$\sum_{n} y(n) = \infty$, $\sum_{n} h(n) = \infty$, $\sum_{n} x(n) = 4$

9) $x(n) = \{1, 1, 0, 1, 1\}$, $h(n) = \{1, 2, -3, 4\}$

$x(n)$ \ $h(n)$	1	1	0	1	1
1	1	1	0	1	1
-2	-2	-2	0	-2	-2
-3	-3	-3	0	-3	-3
4	-4	4	0	4	4

$y(n) = \{1, -1, -5, 2, 3, -5, 4\}$



$$\sum_{n=0}^{n} y(n) = \left(\sum_K x(K)\right)\left(\sum_K h(K)\right) = 4 \times 0 = 0$$

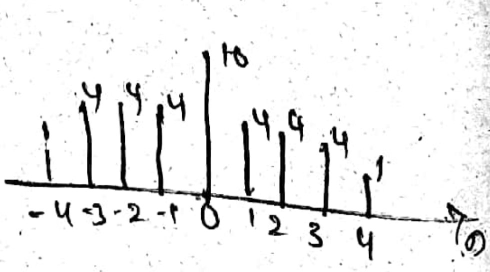
$$\sum_{n=-5}^{2} y(n) = y(-5) + y(-4) + y(-3) + y(-2) + y(-1) + y(0) + y(1) + y(2)$$

$$= 1 - 1 + 5 + 2 + 3 - 5 + 1 + 4 = 0$$

10) $x(n) = \{1, 2, 0, 2, 1\}$, $h(n) = x(n)$

$x(n)$ \ $h(n)$	1	2	0	2	1
1	1	2	0	2	1
2	2	4	0	4	2
0	0	0	0	0	0
2	2	4	0	4	2
1	1	2	0	2	1

$y(n) = \{1, 4, 4, 4, 10, 4, 4, 4\}$



$$\sum_n y(n) = \sum_K x(K) \sum_K h(K)$$

$$= 6 \times 6 = 36$$

$$\sum_{n=-4}^{4} y(n) = 1+4+4+4+10+4+4+4+1 = 36$$

11) $x(n) = (1/2)^n u(n), \quad h(n) = \left(\frac{1}{4}\right)^n u(n)$

$$y(n) = x(n) * h(n) = \sum_{h=-\infty}^{\infty} h(k)\, x(n-k)$$

$$y(n) = (1/2)^n u(n) * \left(\frac{1}{4}\right)^n u(n)$$

$$y(n) = (1/2)^n * \left(\frac{1}{4}\right)^n$$

$$= \sum_{k=0}^{n} (1/2)^k \left(\frac{1}{4}\right)^{n-k}$$

$$= \left(\frac{1}{4}\right)^n \sum_{k=0}^{n} (1/2)^k \left(\frac{1}{4}\right)^{-k}$$

$$= \left(\frac{1}{4}\right)^n \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k \cdot 4^k \qquad \sum_{k=0}^{n} a^k = \frac{1-a^{n+1}}{1-a}$$

$$= \left(\frac{1}{4}\right)^n \sum_{k=0}^{n} 2^k$$

$$= \left(\frac{1}{4}\right)^n \left(\frac{1-2^{n+1}}{1-2}\right)$$

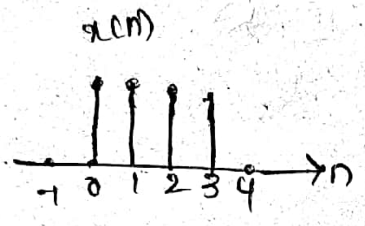
$$= \left(\frac{1}{4}\right)^n \frac{(1-2^n.2)}{-1} = \left(\frac{1}{4}\right)^n (2^n) \frac{\left(\left(\frac{1}{2}\right)^n - 2\right)}{-1}$$

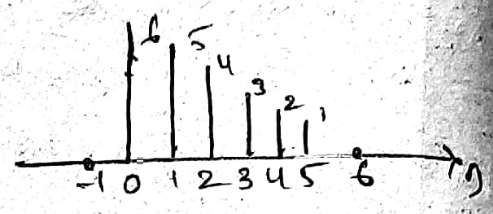
$$= (1/2)^n \left(2 - (1/2)^n\right) \text{ for } n \geq 0$$

$$\sum_n y(n) = 8/3, \quad \sum_n h(n) = \frac{4}{3}, \quad \sum_n x(n) = 2$$

2.17) Compute and plot the convolutions $x(n) * h(n)$ and $h(n) * x(n)$ for the pairs of signals shown in figure.

a)

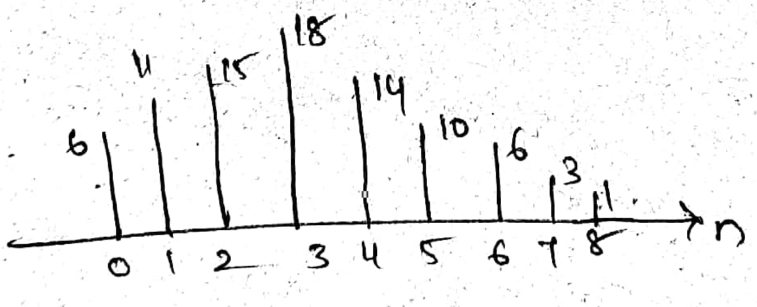




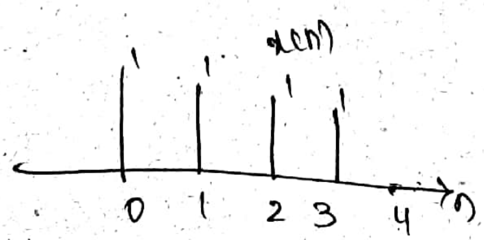
$y(n) = x(n) * h(n)$

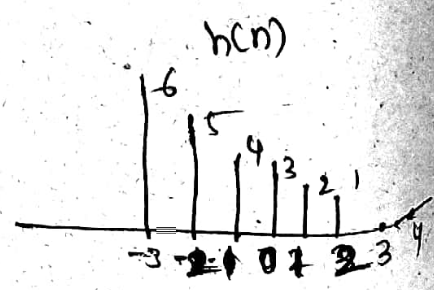
x(n) \ h(n)	6	5	4	3	2	1
1	6	5	4	3	2	1
1	6	5	4	3	2	1
1	6	5	4	3	2	1
1	6	5	4	3	2	1

$y(n) = \{6, 11, 15, 18, 14, 10, 6, 3, 1\}$



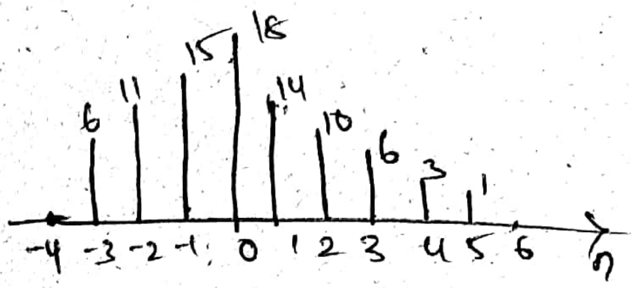
b)



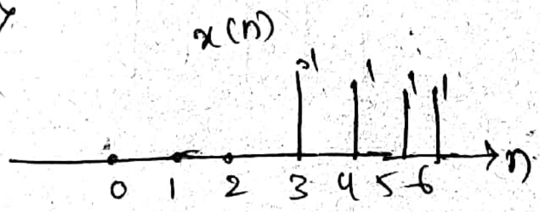


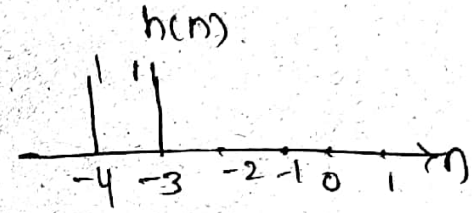
$x(n) \backslash h(n)$	1	1	1	1
6	6	6	6	6
5	5	5	5	5
4	4	4	4	4
3	3	3	3	3
2	2	2	2	2
1	1	1	1	1

$y(n) = \{6, 11, 15, 18, 14, 10, 6, 3, 1\}$



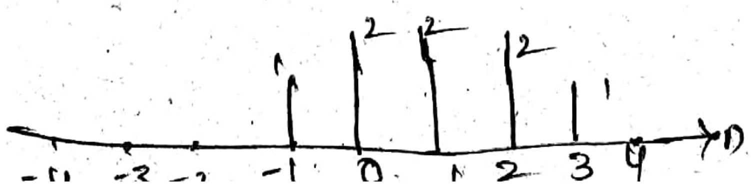
c)





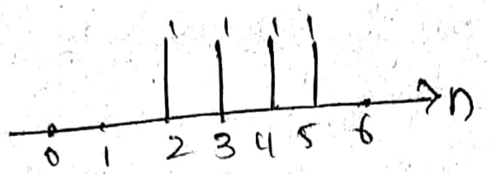
$x(n) \backslash h(n)$	1	1	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
1	1	1	0	0	0	0
1	1	1	0	0	0	0
1	1	1	0	0	0	0
1	1	1	0	0	0	0

$y(n) = \{0, 0, 0, 1, 2, 2, 2, 1, 0, 0, 0\}$

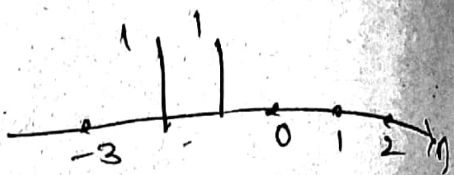


d)

$x(n)$

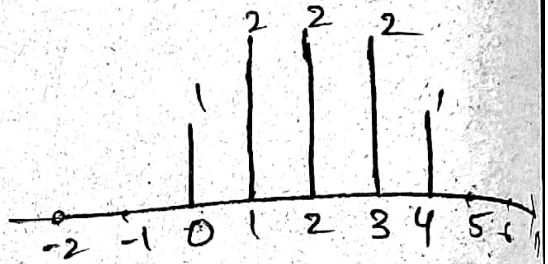


$h(n)$



	1	1	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
1	1	1	0	0	0
1	1	1	0	0	0
1	1	1	0	0	0
1	1	1	0	0	0

$y(n) = \{0, 0, 1, 2, 2, 2, 1, 0, 0, 0\}$



2.18) Determine and sketch the convolution $y(n)$ of the signals.

$$x(n) = \begin{cases} \frac{1}{3}n, & 0 \le n \le 6 \\ 0, & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 1; & -2 \le n \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

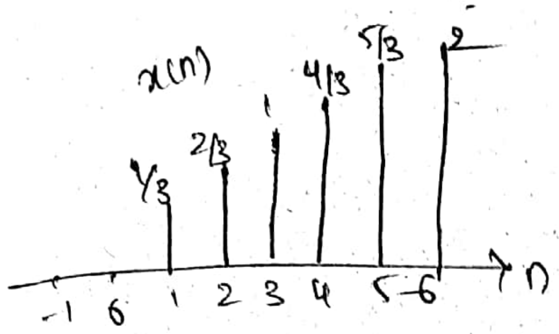
a) Graphically

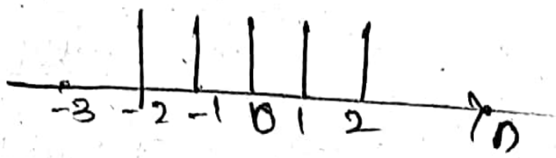
b) Analytically.

a) $x(n) = \{0, \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{5}{3}, 2\}$

$h(n) = \{1, 1, 1, 1, 1\}$

Scanned by CamScanner

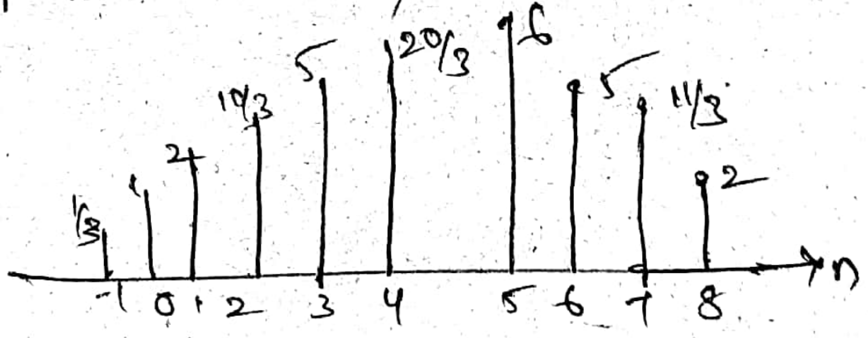




	1	1	1	1	1
0	0	0	0	0	0
1/3	1/3	1/3	1/3	1/3	1/3
2/3	2/3	2/3	2/3	2/3	2/3
1	1	1	1	1	1
4/3	4/3	4/3	4/3	4/3	4/3
5/3	5/3	5/3	5/3	5/3	5/3
6/3	6/3	6/3	6/3	6/3	6/3

$$y(n) = \left\{0, \frac{1}{3}, \frac{2}{3}, \frac{6}{3}, \frac{10}{3}, \frac{15}{3}, \frac{20}{3}, 6, 5, \frac{11}{3}, 2\right\}$$

$$y(n) = \left\{1/3, 1, 2, \frac{10}{3}, 5, \frac{20}{3}, 6, 5, \frac{11}{3}, 2\right\}$$



b) $x(n) = \{\underset{\uparrow}{0}, 1/3, 2/3, 1, 4/3, 5/3, 2\}$

$= 1/3\,\{\underset{\uparrow}{0}, 1, 2, 3, 4, 5, 6\}$

$= 1/3\, n\,\{u(n) - u(n-7)\}$

$h(n) = \{1, 1, 1, 1, 1\} = u(n+2) - u(n-3)$

$y(n) = x(n) * h(n)$

$$y(n) = \left[\tfrac{1}{3} n\, u(n) - \tfrac{1}{3} n\, u(n-7)\right] * \left[u(n+2) - u(n-3)\right]$$

$$y(n) = \tfrac{1}{3}\left[n\,u(n) * u(n+2) - n\,u(n) * u(n-3) + n\,u(n-7) * u(n-3)\ n\,u(n-7) * u(n+2)\right].$$

$$y(n) = \tfrac{1}{3}\delta(n+1) + \delta(n) + 2\delta(n-1) + \tfrac{10}{3}\delta(n-2) + 5\delta(n-3) + \tfrac{20}{3}\delta(n-4) + 6\delta(n-5) + 5\delta(n-6) + 5\delta(n-6') + \tfrac{11}{3}\delta(n-7) + \delta(n-8)$$

2.19) Compute the following three operations

a) multiply the integer number ; 131 & 122

$131 \times 122 = 15982$

b) compute the convolution of signals

$\{1,3,1\} * \{1,2,2\} = \{1,5,9,8,2\}$

```
  262
 262
131
-----
15982
```

$x(n)$ \ $h(n)$	1	2	2
1	1	2	2
3	3	6	6
1	1	2	2

$y(n) = \{1,5,9,8,2\}$

c) multiply the polynomials ; $1+3z+z^2$ and $1+2z+2z^2$

$(1+3z+z^2)(1+2z+2z^2) = 1+5z+9z^2+8z^3+2z$

d) repeat part (a) for the numbers 1.31 and 12.2

$1.31 \times 12.2 = 15.982$

c) comment on your result

These are different ways to perform convolution.

2.20) compute the convolution $y(n)$ of the signals

$$x(n) = \begin{cases} \alpha^n, & -3 \le n \le 5 \\ 0, & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & \text{elsewhere} \end{cases}$$

$$x(n) = \{\alpha^{-3}, \alpha^{-2}, \alpha^{-1}, \underset{\uparrow}{1}, \alpha, \dots \alpha^5\}$$

$$h(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$$

$$y(n) = x(n) * h(n)$$

$x(n) \backslash h(n)$	1	1	1	1	1
α^{-3}	α^{-3}	α^{-3}	α^{-3}	α^{-3}	α^{-3}
α^{-2}	α^{-2}	α^{-2}	α^{-2}	α^{-2}	α^{-2}
α^{-1}	α^{-1}	α^{-1}	α^{-1}	α^{-1}	α^{-1}
1	1	1	1	1	1
α	α	α	α	α	α
α^2	α^2	α^2	α^2	α^2	α^2
α^3	α^3	α^3	α^3	α^3	α^3
α^4	α^4	α^4	α^4	α^4	α^4
α^5	α^5	α^5	α^5	α^5	α^5

$y(n) = \{ \alpha^{-3}, (\alpha^{-3}+\alpha^{-2}), (\alpha^{-3}+\alpha^{-2}+\alpha^{-1}), (1+\alpha^{-1}+\alpha^{-2}+\alpha^{-3}),$
$(1+\alpha+\alpha^{-1}+\alpha^{-2}+\alpha^{-3}), (\alpha^{2}+\alpha+1+\alpha^{-1}+\alpha^{-2}), (\alpha^{3}+\alpha^{2}+\alpha+$
$1+\alpha^{-1}), (1+\alpha^{4}+\alpha^{3}+\alpha^{2}+\alpha), (\alpha^{5}+\alpha^{4}+\alpha^{3}+\alpha^{2}+\alpha),$
$(\alpha^{5}+\alpha^{4}+\alpha^{3}+\alpha^{2}), (\alpha^{5}+\alpha^{4}+\alpha^{3}), (\alpha^{5}+\alpha^{4}), \alpha^{5} \}$

2.21) compute the convolution $y(n) = x(n) * h(n)$ of the following pairs of signals.

a) $x(n) = a^n u(n)$, $h(n) = b^n u(n)$ when $a \neq b$ and when $a = b$

$x(n) = a^n u(n)$

$h(n) = b^n u(n)$

$$y(n) = \sum_{k=0}^{n} a^k u(k)\, b^{n-k} u(n-k)$$

$$y(n) = b^n \sum_{k=0}^{n} (ab^{-1})^k = b^n \sum_{k=0}^{n} \left(\frac{a}{b}\right)^k$$

If $a = b$

$$y(n) = b^n \sum_{k=0}^{n} 1^k$$

$$\sum_{k=0}^{n} a^k = \frac{1-a^{n+1}}{1-a}$$

$$\lim_{a\to 1} \frac{1-a^{n+1}}{1-a} = \lim_{a\to 1} \frac{n+1}{1}$$

$$= \lim_{a\to 1} (n+1)$$

$$= n+1$$

$$\boxed{y(n) = b^n (n+1)\, u(n)}$$

b) $x(n) = \begin{cases} 1, & n = 2, 0, 1 \\ 2, & n = -1 \\ 0, & \text{elsewhere} \end{cases}$

$h(n) = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-5)$

$x(n) = \{1, 2, 1, 1\}$

$h(n) = \{1, -1, 0, 0, 1, 1\}$

$y(n) = \{1, 1, -1, 0, 0, 3, 3, 2, 1\}$

$x(n) \backslash h(n)$	1	-1	0	0	1	1
1	1	-1	0	0	1	1
2	2	-2	0	0	2	2
1	1	-1	0	0	1	1
1	1	-1	0	0	1	1

c) $x(n) = u(n+1) - u(n-4) - \delta(n-5)$

$h(n) = [u(n+2) - u(n-3)]\,(3 - |n|)$

$x(n) = \{1, \underset{\uparrow}{1}, 1, 1, 1, 0, -1\}$

$h(n) = \{1, 2, \underset{\uparrow}{3}, 2, 1\}$

$y(n) = \{1, 3, 6, 8, 9, 8, 5, 1, -2, -2, -1\}$

	1	2	3	2	1
1	1	2	3	2	1
1	1	2	3	2	1
1	1	2	3	2	1
1	1	2	3	2	1
1	1	2	3	2	1
0	0	0	0	0	0
-1	-1	-2	-3	-2	-1

d) $x(n) = u(n) - u(n-5)$

$h(n) = u(n-2) - u(n-8) + u(n-11) - u(n-17)$

$x(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$. $h(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1, 1, 1\}$

$y(n) = \{\underset{\uparrow}{0}, 0, 1, 2, 3, 4, 5, 5, 4, 3, 2, 1\}$

2.22) Let $x(n)$ be the input signal to a discrete time filter with impulse response $h(n)$ and let $y_i(n)$ be the corresponding output.

a) compute and sketch $x(n)$ & $y_i(n)$ in the following cases using the same scale in all figures:

$x(n) = \{1, 4, 2, 3, 5, 3, 4, 5, 7, 6, 9\}$

$h_1(n) = \{1, 1\}$

$h_2(n) = \{1, 2, 1\}$

$h_3(n) = \{\frac{1}{2}, \frac{1}{2}\}$

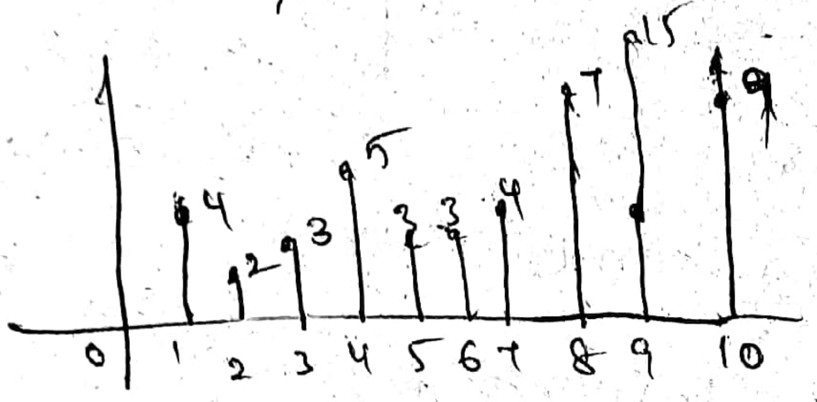
$h_4(n) = \{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\}$

$h_5(n) = \{\frac{1}{4}, \frac{-1}{2}, \frac{1}{4}\}$

sketch $x(n)$; $y_1(n)$, $y_2(n)$ on the graph and $x(n)$, $y_3(n)$, $y_4(n)$, $y_5(n)$ on another graph:

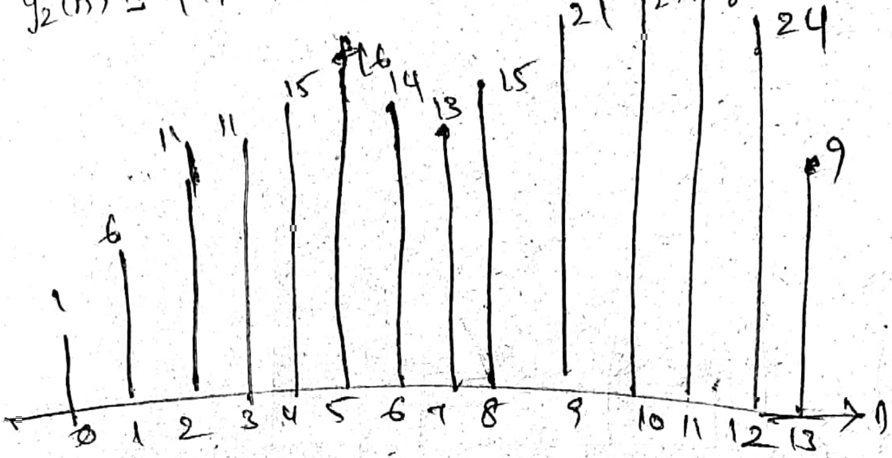
$y(n) = x(n) * h_1(n)$

$y_1(n) = \{\underset{\uparrow}{1}, 5, 6, 5, 8, 6, 7, 9, 12, 12, 15, 9\}$



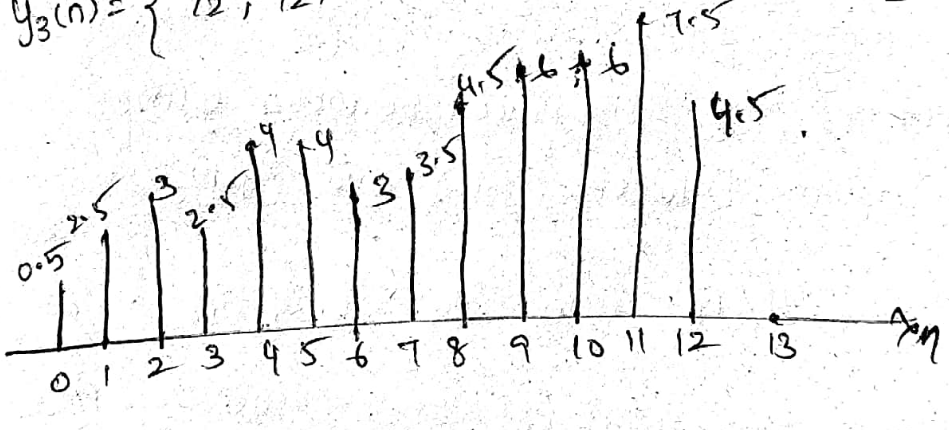
$y_2(n) = x(n) * h_2(n)$

$y_2(n) = \{1, 6, 11, 11, 13, 16, 14, 13, 15, 21, 25, 28, 24, 9\}$



$\Rightarrow y_3(n) = x(n) * h_3(n)$

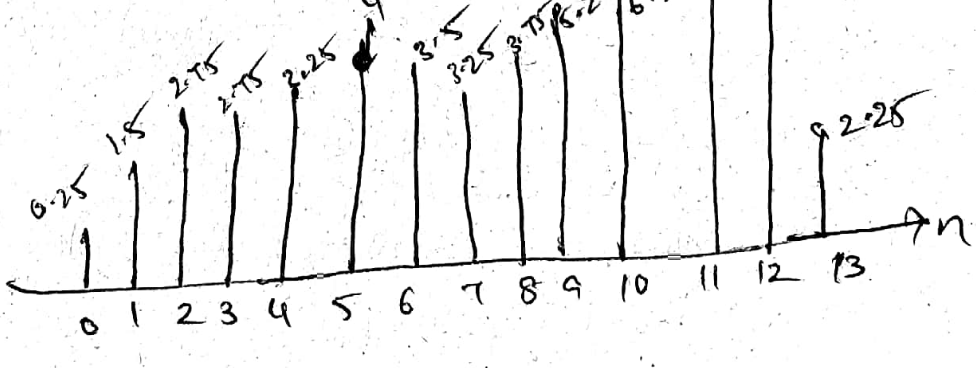
$y_3(n) = \{ 1/2, 5/2, 3, 5/2, 4, 4, 3, 7/2, 9/2, 6, 6, \frac{15}{2}, \frac{9}{2} \}$



$\Rightarrow y_4(n) = x(n) * h_4(n)$

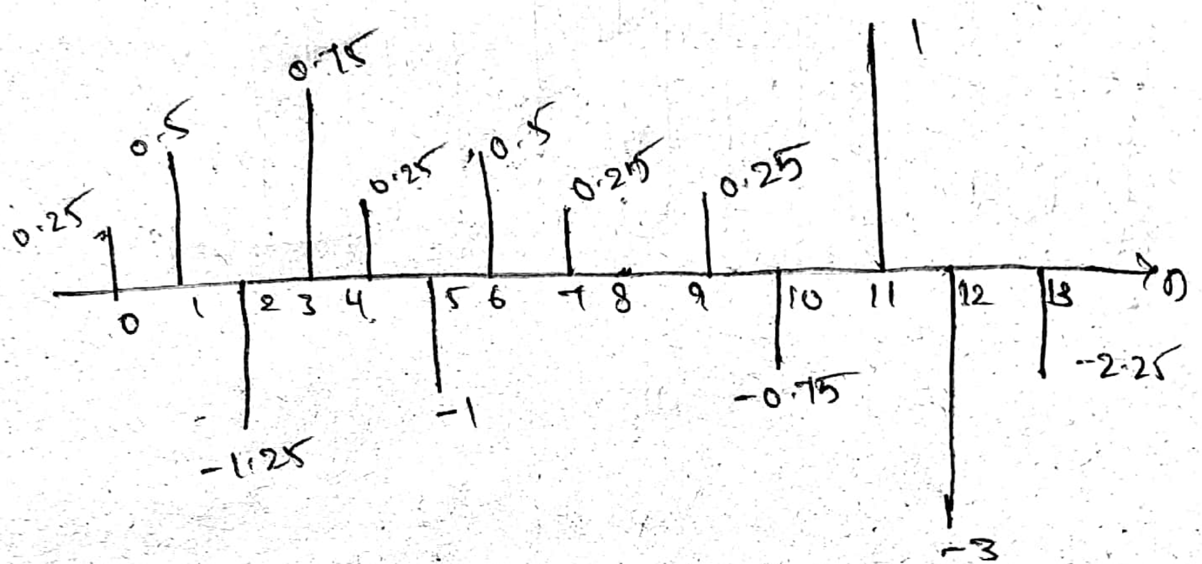
$y_4(n) = \{0.25, 1.5, 2.75, 2.75, 3.25, 4, 3.5, 3.25, 3.75,$

$5.25, 6.25, 7, 6, 2.25\}$



$\Rightarrow y_5(n) = x(n) * h_5(n)$

$y_5(n) = \{0.25, 0.5, -1.25, 0.75, 0.25, -1, 0.5, 0.25, 0, 0.25, -0.75, 1, -3, -2.25\}$



b) what is the difference between $y_1(n)$ and $y_2(n)$ and between $y_3(n)$ and $y_4(n)$?

$y_3 = \frac{1}{2} y_1(n)$ and $y_4 = \frac{1}{4} y_2(n)$

because $h_3(n) = \frac{1}{2} h_1(n)$ and $h_4(n) = \frac{1}{4} h_2(n)$.

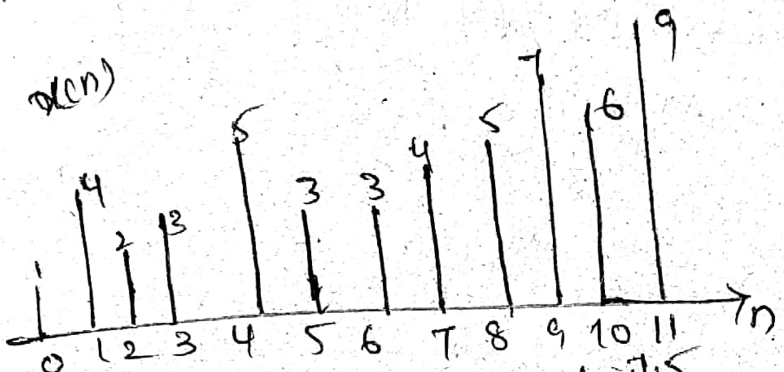
c) Comment on the smoothness of $y_2(n)$ and $y_4(n)$. which factors effect the smoothness?

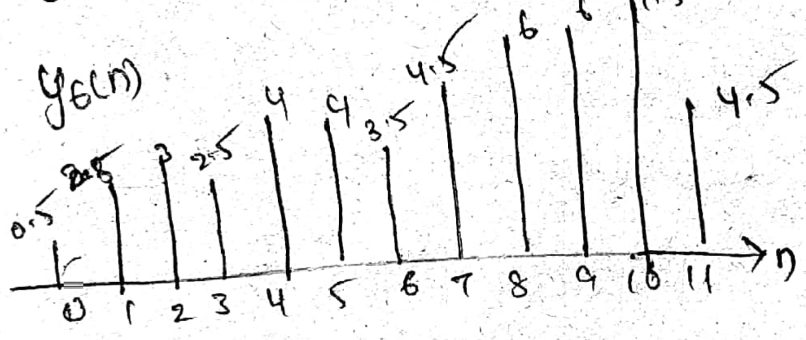
$y_2(n)$ and $y_4(n)$ are smoother than $y_1(n)$, but $y_4(n)$ will appear even smoother because of the smaller scale factor.

d) compare $y_4(n)$ with $y_5(n)$. what is the difference? Can you explain it?

s/m 4 is a smoother o/p. the negative value of $h_5(0)$ is responsible for the non-smooth characteristic of $y_5(n)$.

e) Let $h_6(n) = \{\frac{1}{2}, \frac{1}{2}\}$. compute $y_6(n)$. sketch $x(n)$, $y_2(n)$, $y_6(n)$ on the same figure and comment on the results?





$y_6(n) = x(n) * h_6(n)$

$y_6(n) = y_3(n) = \{\underset{\uparrow}{0.5}, 2.5, 3, 2.5, 4, 4, 3, 3.5, 4.5, 6, 6, 7.5, 4.5\}$

2.23) The discrete time system

$y(n) = n\,y(n-1) + x(n)$, $n \geq 0$

it at rest [i.e. $y(-1)=0$]. check if the system is linear time invariant and BIBO stable.

$y_1(n) = n\,y_1(n-1) + x_1(n)$

$y_2(n) = n\,y_2(n-1) + x_2(n)$

$y(n) = a y_1(n) + b y_2(n)$

The S/m is linear. If the input is $x(n-1)$

$y(n-1) = (n-1)\, y(n-2) + x(n-1)$

$y(n-1) = n\, y(n-2) + x(n-1)$

Hence, the system is time variant. If $x(n) = u(n)$

then $|x(n)| \leq 1$ for bounded input the o/p is

$y(0) = 1$, $y(1) = 1+1 = 2$, $y(2) = 2 \times 2 + 1 = 5$

which is unbounded

$\therefore$ The system is Astable.

2.24) consider the signal $\gamma(n) = a^n u(n)$, $0 \leq a < 1$

a) Show that any sequence $x(n)$ can be decomposed as

$$x(n) = \sum_{n=-\infty}^{\infty} c_k \gamma(n-k)$$

and express c_k in terms of $x(n)$.

$\delta(n) = \gamma(n) - a\gamma(n-1)$ and

$\delta(n-k) = \gamma(n-k) - a\gamma(n-k-1)$ Then,

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\, \delta(n-k)$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k) \left[\gamma(n-k) - a\gamma(n-k-1)\right]$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\, \gamma(n-k) - a \sum_{k=-\infty}^{\infty} x(k)\, \gamma(n-k-1)$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\, \gamma(n-k) - a \sum_{k=-\infty}^{\infty} x(k-1)\, \gamma(n-k)$$

$$x(n) = \sum_{k=-\infty}^{\infty} \left[x(k) - a x(k-1)\right] \gamma(n-k)$$

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Thus, $C_K = x(K) - ax(K-1)$

b) Use the properties of linearity and time invariance to express the o/p $y(n) = T[x(n)]$ in terms of the i/p and the signal $g(n) = T[\delta(n)]$ where $T[\cdot]$ is an LTI Slm.

$$y(n) = T[x(n)]$$

$$= T\left[\sum_{K=-\infty}^{\infty} C_K \delta(n-K)\right]$$

$$= \sum_{K=-\infty}^{\infty} C_K T[\delta(n-K)]$$

$$= \sum_{K=-\infty}^{\infty} C_K g(n-K)$$

c) Express the impulse response $h(n) = T[\delta(n)]$ in terms of $g(n)$.

$$h(n) = T[\delta(n)]$$

$$= T[\delta(n) - a\delta(n-1)]$$

$$= g(n) - ag(n-1)$$

2.25) Determine the zero-input response of the system described by the second-order difference equation

$$x(n) - 3y(n-1) - 4y(n-2) = 0$$

$$x(n) = 0$$

$$0 - 3y(n-1) - 4y(n-2) = 0$$

$$-[3y(n-1) + 4y(n-2)] = 0$$

$$3y(n-1) + 4y(n-2) = 0$$

$y(n-1) = -\frac{4}{3} y(n-2)$

$y(-1) = \left(\frac{-4}{3}\right)^{1} y(-2)$

$y(0) = \left(\frac{-4}{3}\right)^{2} y(-2)$

$\vdots$

$y(k) = \left(\frac{-4}{3}\right)^{k+2} y(-2)$ zero i/p response

2.26) Determine the particular solution of the difference equation.

$y(n) = 5/6\, y(n-1) - 1/6\, y(n-2) + x(n)$

when the forcing function is $x(n) = 2^n u(n)$

$y(n) - 5/6\, y(n-1) + 1/6\, y(n-2) = 0$

the characteristic equation is

$\lambda^2 - 5/6\,\lambda + 1/6 = 0.\lambda$

$(\lambda - 1/2)(\lambda - 1/3) = 0$

$\lambda = 1/2, 1/3$

Hence $y_h(n) = C_1 (1/2)^n + C_2 (1/3)^n$

$x(n) = 2^n u(n)$

$y_p(n) = k(2^n) u(n)$

$k(2^n) u(n) - k(5/6) 2^{n-1} u(n-1) + k\left(\frac{1}{6}\right) 2^{n-2} u(n-2)$
$= 2^n u(n)$

for $n=2$ $\quad 4k - \frac{5k}{3} + \frac{k}{6} = 4$

$k = 8/5$

$y(n) = y_p(n) + y_h(n)$

$y(n) = 8/15 \, 2^n \, u(n) + c_1 (1/2)^n u(n) + c_2 (1/3)^n u(n)$

c_1 and c_2, assume that $y(-2) = y(-1) = 0$ then

$y(0) = 1$ and

$y(1) = 5/6 \, y(0) + 2 = 17/6$

$\frac{8}{5} + c_1 + c_2 = 1 \Rightarrow c_1 + c_2 = -3/5$

$\frac{16}{5} + 1/2 \, c_1 + 1/3 \, c_2 + \frac{17}{6} \Rightarrow 3c_1 + 2c_2 = \frac{-11}{5}$

$\therefore c_1 = -1, \; c_2 = \frac{2}{5}$

$\Rightarrow$ total solution is $y(n) = \left[\frac{8}{5}(2)^n - \left(\frac{1}{2}\right)^n + \frac{2}{5}\left(\frac{1}{3}\right)^n\right] u(n)$

2.27) Determine the response $y(n)$, $n \geq 0$, of the s/m described by the second order difference equation

$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$ to the input $x(n) = 4^n u(n)$

$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

$\lambda = 4, -1.$

$y_h(n) = c_1(n) \, 4^n + c_2 (-1)^n$

$x(n) = 4^n u(n)$

$y_p(n) = K n \, 4^n u(n)$

$Kn\,4^n u(n) - 3K(n-1)4^{n-1} u(n-1) - 4K(n-2)\,4^{n-2}$

$u(n-2) = 4^n u(n) + 2(4)^{n-1} u(n-1)$

for $n=2$

$K(32-12) = 4^2+8 = 24 \rightarrow K = \frac{6}{5}$

The total solution is

$y(n) = y_p(n) + y_h(n)$

$y(n) = \left[\frac{6}{5} n\, 4^n + c_1 4^n + c_2(-1)^n\right] u(n)$

we assume $y(-1) = y(-2) = 0$

$y(0) = 1$ and $y(1) = 9$

Hence $c_1 + c_2 = 1$ and $\frac{24}{5} + 4c_1 - c_2 = 9$

$4c_1 - c_2 = \frac{21}{5}$

$\therefore c_1 = \frac{26}{25}$ and $c_2 = \frac{-1}{25}$

$\therefore y(n) = \left[\frac{6}{5} n 4^n + \frac{26}{25} 4^n - \frac{1}{25}(-1)^n\right] u(n)$

2.28) Determine the impulse response of the following causal system

$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

$\lambda = 4, -1$

$y_h(n) = c_1 4^n + c_2 (-1)^n$

when $x(n) = \delta(n)$, we find that

$y(0)=1$ and $y(1)-3y(0)=2$ or $y(1)=5$

$c_1+c_2=1$ and $4c_1-c_2=5$

$$\begin{array}{r} c_1+c_2=1 \\ 4c_1-c_2=5 \\ \hline 5c_1=6 \end{array}$$

$c_1=6/5$

$\frac{6}{5}+c_2=1$

$c_2=1-\frac{6}{5}=-1/5$

$$\therefore h(n)=\left[\frac{6}{5}4^n-\frac{1}{5}(-1)^n\right]u(n)$$

2.29) Let $x(n)$, $N_1 \leq n \leq N_2$ and $h(n)$. $M_1 \leq n \leq M_2$ be two finite-duration signals.

a) Determine the range $L_1 \leq n \leq L_2$ of their convolution in terms of N_1 and N_2, M_1 and M_2

$L_1=N_1+M_1$ and $L_2=N_2+M_2$

b) Determine the limits of the cases of partial overlapping from the left, full overlap, and partial overlap from the right. for convenience, assume that $h(n)$ has shorter duration than $x(n)$

partial overlap from left

low N_1+M_1 high N_1+M_2-1

full overlap: low N_1+M_2 high N_2+M_1

partial overlap from right

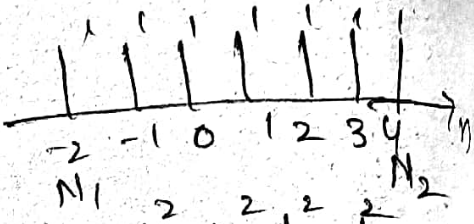
low N_2+M_1+1 high N_2+M_2

c) Illustrate the validity of yours results by computing the convolution of the signals

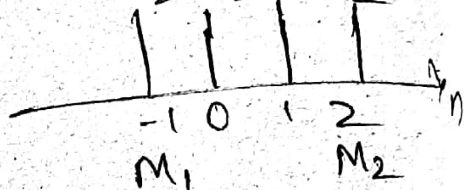
$$x(n) = \begin{cases} 1, & -2 \le n \le 4 \\ 0, & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 2, & -1 \le n \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

$x(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1, 1, 1\}$



$h(n) = \{2, \underset{\uparrow}{2}, 2, 2\}$



$N_1 = -2$

$N_2 = 4$

$M_1 = -1$

$M_2 = 2$

Partial overlap from left

$n = -3,$

$n = -1, L_1 = -3$

full overlap $n = 0, n = 3$

Partial overlap from right: $n = 4, n = 6$ $L_2 = 6$

2.30) Determine the impulse response and unit step response of the systems described by the difference equation.

a) $y(n) = 0.6\,y(n-1) - 0.08\,y(n-2) + x(n)$

b) $y(n) = 0.7\,y(n-1) - 0.1\,y(n-2) + 2x(n) - x(n-2)$

$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$ to the i/p

$x(n) = 4^n u(n)$

$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

$\lambda^2 - 3\lambda - 4 = 0 \qquad \lambda = 4, -1$ }

$y(n) = 0.6\, y(n-1) - 0.08\, y(n-2) + x(n)$

$x(n) = y(n) - 0.6\, y(n-1) - 0.08\, y(n-2)$

$\lambda^2 - 0.6\lambda + 0.08 = 0$

$\lambda = 1/2, 2/3$

$y_h(n) = c_1 \left(\frac{1}{5}\right)^n + c_2 \left(\frac{2}{5}\right)^n$

impulse response $x(n) = \delta(n)$ with $y(0) = 1$

$y(1) = 0 \qquad 6y(0) = 0 \Rightarrow y(1) = 0.6$

so, $c_1 + c_2 = 1$ —①

$\frac{1}{5} c_1 + \frac{2}{5} c_2 = 0.6$ —②

from ① and ② $c_1 = -1;\ c_2 = 3$

$h(n) = \left[-\left(\frac{1}{5}\right)^n + 2\left(\frac{2}{5}\right)^n\right] u(n)$

step response $x(n) = u(n)$

$$s(n) = \sum_{k=0}^{n} h(n-k) \quad n \geq 0$$

$$= \sum_{k=0}^{n} \left(2\left(\frac{2}{5}\right)^{n-k} - \left(\frac{1}{5}\right)^{n-k}\right)$$

$$= 2\left(\frac{2}{5}\right)^n \sum_{k=0}^{n} \left(\frac{5}{2}\right)^k - \left(\frac{1}{5}\right)^n$$

$$= 2\left(\frac{2}{5}\right)^n \left(\frac{5}{2}\right)^{n+1} - 1$$

b) $y(n) = 0.7\,y(n-1) - 0.1\,y(n-2) + 2x(n) = x(n)$

$2x(n) - x(n-2) = y(n), \quad 0.7\,y(n-1) + 0.1y$

$\lambda^2 - 0.7\lambda + 0.1 = 0$

$\lambda = 1/2,\ 1/5$

$y_h(n) = c_1 \left(\frac{1}{2}\right)^n + c_2 \left(\frac{1}{5}\right)^n$

Impulse response $x(n) = \delta(n)$, $y(0) = 2$

$y(1) = 0.7\,y(0) = 0 \Rightarrow y(1)$

$c_1 + c_2 = 2$

$\frac{1}{2}c_1 + \frac{1}{5}c_2 = \frac{7}{5}$ — ①

$c_1 + \frac{2}{5}c_2 = \frac{14}{5}$ — ②

solving ① and ②

$c_1 = \frac{10}{3}; \quad c_2 = \frac{-4}{3}$

So $h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right]u(n)$

Step response $s(n) = \sum_{k=0}^{n} h(n-k)$

$= \frac{10}{3}\sum_{k=0}^{n}\left(\frac{1}{2}\right)^{n-k} - \frac{4}{3}$

$= \frac{10}{3}\left[\frac{1}{2}^n\left(2^{n+1} - 1\right)u(n)\right]u(n)$

3.31) Consider a system with impulse response

$$h(n) = \begin{cases} \left(\frac{1}{2}\right)^n, & 0 \le n \le 4 \\ 0, & \text{elsewhere} \end{cases}$$

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determine the input $x(n)$ for $0 \le n \le 8$ that will generate the o/p sequence.

$y(n) = \{\underset{\uparrow}{1}, 2, 2, 5, 3, 3, 3, 2, 1, 0\}$.

$h(n) = \{\underset{\uparrow}{1}, 1/2, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}\}$

$x(0)\,h(0) = y(0) \Rightarrow x(0) = 1$

$1/2\, x(0) + x(1) = y(1) \Rightarrow x(1) = 3/2$

by continuing this process

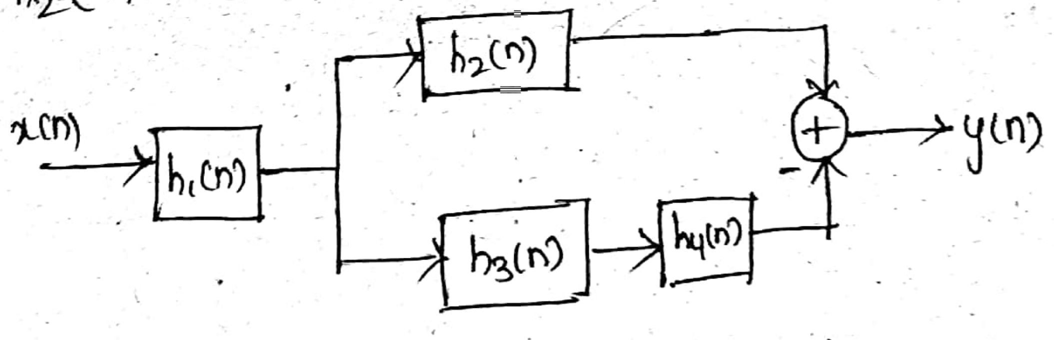
$x(n) = \{1, \frac{3}{2}, \frac{3}{2}, \frac{7}{4}, \frac{3}{2}, \ldots\}$

2.32) consider the interconnection of LTI systems as shown in figure

a) Express the overall impulse response in terms of $h_1(n)$, $h_2(n)$, $h_3(n)$ and $h_4(n)$

b) determine $h(n)$ when $h_1(n) = \{1/2, 1/4, 1/2\}$

$h_2(n) = h_3(n) = (n+1)\, u(n)$ $h_4(n) = \delta(n-2)$



c) determine the response of the s/m in part (b) if $x(n) = \delta(n+2) + 3\,\delta(n-1) - 4\,\delta(n-3)$

a) $h(n) = h_1(n) * [h_2(n) - h_3(n) * h_4(n)]$

b) $h_3(n) * h_4(n) = (n-1)\, u(n-2)$

$h_2(n) - h_3(n) * h_4(n) = 2u(n) - \delta(n)$

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$h_1(n) = \frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)$

Hence

$h(n) = \left[\frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)\right] * \left[2u(n) - \delta(n)\right]$

$h(n) = \frac{1}{2}\delta(n) + \frac{5}{4}\delta(n-1) + 2\delta(n-2) + \frac{5}{2}u(n-3)$

c) $x(n) = \{1, 0, \underset{\uparrow}{0}, 3, 0, -4\}$

$y(n) = \{\frac{1}{2}, \underset{\uparrow}{\frac{5}{4}}, 2, \frac{25}{4}, \frac{13}{2}, 5, 2, 0, 0\}$

$s(n) = u(n) * h(n)$

$s(n) = \sum_{k=0}^{\infty} u(k)\, h(n-k) = \sum_{k=0}^{n} h(n-k)$

$= \sum_{k=0}^{\infty} a^{n-k} = \frac{a^{n+1}-1}{a-1}, \quad n \geq 0$

for $x(n) = u(n+5) - u(n-10)$

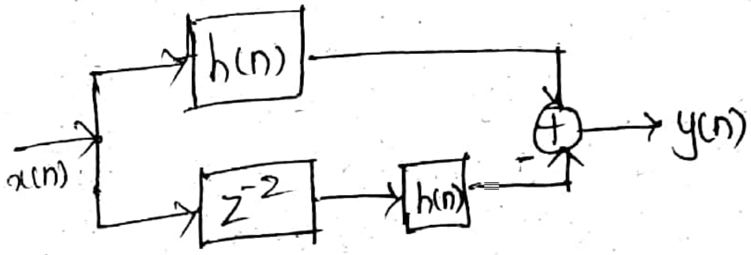
$s(n+5) - s(n-10) = \frac{a^{n+6}-1}{a-1}u(n+5) - \frac{a^{n-9}-1}{a-1}u(n-10)$

$y(n) = x(n) * h(n) - x(n) * h(n-2)$

$$y(n) = \frac{a^{n+6}-1}{a-1}u(n+5) - \frac{a^{n-9}-1}{a-1}u(n-10) - \frac{a^{n+4}-1}{a-1}u(n+3) + \frac{a^{n-11}-1}{a-1}u(n-12)$$

2.33) consider the system in figure with $h(n) = a^n u(n)$ $-1 < a < 1$. Determine the response $y(n)$ of the s/m to the excitation

$x(n) = u(n+5) - u(n-10)$



$$x(n) = u(n+5) - u(n-10)$$

$$s(n+5) - s(n-10) = \frac{a^{n+6}-1}{a-1} u(n+5) - \frac{a^{n-9}-1}{a-1} u(n-10)$$

$$y(n) = x(n) * h(n) - x(n) * h(n-2)$$

$$y(n) = \frac{a^{n+6}-1}{a-1} u(n+5) - \frac{a^{n+9}-1}{a-1} u(n-10) - \frac{a^{n+4}-1}{a-1}$$

$$u(n+3) + \frac{a^{n-11}-1}{a-1} u(n-2)$$

2.34) Compute and sketch the step response of the s/m $y(n) = \frac{1}{M} \sum_{k=0}^{M-1} x(n-k)$

$$h(n) = [u(n) - u(n-M)] / M$$

$$s(n) = \sum_{k=-\infty}^{\infty} u(k)\, h(n-k)$$

$$s(n) = \sum_{k=0}^{n} h(n-k)$$

$$s(n) = \begin{cases} \frac{n+1}{M}, & n < M \\ 1, & n \geq M \end{cases}$$

2.35) Determine the range of values of the parameter a for which the linear time-invariant system with impulse response

$$h(n) = \begin{cases} a^n, & n \geq 0, n\,\text{even} \\ 0, & \text{otherwise} \end{cases}$$

is stable

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=0,\, n\, even}^{\infty} |a|^{n}$$

$$= \sum_{n=0}^{\infty} |a|^{2n} = \frac{1}{1-|a|^2}$$

Stable if $|a| < 1$

2.36) Determine the determine of the system with impulse response $h(n) = a^n u(n)$ to the input signal $x(n) = u(n) - u(n-10)$

$h(n) = a^n u(n)$, the response to $u(n)$ is

$$y_1(n) = \sum_{k=0}^{\infty} u(k) h(n-k) = \sum_{k=0}^{n} a^{n-k}$$

$$= a^n \sum_{k=0}^{n} a^{-k} = \frac{1-a^{n+1}}{1-a} u(n)$$

then, $y(n) = y_1(n) - y_1(n-10)$

$$= \frac{1}{1-a}\left[(1-a^{n+1}) u(n) - (1-a^{n-9}) u(n-10)\right]$$

2.38) Determine the response of the relaxed system characterised by the impulse response

$$h(n) = (1/2)^n u(n)$$

to the input signals

a) $x(n) = 2^n u(n)$ b) $x(n) = u(-n)$

$h(n) = (1/2)^n u(n)$ $\quad y_1(n) = \sum_{k=0}^{\infty} u(k) h(n-k)$

$$= \sum_{k=0}^{n} (1/2)^{n-k}$$

$$= (1/2)^n \sum_{k=0}^{n} \left(\frac{1}{a}\right)^{-k} = \frac{1-(1/2)^{n+1}}{1-1/a} u(n)$$

$$y(n) = y_1(n) - y_1(n-10)$$

$$= \frac{1}{1-1/2}\left[(1-(1/2)^{n+1})\,u(n) - (1-(1/2)^{n-9})\,u(n-10)\right]$$

$$= 2\left[1-\left(\frac{1}{2}\right)^{n+1}\right]u(n) - 2\left[1-\left(\frac{1}{2}\right)^{n-9}u(n-10)\right]$$

$a=2$

$$y(n) = \frac{1}{1-2}\left[(1-(2)^{n+1})\,u(n) - (1-(2)^{n-9})\,u(n-10)\right]$$

$$y(n) = -\left[(1-2^{n+1})\,u(n)\right] + \left[(1-(2)^{n-9})\,u(n-10)\right]$$

2.37) Determine the response of the system characterised by the impulse response $h(n) = (1/2)^n u(n)$ to the i/p signal

$$x(n) = \begin{cases} 1, & 0 \le n \le 10 \\ 0, & \text{otherwise} \end{cases}$$

$$h(n) = (1/2)^n u(n)$$

$$y_1(n) = \sum_{k=0}^{\infty} u(k)\, h(n-k) = \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{n-k}$$

$$= \left(\frac{1}{2}\right)^n \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{-k}$$

$$= \left[\frac{1-(1/2)^{n+1}}{1-1/2}\right] u(n)$$

$y(n) = y_1(n) - y_1(n-10)$

$$= \frac{1}{1-\frac{1}{2}}\left[\left(1-\left(\tfrac{1}{2}\right)^{n+1}\right)u(n) - \left(1-\left(\tfrac{1}{2}\right)^{n-9}\right)u(n-10)\right]$$

$$= 2\left[1-\left(\tfrac{1}{2}\right)^{n+1}\right]u(n) - 2\left[\left(1-\left(\tfrac{1}{2}\right)^{n-9}\right)u(n-10)\right].$$

2-39) Three systems with impulse response $h(n) = \delta(n) - \delta(n-1)$, $h_2(n) = h(n)$ and $h_3(n) = u(n)$, are connected in cascade

a) what is the impulse response, $h_c(n)$ of the overall s/m?

b) Does the order of the interconnection affect the overall s/m?

a) $h(n) = h_1(n) * h_2(n) * h_3(n)$

$$= [\delta(n) - \delta(n-1)] * u(n) * h(n)$$

$$= [u(n) - u(n-1)] * h(n)$$

$$= \delta(n) * h(n) = h(n)$$

b) not effected

2-40)

a) prove and explain graphically the difference b/w the relations

$x(n)\,\delta(n-n_0) = x(n_0)\,\delta(n-n_0)$ and

$x(n) * \delta(n-n_0) = x(n-n_0)$

$x(n)\,\delta(n-n_0) = x(n_0)$ Thus, only the value of $x(n)$ at $n = n_0$ is of interest

$x(n) * \delta(n-n_0) = x(n-n_0)$

Thus, we obtain the shifted version of the sequence $x(n)$

b) show that the discrete-time system which is described by a convolution summation is LTI and relaxed.

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$y(n) = h(n) * x(n)$$

Linearity :- $x_1(n) \rightarrow y_1(n) = h(n) * x_1(n)$

$x_2(n) \rightarrow y_2(n) = h(n) * x_2(n)$

$x(n) = \alpha x_1(n) + \beta x_2(n) \rightarrow y(n) = h(n) * x(n)$

$y(n) = h(n) * [\alpha x_1(n) + \beta x_2(n)]$

$y(n) = \alpha\, h(n) * x_1(n) + \beta\, h(n) * x_2(n)$

$y(n) = \alpha y_1(n) + \beta y_2(n)$

Time invariance

$x(n) \rightarrow y(n) = h(n) * x(n)$

$x(n-n_0) \rightarrow y_1(n) = h(n) * x(n-n_0)$

$= \sum_{k} h(k)\, x(n-n_0-k) = y(n-n_0)$

c) what is the impulse response of the cascade interconnection of systems described by $y(n) = x(n-n_0)$?

$h(n) = \delta(n-n_0)$

2.41) Two signals $s(n)$ and $u(n)$ are related through the following difference equations

$s(n) + a_1 s(n-1) + \cdots + a_N s(n-N) = b_0 v(n)$

Design the block diagram realization of:

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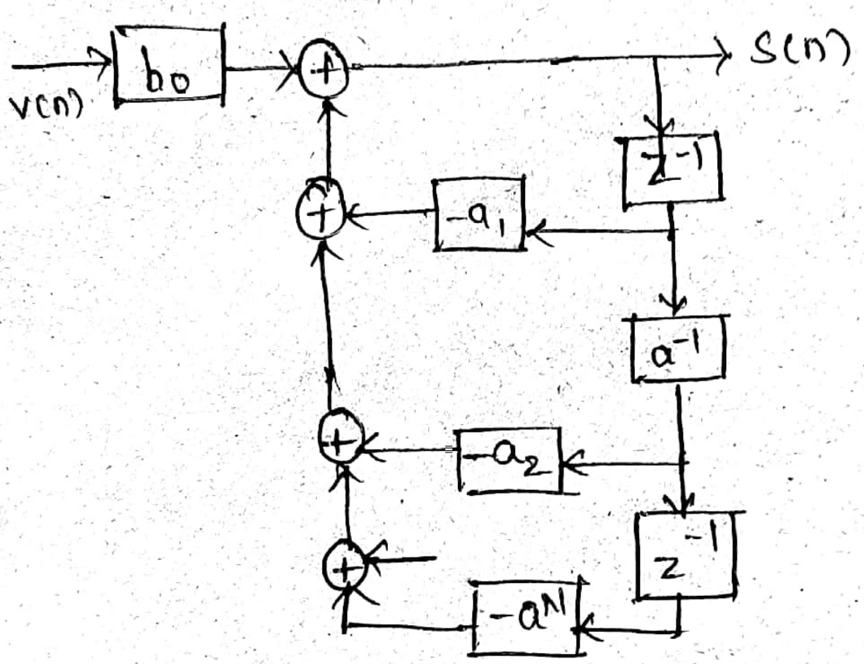
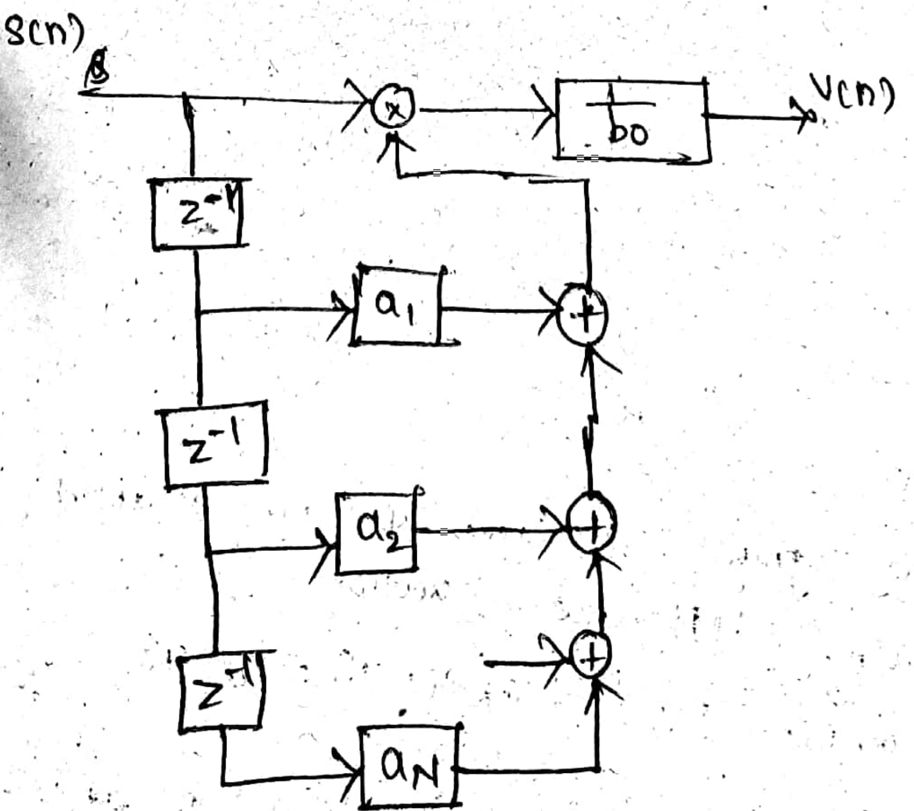
a) The s/m that generates $s(n)$ when excited by $v(n)$
b) the s/m that generate $v(n)$ when excited by $s(n)$
c) what is the impulse response of the cascade interconnection of s/m's in parts (a) and (b)

a) $s(n) = -a\, s(n-1) - a_2 s(n-2) - \cdots a_N s(n-N) + b_0 v(n)$

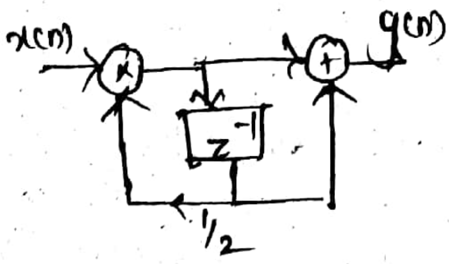
b) $v(n) = \frac{1}{b_0}\left[s(n) + a_1 s(n-1) + a_2 s(n-2) + \cdots + a_N s(n-N)\right]$



c)



2.44) consider the discrete-time system should in figure



a) compute the 10 first samples of its impulse response

b) Find the i/p- o/p relation

c) Apply the input $x(n) = \{\underset{\uparrow}{1}, 1, 1, \ldots\}$ and compute the first 10 samples of the output.

d) compute the first 10 samples of the output for the input given in part (c) by using convolution

e) Is the system causal? Is it stable?

$x(n) = \{1, 0, 0, \ldots\}$

$y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$

$y(0) = x(0) = 1$

$y(1) = \frac{1}{2} y(0) + x(1) + x(0) = \frac{1}{2} + 0 + 1 = 3/2$

$y(2) = \frac{1}{2} y(1) + x(2) + x(1) = \frac{1}{2}(3/2) + 0 + 0) = 3/4$

$y(3) = \frac{1}{2} y(2) + x(3) + x(2) = \frac{1}{2}\left(\frac{3}{4}\right) + 0 + 0 = \frac{3}{8}$

$y(4) = \frac{1}{2} y(3) + x(4) + x(3) = \frac{1}{2}\left(\frac{3}{8}\right) + 0 + 0 = \frac{3}{16}$

$y(5) = \frac{3}{32}$, $y(6) = \frac{3}{64}$, $y(7) = \frac{3}{128}$, $y(8) = \frac{3}{256}$

$y(9) = \frac{3}{512}$, $y(10) = \frac{3}{1024}$

$\therefore y(n) = \left\{1, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \frac{3}{32}, \frac{3}{64}, \frac{3}{128}, \frac{3}{256}, \frac{3}{512}, \frac{3}{1024}\right\}$

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b) $y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$

c) $x(n) = \{1, 1, 1, 1, \dots\}$

$y(0) = x(0) = 1.$

$y(1) = \frac{1}{2} y(0) + x(1) + x(0) = \frac{1}{2}(1) + 1 + 1 = 5/2$

$y(2) = \frac{13}{4}$, $y(3) = \frac{29}{8}$, $y(4) = \frac{61}{16}$, $y(5) = \frac{125}{32}$

$y(6) = \frac{253}{64}$, $y(7) = \frac{509}{128}$, $y(8) = \frac{1021}{256}$, $y(9) = \frac{2045}{512}$

$y(10) = 3.9970$

$y(n) = \{1, 5/2, \frac{13}{4}, \frac{29}{8}, \frac{61}{16}, \frac{125}{32}, \frac{253}{64}, \frac{509}{128}, \frac{1021}{256},$

$\frac{2045}{512}, 3.9970, \dots\}$

d) $y(n) = u(n) * h(n)$

$y(n) = \sum_{K} u(K)\, h(n-K) = \sum_{K=0}^{n} h(n-K)$

$y(0) = h(0) = 1$

$y(1) = h(0) + h(1) = 5/2$

$y(2) = h(0) + h(1) + h(2) = \frac{13}{4}$

$y(3) = \frac{29}{8}$, $y(4) = \frac{61}{16}$, $y(5) = \frac{125}{32}$, $y(6) = \frac{253}{64}$

$y(7) = \frac{509}{128}$, $y(8) = \frac{1021}{256}$, $y(9) = \frac{2045}{512}$,

$y(10) = 3.997$

e) $h(n) = 0$ for $n < 0 \Rightarrow$ the system is causal

$\sum_{n=0}^{\infty} |h(n)| = 1 + 3/2\,(1 + 1/2 + 1/4)$

$= 1 + \frac{3}{2} \times 2 = 4 \Rightarrow$ the system is stable.

2.45) consider the system described by the difference equation $y(n) = a\,y(n-1) + b\,x(n)$

a) Determine b in terms of a so that

$$\sum_{n=-\infty}^{\infty} h(n) = 1$$

b) compute the zero-state step response s(n) of the system and close b so that $s(\infty) = 1$

c) compare the values of b obtained in parts (a) and b what did you notice?

a) $y(n) = a\,(y(n-1)) + b\,x(n)$

$$h(n) = b a^n u(n)$$

$$\sum_{n=0}^{\infty} h(n) = \frac{b}{1-a} = 1 \qquad (\therefore b = 1-a)$$

b) $$s(n) = \sum_{k=0}^{n} h(n-k) = b\left[\frac{1-a^{n+1}}{1-a}\right] u(n)$$

$$s(\infty) = \frac{b}{1-a} = 1$$

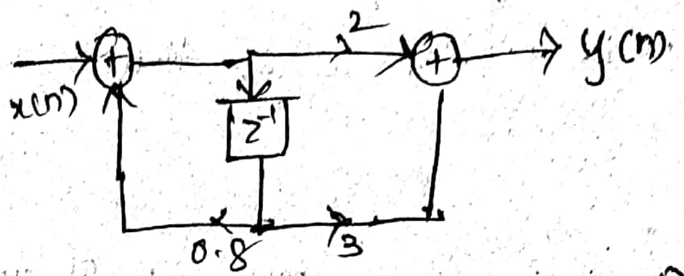
$$b = 1-a$$

c) $b = 1-a$ in both cases

2.46) A discrete-time system is realized by the structure shown in figure

a) Determine the impulse response.

b) Determine a realization for its inverse s/m that is, the s/m which produces $x(n)$ as a o/p when $y(n)$ is used as an input.



a) $y(n) = 0.8\, y(n-1) + 2x(n) + 3x(n-1)$

$y(n) - 0.8\, y(n-1) = 2x(n) + 3x(n-1)$

The characteristic equation is

$\lambda - 0.8 = 0 \quad \lambda = 0.8$

$y_h(n) = c(0.8)^n$

let us first consider the response of the s/m $y(n) - 0.8y(n-1) = x(n)$

$x(n) = \delta(n)$

since $y(0) = 1$, it follows that $c = 1$

Then, the impulse response of the original s/m is

$h(n) = 2(0.8)^n u(n) + 3(0.8)^{n-1} u(n-1)$

$h(n) = 2\delta(n) + 4.6(0.8)^{n-1} u(n-1)$

b) The inverse s/m is characterized by the difference equation

$x(n) = -1.5x(n-1) + \frac{1}{2}y(n) - 0.4\, y(n-1)$

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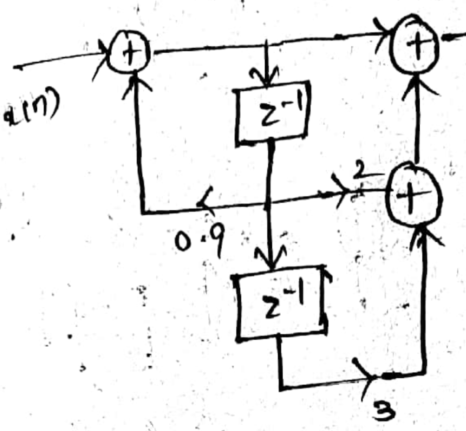
2.47 consider the discrete-time s/m shown in fig



a) compute the first 6 values of the impulse response of the s/m

b) compute the first 6 values of the zero-state step response of the s/m.

c) Determine an analytical expression for the impulse response of the s/m.

a) $y(n) = 0.9\,y(n-1) + x(n) + 2x(n-1) + 3x(n-2)$

a) for $x(n) = \delta(n) = \{-0, 1, 0, 0, 0 \ldots\}$

$y(0) = 0.9\,y(-1) + x(0) + 2x(-1) + 3x(-2) = 0.9(0) + 1 + 0 + 0$
$= 1$

$y(1) = 2.9$, $y(2) = 5.61$, $y(3) = 5.049$, $y(4) = 4.5441$

$y(5) = 4.08969$, $y(6) = 3.680$

b) $s(0) = y(0) = 1$

$s(1) = y(0) + y(1) = 3.91$

$S(2) = 9.51$, $S(3) = 14.56$, $S(4) = 19.10$, $S(5) = 23.19$

$S(6) = 26.87$.

c) $h(n) = (0.9)^n u(n) + 2(0.9)^{n-1} u(n-1) + 3(0.9)^{n-2} u(n-2)$

$h(n) = \delta(n) + 2.9\,\delta(n-1) + 5.61(0.9)^{n-2} u(n-2)$

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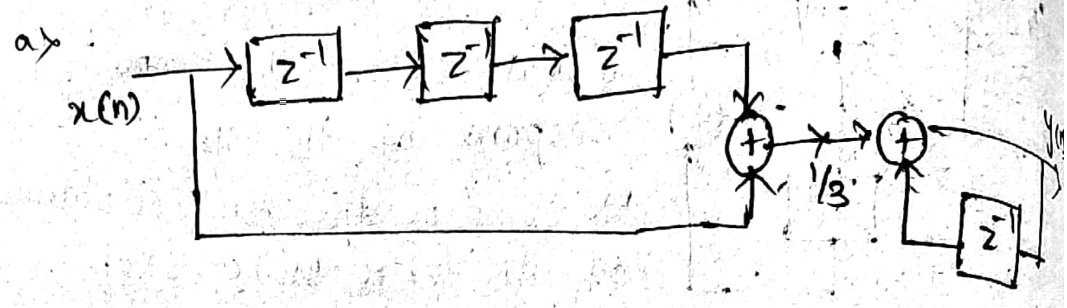
2.48) Determine and sketch the impulse response of the following systems for $n=0,1,\dots,9$

a)



$$y(n) = \tfrac{1}{3}x(n) + \tfrac{1}{3}x(n-3) + y(n-1)$$

for $x(n) = \delta(n)$

$$h(n) = \left\{ \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{2}{3}, \tfrac{2}{3}, \dots \right\}$$

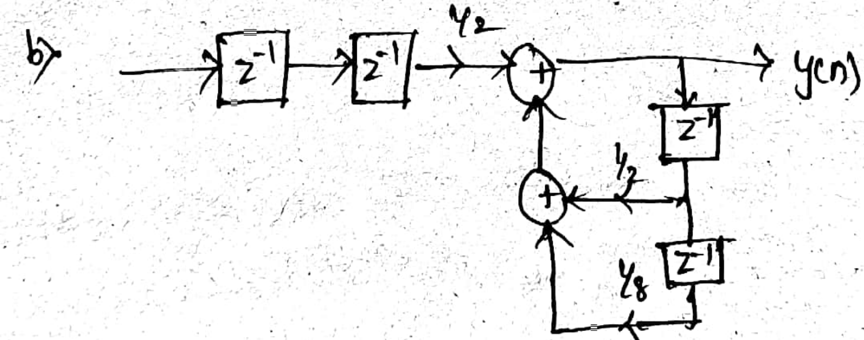
$y(0) = \tfrac{1}{3}x(0) + \tfrac{1}{3}x(-3) + y(-1) = \tfrac{1}{3}(1) + 0 + 0 = \tfrac{1}{3}$

$y(1) = \tfrac{1}{3}$, $y(2) = \tfrac{1}{3}$, $y(3) = 2/3$, $y(4) = 2/3$

$y(5) = \tfrac{2}{3}$, $y(6) = \tfrac{2}{3}$, $y(7) = 2/3$, $y(8) = 2/3$,

$y(9) = \tfrac{2}{3}$

b)



$$y(n) = \tfrac{1}{2}y(n-1) + \tfrac{1}{8}y(n-2) + \tfrac{1}{2}x(n-2)$$

let $x(n) = \delta(n)$

$y(-1) = y(-2) = 0$

$$h(n) = \left\{ 0, 0, \tfrac{1}{2}, \tfrac{1}{4}, \tfrac{3}{16}, \tfrac{1}{8}, \tfrac{11}{128}, \tfrac{15}{256}, \tfrac{41}{1024}, \dots \right\}$$

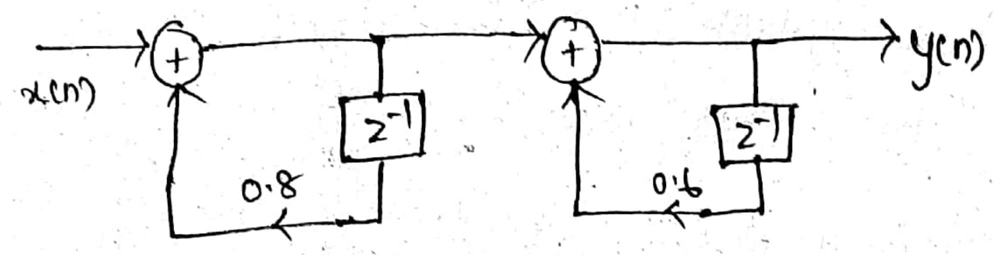
$y(0) = \tfrac{1}{2}y(-1) + \tfrac{1}{8}y(-2) + \tfrac{1}{2}x(-2) = 0$

$y(1) = \tfrac{1}{2}y(0) + \tfrac{1}{8}y(-1) + \tfrac{1}{2}x(-1) = 0$

$y(2) = 1/2,\ y(3) = 1/4,\ y(4) = 3/16,\ y(5) = 1/8$

$y(6) = \frac{11}{128},\ y(7) = \frac{15}{256},\ y(8) = \frac{41}{1024},\ y(9) = \frac{7}{256}$

c)



$y(n) = 1.4\,y(n-1) + x(n) - 0.48\,y(n-2)$

let $x(n) = \delta(n)$

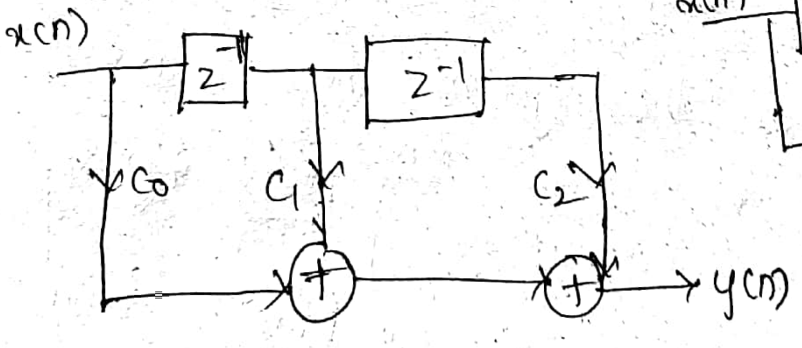
$y(-1) = y(-2) = 0$

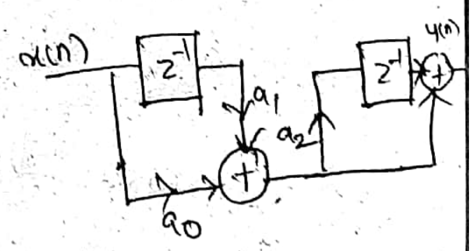
$h(n) = \{1, 1.4, 1.48, 1.4, 1.2996, 1.0774, 0.9086, \ldots\}$

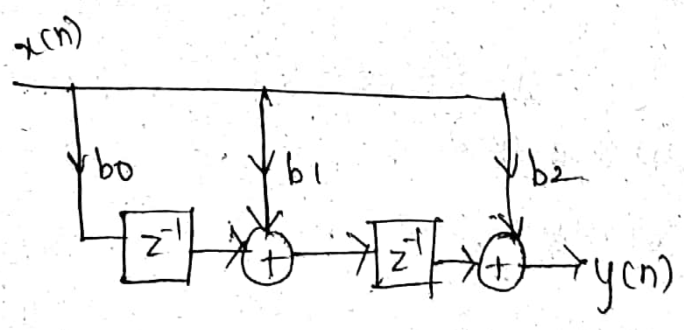
d) classify the systems above as FIR or IIR

all these systems are IIR.

2.49) consider the s/m's shown in figures.







a) Determine & sketch their impulse responses $h_1(n)$, $h_2(n)$, $h_3(n)$.

b) Is it possible to choose the coefficients of these systems in such a way that

$$h_1(n) = h_2(n) = h_3(n)$$

a) $h_1(n) = c_0\,\delta(n) + c_1\,\delta(n-1) + c_2\,\delta(n-2)$

$h_2(n) = b_2\,\delta(n) + b_1\,\delta(n-1) + b_0\,\delta(n-2)$

$h_3(n) = a_0\,\delta(n) + (a_1 + a_0 a_2)\,\delta(n-1) + a_1 a_2\,\delta(n-2)$

b) Let $a_0 = c_0$

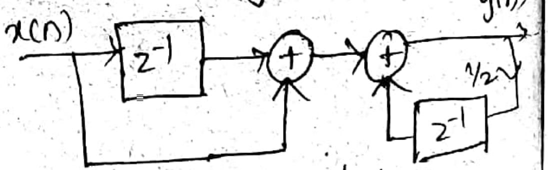
$a_1 + a_2 c_0 = c_1$ $\quad a_2 a_1 = c_2$

Hence $\dfrac{c_2}{a_2} + a_2 c_0 - c_1 = 0$

$\Rightarrow c_0 a_2^2 - c_1 a_2 + c_2 = 0$

for $c_0 \neq 0$, the quadratic has a real solutions if and only if $c_1^2 - 4c_0 c_2 \geq 0$

2.50) Consider the system shown figure



a) Determine it's impulse response $h(n)$

b) Show that $h(n)$ is equal to the convolution of the following signals.

$h_1(n) = \delta(n) + \delta(n-1)$

$h_2(n) = (1/2)^n\,u(n)$

a) $y(n) = \frac{1}{2}\,y(n-1) + x(n) + x(n-1)$

$y(n) - \frac{1}{2}y(n-1) = \delta(n)$, the solution of the s/m is

$h(n) = (1/2)^n u(n) - (1/2)^{n-1} u(n-1)$

b) $h_1(n) * [\delta(n) + \delta(n-1)] = (1/2)^n u(n) + (1/2)^{n-1} u(n-1)$

2.51) compute the sketch the convolution $y_1(n)$ and correlation $r_1(n)$ sequences for the following pair of signals and comment on the results obtained

a) $x(n) = \{\underset{\uparrow}{1}, 2, 4\}$, $h_1(n) = \{1, \underset{\uparrow}{1}, 1, 1, 1\}$

a) convolution : $y_1(n) = x_1(n) * h_1(n)$

x(n) \ h(n)	1	1	1	1	1
1	1	1	1	1	1
2	2	2	2	2	2
4	4	4	4	4	4

$y_1(n) = \{\underset{\uparrow}{1}, 3, 7, 7, 7, 6, 4\}$

correlation

$r_1(n) = x(n) * h(-n)$

$h(-n) = \{1, 1, 1, 1, \underset{\uparrow}{1}\}$

$x(n) = \{\underset{\uparrow}{1}, 2, 4\}$

x(n) \ h(n)	1	1	1	1	1
1	1	1	1	1	1
2	2	2	2	2	2
4	4	4	4	4	4

$r_1(n) = \{1, 3, 7, 7, \underset{\uparrow}{7}, 6, 4\}$

b) $x_2(n) = \{0, 1, -2, 3, -4\}$, $h_2(n) = \{1/2, 1, \underset{\uparrow}{2}, 1, 1/2\}$

convolution: $y_2(n) = x_2(n) * h_2(n)$

x(n) \ h(n)	1/2	1	2	1	1/2
0	0	0	0	0	0
1	1/2	1	2	1	1/2
-2	-1	-2	-4	-2	-1
3	3/2	3	6	3	3/2
-4	-2	-4	-8	-4	-2

$y_2(n) = \{0, 1/2, 0, 5/2, -2, 1/2, -6, -5/2, -2\}$

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correlation

$r_2(n) = x_2(n) * h_2(n)$

$r_2(n) \Rightarrow h_2(-n) = \{1/2, 1, 2, 1, 1/2\}$

$x_2(n) = \{0, 1, -2, 3, -4\}$

$r_2(n) = \{0, 1/2, 0, 3/2, -2, 1/2, -6, -5/2, -2\}$

c) $x_3(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$, $h_3(n) = \{\underset{\uparrow}{4}, 3, 2, 1\}$

Convolution $\Rightarrow y_3(n) = x_3(n) * h_3(n)$

$y_3(n) = \{4, 11, 20, 30, 20, 11, 4\}$

Correlation $\Rightarrow r_3(n) = x_3(n) * h_3(-n)$

x(n) \ h(n)	4	3	2	1
1	4	3	2	1
2	8	6	4	2
3	12	9	6	3
4	16	12	8	4

$h_3(-n) = \{1, 2, 3, \underset{\uparrow}{4}\}$

$x_3(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

$r_3(n) = \{1, 4, 10, \underset{\uparrow}{20}, 25, 24, 16\}$

x(n) \ h(n)	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

d) $x_4(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$, $h_4(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

Convolution

$y_4(n) = x_4(n) * h_4(n)$

$y_4(n) = \{\underset{\uparrow}{1}, 4, 10, 20, 25, 24, 16\}$

x(n) \ h(n)	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

correlation:

$r_4(n) = x_4(n) * h_4(-n)$

$h_4(-n) = \{4, 3, 2, \underset{\uparrow}{1}\}$

$x_4(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

$x(n)$ \ $h(n)$	4	3	2	$\underset{\downarrow}{1}$
$\rightarrow 1$	4	3	2	1
2	8	6	4	2
3	12	9	6	3
4	16	12	8	4

$r_4(n) = \{4, 11, 20, 30, 20, 11, 4\}$

2.52) The zero-state response of a causal LTI s/m to the input $x(n) = \{\underset{\uparrow}{1}, 3, 3, 1\}$ is $y(n) = \{\underset{\uparrow}{1}, 4, 6, 4, 1\}$ Determine it's impulse response.

$x(n) = \{\underset{\uparrow}{1}, 3, 3, 1\}$ $y(n) = \{\underset{\uparrow}{1}, 4, 6, 4, 1\}$

the length of the input is $L_1 = 4$

the length of o/p is $N = 5$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

length of the $h(n)$ is L_2

$L_1 + L_2 - 1 = N \Rightarrow 4 + L_2 - 1 = 5$

$3 + L_2 = 5 \Rightarrow L_2 = 5 - 3 = 2$

$\therefore h(0) = y(0) = x(0) = 1$ $h(n) = \{h(0), h(1)\}$

$3h(0) + h(1) = y(1) = 4$

$\Rightarrow 3(1) + h(1) = 4 \Rightarrow 3 + h(1) = 4$

$h(1) = 4 - 3 = 1$ $\therefore h(n) = \{1, 1\}$

2.54) Determine the response $y(n)$, $n \geq 0$ of the s/m described by the second-order difference equation.

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$ when input is

$x(n) = (-1)^n u(n)$

and the initial conditions are $y(-1) = y(-2) = 0$

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

$\lambda^2 - 4\lambda + 4 = 0$

$\lambda^2 - 2\lambda - 2\lambda + 2\times 2 = 0$

$\lambda(\lambda-2) + (-2)(\lambda-2) = 0$

$(\lambda-2)(\lambda-2) = 0 \quad \lambda = 2, 2$

Hence $y_h(n) = C_1 2^n + C_2 n 2^n$

the particular solution is $y_1(n) = k(-1)^n u(n)$

$k(-1)^n u(n) - 4k(-1)^{n-1} u(n-1) + 4k(-1)^{n-2} u(n-2) =$
$(-1)^n u(n) - (-1)^{n-1} u(n-1)$

for $n=2$

$k - (-4k) + 4k = 1 - (-1) = 2$

$k(1+4+4) = 2 \Rightarrow k = 2/9$

$\therefore y(n) = \left[C_1 2^n + C_2 n\, 2^n + \frac{2}{9}(-1)^n\right] u(n)$

from initial conditions: $y(-1) = y(-2) = 0$

$y(0) = 1; \; y(1) = 2$

$C_1 + \frac{2}{9} = 0$

$C_1 = 1 + \frac{2}{9} = 7/9$

$2C_1 + 2C_2 - \frac{2}{9} = 2$

$2\left(\frac{7}{9}\right) + 2C_2 = 2 + \frac{2}{9}$

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$$2C_2 = \frac{20}{9} - \frac{14}{9} = \frac{6}{9} = \frac{2}{3}$$

$$C_2 = \frac{2}{6} = 1/3$$

$$\therefore y(n) = \left[\frac{7}{9} 2^n + 1/3 n 2^n + \frac{2}{9}(-1)^n\right] u(n)$$

2.55) Determine the impulse response $h(n)$ for the s/m described by the second-order difference equation.

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

the characteristic equation is

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda^2 - 2\lambda - 2\lambda + 2\times2 = 0$$

$$\lambda(\lambda-2) - 2(\lambda-2) = 0$$

$$(\lambda-2)(\lambda-2) = 0 \quad \therefore \lambda = 2,2$$

$$y_h(n) = 2^n C_1 + n2^n C_2$$

when $x(n) = \delta(n)$

with $y(0) = 1$ and $y(1) = 3$

$$C_1 = 1 \Rightarrow 2C_1 + 2C_2 = 3$$

$$2(C_1 + C_2) = 3 \qquad 1 + C_2 = 3/2$$

$$C_2 = 3/2 - 1 = 1/2$$

$$\therefore h(n) = \left[2^n + 1/2\, n 2^n\right] u(n)$$

2.56) Show that any discrete-time signal $x(n)$ can be expressed as $x(n) = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] u(n-k)$

where $u(n-k)$ is a unit step delayed by k units in time. that is $u(n-k) = \begin{cases} 1, & n=k \\ 0, & \text{otherwise} \end{cases}$

$$x(n) = x(n) * \delta(n)$$

$$x(n) = x(n) * [u(n) - u(n-1)]$$

$$x(n) = x(n) * [u(n) - u(n-1)] \quad \therefore \delta(n) = u(n) - u(n-1)$$

$$x(n) = u(n) * [x(n) - x(n-1)]$$

$$= \sum_{k=-\infty}^{\infty} [x(k) - x(n-k)] \, u(n-k)$$

2.57) Show that the output of an LTI system can be expressed in terms of ~~the~~ unit step response $s(n)$ as follows;

$$y(n) = \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)] \, x(n-k)$$

$$y(n) = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] \, s(n-k)$$

Let $h(n)$ be the impulse response of the s/m

$$s(k) = \sum_{k=-\infty}^{\infty} h(n)$$

$$h(k) = s(k) - s(k-1)$$

$$\Rightarrow y(n) = \sum_{k=-\infty}^{\infty} h(k) x(n-k)$$

$$\Rightarrow y(n) = \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)] \, x(n-k)$$

2.58) Compute the correlation sequences $r_{xx}(l)$ and r_{xy} for the following signal sequences

$$x(n) = \begin{cases} 1, & n_0 - N \le n \le n_0 + N \\ 0, & \text{otherwise} \end{cases}$$

$$y(n) = \begin{cases} +1, & -N \le n \le N \\ 0, & \text{otherwise} \end{cases}$$

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$$

the range of non-zero values of $r_{xx}(l)$ is determined by

$$n_0 - N \le n \le n_0 + N$$

$$n_0 - N \le n - l \le n_0 + N$$

$$-2N \le l \le 2N$$

for a given shift l, the number of terms in the summation for which both $x(n)$ and $x(n-l)$ are non-zero is $2N+1-|l|$ and the value of each term is 1

$$r_{xx}(l) = \begin{cases} 2N+1-|l|, & -2N \le l \le 2N \\ 0, & \text{otherwise} \end{cases}$$

for $r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n)\, y(n-l)$

$$r_{xy}(l) = \begin{cases} 2N+1-|l-n_0|, & n_0 - 2N \le l \le n_0 + 2N \\ 0, & \text{otherwise} \end{cases}$$

2.59) Determine the autocorrelation sequence of the following signals

a) $x(n) = \{\underset{\uparrow}{1}, 2, 1, 1\}$

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what is your conclusion?

b) $y(n) = \{1, 1, 2, 1\}$

a) $x(n) = \{\underset{\uparrow}{1}, 2, 1, 1\}$ $\quad x(0)=1, \quad x(1)=2$

$x(2)=1, \quad x(3)=1$

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\,x(n-l) = \sum_{n=0}^{4-l-3} x(n)\,x(n-l)$$

$r_{xx}(-3) = x(0)\,x(0-(-3)) = x(0)\,x(3) = (1)(1) = 1$

$r_{xx}(-2) = x(0)\,x(0-(-2)) = x(0)\,x(2) = 1\times1 + 1\times2$

$= 1+2 = 3$

$r_{xx}(-1) = 5$

$$r_{xx}(0) = \sum_{n=0}^{3} x^2(n) = 7$$

also $r_{xx}(-l) = r_{xx}(l)$

$\therefore r_{xx}(l) = \{1, 3, 5, 7, 5, 3, 1\}$

b) $y(n) = \{\underset{\uparrow}{1}, 1, 2, 1\}$

$$r_{yy}(l) = \sum_{n=-\infty}^{\infty} y(n)\,y(n-l)$$

$r_{yy}(l) = \{1, 3, 5, 7, 5, 3, 1\}$

$y(n) = x(-n+3)$, which is equivalent to reversing the sequence $x(n)$ this has not changed the autocorrelation sequence

2.60) What is the normalised autocorrelation sequence of the signal $x(n)$ given by

$$x(n) = \begin{cases} 1, & -N \le n \le N \\ 0, & \text{otherwise} \end{cases}$$

$$r_{xx}(l) = \sum_{n=-\infty} x(n)\, x(n-l)$$

$$r_{xx}(l) = \begin{cases} 2N+1-|l|, & -2N \le l \le 2N \\ 0, & \text{otherwise} \end{cases}$$

$$r_{xx}(0) \Rightarrow r_{xx}(0) = 2N+1-|0| = 2N+1$$

The normalized autocorrelation is

$$\rho_{xx}(l) = \begin{cases} \frac{1}{2N+1}(2N+1-|l|); & -2N \le l \le 2N \\ 0, & \text{otherwise} \end{cases}$$

2.61) An audio signal $s(t)$ generated by a loudspeaker is reflected at two different walls with reflection coefficients r_1 and r_2. The signal $x(t)$ recorded by a microphone close to the loudspeaker, after sampling, is

$$x(n) = s(n) + r_1 s(n-k_1) + r_2 s(n-k_2)$$

where k_1 and k_2 are the delays of the two echos

a) Determine the autocorrelation $r_{xx}(l)$ of the signal $x(n)$

b) Can we obtain r_1, r_2, k_1 and k_2 by observing $r_{xx}(l)$?

c) What happens if $r_2 = 0$?

a) $$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$$

$$= \sum_{n=-\infty}^{\infty} [s(n) - r_1 s(n-k_1) + r_2 s(n-k_2)] * [s(n-l) + r_1 s(n-l-k_1) + r_2 s(n-k_2-l)]$$

$= [1+\gamma_1^2+\gamma_2^2]\gamma_{ss}(l)+\gamma_1[\gamma_{22}(l+k_1)+\gamma_{22}(l+k_2)+$

$\gamma_2[\gamma_{22}(l+k_2)+\gamma_{33}(l-k_2)]+\gamma_1\gamma_2[\gamma_{33}(l+k_1-k_2)$

$+\gamma_{33}(l+k_2-k_1)]$

b) $\gamma_{xx}(l)$ has peaks at $l=0$, $\pm k_1$, $\pm k_2$ and $\pm(k_1+k_2)$ suppose that $k_1<k_2$. then we can determine n and k_1. the problem is to determine γ_2 and k_2 from the other peaks

c) If $\gamma_2=0$, the peaks occur at $l=0$ and $l=\pm k_1$. then, it is easy to obtain γ_1 and k_1.