Support Vector Machine (SVM)

Group 23

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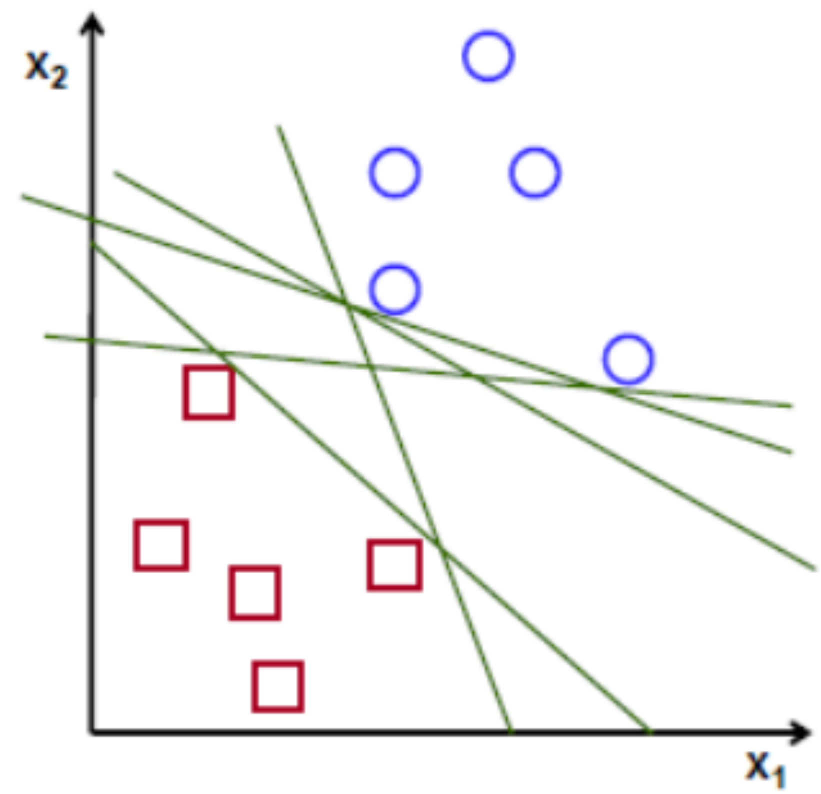

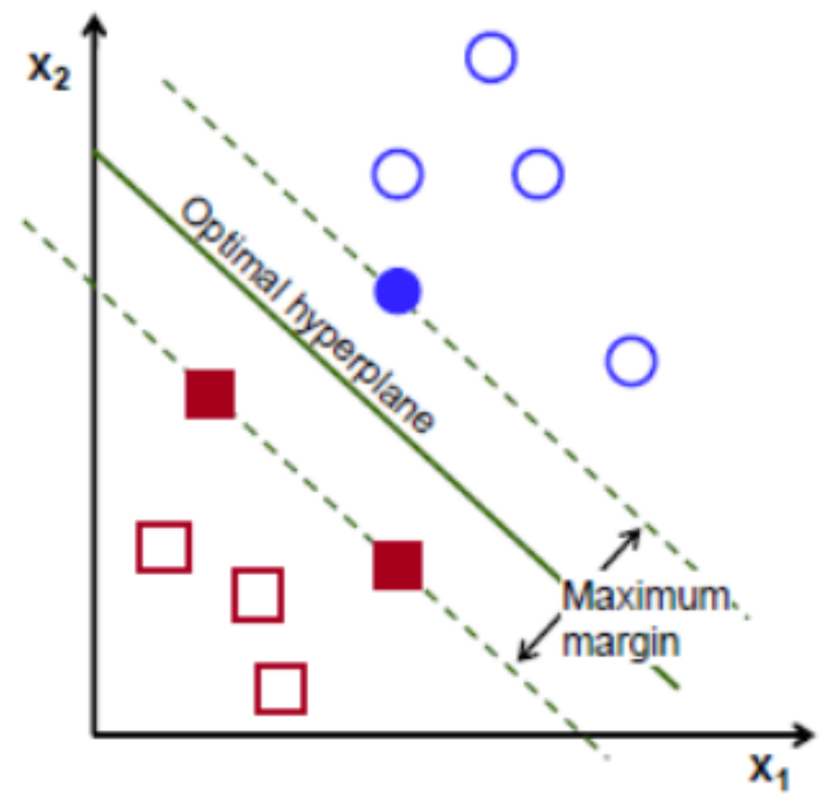

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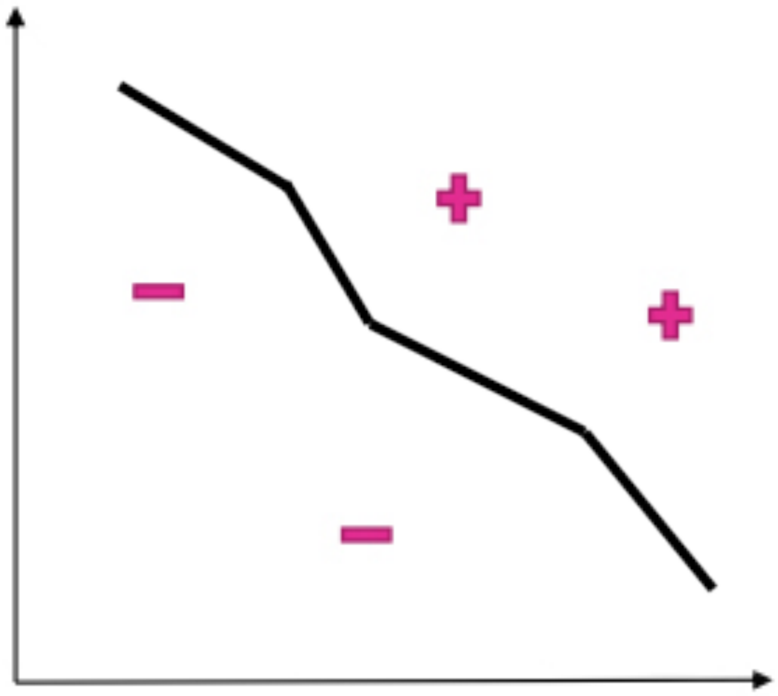
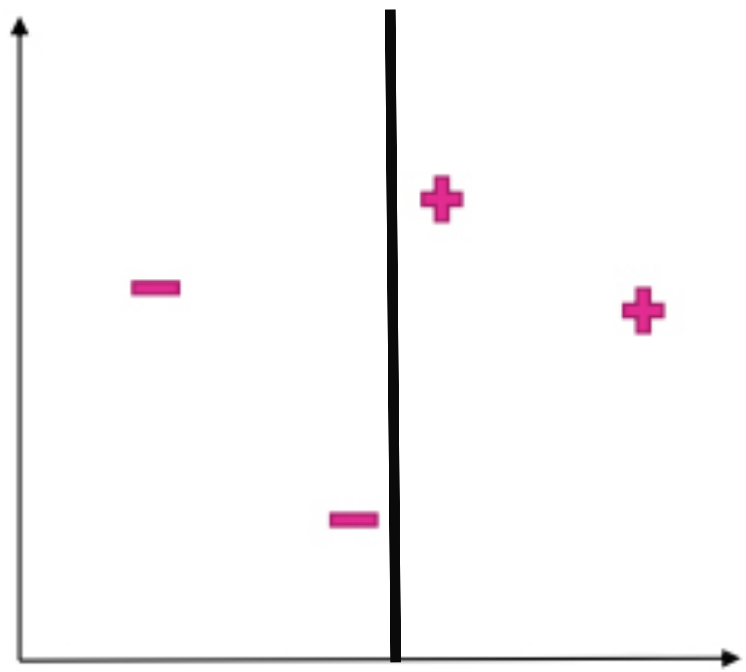
Why we need SVM

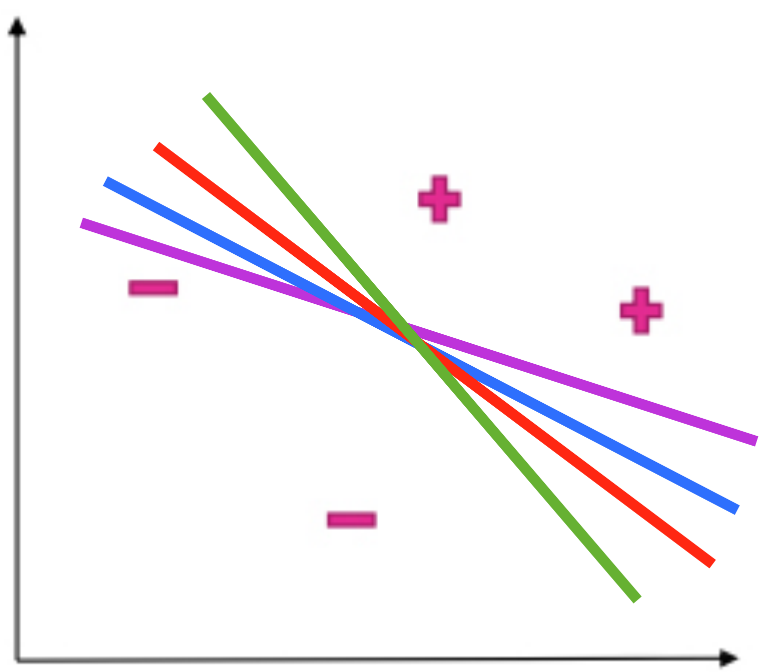
kNN when k=1



Decision tree



Which is the best fit line?



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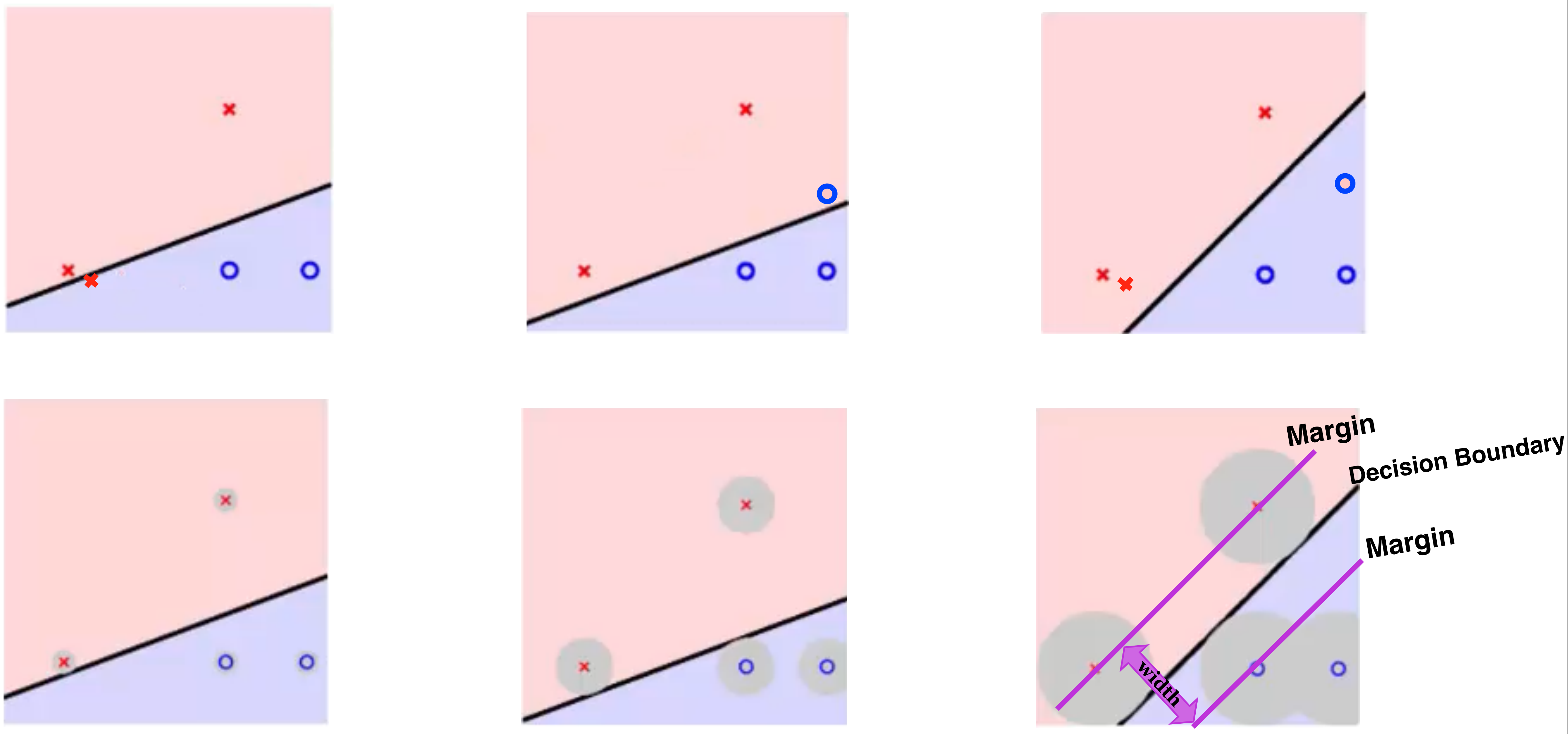
Introduction of SVM

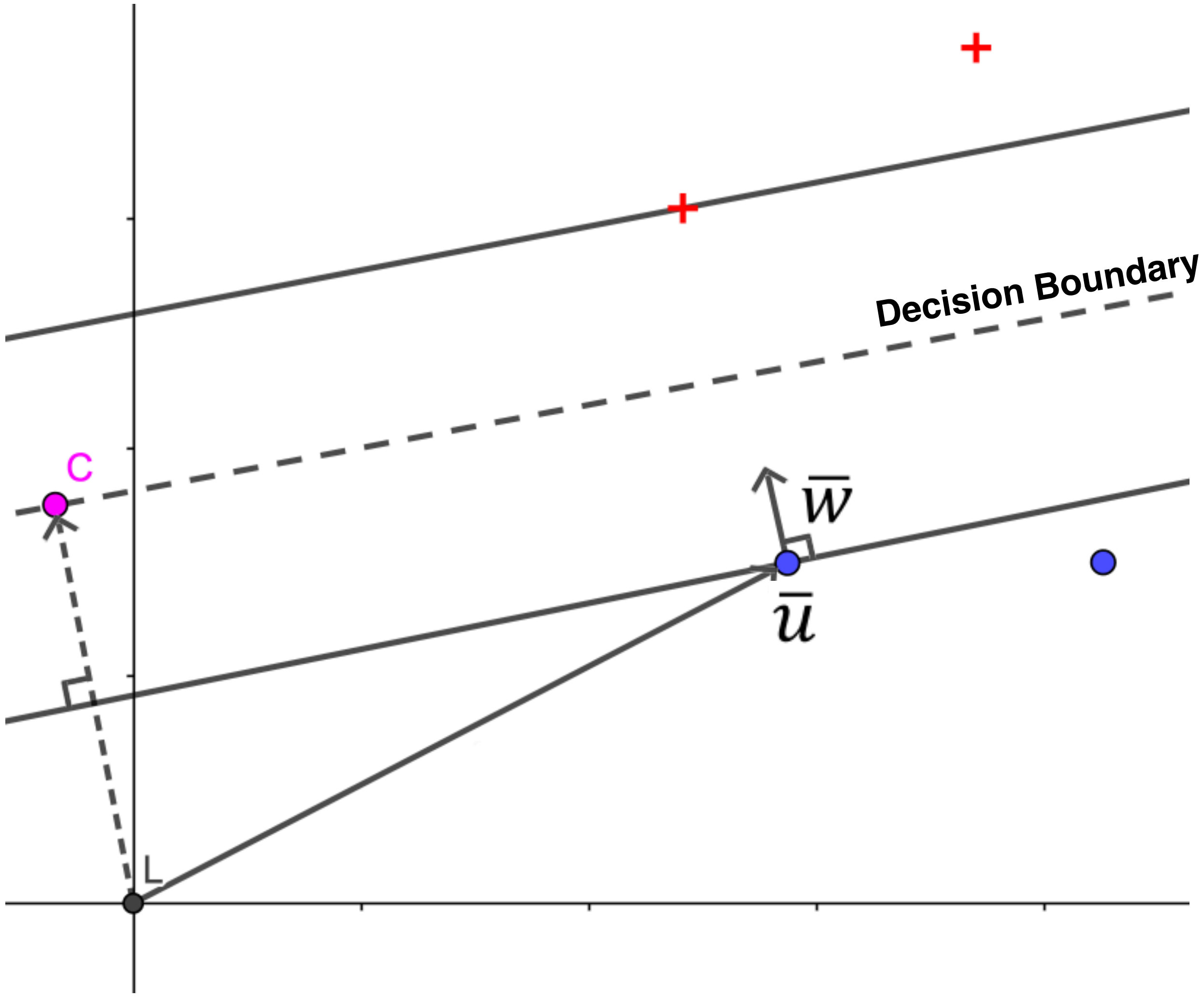
How to find the best decision boundary ?

The goal of SVM



- The goal for SVM is to find a decision boundary which maximizes the distance between decision boundary and the closest data to it.

Optimal Decision Boundary



- $\overline{w}$ is a unit vector perpendicular to decision boundary.

- The sample $\overline{u}$ is positive when $\overline{w}.\overline{u} \geq c$
- This equals $\overline{w}.\overline{u} + \mathrm{b} \geq 0$

Optimal Decision Boundary

- A better restriction

$\bar{w}.\overline{x_+} + \mathrm{b} \geq 1$ then the sample is positive

$\bar{w}.\overline{x_-} + \mathrm{b} \leq -1$ then the sample is negative

To combine the two equations:

y_i s.t. $y_i = +1$ for positive samples

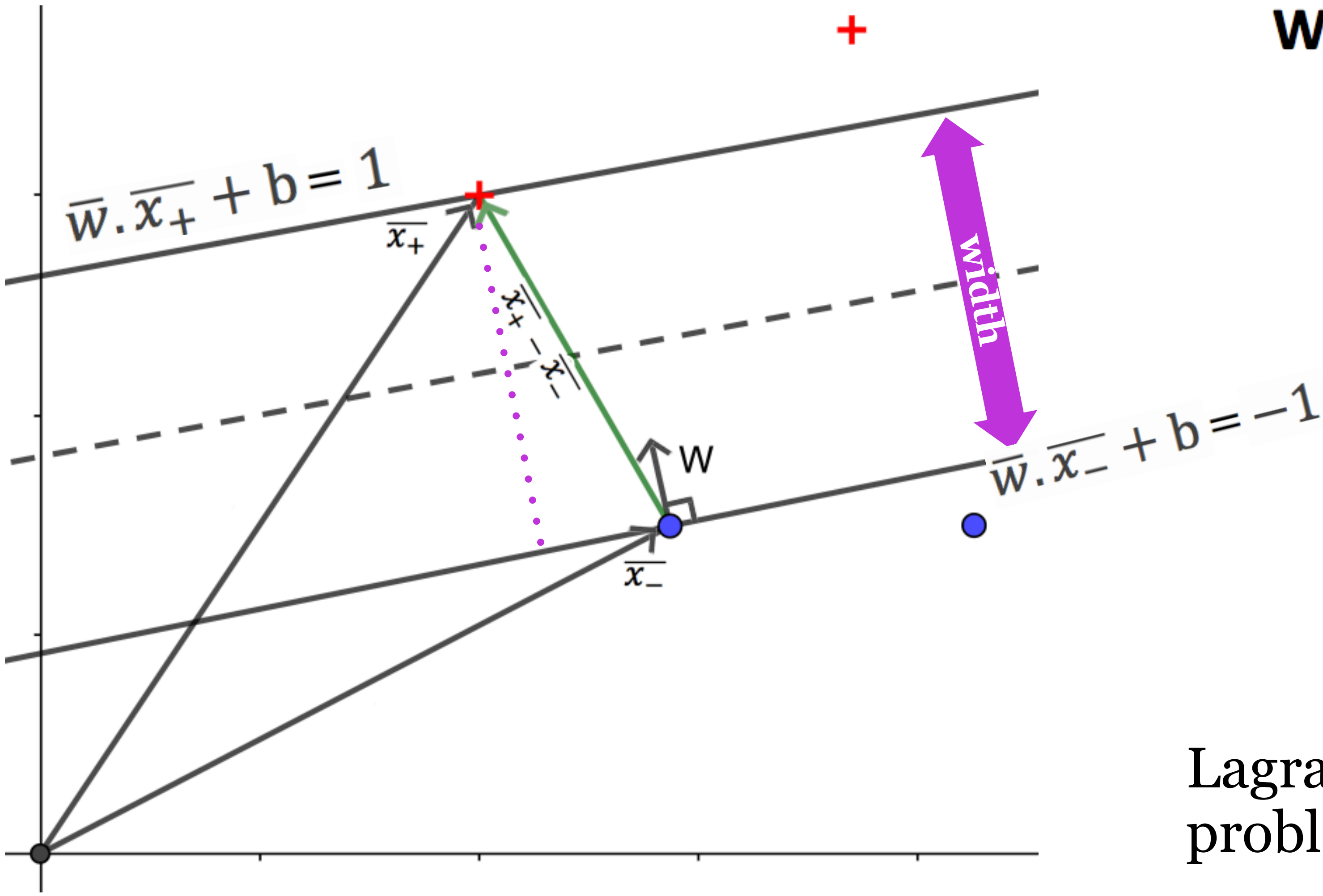
$y_i = -1$ for negative samples

$$y_i(\bar{w}.\overline{x_+} + \mathrm{b}) \geq 1$$
$$y_i(\bar{w}.\overline{x_-} + \mathrm{b}) \geq 1$$

$$\rightarrow$$

$$y_i(\bar{w}.\bar{x_i} + \mathrm{b}) - 1 \geq 0$$

SVM



$$\text{Width} = \frac{w}{\|w\|} \cdot (\overline{x_+} - \overline{x_-})$$

$$\text{Width} = \frac{1\text{-b-(-1-b)}}{\|w\|} = \frac{2}{\|w\|}$$

$$\text{Max}\ \frac{2}{\|w\|} \rightarrow \text{Max}\ \frac{1}{\|w\|}$$

$$\rightarrow \text{Min}\ \frac{1}{2} \| w \|^2 \checkmark$$

$$y_i(\overline{w}.\overline{x_i} + \text{b}) - 1 = 0 \checkmark$$

Lagrange multiplier to solve this optimization problem

Optimization

$$\boxed{\mathbf{J(w,b) = L = \frac{1}{2} \parallel w \parallel^2 - \sum \alpha^i \left[y_i(\bar{w}.\bar{x}_i + b) - 1\right]}}$$

$$\frac{\partial L}{\partial \bar{w}} = \bar{w} - \sum \alpha^i y_i x_i = 0 \longrightarrow \bar{w} = \sum \alpha^i y_i x_i$$

$$\frac{\partial L}{\partial b} = -\sum \alpha^i y_i = 0 \longrightarrow \sum \alpha^i y_i = 0$$

$$\boxed{L = \sum \alpha^i - \frac{1}{2}\left(\sum\sum \alpha^i \alpha^j y_i y_j x_i \cdot x_j\right)}$$

- We are able to solve the equation with numerical analytic method, so that we get all the α_i α_j
- So for the new unknown sample $\bar{u}$

$$\bar{w}.\bar{u} + \mathrm{b}$$

$\sum \alpha^i y_i \bar{x}_i \cdot \bar{u} + b \geq 0$ Then class label is positive

≤ 0 class label is negative

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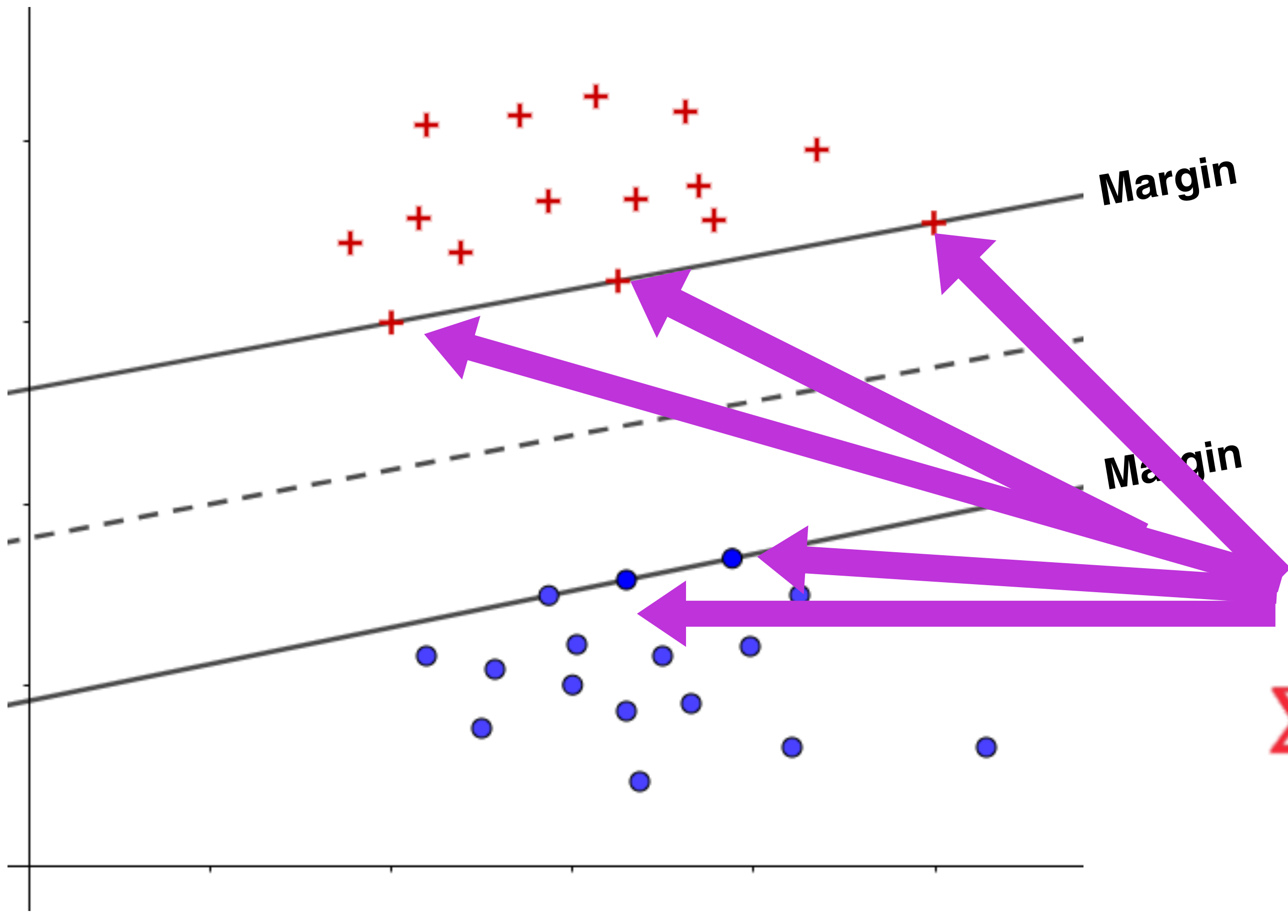
Support Vectors



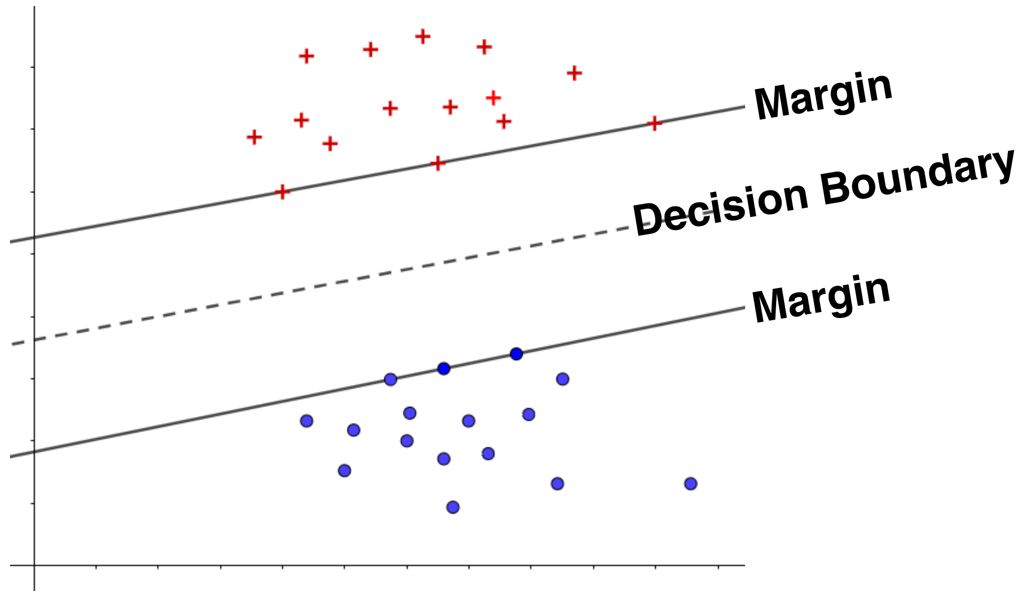
$\alpha^i > 0$ For support vector samples

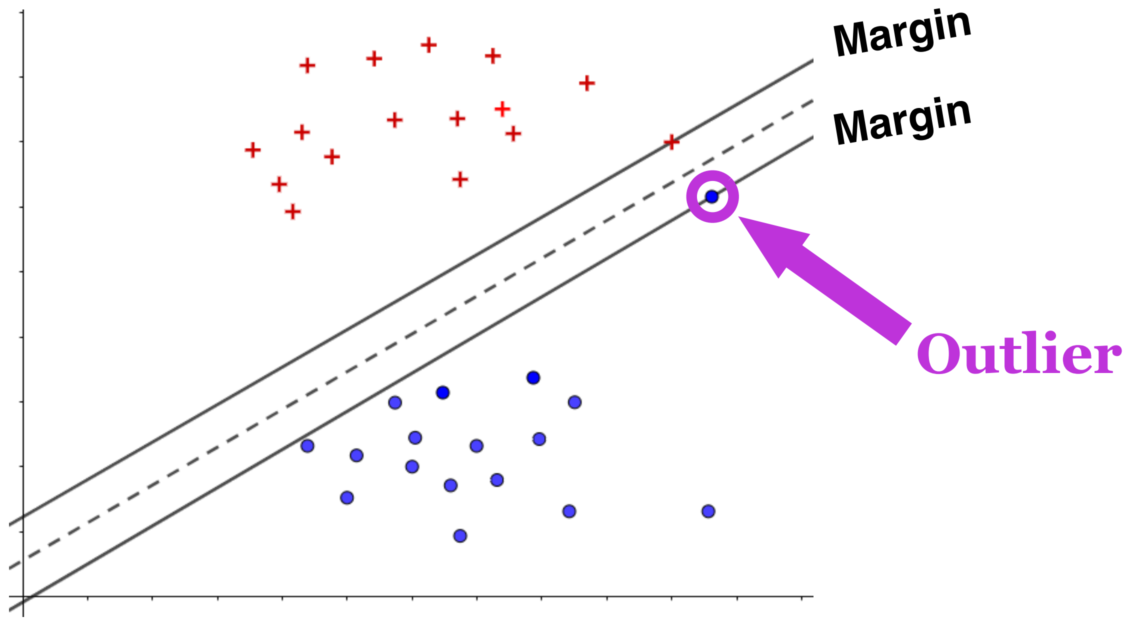
$\alpha^i = 0$ For other samples

Support Vectors

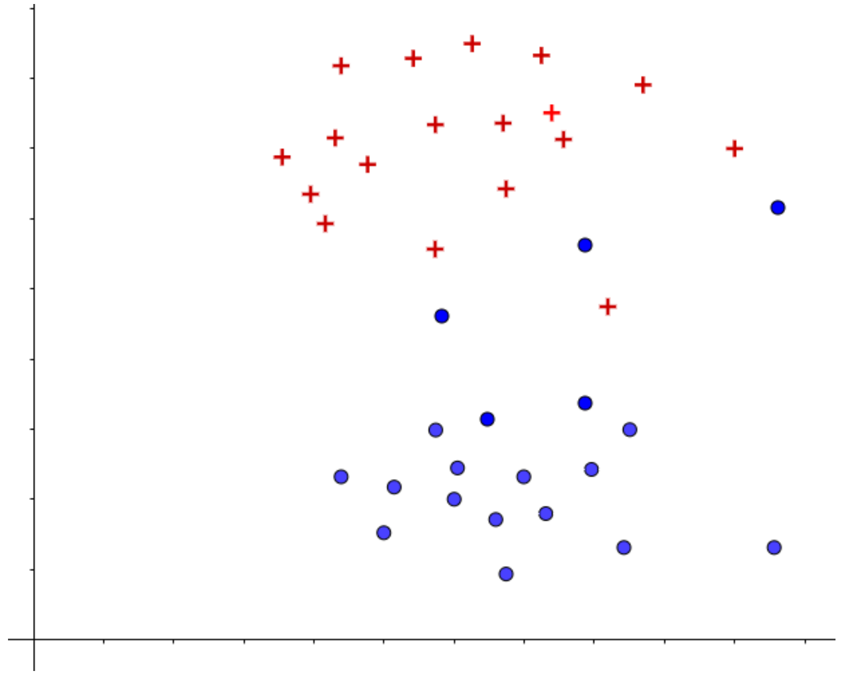
$$\sum \alpha^i y_i \bar{x}_i \cdot \bar{u} + b \geq 0 \quad \text{Then class label is positive}$$
$$\leq 0 \quad \text{class label is negative}$$

Hard-Margin SVM





An example could not be linearly separable



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Soft-Margin SVM

- Recall one constraints we draw to deal with the Hard-margin SVM problem
  $y^{(i)}(w^T x^{(i)} + b) \geq 1$
- Soft-Margin SVM introduces slack variables ξ_i that relax the constraint
  $y^{(i)}(w^T x^{(i)} + b) \geq 1 - \xi_i \qquad \forall i = 1 \ldots N$
- Then the previous optimization problem is defined as:

$$\text{Minimize} \quad J(w) = \frac{1}{2}||w||^2 + \boldsymbol{C}\sum_{i=1}^{N}\xi_i$$

$$\text{Subject to} \quad \begin{cases} y^{(i)}(w^T x^{(i)} + b) \geq 1 - \xi_i & \forall i \\ \xi_i \geq 0 & \forall i \end{cases}$$

- Parameter C:
  It controls the trade-off between complexity and miss-classification

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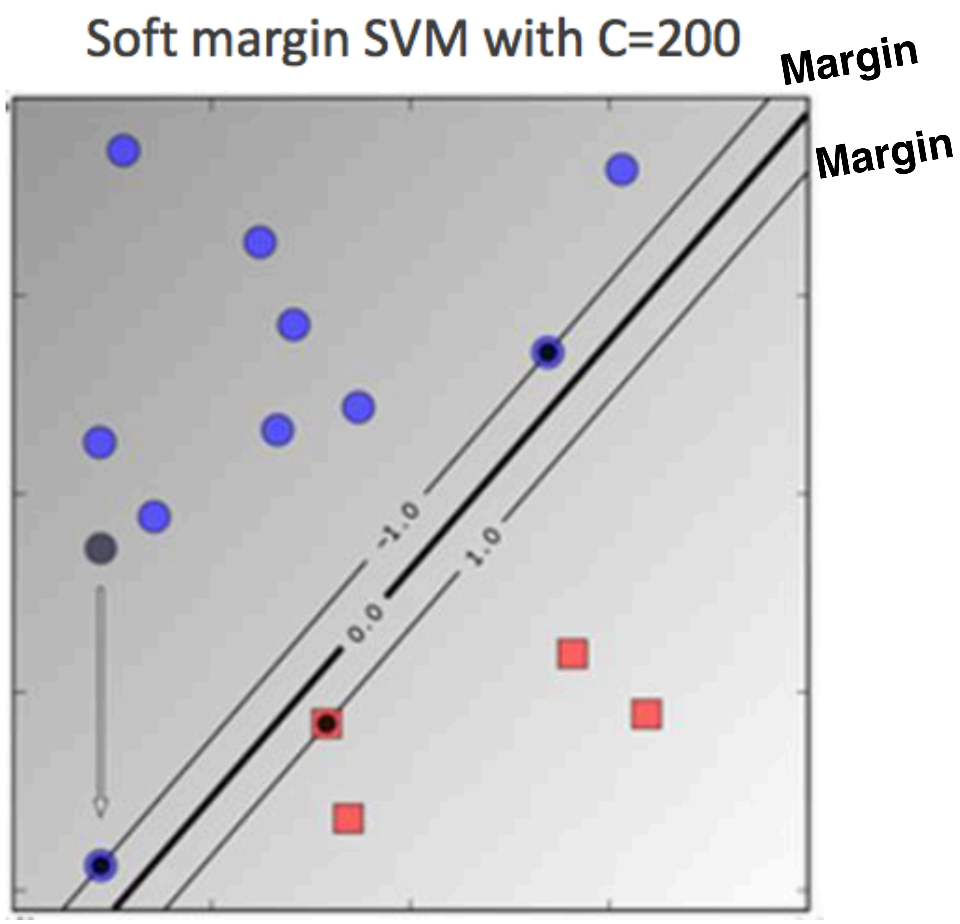
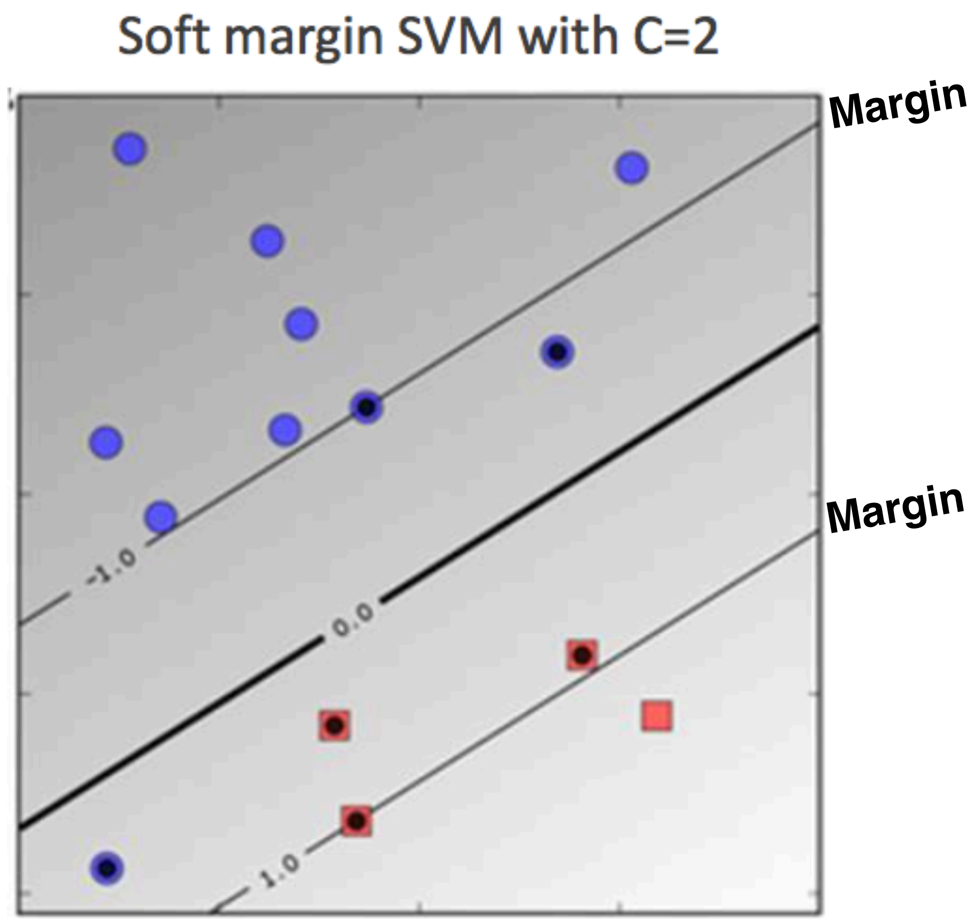
Effect of Penalization Factor C

Larger C: solutions with lower miss-classification error



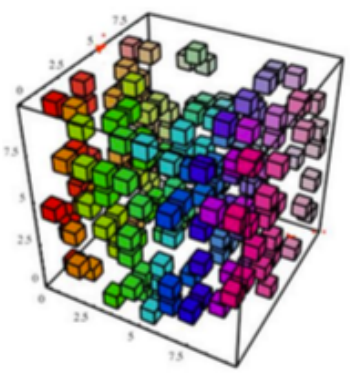
Smaller C: solutions with some miss-classification



[Figure by Ben-Hur, et. al. 2008]

Non-linear SVM

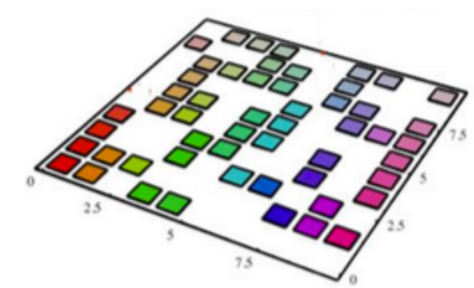
3D



$\boldsymbol{x}$: inpute samples in a 3D feature space

Feature extraction

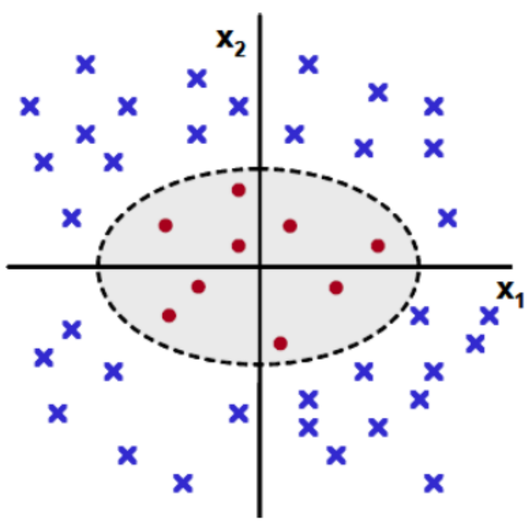
$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_d \end{bmatrix} = \begin{bmatrix} w_{11} & w_{12} & \cdots & & w_{1D} \\ w_{21} & w_{22} & \cdots & & w_{1D} \\ \vdots & \vdots & & \ddots & \vdots \\ w_{d1} & w_{d2} & \cdots & \cdots & w_{dD} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \\ x_D \end{bmatrix}$$



2D

$\boldsymbol{y}$: transformed Samples in a 2D feature space

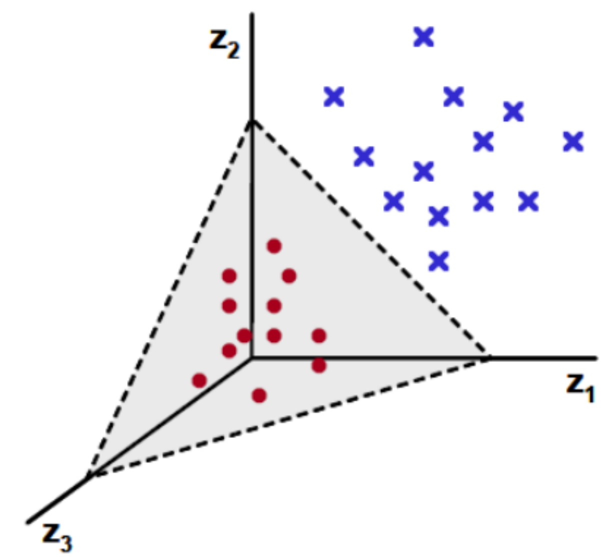
2D



Non-linear function

$$\varphi: \; R^2 \rightarrow R^3$$

$$(x_1, x_2) \;\mapsto (z_1, z_2, z_3) = (x_1^2, \sqrt{2}x_1x_2, x_2^2)$$



3D

Kernel SVM = Non-linear SVM

- Decision Function of Non-linear SVM will be

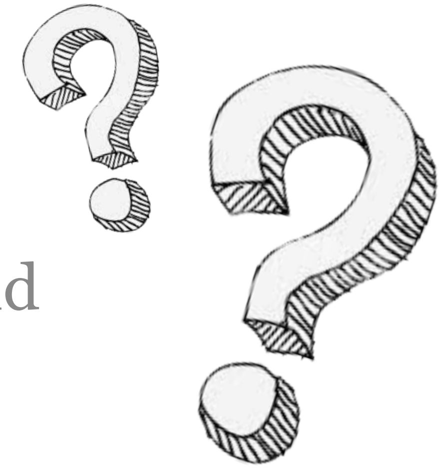
$$h(x) = sign(w^T \varphi(x) + b) = sign\left(\sum_{i=1}^{m} \alpha_i y^{(i)} \varphi\left(x^{(i)}\right)^T \varphi(x) + b\right)$$

- Kernel Function

$$K\left(x^{(i)}, x^{(j)}\right) = \varphi(x^{(i)})^T . \varphi(x^{(j)})$$

$$h(x) = sign\left(\sum_{i=1}^{m} \alpha_i y^{(i)} K(x^{(i)}, x) + b\right)$$

Extensions of SVM



what issues when apply SVM to real world

Advantages

- Can solve linear non-separate problem by mapping datas to high dimensions via kernel functions

- Deleting and adding the datas which are not support vectors do not affect modeling

Disadvantages

- Very difficult to choose Kernel Functions and parameter (c, g)

- Perform poorly in imbalanced samples

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Development

- Deal with unsupervised learning based on SVM

Example: Marketing segmentation of a drink company

Support vectors clustering(unsupervised learning)

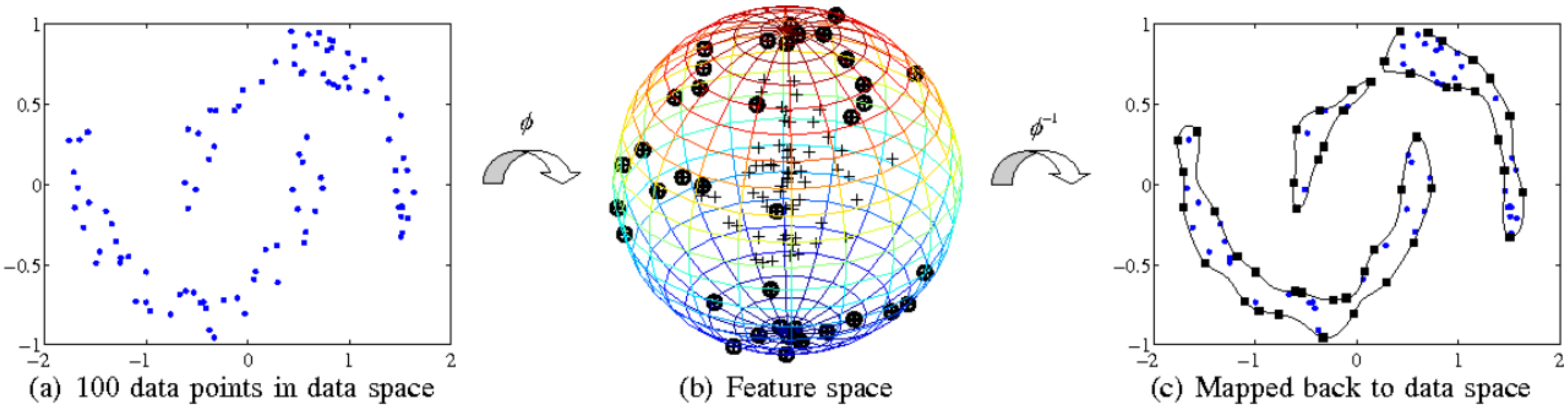

Figure 1. (a) Original data (b) Map to high dimension and minimal enclosing sphere (c) Map back to achieve clustering

Development

- Deal with semi-supervised learning based on SVM

Example: Hand written recognition

Figure 2. Hand-written digits data sets(0, 6, 8, 9)

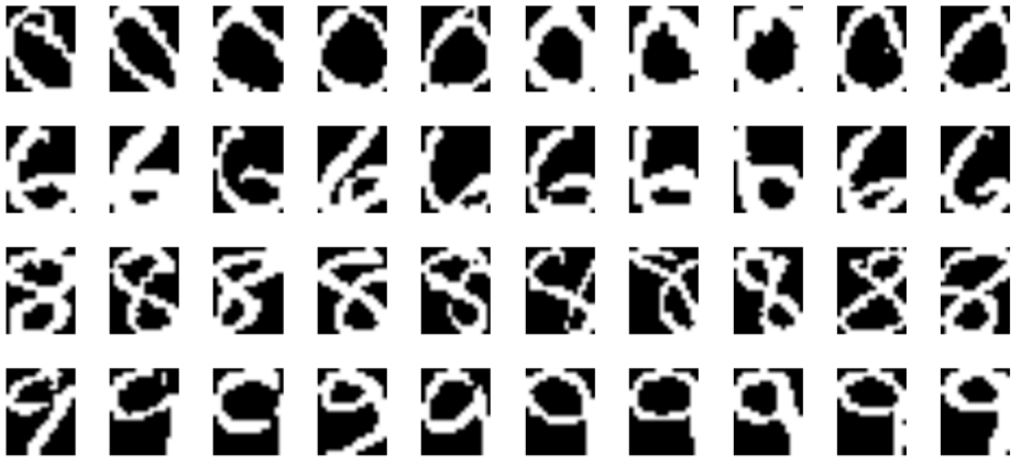

Figure 3. Unlabelled hand written digits data sets

Transductive support-vector machines(semi-supervised learning)

Development

- Deal with multi-classification problem based on SVM

Classes of fruits' images

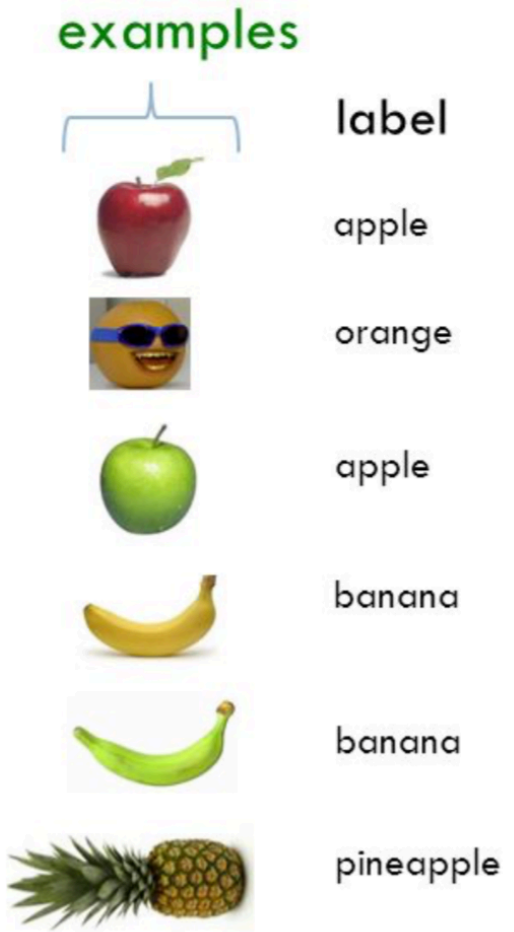

Figure 4. multi-classification problem of fruits

Multi-class SVM

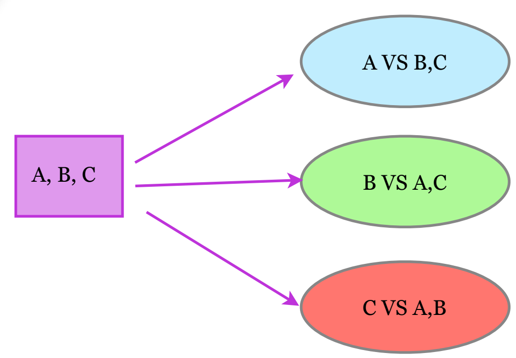

Figure 5. **one-versus-rest(OVR)** for a problem with three classes

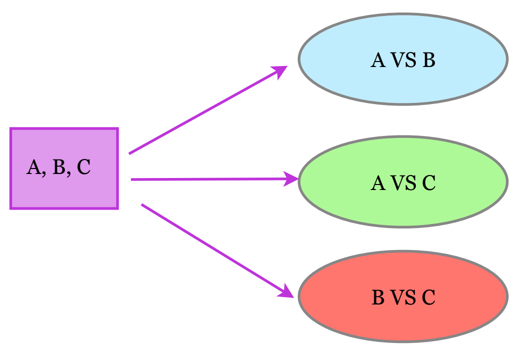

Figure 6. **one-versus-one(OVO)** for a problem with three classes

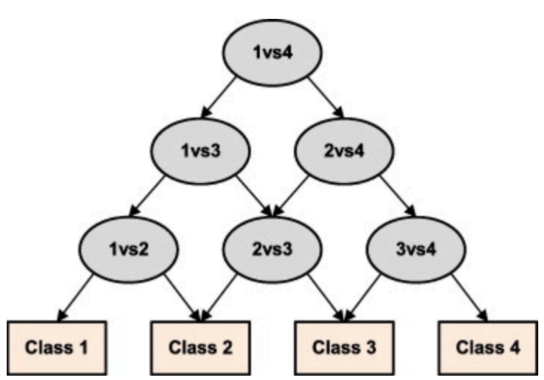

Figure 7. **DAGSVM** with four different classes

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Q & A :

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thank you !

Disadvantages

- Perform poorly in unbalanced samples

Solution:

Example

Disadvantages

- Very difficult to choose Kernel Functions and parameter (c, g)

Linear kernel: $K(x, x') = \left(\mathrm{x}^{\mathrm{T}} x'\right)^{2}$

Polynomial kernel: $K(x, x') = \left(\mathrm{x}^{\mathrm{T}} x' + 1\right)^{\mathrm{d}}$

Radial basis function kernel: $K(x, x') = \exp\left(-\frac{||x - x'||^{2}}{2\sigma^{2}}\right)$

Sigmoid kernel: $k(\vec{u}, \vec{v}) = \tanh(\gamma \vec{u}^{T} \cdot \vec{v} + coef0)$

$c \to \infty$: model $\to$ hard-margin
$c \to 0$: model $\to$ invalid

For same samples, different kernel functions perform similarly.
(c,g) depends on cross-validation

Why we need SVM?

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TWO COLUMN CONTENT SLIDE LAYOUT

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$$\bar{w}.\bar{u} + \mathrm{b} \geq 0$$

$$\sum \boldsymbol{\alpha^{i}}\, \boldsymbol{y_i} \boldsymbol{\bar{x}_i} \cdot \bar{u} + \boldsymbol{b}$$

TWO COLUMN COMPARISON CONTENT SLIDE LAYOUT

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EXAMPLE.2

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EXAMPLE.3

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CONTEXT OR THEME

“Lorem ipsum dolor sit amet, consectetur adipiscing elit. Phasellus non sem semper, rhoncus quam nec, suscipit erat. In non scelerisque est. Cras eu sodales mauris.”

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NAME HERE

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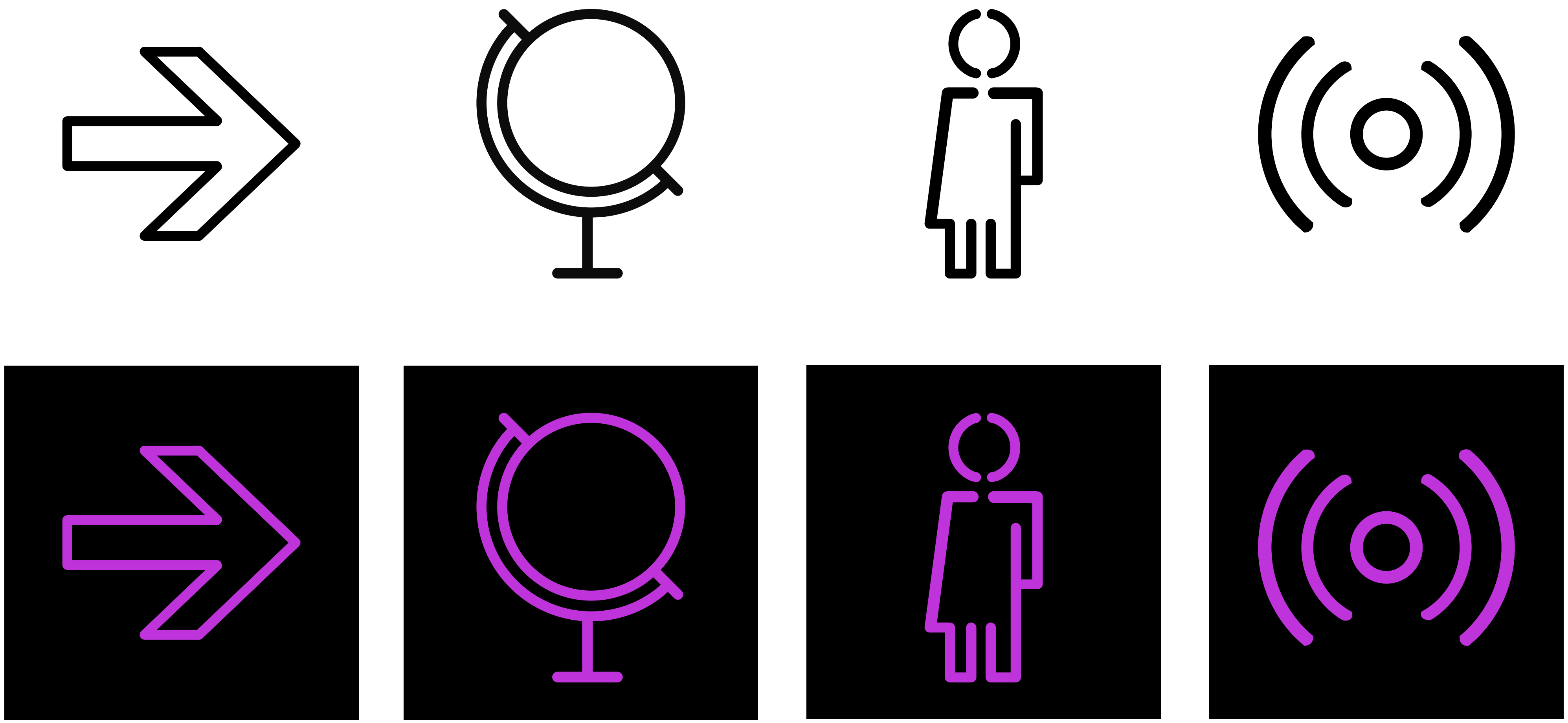

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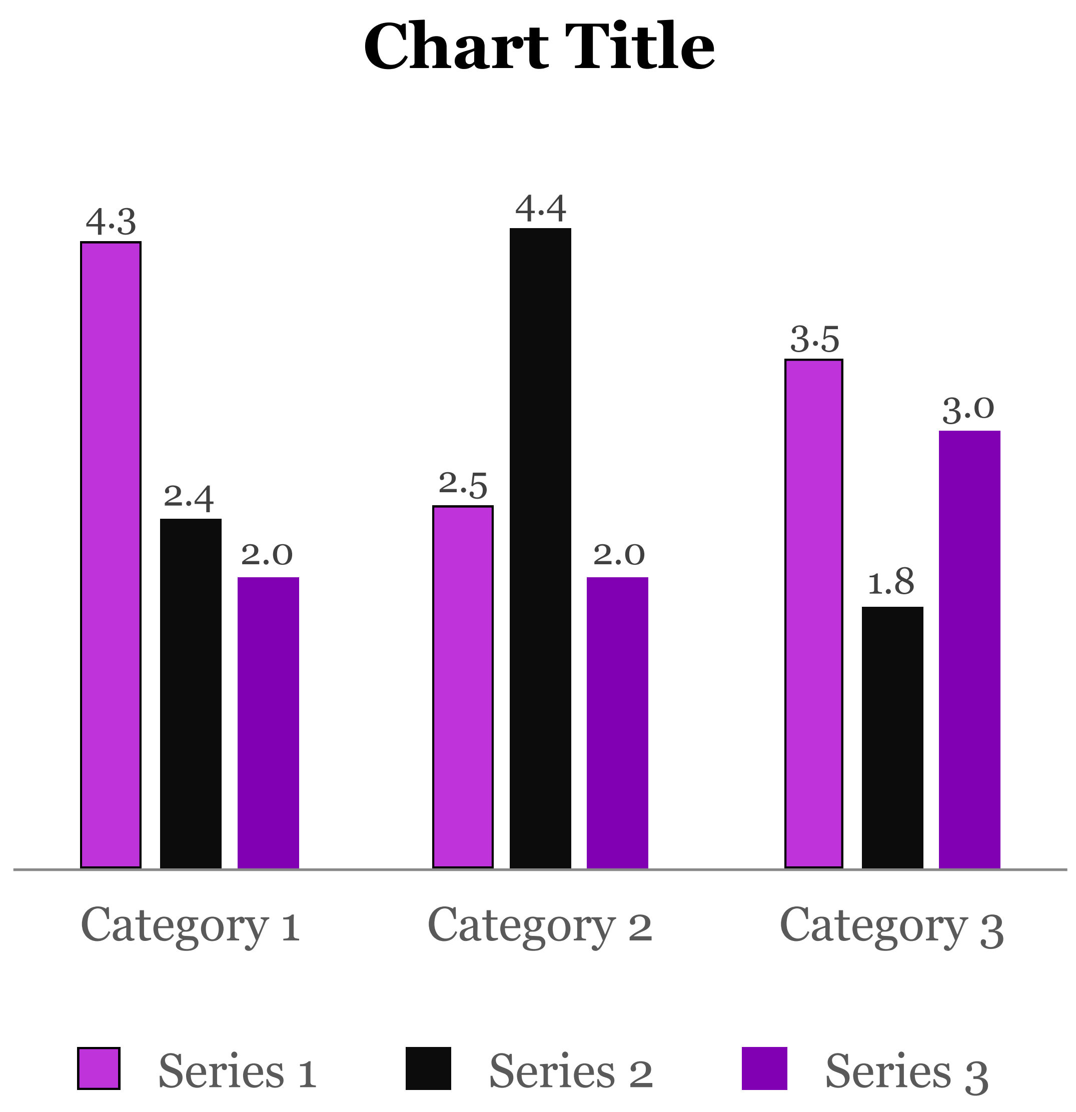
MASTER SLIDE CHART OPTION 1



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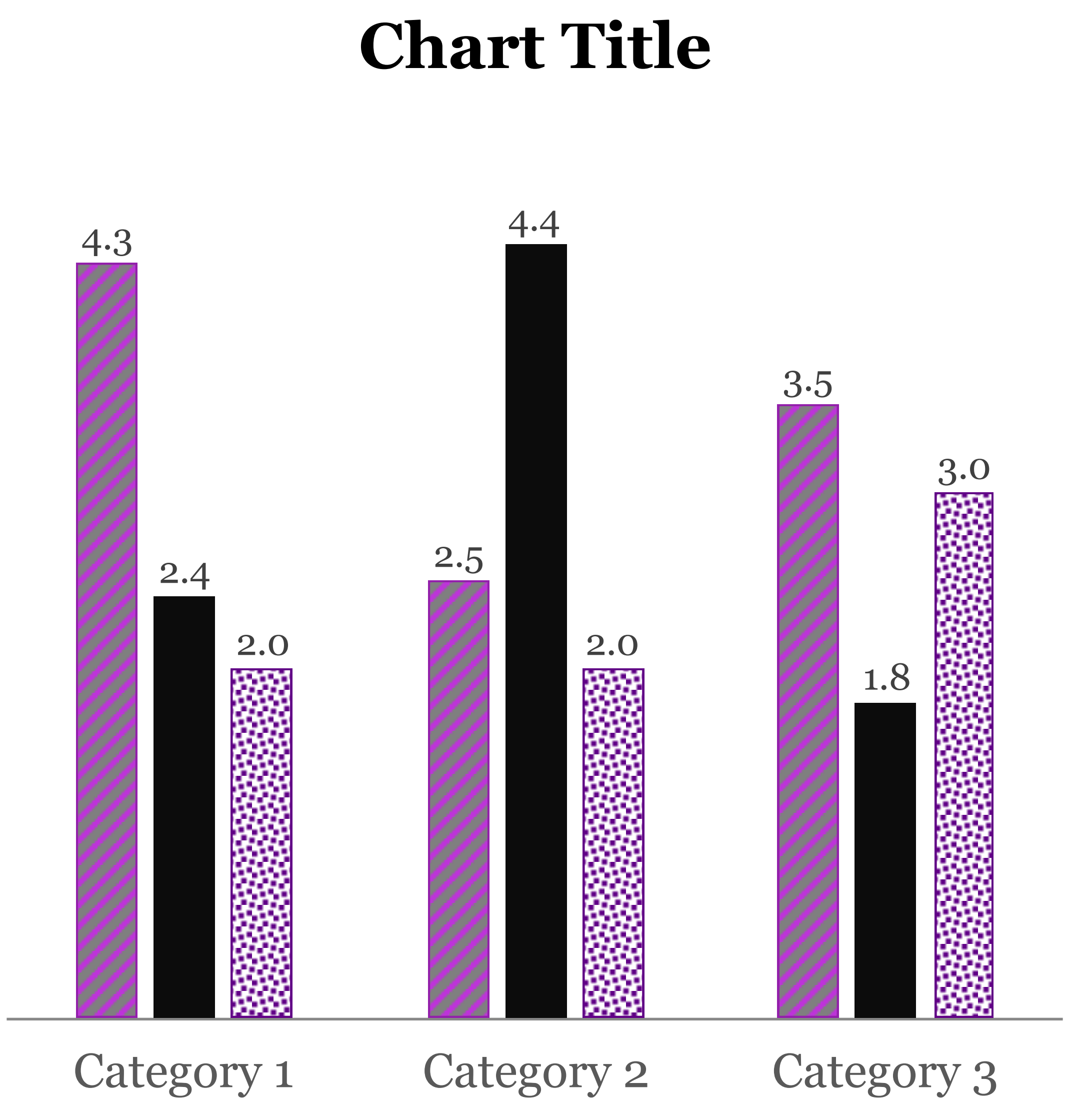

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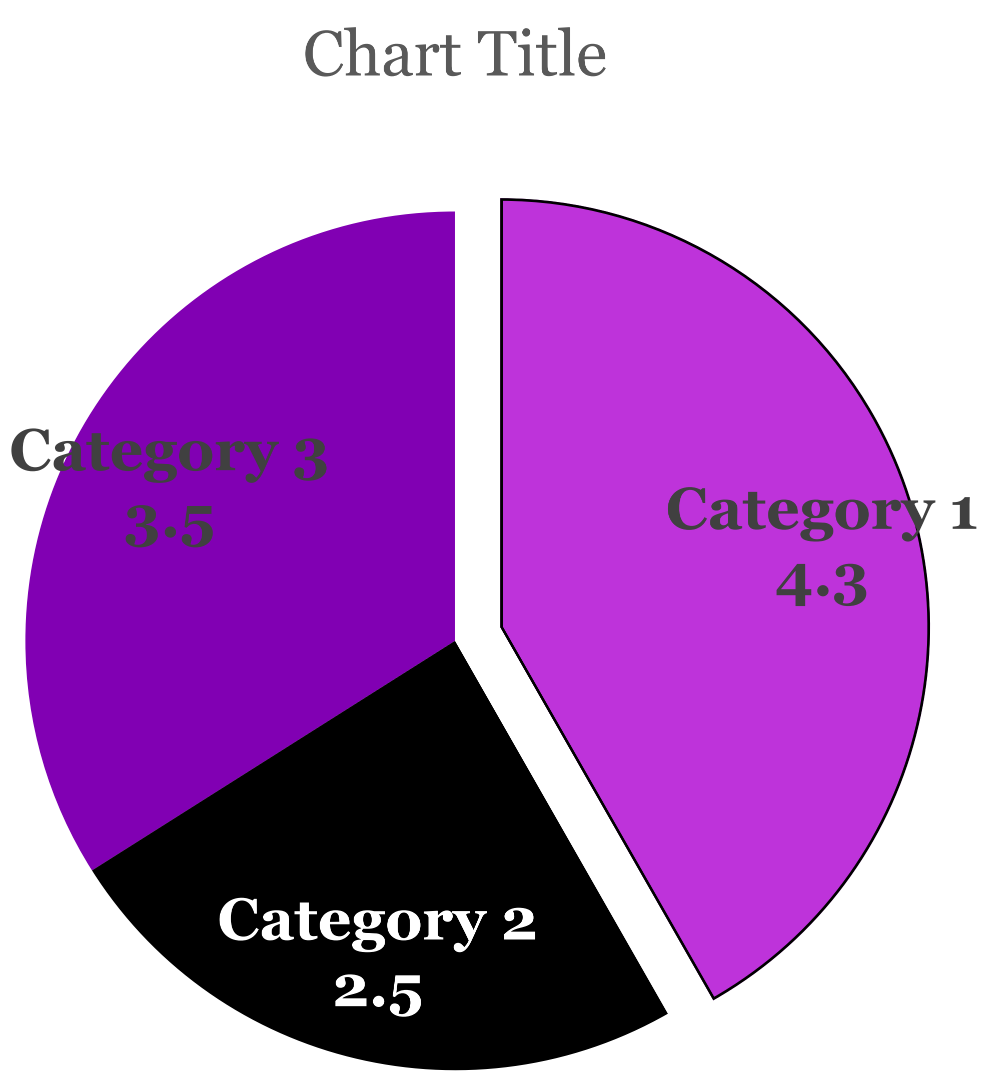

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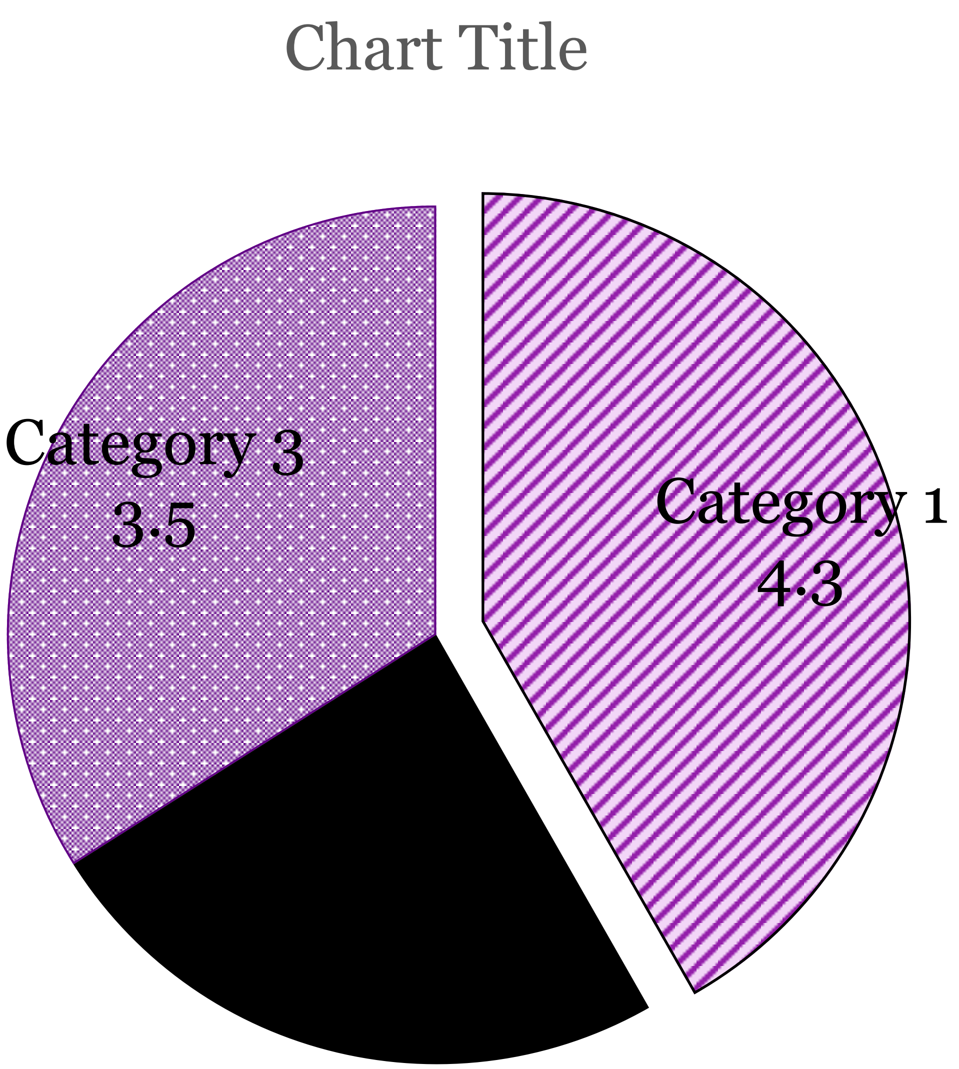

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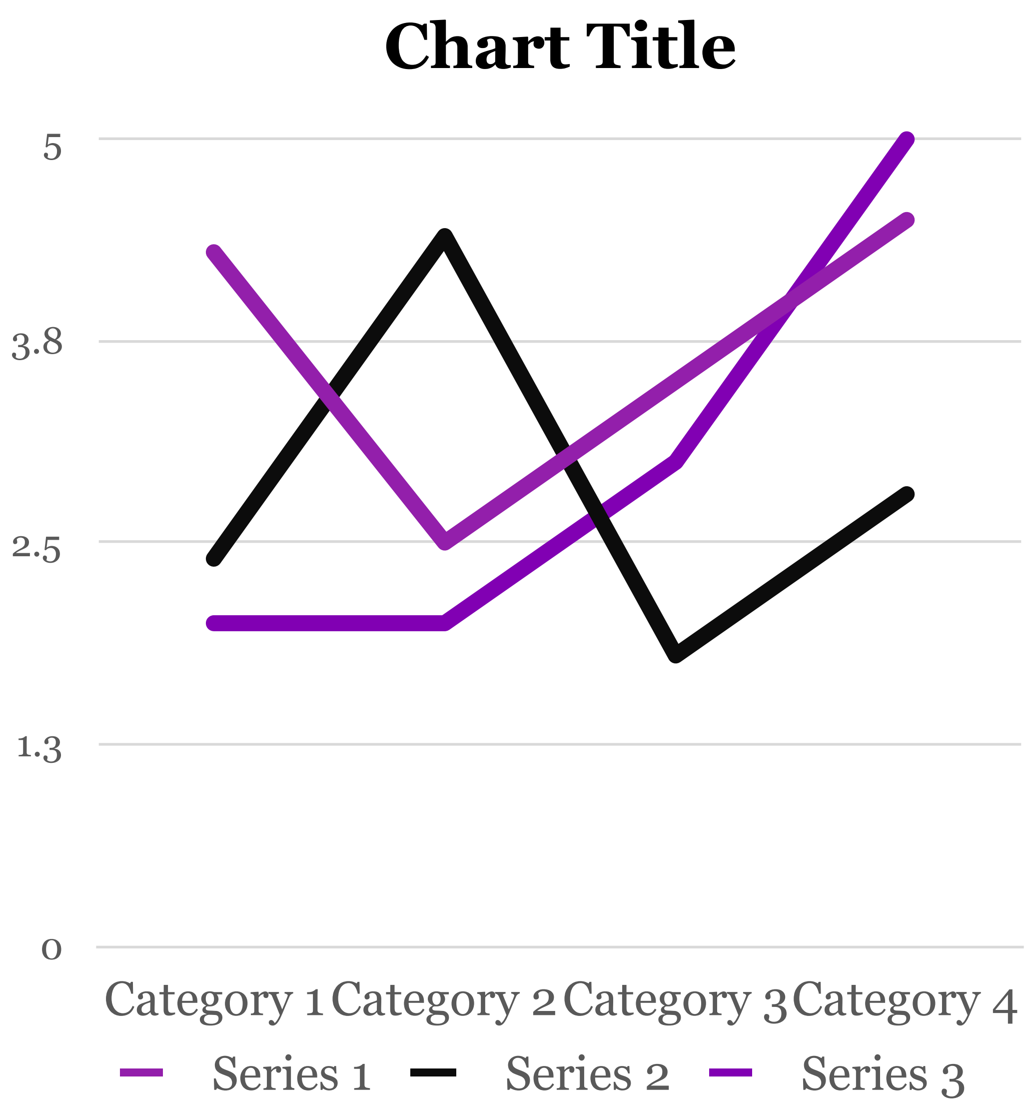
MASTER SLIDE CHART OPTION 3



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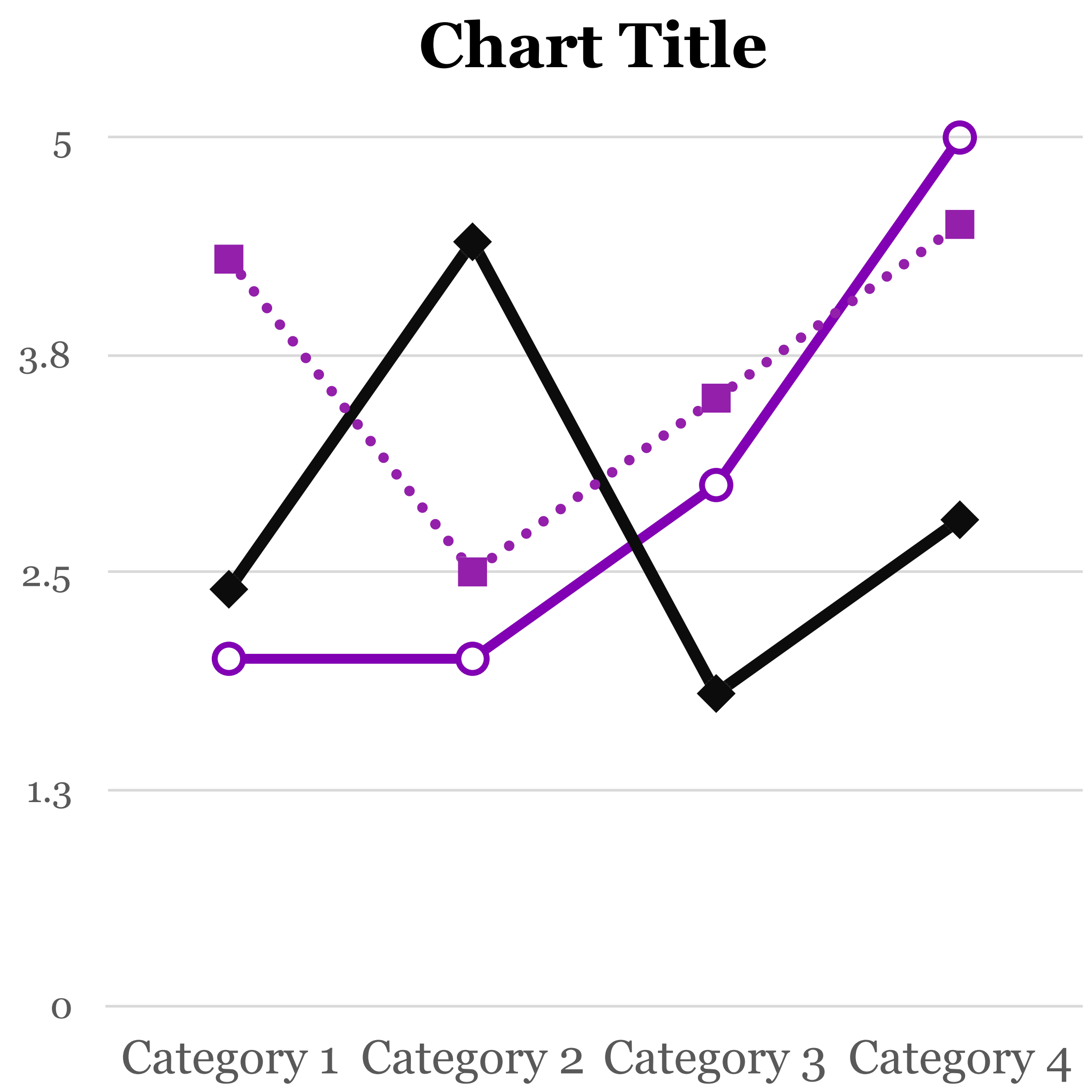
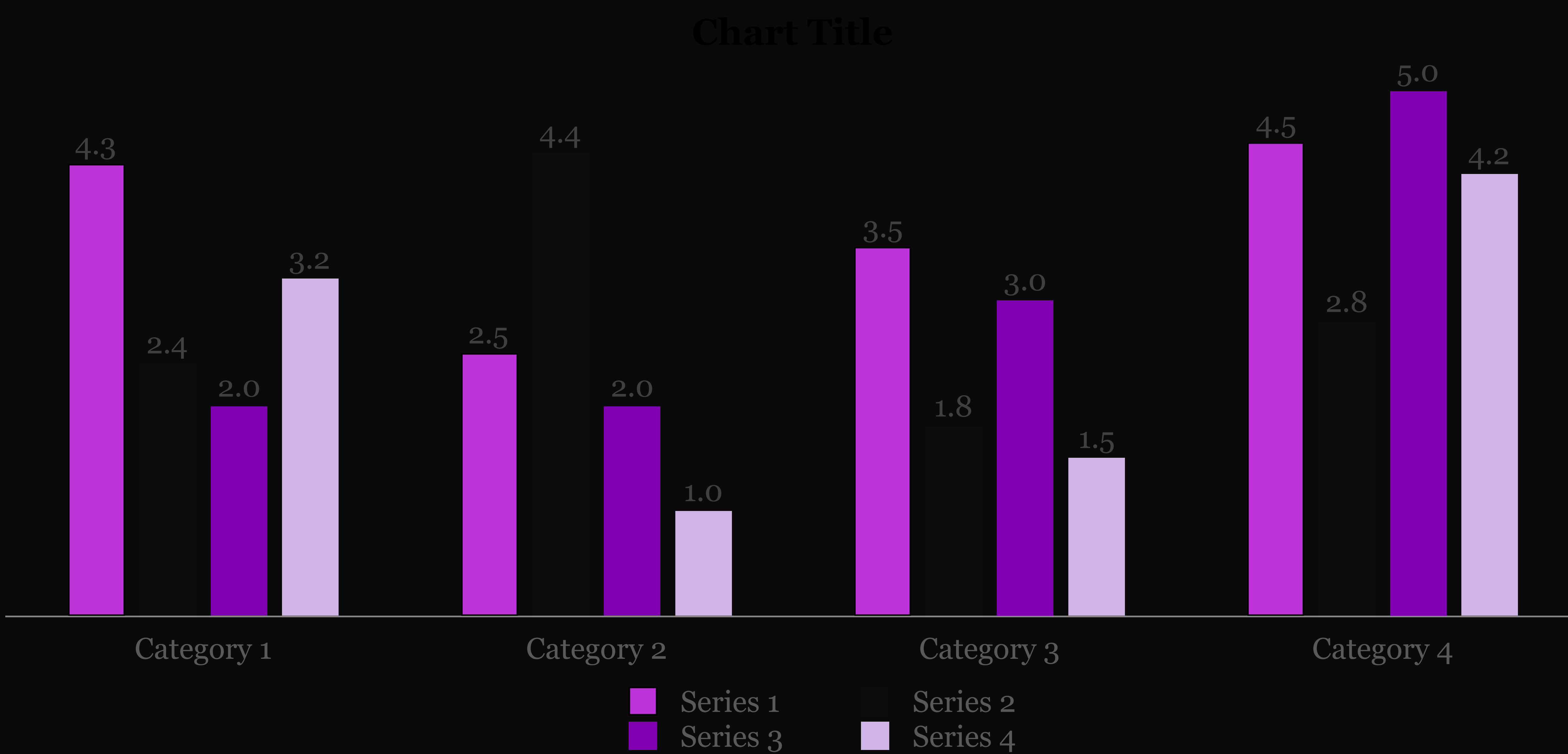
MASTER SLIDE CHART OPTION 3 (accessible version)



- Lorem ipsum dolor
  sit amet, consectetuer adipiscing elit.
  Maecenas porttitor congue massa.
- Fusce posuere, magna sed pulvinar ultricies, purus lectus malesuada libero, sit amet commodo.

MASTER SLIDE CHART OPTION 4



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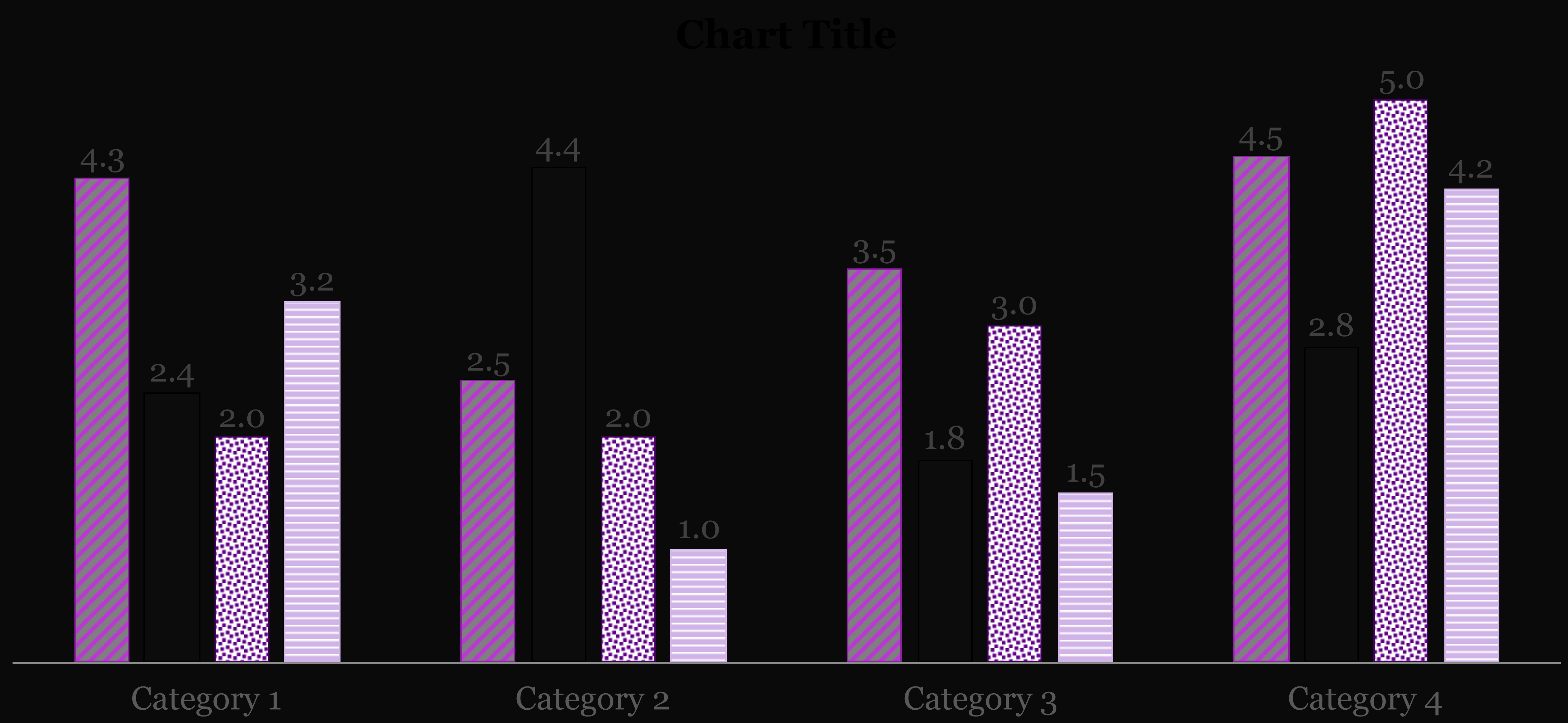
MASTER SLIDE CHART OPTION 4 (accessible version)



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MENU SYSTEM EXAMPLE

MENU SYSTEM EXAMPLE

MENU SYSTEM EXAMPLE

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