Name : Run Zeng

Student ID : 20794087

Email : r24zeng@uwaterloo.ca

Question 1

According to assignment rule and consequence rule, we can infer the program to be three constrains (as the read three parts) as follows:

$$\dfrac{\dfrac{\vdash n \geq 0 \Longrightarrow \boldsymbol{I}[0/r, 0/i, 1/p]}{\vdash \{n \geq 0\} r \coloneqq 0; i \coloneqq 0; p \coloneqq 1 \{\boldsymbol{I}\}} \qquad \dfrac{\dfrac{\dfrac{\vdash \boldsymbol{I} \wedge i \neq n \Longrightarrow \boldsymbol{I}[(r+p)/r, 2p/p, (i+1)/i]}{\vdash \{\boldsymbol{I} \wedge i \neq n\} r \coloneqq r - p; p \coloneqq 2p; r \coloneqq r + p; i \coloneqq i + 1 \{\boldsymbol{I}[r + p/r, 2p/p, i + 1/i]\}}}{\vdash \{\boldsymbol{I}\} while\ i \neq n\ do\ (r := r - p; p := 2p; i := i + 1)\{\boldsymbol{I} \wedge x = n\}} \qquad \vdash \boldsymbol{I} \wedge i = n \Longrightarrow r = 2^n - 1}{\vdash \{\boldsymbol{I}\} while\ i \neq n\ do (r \coloneqq r - p; p \coloneqq 2p; r \coloneqq r + p; i \coloneqq r + 1)\{r = 2^n - 1\}}}{\vdash \{n \geq 0\} r \coloneqq 0; i \coloneqq 0; p \coloneqq 1; while\ i \neq n\ do\ (r \coloneqq r - p; p \coloneqq 2p; r \coloneqq r + p; i \coloneqq r + 1)\{r = 2^n - 1\}}$$

where $\boldsymbol{I}$ is invariant for while loop

If $\boldsymbol{I}$ is valid to all three constrains, then this program is proved to be correct. Three constrains are as:

1. $\vdash n \geq 0 \Longrightarrow I[0/r, 0/i, 1/p]$
2. $\vdash I \wedge i \neq n \Longrightarrow I[(r+p)/r, 2p/p, (i+1)/i]$
3. $\vdash I \wedge i = n \Longrightarrow r = 2^n - 1$

Let $\boldsymbol{I} = (p = 2^i \wedge r = 2^i - 1 \wedge i \leq n)$, then:

1. $\vdash n \geq 0 \Longrightarrow p = 1 \wedge r = 0 \wedge n \geq 0$
2. $\vdash (p = 2^i \wedge r = 2^i - 1 \wedge i \leq n \wedge i \neq n \Longrightarrow p = 2^{(i+1)} \wedge r = 2^{(i+1)} - 1 \wedge (i+1) \leq n \wedge (i+1) \neq n$
3. $\vdash (p = 2^i \wedge r = 2^i - 1 \wedge i \leq n \wedge i \neq n \wedge i = n \Longrightarrow r = 2^n - 1$

All constrains are valid!

This program is correct.