Mathematical Model for Multi-Commodity Flow Problem

Sets

$$
\begin{aligned}
H &= \{\text{Pencils}, \text{Pens}\} \\
N &= \{\text{Detroit}, \text{Denver}, \text{Boston}, \text{New York}, \text{Seattle}\} \\
A &= \{(i, j) \mid i, j \in N, i \neq j\}
\end{aligned}
$$

Parameters

Capacity: $\text{capacity}_{i,j}$ represents the maximum flow that can pass through arc (i, j)
Cost: $\text{cost}_{h,i,j}$ is the cost of transporting commodity h from node i to node j
Inflow: $\text{inflow}_{h,i}$ is the supply (if positive) or demand (if negative) of commodity h at node i

Decision Variables

Flow: $\text{flow}_{h,i,j}$ represents the flow of commodity h from node i to node j

Objective Function

Minimize the total transportation cost:

$$\min \sum_{h \in H} \sum_{(i,j) \in A} \text{cost}_{h,i,j} \cdot \text{flow}_{h,i,j}$$

Constraints

$$
\begin{aligned}
&\text{Capacity Constraints:} \\
&\text{Flow Conservation Constraints:}
\end{aligned}
$$

1. Capacity Constraints:

$$\text{flow}_{h,i,j} \leq \text{capacity}_{i,j} \quad \forall h \in H, \forall (i, j) \in A$$

2. Flow Conservation Constraints:

$$\sum_{i \in N} \text{flow}_{h,i,j} + \text{inflow}_{h,j} = \sum_{k \in N} \text{flow}_{h,j,k} \quad \forall h \in H, \forall j \in N$$