Computation Economics
3E103
Doctoral Programme in Economics
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Exercise nº 1

António Neto,
PhD Student
110421011@fep.up.pt

Note: The following exercise is part of an evaluation moment of the first content in Computational Economics. The next resolution follows lecture's notes and the software used is *MatlabR2010a*.

Introduction

The Solow-Swan Model is the baseline of the Neoclassical Economic Growth Models. With simple but powerful assumptions the Solow-Swan model results are clear and consistent. Therefore this model is the common starting point for studying Economic Growth. In this paper we introduce two extensions of the model: Human Capital and Government. In the first section we explain the main assumptions and derive the differential equations. In the next section we implement the model in a computational software – *Matlab* – and solve the model for several parameters. Then (section three) we make some simulations, interpret the results and conclude the model (section four).

1. The Solow-Swan Model with Human Capital and Government[1]

1.1. Assumptions

The Solow-Swan Model with Human Capital and Government focuses in three inputs: Labor (L); Physical Capital (K) and Human Capital (H). As in the typical Solow-Swan Model, we also have *knowledge* or the *effectiveness of labor* (A). Therefore, the production function has the following form:

$$Y(t) = F\big(K(t), H(t), A(t)L(t)\big)$$

Equation 1

Concerning the production function one of the assumptions is that it has constant returns to scale in its three arguments: physical capital, human capital and effective labor. This means that:

$$F(cK, cH, cAL) = cF(K, H, AL) \; for \; all \; c \geq 0.$$

Equation 2

This assumption allows us to work with the production function in *intensive form*. Setting *c = 1/AL* in the previous equation we get:

$$F\left(\frac{K}{AL}, \frac{H}{AL}, 1\right) = \frac{1}{AL} F(K, H, AL).$$

Equation 3

Defining k = *K/AL*, *h=H/AL*, *y=Y/AL* and *f(k,h) = F(k,h,1)*, we can rewrite *equation 3* as:

$$y = f(k, h)$$

Equation 4

where *k* is the amount of physical capital per unit of effective labor; *h* is the amount of human capital per unit of effective labor and *y* is the output per unit of effective labor.

[1] The next analysis follows the one that was made by David Romer for the typical Solow-Swan Model with only one type of capital and without government.

Moreover the intensive-form production function is assumed to satisfy the following typical assumptions:

a) $f(0) = 0$
b) $f'(k) > 0\,; f'(h) > 0$
c) $f''(k) < 0\,; f''(h) < 0$
d) $\lim_{k\to 0} f'(k) = \infty\,;\ \lim_{h\to 0} f'(h) = \infty\,;$
e) $\lim_{k\to \infty} f'(k) = 0\,;\ \lim_{h\to \infty} f'(h) = 0\,;$

The first three assumptions imply that the marginal product of the physical (and human) capital is positive, but that it declines as physical (and human) per unit of effective labor rises. The last two ones are known as the *Inada Conditions* and ensure that the path of the economy does not diverge.

In our model we use the typical Cobb-Douglas production function:

$$F(K,H,AL) = K^{\alpha}H^{\beta}[AL]^{1-\alpha-\beta}, \quad 0<\alpha<1; \quad 0<\beta<1; \quad \alpha+\beta<1;$$

Equation 5

This function has constant returns to scale,

$$F(cK,cH,cAL) = (cK)^{\alpha}(cH)^{\beta}(cAL)^{1-\alpha-\beta} = c^{\alpha}c^{\beta}c^{1-\alpha-\beta}F(K,H,AL)$$
$$\leftrightarrow$$
$$F(cK,cH,cAL) = cF(K,H,AL)$$

Equation 6

And the intensive form of the production function is the following:

$$\frac{Y}{AL} = \frac{K^{\alpha}H^{\beta}[AL]^{1-\alpha-\beta}}{AL} = K^{\alpha}H^{\beta}[AL]^{-\alpha-\beta} = \frac{K^{\alpha}H^{\alpha}}{[AL]^{\alpha}[AL]^{\beta}}$$

$$y = k^{\alpha}h^{\beta}$$

Equation 7

The previous equation implies that:

$$\begin{cases} f'(k) = \alpha k^{\alpha-1}h^{\beta} > 0 \\ f'(h) = \beta k^{\alpha}h^{\beta-1} > 0 \\ f''(k) = -(1-\alpha)\alpha k^{\alpha-2}h^{\beta} < 0 \\ f''(k) = -(1-\beta)\alpha k^{\alpha}h^{\beta-2} < 0 \end{cases}$$

Therefore the Cobb-Douglas production function fulfills all the necessary assumptions that we need to study the model.

1.2. The Evolution of the Inputs into Production

The next step is to understand how the stocks of labor, knowledge, physical capital and human capital. The model is set in continuous time and the initial levels of physical capital, human capital, labor and knowledge are taken as given. Labor and knowledge grow at constant rates, where *n* and *g* are exogenous parameters:

$$\dot{L}(t) = nL(t),$$

Equation 8

$$\dot{A}(t) = aA(t).$$

Equation 9

Output is divided between consumptions, investment (in physical capital and human capital) and government spending:

$$Y(t) = C(t) + I(t) + G(t)$$

Equation 10

Moreover the fraction of output devoted to investment in physical capital, *s*, and human capital, *w*, are exogenous and constant. The same rule is applied to the physical capital tax rate, *g*, and to the human capital tax rate, *j*. Additionally we also consider a depreciation rate for the physical capital, *d*, and for the human capital, *c*. Then:

$$\dot{K}(t) = sF(K,L) - dK(t) = s\big(Y(t) - T(t)\big) - dK(t) = s\big(Y(t) - gY(t)\big) - dK(t)$$

$$\dot{K}(t) = s(1-g)Y - dK$$

Equation 11

$$\dot{H}(t) = wF(K,L) - cH(t) = w\big(Y(t) - T(t)\big) - cH(t) = w\big(Y(t) - jY(t)\big) - cK(t)$$

$$\dot{H}(t) = w(1-j)Y - cH$$

Equation 12

1.3. The Dynamics of the Model

As in the typical Solow-Swan model we assume that the evolution of labor and knowledge is exogenous. Thus to characterize the behavior of the economy we only need to analyze the behavior of the physical and human capital.

1.3.1. The Dynamics of k and h

In intensive form, the equation of motion for physical capital is given by:

$$\dot{k(t)} = \frac{dk(t)}{dt} = \frac{\frac{dK(t)}{dt}A(t)L(t) - \frac{dA(t)L(t)}{dt}K(t)}{(A(t)L(t))^2}$$

$$\dot{k(t)} = \frac{\dot{K}(t)A(t)L(t) - \left(\frac{dA(t)}{dt}L(t) + \frac{dL(t)}{dt}A(t)\right)K(t)}{(A(t)L(t))^2}$$

$$\dot{k(t)} = \dot{K}(t)\frac{1}{A(t)L(t)} - \left[\frac{\dot{A}(t)L(t) + \dot{L(t)}A(t)}{(A(t)L(t))^2}\right]K(t)$$

$$\dot{k(t)} = \frac{\dot{K(t)}}{K(t)}\frac{K(t)}{A(t)L(t)} - \frac{K(t)}{(A(t)L(t))^2}\left[\frac{\dot{A(t)}}{A(t)}A(t)L(t) + \frac{\dot{L(t)}}{L(t)}A(t)L(t)\right]$$

$$\dot{k(t)} = \frac{\dot{K}(t)}{K(t)}\frac{K(t)}{A(t)L(t)} - \frac{K(t)}{A(t)L(t)}\left[\frac{\dot{A(t)}}{A(t)} + \frac{\dot{L(t)}}{L(t)}\right]$$

$$\dot{k(t)} = \frac{\dot{K(t)}}{A(t)L(t)} - (a+n)k(t)$$

Equation 13

Then, replacing *equation 11* in *equation 13* we get:

$$\dot{k}(t) = \frac{s(1-g)Y(t) - dK(t)}{A(t)L(t)} - (a+n)k(t) = s(1-g)y(t) - dk(t) - (a+n)k(t)$$

$$\dot{k}(t) = s(1-g)y(t) - (a+n+d)k(t)$$

Equation 14

Similarly, the equation of motion for human capital is:

$$\dot{h}(t) = \frac{w(1-j)Y(t) - cH(t)}{A(t)L(t)} - (a+n)h(t) = w(1-j)y(t) - ch(t) - (a+n)h(t)$$

$$\dot{h}(t) = w(1-j)y(t) - (a+n+c)h(t)$$

Equation 15

Finally, replacing *equation 4* in *equations 14* and *15*, we obtain the following results:

$$\dot{k}(t) = s(1-g)f(k(t),h(t)) - (a+n+d)k(t)$$

Equation 16

$$\dot{h}(t) = w(1-j)f(k(t),h(t)) - (a+n+c)h(t)$$

Equation 17

Equations 16 and *17* are the *key equations* of the Solow-Swan Model with Human Capital and Government. They state that the rate of change of the physical capital / human capital stock per unit of effective labor is the difference between two terms. The first, $s(1-g)f(k,h)$, [or $w(1-j)f(k,h)$ for the human capital] is the actual investment per unit of effective labor; output net per unit of effective labor is $(1-g)f(k,h)$, [or $(1-j)f(k,h)$ for the human capital] and the fraction of that output that is invested is s [or w for the human capital]. The second term, $(a+n+d)k$ [or $(a+n+c)h$ for the human capital], is the *break-even investment*, the amount of investment that must be done just to keep at its existing level.

1.4. The Steady-State

In a steady-state, physical and human capital per effective worker must be constant. This implies that we can solve for the steady-state by finding the values for k and h which set the above equations of motion to zero (other than the trivial steady-state given by setting either k or h equal to zero).[2] This means:

$$\begin{cases} \dot{k}(t) = 0 <=> s(1-g)f\big(k(t),h(t)\big) = (a+n+d)k(t) \\ \dot{h}(t) = 0 <=> w(1-j)f\big(k(t),h(t)\big) = (a+n+c)h(t) \end{cases}$$

Equation 18

Therefore, by using the Cobb-Douglas production function we obtain:

$$w(1-j)k(t)^{\alpha}h(t)^{\beta} = (a+n+c)h(t)$$

$$h(t)^{\beta-1} = \left[\frac{a+n+c}{w(1-j)}\right]k(t)^{-\alpha}$$

$$h(t) = \left[\frac{w(1-j)}{a+n+c}\right]^{\left(\frac{1}{1-\beta}\right)} k(t)^{\left(\frac{\alpha}{1-\beta}\right)}$$

Equation 19

[2] Bluedorn, J. , *Macroeconomic Theory*

Then, we substitute this expression into the other steady-state condition, and solve for *k*.

$$s(1-g)f\big(k(t),h(t)\big) = (a+n+d)k(t)$$

$$s(1-g)k(t)^{\alpha}\left[\left[\frac{w(1-j)}{a+n+c}\right]^{\frac{1}{1-\beta}} k(t)^{\frac{\alpha}{1-\beta}}\right]^{\beta} = (a+n+d)k(t)$$

$$k(t)^{\alpha-1}\left[\frac{w(1-j)}{a+n+c}\right]^{\frac{\beta}{1-\beta}} k(t)^{\frac{\alpha\beta}{1-\beta}} = \left[\frac{a+n+d}{s(1-g)}\right]$$

$$k(t)^{\frac{\alpha+\beta-1}{1-\beta}} = \left[\frac{s(1-g)}{a+n+d}\right]^{-1}\left[\frac{w(1-j)}{a+n+c}\right]^{\frac{-\beta}{1-\beta}}$$

$$k^{*}(t) = \left[\frac{s(1-g)}{a+n+d}\right]^{\frac{1-\beta}{1-\alpha-\beta}}\left[\frac{w(1-j)}{a+n+c}\right]^{\frac{\beta}{1-\alpha-\beta}}$$

Equation 20

The asterisk denotes the steady-state value of a variable. Now we can substitute this result into the expression for h (*equation 19*).

$$h^{*}(t) = \left[\frac{w(1-j)}{a+n+c}\right]^{\frac{1}{1-\beta}}\left[\left[\frac{s(1-g)}{a+n+d}\right]^{\frac{1-\beta}{1-\alpha-\beta}}\left[\frac{w(1-j)}{a+n+c}\right]^{\frac{\beta}{1-\alpha-\beta}}\right]^{\frac{\alpha}{1-\beta}}$$

$$h^{*}(t) = \left[\frac{w(1-j)}{a+n+c}\right]^{\frac{1}{1-\beta}}\left[\frac{s(1-g)}{a+n+d}\right]^{\frac{\alpha(1-\beta)}{(1-\alpha-\beta(1-\beta))}}\left[\frac{w(1-j)}{a+n+c}\right]^{\frac{\alpha\beta}{(1-\alpha-\beta)(1-\beta)}}$$

$$h^{*}(t) = \left[\frac{w(1-j)}{a+n+c}\right]^{\frac{1-\alpha-\beta}{(1-\alpha-\beta)(1-\beta)}+\frac{\alpha\beta}{(1-\alpha-\beta)(1-\beta)}}\left[\frac{s(1-g)}{a+n+d}\right]^{\frac{\alpha}{1-\alpha-\beta}}$$

$$h^{*}(t) = \left[\frac{w(1-j)}{a+n+c}\right]^{\frac{1-\alpha}{1-\alpha-\beta}}\left[\frac{s(1-g)}{a+n+d}\right]^{\frac{\alpha}{1-\alpha-\beta}}$$

Equation 21

Therefore, *equations 21* and *22* stand for the steady-state values of physical and human capital.

By the relation between the two variables we expected that the *Phase Diagram* take the following graph:

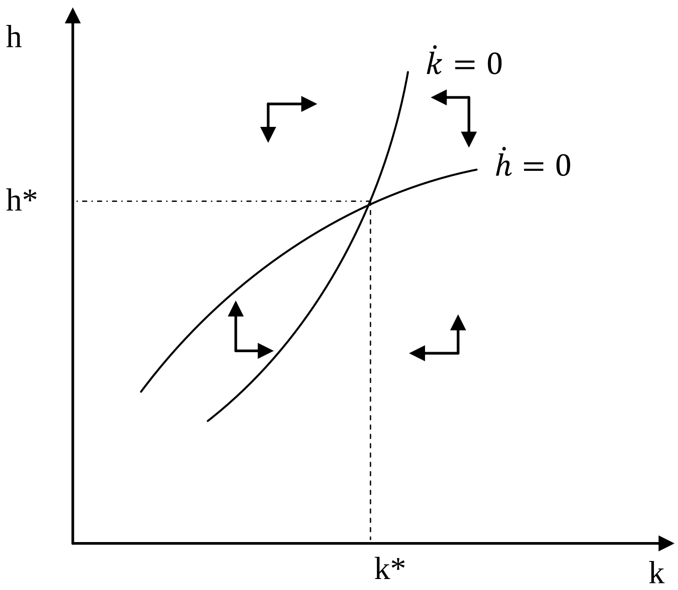


Figure 1

Finally, we can determinate y*:

$$y(t)^* = k^*(t)^\alpha h^*(t)^\beta$$

$$y^*(t) = \left[\left[\frac{s(1-g)}{a+n+d}\right]^{\frac{1-\beta}{1-\alpha-\beta}} \left[\frac{w(1-j)}{a+n+c}\right]^{\frac{\beta}{1-\alpha-\beta}}\right]^\alpha \left[\left[\frac{w(1-j)}{a+n+c}\right]^{\frac{1-\alpha}{1-\alpha-\beta}} \left[\frac{s(1-g)}{a+n+d}\right]^{\frac{\alpha}{1-\alpha-\beta}}\right]^\beta$$

$$y(t)^* = \left[\frac{s(1-g)}{a+n+d}\right]^{\frac{\alpha}{1-\alpha-\beta}} \left[\frac{w(1-j)}{a+n+c}\right]^{\frac{\beta}{1-\alpha-\beta}}$$

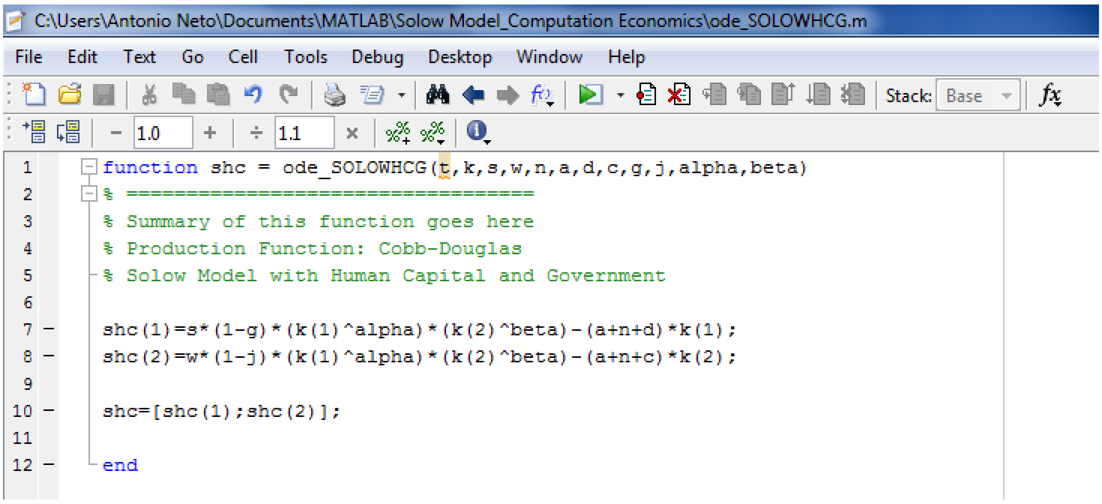
Equation 22

This last expression finishes the section about the explanation and derivation of the Solow-Swan Model with Human Capital and Government. We are now able to understand the model and implement it in the software.

2. The Model in *Matlab*

2.1. The *ode* function

In order to introduce the model first we need to create a specific type of function: the *ode* function. This function allows us to get the steady-state values for the physical and human capital. For this we only need to compute the equations *16* and *17*, the differential equations.




```
function shc = ode_SOLOWHCG(t,k,s,w,n,a,d,c,g,j,alpha,beta)
% ===================================
% Summary of this function goes here
% Production Function: Cobb-Douglas
% Solow Model with Human Capital and Government

shc(1)=s*(1-g)*(k(1)^alpha)*(k(2)^beta)-(a+n+d)*k(1);
shc(2)=w*(1-j)*(k(1)^alpha)*(k(2)^beta)-(a+n+c)*k(2);

shc=[shc(1);shc(2)];

end
```

Figure 2

Notice that *k(1)* stands for the physical capital (k) and *k(2)* stands for the human capital (h). All the other variables have the same name.

2.2. The *script*

The *ode* function is all we need if we only want to find the steady-state values for one specific economy. We just give values for the parameters and the function will do all the work. However, if we want to study several situations with different parameters it is more efficient to create also a *script*. Basically the *script* allows us to interact with the function and choose different parameters, do different graphs / plots among other things. In the next section we present some of the functionalities of our *script*. In other words, we show how the model works in computational terms[3].

2.3. The Model

When we execute the *script* (named by *Solow_EC*) the first thing we get is the *Main Menu* and a short introduction to the model.

[3] In appendix we also present all the commands we use to build the *script*.

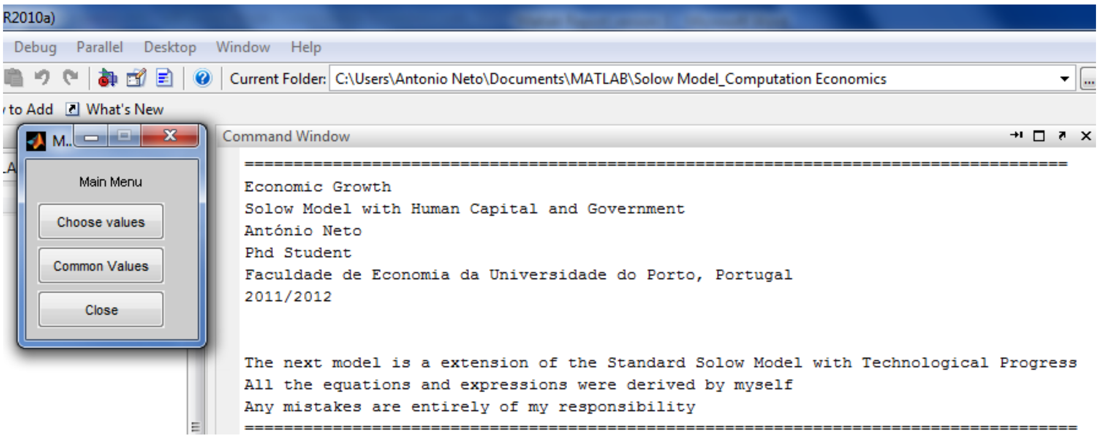


Figure 3

Then, in our to proceed we can selected one of two options: if we want to study a particular case with specific values for the parameters we select the option *Choose Values*; however, if we want to see how the model works in general we can select the second option (*Common values*) and accept the *default values*. Independently of the option we choose, the next *Menu* is exactly the same. *Figure 4* shows us the first option (where we input some random values) and *figure 5* shows us the second option.

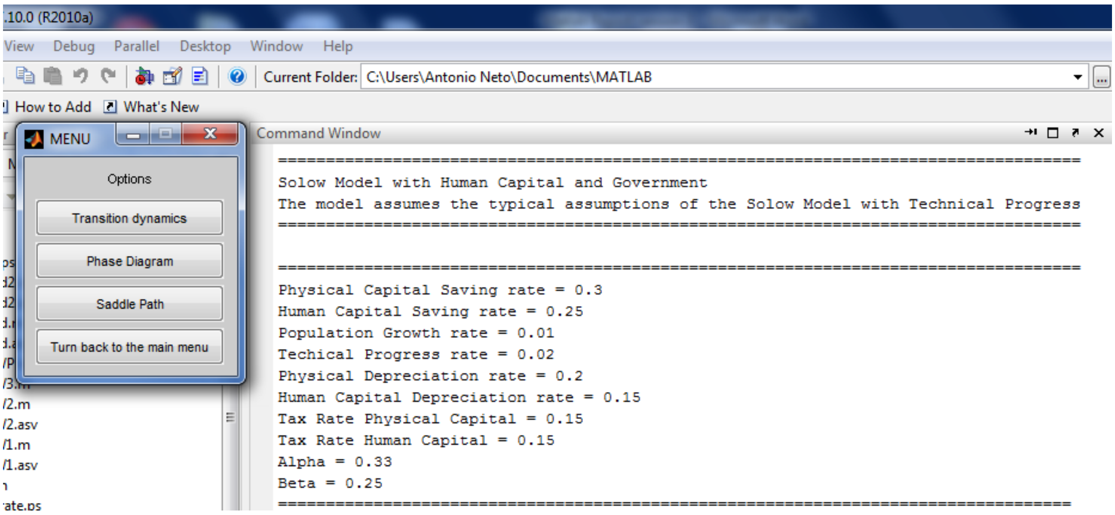


Figure 4

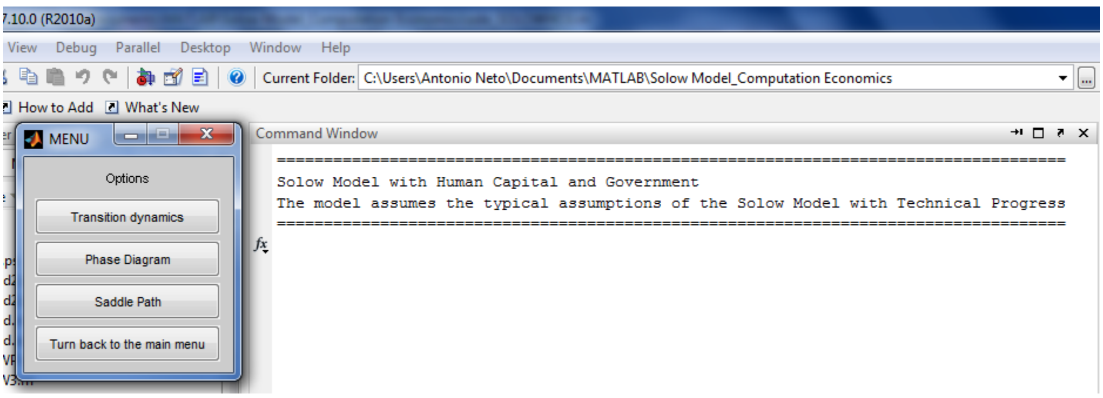


Figure 5

As we already said, the *Options Menu* is the same for both possibilities. Therefore, in the following analyzes we only present the results of the *Common Values*. The first option shows us the *plot* that the *ode function* would provide us without any *script*. This plot is called the *Transition Dynamics*.

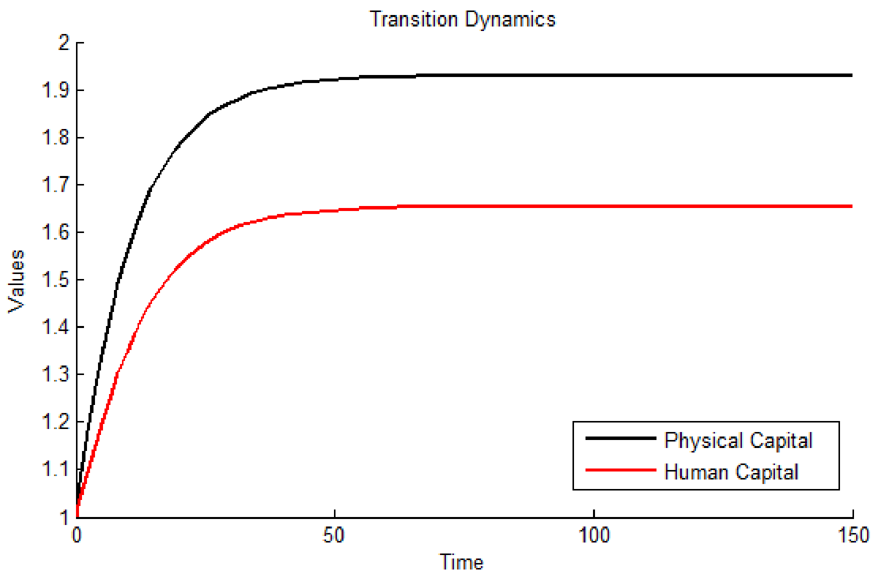


Figure 6

The second option corresponds to the *Phase Diagram* that we derived in the previous section (*figure 1*).

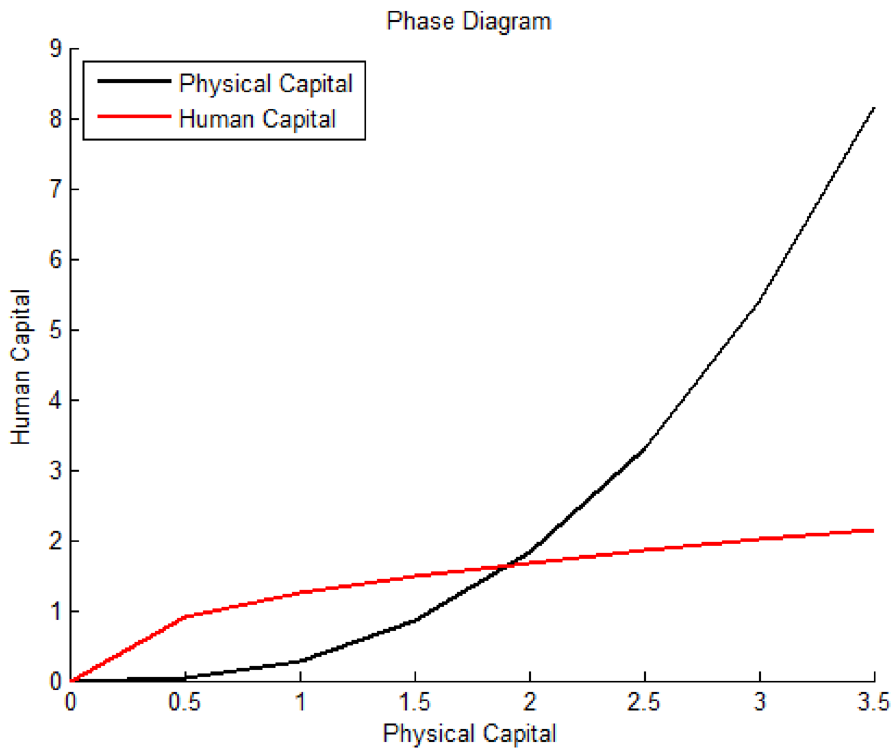


Figure 7

Finally, the third option shows us the *Saddle Path* for the inputs (physical capital per unit of effective labor and human capital per unit of effective labor) and for the output per unit of effective labor. Since we have two inputs, the last plot is three-dimensional.

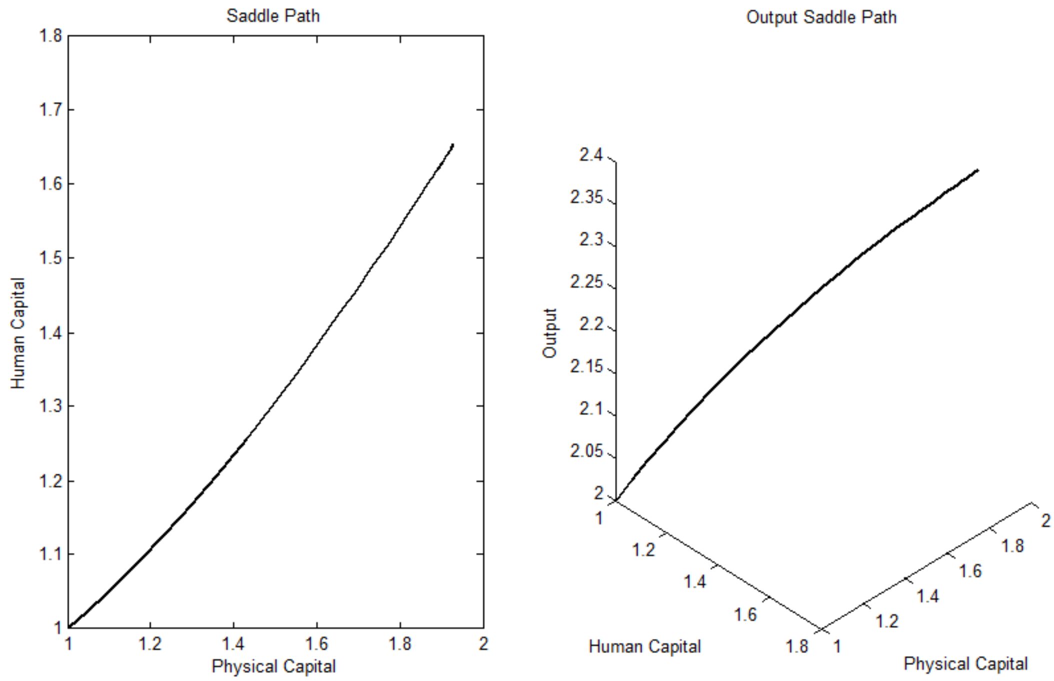


Figure 8

3. Simulations and shocks

In the last section we did not tell the entire story. In fact, after we select an option, a plot and a question appears.

```
Command Window
================================================================================
Solow Model with Human Capital and Government
The model assumes the typical assumptions of the Solow Model with Technical Progress
================================================================================

fx Would you like to perform a shock in one of the parameters? (y/n) - |
```

Figure 9

This question allows us to change some parameters in our initial economy and analyze the results. In the typical Solow-Swan Model the most typical examples are changes in the saving and depreciation rate. Therefore, we will use these examples in our simulations: we consider a change in the physical capital saving rate and a change in the human capital depreciation rate.

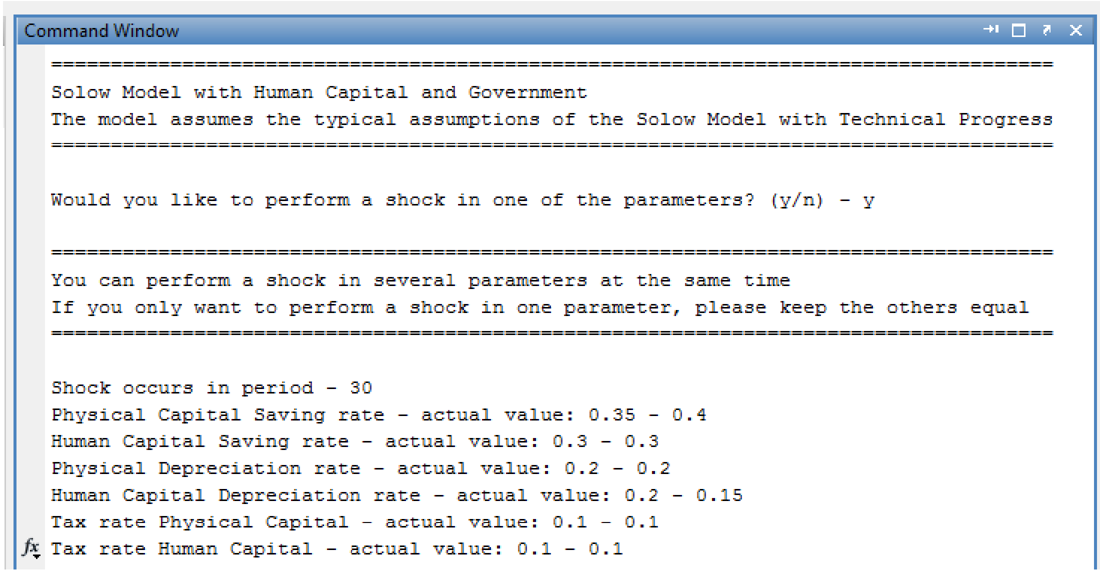


Figure 10

Notice that we can also choose the period when the shock occurs. In our case, since we choose the *period 30* the economy is still not in the steady-state (*figure 6*). This means that the shock will lead to a new steady-state without even reaching the previous one. The results are shown in the following figure.

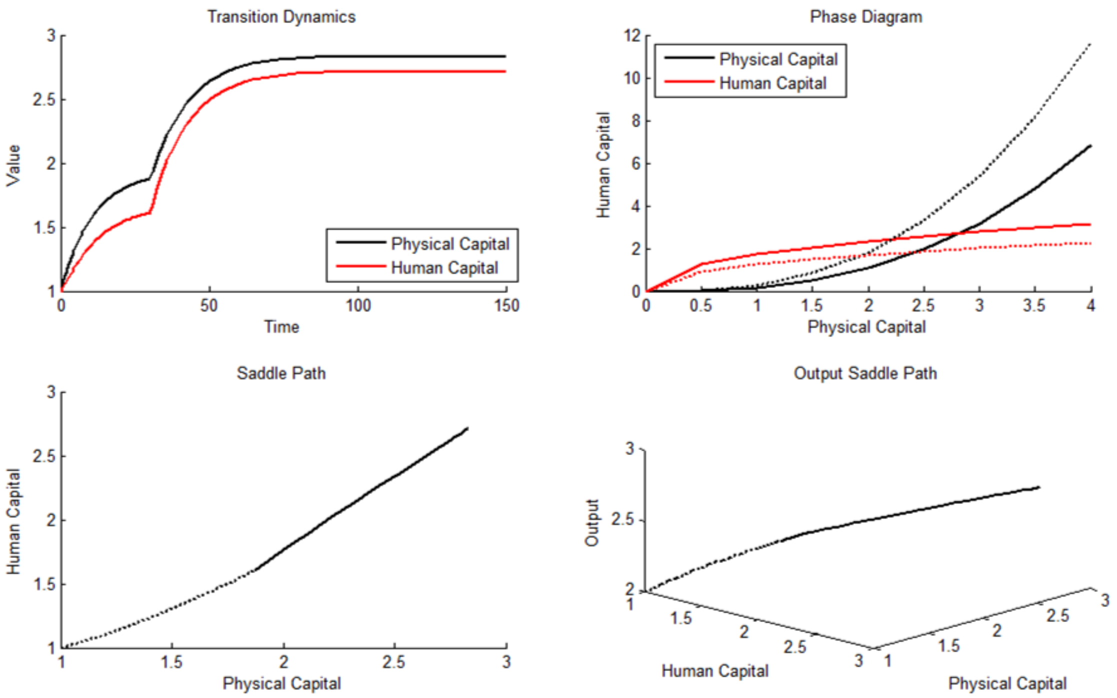


Figure 11

Notice that in the first plot (*Transition Dynamics*) in the moment when the shock occurs both lines were not horizontal which means that physical and human capital were not in the steady-state. The second one is more similar to the typical graph. The solid lines represent the new equilibrium. Finally, the last two plots show us the old (dot line) and the new (solid line) Saddle Path for the inputs and output.

4. Conclusion

The Solow-Swan Model with Human Capital and Government is a powerful extension of the typical Solow-Swan model that allows us to understand and fit better the empirical evidence. Therefore, the introduction of the model in a computational software (*matlab*) is a helpful tool in Economic Growth research since we can analyze different economies and simulate different shocks in a faster and efficient way.

5. Appendix

```
clc
disp('==============================================================')
disp('Economic Growth')
disp('Solow Model with Human Capital and Government')
disp('António Neto')
disp('Phd Student')
disp('Faculdade de Economia da Universidade do Porto, Portugal')
disp('2011/2012')
disp(' ')
disp(' ')
disp('The next model is a extension of the Standard Solow Model with
Technological Progress')
disp('All the equations and expressions were derived by myself')
disp('Any mistakes are entirely of my responsibility')
disp('==============================================================')
fij = 2;
while fij~=1;
Menu = menu ('Main Menu',...
    'Choose values','Common Values','Close');
    if Menu == 1
    clc
disp('==============================================================')
    disp('Solow Model with Human Capital and Government');
    disp('The model assumes the typical assumptions of the Solow Model
with Technical Progress')
disp('==============================================================')
    %Input the values of the parameters
    disp(' ')
    disp
('==================================================================')
    s = input('Physical Capital Saving rate = ');
    w = input('Human Capital Saving rate = ');
    n = input('Population Growth rate = ');
    a = input('Techical Progress rate = ');
    d = input('Physical Depreciation rate = ');
    c = input('Human Capital Depreciation rate = ');
    g = input('Tax Rate Physical Capital = ');
    j = input('Tax Rate Human Capital = ');
    alpha = input('Alpha = ');
    beta = input('Beta = ');
    disp
('==================================================================')
    %initial values and time (defining tow the initial values k=1)
    init_values=[1;1];
    %Solve the system of ODE's
    time_period=[0 150];
```

```
    [t , k] =
ode45(@ode_SOLOWHCG,time_period,init_values,[],s,w,n,a,d,c,g,j,alpha,b
eta);
      fi=2;
        while fi~=1
            M1= menu('Options','Transition dynamics','Phase
Diagram','Saddle Path','Turn back to the main menu');
            if M1 == 1
                hold on
                plot(t,k(:,1),'k-','LineWidth',2);
                plot(t,k(:,2),'r-','LineWidth',2);
                legend('Physical Capital','Human Capital',2);
                disp(' ');
                i = input('Would you like to perform a shock in one of
the parameters? (y/n) - ','s');
            elseif M1 == 2
                hold off
               x = 0:0.5:3.5;
               z = ((((w*(1-j)))/(a+n+c))^(1/(1-beta)))*x.^(alpha/(1-beta));
               y = ((a+n+d)/(s*(1-g)))^(1/beta)*x.^((1-alpha)/beta);
               hold on
               plot(x,y,'k-','LineWidth',2);
               plot(x,z,'r-','LineWidth',2);
               legend('Physical Capital','Human Capital',2);
               xlabel('Physical Capital')
               ylabel('Human Capital')
               title('Phase Diagram')
               disp(' ');
               i = input('Would you like to perform a shock in one of
the parameters? (y/n) - ','s');
            elseif M1 == 3
                hold off
                subplot (1,2,1)
                plot(k(:,1),k(:,2),'k-','LineWidth',2);
                xlabel('Physical Capital')
                ylabel('Human Capital')
                title('Saddle Path')
                yy = (k(:,1).^alpha + k(:,2).^beta);
                subplot (1,2,2)
                plot3(k(:,1),k(:,2),yy,'k-','LineWidth',2);
                xlabel('Physical Capital')
                ylabel('Human Capital')
                zlabel('Output')
                title('Output Saddle Path')
                disp(' ');
                i = input('Would you like to perform a shock in one of
the parameters? (y/n) - ','s');
            else
                i = 'n';
                fi = 1;
            end
        if i == 'y'
            disp ('  ')
            disp
('===================================================================')
            disp ('You can perform a shock in several parameters at
the same time')
            disp ('If you only want to perform a shock in one
parameter, please keep the others equal')
            disp
('===================================================================')
```

```
disp ('  ')
f = input ('Shock occurs in period - ');
ss = input ('Physical Capital Saving rate - ');
ww = input ('Human Capital Saving rate - ');
dd = input ('Physical Depreciation rate - ');
cc = input ('Human Capital Depreciation rate - ');
gg = input ('Tax rate Physical Capital - ');
jj = input ('Tax rate Human Capital - ');
%Phase Diagram
x = 0:0.5:4;
y = ((a+n+dd)/(ss*(1-gg)))^(1/beta)*x.^((1-alpha)/beta);
z = ((((ww*(1-jj)))/(a+n+cc))^(1/(1-beta)))*x.^(alpha/(1-beta));
yy = ((a+n+d)/(s*(1-g)))^(1/beta)*x.^((1-alpha)/beta);
zz = ((((w*(1-j)))/(a+n+c))^(1/(1-beta)))*x.^(alpha/(1-beta));
subplot(2,2,2)
hold on
plot(x,y,'k-','LineWidth',2);
plot(x,z,'r-','LineWidth',2);
plot(x,yy,'k:','LineWidth',2);
plot(x,zz,'r:','LineWidth',2);
legend('Physical Capital','Human Capital', 2);
xlabel('Physical Capital')
ylabel('Human Capital')
title('Phase Diagram')
%Transition Dinamics
init_values = [1;1];
time_period=[0 f];
[t,k] =
ode45(@ode_SOLOWHCG,time_period,init_values,[],s,w,n,a,d,c,g,j,alpha,beta);
subplot(2,2,1)
hold on
plot(t,k(:,1),'k-','LineWidth',2);
plot(t,k(:,2),'r-','LineWidth',2);
s = ss;
w = ww;
d = dd;
c = cc;
g = gg;
j = jj;
zz=k(end,1);
ff=k(end,2);
init_values=[zz;ff];
time_period=[f 150];
[t,kk] =
ode45(@ode_SOLOWHCG,time_period,init_values,[],s,w,n,a,d,c,g,j,alpha,beta);
hold on
plot(t,kk(:,1),'k-','LineWidth',2);
plot(t,kk(:,2),'r-','LineWidth',2);
legend('Physical Capital','Human Capital',2);
xlabel('Time')
ylabel('Value')
title('Transition Dynamics')
subplot (2,2,3)
hold on
plot(k(:,1),k(:,2),'k:','LineWidth',2);
plot(kk(:,1),kk(:,2),'k-','LineWidth',2);
xlabel('Physical Capital')
ylabel('Human Capital')
title('Saddle Path')
yy = (k(:,1).^alpha + k(:,2).^beta);
yyy = (kk(:,1).^alpha + kk(:,2).^beta);
subplot (2,2,4)
```

```
            plot3(k(:,1),k(:,2),yy,'k:','LineWidth',2);
            hold on
            plot3(kk(:,1),kk(:,2),yyy,'k-','LineWidth',2);
            xlabel('Physical Capital')
            ylabel('Human Capital')
            zlabel('Output')
            title('Output Saddle Path')
        end
        if i == 'n'
        end
        end
    elseif Menu == 2
      fj = 2;
        while fj ~= 1
        clc
disp('==============================================================')
        disp('Solow Model with Human Capital and Government');
        disp('The model assumes the typical assumptions of the Solow
Model with Technical Progress')

disp('==============================================================')
        s = 0.35;
        w = 0.3;
        n = 0.01;
        a = 0.02;
        d = 0.2;
        c = 0.2;
        g = 0.1;
        j = 0.1;
        alpha = 0.33;
        beta = 0.25;
        %initial values and time (defining tow the initial values k=1)
        init_values=[1;1];
        %Solve the system of ODE's
        time_period=[0 150];
        [t , k] =
ode45(@ode_SOLOWHCG,time_period,init_values,[],s,w,n,a,d,c,g,j,alpha,beta);
        M2= menu('Options','Transition dynamics','Phase
Diagram','Saddle Path','Turn back to the main menu');
            if M2 == 1
                hold on
                p=plot(t,k(:,1),'k-','LineWidth',2);
                plot(t,k(:,2),'r-','LineWidth',2);
                legend('Physical Capital','Human Capital',2);
                xlabel('Time')
                ylabel('Values')
                title('Transition Dynamics')
                disp(' ');
                i = input('Would you like to perform a shock in one of
the parameters? (y/n) - ','s');
            elseif M2 == 2
                hold off
                x = 0:0.5:3.5;
                y =((a+n+d)/(s*(1-g)))^(1/beta)*x.^((1-alpha)/beta);
                z=((((w*(1-j)))/(a+n+c))^(1/(1-beta)))*x.^(alpha/(1-beta));
                hold on
                plot(x,y,'k-','LineWidth',2);
                plot(x,z,'r-','LineWidth',2);
                legend('Physical Capital','Human Capital',2);
                xlabel('Physical Capital')
                ylabel('Human Capital')
                title('Phase Diagram')
```

```
                disp(' ');
                i = input('Would you like to perform a shock in one of
the parameters? (y/n) - ','s');
            elseif M2 == 3
                hold off
                subplot (1,2,1)
                plot(k(:,1),k(:,2),'k-','LineWidth',2);
                xlabel('Physical Capital')
                ylabel('Human Capital')
                title('Saddle Path')
                yy = (k(:,1).^0.33 + k(:,2).^0.25);
                subplot (1,2,2)
                plot3(k(:,1),k(:,2),yy,'k-','LineWidth',2);
                xlabel('Physical Capital')
                ylabel('Human Capital')
                zlabel('Output')
                title('Output Saddle Path')
                disp(' ');
                i = input('Would you like to perform a shock in one of
the parameters? (y/n) - ','s');
            else
                i = 'n';
                fj = 1;
            end
        if i == 'y'
            disp ('  ')
            disp
('==================================================================')
            disp ('You can perform a shock in several parameters at
the same time')
            disp ('If you only want to perform a shock in one
parameter, please keep the others equal')
            disp
('==================================================================')
            disp ('  ')
        f = input ('Shock occurs in period - ');
        ss = input ('Physical Capital Saving rate - actual value: 0.35 - ');
        ww = input ('Human Capital Saving rate - actual value: 0.3 - ');
        dd = input ('Physical Depreciation rate - actual value: 0.2 - ');
        cc = input ('Human Capital Depreciation rate - actual value: 0.2 - ');
        gg = input ('Tax rate Physical Capital - actual value: 0.1 - ');
        jj = input ('Tax rate Human Capital - actual value: 0.1 - ');
            %Phase Diagram
            x = 0:0.5:4;
            y = ((a+n+dd)/(ss*(1-gg)))^(1/beta)*x.^((1-alpha)/beta);
            z = ((((ww*(1-jj)))/(a+n+cc))^(1/(1-beta)))*x.^(alpha/(1-beta));
            yy = ((a+n+d)/(s*(1-g)))^(1/beta)*x.^((1-alpha)/beta);
           zz = ((((w*(1-j)))/(a+n+c))^(1/(1-beta)))*x.^(alpha/(1-beta));
            subplot(2,2,2)
            hold on
            plot(x,y,'k-','LineWidth',2);
            plot(x,z,'r-','LineWidth',2);
            plot(x,yy,'k:','LineWidth',2);
            plot(x,zz,'r:','LineWidth',2);
            legend('Physical Capital','Human Capital', 2);
            xlabel('Physical Capital')
            ylabel('Human Capital')
            title('Phase Diagram')
            %Transition Dinamics
            init_values = [1;1];
            time_period=[0 f];
```

```
            [t,k] =
ode45(@ode_SOLOWHCG,time_period,init_values,[],s,w,n,a,d,c,g,j,alpha,b
eta);
            subplot(2,2,1)
            hold on
            plot(t,k(:,1),'k-','LineWidth',2);
            plot(t,k(:,2),'r-','LineWidth',2);
            s = ss;
            w = ww;
            d = dd;
            c = cc;
            g = gg;
            j = jj;
            zz=k(end,1);
            ff=k(end,2);
            init_values=[zz;ff];
            time_period=[f 150];
            [t,kk] =
ode45(@ode_SOLOWHCG,time_period,init_values,[],s,w,n,a,d,c,g,j,alpha,b
eta);
            hold on
            plot(t,kk(:,1),'k-','LineWidth',2);
            plot(t,kk(:,2),'r-','LineWidth',2);
            legend('Physical Capital','Human Capital',2);
            xlabel('Time')
            ylabel('Value')
            title('Transition Dynamics')
            subplot (2,2,3)
            hold on
            plot(k(:,1),k(:,2),'k:','LineWidth',2);
            plot(kk(:,1),kk(:,2),'k-','LineWidth',2);
            xlabel('Physical Capital')
            ylabel('Human Capital')
            title('Saddle Path')
            yy = (k(:,1).^0.33 + k(:,2).^0.25);
            yyy = (kk(:,1).^0.33 + kk(:,2).^0.25);
            subplot (2,2,4)
            plot3(k(:,1),k(:,2),yy,'k:','LineWidth',2);
            hold on
            plot3(kk(:,1),kk(:,2),yyy,'k-','LineWidth',2);
            xlabel('Physical Capital')
            ylabel('Human Capital')
            zlabel('Output')
            title('Output Saddle Path')
        end
        if i == 'n'
        end
        end
     else
        clc
        fij=1;
        disp ('===============================================')
        disp ('Acknowledgments')
        disp ('Prof. Óscar Afonso; Prof. Paulo Vasconcelos;')
        disp ('Faculdade de Economia da Universidade do Porto;')
        disp ('===============================================')
    end
end
```