Econometrics II
Time Series #1

Thursday, 13.04.2016

1. (Stock & Watson) Let us define the difference operator $\Delta = (1 - L)$, where L is the lag operator, such that $L^j Y_t = Y_{t-j}$. Generally, $\Delta^i{}_j = (1 - L^j)^i$, where the indices i and j are typically omitted when they both take the value of 1 (*first differences*). Show the expressions in Y only when applying the difference operator to the following expressions, and give the resulting expression an economic interpretation, assuming that you are working with quarterly data.
   (a) $\Delta_4 Y_t$
   (b) $\Delta^2 Y_t$
   (c) $\Delta_1 \Delta_4 Y_t$
   (d) $\Delta_4^2 Y_t$

2. (Stock & Watson) Consider the standard AR(1) model $Y_t = \beta_0 + \beta_1 Y_{t-1} + u_t$ where the usual assumptions are valid (stationarity, disturbances $u_t \overset{iid}{\sim} (0, \sigma_u^2)$).
   (a) Define $y_t = \beta_1 y_{t-1} + u_t$, where y_t is Y_t with mean removed (*demeaned*), i.e., $y_t = Y_t - E[Y_t]$. Show that $E[y_t] = 0$.
   (b) Calculate $E[Y_t]$.
   (c) Calculate $Var(y_t)$ and $Var(Y_t)$.
   (d) Show that the $r-$period ahead forecast $E[y_{T+r|T}] = \beta_1^r y_T$. If $\beta_1 \in (0, 1)$, how does the forecast behave as r becomes increasingly larger? What is the forecast of $Y_{T+r|T}$ for large r?
   (e) The median lag τ is the number of periods it takes a time series with zero mean to halve its current value (in expectation), i.e., it is the solution τ to $E[y_{T+\tau|T}) = 0.5 y_T$. Show that in the present case this is given by $\tau = -\frac{\log(2)}{\log(\beta_1)}$ Based on this result provide an intuitive interpretation why the coefficient β_1 measures the persistence of y_t.

3. The attached SBITOP time series (Slovenian "Blue Chip"[1] stock exchange index values for the period from 03.04.2006 to 22.03.2013) is non-stationary. Confirm this and try to find an appropriate transformation to make the time series stationary. Can we identify the SBITOP time series as one type from the ARIMA family? How can we check whether there is conditional heteroskedasticity present in the residuals?

[1] Note: the quotes are "ironic".