Symmetric Nonnegative Matrix Factorization-Based Community Detection Models and Their Convergence Analysis

Project Review

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Presentation Outline

Introduction

- Community detection is a popular issue in social network analysis. This study proposes four novel community detection models based on symmetric non-negative matrix factorization (SNMF) and graph-regularized SNMF (GSNMF) with scaling-factor adjustment. Theoretical analysis shows that the proposed models achieve non-increasing loss functions during training and converge to a stationary point.
- A community consists of nodes with similar patterns in their related links, which are vital in describing knowledge hidden in a complex network, e.g., node clusters, edge distributions, and potential links

Problem definition

- Given a symmetric matrix $A_{n\times n}$, an SNMF model seeks for its low-rank approximation $\hat{A}$ on the latent factor (LF) matrix $X_{n\times K}$ with K denoting the dimension of the latent feature space and $K \leq n$, i.e., $\hat{A} = XX^T$. To obtain X, an objective function describing the difference between A and $\hat{A}$ is necessary, as the following Euclidean distance function:

$$\min_X \text{JSNMF} = \min_X \|A - XX^T\|_F^2, \quad \text{s.t. } X \geq 0 \tag{1}$$

where the operator $\|\cdot\|_F$ computes the Frobenius norm of an enclosed matrix.

Objective

- Our main objective is to minimize the objective function min

$$\min_X \mathrm{JSNMF} = \min_X \operatorname{tr}(A - XXT) \operatorname{tr}(A - XXT)^T \qquad (2)$$

subject to,

$$X_{ik} \leftarrow X_{ik} \frac{(AX)_{ik}}{(XX^T X)_{ik}}$$

This formulation gives a standard NMU scheme for an SNMF model, extending the algorithm to consider a target network's topological symmetry. However, prior work indicates that frequently makes an SNMF model suffer from training fluctuations caused by the scaling factor $(AX)_{ik}/(XX\ \mathrm{T}X)_{ik}$.

Methods to achieve the objective I

- This study proposes scaling-factor-adjusted NMU (SNMU) schemes for SNMF and graph-regularized SNMF (GSNMF) models, thereby achieving highly accurate community detectors with theoretically guaranteed convergence behaviors.
- Four novel community detection models have been presented, i.e., α-SNMF, β-SNMF, α-GSNMF, and β-GSNMF.
- The SNMF-based detector's accuracy is be improved by integrating graph-regularization terms into object function to achieve a GSNMF model,

$$\min_{X} J_{\mathrm{GSNMF}} = \min_{X} \left\| A - XX^T \right\|_{\mathrm{F}}^2 + \lambda \operatorname{tr}\left(X^T L X \right) \quad \text{s.t. } X \geq 0 \tag{3}$$

Methods to achieve the objective II

- Let $M^{n \times K}$ be a matrix caching the scaling factors for all parameters in X, this study proposes the following tuning rules for learning rate equation of SNMF:

  Non-linear tuning: $M_{ik} = \left(\frac{(AX)_{ik}}{(XX^T X)_{ik}} \right)^{\alpha}, \quad 0 < \alpha \leq 1$

  Linear tuning: $M_{ik} = \beta \frac{(AX)_{ik}}{(XX^T X)_{ik}}, \quad 0 < \beta \leq 1.$

- Similarly for GSNMF, let λ is the graph regularization constant.

  Non-linear tuning: $M_{ik} = \left(\frac{(1+\lambda)(AX)_{ik}}{(XX^T X+\lambda DX)_{ik}} \right)^{\alpha}, \quad 0 < \alpha \leq 1$

  Linear tuning: $M_{ik} = \beta \frac{(1+\lambda)(AX)_{ik}}{(XX^T X+\lambda DX)_{ik}}, \quad 0 < \beta \leq 1.$

Action Plan

1. Understand the problem of community detection and the limitations of existing methods.
2. Study the proposed scaling-factor-adjusted NMU schemes and choose the appropriate one for SNMF-based community detection models.
3. Apply the chosen scheme to SNMF or GSNMF models to achieve a novel SNMF-based community detector.
4. Set the hyperparameters of the model and validate its performance using a set of experiments.
5. Conduct theoretical studies to show that the model can keep its loss function non-increasing during its training process and converge to a stationary point.
6. Conduct empirical studies on real-world social networks to show the effectiveness of the proposed methods.