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Configuring an Intelligent Reflecting Surface for Wireless Communications

Team Wireless Shark – Solution Description

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Determining Channel Response

First, it is assumed that the two states of the IRS, do not give much of an amplitude difference except for the phase difference of π. This assumption is justified later in this document with valid reasoning. Then the N paths via the IRS are modelled to have responses $\bar{h}_1, \bar{h}_2, \bar{h}_3, \ldots \bar{h}_N$ when each IRS element configured to the +1 state. Since each IRS element only gives a phase difference of π when configured to the -1 state, the frequency responses of the IRS elements at -1 configuration can be written as $-\bar{h}_1, -\bar{h}_2, -\bar{h}_3, \ldots -\bar{h}_N$ (by multiplying by -1). The direct NLOS path to the user can also be modelled as $\bar{h}_0[v]$. Altogether we have N+1 paths to be determined in the channel response. Since we only have N received signals in dataset2, it is necessary to find a pattern in the channel response and the dataset.

Now, each received signal $\bar{z}$ can be modelled as $\bar{z} = \left[\bar{h}_0 + \sum_{i=1}^{N} \bar{h}_i \theta_i\right] \odot \bar{x} + \bar{n}$ where $\theta_i \in \{+1, -1\}$ corresponds to the configuration of i^{th} IRS element.
The $\bar{h}_0 + \sum_{i=1}^{N} \bar{h}_i \theta_i$ term can be approximated by taking $\bar{z} = \left[\bar{h}_0 + \sum_{i=1}^{N} \bar{h}_i \theta_i\right] \odot \bar{x}$ with some noise included. Then we have N equations with N+1 unknowns. Considering the first 4096 configurations of the pilotMatrix4N, it can be seen that the first IRS element is always 1. This means θ_1 is always 1 for the first 4096 as the Hadamard matrix has 1's in the first column. Hence, taking $\bar{h}_1$ out the of the summation, the $\bar{h}_0 + \bar{h}_1$ term is considered together in the resulting equations. Thereby we can obtain N equations with N unknowns from the received signals. Then the N equations can be solved for $\bar{h}_0 + \bar{h}_1, \bar{h}_2, \bar{h}_3, \ldots \bar{h}_N$ as a usual system of equations.

Finding the Relation Between the IRS Elements

In dataset1.mat, a nice pattern can be seen among the configurations in pilotMatrix4N. The opposite configuration of the first 4096 configurations (first N) can be seen from 4097 to 8192 (second N), in proper order. That means the i^{th} and the $i+4096^{th}$ configurations are opposite configurations. Similarly, the configurations from 12289 to 16384 (fourth N) contain the opposite configuration of the configurations from 8193 to 12288 (third N). (Note: - The opposite configuration of configuration vector Θ is -Θ) Therefore adding the 4N received signals and taking the average will give us $\bar{z}_{avg} = \bar{h}_0 \odot \bar{x} + \bar{n}$ as the opposite terms get cancelled off. This expression can be used to determine $\bar{h}_0$.

Then another set of equations can be obtained by subtracting the average signal from every received signal which can be modelled as $\bar{z}_{IRS} = \left[\sum_{i=1}^{N} \bar{h}_i \theta_i\right] \odot \bar{x} + \bar{n}$ .This has only N unknowns. Now this can be solved to come up with a pattern to be used in dataset2. After solving the channel responses of the paths via the N IRS elements ($\bar{h}_1, \bar{h}_2, \bar{h}_3, \ldots \bar{h}_N$) and taking the correlation matrix, it can be observed that the i^{th}, $i+64^{th}$, $i+128^{th}$, … IRS elements have a very high correlation in dataset 1. (Correlation coefficient close to 1) This idea is very helpful since we cannot determine the direct path response straight forward in dataset2, as in dataset1. [*refer to the figure 1 in the appendix*]

Applying the Above Results to Dataset2

In dataset2 the N variables $\left(\bar{h}_0 + \bar{h}_1, \bar{h}_2, \bar{h}_3, \ldots \bar{h}_N\right)$can be determined for each user. $\bar{h}_1$ is approximated by taking the average of $\bar{h}_{65}, \bar{h}_{129} \ldots, \bar{h}_{4033}$ using the above relationship obtained using the correlation in dataset1. (The user in dataset1 has a LOS path from IRS. But for NLOS users in dataset2, these coefficients can differ. As a reasonable approximation, we can take the average of the highly correlated IRS elements to calculate the response of the 1^{st} IRS element.) We use this approach to determine $\bar{h}_1$ which in turn can be written as, $\bar{h}_1 \simeq \frac{1}{63} \sum_{i=1}^{63} \bar{h}_{64*i+1}$.

Thereafter $\bar{h}_0$ can be computed as,

$\bar{h}_0 = [\bar{h}_0 + \bar{h}_1] - \frac{1}{63}\sum_{i=1}^{63} \bar{h}_{64*i+1}$.

Note: - We can justify our assumption of constant amplitude of the two states of the IRS elements. We solved the channel response using the assumption for the first 4096 signals in dataset1. It also gives very close approximations to the received signal energy of all the remaining transmissions in the dataset1. When considering the ratios of the norms of the two opposite configuration states (i.e., i^{th} and the $i+4096^{th}$) of the channel response, almost everyone was within the range of 0.9 and 1.1. (Only 0.7% falls outside this range) Thus, it is reasonable in assuming so. The slight variation in amplitude is due to noise which can be reasonably neglected.

<u>Determining Best IRS Configuration for Users in Dataset2</u>

The N+1 x N+1 correlation matrix which corresponds to the N+1 complex vectors $\bar{h}_0, \bar{h}_1, \bar{h}_2, \bar{h}_3, \dots \bar{h}_N$ is considered. The eigen values of this correlation matrix are calculated. Then the eigen vectors which correspond to the largest eigen value of the matrix is obtained. (These vectors are the coordinate vectors for the N+1 vectors.) [*refer appendix 2.1*]

Note: - We can define the correlation coefficient $\cos\alpha$, as $\cos\alpha = \frac{Re(\bar{u}\cdot\bar{v})}{\|u\|\|v\|}$ where α is the angle between two complex vectors. Although there are components from noise in our approximation for the responses of the N+1 paths, the impact of noise will be less when we take the correlations between those N+1 complex vectors.

Then the following approach is used to determine the optimal configuration.
The following equation was given to calculate the channel rate.

$$R = \frac{B}{K+M-1}\sum_{v=0}^{K-1}\log_2\left(1 + \frac{P\left|\bar{h}_\theta[v]\right|^2}{BN_0}\right)$$

It is clear that when B, N_0 and P are constant, the channel responses with higher $\left|\bar{h}_\theta[v]\right|^2$ values for all its components will lead to higher rates. Note that the log function is positive and increases when $\left|\bar{h}_\theta[v]\right|^2$ increases, at higher SNRs. (To maximize the channel rate, the channel SNR needs to be maximized.)

Let the vectors $\bar{w}_1, \bar{w}_2, \bar{w}_3, \bar{w}_4$ be the eigenvectors which corresponds to the largest four eigenvalues of the correlation matrix.

Then a vectors $\varphi_1, \varphi_2, \varphi_3, \varphi_4$ will be calculated as follows.

$$\varphi_k[i] = \begin{cases} 1 \; if \; \bar{w}_k[i] \geq 0 \\ -1 \; if \; \bar{w}_k[i] < 0 \end{cases}$$

Note that these vectors φ_k act as coordinate vectors for N+1 complex vectors which are corresponds to the channel responses of N+1 paths. So that $\varphi_k[1]$ = 1 for each k, since the since $\bar{h}_0$ term cannot be negative. Note that the term $\varphi_k[1]$ corresponds to the $\bar{h}_0$ term in the coordinate vector which should be constant with value 1. So, If $\varphi_k[1]$ = -1 in any of our our vectors φ_k, We multiply those vectors by -1 to make $\varphi_k[1]$ = 1 (for any vector φ_k, the same rate will be given for $-\varphi_k$). Since we need to find the best IRS configuration, we only take the $\varphi_k[2{:}N+1]$ as our IRS configuration for the initial step (we refer this configuration as approach1).

Then we do the following iteration procedure on each vector φ_k to maximize the rate further. For each iteration,[*refer appendix 2.2]*

1. We start with the vector φ_k^jwhich was obtained previously. (In the first iteration we assign $\varphi_k^0 = \varphi_k$).
2. Then we swap the configuration of each IRS element per once (replace $\varphi_k^j[i]$ with $-\varphi_k^j[i]$ for each $i \in (2, N+1)$) and calculate the channel rate for each resulting coordinate vector.
3. Then we get our vector φ_k^{j+1} by swapping the all IRS elements configuration in φ_k^j which results an increase in channel rate in the previous step.

This iteration process will continue for 50 iterations at max. (we will stop the iteration if the swapping of any IRS element isn't producing any increase in the channel rate).

After we complete iterations on every vector φ_k we get the resulting vectors. Let's call them $\beta_1, \beta_2, \beta_3, \beta_4$ where each β_k is the vector after applying the above iteration on the vector φ_k.

Then we calculate the channel rate for each of the vectors β_k and the vector gives the maximum channel rate is chosen. (Let's call it α)

Now we can have our desired configuration vector $\theta_{optimal}$ as follows.

$$\theta_{optimal} = \alpha[2{:}N+1]$$

Note: - The norm of the channel response is defined as $\|\bar{h}_\theta\| = \sqrt{Re(\bar{h}_\theta \cdot \bar{h}_\theta)}$

According to the definition of the dot product for the complex vectors, this can be written as follows.

$$\|\bar{h}_\theta\| = \sqrt{\sum_{v=1}^{K-1} |\bar{h}_\theta[v]|^2}$$

Higher norms for the channel response mean higher values of $|\bar{h}_\theta[v]|^2$, which are more likely to give higher data rates. (Which is obviously a maximum rate if the frequency response is flat.) [*refer to the figure 4 in the appendix*]

The solution always produces an IRS configuration with higher or equal rate than the maximum rate within the configurations of the pilot matrix for each user, when we approximate the rate using a certain N_0 value. [*refer to the figure 2 in the appendix*] This algorithm can identify the optimal configuration even if the pilot matrix does not contain it. Also, this approach will be very efficient since it reduces the iterations of the search by first taking the correlation matrix and then choosing the eigen vectors corresponds to the highest eigen values and applying iteration procedure on those vectors calculated based on these eigen vectors to maximize the rate further.

Note: - Even though, If we chose the coordinate vector α as the vector which gives the maximum channel rate from the vectors φ_k^0, the resulting rates will be very close to the rates obtained with the vector we obtained after 50 iterations. which can be obtained after 50 iterations. If the intention is to reduce the running time further, this approach of considering φ_k^0 without running any iterations on them can be considered as it will give a reasonably close value. [*refer to the figure 3 in the appendix*]