DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY MADRAS

PH1010-2020 **Mini-Test 2 (21 Marks)**

Undertaking of ACADEMIC HONESTY:

By submitting this test, it is understood that you promise to uphold academic honesty by neither giving nor receiving any unauthorised help on this mini-test. The answers should be based on your individual effort. Any deviation, if identified, may lead to reduction in marks.

Important instructions:

1. Write your answers with ALL required steps in good quality A4 sheets.
2. Begin your answers for every question on a fresh A4 sheet.
3. Use only dark blue or black ink for writing answers (strictly no pencil).
4. Write the **name and IITM roll no** at the top right corner of the A4 sheet. Number the pages in order.
5. Use of calculator, books, and online resources are permitted.
6. Digitise/scan your answer scripts neatly taking into account proper cropping and contrast.
7. Upload only a single, combined PDF file.
8. Please check your PDF file completely before uploading the same.
9. DO NOT FORGET to click **SUBMIT** button after uploading your answer file.
10. The marks you earn in this test will count towards your aggregate in the course.
11. If your answer is a vector quantity, make sure that its direction is clearly indicated (e.g., by using appropriate unit vectors).

Submission deadline: Monday, 21-12-2020, 9:00 AM.
Problems 1 and 2 : 7 marks each.
Problem 3 : 2 marks.
Problem 4 : 5 marks.
TOTAL MARKS: 21

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1. Consider a particle of mass m and energy E moving in a one-dimensional region with potential energy

$$U(x) = U_0\left[\left(\frac{x}{a}\right)^2 - \left(\frac{x}{b}\right)^4\right]$$

where $U_0 > 0$ has the dimensions of energy, and $a, b > 0$.

(a) Identify the locations of maxima and minima of $U(x)$ (1 MARK).

(b) Graphically sketch $U(x)$ versus x in the range $x \in (-\infty, \infty)$ (1 MARK).

(c) Find the range of energy E corresponding to bounded motion in the form $E_1 < E < E_2$ (i.e., find E_1 and E_2) (1 MARK).

(d) Sketch the complete phase portrait for the system, including bounded and unbounded trajectories as well as separatrix curves (2 MARKS).

(e) Write down the equation for the separatrix curve (1 MARK).

(f) Sketch the force $F(x)$ on the particle as a function of x in the range $x \in (-\infty, \infty)$ (1 MARK).

2. A toy train consists of an engine and wagon of equal mass m each, connected by a spring with spring constant k. The relaxed length of the spring may be considered to be zero. The train is initially placed at the centre of a horizontal, circular turntable (see Fig. 1), and is free to move on a radial frictionless track on the turntable. The engine (alone) is now given an initial (radial) velocity v_0, and the turntable is independently set in motion to rotate counterclockwise with an angular speed ω. Neglect the physical dimensions of the train.

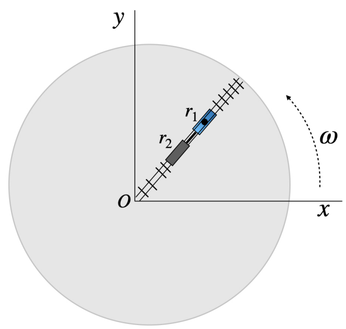

Figure 1:

(a) Write down the equations of motion for the radial coordinates of the engine and the wagon, denoted by r_1 and r_2 (1 MARK).

(b) Using (a), write down the equation of motion for the radial coordinate $R(t)$ of the centre of mass (COM) of the train. Solve this equation subject to the given initial conditions and determine $R(t)$ (2 MARKS).

(c) Using (a), write down the equation of motion for the separation $r = r_1 - r_2$ between the engine and the wagon. Solve the equation and find $r(t)$ subject to the given initial conditions (assume that $\omega^2 < 2k/m$) (2 MARKS).

(d) Find $r(t)$ if $\omega^2 > 2k/m$. Speculate about what would happen to the train[1] in this case, if the table is infinite in extent (2 MARKS).

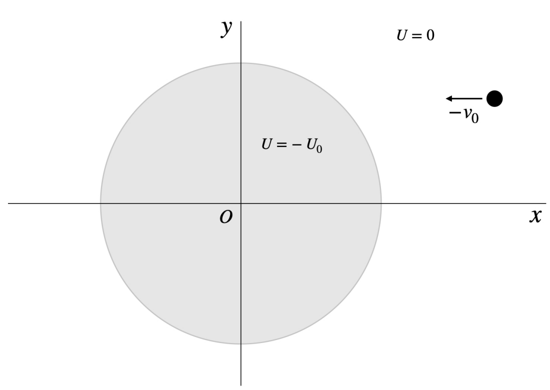

Figure 2:

[1] i.e., to a "real" train, with a real spring!

3. Consider a two-dimensional region (say, the $x - y$ plane), in which a particle (mass m) experiences a force-field, characterised by potential energy

$$U(\mathbf{r}) = -U_0 \qquad |\mathbf{r}| \leq a$$
$$U(\mathbf{r}) = 0 \qquad |\mathbf{r}| > a \tag{1}$$

where $U_0 > 0$ and $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$ (see Fig. 2). [2] The particle approaches the well from $x = \infty$ with velocity $-v_0\hat{\mathbf{i}}$ ($v_0 > 0$), energy $E = mv_0^2/2$ and angular momentum $\mathbf{L} = \ell\ \hat{\mathbf{k}}$, where $\ell > 0$.[3]

(a) Identify all conserved dynamical quantities associated with the motion of this particle (1 MARK).

(b) Determine the condition on ℓ such that the particle will eventually enter the potential well (1 MARK).

4. In problem 3 above, consider specific values $\ell = mv_0a/\sqrt{2}$ and $U_0 = E/2$.

(a) Determine the velocity of the particle, immediately AFTER it enters the well, **in plane polar coordinates** (1 MARK).

(b) Use the relevant conservation laws (refer to 3 (a) above) to express the radial speed $\dot{r}$ in terms of the radial coordinate r (2 MARKS).

(c) By solving the equation in (b) or otherwise (show details), determine the time T it takes for the particle to escape from the well (2 MARKS).

[2] Imagine a circular "potential well" of depth U_0 and radius a, see Fig.2.

[3] measured with respect to the centre of the well (the origin here).