DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY MADRAS

PH1010 Physics I Quiz I 11/09/2013 8:00AM-8:50AM Max. Marks 20

Instructions: First read these instructions. Not following them will result in the paper not being evaluated.

- **There are FOUR questions in all.**
- **Use ONLY the answer sheet provided. NO additional sheets will be provided.**
- **Write your NAME, ROLL No. and BATCH.**
  Note that the BATCH (A to J) MUST be clearly indicated next to the Course No. box.
- **Use a black or blue pen ONLY.**
- **Answers to part-A and part-B questions must NOT be mixed up. Clearly mark ONLY TWO sections of your answer booklet where part-A and part-B are attempted.**
- **NO cellphones, calculators etc.**

Good Luck!

Part A

1. Indicate clearly your answer, which should be **a number or an expression.** Highlight your answer by either underlining or enclosing it in a box. **(4)**

   (a) Of the 27 components ϵ_{ijk}, how many are nonzero? (ϵ_{ijk} is the Levi-Civita symbol).

   (b) A particle with charge q and mass m, moving with a velocity $\boldsymbol{v}$, is subjected to an electric field $\boldsymbol{E}$ and magnetic field $\boldsymbol{B}$. The force experienced by the particle is $\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$. Give the expression for F_i, the i^{th} component of the force, using index notation.

   (c) Along some trajectory of a particle its position vector is $\boldsymbol{r} = 2\hat{\boldsymbol{e}}_{\boldsymbol{x}} - \hat{\boldsymbol{e}}_{\boldsymbol{y}} + \hat{\boldsymbol{e}}_{\boldsymbol{z}}$ and velocity is $\boldsymbol{v} = -\hat{\boldsymbol{e}}_{\boldsymbol{x}} - \hat{\boldsymbol{e}}_{\boldsymbol{y}} + v_z\hat{\boldsymbol{e}}_{\boldsymbol{z}}$ at time t_0. Then at this instance, the distance from the origin is a local extremum if $v_z =$? (arbitrary units in the problem).

   (d) if $\hat{\boldsymbol{r}}$ and $\hat{\boldsymbol{\theta}}$ are polar coordinate unit vectors, then $d^2\hat{\boldsymbol{\theta}}/dt^2$ is equal to what?

2. Answer TRUE or FALSE (no explanation needed), again highlighting your answer, along with the question number **in a box.** **(6)**

(a) **Both** angular momentum and magnetic fields **are** examples of pseudovectors. TRUE or FALSE?

(b) The force between two infinite straight and parallel wires carrying steady currents **obeys** Newton's third law. TRUE or FALSE?

(c) In the absence of external forces, the total angular momentum for a collection of many particles exerting mutually equal and opposite, but **non-central**, forces is constant. TRUE or FALSE?

(d) For a given system, phase space trajectories cannot cross each other, **even if** forces are explicitly time dependent. TRUE or FALSE?

(e) The matrix $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ is a **valid** rotation matrix. TRUE or FALSE?

(f) For a damped unforced harmonic oscillator, the amplitude decays **fastest** at critical damping. TRUE or FALSE?

Part B

3. Consider a critically damped and forced oscillator obeying

$$\ddot{x} + 2\gamma\dot{x} + \gamma^2 x = f_0 \cos(\omega t).$$

(Note that the restoring force is $-\gamma^2 x$).

(a) Show that both $e^{-\gamma t}$ and $te^{-\gamma t}$ are solutions of the homogeneous equation. **(1)**

(b) Derive the particular solution $x_p(t)$ which is the response after the transients have died out, paying attention to the amplitude and phase. Using the previous part write the general solution. **(2)**

(c) Sketch the amplitude of the response as a function of ω for a fixed γ. Is there resonance when ω equals the oscillator's natural frequency? **(1)**

4. Let a particle (mass m) on a plane be subjected to the force $\boldsymbol{F} = -k\boldsymbol{r}$ where $k > 0$ is a constant, and $\boldsymbol{r}$ is the position vector.

(a) Write Newton's second law in polar coordinates for this particle, and using the angular part show that $r^2\dot{\theta} = C$ where C is a constant. **(2)**

(b) Define a function

$$V(r) = \frac{kr^2}{2} + \frac{mC^2}{2r^2}.$$

Show that $m\ddot{r} = -dV(r)/dr$. **(2)**

(c) Deduce, **solely** from the result of the previous part (and what you know about the time period of a harmonic oscillator), the time period for a particle (mass m) trapped in a **one-dimensional** potential given by $U(x) = kx^2/2 + \alpha/(2x^2)$, where $k, \alpha > 0$ are constants. (Do **not** set up or perform any integrals!) **(2)**