Department of Physics
Indian Institute of Technology Madras
Quiz I

PH1010 Physics I **12.09.2019**

Time: 8:00 to 8:50 AM **Max. Marks: 20**

Name	Roll No.	Batch No. and Teacher's Name

Instructions

1. **Please write your complete name, your roll number, the name of your teacher, and your batch number.**
2. This question paper cum answer sheet booklet contains 12 pages. Please check that all the pages are present. You are expected to answer all the questions. There are no negative marks.
3. Write the answers only in the allotted box for Q1 and Q2. All vectors must be indicated clearly.
4. You can use pages 11-12 for rough work. No extra sheets will be provided.
5. All symbols have their usual meaning unless stated otherwise. **All constants are positive and of appropriate dimensions.**
6. You must use only black or blue ink for writing the answers.
7. **Do not keep calculators, cell phones or any internet connectable device with you during the examination.**

For use by examiners
(Do not write in this space)

Q1	Q2	Q3	Q4	Q5	**Total Marks**

1. Indicate whether the following statement is true or false (write **True** or **False** in the box provided.) **[1 × 3 = 3]**

(i) If α, β, γ are the direction cosines of a vector $\vec{A}$ along x, y and z axes respectively, then $\alpha\hat{x} + \beta\hat{y} + \gamma\,\hat{z}$ is a unit vector.

True (+1)

(ii) For a conservative force, the work done in bringing a particle from a point P to a point Q depends on the specific path chosen.

False (+1)

(iii) The equipotential curves for the force $\vec{F}(x,y) = x\hat{x} + 2y\hat{y}$ are of the form $x^2 + 2y^2 = C$, where C is a constant quantity.

True (+1)

2. Write the final answer in the box provided: Derivation is NOT required.

(i) Suppose the temperature of this room at a point (x,y,z) is described by T(x,y,z)= $2x^2 + y^2 + 3z^2$. The unit vector along the direction in which the temperature increases most rapidly at the point (0,1,1) is **[1 mark]**

$\hat{U} = \frac{\hat{j} + 3\hat{k}}{\sqrt{10}}$ (+1)

(ii) A particle of mass m is moving on a plane. The radial distance of the particle from the origin is given by $r(t) = r_0 \frac{\theta(t)}{\pi}$, where $\theta(t) = \frac{\alpha t^2}{2}$ with r_0 and α as constants. (a) Find the force $\vec{F}$ on the particle as a function of time. (b) For what value (s) of θ the radial component of the force vanishes? **[1+1=2 marks]**

(a) $\vec{F} = \frac{m r_0 \alpha}{\pi} \cdot \left(\left(1 - \frac{\alpha^2 t^4}{2}\right)\hat{r} + \left(\frac{5\alpha t^2}{2}\right)\hat{\theta}\right)$ (+1)

(b) $\theta = \pm \frac{1}{\sqrt{2}}$ (+1)

(iii) A particle of unit mass moves along a circle of radius R anti-clockwise under the influence of a force given by $\vec{F}(r,\theta) = -\frac{a}{r^2}\hat{r} + b\hat{\theta}$, where a and b are constants. The work done by the force in moving the particle for one complete circle is **[1 mark]**

$W = 2\pi bR$ (+1)

(iv) If $f(r) = e^{-r}$, where $r = \sqrt{x^2 + y^2 + z^2}$, then $\vec{\nabla} f$ at (0,0,1) is **[1 mark]**

$\frac{1}{e}\hat{k}$ or $e^{-1}\hat{k}$ ✓ +1

(v) If $U(x) = \frac{A}{x^{12}} - \frac{B}{x^6}$ describes the potential energy of an ideal diatomic molecule, for instance HCl, where x is the interatomic distance, and A, B are positive constants, then the equilibrium bond length of the molecule is **[1 mark]**

$x = \left(\frac{2A}{B}\right)^{1/6}$ ✓ +1

(vi) A damped driven oscillator with natural frequency ω_0 satisfies the following equation of motion

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = f_0 \cos\omega t$$

The frequency (ω) at which the amplitude of oscillations is maximum is **[1 mark]**

$\omega = \sqrt{\omega_0^2 - 2\beta^2}$ ✓ +1

3. A cricket ball of mass m is thrown vertically up with an initial speed v_0 and is subjected to air resistance of the form $f = -bv^2$, where b is a positive constant of appropriate dimension.

(i) Find the terminal speed.

(ii) Determine the time taken by the ball to reach the maximum height. **[1+2 =3 marks]**

4. A conservative force is given by $\vec{F}(x, y, z) = (2xy + z^3)\hat{x} + x^2\hat{y} + 3xz^2\hat{z}$.

Find the potential $U(x, y, z)$ corresponding to the given force. **[3 marks]**

5. An undamped driven oscillator of unit mass with natural frequency ω_0 is subjected to an external force $f(t) = f_0 \cos\omega t$, where f_0 and ω ($\omega \neq \omega_0$) are constants. The oscillator satisfies the following equation of motion

$$\ddot{x} + \omega_0^2 x = f(t)$$

(i) Find the particular solution $x_p(t)$.

(ii) Write down the general solution $x_h(t)$ to the corresponding homogeneous differential equation.

(iii) Determine the complete solution $x(t)$ to the equation of motion using the initial conditions $x\ (t = 0) = 0$, $\dot{x}\ (t = 0) = 0$.

[1+1+2=4 marks]