Quiz 2, Solutions. PH1010, 17/10/2013.

PART-A

1 $\boxed{\text{a) } 0}$ or $\boxed{\text{a) } \vec{0}}$

$\boxed{\text{b) } \pi + \frac{\pi^2}{2}}$

$\boxed{\text{c) } -\hat{e}_x}$ or $\boxed{\text{c) } -\hat{i}}$ or $\boxed{\text{c) } -\hat{x}}$

d) $\boxed{L = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - mgz}$

Cyclic coord: x, y

also $L = \frac{1}{2} m v^2 - mgz$ is OK.

e) $\boxed{P + x = 0}$ or $\boxed{\dot{x} + x = 0}$

2

a) FALSE

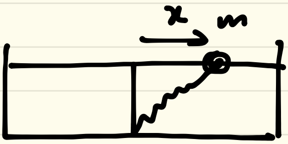
b) FALSE

c) TRUE

d) FALSE

e) FALSE

Part-B



3 $V(x) = \frac{1}{2}k\left[\sqrt{h^2+x^2} - L_0\right]^2$

Equil. points $\frac{dV}{dx} = 0$; $\frac{dV}{dx} = k\left[\sqrt{h^2+x^2} - L_0\right]\frac{x}{\sqrt{x^2+h^2}}$

a $\therefore \frac{dV}{dx} = 0 \Rightarrow \quad x = 0$ or $x = \pm\sqrt{L_0^2 - h^2}$.

$\therefore$ If $h < L_0$ spring is compressed when $x = 0$

$\underline{x_1 = 0}$, $\underline{x_2 = +\sqrt{L_0^2 - h^2}}$, $\underline{x_3 = -\sqrt{L_0^2 - h^2}}$

are 3 equil. points.

& If $h > L_0$ Spring is stretched when $x > 0$

and $\underline{x_1 = 0}$ is THE ONLY Equil. pt.

STABILITY : $\frac{d^2V}{dx^2} = k\frac{d}{dx}\left[x\left(1 - \frac{L_0}{\sqrt{x^2+h^2}}\right)\right]$

$= k\left(1 - \frac{L_0}{\sqrt{x^2+h^2}}\right) + \frac{kx^2L_0}{(x^2+h^2)^{3/2}}$; $V''(0) = k\left(1 - \frac{L_0}{h}\right)$

$V''\left(\pm\sqrt{L_0^2 - h^2}\right) = k(L_0^2 - h^2)\frac{L_0}{L_0^3} = k\left(1 - \frac{h^2}{L_0^2}\right)$

If $h < L_0$: $V''(0) = k\left(1 - \frac{L_0}{h}\right) < 0$

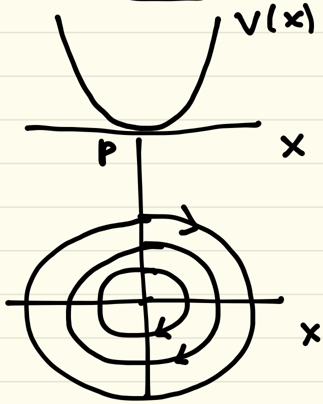
$\boxed{x_1 = 0 \therefore \text{UNSTABLE}}$

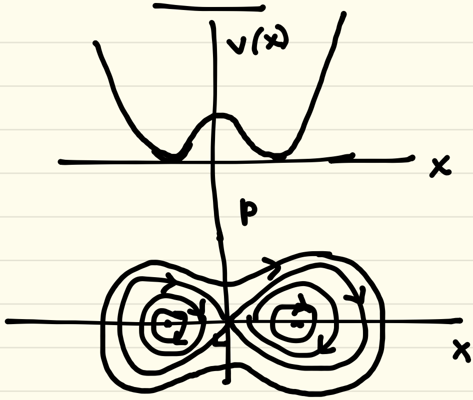
$V''\left(\pm\sqrt{L_0^2 - h^2}\right) = k\left(1 - \frac{h^2}{L_0^2}\right) > 0$ $\boxed{x_2, x_3 = \pm\sqrt{L_0^2 - h^2} \quad \text{STABLE}}$

If $h > L_0$: only $x_1 = 0$ exists and

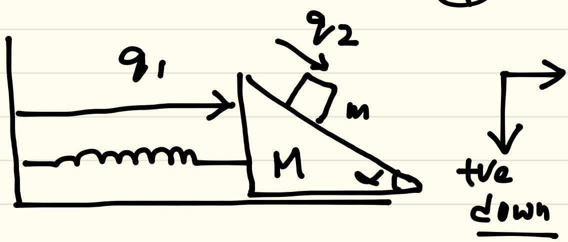
$V''(0) > 0 \Rightarrow \boxed{x_1 = 0 \text{ is STABLE}}$

(b) $\underline{h > L_0}$ $\underline{h < L_0}$





(4) See next page



④

$$T = \frac{1}{2} M U_M^2 + \frac{1}{2} m U_m^2$$

$$x_m = q_1 + q_2 \cos\alpha \qquad y_m = q_2 \sin\alpha$$

$$U_m^2 = \dot{x}_m^2 + \dot{y}_m^2 = \dot{q}_1^2 + \dot{q}_2^2 + 2\dot{q}_1 \dot{q}_2 \cos\alpha$$

$$\therefore \; T = \frac{1}{2}(M+m)\dot{q}_1^2 + \frac{1}{2} m \left(\dot{q}_2^2 + 2\dot{q}_1\dot{q}_2 \cos\alpha\right)$$

$$V = \frac{1}{2} k (q_1 - L_0)^2 - m g q_2 \sin\alpha$$

(a)
$$L = T - V = \frac{1}{2}(M+m)\dot{q}_1^2 + \frac{1}{2} m \left(\dot{q}_2^2 + 2\dot{q}_1\dot{q}_2 \cos\alpha\right) - \frac{1}{2} k (q_1 - L_0)^2 + m g q_2 \sin\alpha$$

(b)
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_1}\right) - \frac{\partial L}{\partial q_1} = 0 \Rightarrow (M+m)\ddot{q}_1 + m\ddot{q}_2 \cos\alpha = -k(q_1 - L_0) \quad -①$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_2}\right) - \frac{\partial L}{\partial q_2} = 0 \Rightarrow m\ddot{q}_2 + m\ddot{q}_1 \cos\alpha = m g \sin\alpha \quad -②$$