Department of Physics
Indian Institute of Technology Madras
Quiz II

PH1010 Physics I 2019 **17.10.2019**

Time: 8:00 to 8:50 AM **Max. Marks: 20**

Name	Roll No.	Batch No. and Teacher's name

Instructions

1. Begin by completing the information requested above. Please write your complete name, your roll number the name of your instructor, and your batch number.
2. This question paper cum answer sheet booklet contains **14** pages. Please check right away that all the pages are present. You are expected to answer all the questions. There are no negative marks.
3. Write the answers only in the allotted box. All vectors must be indicated clearly.
4. You can use the empty reverse sides for rough work. No extra sheets will be provided.
5. All symbols have their usual meaning unless stated otherwise. **All constants are of appropriate dimensions. Further assume that unless otherwise stated all constants are positive.**
6. You must use only black or blue ink for writing the answers. Pencil must not be used even in sketches.
7. Calculators, cell phones or any internet connectable device must not be in your possession during the examination.

For use by examiners (Don't write in this space)

Q1	Q2	Q3	Q4	Q5	Total Marks

1. Indicate whether the following statement is true or false (write **True** or **False** in the box provided.) **[1 × 3 = 3 marks]**

(i) The time period of the oscillations is independent of energy for a particle of mass m moving with potential energy $U(x) = A\,x^8$ $(A > 0)$.

False

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(ii) Two phase trajectories of an undamped simple harmonic oscillator don't cross each other.

True

(iii) If the orbit of a particle in a central force described by the spiral path of the form $r(\theta) = r_0 e^{-\alpha\theta}$ $(r_0 > 0;\ \alpha > 0)$, then the force is proportional to $\frac{1}{r^4}$.

False

2. Write the final answer in the box provided: Derivation is NOT required.

(i) A particle of mass m is moving with potential energy $U(x) = -U_0 x^2 e^{-\alpha x}$ $(U_0 > 0;\ \alpha > 0)$.

(a) What is the point of stable equilibrium position?

(b) Find the frequency of small oscillations around the stable equilibrium point.

[1+1=2 marks]

(a) $x = \frac{2}{\alpha}$ ✓ (1)

(b) $f = \frac{1}{2\pi}\sqrt{\frac{\frac{d^2U}{dx^2}\Big|_{x=\frac{2}{\alpha}}}{m}} = \frac{1}{2\pi}\cdot\sqrt{\frac{2U_0 e^{-2}}{m}}$ ✗ (2)

(ii) The total energy E of a particle moving in a circular orbit of radius R under the influence of an attractive central force with potential energy $U(r) = -\frac{k}{r}$ is (express E in terms of k and R only). **[1 mark]**

$E = -\frac{k}{2R}$ ✓ (1)

(iii) A particle of mass μ and angular momentum l is moving under the influence of a central force of the form $\vec{F}(r) = -\alpha\frac{e^{-\beta r}}{r^2}\hat{r}$ $(\alpha > 0;\ \beta > 0)$. Find the condition for which the circular orbit of radius r_0 is stable. **[2 marks]**

$r_0 = \frac{\beta}{2}$ $\quad r_0 < \frac{1}{\beta}$

(iv) A particle of mass μ is moving in a central force $\vec{F}(r) = -\frac{k}{r^4}\hat{r}$. The value of angular momentum for a circular orbit in terms of its radius r_0 is [1 **mark**]

$L = \sqrt{\frac{\mu k}{r_0}}$

3. A particle of mass m is moving in one dimension has potential energy given by $U(x) = U_0\left[-\frac{1}{2}\left(\frac{x}{a}\right)^2 + \frac{1}{3}\left(\frac{x}{a}\right)^3\right]$, where U_0 and a are positive constants of appropriate dimensions.

(i) Sketch $U(x)$ versus x, suitably indicating the values of the extrema.

(ii) Determine the frequency of small oscillations about the point of stable equilibrium.

(iii) Sketch the corresponding phase space trajectories for energies, $E < 0$, $E = 0$, and $E > 0$. Indicate the direction of motion with arrows. **[1+1+2 = 4 marks]**

4. A particle of mass μ moves in a central force field with potential energy of the form $U(r) = -kr^{-\beta}$ $(k > 0;\ \beta > 0)$. Let the angular momentum be l.

(i) For what value of β are stable circular orbits possible?

(ii) Find the value of radius, r_0, of the stable circular orbit.

(iii) If the particle is given a tiny kick so that the radius oscillates around r_0, then determine the time period of these small oscillations. **[1+1+2 = 4 marks]**

5. A satellite of mass m_s orbiting around the earth (of mass M_e) under the influence of an inverse square central force. The maximum and minimum orbital speeds of the satellite are v_{max} and v_{min} respectively.

(a) Express the eccentricity of the orbit in terms of v_{max} and v_{min}.

(b) If the time period of the satellite is T and the satellite moves in an elliptical path, then find the length of the semi-major axis in terms of v_{max}, v_{min} and T.

[1+2 = 3 marks]