Mini Test 1 - Mock

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1. A particle of mass m is subject to a force (F). Find the time dependent position x(t) of the particle for the following two cases:

   (a) F(x) = kx, with k > 0, the initial position is x_0, and the initial speed is zero.

   (b) F(v) = $-bv^2$ and the initial position is zero, and the initial speed is v_0.

a) $F(x) = Kx$

$$\Rightarrow M\frac{d^2x}{dt^2} - Kx = 0$$

Auxillary Equation : $D^2 - \frac{K}{M} = 0 \quad \left(+\sqrt{\frac{K}{M}}, -\sqrt{\frac{K}{M}}\right)$

$$\Rightarrow x(t) = c_1 e^{\sqrt{\frac{K}{M}}t} + c_2 e^{-\sqrt{\frac{K}{M}}t}$$

$$\dot{x}(t) = c_1\sqrt{\frac{K}{M}} e^{\sqrt{\frac{K}{M}}t} - c_2\sqrt{\frac{K}{M}} e^{-\sqrt{\frac{K}{M}}t}$$

$\dot{x}(0) = 0$

$$\Rightarrow c_1 - c_2 = 0 \Rightarrow c_1 = c_2$$

$x(0) = x_0$

$$\Rightarrow c_1 + c_2 = x_0$$

$$\Rightarrow x(t) = \frac{x_0}{2} e^{\sqrt{\frac{K}{M}}t} + \frac{x_0}{2} e^{-\sqrt{\frac{K}{M}}t}$$

b) $F(V) = -bV^2$

$$\Rightarrow m\frac{dV}{dt} = -bV^2$$

$$= -\frac{1}{V^2}dV = \frac{b}{m}dt$$

$$\int_{V_0}^{V} -\frac{1}{V^2}dV = \int_0^t \frac{b}{m}dt$$

$$= \frac{1}{V} - \frac{1}{V_0} = \frac{b}{m}t \Rightarrow V = \frac{mV_0}{m+bV_0t}$$

$$\Rightarrow \frac{dx}{dt} = \frac{mV_0}{m+bV_0t} \Rightarrow dx = \frac{mV_0}{m+bV_0t}dt$$

$$\Rightarrow \int_0^x dx = \int_0^t \frac{mV_0}{m+bV_0t}dt$$

$$\Rightarrow x(t) = \frac{m}{b}\ln\left|\frac{m+bV_0t}{m}\right|$$