DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY MADRAS

PH1010-2020 **Mini-Test 3 ($3\times$ 7= 21 Marks)**

Undertaking of ACADEMIC HONESTY:

By submitting this test, it is understood that you promise to uphold academic honesty by neither giving nor receiving any unauthorised help on this mini-test. The answers should be based on your individual effort. Any deviation, if identified, may lead to reduction in marks.

Important instructions:

1. Write your answers with ALL required steps in good quality A4 sheets.
2. Begin your answers for every question on a fresh A4 sheet.
3. Use only dark blue or black ink for writing answers (strictly no pencil).
4. Write the **name and IITM roll no** at the top right corner of the A4 sheet. Number the pages in order.
5. Use of calculator, books, and online resources are permitted.
6. Digitise/scan your answer scripts neatly taking into account proper cropping and contrast.
7. Upload only a single, combined PDF file.
8. Please check your PDF file completely before uploading the same.
9. DO NOT FORGET to click **SUBMIT** button after uploading your answer file.
10. The marks you earn in this test will count towards your aggregate in the course.
11. If your answer is a vector quantity, make sure that its direction is clearly indicated (e.g., by using appropriate unit vectors).

Submission deadline: Monday, 28-12-2020, 9:00 AM.

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1. A rocket of mass m is launched "horizontally" from a tower of height h on the earth's surface (see Fig.1). The initial speed of the rocket is $v_0 = \alpha\sqrt{2GM/R}$, where α is a positive number. Here, R the radius of earth and M, its mass. Assume that the earth is a solid, smooth sphere. Neglect the height of the tower in comparison with R. Ignore air resistance.

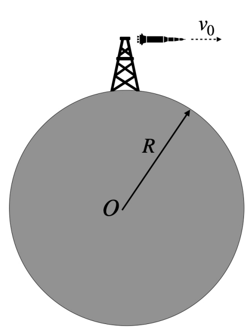

Figure 1: (not drawn to scale)

(a) Write down expressions for the energy and angular momentum of the rocket in terms of the given parameters (1 MARK).

(b) Using (a), determine r_0, the "radius of the circular orbit" for the rocket, and E_0, the energy of the circular orbit (1 MARK).

(c) Using (a) and (b), determine the eccentricity ϵ of the orbit of the rocket (1 MARK).

(d) Using (b) and (c), determine the conditions on α such that the rocket will become a satellite for earth (2 MARKS).

(e) For $\alpha = 3/4$, determine the maximum "height" reached by the rocket, with respect to the earth's surface (1 MARK).

(f) Determine the nature (i.e., shape) of the orbit for $\alpha = 1$, and give your reasons for the same (1 MARK).[1]

2. Consider the vector field,

$$\mathbf{F} = \rho \sin^2 \phi \hat{\boldsymbol{\rho}} + \rho \sin \phi \cos \phi \hat{\boldsymbol{\phi}} + 4z\hat{\mathbf{z}}$$

and the seamless structure shown in Figure 2. A cone is placed on the top surface of a cylinder. The origin is located at the centre of the bottom flat surface of the cylinder. Use cylindrical polar coordinates.

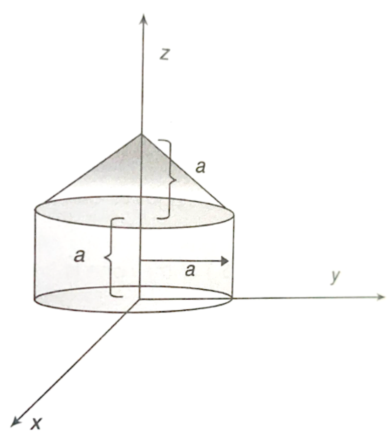

Figure 2:

(a) Write down the surface area elements for (i) the curved surface of the cylinder ($d\mathbf{S_1}$), (ii) the bottom flat surface of the cylinder ($d\mathbf{S_2}$), (iii) the curved surface of the cone placed on top of the cylinder ($d\mathbf{S_3}$) indicating the sense(direction) of the surface normals. Also write down the expression for the volume element dV (2 MARKS).

(b) Determine the flux of the vector field $\mathbf{F}$ passing through the

i. curved surface of the cylinder (1 MARK).

ii. bottom flat surface of the cylinder (1 MARK).

iii. curved surface of the cone placed on top of the cylinder (2 MARKS).

(c) Calculate the divergence of the vector field and verify Gauss' divergence theorem using the seamless structure shown in Figure 2 (1 MARK).

[1] The reasoning here should be based on the rigorous classification of orbits on the basis of.............., and not just a simple statement.

3. **Give ONLY THE FINAL ANSWERS to the questions below in your answer sheet, against the question number. Each question is worth 1 MARK, NO partial marks here.**

   (a) A particle of mass m is constrained to move on the inner surface of a paraboloid, described by the equation $z = \rho^2/4a$ (where $\rho = \sqrt{x^2 + y^2}$). Here, the gravitational force on the particle is $-mg\hat{\mathbf{z}}$. The initial angular momentum is $\boldsymbol{\ell} = \ell_\rho \hat{\boldsymbol{\rho}} + \ell_\phi \hat{\boldsymbol{\phi}} + \ell_z \hat{\mathbf{z}}$. The total energy of the particle, in cylindrical coordinates, can be expressed as

$$E = \frac{m_{\text{eff}}(\rho)}{2}\dot{\rho}^2 + U_{\text{eff}}(\rho)$$

   where m_{eff}=...................... and U_{eff} =..................................

   (b) A satellite in a circular orbit is given a tangential thrust at its apogee, so that its speed is doubled. The eccentricity of the resulting orbit is $\epsilon =$ ....

   (c) For a satellite in elliptical orbit around earth with period T, the orbital speed at perigee is 4 times the same at apogee. If the orbital speed at apogee is v, the semi-major axis of the orbit is $a =$.....................

   (d) A particle of mass m is moving in a central force field with $F(r) = -\alpha \frac{e^{-\lambda r}}{r^2}$ where $\alpha, \lambda > 0$. The condition on its angular momentum ℓ such that at least one circular orbit is possible is ....

   (e) State whether **TRUE** or **FALSE**: $\nabla \cdot \hat{\mathbf{r}} = 0$ everywhere in space, except at the origin.

   (f) An incompressible fluid (of uniform and constant density, denoted by ρ_m) leaks out of a point source at the origin, and spreads isotropically in space, in the absence of gravity. If the rate of mass efflux from the container is $\dot{M}$, the velocity field of the fluid, in spherical polar coordinates, after the fluid has spread in all available space, is $\mathbf{v}(r)$ =...............................

   (g) State whether **TRUE** or **FALSE** : For *any* closed loop $\mathcal{C}$ lying in the $x - y$ plane, $\oint_{\mathcal{C}} \hat{\mathbf{n}} dl = 0$, where $\hat{\mathbf{n}}$ is the local unit outward normal to the loop, also lying in the $x - y$ plane (see Fig.3), and dl is the infinitesimal arc-length along the loop.

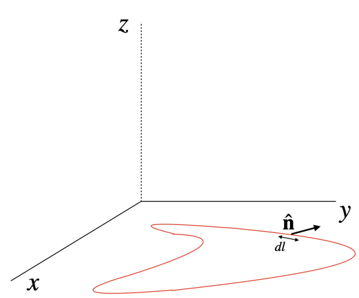

Figure 3: