DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY MADRAS

PH1010 Physics I Quiz 2 17/10/2013 8:00AM-8:50AM Max. Marks 20

Instructions: First read these instructions. Not following them will result in the paper not being evaluated.

- **There are FOUR questions in all.**
- **Use ONLY the answer sheet provided. NO additional sheets will be provided.**
- **Write your NAME, ROLL No. and BATCH.**
  Note that the BATCH (A to J) MUST be clearly indicated next to the Course No. box.
- **Use a black or blue pen ONLY.**
- **Answers to part-A and part-B questions must NOT be mixed up. Clearly mark ONLY TWO sections of your answer booklet where part-A and part-B are attempted.**
- **NO cellphones, calculators etc.**
- **All symbols, unless otherwise specified, have their usual meanings.**

Good Luck!

Part A

1. Indicate clearly your answer, which should be **a number or an expression.** Highlight your answer by enclosing it in a box. **No need to show any working or explanations.** **(5)**

   (a) Evaluate $\boldsymbol{\nabla} \times \boldsymbol{r}$. ($\boldsymbol{r}$ is the position vector).

   (b) given a force field $\boldsymbol{F} = -y\hat{\boldsymbol{e}}_{\boldsymbol{x}} + x\hat{\boldsymbol{e}}_{\boldsymbol{y}} + z\hat{\boldsymbol{e}}_{\boldsymbol{z}}$, calculate the work done (arbitrary units) in moving the particle from $(1, 0, 0)$ to $(-1, 0, \pi)$ along a helix parametrized using the variable u, $(-\infty < u < \infty)$, as $x = \cos u,\ y = \sin u,\ z = u$.

   (c) If $\phi = x^2 - y^2 + 2xy$, is a function on the $x - y$ plane, what is the direction of **maximum decrease** of ϕ at the point $(1, 1)$?

   (d) Write the **Lagrangian** for a particle (mass m) in three-dimensions, subjected to only the gravitational potential mgz, using Cartesian coordinates, and indicate the **cyclic coordinates**.

   (e) A particle of unit mass is subjected to the one-dimensional potential which in appropriate units is $-x^2/2$. What is the equation of the phase-space curve on which any initial condition **approaches the equilibrium point** arbitrarily closely as $t \to \infty$?

2. Answer TRUE or FALSE (**No explanation/working needed**). Write your answer, along with the question number, **in a box.** **(5)**

   (a) $\boldsymbol{F} = -y\hat{\boldsymbol{e}}_{\boldsymbol{x}} + x\hat{\boldsymbol{e}}_{\boldsymbol{y}} + z\hat{\boldsymbol{e}}_{\boldsymbol{z}}$ is a **conservative force** field. TRUE or FALSE?

   (b) $\boldsymbol{F} = \hat{\boldsymbol{e}}_{\boldsymbol{r}}\dfrac{\cos\theta}{r}$ is a **central and conservative force**. TRUE or FALSE?

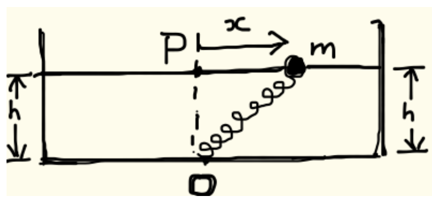
   (c) The time-period of the simple pendulum tends to **infinity** as the amplitude tends to π. TRUE or FALSE?

   (d) If $V(\rho, \phi)$ is the potential energy for a particle on a plane, **the force in the $\hat{\boldsymbol{e}}_{\boldsymbol{\phi}}$** direction is $-\dfrac{\partial V}{\partial \phi}$. TRUE or FALSE?

   (e) In **spherical polar** coordinates $\dfrac{\partial \hat{\boldsymbol{e}}_{\boldsymbol{r}}}{\partial \phi} = 0$. TRUE or FALSE?

Part B

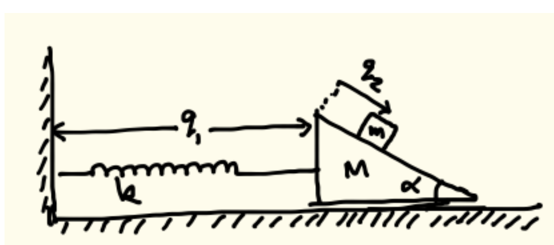
3. A bead of mass m is constrained to slide without friction along a rigid horizontal wire. The mass is attached to a spring of stiffness k and relaxed length L_0. Choose coordinates along the wire so that $x = 0$ occurs at the point P vertically above the support point of the spring O; let h be the distance between this support point and the wire. **(5)**



   (a) Write down the potential energy and hence find all the **equilibrium points and determine if they are stable or unstable**. Pay attention to the two cases $h < L_0$ and $h > L_0$.

   (b) Schematically sketch the potential energy and typical **phase space curves for the two cases**, including local behaviors around equilibrium points.

4. A block of mass m slides without friction down a wedge of mass M (angle α) which in turn is connected by a spring of stiffness k and natural length L_0, to a wall. The wedge can slide without friction on a horizontal surface. **(5)**



   (a) Using the indicated q_1 and q_2 as generalized coordinates write the Lagrangian of the system.

   (b) Using the Euler-Lagrange equation, find the equations of motion for the above system.