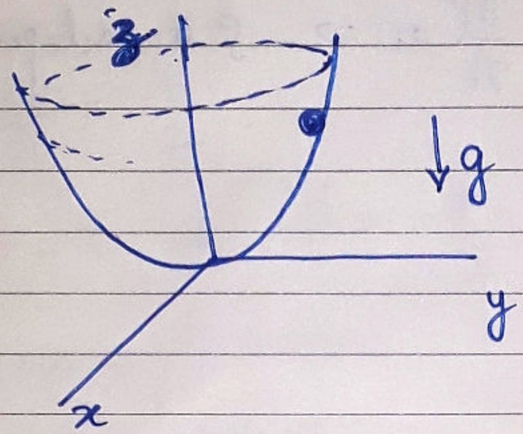
Q2 DETAILED ~~ROUGH~~ SOLUTIONS

Date No.

1. 

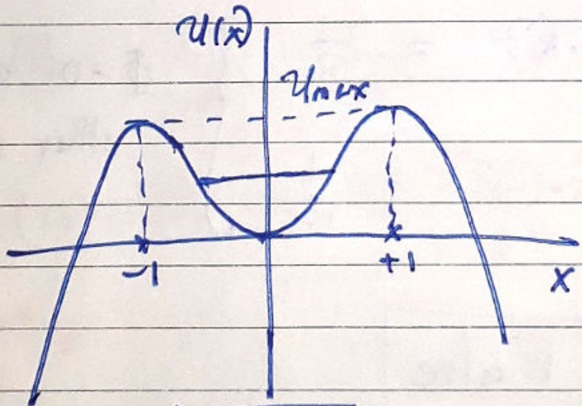
Both L_z and E are conserved.
$\downarrow$
since the system is invariant w.r.t rotations about z-axis

P_z not conserved because of grav

P_x, P_y not conserved because of normal reaction force from the glass

$\therefore$ $\boxed{\text{Ans} = [C]}$

2. [C] is incorrect, hence $\boxed{\text{Ans} = \cancel{[C]}}$ [C, B] (both or one). All other statements are correct.
[B] is incorrect, since $\dot{\hat{\rho}} = \dot{\phi}\hat{\phi}$
$a_r = \ddot{r} - r\dot{\phi}^2$.

3. 

Maxima at $x^2 = 1 \Rightarrow x = \pm 1$.

$U_{max} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$.

$E = \frac{1}{2} \times 1 \times \left(\frac{1}{2}\right)^2 = \frac{1}{8} < \frac{1}{4}$.

$\therefore$ Particle can only perform bounded motion.

$\boxed{\text{False}}$

4. $U_{eff}(r) = \frac{L^2}{2mr^2} + \underbrace{U(r)}_{0}$ for free particle.

$\therefore$ $\boxed{\text{False}}$

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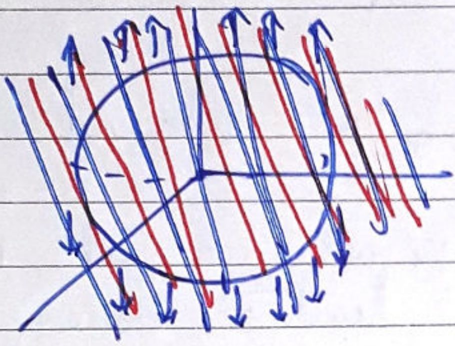
5. $\nabla\cdot\vec{J} = 0$, hence $\frac{\partial \rho}{\partial t} = 0 \Rightarrow \rho$ is independent of

$\boxed{\text{True}}$

6.



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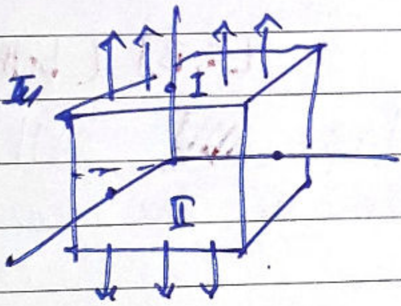

$z = \frac{1}{2}$ on surface I,

$= -\frac{1}{2}$ on surface II.

$$\left.\begin{aligned} \Phi_{I} &= \int \underbrace{z}_{1/2}\, dx\, dy\, (\hat{k}\cdot\hat{k}) = \frac{1}{2} \\ \Phi_{II} &= \int \underbrace{z}_{-\frac{1}{2}}\, dx\, dy\, \hat{k}\cdot(-\hat{k}) = \frac{1}{2} \end{aligned}\right\}$$

$\Phi = 0$ on every other surface.

$\therefore \Phi = 1$.

$\boxed{\text{False}}$

⑦ $\epsilon = \frac{r_{max} - r_{min}}{r_{max} + r_{min}} = \frac{12K - 8K}{12K + 8K} = \frac{4}{20} = \boxed{\frac{1}{5} \text{ or } 0.2}$

⑧ $I = \frac{MR^2}{2} = \frac{1}{2}$

$I\ddot{\theta} = -\tau \Rightarrow \dot{\theta} = -\frac{\tau}{I}t + \omega_0$

$\dot{\theta} = 1200$ rpm @ $t = 0$

$\Rightarrow \omega_0 = 1200$ rpm

$\Rightarrow \omega_0 = \frac{1200}{60}$ rps $= 20$ rotations/sec $= 20 \times 2\pi$ rad/sec
$= 40\pi$ rad/sec.

$0 = -\frac{\tau . 40}{(1/2)} + 20 \times 2\pi \Rightarrow \quad -80\tau = -40\pi$

$\boxed{\tau = \frac{\pi}{2}}$

(Some students who don't convert 20 rot./sec to 40π rad./sec will answer this as $\frac{1}{4}$) $\rightarrow$ No marks for this

⑨ $U_{eff}(r) = \frac{l^2}{2mr^2} - \frac{C}{3r^3}$

$U'_{eff}(r_0) = 0 \Rightarrow r_0 = \frac{mC}{l^2}$

$U_{eff}^{max.} = \frac{l^6}{6m^3C^2}$

⑩ $m(\ddot{r} - r\dot{\theta}^2) = 0$

$m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = f_\theta = m\dot{r}\dot{\theta}$

$r\ddot{\theta} = -\dot{r}\dot{\theta}$

$\Rightarrow \frac{\ddot{\theta}}{\dot{\theta}} = -\frac{\dot{r}}{r} \Rightarrow \ln\dot{\theta} = C - \ln r$

$\dot{\theta} = D/r$

$l = mr^2\dot{\theta} = mr^2\frac{D}{r} = mrD \Rightarrow \boxed{l \propto r}$

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11. $\rho = \sqrt{x^2+y^2} = R, \quad \phi = \tan^{-1} y/x = \omega t \Rightarrow \dot{\phi} = \omega$

$\therefore \vec{r} = R\,\hat{\rho} + z\,\hat{K}$ — (1)

$\vec{V} = \dot{\vec{r}} = \dot{\rho}\,\hat{\rho} + \rho\dot{\phi}\,\hat{\phi} + \dot{z}\,\hat{K}$ (OR $R\dot{\hat{\rho}} + \dot{z}K = R\dot{\phi}\,\hat{\phi} + \dot{z}$ — $(\frac{1}{2})$

$= R\omega\,\hat{\phi} + V_0\,\hat{K}$ — $(\frac{1}{2})$

$$\vec{L} = \vec{r}\times\vec{p} = \begin{vmatrix} \hat{\rho} & \hat{\phi} & \hat{K} \\ R & 0 & z \\ 0 & mR\omega & V_0 \end{vmatrix}$$ — or equivalent $(\frac{1}{2})$

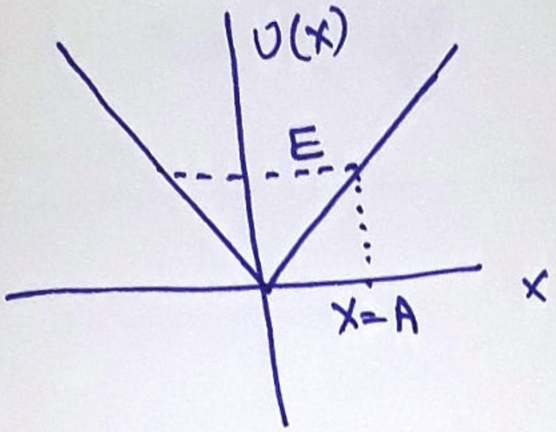
$\vec{L} = -mR\omega z\,\hat{\rho} - mV_0 R\,\hat{\phi} + mR^2\omega\,\hat{K}$ $-(\frac{1}{2})$

$U = -\int F \cdot dr$

For this 1-D case

$U(x) = +F_0 x, \quad x > 0$

$\quad = -F_0 x, \quad x < 0$ where $F_0 > 0$



$E = F_0 A$ [KE=0 at $x = A$]

From conservation of energy,

$\frac{1}{2}mv^2 + U(x) = F_0 A$

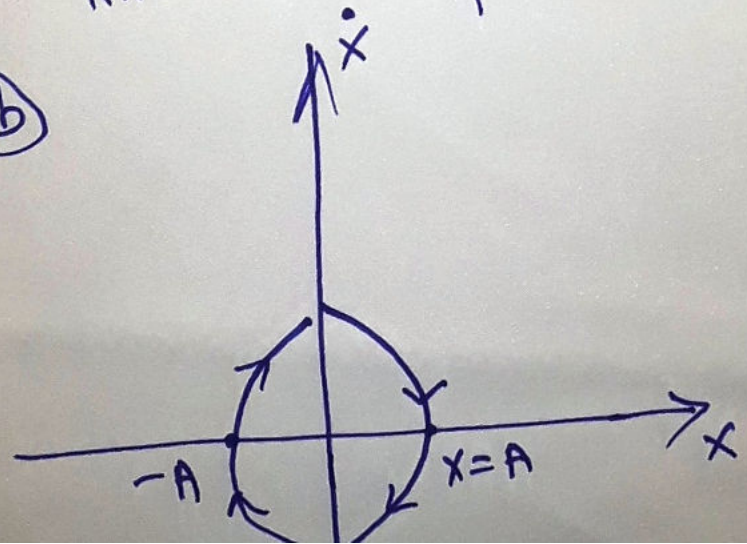
$\Rightarrow \quad v = \pm\sqrt{\frac{2F_0}{m}(A-x)} \quad , x > 0$

$\quad = \pm\sqrt{\frac{2F_0}{m}(A+x)} \quad , x < 0$

LHS is a parabola with vertex at $x = -A$

RHS is a parabola with vertex at $x = +A$.

(b)



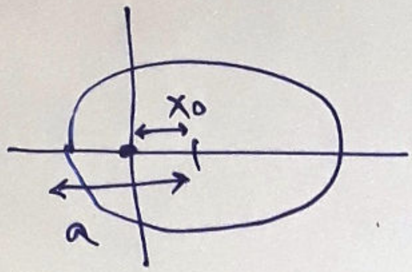
(c) For $x>0$, $m\ddot{x} = -F_0$

$$x(t) = A - \frac{F_0 t^2}{2m} \quad (\text{using I.C., } x(0) = A,\ \dot{x}(0) = 0)$$

$T = 4t_0$ where t_0 is the time taken to go from $x = A$ to $x = 0$

$$T = 4\sqrt{\frac{2mA}{F_0}}$$

(a) $r = r_0/(1-\epsilon\cos\theta)$ — 1 mark



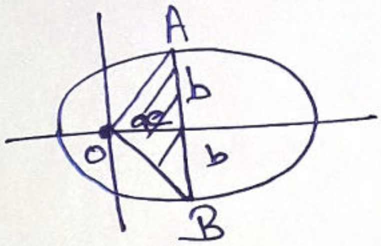
(b) $r_{max} = \dfrac{r_0}{1-\epsilon}$, $r_{min} = \dfrac{r_0}{1+\epsilon}$

$a = \dfrac{A}{2} = \dfrac{r_{max}+r_{min}}{2} = \dfrac{r_0}{1-\epsilon^2}$ — $\frac{1}{2}$ mark

$\tilde{x}_0 = a - r_{min} = r_0\left(\dfrac{1}{1-\epsilon^2} - \dfrac{1}{1+\epsilon}\right) = \dfrac{r_0\epsilon}{1-\epsilon^2}$ — $\frac{1}{2}$ mark either

Note, $\tilde{x}_0 = a\epsilon$ $[x_0 = 2ae, 0]$

(c)



Additional area swept is $= 2(\text{Area of triangle AOB.})$ — 1 mark

$\text{Area}_\Delta = \frac{1}{2} \times 2b \times ae = aeb$

Planet sweeps equal area in equal time-intervals (Kepler's 2nd law)
The areas swept in right and left halves are

$= \frac{1}{2}\pi ab \pm aeb$

$\Rightarrow \Delta t \equiv 2aeb$ area

It sweeps an area πab in 1-year

$\Delta t = 1 \text{ year} \times \left(\dfrac{2e}{\pi}\right) = 365 \text{ days} \times \dfrac{2}{\pi}\dfrac{\pi}{200}$

$= \dfrac{365}{100}$ days

$= \underline{3.65}$ days — 1 mark