DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY MADRAS

PH1010-2020 **Mini-Test 1 (7 Marks)**

Undertaking of ACADEMIC HONESTY:

By submitting this test, it is understood that you promise to uphold academic honesty by neither giving nor receiving any unauthorised help on this mini-test. The answers should be based on your individual effort. Any deviation, if identified, may lead to reduction in marks.

Important instructions:

1. Write your answers with ALL required steps in good quality A4 sheets.
2. Begin your answers for every question on a fresh A4 sheet.
3. Use only dark blue or black ink for writing answers (strictly no pencil).
4. Write the **name and IITM roll no** at the top right corner of the A4 sheet. Number the pages in order.
5. Use of calculator, books, and online resources are permitted.
6. Digitise/scan your answer scripts neatly taking into account proper cropping and contrast.
7. Upload only a single, combined PDF file.
8. Please check your PDF file completely before uploading the same.
9. DO NOT FORGET to click **SUBMIT** button after uploading your answer file.
10. The marks you earn in this test will count towards your aggregate in the course.
11. If your answer is a vector quantity, make sure that its direction is clearly indicated (e.g., by using appropriate unit vectors).

Submission deadline: Monday, 30th of November, 9:00 AM

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1. A *critically* damped, driven oscillator's displacement $x(t)$ satisfies the equation of motion

$$\ddot{x} + 2\omega_0\dot{x} + \omega_0^2 x = f_0 \cos\omega t \tag{1}$$

where ω_0 is the natural frequency, and ω is the "driving frequency".

(i) Find the particular solution to the above equation, in the form $x_p(t) = A\cos(\omega t + \phi)$. Your answer should clearly give the expressions for A and ϕ.

(ii) The *homogeneous* equation $\ddot{x} + 2\omega_0\dot{x} + \omega_0^2 x = 0$ has $e^{-\omega_0 t}$ as one solution. Show, by substitution, that the function $te^{-\beta t}$ can be the second solution. Find β in terms of ω_0.

(iii) Use the above results to construct the complete solution to Eq. 1, subject to the initial conditions $x(0) = 0 = \dot{x}(0)$. (**3 MARKS**)

2. Consider a particle of unit mass, moving in a region of space where its potential energy is

$$U(\mathbf{r}) = U_0 \frac{e^{-\mathbf{p}\cdot\mathbf{r}}}{r}, \tag{2}$$

where $\mathbf{p}$ is a constant vector and U_0 is a constant.

(i) Find the force $\mathbf{F}$ acting on the particle.

(ii) Assume $\mathbf{p} = p\hat{\mathbf{j}}$. Imagine that the particle is released in a state of rest, at the point $(0, 1/p, 0)$. Find the velocity $\mathbf{v}_\infty$ of the particle at infinity. (**2 MARKS**)

3. A particle is moving in the $x - y$ plane such that its position vector is

$$\mathbf{r}(t) = R\left\{(\omega t - \sin\omega t)\hat{\mathbf{i}} + (1 - \cos\omega t)\hat{\mathbf{j}}\right\} \qquad (R > 0) \tag{3}$$

(i) Derive an expression for the speed $v(t)$ of the particle at time t, and find its maximum value $v_{\max}$.

(ii) Find the mean values of the x and y components of velocity over one cycle, viz.,

$$\overline{v_x} = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} v_x dt \quad ; \quad \overline{v_y} = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} v_y dt.$$

(**2 MARKS**)