Session-20

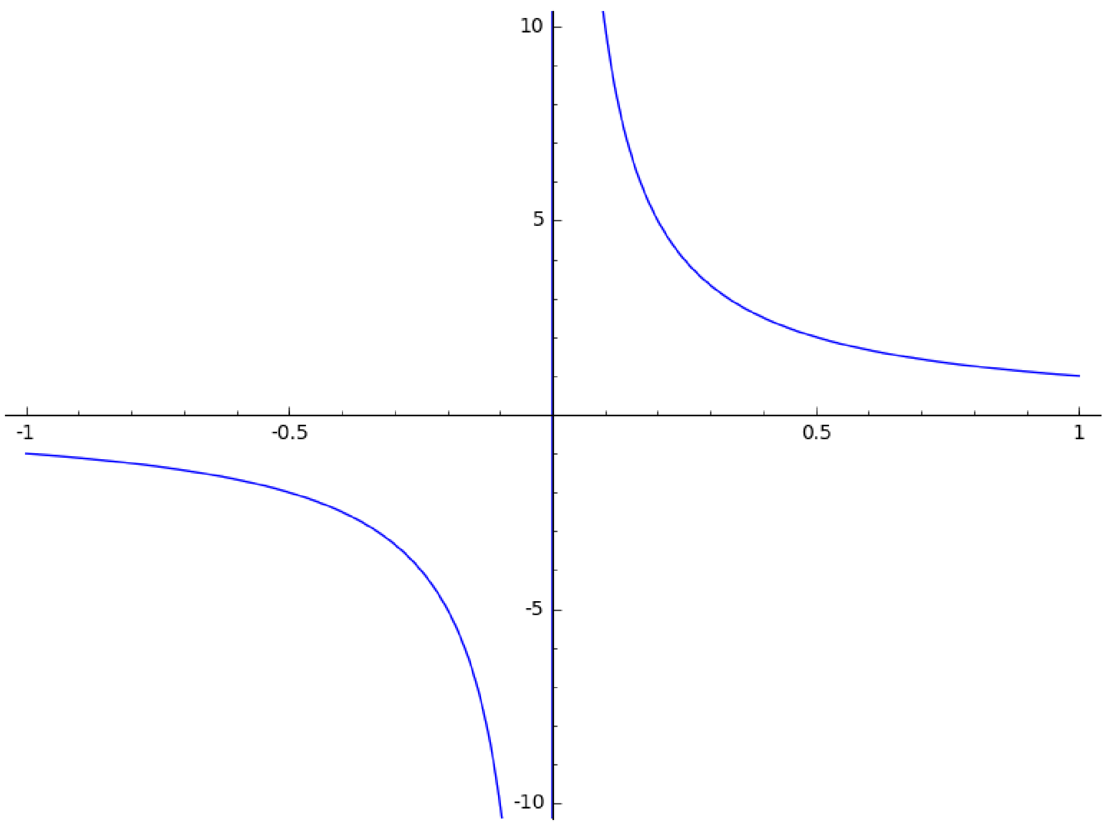
October 09, 2019

1 Limits, Series and Transforms

1.1 Limits

In [2]:
```
var('x')
f(x) = 1/x
plot(f,x,-1,1,ymin=-10, ymax=10)
```

Out[2]:



In [3]:
```
f.limit(x=0,dir='minus')
```

Out[3]: x |--> -Infinity

In [4]:
```
f.limit(x=0,dir='plus')
```

Out[4]: x |--> +Infinity

In [5]:
```
g(x) = (2*x + 8)/(x^2 + x - 12)
g.show()
```

x |--> 2*(x + 4)/(x^2 + x - 12)

Try evaluating g(-4)

In [8]:
```
g.limit(x=4)
```

Out[8]: x |--> 2

In [9]:
```
h(x)=(x^2-4)/(x-2)
h.show()
```

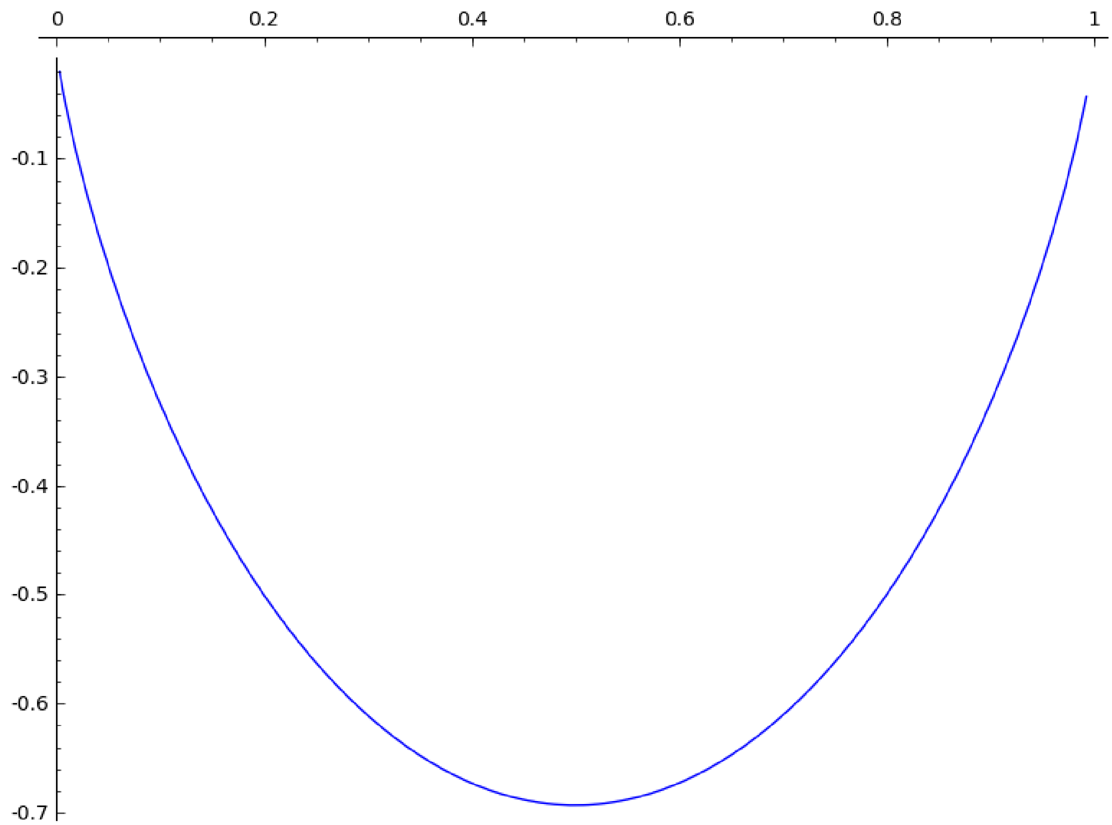
x |--> (x^2 - 4)/(x - 2)

In [11]:
```
h.limit(x=2)
```

Out[11]: x |--> 4

In [13]:
```
ga(x)=x*log(x)+(1-x)*log(1-x)
plot(ga,x,0,1)
```

Out[13]:




```
In [14]: ga.limit(x=0)

Out[14]: x |--> 0

In [15]: ga.limit(x=1)

Out[15]: x |--> 0

In [17]: dga(x)=diff(ga,x)
         dga.show()

x |--> log(x) - log(-x + 1)

In [19]: dga.limit(x=0)

Out[19]: x |--> Infinity

In [20]: dga.limit(x=1)

Out[20]: x |--> Infinity
```

1.2 Series

```
In [21]: s(x)=sin(x)/(x^2)
         s.show()
```

```
x |--> sin(x)/x^2
```

Expanding around x=1 using power series

```
In [23]: s1(x)=s.series(x==1,4)
         s1.show()
```

```
x |--> (sin(1)) + (cos(1) - 2*sin(1))*(x - 1) + (-2*cos(1) + 5/2*sin(1))*(x - 1)^2 + (17/6*cos(1
```

Expanding using Taylor series

```
In [24]: s2(x)=s.taylor(x,1,4)
         s2.show()
```

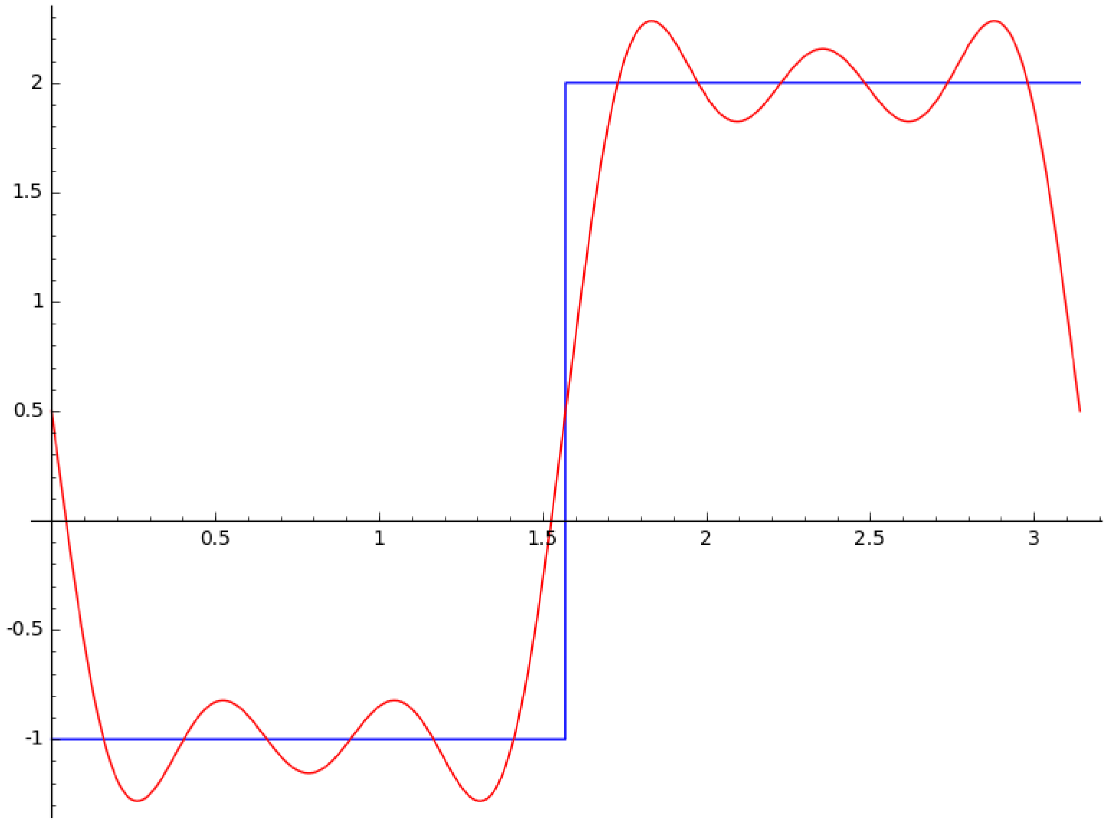
```
x |--> -1/24*(x - 1)^4*(88*cos(1) - 85*sin(1)) + 1/6*(x - 1)^3*(17*cos(1) - 18*sin(1)) - 1/2*(x
```

Fourier Series

```
In [36]: f = piecewise([((0,pi/2), -1), ((pi/2,pi), 2)])
         f.fourier_series_cosine_coefficient(0)
         f.fourier_series_sine_coefficient(5)
         s5 = f.fourier_series_partial_sum(5);
         s5.show()
         plot(f, (0,pi)) + plot(s5, (x,0,pi), color='red')
```

```
-6/5*sin(10*x)/pi - 2*sin(6*x)/pi - 6*sin(2*x)/pi + 1/2
```

```
Out[36]:
```



1.3 Sum of Series

```
In [27]: var('n,k')
```

```
Out[27]: (n, k)
```

Alternating harmonic series

```
In [28]: ss1(k) = (-1)^(k+1)*1/k
         ss1.show()
```

```
k |--> (-1)^(k + 1)/k
```

```
In [29]: ss1.sum(k,1,infinity)
```

```
Out[29]: log(2)
```

Binomial series

```
In [30]: ss2(k)=binomial(n,k)
         ss2.show()
```

```
k |--> binomial(n, k)
```

```
In [31]: ss2.sum(k,1,infinity)
```

```
Out[31]: 2^n - 1
```

Harmonic series

```
In [32]: ss3 = 1/k
         ss3.sum(k,1,infinity)
```

```
        ---------------------------------------------------------------------------

        ValueError                                Traceback (most recent call last)

        <ipython-input-32-b9a86d7ca9a1> in <module>()
          1 ss3 = Integer(1)/k
    ----> 2 ss3.sum(k,Integer(1),infinity)


        /opt/SageMath/sage-8.2/local/lib/python2.7/site-packages/sage/symbolic/expression.pyx in
      12200         """
      12201         from sage.calculus.calculus import symbolic_sum
    > 12202         return symbolic_sum(self, *args, **kwds)
      12203
      12204     def prod(self, *args, **kwds):


        /opt/SageMath/sage-8.2/local/lib/python2.7/site-packages/sage/calculus/calculus.pyc in s
        609
        610     if algorithm == 'maxima':
    --> 611         return maxima.sr_sum(expression,v,a,b)
        612
        613     elif algorithm == 'mathematica':


        /opt/SageMath/sage-8.2/local/lib/python2.7/site-packages/sage/interfaces/maxima_lib.pyc
        896 # could not find an example where 'Pole encountered' occurred, though
        897 #            if "divergent" in s or 'Pole encountered' in s:
    --> 898                 raise ValueError("Sum is divergent.")
        899             elif "Is" in s: # Maxima asked for a condition
        900                 self._missing_assumption(s)


        ValueError: Sum is divergent.
```

1.4 Transforms

Laplace Transform

```
In [33]: var('t, a, k, s')
         f(t) = sin(k*t)
         F(s) = f.laplace(t,s)
         F.show()
```

```
s |--> k/(k^2 + s^2)
```

```
In [34]: G(s) = 1/((s-1)*(s+2)*(s+4))
         G.show()
         g(t) = G.inverse_laplace(s,t)
         g.show()
```

```
s |--> 1/((s + 4)*(s + 2)*(s - 1))
```

```
t |--> -1/6*e^(-2*t) + 1/10*e^(-4*t) + 1/15*e^t
```