# Session-18

September 25, 2019

## 1 Symbolic Expressions

In this notebook we explore things that can be done with expressions.

```
In [1]: var('x,y')
```

```
Out[1]: (x, y)
```

```
In [2]: poly1 = (x+3)^3
        poly1.show()
```

```
(x + 3)^3
```

```
In [3]: type(poly1)
```

```
Out[3]: <type 'sage.symbolic.expression.Expression'>
```

```
In [4]: poly1.expand().show()
```

```
x^3 + 9*x^2 + 27*x + 27
```

```
In [5]: poly2 = x^2+2*x-3
        poly2.show()
```

```
x^2 + 2*x - 3
```

```
In [6]: poly2.factor().show()
```

```
(x + 3)*(x - 1)
```

## 1.1 Equations as expressions

```
In [7]: expr1 = (y-3)/(x^2+1)==x^3-5
        expr1.show()
```

(y - 3)/(x^2 + 1) == x^3 - 5

```
In [8]: expr1.lhs().show()
```

(y - 3)/(x^2 + 1)

```
In [9]: expr1.rhs().show()
```

x^3 - 5

```
In [10]: expr2=expr1.multiply_both_sides(x^2+1)
         expr2.show()
```

y - 3 == (x^3 - 5)*(x^2 + 1)

```
In [11]: expr3=expr2.add_to_both_sides(3)
         expr3.show()
```

y == (x^3 - 5)*(x^2 + 1) + 3

```
In [12]: expr3a=expr3.expand().show()
```

y == x^5 + x^3 - 5*x^2 - 2

## 1.2 Analytical solutions

```
In [13]: solve(expr1,y)
```

Out[13]: [y == x^5 + x^3 - 5*x^2 - 2]

```
In [14]: expr4 = x^2/2-x-2 == 0
         solve(expr4,x)
```

Out[14]: [x == -sqrt(5) + 1, x == sqrt(5) + 1]

```
In [15]: solve(x^2+9*x+15==0,x)
```

Out[15]: [x == -1/2*sqrt(21) - 9/2, x == 1/2*sqrt(21) - 9/2]

```
In [16]: var('a,b,c')
         [x1,x2]=solve(a*x^2+b*x+c==0,x)
         x1.show()
         x2.show()
```

x == -1/2*(b + sqrt(b^2 - 4*a*c))/a


x == -1/2*(b - sqrt(b^2 - 4*a*c))/a


## 1.3  Manipulation of polynomials

```
In [17]: gcdpart1 = x^2-8*x+15
         gcdpart2 = x^2-5*x+6
         factor(gcdpart1).show()
         factor(gcdpart2).show()
         gcd(gcdpart1, gcdpart2)
```

(x - 3)*(x - 5)


(x - 2)*(x - 3)


Out[17]: x - 3

## 1.4  Rational functions

```
In [18]: expr5 = (x+1)/(x^2*(x+2)^3)
         expr5.show()
```

(x + 1)/((x + 2)^3*x^2)


```
In [19]: expr5.numerator().show()
```

x + 1


```
In [20]: expr5.denominator().show()
```

(x + 2)^3*x^2


```
In [21]: expr5.expand_rational().show()
```

1/((x^3 + 6*x^2 + 12*x + 8)*x) + 1/((x^3 + 6*x^2 + 12*x + 8)*x^2)


```
In [22]: expr5.partial_fraction().show()
```

1/16/(x + 2) - 1/16/x + 1/8/x^2 - 1/4/(x + 2)^3

## 1.5 Substitutions

```
In [23]: expr6 = 1/x+ 3*x^2 + cos(x)

In [24]: var('t')
         expr6.substitute(x==5*t).show()
```

```
75*t^2 + 1/5/t + cos(5*t)
```

## 1.6 Manipulating Trigonometric Expressions

```
In [25]: texp1 = sin(x)^2*cos(x)^2+sin(y)^2
         texp1.show()
```

```
cos(x)^2*sin(x)^2 + sin(y)^2
```

```
In [26]: texp2 = texp1.trig_reduce()
         texp2.show()
```

```
-1/8*cos(4*x) - 1/2*cos(2*y) + 5/8
```

```
In [27]: texp2.trig_simplify().show()
```

```
-cos(x)^4 + cos(x)^2 + sin(y)^2
```

## 1.7 Logarithms, rational functions and radicals

```
In [28]: gexp1 = log(x^2*sin(x)/sqrt(1+x))
         gexp1.show()
```

```
log(x^2*sin(x)/sqrt(x + 1))
```

```
In [29]: gexp2 = gexp1.expand_log()
         gexp2.show()
```

```
-1/2*log(x + 1) + 2*log(x) + log(sin(x))
```

```
In [30]: gexp3=gexp2.simplify_log()
         gexp3.show()
```

```
log(x^2*sin(x)) - 1/2*log(x + 1)
```

## 1.8    Rational functions

```
In [31]: gexp4 = (1+x)/(x+x^2)
         gexp4.show()
```

(x + 1)/(x^2 + x)

```
In [32]: gexp5 = gexp4.simplify_rational()
         gexp5.show()
```

1/x

## 1.9    Radicals

```
In [33]: gexp6 = sqrt(x^2+x)/sqrt(x)
         gexp6.show()
```

sqrt(x^2 + x)/sqrt(x)

```
In [34]: gexp7 = gexp6.canonicalize_radical()
         gexp7.show()
```

sqrt(x + 1)