Session-19

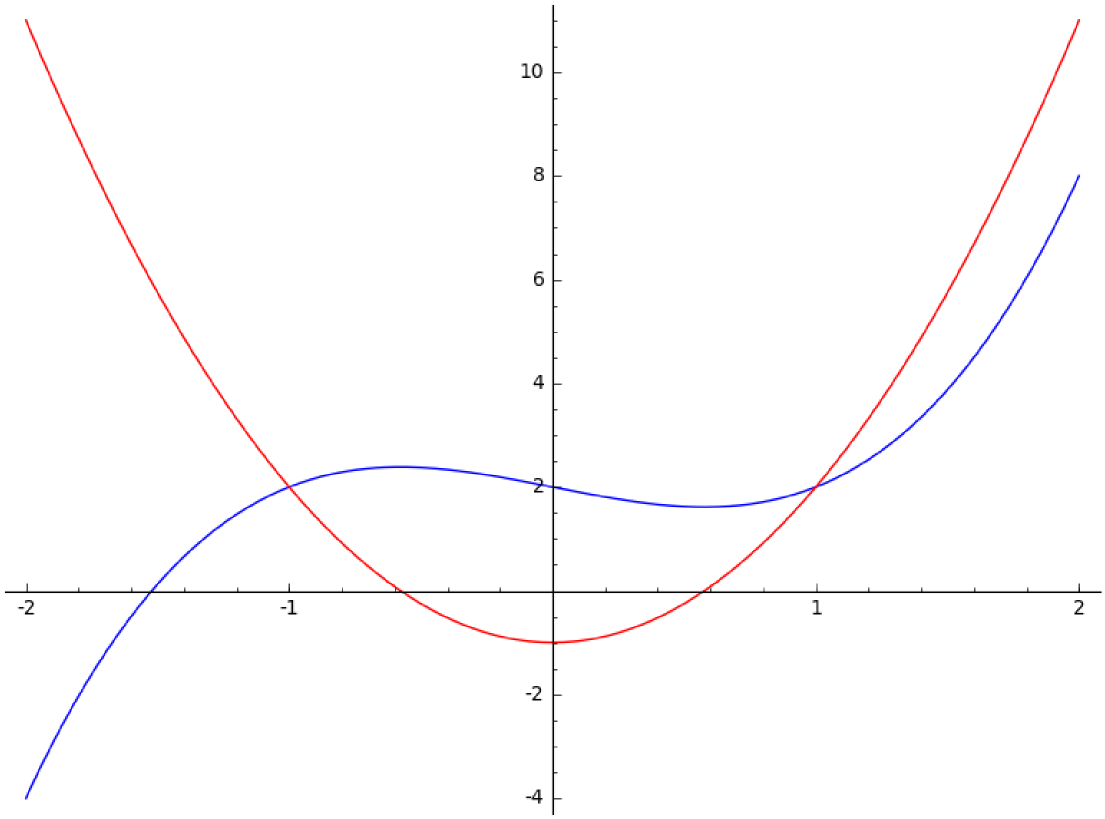
September 26, 2019

1 Functions, Differentials and Integrals

```
In [1]: var('a,b,c,x')

Out[1]: (a, b, c, x)
```

1.1 Derivatives

```
In [6]: f(x)=a*x^2+b*x+c
        f.show()

x |--> a*x^2 + b*x + c

In [7]: df(x)=f.derivative(x)
        df.show()

x |--> 2*a*x + b

In [23]: diff(e^sin(x^2),x).show()

2*x*cos(x^2)*e^(sin(x^2))

In [25]: g(x)=x^3-x+2
         gprime(x)=g(x).derivative()
         gprime.show()
         plot(g(x),-2,2,color='blue') + plot(gprime(x),-2,2,color='red')

x |--> 3*x^2 - 1

Out[25]:
```



1.2 Partial derivatives

```
In [9]: f1(x,y) = 3*x^4*y^3 + 9*y*x^2 - 4*x + 8*y
        f1.show()
```

```
(x, y) |--> 3*x^4*y^3 + 9*x^2*y - 4*x + 8*y
```

```
In [10]: df1dx(x,y) = diff(f1,x)
         df1dx.show()
```

```
(x, y) |--> 12*x^3*y^3 + 18*x*y - 4
```

```
In [11]: df1dy(x,y)=diff(f1,y)
         df1dy.show()
```

```
(x, y) |--> 9*x^4*y^2 + 9*x^2 + 8
```

```
In [12]: d2f1dxy(x,y)=diff(f1,x,y)
         d2f1dxy.show()
```

```
(x, y) |--> 36*x^3*y^2 + 18*x
```

```
In [13]: d2f1dyx(x,y)=diff(f1,y,x)
         d2f1dyx.show()
```

```
(x, y) |--> 36*x^3*y^2 + 18*x
```

1.3 Double differentiation

```
In [14]: d2f1dx2(x,y)=diff(f1,x,2)
         d2f1dx2.show()
```

```
(x, y) |--> 36*x^2*y^3 + 18*y
```

1.4 Implicit differentiation

```
In [16]: h(x)=function('h')(x)
         expr = 5*h^2 + sin(h) == x^2
         expr.show()
```

```
x |--> 5*h(x)^2 + sin(h(x)) == x^2
```

```
In [17]: dhdx = solve(diff(expr),diff(h))
```

```
In [18]: dhdx[0].show()
```

```
diff(h(x), x) == 2*x/(cos(h(x)) + 10*h(x))
```

1.5 Indefinite Integrals

```
In [27]: integral(x*sin(x^2),x).show()
```

```
-1/2*cos(x^2)
```

```
In [28]: integral(x/(x^2+1),x).show()
```

```
1/2*log(x^2 + 1)
```

```
In [31]: integral(1/(1+x^2),x).show()
```

```
arctan(x)
```

```
In [34]: integral(e^(-x^2),x).show()
```

```
1/2*sqrt(pi)*erf(x)
```

1.6 Integration by partial fractions

```
In [39]: expr2 = (x^3-x)/(x^2+5*x+6)
         expr2.show()
```

```
(x^3 - x)/(x^2 + 5*x + 6)
```

```
In [41]: expr2.partial_fraction().show()
```

```
x + 24/(x + 3) - 6/(x + 2) - 5
```

```
In [40]: integral(expr2, x)
```

```
Out[40]: 1/2*x^2 - 5*x + 24*log(x + 3) - 6*log(x + 2)
```

1.7 Definite Integrals

```
In [35]: integral(x/(x^2+1),x,0,1)
```

```
Out[35]: 1/2*log(2)
```

```
In [38]: integral(x^2*e^x, x,0,1)
```

```
Out[38]: e - 2
```

```
In [48]: integral(1/x^2,x,2,oo)
```

```
Out[48]: 1/2
```

```
In [47]: integral(e^(-x^2),x,-oo,oo)
```

```
Out[47]: sqrt(pi)
```

```
In [53]: ans = integral((2/sqrt(pi))*exp(-x^2),x,-oo,2)
         ans.show()
         ans.full_simplify()
```

```
(sqrt(pi)*erf(2) + sqrt(pi))/sqrt(pi)
```

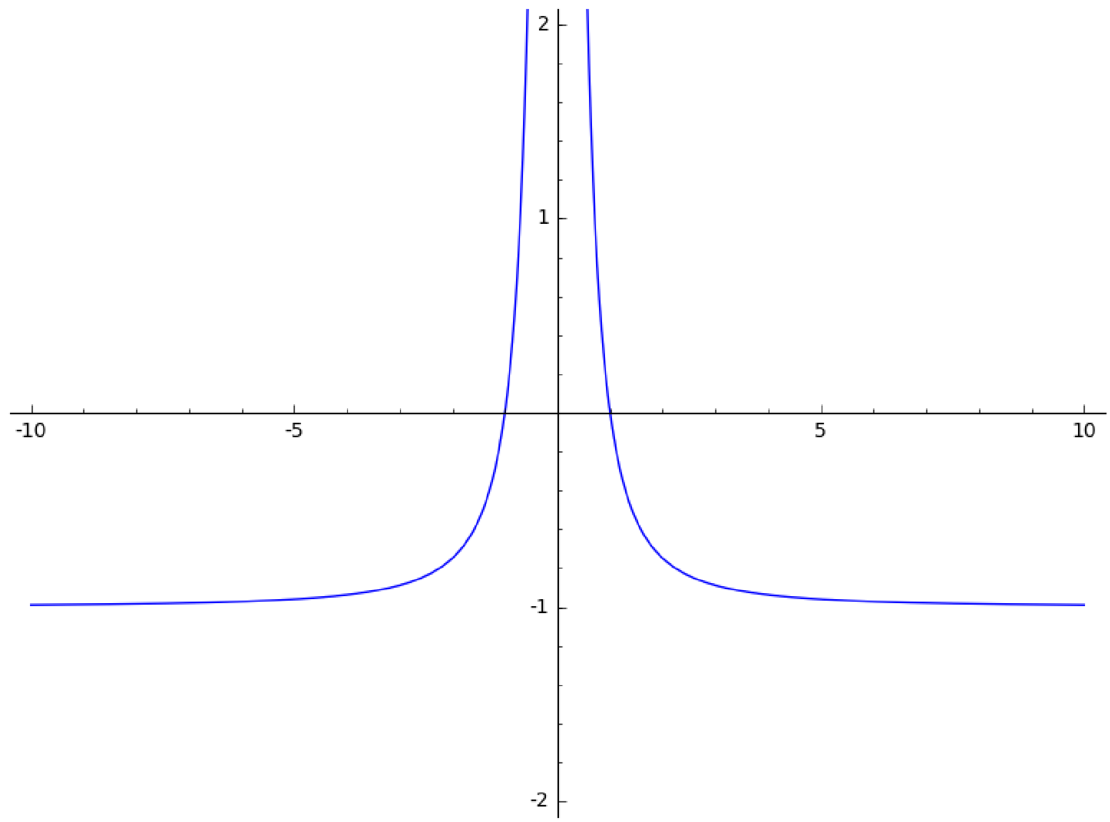
```
Out[53]: erf(2) + 1
```

1.8 Improper Integrals

```
In [43]: expr3 = -1 + 1/x^2
         expr3.show()
         plot(expr3, -10, 10, ymin=-2, ymax=2)
```

```
1/x^2 - 1
```

Out[43]:




```
In [46]: integral(expr3,x)
```

```
Out[46]: -x - 1/x
```

Try the above integral from -1 to 1

```
In [54]: integral(1/(1+x^2),x,-oo,oo)
```

```
Out[54]: pi
```