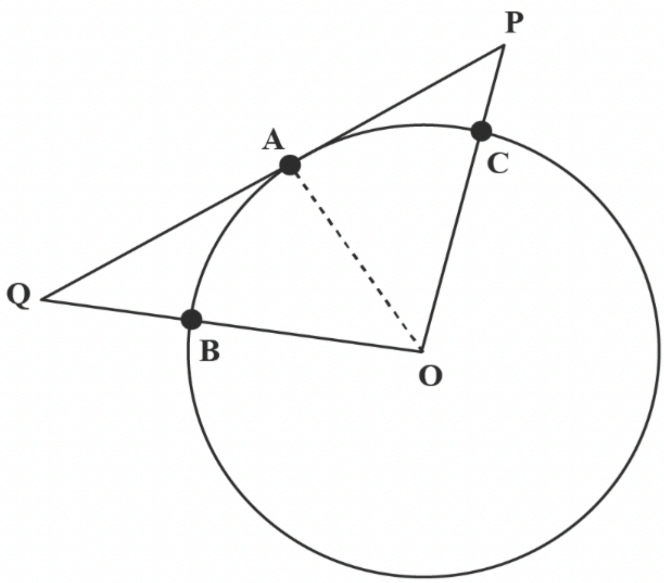
Question



A circle with centre O has radius 3 cm. PQ is a tangent through a point A on the circle. The area of the triangle OPQ is 9 cm^2.

(a) Find the length of the side PQ.

[3 marks]

(b) If the ratios of areas of $\frac{\Delta OAP}{\Delta OAQ} = \frac{3}{2}$, find the lengths of QA and PA.

[4 marks]

(c) Find the angle of the sector OBAC in radians.

[4 marks]

(d) Hence, or otherwise, find the ratio of the area of sector OBAC to area of ΔOPQ.

[3 marks]

(e) Find the relation between the above ratio and the angle of the sector.

[1 mark]

Question	Example answer	Marks	Commentary
(a)	Area of triangle OPQ: $\frac{1}{2} \times PQ \times OA$ and $OA = 3$ cm (radius) **[3]** $PQ = 6$ cm	3	Award mark for using PQ as perpendicular to OA. Award mark for using the area formula. Award mark for correct answer
(b)	$\frac{\Delta OAP}{\Delta OAQ} = \frac{3}{2}$ Let $QA = x$, then $AP = 6 - x$. **[2]** $\frac{\left(\frac{1}{2}(6-x)\,3\right)}{\frac{1}{2}x\,(3)} = \frac{3}{2} \Rightarrow x = 2.4$ Hence, $QA = 2.4$ cm and $AP = 3.6$ cm **[2]**	4	Award marks for using correct ratio of lengths.
(c)	Angles: $\angle QOA = \tan^{-1}\left(\frac{2.4}{3}\right)$ $= 0.6747 \ldots$ rad **[2]** $\angle POA = \tan^{-1}\left(\frac{3.6}{3}\right) = 0.87605 \ldots$ rad **[1]** Angle of the sector $= 1.5507..$ $= 1.55$ rad (3 s.f.) **[1]**	4	
(d)	Area of sector OBAC $= 6.9785 \ldots$ cm^2 **[2]** Ratio : $\frac{6.9785..}{9} = 0.7753 \ldots = 0.775$ (3 s.f.) **[1]**	3	
(e)	Ratio of the areas = half of the angle of the sector **[1]**	1	

Question

The area of a sector is $\frac{5\pi}{3}$ and the length of its arc is $\frac{5\sqrt{2}\pi}{3}$.

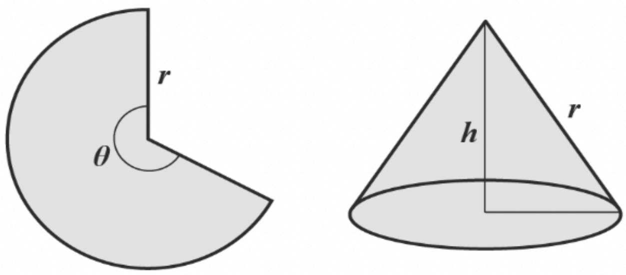
(a) Find the exact value of the radius of the sector.

[2 marks]

(b) Find the angle of the sector in radians.

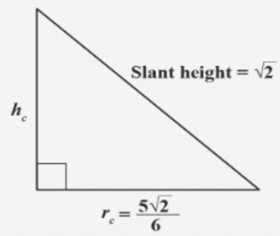
[2 marks]

(c) The sector is folded to form a cone. Find the height of the cone.



[5 marks]

Answers

Question	Example answer	Marks
(a)	$\frac{1}{2}r^2\theta = \frac{5\pi}{3}$ **[2]** $r\theta = \frac{5\sqrt{2}\,\pi}{3}$ Dividing the above two equations and multiplying the result by 2 gives $r = \sqrt{2}$	2
(b)	$r\theta = \frac{5\sqrt{2}\pi}{3}$ $\sqrt{2}\,\theta = \frac{5\sqrt{2}\pi}{3}$ $\theta = \frac{5\pi}{3}$	2
(c)	The length of arc = circumference of the base circle of the cone. **[1]** $2\pi r_c = \frac{5\sqrt{2}\pi}{3} \Rightarrow r_c = \frac{5\sqrt{2}}{6}$ The slant height of the cone is the radius of the sector. **[2]**  $h_c = \sqrt{(\sqrt{2})^2 - \left(\frac{5\sqrt{2}}{6}\right)^2}$ $= \sqrt{2 - \frac{25 \times 2}{36}} = 0.782$ to 3 d.p. **[2]**	5

Question 20

Suppose A is an obtuse angle and $\sin A = p$. Find the value of each of the following in terms of p.

(a) $\cos A$

$\cos A^2 + \sin A^2 = 1$

$\cos A^2 + p^2 = 1$

$\cos A = \pm\sqrt{1-p^2} \rightarrow -\sqrt{1-p^2}$

[3]

(b) $\tan(-A)$

$\tan(-A) = -\tan(A)$

$-\tan(A) = -\dfrac{p}{\sqrt{1-p^2}}$

$\tan(-A) = \dfrac{-p}{\sqrt{1-p^2}}$

[3]

(c) $\sin(\pi - A)$

[2]

[Maximum mark: 8]

Answers

	Answer	Mark	Guidance
(a)	$\cos^2 A + p^2 = 1$ $\cos^2 A = 1 - p^2$ $\cos A = -\sqrt{1 - p^2}$	*M1* *A1* *A1*	Use the Pythagorean Identity and solve for $\cos A$. The answer is negative because A is obtuse.
(b)	$\tan A = \dfrac{p}{-\sqrt{1 - p^2}}$ $\tan(-A) = \dfrac{p}{\sqrt{1 - p^2}}$	*M1* *A1* *A1*	Use the identity $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and the property of the tangent function: $\tan x = -\tan(-x)$.
(c)	$\sin(\pi - A) = p$	*R1* *A1*	Since A is obtuse, $\pi - A$ is acute and is the reference angle for A. Thus $\sin(\pi - A) = \sin A$.

Paper: 2 Marks: 11

Question

The distance, d, in millions of kilometres, between Earth and Mars is modelled by the function $d = 173.2\cos\left(0.245(t-5)\right) + 227.8$, where t is measured in months and $t = 0$ represents January 2018.

(a) (i) Find the range of the function d.

(ii) In the context of the question, explain the meaning of the range.

[4]

(b) Calculate the period of the function d. Round your answer to the nearest whole number of months.

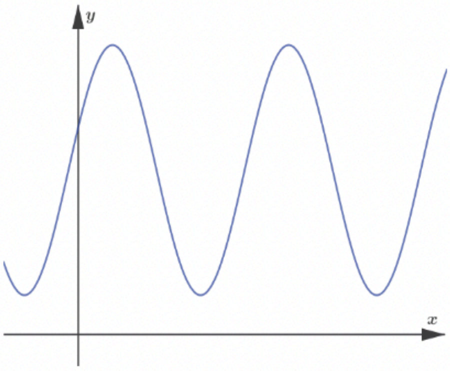
[3]

(c) An aerospace engineering company wants to launch a probe to Mars in the year 2021. Determine the least distance between Earth and Mars in 2021, and state the month in which it will occur.

[4]

[Maximum mark: 11]

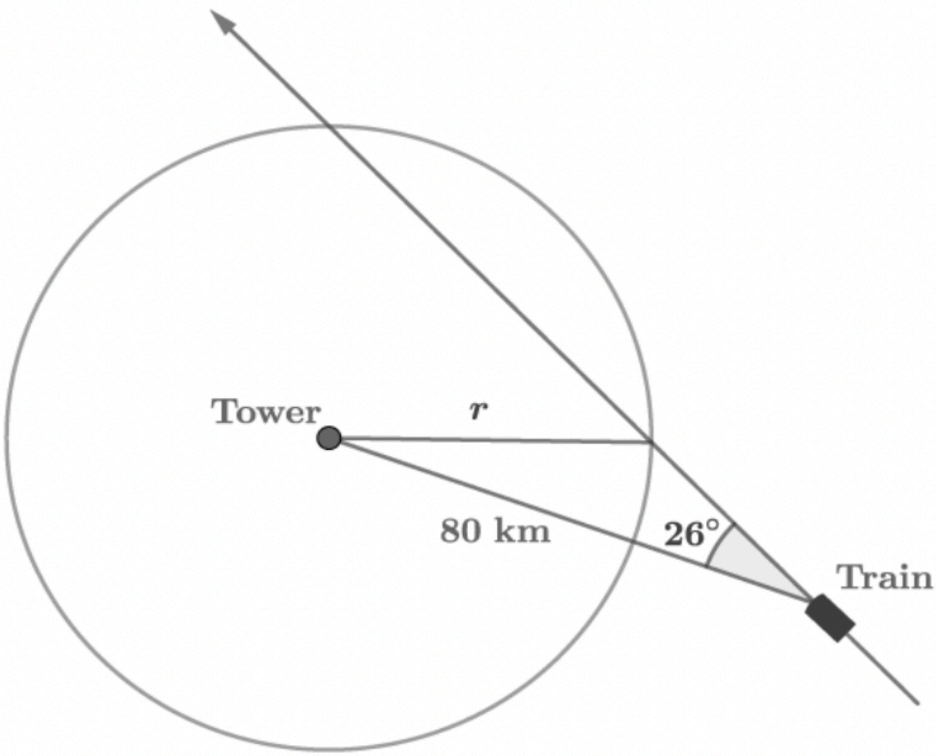
Answers

	Answer	Mark	Guidance
(a) (i)	**Method One – Graphical**  (Click on the image to enlarge it) Range is $54.6 \leq d \leq 401$. **Method Two – Analytical** Minimum $= 227.8 - 173.2 = 54.6$ Maximum $= 227.8 + 173.2 = 401$ Range is $54.6 \leq d \leq 401$.	*M1* *A1* *A1*	**Method One – Graphical** Use the graph analysis features on your GDC to find the minimum and maximum y-values on the graph of the function. Write the range as an interval of values and show your work by drawing a sketch. **Method Two – Analytical** From the equation, the midline of the graph is 227.8 and the amplitude is at 173.2. Add these to get the maximum, and subtract to get the minimum, then write the range as an interval.
(ii)	The range represents the varying distance between Earth and Mars from least to greatest.	*A1*	
(b)	$\frac{2\pi}{0.245} = 25.645\ldots$ The period is approximately 26 months.	*M1* *A1* *A1*	Use the formula period $= \frac{2\pi}{b}$. Be sure to round your answer to the nearest whole number.

(c)	For $36 \leq t < 48$ the minimum point is $(43.5, 54.6)$. $43.5 - 36 = 7.5$ The least distance is 54.6 million kilometres and it occurs during the eighth month (August).	***R1*** ***M1*** ***A1*** ***A1***	The year 2021 corresponds to $36 \leq t < 48$. Use the graph analysis features on your GDC to find the coordinates of the minimum point within this interval. Then interpret the units for your final answer.

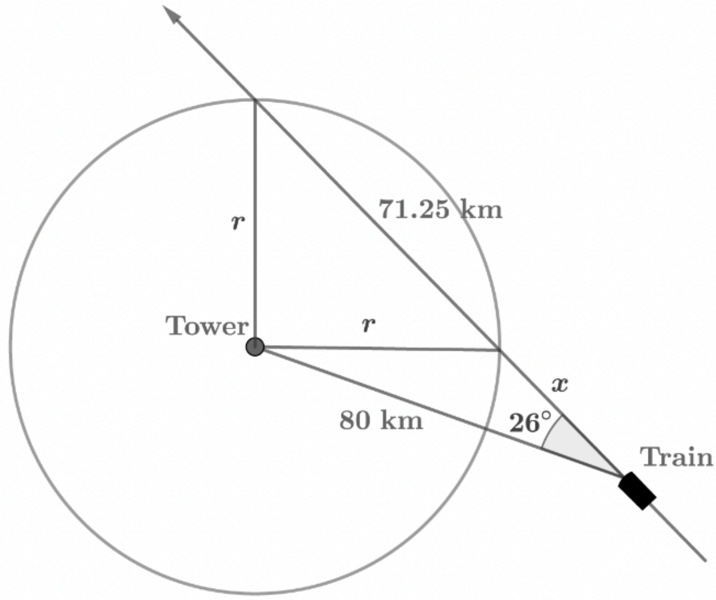
Question

A train travels at a constant speed of $95\ \text{km h}^{-1}$ along a straight track. The train operator locates a communications tower 80 km away at an angle of 26° from the track. The tower broadcasts a signal within a radius of r km, as shown in the diagram below.

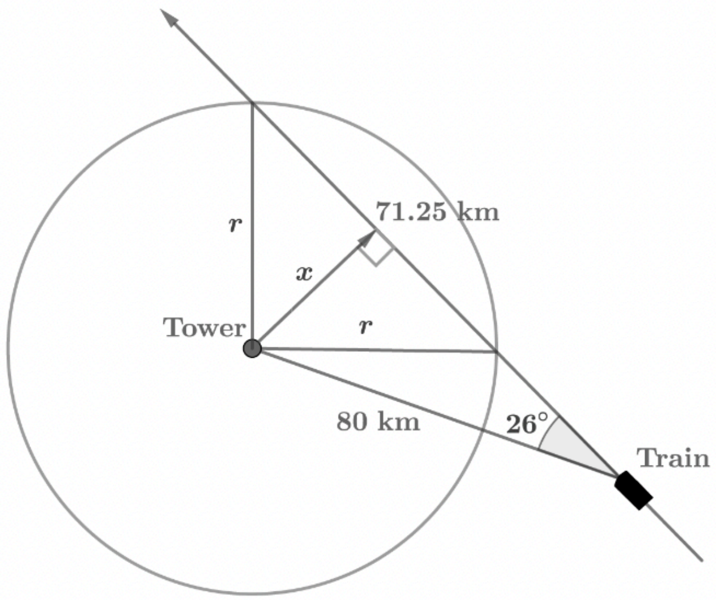


The conductor knows that the train will be within range of the signal from the tower for 45 minutes. Find the value of r.

[Maximum mark: 8]

Answer	Mark	Guidance
Method One – Using the cosine rule	***A1***	**Method One**
Distance $= 95(0.75) = 71.25$	***R1***	Start by finding the distance the train
	***M1*** ***A1*** ***A1*** ***M1*** ***A1*** ***A1*** ***A1*** ***M1*** ***A1***	travels in 45 minutes. Then use the cosine rule in the two triangles shown to write two equations in terms of x and r. Combine the equations to eliminate r and solve for x. Then use the cosine rule again to find the value of r. **Method Two**
$r^2 = 80^2 + x^2 - 2(80)(x)\cos 26^\circ$ $r^2 = 80^2 + (71.25 + x)^2 - 2(80)(71.25 + x)\cos 26^\circ$ $x^2 - 160x\cos 26^\circ = (71.25 + x)^2 - 160(71.25 + x)\cos 26^\circ$ $x = 36.2785\ldots$ $r = 49.9902\ldots$ About 50.0 km	***M1*** ***A1*** ***M1*** ***A1*** ***A1***	Start by finding the distance the train travels in 45 minutes. Then construct a perpendicular bisector from the tower to the track, shown as x in the diagram. Use right-angled triangle

Method Two – Using right-angled triangles



Distance $= 95(0.75) = 71.25$

$$\sin 26^\circ = \frac{x}{80}$$

$$x = 80\sin 26^\circ$$

$$r^2 = (80\sin 26^\circ)^2 + \left(\frac{71.25}{2}\right)^2$$

$$r^2 = 1229.8832\cdots + 1269.1406\ldots$$

$$r^2 = 2499.0239\ldots$$

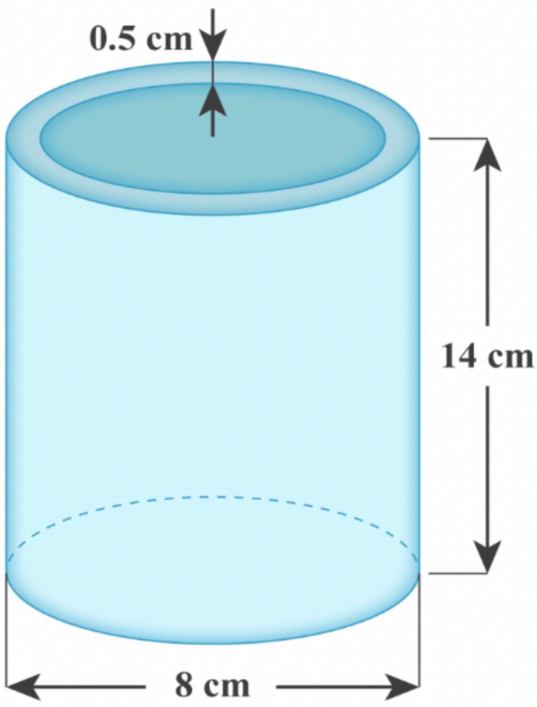
$$r = 49.9902\ldots$$

About 50.0 km

trigonometry and Pythagoras' theorem to solve for r.

Question

An insulated coffee mug is designed in the shape of an open top cylinder, as shown in the diagram below. The exterior of the mug has a height of 14 cm and a base diameter of 8 cm. The sides and base of the mug have a constant thickness of 0.5 cm.



(a) Calculate the volume of the inside of the mug. Round your answer to the nearest ten millilitres. (Note that one cubic centimetre equals one millilitre.)

[4]

(b) The exterior of the mug will be painted using paint that costs 0.06 cents per square centimetre. Determine the cost to paint the mug, rounded to the nearest hundredth of a cent.

[3]

[Maximum mark: 7]

Answers

	Answer	Mark	Guidance
(a)	Radius $= \frac{1}{2}(8) - 0.5 = 3.5$ Height $= 14 - 0.5 = 13.5$ $V = \pi(3.5)^2(13.5)$ $= 519.5408\ldots$ ≈ 520 The volume is about 520 ml.	A1 A1 M1 A1	Use the formula $V = \pi r^2 h$ and be sure to round your answer as directed.
(b)	Surface area $= 2\pi(4)(14) + \pi(4^2)$ $= 402.123\ldots$ $(402.123\ldots)(0.06) = 24.127\ldots$ The cost is about 24.13 cents.	M1 A1 A1	Use the formulas $A = 2\pi rh$ for the sides of the cylinder and $A = \pi r^2$ for the base. Multiply the surface area by the cost, and be sure to round your answer as directed. Note that if you consider the top of the wall as part of the exterior, then you will also need a bit of paint for that ring, so the cost is a bit more.

Paper: 1 Marks: 9

Question

(a) Solve $-1 - 5\sin\theta = 2\cos^2\theta$ for $-\pi \le \theta \le 2\pi$.

$2\cos^2\theta + 2\sin^2\theta = 2$

$-1 - 5\sin\theta = 2 - 2\sin^2\theta$

$0 = -2\sin^2\theta + 5\sin\theta + 3$

Let $\sin\theta = y$

$0 = -2y^2 + 5y + 3$

$0 = (-2y^2 + 6y)(-y + 3)$

$0 = -2y(y-3) - 1(y-3)$

$0 = (y-3)(-2y-1)$

$\sin\theta = 3$, OOR

$\sin\theta = -\frac{1}{2}$

$\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

[5]

(b) Solve $\tan^3 x = \tan x$ for $0 < x \le 2\pi$.

$\tan x(\tan^2 x - 1) = 0$

$\tan x = 0$, $\tan x = \pm 1$

$x = \cancel{0}, \pi, 2\pi$ $\quad x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

[4]

[Maximum mark: 9]

	Answer	Mark	Guidance
(a)	$-1 - 5\sin\theta = 2\cos^2\theta$ $-1 - 5\sin\theta = 2(1 - \sin^2\theta)$ $-1 - 5\sin\theta = 2 - 2\sin^2\theta$ $2\sin^2\theta - 5\sin\theta - 3 = 0$ $(2\sin\theta + 1)(\sin\theta - 3) = 0$ $\sin\theta = -\frac{1}{2}$ or $\sin\theta = 3$ $\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$	***M1*** ***A1*** ***M1*** ***R1*** ***A1***	Use the Pythagorean identity to write the equation in terms of sine only. Then factor and solve using the unit circle. Note that $\sin\theta = 3$ has no solution because 3 is not in the range of the sine function.
(b)	$\tan^3 x = \tan x$ $\tan^3 x - \tan x = 0$ $\tan x(\tan^2 x - 1) = 0$ $\tan x = 0$ or $\tan x = \pm 1$ $x = \pi, 2\pi, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$	***M1*** ***A1*** ***A1*** ***A1***	Factorise and solve using the unit circle.

Question

(a) Line L passes through the point P(4, – 1) and is perpendicular to the line $y = \frac{8}{3}x + 2$. Find the equation of line L.

[3]

(b) Consider the points Q(7, – 2) and R(– 3, 5). Line M is the perpendicular bisector of segment [QR]. Find the equation of line M. Use exact values in your answer.

[7]

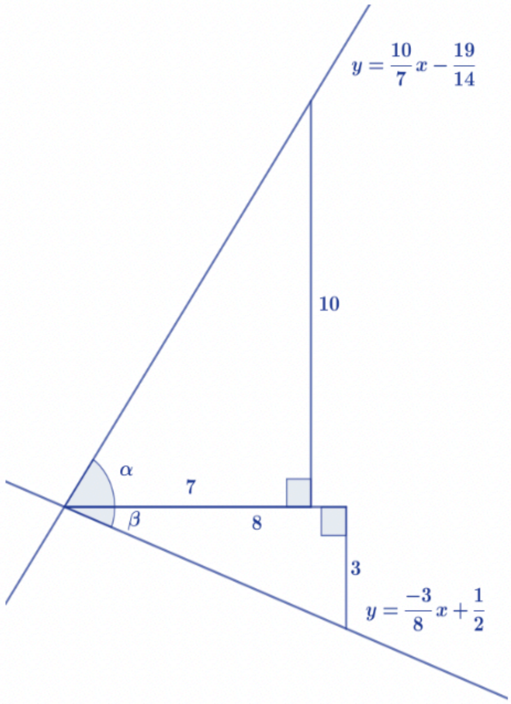
(c) Find the degree measure of the acute angle formed when lines L and M intersect.

[6]

[Maximum mark: 16]

Answers

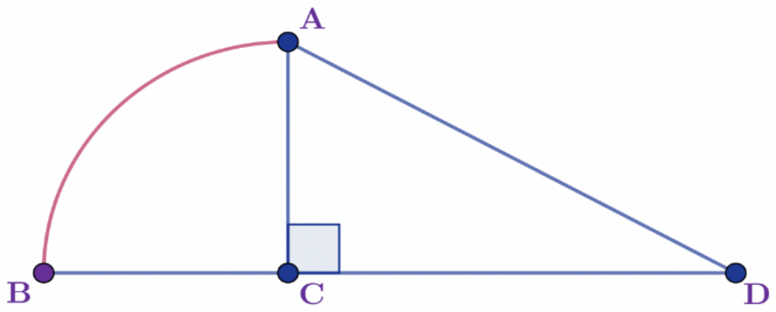
	Answer	Mark	Guidance
(a)	$-1 = -\frac{3}{8}(4) + b$ $-1 = -\frac{3}{2} + b$ $b = \frac{1}{2}$ $y = -\frac{3}{8}x + \frac{1}{2}$	***R1*** ***M1*** ***A1***	The slopes of perpendicular lines are opposite reciprocals. Substitute the slope along with the given point into $y = mx + b$ and solve for b. Then write the equation.
(b)	$\text{Midpoint} = \left(\frac{7+(-3)}{2}, \frac{-2+5}{2}\right)$ $= \left(2, \frac{3}{2}\right)$ Slope of $[\text{QR}] = \frac{5-(-2)}{-3-7} = -\frac{7}{10}$ Slope of $\text{M} = \frac{10}{7}$ $\frac{3}{2} = \frac{10}{7}(2) + b$ $\frac{3}{2} = \frac{20}{7} + b$ $b = -\frac{19}{14}$ $y = \frac{10}{7}x - \frac{19}{14}$	***M1*** ***A1*** ***M1*** ***A1*** ***M1*** ***A1*** ***A1***	Find the midpoint and the slope of $[\text{QR}]$. Substitute these values into $y = mx + b$ and solve for b. Then write the equation.

(c)		***R1*** ***M1*** ***A1*** ***M1*** ***A1*** ***A1***	Use inverse tangent of each slope to find the angle formed by each line with the horizontal. Then add these angles to find the final answer.
	$\tan\beta=\frac{3}{8}$ $\beta=\tan^{-1}\left(\frac{3}{8}\right)$ $=20.556\ldots^\circ$ $\tan\alpha=\frac{10}{7}$ $\alpha=\tan^{-1}\left(\frac{10}{7}\right)$ $=55.007\ldots^\circ$ $\theta=\alpha+\beta\approx 75.6^\circ$		

Paper: 2 Marks: 7

Question

In the figure below, ABC is a sector of circle centred at C with radius 3 metres. Triangle ACD is right angled at C. Points B, C, and D are collinear. The length of AD is 7 metres.



(a) Find the perimeter of the figure.

[4]

(b) Find the area of the figure.

[3]

[Maximum mark: 7]

Answers

	Answer	Mark	Guidance
(a)	$CD = \sqrt{7^2 - 3^2} = \sqrt{40} = 2\sqrt{10}$ Arc $AB = 3\left(\frac{\pi}{2}\right) = 1.5\pi$ $\text{Perimeter} = 3 + 7 + 2\sqrt{10} + 1.5\pi$ $= 10 + 2\sqrt{10} + 1.5\pi$ $\approx 21.0\ (\text{m})$	***M1*** ***A1*** ***M1*** ***A1***	Use Pythagoras' theorem to find the length CD. Use the formula $l = r\theta$ to find the length of the arc AB. Add the lengths BC, CD, AD, and the arc length to find the perimeter.
(b)	Area of triangle: $\frac{1}{2}(3)\left(2\sqrt{10}\right) = 3\sqrt{10}$ Area of sector: $\frac{1}{2}\left(\frac{\pi}{2}\right)\left(3^2\right) = \frac{9\pi}{4}$ Area of the figure: $3\sqrt{10} + \frac{9\pi}{4} \approx 16.6\ (\text{m}^2)$	***M1*** ***M1*** ***A1***	Use the formula $A = \frac{1}{2}bh$ to find the area of the triangle. Use the formula $A = \frac{1}{2}r^2\theta$ to find the area of the sector.

Paper: 2 Marks: 5

Question

Given that $\sin \dfrac{563\pi}{36} = \sin A$ for $0 < A \leq 2\pi$, find the possible values of A. Give your answers in exact form, in terms of π.

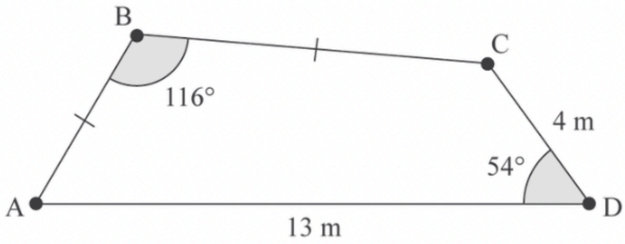
[Maximum mark: 5]

Answers

Answer	Mark	Guidance
$\dfrac{563\pi}{36} \times \dfrac{180^\circ}{\pi} = 2815^\circ$ $\sin 2815^\circ = -0.906\ldots$ $\sin^{-1}(-0.906\ldots) = -65^\circ$ The angle in Q3 is $180^\circ - (-65^\circ) = 245^\circ$, so $A = 245^\circ \times \dfrac{\pi}{180^\circ} = \dfrac{49\pi}{36}$. The angle in Q4 is $360^\circ + (-65^\circ) = 295^\circ$, so $A = 295^\circ \times \dfrac{\pi}{180^\circ} = \dfrac{59\pi}{36}$. The possible values are $\dfrac{49\pi}{36}$ and $\dfrac{59\pi}{36}$.	***M1*** ***A1*** ***R1*** ***A1*** ***A1***	Since the answers need to be exact, the easiest way to solve this problem is to work in degrees. Use your GDC to find the sine of the given angle. Since the result is negative, angle A will be in quadrants 3 and 4. Use inverse sine of the result to find the two possible values of A. Then convert back to radians for your final answers.

Question

Quadrilateral ABCD is shown in the following diagram:



Note – diagram is <u>not</u> drawn to scale.

[AB] and [BC] have the same length. $AD = 13\,\text{m}$, $CD = 4\,\text{m}$, $\angle ADC = 54^\circ$ and $\angle ABC = 116^\circ$.

(a) Write down the size of angle BAC.

[1]

(b) Find AC.

[3]

(c) Find BC.

[3]

(d) Find the area of the quadrilateral ABCD.

[4]

[Maximum mark: 11]

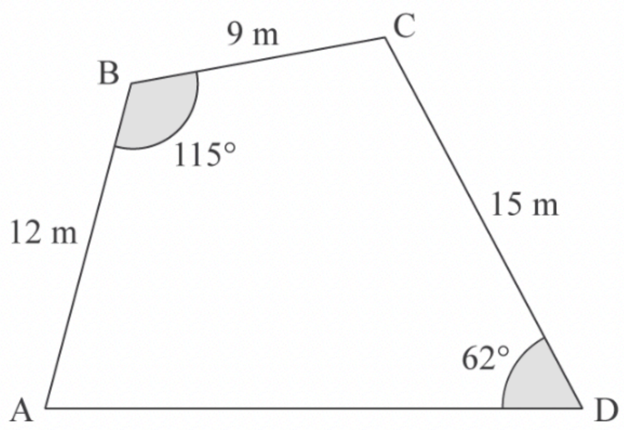
Answers

	Answer	Mark	Guidance
(a)	$\angle BAC = \frac{1}{2} \times (180^\circ - 116^\circ) = 32^\circ$	*A1*	Since $AB = BC$ we know that ABC is an isosceles triangle.
(b)	$AC^2 = 13^2 + 4^2 - 2 \times 13 \times 4 \times \cos 54^\circ$ $AC^2 = 123.8703...$ $AC = 11.1297...$ $AC \approx 11.1 \text{ m}$	*M1* *A1* *A1*	Use the cosine rule in triangle ADC.
(c)	$\frac{BC}{\sin 32^\circ} = \frac{11.1297...}{\sin 116^\circ}$ $BC = \frac{(11.1297...) \sin 32^\circ}{\sin 116^\circ}$ $BC = 6.5619...$ $BC \approx 6.56 \text{ m}$	*M1* *A1* *A1*	Use the sine rule.

(d)	Area of a triangle, $A = \frac{1}{2}ab \sin C$ Area of triangle ABC: $A_{ABC} = \dfrac{1}{2}(6.5619...)(6.5619...) \times \sin 116°$ $A_{ABC} = 19.3506...$ Area of triangle ACD: $A_{ACD} = \dfrac{1}{2}(4)(13) \sin 54°$ $A_{ACD} = 21.0344...$ Area of ABCD: $A_{ABCD} = A_{ABC} + A_{ACD}$ $A_{ABCD} = 19.3506... + 21.0344...$ $A_{ABCD} = 40.3851...$ $A_{ABCD} \approx 40.4 \text{ m}^2$	**A1** **A1** **M1** **A1**	Use the formula to find the area of each triangle. Then add the two triangle areas to obtain the area of quadrilateral ABCD.

Question

Julian buys a piece of land ABCD in the shape of a quadrilateral, as shown in the following diagram:



$AB = 12\,\text{m}$, $BC = 9\,\text{m}$, $CD = 15\,\text{m}$ and the angles $\angle ABC = 115^{\circ}$ and $\angle ADC = 62^{\circ}$.

(a) Julian builds a straight fence across the land from point A to point C. Determine the length of the fence.

[3]

(b) Calculate the angle formed by the fence AC and the line AD.

[3]

(c) Julian leases the trianglar piece of land ABC to a farmer. Show that the area of the parcel of land is 49 m^2, correct to 2 significant figures.

[2]

(d) Julian charges the farmer a monthly rent of 15 EUR per m^2. Using 49 m^2 as the area of the land, calculate the amount of rent the farmer pays in one year.

[2]

(e) Julian invests the first year's rent in an account that pays 2.5% annual interest compounded monthly. Determine the money he will have in the account after 5 years. Round your answer to the nearest whole number of Euros.

[3]

[Maximum mark: 13]

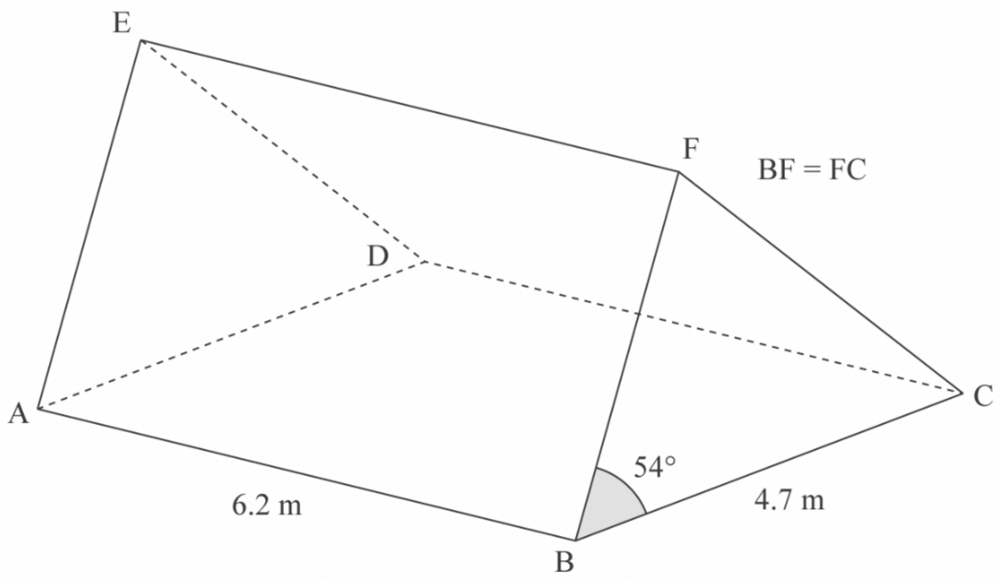
Answers

	Answer	Mark	Guidance
(a)	$AC^2 = 12^2 + 9^2 - 2(12)(9)\cos 115°$ $AC^2 = 316.2855...$ $AC = 17.7844...$ $AC \approx 17.8$	***M1*** ***A1*** ***A1***	Use the cosine rule.
(b)	$\dfrac{\sin C\hat{A}D}{15} = \dfrac{\sin 62°}{17.7844...}$ $\sin C\hat{A}D = \dfrac{15 \sin 62°}{17.7844...}$ $\sin C\hat{A}D = 0.7447...$ $C\hat{A}D = 48.134...$ $C\hat{A}D \approx 48.1°$	***M1*** ***A1*** ***A1***	Use the sine rule.
(c)	$A = \dfrac{1}{2}(12)(9)\sin 115°$ $A = 48.9406...$ To 2 s.f. this is 49 m.	***A1*** ***A1***	Use the triangle area formula, $A = \frac{1}{2}ab\sin C$.
(d)	Cost = $49 \times 15 \times 12$ Cost = 8820 EUR	***M1*** ***A1***	Multiply the area times the cost times 12 months in a year.

(e)	$FV = 8820\left(1 + \frac{2.5}{100 \times 12}\right)^{12 \times 5}$ $FV = 9993.069...$ $FV \approx 9993 \text{ EUR}$	***M1*** ***A1*** ***A1***	Use the compound interest formula.

Question

A roof is in the shape of a right triangular prism ABCDEF. One side of this prism is the rectangle ABCD with $AB = 6.2$ m and $BC = 4.7$ m. The cross-section BCF is an isosceles triangle with $BF = FC$ and the angle $FBC = 54^{\circ}$.



(a) Find the length of BF.

[4]

(b) Calculate the length of the diagonal AC.

[2]

(c) Determine the area of triangle BFC.

[3]

(d) Show that the enclosed volume of the roof is 47 m^3, correct to the nearest whole number.

[2]

(e) The external surfaces of the roof are painted. One tin of paint covers a surface area of 10 m^2. Determine the number of tins of paint that must be bought to paint the roof.

[3]

[Maximum mark: 14]

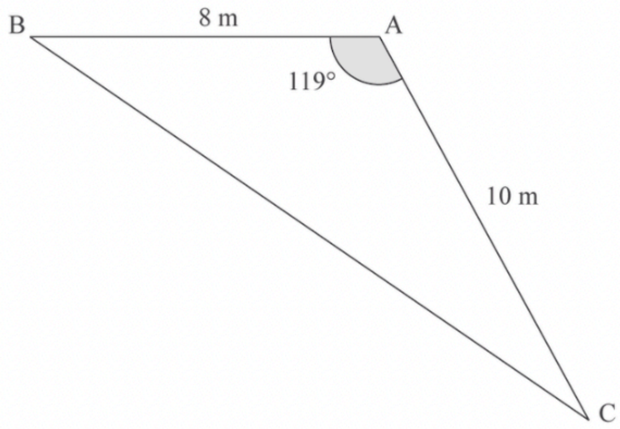
Answers

	Answer	Mark	Guidance
(a)	**Method One:** Triangle BFC is isosceles, so $B\hat{F}C = 180^\circ - 2 \times 54^\circ = 72^\circ$. $\dfrac{BF}{\sin 54^\circ} = \dfrac{4.7}{\sin 72^\circ}$ $BF = \dfrac{(4.7)\sin 54^\circ}{\sin 72^\circ}$ $BF = 3.998...$ $BF \approx 4.00$ **Method Two:** A perpendicular from F to [BC] makes 2 right angled triangles with base 2.35 m. $\cos 54^\circ = \dfrac{2.35}{BF}$ $BF = \dfrac{2.35}{\cos 54^\circ}$ $BF = 3.998...$ $BF \approx 4.00 \text{ m}$	*A1* *M1* *A1* *A1* *M1* *A2* *A1*	**Method One:** Find $B\hat{F}C$ from the isosceles triangle, and then use the sine rule. **Method Two:** Form a right triangle by constructing the perpendicular bisctor from F to [BC]. Then the base is 2.35 m in length. Then use the cosine function to find BF.

(b)	$AC^2 = 6.2^2 + 4.7^2$ $AC^2 = 60.53$ $AC = 7.7801...$ $AC \approx 7.78 \text{ m}$	***M1*** ***A1***	Use Pythagoras' theorem.
(c)	$A = \frac{1}{2}ab \sin C$ $A = \frac{1}{2}(3.998...)(4.7) \sin 54°$ $A = 7.6010...$ $A \approx 7.60 \text{ m}^2$	***M1*** ***A1*** ***A1***	Use the triangle area formula.
(d)	$V = Ah$ $V = (7.6010...)(6.2)$ $= 47.126...$ $V = 47 \text{ m}^3$ to the nearest whole number.	***M1*** ***A1***	Use the formula for the volume of a prism, $V = Ah$, where A is the answer from part **(c)** and h is the length of side AB.
(e)	Area of rectangle ABFE: $A = bh = (6.2)(3.998...) = 24.7879...$ Total surface area of roof: $SA = 2(24.7879...) + 2(7.6010...)$ $= 64.778...$ Amount of paint: $\frac{64.778...}{10} = 6.4778...$ $\approx 7 \text{ tins}$	***A1*** ***A1*** ***A1***	Find the area of rectangle ABFE using the height found in part **(a)**. The area of triangle BFC was found in part **(c)**. The external surface of the roof is made up of two rectangles and two triangles.

Question

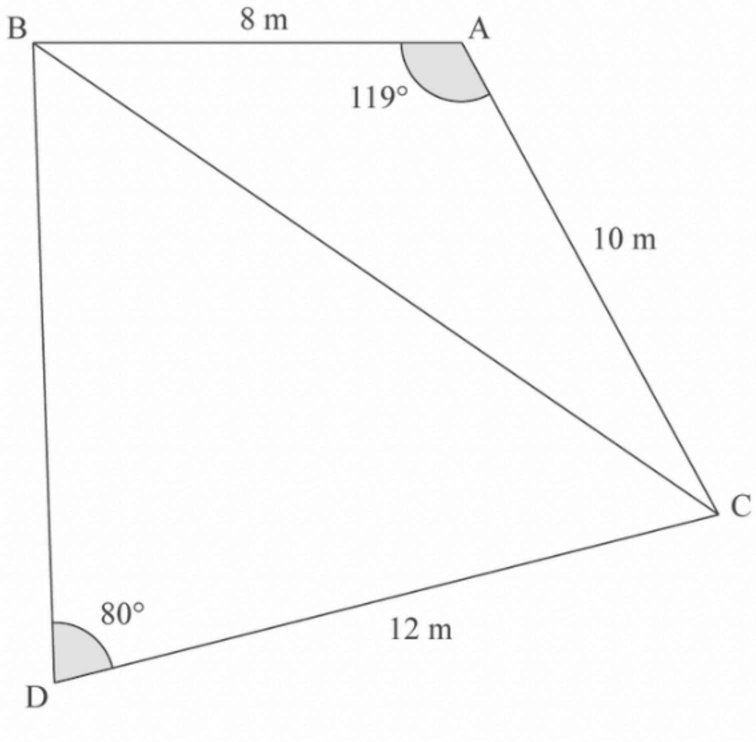
A triangular field ABC has $AB = 8$ m, $AC = 10$ m and angle $B\hat{A}C = 119°$, as shown in the diagram below.



(a) Find BC.

[3]

The field ABC is connected to another triangular field BCD, with $DC = 12$ m and angle $C\hat{D}B = 80°$. Together these fields form a quadrilateral ABCD, as shown in the following diagram.



(b) Show that $D\hat{C}B = 50.5^\circ$, correct to 3 significant figures.

[4]

(c) Find the area of the quadrilateral ACDB.

[4]

(d) The farmer covers the field ABCD in fertilizer to a depth of 1 cm. Determine the volume of fertilizer needed.

[2]

(e) The fertilizer is delivered in packets which are cuboid in shape, of length 30 cm, width 40 cm and height 50 cm. Determine the number of packets of fertilizer the farmer must buy to fertilize the field ABCD.

[3]

[Maximum mark: 16]

Answers

	Answer	Mark	Guidance
(a)	$BC^2 = 8^2 + 10^2 - 2(8)(10)\cos 119^\circ$ $BC^2 = 241.5695...$ $BC = 15.5425...$ $BC \approx 15.5$	M1 A1 A1	Use the cosine rule.
(b)	$\dfrac{\sin D\hat{B}C}{12} = \dfrac{\sin 80^\circ}{15.5425...}$ $\sin D\hat{B}C = \dfrac{12 \sin 80^\circ}{15.5425...}$ $\sin D\hat{B}C = 0.7603...$ $D\hat{B}C = 49.4976..^\circ$ $D\hat{B}C \approx 49.5^\circ$ $D\hat{C}B = 180^\circ - D\hat{B}C - B\hat{D}C$ $D\hat{C}B = 180^\circ - 49.4976...^\circ - 80^\circ$ $D\hat{C}B = 50.5052...^\circ$ $D\hat{C}B \approx 50.5^\circ$	M1 A1 M1 A1	Start by using the sine rule to find angle $D\hat{B}C$. Then subtract to find angle $D\hat{C}B$ Use the unrounded value of $D\hat{B}C$for the calculation.

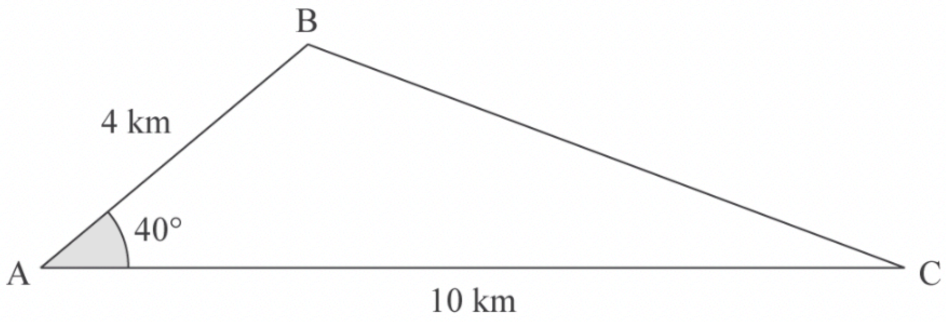
(c)	$A_{\text{ABC}} = \frac{1}{2}(8)(10)\sin 119°$ $A_{\text{ABC}} = 34.9847...$ $A_{\text{BCD}} = \frac{1}{2}(15.5425...)(12) \times \sin 50.5052...°$ $A_{\text{BCD}} = 71.9633...$ Total area: $A_{\text{ABCD}} = A_{\text{ABC}} + A_{\text{BCD}}$ $A_{\text{ABCD}} = 34.9847... + 71.9633...$ $A_{\text{ABCD}} = 106.948...$ $A_{\text{ABCD}} \approx 107 \text{ m}^2$	***M1*** ***A1*** ***A1*** ***A1***	Use the triangle area formula $A = \frac{1}{2}ab \sin C$ to find the areas of ABC and BCD. Add the areas of the two triangles to obtain the area of quardillateral ABCD.
(d)	$V = Bh$ $V = (106.948...)(0.01)$ $V = 1.0694...$ $V \approx 1.07 \text{ m}^3$	***A1*** ***A1***	Use the formula $V = Bh$ where B is the answer from part **(c)** and h is the depth of the fertilizer. Remember to convert h from centimetres to metres.

(e)	Volume of packet: $(0.3)(0.4)(0.5) = 0.06 \text{ m}^3$ $\dfrac{1.0694...}{0.06} = 17.8246... \approx 18 \text{ packets}$	***M1*** ***M1*** ***A1***	The volume in part **(d)** is in m^3, so you need the volume of fertilizer in a packet in the same units. First convert all the dimensions of the packet to metres. Then divide the volume needed from part **(d)** by the volume in one packet, and round up to the nearest whole number.

Paper: 2 Marks: 6

Question

The course for a boat race follows the perimeter of triangle ABC as shown in the diagram below. AB is 4 km, AC is 10 km and the measure of angle $B\hat{A}C$ is 40°.



(a) Find BC.

[3]

(b) Write down the length of the course.

[1]

Billy has a boat which travels at an average speed of 19 kmh^{-1}.

(c) Determine the time required for Billy to complete the course. Give your answer correct to the nearest minute.

[2]

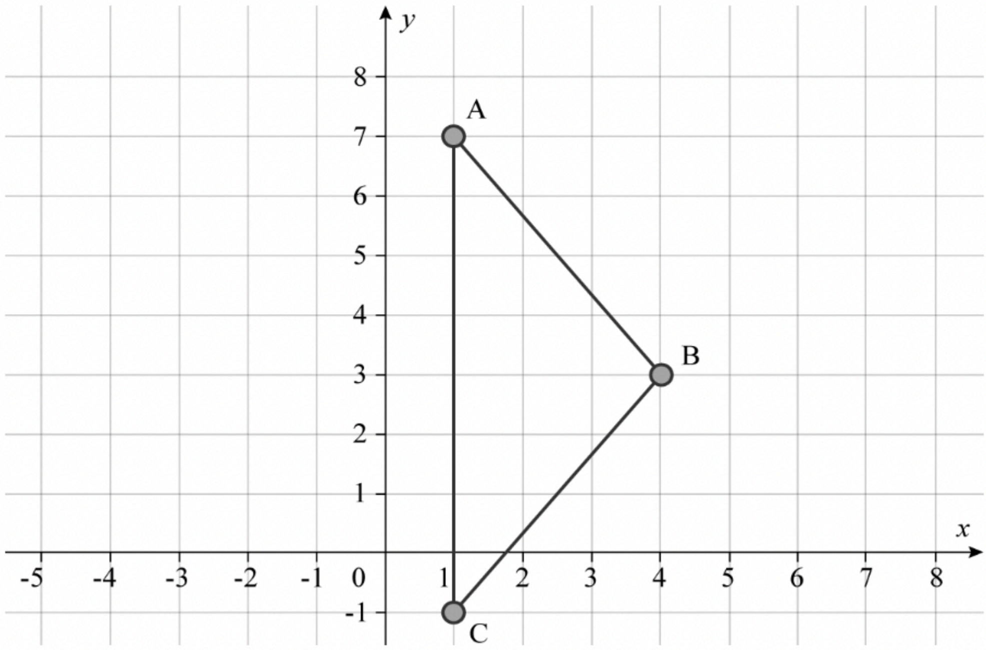
[Maximum mark: 6]

Answers

	Answer	Mark	Guidance
(a)	$BC^2 = 10^2 + 4^2 - (2 \times 10 \times 4 \cos 40^\circ)$ $BC^2 = 54.716...$ $BC = 7.397...$ $BC \approx 7.40 \text{ km}$	***M1*** ***A1*** ***A1***	Substitute the given values into the cosine rule formula. Round your answer to 3 significant figures.
(b)	$4 + 10 + 7.40 = 21.4 \text{ km}$	***A1***	Add the lengths of the three sides of triangle ABC to determine the length of the course:
(c)	$21.4 = 19t$ $t = \dfrac{21.4}{19}$ $t = 1.126...$ $t = 1.13 \text{ hours}$ $1.126 \times 60 = 67.578... \approx 68$ Time rounded to the nearest minute is 1 hour 8 minutes.	***A1*** ***A1***	Use the formula: Distance = Speed × Time to find the time. Note that this result is in hours because the rate is given in km per hour. Multiply by 60 to obtain the answer in minutes. Be sure to round your final answer to the nearest minute, as instructed in the question.

Question

The points A(1, 7), B(4, 3), and C(1, −1) are the vertices of a triangle, as shown in the figure below.



(a) Find AB.

[2]

(b) M is the midpoint of [AC]. Write down the coordinates of M.

[1]

(c) Hence, calculate the size of the angle ABC.

[3]

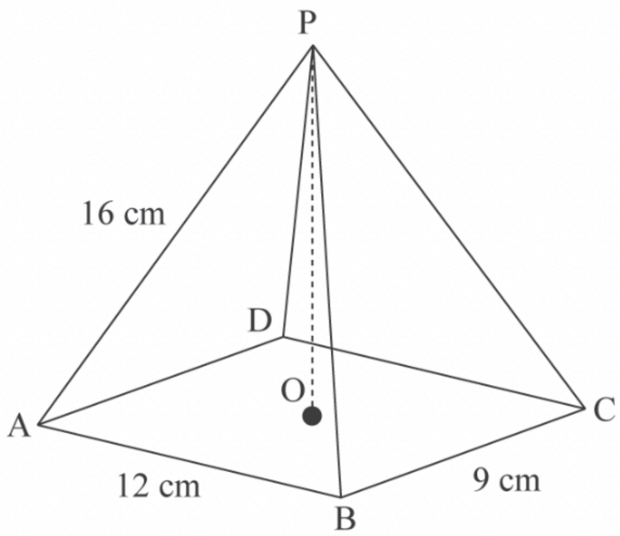
[Maximum mark: 6]

Answers

	Answer	Mark	Guidance
(a)	$AB = \sqrt{(4-1)^2 + (3-7)^2}$ $AB = \sqrt{9+16}$ $AB = \sqrt{25}$ $AB = 5$	*M1* *A1*	Substitute the coordinates of point A and point B into the distance formula or think of it as Pythagoras.
(b)	$\left(\frac{1+1}{2}, \frac{7-1}{2}\right) = (1, 3)$	*A1*	Use the midpoint formula, or just count the squares.
(c)	$\tan A\hat{B}M = \frac{4}{3}$, or $\sin A\hat{B}M = \frac{4}{5}$, or $\cos A\hat{B}M = \frac{3}{5}$ $A\hat{B}M \approx 53.1°$ $A\hat{B}C = 2 \times A\hat{B}M$ $A\hat{B}C = 2 \times 53.1301...$ $A\hat{B}C = 106.2602...$	*M1* *A1* *A1*	From the given figure, we can see that triangle ABM is a right angled at M, and that $AM = 4$ and $BM = 3$. Thus, we use a trigonometric ratio to calculate angle $A\hat{B}M$. Then find angle $A\hat{B}C$ by doubling the measure of angle $A\hat{B}M$.
	$A\hat{B}C \approx 106°$		

Question

A right pyramid has a rectangular base ABCD where $AB = 12$ cm and $BC = 9$ cm. The centre of the base, O, lies vertically below the apex P, and $AP = 16$ cm.



(a) Calculate the vertical height of the pyramid, OP.

[4]

(b) Calculate the volume of the pyramid.

[2]

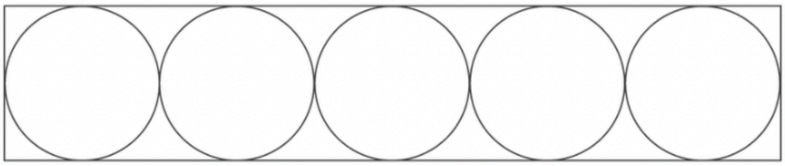
[Maximum mark: 6]

Answers

	Answer	Mark	Guidance
(a)	$AC^2 = 12^2 + 9^2$ $AC = \sqrt{144 + 81}$ $AC = \sqrt{225}$ $AC = 15 \text{ cm}$ $AO = \frac{1}{2} \times 15 = 7.5$ From triangle OPA: $16^2 = 7.5^2 + OP^2$ $OP^2 = 16^2 - 7.5^2$ $OP^2 = 199.75$ $OP = 14.1332...$ $OP \approx 14.1 \text{ cm}$	***M1*** ***A1*** ***M1*** ***A1***	Use Pythagoras' theorem to find the length AC The length AO is half of the length AC. Then use Pythagoras' theorem again to find the length OP.
(b)	$V = \frac{1}{3} \times Ah$ $V = \frac{1}{3} \times 12 \times 9 \times 14.1332...$ $V = 509 \text{ cm}^3$	***M1*** ***A1***	A is the area of the base, which is 12×9.

Question

Spherical sweets are sold in a tube containing 5 sweets. The tube is cylindrical in shape and the sweets are placed next to each other, each sweet exactly touching the sides.



The radius of a sweet is 0.5 cm.

(a) Find the volume of a sweet.

[2]

(b) Determine the volume of air in the tube.

[4]

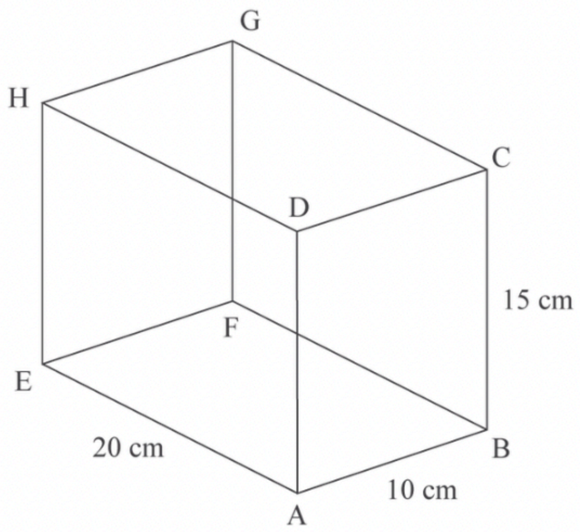
[Maximum mark: 6]

Answers

	Answer	Mark	Guidance
(a)	$V = \frac{4}{3}\pi r^3$ $V = \frac{4}{3}\pi(0.5)^3$ $V = 0.524 \text{ cm}^3$	*M1* *A1*	Use the formula for volume of a sphere, $V = \frac{4}{3}\pi r^3$.
(b)	Total volume of the sweets: $V_{\text{sweets}} = 5 \times 0.5235... = 2.6179...$ Volume of the tube: $V = \pi r^2 h$ Height of the tube is $h = 5 \times 2 \times 0.5 = 5$ So for the tube: $V_{\text{cylinder}} = \pi \times 0.5^2 \times 5$ $V_{\text{cylinder}} = 3.9269...$ Therefore $V_{\text{air}} = V_{\text{cylinder}} - V_{\text{sweets}}$ $V_{\text{air}} = 3.9269... - 2.6179...$ $V_{\text{air}} = 1.3089...$ $V_{\text{air}} \approx 1.31$	*A1* *A1* *M1* *A1*	You need to find the volume of the sweets and subtract this from the volume of the tube. The height of the tube is 5 times the diameter of the sweets. The tube is a cylinder, so use the formula for volume of a cylinder. The volume of air is the difference between these two volumes.

Question

A box is in the shape of a cuboid of dimensions 15 cm by 10 cm by 20 cm.



(a) Find AF.

[2]

(b) Find AG.

[2]

(c) Calculate the angle between [AG] and the base of the box.

[2]

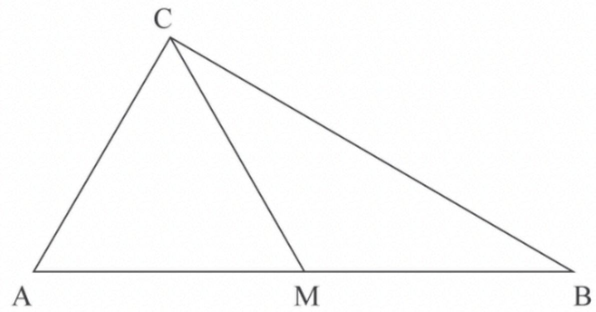
[Maximum mark: 6]

Answers

	Answer	Mark	Guidance
(a)	$AF^2 = 10^2 + 20^2$ $AF^2 = 500$ $AF = \sqrt{500}$ $AF = 22.3606...$ $AF \approx 22.4$ cm	*M1* *A1*	Use Pythagoras' theorem.
(b)	$AG^2 = AF^2 + FG^2$ $AG^2 = 500 + 15^2$ $AG^2 = 725$ $AG = \sqrt{725}$ $AG = 26.9258...$ $AG \approx 26.9$	*M1* *A1*	This question also uses Pythagoras' theorem.
(c)	$\sin G\hat{A}F = \frac{15}{\sqrt{725}}$ or $\cos G\hat{A}F = \frac{\sqrt{500}}{\sqrt{725}}$ or $\tan G\hat{A}F = \frac{15}{\sqrt{500}}$ $G\hat{A}F = 33.8545...$ $G\hat{A}F \approx 33.9^\circ$	*M1* *A1*	The angle we need to find is $\angle GAF$. Notice that triangle GAF is right angled at F, so we can use a trigonometric ratio to calculate the angle.

Question

In the diagram below, ABC is a triangle with $AB = 10$ cm. M is the midpoint of [AB] and ACM is an equilateral triangle.



(a) Find:

(i) the length of [CM].

(ii) the measure of angle BMC.

(iii) the measure of angle CBM.

[4]

(b) Find the length of [BC].

[3]

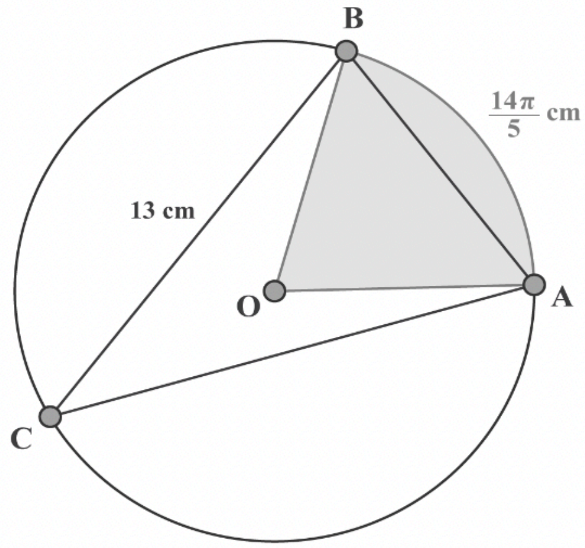
[Maximum mark: 7]

Answers

	Answer	Mark	Guidance
(a) (i)	$CM = AM = 5$ cm	*A1*	Because M is the midpoint of [AB], we know that $AM = \frac{1}{2} \times AB = 5$. And because ACM is an equilateral triangle, we know that $CM = AM$.
(ii)	$\angle BMC = 180 - \angle AMC$ $= 180 - 60$ $= 120°$	*A1*	Since ACM is an equilateral triangle, all three of its angles measure $60°$.
(iii)	$\angle CBM = \frac{1}{2} \times (180 - \angle BMC)$ $= 30°$	*M1* *A1*	Because M is the midpoint of [AB], we know that $BM = CM$. Thus CMB is an isoceles triangle.
(b)	$\frac{BC}{\sin 120°} = \frac{5}{\sin 30°}$ $BC = \frac{5 \sin 120°}{\sin 30°}$ $BC = 8.660...$ $BC \approx 8.66$ cm	*M1* *A1* *A1*	Use the sine rule in triangle CMB.

Question

Points A, B, and C lie on the circle centered at O, as shown in the following diagram:



The length of the minor arc between A and B is $\frac{14\pi}{5}$ cm,

the area of the shaded sector is $\frac{49\pi}{5}$ cm^2, and BC = 13 cm.

(a) Find the length of the radius of the circle.

[5]

(b) Find the exact size of angle AOB.

[2]

(c) Find the length of AB.

[3]

(d) Find the size of angle ABC.

[5]

(e) Find the area of triangle ABC.

[3]

[Maximum mark: 18]

Answers

	Answer	Mark	Guidance
(a)	$r\theta = \dfrac{14\pi}{5}$ $\dfrac{1}{2}r^2\theta = \dfrac{49\pi}{5}$ $\dfrac{1}{2}r \times r\theta = \dfrac{49\pi}{5}$ $\dfrac{1}{2}r \times \dfrac{14\pi}{5} = \dfrac{49\pi}{5}$ $r = \dfrac{49\pi}{5} \times \dfrac{5}{14\pi} \times 2$ $r = 7 \text{ (cm)}$	M1 A1 A1 M1 A1	Use the formulas for the length of an arc and the area of a sector to set up a system of equations in terms of the radius, r, and the angle $\theta = \mathrm{A\hat{O}B}$.
(b)	$7\theta = \dfrac{14\pi}{5}$ $\theta = \dfrac{2\pi}{5}$ Note: accept 72°	M1 A1	Use the arc length formula and the value of r from part a.
(c)	$AB^2 = OA^2 + OB^2 - 2(OA)(OB)\cos A\hat{O}B$ $AB^2 = 7^2 + 7^2 - 2(7)(7)\cos\left(\dfrac{2\pi}{5}\right)$ $AB^2 = 67.7163...$ $AB \approx 8.23 \text{ (cm)}$	M1 A1 A1	Use the cosine rule in triangle AOB.

(d)	$\hat{ABO} = \dfrac{\pi - \hat{AOB}}{2}$ $= \dfrac{\pi - \dfrac{2\pi}{5}}{2}$ $= \dfrac{3\pi}{10} = 0.9424...$ $\cos \hat{OBC} = \dfrac{OB^2 + BC^2 - OC^2}{2(OB)(BC)}$ $\cos \hat{OBC} = \dfrac{7^2 + 13^2 - 7^2}{2(7)(13)}$ $\hat{OBC} = \cos^{-1}\left(\dfrac{13}{14}\right)$ $\hat{OBC} = 0.3802...$ $\hat{ABC} = 0.9424... + 0.3802... \approx 1.32$ Note: accept 75.8°	*M1* *A1* *M1* *A1* *A1*	Note that $\hat{ABC}$ is comprised of two angles, $\hat{ABO}$ and $\hat{OBC}$. Use the fact that triangle AOB is isosceles to find $\hat{ABO}$. To find $\hat{OBC}$, use the cosine rule in triangle OBC.
(e)	$\text{Area} = \dfrac{1}{2} \times AB \times BC \times \sin \hat{ABC}$ $= \dfrac{1}{2} \times 8.23 \times 13 \times \sin 1.32$ $\approx 51.9\ (\text{cm}^2)$	*M1* *A1* *A1*	Use the triangle area formula $\text{Area} = \dfrac{1}{2}ab\sin C.$

Question

The obtuse angle θ is such that $\sin\theta = \frac{2}{3}$.

Find the exact value of

(a) $\cos\theta$

$\left(\frac{2}{3}\right)^2 + \cos^2\theta = 1$

$\cos\theta = \sqrt{\frac{5}{9}} = -\frac{\sqrt{5}}{3}$

[3]

(b) $\tan\theta$

$\tan\theta = \frac{2}{\cancel{3}} \cdot \frac{-\cancel{3}}{\sqrt{5}} = \frac{-2}{\sqrt{5}}$

[2]

(c) $\sin 2\theta$

$2\left(\frac{2}{3}\right)\left(-\frac{\sqrt{5}}{3}\right) = -\frac{4\sqrt{5}}{9}$

[2]

(d) $\cos 2\theta$

$\left(-\frac{\sqrt{5}}{3}\right)^2 - \left(\frac{2}{3}\right)^2 = \frac{5}{9} - \frac{4}{9} = \frac{1}{9}$

[2]

(e) $\tan 2\theta$

$\frac{2\left(\frac{-2}{\sqrt{5}}\right)}{\left(1 - \left(\frac{-2}{\sqrt{5}}\right)^2\right)} = \frac{-4}{\sqrt{5}} \cdot \frac{5}{1} = \frac{-20}{\sqrt{5}} = \frac{-20\sqrt{5}}{5} = \frac{-4\sqrt{5}}{3}$

[2]

OR

$\frac{-4\sqrt{5}}{\cancel{9}} \cdot \frac{\cancel{3}}{1} = -\frac{4\sqrt{5}}{3}$

(f) $\sin 4\theta = 2\sin 2\theta \cos 2\theta$

$2\left(-\frac{4\sqrt{5}}{9}\right)\left(\frac{1}{9}\right) = \frac{-8\sqrt{5}}{81}$

[2]

(g) $\cos 4\theta = \cos^2(2\theta) - \sin^2(2\theta)$

$\left(\frac{1}{9}\right)^2 - \left(-\frac{4\sqrt{5}}{9}\right)^2 = \frac{1}{81} - \frac{80}{81} = \frac{-79}{81}$

[2]

(h) $\tan 4\theta = \frac{\sin 4\theta}{\cos 4\theta}$

$\frac{-8\sqrt{5}}{\cancel{81}} \cdot \frac{\cancel{81}}{-79} = \frac{8\sqrt{5}}{79}$

[2]

[Maximum mark: 17]

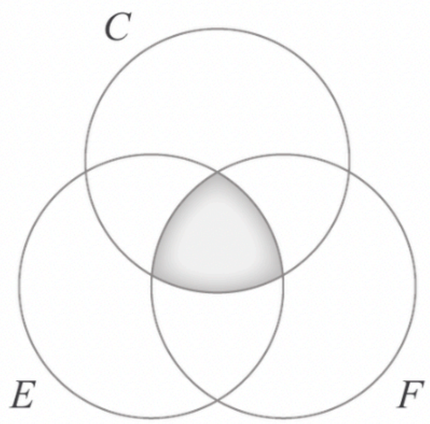
Answers

	Answer	Mark	Guidance
(a)	$\sin^2\theta + \cos^2\theta = 1$ $\frac{4}{9} + \cos^2\theta = 1$ $\cos^2\theta = 1 - \frac{4}{9}$ $cos^2\theta = \frac{5}{9}$ $cos\theta = \pm\sqrt{\frac{5}{9}}$	*M1* *A1* *A1*	Use the Pythagorean identity. It is given that θ is an obtuse angle, meaning it terminates in the second quadrant, thus $\cos\theta$ is negative.
	$\cos\theta = -\frac{\sqrt{5}}{3}$		
(b)	$\tan\theta = \frac{2/3}{-\sqrt{5}/3}$ $= \frac{2}{3} \times \frac{-3}{\sqrt{5}}$ $= -\frac{2}{\sqrt{5}}$	*M1* *A1*	Use the identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$, then simplify your answer.
(c)	$\sin 2\theta = 2 \times \frac{2}{3} \times \left(-\frac{\sqrt{5}}{3}\right)$ $= -\frac{4\sqrt{5}}{9}$	*M1* *A1*	Use the identity $\sin 2\theta = 2\sin\theta\cos\theta$, then simplify your answer.

(d)	$\cos 2\theta = 1 - 2 \times \frac{4}{9}$ $= 1 - \frac{8}{9}$ $= \frac{1}{9}$	***M1*** ***A1***	Use the identity $\cos 2\theta = 1 - 2\sin^2\theta$, then simplify your answer.
(e)	$\tan 2\theta = \frac{-4\sqrt{5}/9}{1/9}$ $= \frac{-4\sqrt{5}}{9} \times \frac{9}{1}$ $= -4\sqrt{5}$	***M1*** ***A1***	Use the identity $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}$, then simplify your answer.
(f)	$\sin 4\theta = 2 \times \left(-\frac{4\sqrt{5}}{9}\right) \times \frac{1}{9}$ $= -\frac{8\sqrt{5}}{81}$	***M1*** ***A1***	Use the identity $\sin 4\theta = 2\sin 2\theta \cos 2\theta$, then simplify your answer.
(g)	$\cos 4\theta = 2 \times \frac{1}{81} - 1$ $= \frac{2}{81} - 1$ $= -\frac{79}{81}$	***M1*** ***A1***	Use the identity $\cos 4\theta = 2\cos^2 2\theta - 1$, then simplify your answer.
(h)	$\tan 4\theta = \frac{-8\sqrt{5}/81}{-79/81}$ $= \frac{8\sqrt{5}}{81} \times \frac{81}{79}$ $= \frac{8\sqrt{5}}{79}$	***M1*** ***A1***	Use the identity $\tan 4\theta = \frac{\sin 4\theta}{\cos 4\theta}$, then simplify your answer.

Question

Three circles with radius r are drawn, each with its centre on the circumference of the other two circles, as shown in the figure below.



(a) Show that the area of the shaded region may be written as

$$A = \frac{r^2}{2}\left(\pi - \sqrt{3}\right)$$

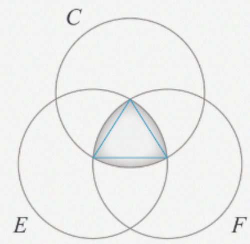
[6]

(b) Given that $r = 4$, find the area of the shaded region.

[2]

[Maximum mark: 8]

Answers

	Answer	Mark	Guidance
(a)	$\frac{1}{2} \times r \times r \times \sin\frac{\pi}{3} = \frac{1}{2}r^2 \sin\frac{\pi}{3}$ $\frac{1}{2}r^2\frac{\pi}{3} = \frac{\pi r^2}{6}$ $A = \frac{1}{2}r^2 \sin\frac{\pi}{3} + 3(\frac{\pi r^2}{6} - \frac{1}{2}r^2 \sin\frac{\pi}{3})$ $= \frac{1}{2}r^2 \sin\frac{\pi}{3} + \frac{\pi r^2}{2} - \frac{3}{2}r^2 \sin\frac{\pi}{3}$ $= \frac{\pi r^2}{2} - r^2 \sin\frac{\pi}{3}$ $= \frac{\pi r^2}{2} - \frac{r^2\sqrt{3}}{2}$ $= \frac{r^2}{2}(\pi - \sqrt{3})$	***A1*** ***A1*** ***M1*** ***M1*** ***A1*** ***A1***	Connecting the three points of intersection, as shown, gives an equilateral triangle with side lengths r:  Use the formula $\frac{1}{2}ab\sin C$ to find the area of the triangle, and use the formula $\frac{1}{2}r^2\theta$ to find the area of the sector. Then subtract to find the area of the shaded region.
(b)	$A = \frac{4^2}{2}(\pi - \sqrt{3})$ $= 8(\pi - \sqrt{3})$ $= 11.276...$ ≈ 11.3	***M1*** ***A1***	Substitute 4 for r in the equation and simplify.

Paper: 1 Marks: 6

Question

Show that

$$\tan 4x = \frac{4\tan x - 4\tan^3 x}{1 - 6\tan^2 x + \tan^4 x}$$

[Maximum mark: 6]

Answers

Answer	Mark	Guidance
$\sin 4x = \sin(2 \times 2x)$ $= 2\sin 2x \cos 2x$ $= 4\sin x \cos x(\cos^2 x - \sin^2 x)$ $= 4\sin x \cos^3 x - 4\sin^3 x \cos x$ $\cos 4x = \cos(2 \times 2x)$ $= \cos^2 2x - \sin^2 2x$ $= (\cos^2 x - \sin^2 x)^2 - (2\sin x \cos x)^2$ $= \cos^4 x + \sin^4 x - 6\sin^2 x \cos^2 x$ $\tan 4x = \dfrac{4\sin x \cos^3 x - 4\sin^3 x \cos x}{\cos^4 x - 6\sin^2 x \cos^2 x + \sin^4 x}$ Dividing both numerator and denominator by $\cos^4 x$ gives the result required. $\tan 4x = \dfrac{4\tan x - 4\tan^3 x}{1 - 6\tan^2 x + \tan^4 x}$	M1 A1 M1 A1 M1 A1	Use that $\tan 4x = \dfrac{\sin 4x}{\cos 4x}$ and use the double angle idetities to rewrite $\sin 4x$ and $\cos 4x$ first. Use that $\dfrac{\sin x \cos^3 x}{\cos^4 x} = \tan x$ $\dfrac{\sin^3 x \cos x}{\cos^4 x} = \tan^3 x$ $\dfrac{\sin^2 x \cos^2 x}{\cos^4 x} = \tan^2 x$ $\dfrac{\sin^4 x}{\cos^4 x} = \tan^4 x$ $\dfrac{\cos^4 x}{\cos^4 x} = 1$

Paper: 1 Marks: 7

Question

Given that $\sin x + \cos x = \dfrac{1}{4}$, use the binomial theorem to find the exact value of $\sin^3 x + \cos^3 x$.

[Maximum mark: 7]

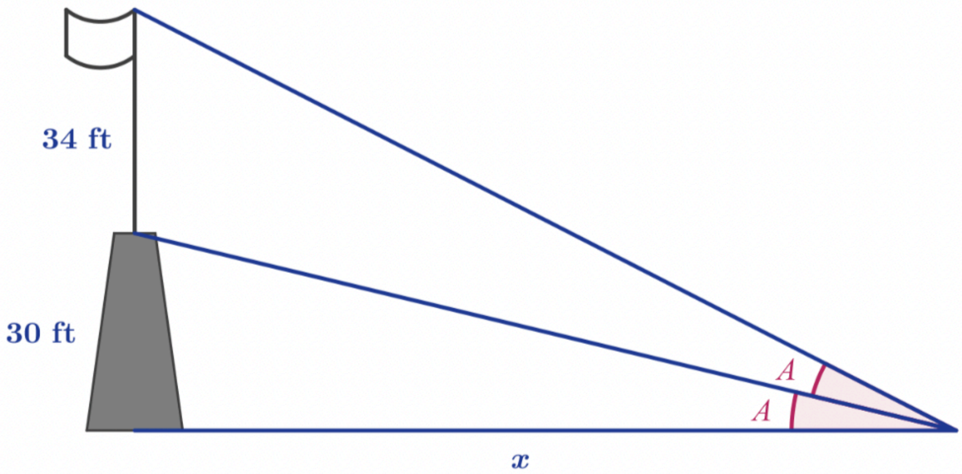
Answers

Answer	Mark	Guidance
$(\sin x + \cos x)^2 = \left(\frac{1}{4}\right)^2$ $\sin^2 x + 2\sin x\cos x + \cos^2 x = \frac{1}{16}$ $1 + 2\sin x\cos x = \frac{1}{16}$ $2\sin x\cos x = -\frac{15}{16}$ $\sin x\cos x = -\frac{15}{32}$ $\sin^3 x + \cos^3 x = (\sin x + \cos x)^3 - 3\sin x\cos x(\sin x + \cos x)$ $= \frac{1}{4^3} - 3 \times \left(-\frac{15}{32}\right) \times \frac{1}{4}$ $= \frac{47}{128}$	*M1* *M1* *A1* *M1* *A1* *A1* *A1*	Square both sides of the given equation and apply the Pythagorean identity to obtain a value of $\sin x \cos x$. From the binomial theorem: $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ $= a^3 + b^3 + 3ab(a+b)$ and thus $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$ You can find the exact value of $\sin^3 x + \cos^3 x$ by substituting $a = \sin x$ and $b = \cos x$.

Question

A flagpole that is 34 feet tall sits on top of a tower that is 30 feet tall. From a point on the horizontal ground, x feet away from the base of the tower, the angle subtended by the flagpole is equal to the angle of elevation of the top of the tower, as shown in the figure below.

Find the value of x.



[Maximum mark: 6]

Answers

Answer	Mark	Guidance
$\frac{64}{x} = \frac{2\left(\frac{30}{x}\right)}{1-\left(\frac{30}{x}\right)^2} = \frac{\frac{60}{x}}{\frac{x^2-900}{x^2}} = \frac{60x}{x^2-900}$	***M1*** ***A1*** ***M1***	For the large right angled triangle: $\tan 2A = \frac{64}{x}$
$64x^2 - 64(900) = 60x^2$ $4x^2 = 64(900)$ $x^2 = 16(900)$ $x = 120 \text{ (ft)}$	***A1*** ***A1*** ***A1***	For the small right angled triangle: $\tan A = \frac{30}{x}$ Substitute these values into the double angle identity, simplfy, and solve for x.

Note that this solution uses the double angle formula for tangent, which is not in the standard level syllabus. A similar approach using Pythagoras' theorem and expressions for $\cos A$ and $cos 2A$ can also be used.

Paper: 1 Marks: 6

Question

Given that $\sin(2x) = \dfrac{2}{3}$, find the exact value of $\sin^6 x + \cos^6 x$

$\sin^6 x + \cos^6 x = (\sin^2 x)^3 + (\cos^2 x)^3$

[Maximum mark: 6]

$= (1-\cos^2 x)^3 + (1-\sin^2 x)^3$

$= 1^3 - 3$

Answers

Answer	Mark	Guidance
$\sin^6 x + \cos^6 x = (\sin^2 x)^3 + (\cos^2 x)^3$ $= 1^3 - 3\sin^2 x\cos^2 x(1)^3$ $= 1 - 3\sin^2 x\cos^2 x$ $= 1 - 3(\sin x\cos x)^2$ $= 1 - \frac{3}{4} \times (2\sin x\cos x)^2$ $= 1 - \frac{3}{4} \times \sin^2 2x$ $= 1 - \frac{3}{4} \times \left(\frac{2}{3}\right)^2$ $= 1 - \frac{1}{3}$ $= \frac{2}{3}$	***M1*** ***M1*** ***M1*** ***A1*** ***A1*** ***A1***	Note that $\sin^6 x + \cos^6 x = (\sin^2 x)^3 + (\cos^2 x)^3$. This means you can use the binomial exansion: $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ $= a^3 + b^3 + 3ab(a+b)$ which can be rearranged as: $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$ Let $a = \sin^2 x$ and $b = \cos^2 x$ $\sin^6 x + \cos^6 x$ $= (\sin^2 x)^3 + (\cos^2 x)^3$ $= \left(\sin^2 x + \cos^2 x\right)^3 - 3\sin^2 x\cos^2 x\left(\sin^2 x + \cos^2 x\right)$ Use the Pythagorean Identity to simplify this expression: $= 1^3 - 3\sin^2 x\cos^2 x(1)^3$ $= 1 - 3\sin^2 x\cos^2 x$ Now apply the Double Angle Identity: $= 1 - 3(\sin x\cos x)^2$ $= 1 - \frac{3}{4} \times (2\sin x\cos x)^2$ $= 1 - \frac{3}{4} \times \sin^2 2x$ Finally, substitute the given value and simplify: $= 1 - \frac{3}{4} \times \left(\frac{2}{3}\right)^2$ $= 1 - \frac{1}{3}$ $= \frac{2}{3}$

Question

Given that $x \in [0, \pi]$, solve

$$\sin x + \tan x - \frac{\sin^2 x}{\cos x} - 1 = 0$$

[Maximum mark: 7]

Answers

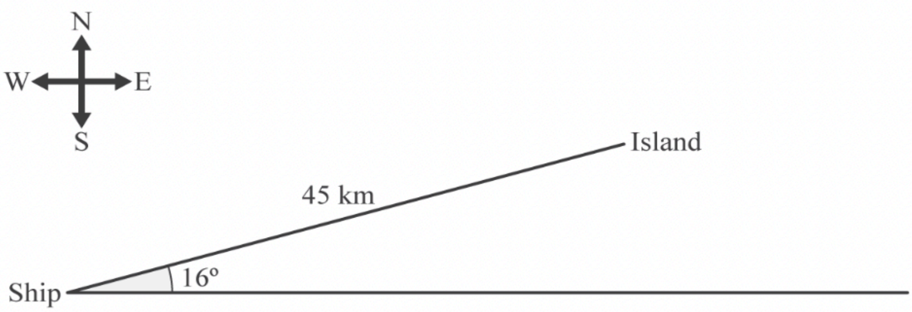
Answer	Mark	Guidance
$$\sin x + \tan x - \frac{\sin^2 x}{\cos x} - 1 = 0$$ $$\sin x + \tan x - \frac{\sin x}{\cos x}\sin x - 1 = 0$$ $$\sin x + \tan x - \tan x \sin x - 1 = 0$$ $$(\tan x - 1) - (\tan x \sin x - \sin x) = 0$$ $$(\tan x - 1) - \sin x(\tan x - 1) = 0$$	***M1*** ***A1*** ***M1*** ***A1*** ***M1*** ***R1*** ***A1***	Simplify the equation by using the identity: $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ Regroup the terms, factorise, and solve. Note that $x = \dfrac{\pi}{2}$ is not a solution to the original equation as $\tan\dfrac{\pi}{2}$ is undefined. Thus, the only solution to the equation in the given domain is $x = \dfrac{\pi}{4}$.
$$(\tan x - 1)(1 - \sin x) = 0$$ $$1 - \sin x = 0$$ $$\sin x = 1$$ $$x = \frac{\pi}{2} \text{ (rejected)}$$ $$\tan x - 1 = 0$$ $$\tan x = 1$$ $$x = \frac{\pi}{4}$$		Note: do not award the final A1 if more than one solution is given

Question

A ship is sailing due east at a constant speed of 30 km h^{-1}.

A sailor on the ship notices an island 45 km away.

The angle between the ship and the island is 16º, as shown in the diagram below:



(a) Determine the amount of time required for the ship to first reach a distance of 25 km from the island. Give your answer rounded to the nearest minute.

[6]

(b) The ship will reach a distance of 25 km from the island for the second time p minutes later. Find the value of p, rounded to the nearest whole number.

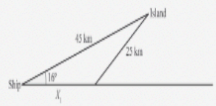
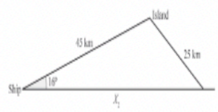
[3]

(c) Find the closest distance the ship will get from the island.

[3]

[Maximum mark: 12]

Answers

	Answer	Mark	Guidance
(a)	$25^2 = 45^2 + x^2 - 2(45)(x)\cos 16$ $0 = x^2 - 90\cos 16x + 1400$ $x = \dfrac{90\cos 16 \pm \sqrt{(90\cos 16)^2 - 4(1)(1400)}}{2(1)}$ $x = \dfrac{86.5135... \pm \sqrt{1884.5947...}}{2}$ $x_1 = 21.5508... \text{ or } x_2 = 64.9627...$ $\dfrac{21.5508...}{30} = 0.7183...$ $0.7183... \times 60 = 43.1016...$ 43 minutes (to the nearest minute)	*M1* *A1* *A1* *A1* *M1* *A1*	 (Click on image to enlarge it) Use the cosine rule to write an equation, and then use your GDC or the quadratic formula to solve for x_1, as shown in the diagram above. To find the time required to travel this distance, divide by the speed of the ship, and then multiply by sixty to obtain the answer in minutes. Note that your final answer must be an integer.
(b)	$x_2 - x_1 = 64.9627... - 21.5508...$ $= 43.4119...$ $\dfrac{43.4119...}{30} = 1.447...$	*M1* *M1* *A1*	 (Click on image to enlarge it)

	$1.447... \times 60 = 86.8238...$ 87 minutes (to the nearest minute)		You can determine how much further the ship must travel to reach a distance of 25 km from the island for the second time by finding the difference between the two solutions from part a. As before, you must find the time required to travel this distance, and write the final answer as an integer.
(c)	$\sin 16 = \dfrac{h}{45}$ $h = 45 \sin 16 \approx 12.4 \text{ (km)}$	***M1*** ***A1*** ***A1***	The closest distance occurs when the ship is directly south of the island, forming a right triangle as shown here:  (Click on image to enlarge it) Solve for h using right triangle trigonometry.

Paper: 2 Marks: 11

Question

Consider the equation $8\sin^2 x\cos x - 2\cos^2 x = 2\cos x$ defined for $x \in [0, 2\pi]$.

(a) Find the exact values of $\cos x$.

[6]

(b) Hence, find the possible values of x.

[5]

[Maximum mark: 11]

	Answer	Mark	Guidance
(a)	$8\sin^2 x\cos x - 2\cos^2 x = 2\cos x$ $8\sin^2 x\cos x - 2\cos^2 x - 2\cos x = 0$ $(\cos x)\left(8\sin^2 x - 2\cos x - 2\right) = 0$ $(\cos x)\left[8\left(1-\cos^2 x\right) - 2\cos x - 2\right] = 0$ $(\cos x)\left(8 - 8\cos^2 x - 2\cos x - 2\right) = 0$ $(\cos x)\left(6 - 2\cos x - 8\cos^2 x\right) = 0$ $(\cos x)(2 + 2\cos x)(3 - 4\cos x) = 0$ $\cos x = 0, \quad \cos x = -1, \quad \cos x = \frac{3}{4}$	***M1*** ***M1*** ***A1*** ***A1*** ***A1*** ***A1***	Move all the terms to one side and factor. Then use the Pythagorean Identity to write the equation in terms of cosine only. Factorise the quadratic expression, set each factor equal to zero, then solve for $\cos x$.
(b)	$\cos x = 0$ $x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\cos x = -1$ $x = \pi$ $\cos x = \frac{3}{4}$ $x \approx 0.723, 5.56$	***A1*** ***A1*** ***A1*** ***A1*** ***A1***	Solve each cosine equation for x, using either the unit circle or your GDC.

Question

Consider the function $f(x) = -\frac{3}{2}\cos\left(3x - \frac{\pi}{4}\right) + 2$ given that $x \in [0, 2\pi]$

(a) Sketch the graph of the function on the given domain.

[3]

(b) Determine the set of values k such that the equation $f(x) = k$ has:

(i) no solution.

[2]

(ii) exactly three solutions.

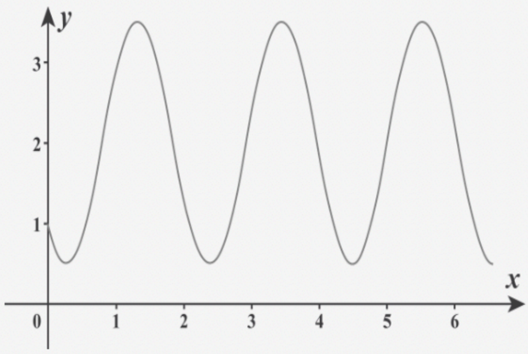
[2]

(iii) more than three solutions.

[2]

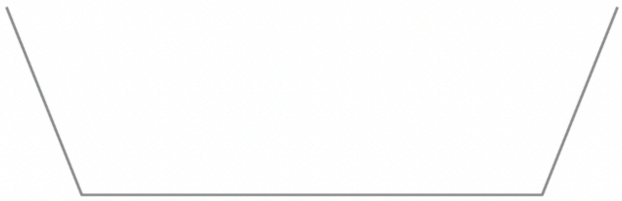
[Maximum mark: 9]

Answers

	Answer	Mark	Guidance
(a)	 (Click on the image to enlarge it)	*A1* *A1* *A1*	Use your GDC to graph the function and draw the graph carefully showing the important features. Award one mark for correct shape, one mark for correct domain, and one mark for correct min/max values.
(b) (i)	$k > 3.5$ or $k < 0.5$	*A1* *A1*	The range of the function is $f0.5 \leq f(x) \leq 3.5$ Therefore, the equation $f(x) = k$ will have no solution when k falls outside the range.
(b) (ii)	$k = 3.5$ or $k = 0.5$	*A1* *A1*	Since there are three maximum points and three minimum points within the given domain, the equation $f(x) = k$ will have exactly three solutions when k is equal to the minimum value or the maximum value.
(b) (iii)	$0.5 < k < 3.5$	*A1* *A1*	Any horizontal line drawn between the maximum and minimum values of $f(x)$ will intersect the graph at more than three points.

Question

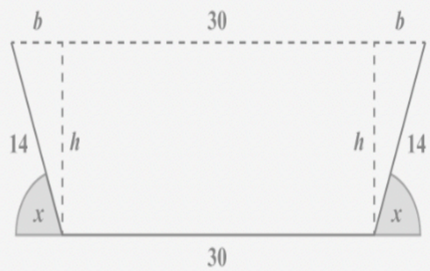
A strip of metal, 58 cm in length, is to be formed into a trough with a trapezoidal cross-section by bending up the edges, as shown in the diagram below:



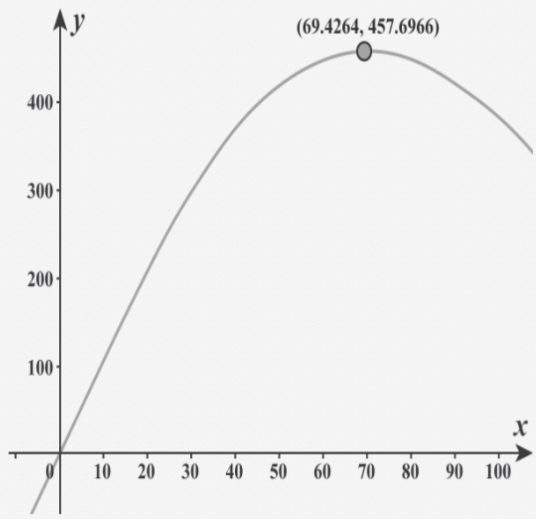
Given that the base is 30 cm wide, determine the width across the top that gives the greatest area.

[Maximum mark: 7]

Answers

Answer	Mark	Guidance
$\text{Area} = \frac{1}{2}(30 + 30 + 2b)h$ $= (30 + b)h$ $= (30 + 14\cos x)(14\sin x)$ $x = 69.4264....$ $b = 14\cos 69.4264... = 4.9197...$ $\text{width} = 30 + 2b$ $= 30 + 2(4.9197...)$ $\approx 39.8 \text{ (cm)}$	*A1* *M1* *M1* *A1* *M1* *A1* *A1*	Since the base is 30 cm, you can calculate the length of the sides: $\text{side lengths} = \frac{58 - 30}{2} = 14$ Start by determining the angle, x, at which the sides will be bent, as shown below:  From the diagram above using right angled triangle trigonometry: $\sin x = \frac{h}{14} \quad \Rightarrow \quad h = 14\sin x$ $\cos x = \frac{b}{14} \quad \Rightarrow \quad b = 14\cos x$

Now write an expression, in terms of x, for the area of the trapezoid. Use your GDC to graph the function and find the coordinates of the maximum point:



The maximum area occurs when $x = 69.4264...^\circ$.

Next find the value of b, and finally, the width across the top.

Paper: 2 Marks: 6

Question

From the top of a vertical cliff 250 metres above sea level, an observer sees two ships due west of the foot of the cliff. The angles of depression of the ships measure 62° and 34°.

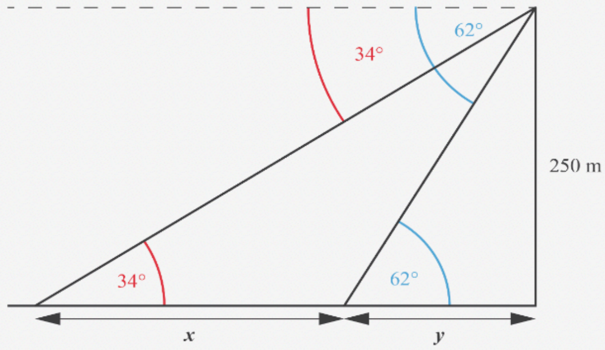
(a) Find, to the nearest metre, the distance of each ship from the foot of the vertical cliff.

[5]

(b) Write down the distance between the two ships.

[Maximum mark: 6]

Answers

	Answer	Mark	Guidance
(a)	 For one ship: $\tan 62^\circ = \dfrac{250}{y}$ $y = \dfrac{250}{\tan 62^\circ}$ $y = 132.927...$ $y \approx 133$ m For the other ship: $\tan 34^\circ = \dfrac{250}{x+y}$ $x + y = \dfrac{250}{\tan 34^\circ}$ $x + y = 370.6402...$ $x + y \approx 371$ m	***M1*** ***M1*** ***A1*** ***M1*** ***A1***	Draw a clear, careful diagram and label everything you can.

(b)	Distance between ships: $x = 370.640... - 132.927...$ $= 237.713$ $\approx 238 \text{ m}$	***A1***	The distance between the ships can be found by subtracting the two distances found in part **(a)**.

Paper: 1 Marks: 6

Question

Solve the equation $\sin\theta - \cos\theta = 1$, where $0^\circ \le \theta \le 180^\circ$.

[Maximum mark: 6]

Answers

	Answer	Mark	Guidance
	$(\sin\theta - \cos\theta)^2 = 1^2$ $\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta = 1$ $(\sin^2\theta + \cos^2\theta) - 2\sin\theta\cos\theta = 1$ $1 - 2\sin\theta\cos\theta = 1$ $2\sin\theta\cos\theta = 0$	***M1*** ***A1*** ***M1*** ***A1*** ***R1*** ***A1***	Solve by squaring both sides of the equation. This may introduce extraneous answers so you will need to check the solutions: • $\sin 0^\circ - \cos 0^\circ = 0 - 1 = -1$, so $\theta = 0^\circ$ is rejected. • $\sin 90^\circ - \cos 90^\circ = 1 - 0 = 1$, so $\theta = 90^\circ$ is a solution. • $\sin 180^\circ - \cos 180^\circ = 0 - (-1) = 1$, so $\theta = 180^\circ$ is also a solution.
	$\sin\theta = 0$ $\theta = 0$ (rejected) $\theta = 180^\circ$ $\cos\theta = 0$ $\theta = 90^\circ$		The solutions are $\theta = 90^\circ$ and 180°

Question

The depth of water in a harbour on a particular day of the year is modelled by the the equation $d = 16 - 2\cos\left(\frac{\pi t}{12}\right)$, where d is measured in metres and t is the time, in hours after midnight.

(a) Find the depth of water in the harbour at midnight.

[2]

(b) Determine the maximum and minimum depth of water in the harbour.

[3]

(c) Find the first time after midnight when the water depth is 17.5 m.

[3]

(d) Show that the depth of water repeats every 24 hours.

[2]

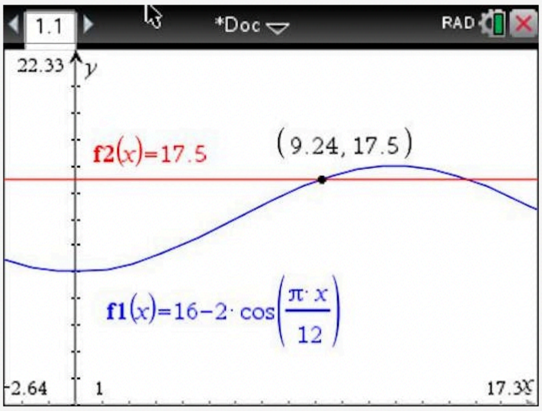
(e) Find the interval of time, within the first 24 hours, when the depth of water is above 14.6 m.

[4]

[Maximum mark: 14]

Answers

	Answer	Mark	Guidance
(a)	$d(0) = 16 - 2\cos\left(\frac{\pi \times 0}{12}\right)$ $= 16 - 2\cos(0)$ $= 16 - 2 \times 1$ $= 14$ (m)	***M1*** ***A1***	At midnight, $t = 0$. Substitute this value into the function and simplify.
(b)	The maximum depth is 18 (m) The minimum depth is 14 (m)	***M1*** ***A1*** ***A1***	Use your GDC to view the graph of the function and find the maximum and minimum y-values. Or, you can calculate them by hand using the fact that $-1 \leq \cos\theta \leq 1$: Maximum value is $16 - 2 \times (-1) = 18$ Minimum value if $16 - 2 \times 1 = 14$

(c)	 $x = 9.24$ 9:14 am	***M1*** ***A1*** ***A1***	Use your GDC to graph the function d and the horizontal line $y = 17.5$. Find the coordinates of the intersection point and then convert the x-value to a time.
(d)	$\text{period} = \frac{2\pi}{b} = \frac{2\pi}{\frac{\pi}{12}} = 2\pi \times \frac{12}{\pi} = 24$	***R1*** ***A1***	To show that the depth of water repeats every 24 hours you need to show that the period of the function d is 24
(e)	$x = 3.04, 20.96$ From 3:02 am to 8:58 pm	***M1*** ***A1A1*** ***A1***	Use your GDC to graph the function d and the horizontal line $y = 14.6$. Find the coordinates of the intersection points and then convert the x-values to times. Note: the final answer must be an interval.

Paper: 2 Marks: 15

Question

Let $f(x) = \sin\left(\frac{\pi}{2}x\right) + \cos\left(\frac{\pi}{2}x\right)$, for $-4 \leqslant x \leqslant 4$.

(a) Sketch the graph of f.

[3]

(b) Find the set of values of x for which the function is decreasing.

[5]

(c) The function f can also be written in the form $f(x) = a\cos(b(x - c))$, where a, b, and c are positive real numbers. Find the value of:

(i) a

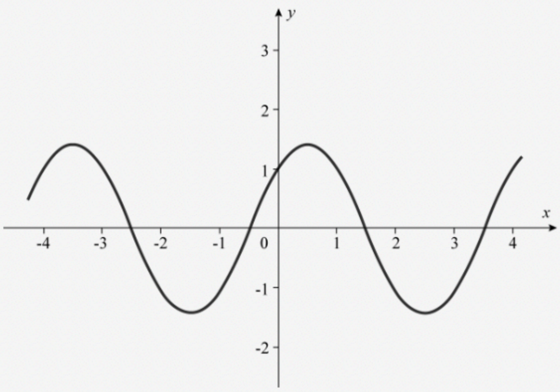
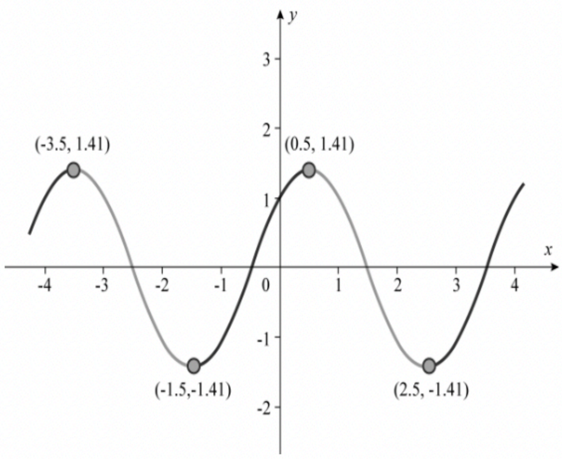
(ii) b

(iii) c

[7]

[Maximum mark: 15]

Answers

	Answer	Mark	Guidance
(a)		*A1* *A1* *A1*	Use your GDC to graph the function and draw the graph carefully showing all the important features. Award one mark for the correct shape, one mark for the correct domain, and one mark for the correct max/min points.
(b)	 $-3.5 < x < -1.5$ and $0.5 < x < 2.5$	*M1* *A2* *A2*	Use your GDC to find the coordinates of the maximum and minimum points.

(c) (i)	$a = \dfrac{1.41 - (-1.41)}{2} = 1.41$	*M1* *A1*	The value of a is the amplitude of the graph, which is defined as half the distance between maximum and minimum y-values. Use your GDC to the required y-values and then calculate the value of a. Note: you can also use calculus to find the amplitude. Accept the exact answer, $\sqrt{2}$.
(ii)	$\text{period} = 0.5 - (-3.5) = 4$ $4 = \dfrac{2\pi}{b}$ $4b = 2\pi$ $b = \dfrac{\pi}{2}$	*A1* *M1* *A1*	The period of the function can be calculated by finding difference between x-coordinates of consecutive maximum points. Then use the formula $\text{period} = \dfrac{2\pi}{b}$ to calculate the value of b.

(iii)	$c = 0.5$	***A2***	The value of c represents the horizontal shift of the graph relative to the cosine parent function. Since the graph of $y = \cos x$ has a maximum point at $x = 0$, the horizontal shift can be represented by the x-coordinate of any maximum point on the graph of f. But since c must be a positive number, we need $c = 0.5$.