INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

MAA

EXERCISES [MAA 4.5-4.7]

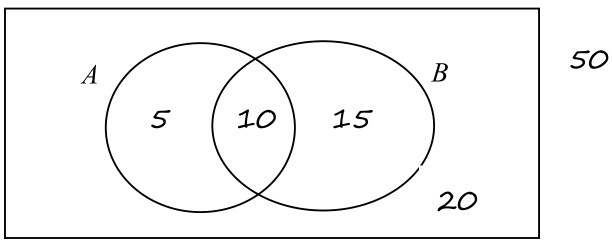
PROBABILITY I (VENN DIAGRAMS – TABLES)

Compiled by Christos Nikolaidis

O. Practice questions

1. [Maximum mark: 18] ***[without GDC]***

The following Venn diagram shows the sample space U and the event A and B together with the numbers of elements in the corresponding regions.



(a) Complete the following table. [6]

$n(A)$		$n(B)$		$n(A \cap B)$	
$n(A')$		$n(B')$		$n(A \cup B)$	
$n(A' \cap B)$		$n(A \cap B')$		$n(A' \cap B')$	
$n(A' \cup B)$		$n(A \cup B')$		$n(A' \cup B')$	

(b) Write down the following probabilities [6]

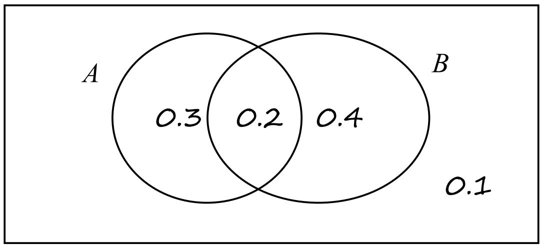
$\mathrm{P}(A)$		$\mathrm{P}(A')$		$\mathrm{P}(A \cup B)$	
$\mathrm{P}(A' \cap B)$		$\mathrm{P}(A' \cup B)$		$\mathrm{P}(B' \cup A)$	

(c) Write down the following **conditional** probabilities [6]

$\mathrm{P}(A \mid B)$		$\mathrm{P}(A' \mid B)$		$\mathrm{P}(B' \mid A)$	
$\mathrm{P}(B \mid A)$		$\mathrm{P}(A \mid B')$		$\mathrm{P}(A' \mid B')$	

2. [Maximum mark: 12] ***[without GDC]***

The following Venn diagram shows the sample space U and the events A and B together with their probabilities in the corresponding regions.



(a) Write down the following probabilities [6]

$P(A)$		$P(A')$		$P(A \cap B)$	
$P(A \cup B)$		$P(A' \cap B)$		$P(A' \cup B)$	

(b) Write down the following **conditional** probabilities [6]

$P(A \mid B)$		$P(A' \mid B)$		$P(B' \mid A)$	
$P(B \mid A)$		$P(A \mid B')$		$P(A' \mid B')$	

3. [Maximum mark: 8] ***[without GDC]***

The following table shows the distribution of a population according to two criteria, gender and group. We select a person at random.

	Group A	**Group B**	**Group C**	**Total**
Boys	5	15	10	**30**
Girls	20	25	5	**50**
Total	**25**	**40**	**15**	**80**

(a) Write down the following probabilities [4]

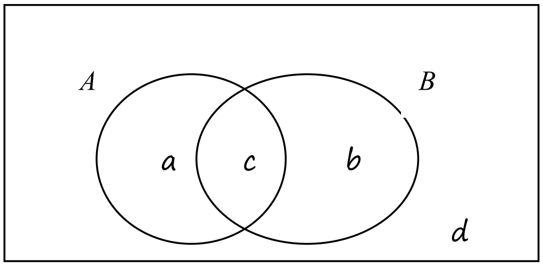
$P(Boy)$		$P(Group\,C)$	
$P(Boy\ AND\ Group\ C)$		$P(Boy\ OR\ Group\ C)$	

(b) Write down the following **conditional** probabilities [4]

$P(Boy \mid Group\,C)$		$P(Group\,C \mid Boy)$	
$P(Boy \mid NOT\,Group\,C)$		$P(NOT\ Group\ C \mid Boy)$	

4. [Maximum mark: 20] ***[without GDC]***

The following Venn diagram shows the universal set U and the sets A and B together with the probabilities of the corresponding regions.



where $a+b+c+d = 1$.

(a) Express in terms of a, b, c, d the following probabilities:

$P(A)$	$a+c$	$P(A'\cap B)$	
$P(A')$		$P(A'\cup B)$	
$P(A\cap B)$		$P(A'\cap B')$	
$P(A\cup B)$		$P(A'\cup B')$	

[7]

(b) Express in terms of a, b, c, d the following probabilities:

$P(A\mid B)$		$P(B'\mid A)$	
$P(B\mid A)$		$P(B\mid A')$	
$P(A'\mid B)$		$P(A'\mid B')$	
$P(A\mid B')$		$P(B'\mid A')$	

[8]

(c) Given that the events A and B are independent and $a=b=c$, find the values of a,b,c and d [5]

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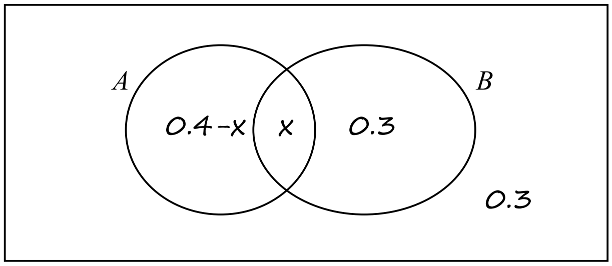
5. [Maximum mark: 12] ***[with GDC]***

The following Venn diagram shows the universal set U and the sets A and B together with the probabilities of the corresponding regions.



(a) Write down the values of $\mathrm{P}(A)$ and $\mathrm{P}(A \cup B)$ [2]

(b) Write down the value of x given that A and B are mutually exclusive. [1]

(c) Find the value of x given that A and B are independent. [3]

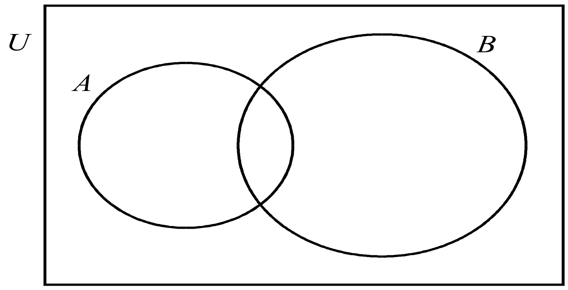
(d) Find the value of x given that $\mathrm{P}(A \mid B) = 0.5$. [3]

(e) Find the value of x given that $\mathrm{P}(B \mid A) = 0.25$. [3]

A. Exam style questions (SHORT)

6. [Maximum mark: 4] ***[without GDC]***

The following Venn diagram shows the universal set U and the sets A and B.



$n(U) = 100$

$n(A) = 30, \ n(B) = 50$

$n(A \cup B) = 65.$

(a) Find $n(B \cap A')$ [2]

(b) An element is selected at random from U. What is the probability that this element is in $\mathrm{P}(B \cap A')$? [2]

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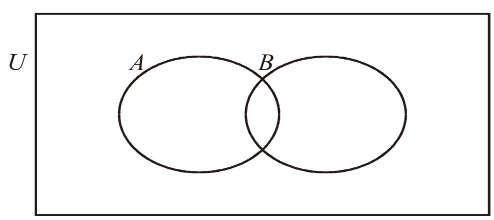
7. [Maximum mark: 4] ***[without GDC]***

The following Venn diagram shows a sample space U and events A and B.



$n(U) = 36$

$n(A) = 11, \ n(B) = 6$

$n\,(A \cup B)' = 21.$

(a) Find (i) $n(A \cap B)$; (ii) $\mathrm{P}(A \cap B)$ [2]

(b) Explain why events A and B are not mutually exclusive. [2]

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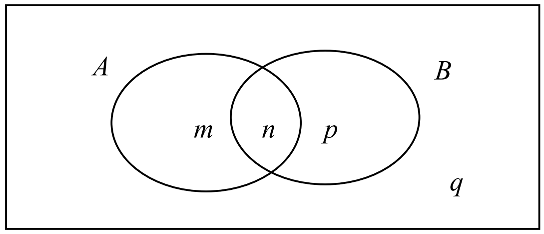
8. [Maximum mark: 6] ***[without GDC]***

The Venn diagram below shows events A and B where $\mathrm{P}(A)=0.3$, $\mathrm{P}(A\cup B)=0.6$ and $\mathrm{P}(A\cap B)=0.1$. The values m, n, p and q are probabilities.



(a) (i) Write down the value of n.

(ii) Find the value of m, of p, and of q. [4]

(b) Find $\mathrm{P}(B')$. [2]

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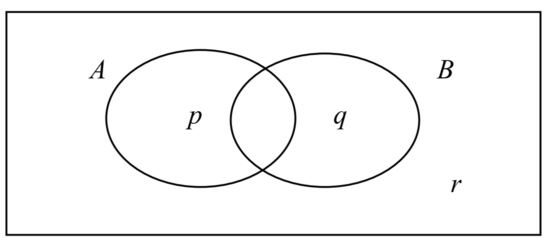
9. [Maximum mark: 6] ***[without GDC]***

Consider the events A and B, where $\mathrm{P}(A)=0.5$, $\mathrm{P}(B)=0.7$ and $\mathrm{P}(A\cap B)=0.3$.

The Venn diagram below shows the events A and B, and the probabilities p, q, r.



(a) Write down the value of (i) p; (ii) q; (iii) r. [3]

(b) Find the value of $\mathrm{P}(A\,|\,B')$. [2]

(c) Hence, or otherwise, show that the events A and B are **not** independent. [1]

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10. [Maximum mark: 4] ***[without GDC]***

For the events A and B, $\mathrm{P}(A) = 0.6$, $\mathrm{P}(B) = 0.8$ and $\mathrm{P}(A \cup B) = 1$.

(a) Find $\mathrm{P}(A \cap B)$; [2]

(b) Find $\mathrm{P}(A' \cup B')$ [2]

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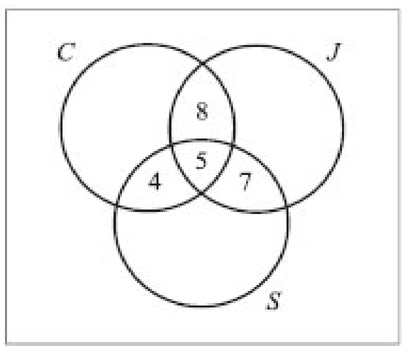
11. [Maximum mark: 6] ***[with GDC]***

The Venn diagram below shows information about 120 students in a school. Of these, 40 study Chinese (*C*), 35 study Japanese (*J*), and 30 study Spanish (*S*).



A student is chosen at random from the group. Find the probability that the student

(a) studies exactly two of these languages; [1]

(b) studies only Japanese; [2]

(c) does not study any of these languages. [3]

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12. [Maximum mark: 6] ***[without GDC]***

In a class, 40 students take chemistry only, 30 take physics only, 20 take both chemistry and physics, and 60 take neither.

(a) Find the probability

(i) that a student takes physics given that the student takes chemistry. [2]

(ii) that a student takes physics given that the student does **not** take chemistry. [4]

(b) State whether the events “taking chemistry” and “taking physics” are mutually exclusive, independent, or neither. Justify your answer. [2]

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13. [Maximum mark: 4] ***[without GDC]***

In a survey, 100 students were asked “do you prefer to watch television or play sport?” Of the 46 boys, 33 said they would choose sport, while 29 girls made this choice.

	Boys	Girls	Total
Television			
Sport	33	29	
Total	46		100

By completing this table or otherwise, find the probability that

(a) a student selected at random prefers to watch television; [2]

(b) a student prefers to watch television, given that the student is a boy. [2]

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14. [Maximum mark: 4] ***[without GDC]***

In a survey of 200 people, 90 of whom were female, it was found that 60 people were unemployed, including 20 males.

(a) Using this information, complete the table below. [2]

	Males	**Females**	**Totals**
Unemployed			
Employed			
Totals			200

(b) If a person is selected at random from this group of 200, find the probability that this person is

(i) an unemployed female;

(ii) a male, given that the person is employed. [2]

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15. [Maximum mark: 6] ***[without GDC]***

The eye colour of 97 students is recorded in the chart below.

	Brown	Blue	Green
Male	21	16	9
Female	19	19	13

One student is selected at random. Write down

(a) the probability that the student is a male. [2]

(b) the probability that the student has green eyes, given that the student is a female [2]

(c) Find the probability that the student has green eyes or is male. [2]

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16. [Maximum mark: 7] ***[with GDC]***

There are 20 students in a classroom. Each student plays only one sport. The table below gives their sport and gender.

	Football	**Tennis**	**Hockey**
Female	5	3	3
Male	4	2	3

(a) One student is selected at random.

(i) Calculate the probability that the student is a male or is a tennis player.

(ii) Given that the student selected is female, calculate the probability that the student does not play football. [4]

(b) Two students are selected at random. Calculate the probability that neither student plays football. [3]

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17. [Maximum mark: 6] ***[without GDC]***

Consider events A, B such that $\mathrm{P}(A) \neq 0$, $\mathrm{P}(A) \neq 1$, $\mathrm{P}(B) \neq 0$, and $\mathrm{P}(B) \neq 1$.

In each of the situations (i), (ii), (iii) below state whether A and B are

mutually exclusive (M); independent (I); neither (N).

(i) $\mathrm{P}(A \mid B) = P(A)$ (ii) $\mathrm{P}(A \cap B) = 0$ (iii) $\mathrm{P}(A \cap B) = P(A)$

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18. [Maximum mark: 6] ***[without GDC]***

In a bilingual school there is a class of 21 pupils. In this class, 15 of the pupils speak Spanish as their first language and 12 of these 15 pupils are Argentine. The other 6 pupils in the class speak English as their first language and 3 of these 6 pupils are Argentine. A pupil is selected at random from the class and is found to be Argentine. Find the probability that the pupil speaks Spanish as his/her first language.

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19. [Maximum mark: 6] ***[without GDC]***

Let A and B be events such that $\mathrm{P}(A)=\frac{1}{2}$, $\mathrm{P}(B)=\frac{3}{4}$ and $\mathrm{P}(A\cup B)=\frac{7}{8}$.

(a) Calculate $\mathrm{P}(A\cap B)$. [2]

(b) Calculate $\mathrm{P}(A\mid B)$. [2]

(c) Are the events A and B independent? Give a reason for your answer. [2]

(a) $P(A\cap B)=\frac{1}{2}\left(\frac{3}{4}\right)=\frac{3}{8}$

(b) $P(A\mid B)=\frac{\cancel{3}}{\cancel{8}_{2}}\cdot\frac{\cancel{4}^{1}}{\cancel{3}}=\frac{1}{2}$

(c) $P(A\cap B)=P(A)\cdot P(B)$

$\frac{3}{8}=\frac{1}{2}\left(\frac{3}{4}\right)$

$\frac{3}{8}=\frac{3}{8}$, Therefore yes, they're independent.

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20*. [Maximum mark: 6] ***[without GDC]***

Given that $(A \cup B)' = \varnothing$, $\mathrm{P}(A'|B) = \frac{1}{3}$ and $\mathrm{P}(A) = \frac{6}{7}$, find $\mathrm{P}(B)$.

21. [Maximum mark: 6] ***[without GDC]***

The letters of the word PROBABILITY are written on 11 cards as shown below.

P	R	O	B	A	B	I	L	I	T	Y

Two cards are drawn at random without replacement.

Let A be the event the first card drawn is the letter A.

Let B be the event the second card drawn is the letter B.

(a) Find $\mathrm{P}(A)$. [1]

(b) Find $\mathrm{P}(B \mid A)$. [2]

(c) Find $\mathrm{P}(A \cap B)$. [3]

(a) $P(A) = \frac{1}{11}$

(b) $P(B|A) = \frac{2}{10} = \frac{1}{5}$

(c) $P(A \cap B) = \frac{1}{11} \cdot \frac{2}{10} = \frac{2}{110} = \frac{1}{55}$

22. [Maximum mark: 6] ***[without GDC]***

Let A and B be independent events such that $\mathrm{P}(A) = 0.3$ and $\mathrm{P}(B) = 0.8$.

(a) Find $\mathrm{P}(A \cap B)$. [2]

(b) Find $\mathrm{P}(A \cup B)$. [2]

(c) Are A and B mutually exclusive? Justify your answer. [2]

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23. [Maximum mark: 6] ***[without GDC]***

Events E and F are independent, with $\mathrm{P}(E) = \frac{2}{3}$ and $\mathrm{P}(E \cap F) = \frac{1}{3}$. Calculate

(a) $\mathrm{P}(F)$; [3]

(b) $\mathrm{P}(E \cup F)$. [3]

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24. [Maximum mark: 6] ***[with GDC]***

Consider the events A and B, where $\mathrm{P}(A)=\frac{2}{5}$, $\mathrm{P}(B')=\frac{1}{4}$ and $\mathrm{P}(A\cup B)=\frac{7}{8}$.

(a) Write down $\mathrm{P}(B)$. [1]

(b) Find $\mathrm{P}(A\cap B)$. [2]

(c) Find $\mathrm{P}(A\mid B)$. [3]

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25. [Maximum mark: 6] ***[without GDC]***

Let A and B be independent events, where $\mathrm{P}(A)=0.6$ and $\mathrm{P}(B)=x$.

(a) Write down an expression for $\mathrm{P}(A\cap B)$. [1]

(b) Given that $\mathrm{P}(A\cup B)=0.8$,

(i) find x; (ii) find $\mathrm{P}(A\cap B)$. [4]

(c) **Hence**, explain why A and B are **not** mutually exclusive. [1]

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26*. [Maximum mark: 6] ***[with GDC]***

The events A and B are independent such that $\mathrm{P}(B) = 3\mathrm{P}(A)$ and $\mathrm{P}(A \cup B) = 0.68$.

Find $\mathrm{P}(B)$.

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27. [Maximum mark: 6] ***[without GDC]***

For events A and B, the probabilities are $\mathrm{P}(A) = \frac{3}{11}$, $\mathrm{P}(B) = \frac{4}{11}$.

Calculate the value of $\mathrm{P}(A \cap B)$

(a) if $\mathrm{P}(A \cup B) = \frac{6}{11}$; [3]

(b) if events A and B are independent. [3]

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28. [Maximum mark: 6] ***[without GDC]***

The events A and B are such that $\mathrm{P}(A)=0.5$, $\mathrm{P}(B)=0.3$, $\mathrm{P}(A\cup B)=0.6$.

(a) (i) Find the value of $\mathrm{P}(A\cap B)$

(ii) Hence show that A and are not independent. [3]

(b) Find the value of $\mathrm{P}(B\mid A)$. [3]

29*. [Maximum mark: 6] ***[without GDC]***

Let A and B be events such that $\mathrm{P}(A)=0.6$, $\mathrm{P}(A\cup B)=0.8$ and $\mathrm{P}(B\mid A)=0.6$. Find $\mathrm{P}(B)$.

$0.8=0.6+P(B)-P(A\cap B)$

$P(B)=0.2+P(A\cap B)$

$0.6=\dfrac{P(A\cap B)}{P(B)}$

$0.6=\dfrac{P(A\cap B)}{0.2+P(A\cap B)}$

$0.03+0.06P(A\cap B)=P(A\cap B)$

$0.03=0.94P(A\cap B)$

$P(A\cap B)=0.0319$

30*. [Maximum mark: 6] ***[without GDC]***

Let A and B be events such that $P(A)=\frac{1}{5}$, $P(B\,|\,A)=\frac{1}{4}$ and $P(A\cup B)=\frac{7}{10}$.

(a) Find $P(A\cap B)$.

(b) Find $P(B)$

(c) Show that A and B are **not** independent.

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31. [Maximum mark: 5] ***[without GDC]***

Given that events A and B are independent with $P(A\cap B)=0.3$, $P(A\cap B')=0.3$, find $P(A\cup B)$.

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32*. [Maximum mark: 6] ***[with GDC]***

The independent events A and B are such that $P(A)=0.4$ and $P(A\cup B)=0.88$. Find

(a) $P(B)$. [3]

(b) the probability that either A occurs or B occurs, but **not** both. [3]

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33*. [Maximum mark: 7] ***[with GDC]***

Consider the independent events A and B.

Given that $\mathrm{P}(B)=2P(A)$ and $\mathrm{P}(A\cup B)=0.52$, find $\mathrm{P}(B)$.

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34. [Maximum mark: 6] ***[without GDC]***

Two unbiased 6-sided dice are rolled, a red one and a black one. Let E and F be the events

E: the same number appears on both dice;

F: the sum of the numbers is 10.

(a) Find $\mathrm{P}(E)$. [2]

(b) Find $\mathrm{P}(F)$. [1]

(c) Find $\mathrm{P}(E \cup F)$. [3]

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35. [Maximum mark: 7] ***[without GDC]***

Two fair dice are thrown and the number showing on each is noted. The sum of these two numbers is *S*. Find the probability that

(a) *S* is less than 8; [2]

(b) at least one die shows a 3; [2]

(c) at least one die shows a 3, given that *S* is less than 8. [3]

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B. Paper 2 questions (LONG)

36. [Maximum mark: 10] ***[without GDC]***

In a survey of 50 people it is found that 40 own a television and 21 own a computer. Four do not own either a computer or a television. A person is chosen at random from this group. Find the probability

(a) that this person owns both a television and a computer. [2]

(b) that this person owns a television, given that he also owns a computer. [2]

(c) that this person owns both a television or a computer. [2]

(d) that this person owns a television, given that he does not own a computer. [2]

(e) that this person owns a computer, given that he does not own a television. [2]

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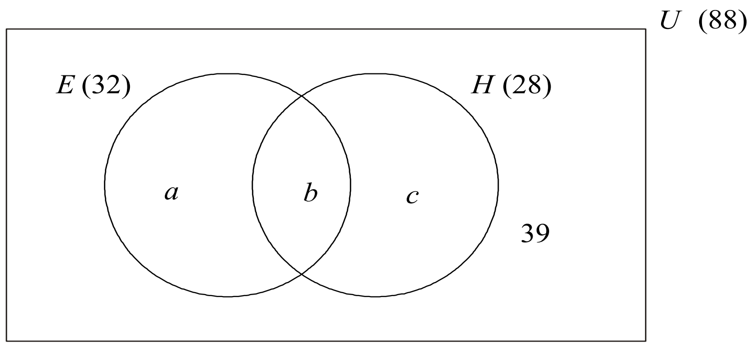
37. [Maximum mark: 12] ***[with GDC]***

In a school of 88 boys, 32 study economics (E), 28 study history (H) and 39 do not study either subject. This information is represented in the following Venn diagram.



(a) Calculate the values a, b, c. [4]

(b) A student is selected at random.

(i) Calculate the probability that he studies **both** economics and history.

(ii) Given that he studies economics, calculate the probability that he does **not** study history. [3]

(c) A group of three students is selected at random from the school.

(i) Calculate the probability that none of these students studies economics.

(ii) Calculate the probability that at least one of these students studies economics. [5]

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38. [Maximum mark: 16] ***[with GDC]***

Two restaurants, *Center* and *New*, sell fish rolls and salads.

Let F be the event a customer chooses a fish roll.

Let S be the event a customer chooses a salad.

Let N be the event a customer chooses neither a fish roll nor a salad.

In the *Center* restaurant $\mathrm{P}(F) = 0.31$, $\mathrm{P}(S) = 0.62$, $\mathrm{P}(N) = 0.14$.

(a) Show that $\mathrm{P}(F \cap S) = 0.07$. [3]

(b) Given that a customer chooses a salad, find the probability the customer also chooses a fish roll. [3]

(c) Are F and S independent events? Justify your answer. [3]

At *New* restaurant, $\mathrm{P}(N) = 0.14$. Twice as many customers choose a salad as choose a fish roll. Choosing a fish roll is **independent** of choosing a salad.

(d) Find the probability that a fish roll is chosen. [7]

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39. [Maximum mark: 12] ***[with GDC]***

The table below shows the subjects studied by 210 students at a college.

	Year 1	Year 2	Totals
History	50	35	85
Science	15	30	45
Art	45	35	80
Totals	110	100	210

(a) A student from the college is selected at random.

Let A be the event the student studies Art.

Let B be the event the student is in Year 2.

(i) Find $\mathrm{P}(A)$.

(ii) Find the probability that the student is a Year 2 Art student.

(iii) Are the events A and B independent? Justify your answer. [6]

(b) Given that a History student is selected at random, calculate the probability that the student is in Year 1. [2]

(c) Two students are selected at random from the college. Calculate the probability that one student is in Year 1, and the other in Year 2. [4]

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40. [Maximum mark: 12] ***[with GDC]***

In a class of 100 boys, 55 boys play football and 75 boys play rugby. Each boy must play at least one sport from football and rugby.

(a) (i) Find the number of boys who play both sports.

(ii) Write down the number of boys who play only rugby. [3]

(b) One boy is selected at random.

(i) Find the probability that he plays only one sport.

(ii) Given that the boy selected plays only one sport, find the probability that he plays rugby. [4]

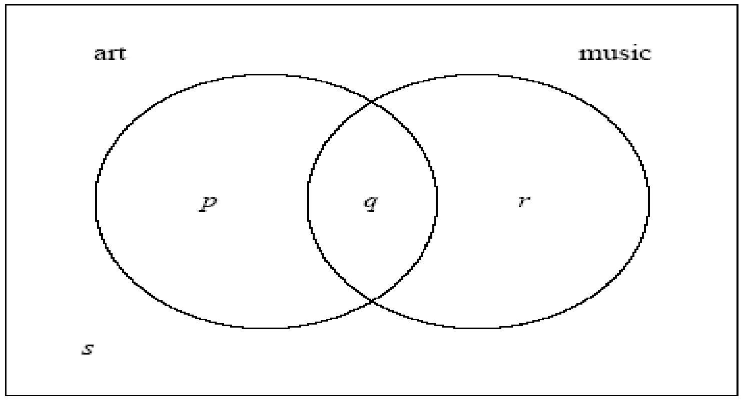
Let A be the event that a boy plays football and B be the event that a boy plays rugby.

(c) Explain why A and B are **not** mutually exclusive. [2]

(d) Show that A and B are **not** independent. [3]

41. [Maximum mark: 13] ***[with GDC]***

In a group of 16 students, 12 take art and 8 take music. One student takes neither art nor music. The Venn diagram below shows the events art and music. The values p, q, r and s represent numbers of students.



(a) (i) Write down the value of s.

(ii) Find the value of q.

(iii) Write down the value of p and of r. [5]

(b) (i) A student is selected at random. Given that the student takes music, write down the probability the student takes art.

(ii) **Hence**, show that taking music and taking art are **not** independent events. [4]

(c) Two students are selected at random, one after the other. Find the probability that the first student takes **only** music and the second student takes **only** art. [4]

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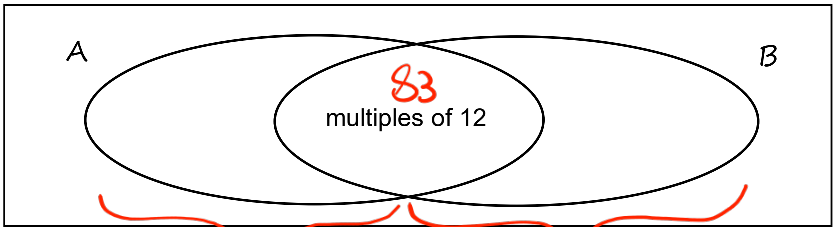
42.** [Maximum mark: 14] ***[with GDC]***

An integer is chosen at random from the first **one thousand** positive integers.

Let $A = \{\text{multiples of 4}\}$ and $B = \{\text{multiples of 6}\}$.

Then $A \cap B = \{\text{multiples of 12}\}$ (since the least common multiple of 4 and 6 is 12)



250 166

(a) Find the number of multiples

(i) of 4 (ii) of 6 (iii) of 12 [4]

(b) Find the probability that the integer chosen is a multiple of 4. [1]

(c) Find the probability that the integer chosen is a multiple of 6. [1]

(d) Find the probability that the integer chosen is a multiple of **both** 4 and 6. [2]

(e) Find the probability that the integer chosen is a multiple of 4 **but not** of 6. [2]

(f) Find the probability that the integer chosen is a multiple of 4 or 6. [2]

(g) Find the probability that the integer chosen is a multiple of 4 or 6 but not both. [2]

(a)

(i) $\frac{1000}{4} = 250$ (ii) $\frac{1000}{6} = 166$ (iii) $\frac{1000}{12} = 83$

(b) $\frac{250}{1000} = \frac{1}{4}$ (c) $\frac{166}{1000} = 0.166$ (d) $\frac{83}{1000}$

(e) $\frac{250-83}{1000} = \frac{167}{1000}$

(f) $\frac{250+166-83}{1000} = \frac{333}{1000}$

(g) $250+\cancel{166}-\cancel{83(2)} = \frac{250}{1000} = \frac{1}{4}$