INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

MAA

EXERCISES [MAA 4.13]

COUNTING AND PROBABILITY

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O. Practice questions

1. [Maximum mark: 12] ***[with / without GDC]***

There are 12 students in a class. Find the number of all possible ways

(a) to select 4 students [2]

(b) to select 8 students [2]

(c) to select 2 students [2]

(d) to select 1 student [2]

(e) to select all 12 students [2]

(f) to split the students into two groups of equal size. [2]

(a) $^{12}C_4 = 495$

(b) $^{12}C_8 = 495$

(c) $^{12}C_2 = 66$

(d) $^{12}C_1 = 12$

(e) 1 OR $^{12}C_{12}$

(f) $^{12}C_6 \cdot {}^{6}C_6 = {}^{12}C_6 \cdot 1 = 924$

2. [Maximum mark: 6] ***[with GDC]***

There are 12 students in a class. Find the number of all possible ways

(a) to select 4 students to sit in a row of 4 seats. [2]

(b) to select 8 students to sit in a row of 8 seats. [2]

(c) to arrange all 12 students in a row of 12 seats. [2]

(a) ${}^{12}P_4 = 11880$

(b) ${}^{12}P_8 = 19958400$

(c) ${}^{12}P_{12} = 479001600$

3. [Maximum mark: 5] ***[with GDC]***

In a waiting room there is a number of students and some available seats in a row.

Find the number of possible arrangements

(a) if we have 12 students but only 3 seats. [2]

(b) if we have only 3 students and 12 available seats. [3]

(a) ${}^{12}P_3 = 1320$

(b) • • • _ _ _ _ _ _ _ _ _

$1+9+9+9 = 28$ slotting arrangements

${}^{28}P_3 = 19656$

${}^{12}C_3 \cdot 3!$

Think of (b) like this: You're not putting 3 students in 12 chairs, you're putting 12 chairs under just 3 students — like that it's the same as (a) so (b) = 1320

4. [Maximum mark: 14] ***[with GDC]***

There are 12 students in a class, 9 boys and only 3 girls. Three students are selected at random to form a **3-member committee**.

(a) Find the number of all possible committees. [2]

(b) Find the number of possible committees, given that that the committee contains

(i) two boys and 1 girl;

(ii) only boys;

(iii) students of the same gender;

(iv) more boys than girls; [8]

(c) Find the probability for each case in question (b). [4]

a) ${}^{12}C_3 = 220$

b.i) ${}^9C_2 \cdot {}^3C_1 = {}^9C_2 \cdot 3 = 108$

b.ii) ${}^9C_3 \cdot {}^3C_0 = 84$

b.iii) ${}^9C_3 \cdot {}^3C_0 + {}^9C_0 \cdot {}^3C_3 = 85$

b.iv) ${}^9C_3 \cdot {}^3C_0 + {}^9C_2 \cdot {}^3C_1 = 84 + 108 = 192$

c)

c.i) $\frac{9}{12} \cdot \frac{8}{11} \cdot \frac{3}{10} = 0.164$

c.ii) $\frac{9}{12} \cdot \frac{8}{11} \cdot \frac{7}{10} = 0.382$

c.iii) $0.382 + \frac{3}{12} \cdot \frac{2}{11} \cdot \frac{1}{10} = 0.387$

c.iv) $0.382 + 0.164 = 0.546$

Just divide the combinations by all possible outcomes, ${}^{12}C_3 = 220$.

c.i) $108/220$ c.ii) $84/220$ c.iii) $85/220$ c.iv) $192/220$

5. [Maximum mark: 8] ***[with GDC]***

There are 12 students in a class, of which 7 are girls and 5 are boys. The students must be arranged in a row of 12 seats.

(a) Find the **number of possible arrangements**

(i) if girls must sit next to each other and boys must sit next to each other.

(ii) if only girls must sit next to each other. [6]

(b) The students are arranged at random. Find the **probability**

(i) that girls sit next to each other, and boys sit next to each other.

(ii) that only girls sit next to each other. [2]

a.i) $^{12}C_7 \cdot {}^{12}C_5 =$

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6. [Maximum mark: 6] ***[with GDC]***

There are 12 students in a class, 5 Greeks, 4 Italians and 3 Germans. The students must be arranged in a row of 12 seats. Find the **number of possible arrangements**

(a) if students of the same nationality must sit next to each other. [3]

(b) if only Greeks must sit next to each other. [3]

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7. [Maximum mark: 5] ***[with GDC]***

There are 12 students in a class, 10 boys and only 2 girls. The students must be arranged in a row of 12 seats. Find the **number of possible arrangements**

(a) if the two girls must sit next to each other. [3]

(b) if the two girls must be separated. [2]

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8*. [Maximum mark: 9] ***[with GDC]***

There are 12 students in a class, 9 boys and only 3 girls. The students must be arranged in a row of 12 seats. Find the **number of possible arrangements**

(a) if the **three girls** must sit next to each other. [3]

(b) if **not all the three girls** sit next to each other (thus two of them may sit together). [2]

(c) if the three girls must be separated. [4]

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9. [Maximum mark: 14] ***[with GDC]***

We construct 5-digit codes using all digits except 0 (that is the digits 1,2,3,4,5,6,7,8,9).

For example, 25439, 25518, 77788 are some of the possible codes.

(a) Find the total number of possible codes. [2]

(b) Find the number of possible codes

(i) of different digits (no digit can be repeated);

(ii) that are even (last digit is 2,4,6 or 8);

(iii) that contain only even digits;

(iv) that **do not** contain the digit 1;

(v) that contain the digit 1. [10]

(c) Find the **probability** that a code contains the digit 1. [2]

10. [Maximum mark: 14] ***[with GDC]***

In this question we consider 5-digit numbers.

(a) Find the total number of 5-digit numbers. [2]

(b) Find the number of 5-digit numbers

(i) of different digits (no digit can be repeated).

(ii) that contain only even digits.

(iii) that contain **two** 3s and **three** 2s .

(iv) that **do not** contain the digit 7.

(v) that contain the digit 7. [10]

(c) Find the **probability** that a 5-digit number contains the digit 7. [2]

A. Exam style questions (SHORT)

11. [Maximum mark: 6] ***[with GDC]***

There are 30 students in a class, of which 18 are girls and 12 are boys. Four students are selected at random to form a committee. Calculate

(a) the total number of possible committees. [2]

(b) the number of possible committees so that two girls and two boys are selected; [2]

(c) the number of possible committees with students all of the same gender. [2]

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12. [Maximum mark: 6] ***[with GDC]***

A team of five students is to be chosen at random to take part in a debate. The team is to be chosen from a group of eight medical students and three law students. Find the number of all possible ways

(a) if there is no restriction for the subject of the students. [2]

(b) that only medical students are chosen; [2]

(c) that all three law students are chosen; [2]

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13. [Maximum mark: 8] ***[with GDC]***

A room has nine desks arranged in three rows of three desks. Three students sit in the room. The students randomly choose a desk.

(a) Find the number of ways that three desks may be chosen. [2]

(b) Find the number of ways that three desks may be chosen so that two out of the front three desks are included. [3]

(c) Find the number of ways that the three students can sit if exactly two of them sit in the first three desks. [2]

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14. [Maximum mark: 6] ***[with GDC]***

A committee of four children is chosen from eight children. The two oldest children cannot both be chosen. Find the number of ways the committee may be chosen.

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15. [Maximum mark: 8] ***[with GDC]***

There are six boys and five girls in a school tennis club. A team of two boys and two girls will be selected to represent the school in a tennis competition.

(a) In how many different ways can the team be selected? [3]

(b) Tim is the youngest boy in the club and Anna is the youngest girl. In how many different ways can the team be selected if it must include both of them? [2]

(c) Fred is the oldest boy in the club. Given that Fred is selected for the team, in how many different ways can the team be selected if it must include Tim or Anna, but not both? [3]

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16. [Maximum mark: 6] ***[with GDC]***

Twelve people travel in three cars, with four people in each car. Each car is driven by its owner. Find the number of ways in which the remaining nine people may be allocated the cars. (the arrangement of people within a particular car is not relevant).

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17. [Maximum mark: 6] ***[with GDC]***

There are 10 seats in a row in a waiting room. There are six people in the room.

(a) In how many different ways can they be seated? [2]

(b) In the group of six people, there are three sisters who must sit next to each other. In how many different ways can the group be seated? [4]

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18. [Maximum mark: 7] ***[without GDC]***

Consider the 26 letters of the Latin alphabet {A, B, …, Z}. Any combination of letters is called a word, for example, ABC, BBC, BCB are different words of 3 letters.

(a) How many words of 5 letters are there? [2]

(b) How many words of 5 letters are there containing at least one A? [4]

(c) Find the probability that a word of 5 letters contains A. [1]

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19. [Maximum mark: 5] ***[with GDC]***

How many four-digit numbers are there which contain at least one digit 3?

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20. [Maximum mark: 7] ***[with GDC]***

Three Mathematics books, five English books, four Science books and a dictionary are to be placed on a student's shelf so that the books of each subject remain together.

(a) In how many different ways can the books be arranged? [4]

(b) In how many of these will the dictionary be next to the Mathematics books? [3]

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21. [Maximum mark: 6] ***[with GDC]***

There is a team of ten people working on a building, including three electricians and two plumbers. The architect called a meeting with five of the team, and randomly selected people to attend. Calculate the probability that **exactly two** electricians and **one** plumber were called to the meeting.

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22. [Maximum mark: 5] ***[with GDC]***

The set $S = \{a, b\}$ has 4 subsets:

the empty set (it contains no elements): $\{\}$,

the single element subsets: $\{a\}$ and $\{b\}$

the set S itself: $\{a, b\}$

Given that the set S contains 10 elements, show that the number of its subsets is 2^{10}

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23. [Maximum mark: 5] ***[with GDC]***

In how many ways can six different coins be divided between two students so that each student receives at least one coin?

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B. Exam style questions (LONG)

24. A deck of 52 playing cards consists of

4 **suits** (♠,♣,♥,♦),

of 13 **ranks** each (**A,2,3,4,5,6,7,8,9,10,J,Q,K**)

We select 2 cards. Find the number of possible combinations if

(a) there is no restriction (i.e. the total number of combinations)

(b) both cards are ♦

(c) the cards are of the same suit

(d) the cards are ♦ and ♣

(e) the cards are of different suits

(f) the cards are of the same rank

(g) the ranks are consecutive (namely **A2**, **23**, 34,…, **QK**, **KA**, regardless the suit)

Further investigation

25. A deck of 52 playing cards consists of

4 **suits** (♠,♣,♥,♦),

of 13 **ranks** each (**A,2,3,4,5,6,7,8,9,10,J,Q,K**)

When playing poker, we select **five** cards. Confirm the following numbers of winning combinations by using techniques of counting.

	Combination	Explanation - Example	Number of combinations
(a)	Royal Straight Flush	10 J Q K A, the same suit	4
(b)	Other Straight Flush	A 2 3 4 5 up to 9 10 J Q K, the same suit	36
(c)	Four of a kind	**XXXX**y e.g. A A A A 2	624
(d)	Full House	**XXXYY** e.g. A A A K K	3744
(e)	Flush	the same suit except (a) and (b)	5108
(f)	Straight	A 2 3 4 5 up to 10 J Q K A, any suits - except (a) and (b)	10200
(g)	Three of a kind	**XXX**yz e.g. A A A 2 3	54912
(h)	Two pairs	**XXYY**z e.g. A A K K 2	123552
(i)	One pair	**XX**yzw e.g. A A 2 5 7	1098240

Notice: The number of all possible combinations is $\binom{52}{5}$ = 2598960

If you divide each result by the total number 2598960, you may also find the probability of the corresponding combination.