

INTERNATIONAL BACCALAUREATE  
Mathematics: analysis and approaches  
**MAA**

**EXERCISES [MAA(HL) 4.1-4.3]**  
**STATISTICS – BASIC CONCEPTS**  
*Compiled by Christos Nikolaidis*

**O. Practice questions**

1. [Maximum mark: 9] **[without GDC]**

Consider the set of data 1, 2, 3, 4, 5.

- (a) Find the mean. [2]  
(b) Find the variance (by using both the formulas for the variance) [4]  
(c) Write down the standard deviation. [1]  
(d) Find the interquartile range. [2]

$$(a) \frac{5+4+3+2+1}{5} = \frac{15}{5} = 3$$

$$(b) \sigma^2 = \frac{(5-3)^2 + (4-3)^2 + (3-3)^2 + (2-3)^2 + (1-3)^2}{5}$$

$$\sigma^2 = \frac{4+1+1+4}{5} = 2$$

OR

$$\sigma^2 = \frac{5^2 + 4^2 + 3^2 + 2^2 + 1^2}{5} - (3)^2 = \frac{25+16+9+4+1}{5} - 9$$
$$= \frac{55}{5} - 9 = 11 - 9 = 2$$

$$(c) \sigma = \sqrt{2}$$

$$(d) Q_1 = \frac{2+1}{2} = 1.5, Q_3 = \frac{5+4}{2} = 4.5$$
$$I_r = 4.5 - 1.5 = 3$$

2. [Maximum mark: 10] **[without GDC]**

Consider the set of data

$x$	frequency
10	2
20	5
30	3

- (a) Find the mean. [2]  
 (b) Find the variance (by using both the formulas for the variance) [5]  
 (c) Write down the standard deviation. [1]  
 (d) Find the interquartile range. [2]

10, 10, 20, 20, 20, 20, 20, 30, 30, 30  
 Q<sub>1</sub> Q<sub>3</sub>

## 3. [Maximum mark: 9] [without GDC]

Consider the data  $x_1, x_2, x_3, \dots, x_n$ , with mean  $\bar{x}$  and standard deviation  $s$ .

(a) If each number is increased by  $k$ ,

(i) show that the new mean is  $\bar{x} + k$  (i.e. it is also increased by  $k$ )

(ii) show that the new standard deviation is  $s$  (i.e. it remains the same) [4]

(b) If each number is multiplied by  $k$

(i) show that the new mean is  $k\bar{x}$  (i.e. it is also multiplied by  $k$ )

(ii) show that the new standard deviation is  $ks$  (i.e. it is also multiplied by  $k$ )

(iii) write down the relation between the original and the new variance. [5]

$$(a) \quad (i) \quad \bar{x} = \frac{\sum_{i=1}^n x_i}{n}, \quad \bar{x}' = \frac{\sum_{i=1}^n (x_i + k)}{n} = \frac{\left( \sum_{i=1}^n x_i + \sum_{i=1}^n k \right)}{n} = \frac{\left( \sum_{i=1}^n x_i + nk \right)}{n}$$

$$\bar{x}' = \bar{x} + \frac{nk}{n}; \quad \bar{x}' = \bar{x} + k$$

$$(ii) \quad s' = \sqrt{\frac{\sum_{i=1}^n (x_i + k - (\bar{x} + k))^2}{n}} = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}} = s$$

**A. Exam style questions (SHORT)**

4. [Maximum mark: 6]
- [without GDC]**

A fair six-sided die, with sides numbered 1, 1, 2, 3, 4, 5 is thrown. Find the mean and variance of the score.

$$\bar{x} = \frac{16}{6} = \frac{8}{3}$$

$$\sigma^2 = \frac{5^2 + 4^2 + 3^2 + 2^2 + 2^2 + 1^2}{6} - \left(\frac{8}{3}\right)^2 = \frac{56}{6} - \frac{64}{9}$$

$$= \frac{168}{18} - \frac{128}{18} = \frac{40}{18} = \frac{20}{9}$$

5. [Maximum mark: 6]
- [with GDC]**

Consider the six numbers 2, 3, 6, 9,  $a$  and  $b$ . The mean of the numbers is 6 and the variance is 10. Find the value of  $a$  and of  $b$ , if  $a < b$ .

$$\frac{20 + a + b}{6} = 6, a + b = 16$$

$$10 = \frac{9^2 + 6^2 + 3^2 + 2^2 + a^2 + b^2}{6} - 6^2 = \frac{130 + a^2 + b^2}{6} - 36$$

$$a^2 + b^2 = 146$$

$$\text{By GDC: } a = 5, b = 11$$

## 6. [Maximum mark: 6] [with GDC]

A teacher drives to school. She records the time taken on each of 20 days. She finds that

$$\sum_{i=1}^{20} x_i = 626 \text{ and } \sum_{i=1}^{20} x_i^2 = 19780.8,$$

where  $x_i$  denotes the time, in minutes, taken on the  $i$ -th day.

For this period, calculate

- the mean time  $\mu$  taken to drive to school;
- the variance  $\sigma^2$  of the time taken to drive to school.
- the sum  $\sum_{i=1}^{20} (x_i - \mu)^2$ .

$$(a) \frac{626}{20} = 31.3 \text{ minutes} = \mu$$

$$(b) \frac{19780.8}{20} - 31.3^2 = 9.35$$

$$(c) (x_i - \mu)^2 = x_i^2 - 2x_i\mu + \mu^2$$

$$\sum_{i=1}^{20} (x_i - \mu)^2 = 19780.8 - 2(626)(20(31.3)) - [20(31.3)]^2$$

= negative  $i-i$

(c)  $-\mu$  doesn't change standard deviation, so neither does it variance.

$$\sum_{i=1}^{20} (x_i - \mu)^2$$

$$= \sigma^2 = 9.35$$

$$\sum_{i=1}^{20} (x_i - \mu)^2 = 187$$

## 7. [Maximum mark: 4] [without GDC]

Ten numbers have mean 9 and standard deviation 2. Find the sum of their squares.

$$\sum_{i=1}^{10} (x_i)^2$$

$$\frac{\sum_{i=1}^{10} (x_i)^2}{10} - 9^2 = 2^2$$

$$\sum_{i=1}^{10} (x_i)^2 = (4 + 81)10 = 850$$

8. [Maximum mark: 6] [with GDC]

Consider the 10 data items  $x_1, x_2, \dots, x_{10}$ . Given that  $\sum_{i=1}^{10} x_i^2 = 1341$  and the standard deviation is 6.9, find the value of  $\bar{x}$ .

$$\frac{1341}{10} - \bar{x}^2 = 6.9^2$$

$$\bar{x} = \sqrt{134.1 - 86.49} = \pm 9.3$$

9. [Maximum mark: 8] [with GDC]

Twenty candidates sat an examination in French. The sum of their marks was 826 and the sum of the squares of their marks was 34 132. Two candidates sat the examination late and their marks were  $a$  and  $b$ . The new mean and variance were calculated, giving the following results:

mean = 42 and variance = 32.

Find a set of possible values of  $a$  and  $b$ .

$$\frac{826 + a + b}{22} = 42, a = 98 - b$$

$$\frac{34132 + a^2 + b^2}{22} - 42^2 = 32, a = \sqrt{5380 - b^2}$$

$$a = 32, b = 66 \text{ or vice versa}$$

10\*. [Maximum mark: 10] [with GDC]

Consider the set of data

$x$	frequency
2	10
5	20
7	30
$a$	15
$b$	25

where  $a$  and  $b$  are **integers**. The mean of the numbers is 7 and the variance is 5.7.Find the value of  $a$  and of  $b$ .

$$7 = \frac{2(10) + 5(20) + 7(30) + 15a + 25b}{100}$$

$$5.7 = \frac{10(2)^2 + 20(5)^2 + 30(7)^2 + 15(a)^2 + 25(b)^2}{100} - 7^2$$

Intersection and solve

11\*. [Maximum mark: 10] **[with GDC]**

Consider the set of data

$x$	frequency
2	10
5	20
7	30
8	$a$
10	$b$

The mean of the numbers is 7 and the variance is 5.7. Find the value of  $a$  and of  $b$ .

Something as Q10 No cap.