INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

MAA

EXERCISES [MAA 3.1-3.3]

3D GEOMETRY – TRIANGLES

Compiled by Christos Nikolaidis

O. Practice questions

3D GEOMETRY

1. [Maximum mark: 7] ***[without GDC]***

Let A(2,-3,5) and B(-1,1,5). Find

(a) the distance between A and B. [2]

(b) the distance between O and B. [1]

(c) the coordinates of the midpoint M of the line segment [AB]. [2]

(d) the coordinates of point C given that B is the midpoint of [AC]. [2]

(a) $d=\sqrt{(-1-2)^2+(1+3)^2+(5-5)^2}=\sqrt{25}=5$

(b) $d=\sqrt{(-1)^2+(1)^2+5^2}=\sqrt{27}=\sqrt{9}\cdot\sqrt{3}=3\sqrt{3}$

(c) $M=\left(\frac{2-1}{2},\frac{-3+1}{2},\frac{5+5}{2}\right)=\left(\frac{1}{2},-1,5\right)$

(d) $(-1,1,5)=\left(\frac{2+x}{2},\frac{-3+y}{2},\frac{5+z}{2}\right)$

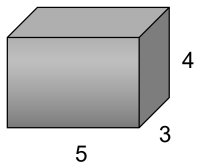
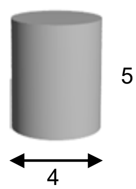
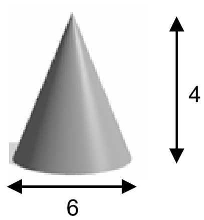
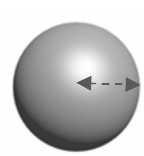
$-1=\frac{2+x}{2}$ | $1=\frac{-3+y}{2}$ | $5=\frac{5+z}{2}$

$x=-4$ | $y=5$ | $z=5$

$C=(-4,5,5)$

2. [Maximum mark: 16] ***[without GDC]***

Complete the table

Solid	Volume	Surface area
cuboid 	$5\times3\times4=60$	$5(4)+5(4)+3(4)+3(4)+5(3)+5(3)$ $=94$
cylinder  (diameter)	$\pi(2)^2(5)$ $=20\pi$	$SA=2\pi rh+2\pi r^2$ $=2\pi(2)(5)+2\pi(2)^2$ $=20\pi+8\pi$ $=28\pi$
cone $l=\sqrt{3^2+4^2}=5$  (diameter)	$\frac{1}{3}\pi(3)^2(4)$ $=12\pi$	$SA=\pi rL+\pi r^2$ $=\pi(3)(5)+\pi(3)^2$ $=15\pi+9\pi$ $=24\pi$
sphere  radius = 3	$\frac{4}{3}\pi(3)^3=36\pi$	$SA=4\pi r^2$ $=4\pi(3)^2$ $=36\pi$

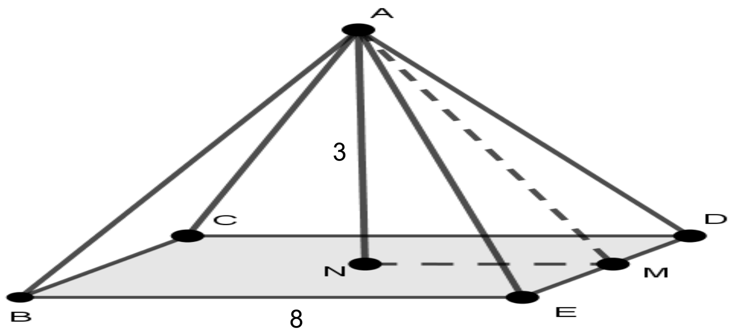
for each shape [1+3]

3. [Maximum mark: 7] ***[without GDC]***

For a right pyramid of square base of side 8 and vertical height 3 find

(a) the volume [2]

(b) the surface area $SA = b^2 + \frac{1}{2}(P_b)(L)$ [5]



$\overline{NM} = \frac{1}{2}\overline{BE} = 4$

$\overline{AM} = 5$

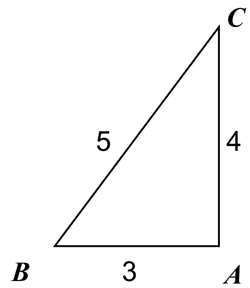
(a) $V = \cancel{\frac{1}{3}}(8)(8)(\cancel{3}) = 192$

(b) $SA = 8^2 + \frac{1}{2}(4 \cdot 8)(5) = 64 + 80 = 144$

TRIANGLES

4. [Maximum mark: 14] ***[without GDC]***

Consider the following right-angled triangle, where $\hat{A}$ = 90º



(a) Complete the tables

$\sin\hat{B}$	$4/5$
$\cos\hat{B}$	$3/5$
$\tan\hat{B}$	$4/3$

$\sin\hat{C}$	$3/5$
$\cos\hat{C}$	$4/5$
$\tan\hat{C}$	$3/4$

[6]

(b) Confirm that the **sine rule** holds. (It is known that $\sin\hat{A}=1$)

$\frac{a}{\sin\hat{A}}=\frac{5}{1}=5$	$\frac{b}{\sin\hat{B}}=\frac{4}{1}\cdot\frac{5}{4}=5$	$\frac{c}{\sin\hat{C}}=\frac{3}{1}\cdot\frac{5}{3}=5$

[2]

(c) Confirm that all three versions of the **cosine rule** hold.
(the first version is given below; it is known that $\cos\hat{A}=0$)

LHS	RHS
5^2	$3^2+4^2-2(3)(4)\cos\hat{A}=9+16-0=25$
3^2	$5^2+4^2-2(\cancel{5})(4)\left(\frac{4}{\cancel{5}}\right)=25+16-32=9$
4^2	$5^2+3^2-2(\cancel{5})(3)\left(\frac{3}{\cancel{5}}\right)=34-18=16$

[4]

(d) Find the area of the triangle, by using all the three versions of the formula

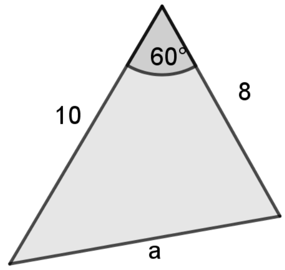
$$Area=\frac{1}{2}ab\sin\hat{C} \quad \text{(the first version is given below)}$$

$Area=\frac{1}{2}\times3\times4\times\sin\hat{A}=6$
$Area=\frac{1}{2}(\cancel{5})(4)\left(\frac{3}{\cancel{5}}\right)=6$
$Area=\frac{1}{2}(\cancel{5})(3)\left(\frac{4}{\cancel{5}}\right)=6$

[2]

5. [Maximum mark: 4] ***[with / without GDC]***

Use the **cosine rule** to find the size of the side a.



$\cos 60 \neq \cos 45$

$\cos 60 = \frac{1}{2}$

$a^2 = 10^2 + 8^2 - 2(10)(8)\cos 60$

$a^2 = 164 - \frac{160\sqrt{2}}{2}$ $\qquad a^2 = 164 - 80 = 84$

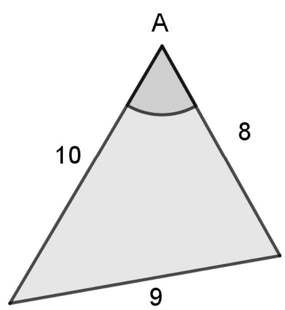
$a \cong 7.13$ $\qquad a = \sqrt{84} = \sqrt{21}\sqrt{4} = 2\sqrt{21}$

By full GDC, $a \cong 9.17$

6. [Maximum mark: 5] ***[with GDC]***

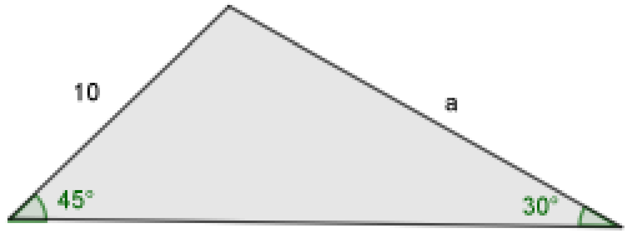
(a) Use the **cosine rule** to find the cosine of the angle A. [4]

(b) Hence find the size of the angle A. [1]



7. [Maximum mark: 4] ***[with / without GDC]***

Use the **sine rule** to find the size of the side a.



$$\frac{\sin 30}{10} = \frac{\sin 45}{a}$$

$$\frac{1}{2}a = 5\sqrt{2}$$

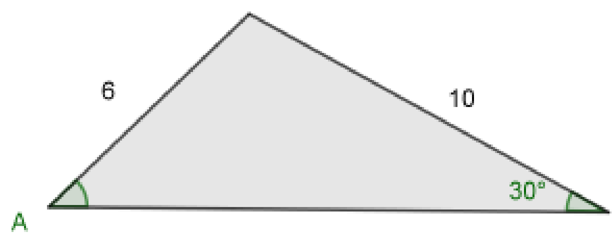
$$a = 10\sqrt{2}$$

8. [Maximum mark: 6] ***[with GDC]***

(a) Use **the sine rule** to find the sine of the angle A. [4]

(b) Hence find the **two possible** values of the angle A. [2]

Ambiguous



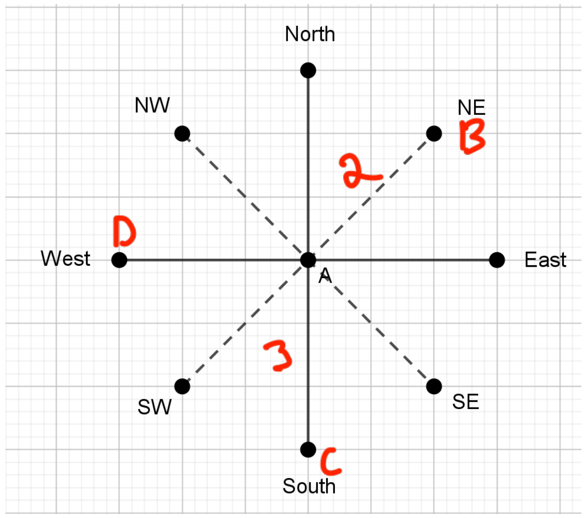
$$\frac{\sin 30}{6} = \frac{\sin \hat{A}}{10}$$

$$5 = 6\sin \hat{A}$$

$$\hat{A} = \arcsin\left(\frac{5}{6}\right) \simeq 56.4 \text{ or } 180 - 56.4 \simeq 124$$

9. [Maximum mark: 10] ***[with GDC]***

Point A is at the center of the following diagram.



Bill and Chris and Dianna are located at point A and start moving,

Bill to the **NE** at point **B**

Chris to the **South** at point **C**

Dianna to the **West** at point **D**

(a) Write down the size of angle BAC. [1]

(b) Write down the bearing of the course of each person. [3]

(c) Find the bearing of the course from B to A. [2]

(d) Given that AB = 2 km and AC is 3km, Find the distance between B and C. [3]

(a) 135°

(b) Bill: 45°, Chris: 180°, Dianna: 270°

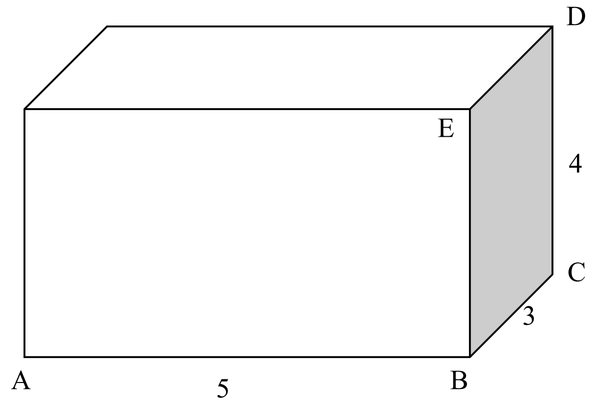
(c) $180 + 90(\frac{1}{2}) = 225^\circ$

(d) $BC^2 = 2^2 + 3^2 - 2(2)(3)\cos(135)$

$BC = 4.64$ km

10. [Maximum mark: 14] ***[with GDC]***

Consider the following cuboid of dimensions 5×3×4, as shown.



(a) Find the length AC. [3]

(b) Find the length AD. [3]

(c) Find the angle of elevation from A to E. [3]

(d) Find the angle of elevation from A to D. [3]

(e) Find the angle of depression from E to A. [2]

(a) $\sqrt{5^2+3^2} \simeq 5.83 = AC$

(b) $AD = \sqrt{(5.83..)^2+4^2} \simeq 7.07$

(c) $\tan E\hat{A}B = \frac{4}{5}$

$E\hat{A}B \simeq 38.7°$

(d) $\tan D\hat{A}C = \frac{4}{5.83..} \simeq 34.4°$

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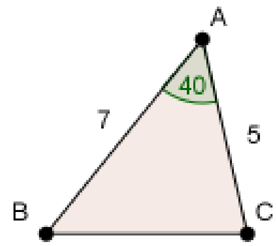
(e) $\tan \hat{E} = \frac{4}{5} \simeq 38.7°$

11*. [Maximum mark: 30] ***[with GDC]***

In each of the following triangles one of the angles has size 40°, two of the sides have lengths 5 and 7 respectively.

(a) For the following triangle



(i) Find the area of the triangle

(ii) Find BC

(iii) Find the size of $\hat{B}$ and **hence** the size of $\hat{C}$. [7]

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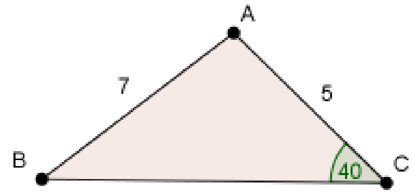
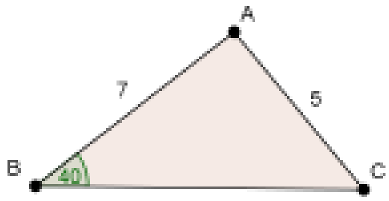
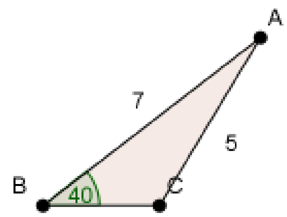
(b) For the following triangle



find the size of $\hat{B}$ and **hence** the size of $\hat{A}$. [5]

(c) For each of the following triangles (ambiguous case)





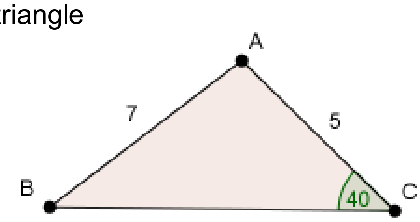
find the size of $\hat{C}$ and **hence** the size of $\hat{A}$. [6]

$\hat{C} \cong 64.1°$ or $180-64.1 \cong 116°$

$\hat{A} = 180-40-64.1$ or $180-40-116 \cong 24.1°$

$\hat{A} \cong 75.9°$ or $\cong 24.1°$

(d) For the following triangle



(i) Use the cosine rule to directly find the side BC.

(ii) **Hence** find the area of the triangle. [5]

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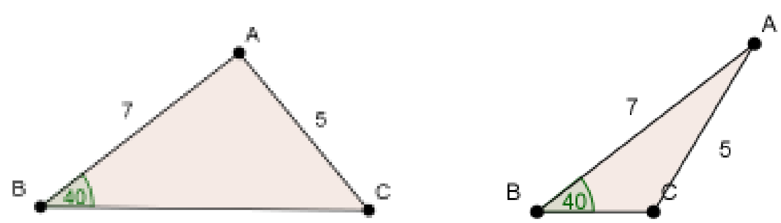
(e) For the following triangles (ambiguous case)



(i) Use the cosine rule to directly find the side BC of each triangle.

(ii) **Hence** find the area of each triangle. [7]

Area doesn't change!?

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A. Exam style questions (SHORT)

12. [Maximum mark: 6] ***[with GDC]***

The following diagram shows triangle ABC.

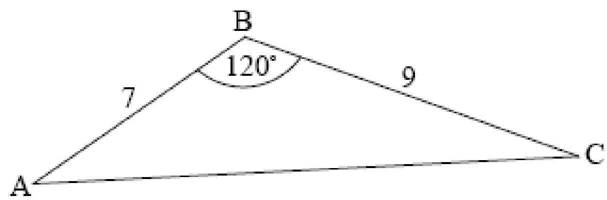


diagram not to scale

AB = 7 cm, BC = 9 cm and $A\hat{B}C = 120°$.

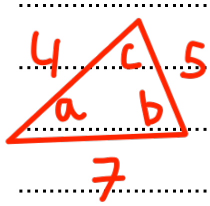
(a) Find AC. [3]

(b) Find $B\hat{A}C$. [3]

13. [Maximum mark: 4] ***[with GDC]***

A triangle has sides of length 4, 5, 7 units. Find, to the nearest tenth of a degree, the size of the largest angle.

$7^2 = 5^2 + 4^2 - 2(5)(4)\cos\hat{C}$



$\cos\hat{C} = \dfrac{5^2+4^2-7^2}{2(5)(4)} = -0.2$

$\hat{C} \simeq 102° =$ Largest

14. [Maximum mark: 4] ***[with GDC]***

The following diagram shows a triangle with sides 5 cm, 7 cm, 8 cm.

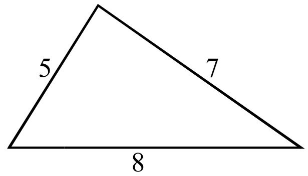


Diagram not to scale

(a) Find the size of the smallest angle, in degrees; [2]

(b) Find the area of the triangle. [2]

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15. [Maximum mark: 6] ***[with GDC]***

In the triangle PQR, PR = 5 cm, QR = 4 cm and PQ = 6 cm. Calculate

(a) the size of $\mathrm{P\hat{Q}R}$; [4]

(b) the area of triangle PQR. [2]

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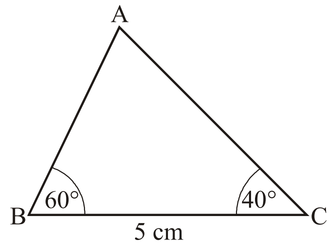
16. [Maximum mark: 6] ***[with GDC]***

The following diagram shows a triangle ABC, where BC = 5 cm, $\hat{B}$ = 60°, $\hat{C}$ = 40°.



(a) Calculate AB. [3]

(b) Find the area of the triangle. [3]

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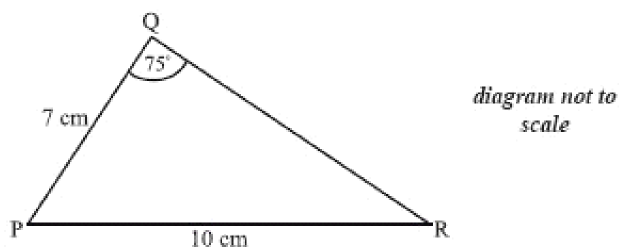
17. [Maximum mark: 6] ***[with GDC]***

The diagram below shows triangle PQR. The length of [PQ] is 7 cm, the length of [PR] is 10 cm, and $P\hat{Q}R$ is 75°.



(a) Find $P\hat{R}Q$ [3]

(b) Find the area of triangle PQR. [3]

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18. [Maximum mark: 6] ***[with GDC]***

In triangle ABC, AC = 5, BC = 7, $\hat{A}$ = 48°, as shown in the diagram.

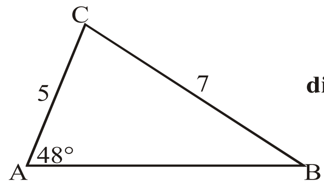

diagram not to scale

Find $\hat{B}$, giving your answer correct to the nearest degree.

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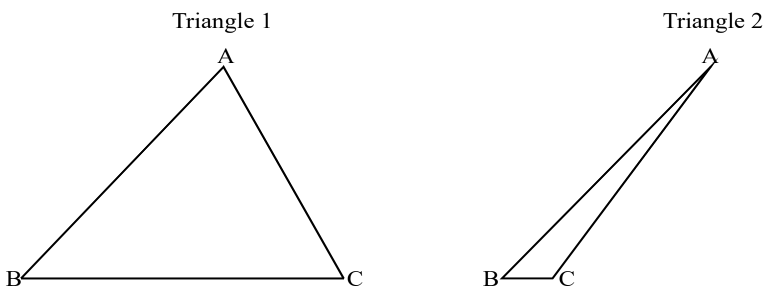
19*. [Maximum mark: 4] ***[with GDC]***

The diagrams below show two triangles both satisfying the conditions

AB = 20 cm, AC = 17 cm, $A\hat{B}C$ = 50°.



Diagrams not to scale

Calculate

(a) the size of $A\hat{C}B$ in **Triangle 2.** [2]

(b) the area of **Triangle 1**. [2]

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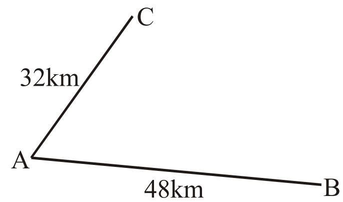
20. [Maximum mark: 4] ***[with GDC]***

Town A is 48 km from town B and 32 km from town C as shown in the diagram.



Given that town B is 56 km from town C, find the size of angle $C\hat{A}B$ to the nearest degree.

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21*. [Maximum mark: 6] ***[with GDC]***

Two boats A and B start moving from the same point P. Boat A moves in a straight line at 20 km h^{-1} and boat B moves in a straight line at 32 km h^{-1}. The angle between their paths is 70°. Find the distance between the boats after 2.5 hours.

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22. [Maximum mark: 6] ***[with GDC]***

The following diagram shows the triangle ABC.

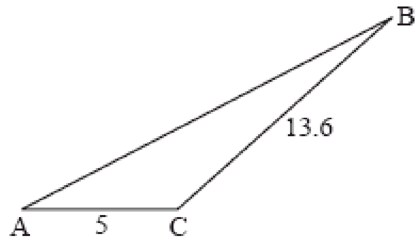

diagram not to scale

The angle at C is obtuse, AC = 5 cm, BC = 13.6 cm and the area is 20 cm^2.

(a) Find $\mathrm{A\hat{C}B}$. [3]

(b) Find AB. [3]

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23. [Maximum mark: 6] ***[with GDC]***

In a triangle ABC, AB = 4 cm, AC = 3 cm and the area of the triangle is 4.5 cm^2.

Find the **two** possible values of the angle $\mathrm{B\hat{A}C}$.

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24. [Maximum mark: 6] ***[without GDC]***

In triangle PQR, PQ is 10 cm, QR is 8 cm and angle PQR is acute. The area of the triangle is 20 cm^2. Find the size of angle $\mathrm{P\hat{Q}R}$.

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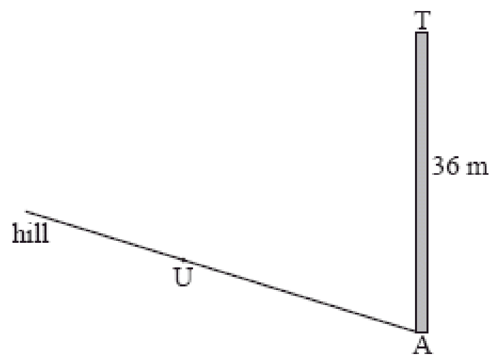
25. [Maximum mark: 7] ***[with GDC]***

There is a vertical tower TA of height 36 m at the base A of a hill. A straight path goes up the hill from A to a point U. This information is represented by the following diagram.



The path makes a 4° angle with the horizontal.

The point U on the path is 25 m away from the base of the tower.

The top of the tower is fixed to U by a wire of length x m.

(a) Complete the diagram, showing clearly all the information above. [3]

(b) Find x. [4]

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26. [Maximum mark: 8] ***[with GDC]***

The diagram below shows a triangle ABD with AB = 13 cm and AD = 6.5 cm.

Let C be a point on the line BD such that BC = AC = 7 cm.

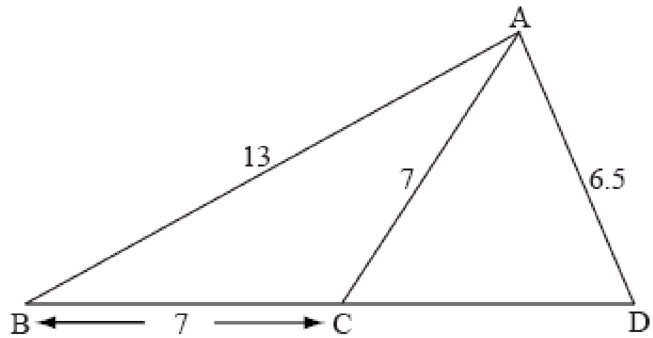

diagram not to scale

(a) Find the size of angle ACB. [3]

(b) Find the size of angle CAD. [5]

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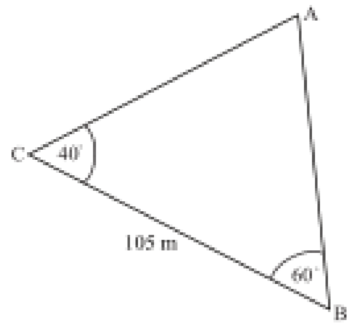
27. [Maximum mark: 6] ***[with GDC]***

The following diagram shows ΔABC, where $BC = 105$ m, $A\hat{C}B = 40°$, $A\hat{B}C = 60°$



Find the area of the triangle.

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28. [Maximum mark: 6] ***[with GDC]***

In the triangle ABC, $\hat{A} = 30°$, BC = 3 and AB = 5. Find the two possible values of $\hat{B}$.

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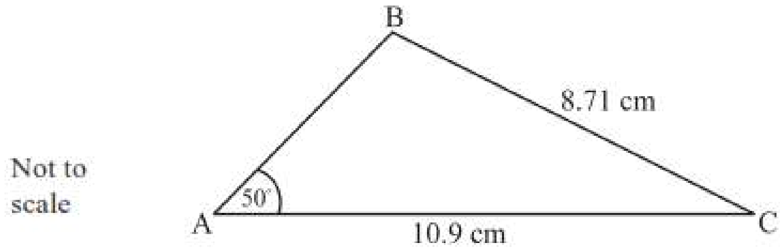
29. [Maximum mark: 6] ***[with GDC]***

In the **obtuse-angled** triangle ABC, $\mathrm{AC} = 10.9\,\mathrm{cm}$, $\mathrm{BC} = 8.71\,\mathrm{cm}$ and $\mathrm{B\hat{A}C} = 50°$.



Find the area of triangle ABC.

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30. [Maximum mark: 6] ***[with GDC]***

Triangle ABC has $\hat{C} = 42°$, $\mathrm{BC} = 1.74\,\mathrm{cm}$, and area $1.19\,\mathrm{cm}^2$.

(a) Find AC. [3]

(b) Find AB. [3]

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31*. [Maximum mark: 6] ***[with GDC]***

In the triangle ABC, $\hat{A} = 30°$, $a = 5$ and $c = 7$. Find the difference in area between the two possible triangles for ABC.

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32*. [Maximum mark: 7] ***[with GDC]***

In a triangle *ABC*, $A\hat{B}C = 30°$, $AB = 6cm$, $AC = 3\sqrt{2}\ cm$. Find the possible areas of the triangle.

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33*. [Maximum mark: 6] ***[with GDC]***

In a triangle ABC, $\text{A}\hat{\text{B}}\text{C}=30°$, $\text{AB}=6\,\text{cm}$, $\text{AC}=3\sqrt{2}\,\text{cm}$. Find the possible lengths of $[\text{BC}]$.

METHOD A: Use Sine rule.

METHOD B: Use Cosine rule.

34*. [Maximum mark: 7] ***[with GDC]***

In a triangle ABC, $\hat{A} = 35°$, $\mathrm{BC} = 4\,\mathrm{cm}$ and $\mathrm{AC} = 6.5\,\mathrm{cm}$. Find the possible values of $\hat{\mathrm{B}}$ and the corresponding values of AB.

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35*. [Maximum mark: 6] ***[with GDC]***

. Triangle ABC has $\mathrm{AB} = 8\,\mathrm{cm}$, $\mathrm{BC} = 6\,\mathrm{cm}$, $\mathrm{B\hat{A}C} = 20°$. Find the smallest possible area of $\Delta\mathrm{ABC}$.

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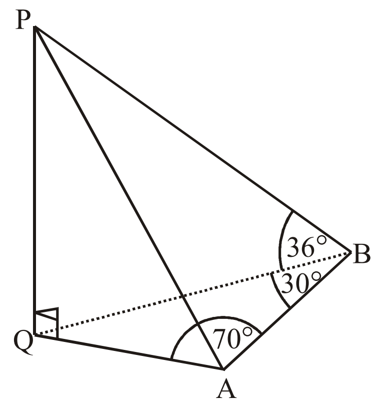
36*. [Maximum mark: 6] ***[with GDC]***

In triangle ABC, $\text{A}\hat{\text{B}}\text{C} = 31°$, $\text{AC} = 3\,\text{cm}$, $\text{BC} = 5\text{cm}$. Calculate the possible lengths of $[\text{AB}]$.

37*. [Maximum mark: 7] ***[with GDC]***

Consider triangle ABC with $\text{B}\hat{\text{A}}\text{C} = 37.8°$, $\text{AB} = 8.75$ and $\text{BC} = 6$. Find AC.

38*. [Maximum mark: 4] ***[with GDC]***
The diagram shows a vertical pole PQ, which is supported by two wires fixed to the horizontal ground at A and B.



$\mathrm{BQ} = 40\text{ m}$
$\mathrm{P\hat{B}Q} = 36°$
$\mathrm{B\hat{A}Q} = 70°$
$\mathrm{A\hat{B}Q} = 30°$

Find

(a) the height of the pole, PQ; [2]

(b) the distance between A and B. [2]

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39*. [Maximum mark: 7] ***[with GDC]***

A ship leaves port A on a bearing of 030°. It sails a distance of 25 km to point B.

At B, the ship changes direction to a bearing of 100°. It sails a distance of 40 km to reach point C. This information is shown in the diagram below.

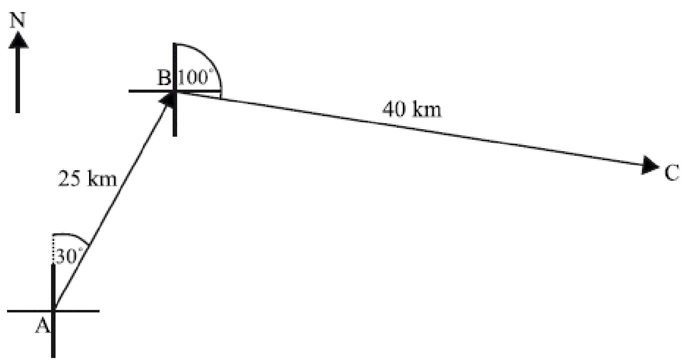

diagram not to scale

A second ship leaves port A and sails directly to C.

(a) Find the distance the second ship will travel. [4]

(b) Find the bearing of the course taken by the second ship. [3]

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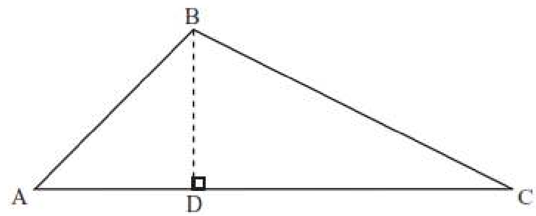
40*. [Maximum mark: 6] ***[without GDC]***

In triangle ABC, $\text{BC}=a$, $\text{AC}=b$, $\text{AB}=c$ and $[\text{BD}]$ is perpendicular to $[\text{AC}]$.



(a) Show that $\text{BD}=c\sin A$. [1]

(b) Show that $\text{CD}=b-c\cos A$. [2]

(c) **Hence**, by using Pythagoras' Theorem in the triangle BCD, prove the cosine rule for the triangle ABC. [3]

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41.** [Maximum mark: 7] ***[without GDC]***

In triangle ABC, $\text{BC}=a$, $\text{AC}=b$, $\text{AB}=c$ and $\text{A}\hat{\text{B}}\text{C}=60°$.

Use the cosine rule to show that $c=\frac{1}{2}a\pm\sqrt{b^2-\frac{3}{4}a^2}$.

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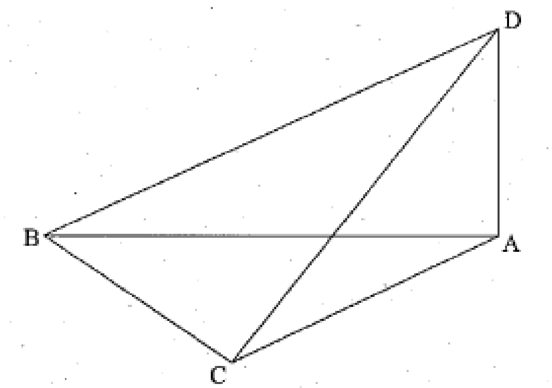
42.** [Maximum mark: 7] ***[with GDC]***

The following three dimensional diagram shows the four points A,B,C and D.

A,B,C are in the same horizontal plane and AD is vertical. The angle ABC is 45^{o} , and BC = 50m. The angle of elevation from point B to point D is 30^{o}, while the angle of elevation from point C to point D is 20^{o}.



Using the cosine rule in the triangle ABC, or otherwise, find AD.

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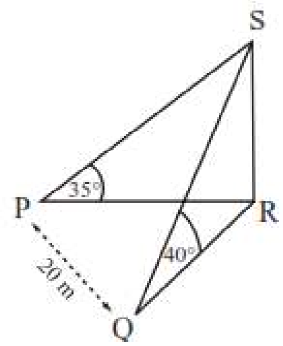
43.** [Maximum mark: 7] ***[with GDC]***



The above 3-dimensional diagram shows the points P and Q which are respectively west and south-west of the base R of a vertical flagpole RS on horizontal ground. The angles of elevation of the top S of the flagpole from P and Q are respectively $35°$ and $40°$, and $\mathrm{PQ} = 20\,\mathrm{m}$.

Determine the height of the flagpole.

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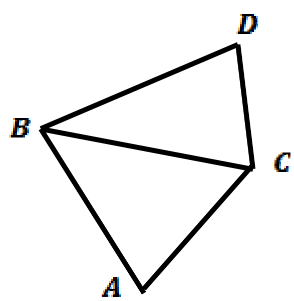
B. Past paper questions (*LONG*)

44. [Maximum mark: x] ***[with GDC]***

Consider the following diagram



$AB = 7$

$AC = 5$

$\hat{A} = 60^\circ$

$\hat{D} = 80^\circ$

$D\hat{B}C = 30^\circ$

Find

(a) Find the length of the side *BD*.

(b) Find the area of the quadrilateral *ABDC*.

(c) Find the perimeter of the quadrilateral *ABDC*.

It is given that the bearing from *B* to *D* is 70º. Find

(d) Find (i) the bearing from *B* to *A*. (ii) Find the bearing from *A* to *B*.

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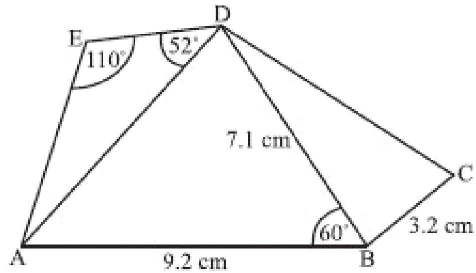
45. [Maximum mark: 21] ***[with GDC]***

The following diagram shows a pentagon ABCDE, with $AB = 9.2$ cm, $BC = 3.2$ cm, $BD = 7.1$ cm, $\hat{AED} = 110°$, $\hat{ADE} = 52°$ and $\hat{ABD} = 60°$.



(a) Find AD. [4]

(b) Find DE. [4]

(c) The area of triangle BCD is 5.68 cm^2. Find $\hat{DBC}$. [4]

(d) Find AC. [4]

(e) Find the area of quadrilateral ABCD. [5]

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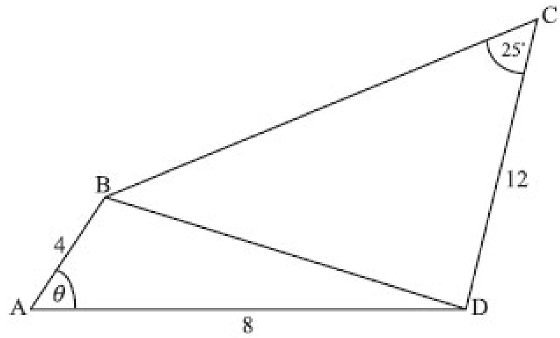
46. [Maximum mark: 16] ***[with GDC]***

The diagram below shows a quadrilateral ABCD. AB = 4, AD = 8, CD =12, $B\hat{C}D = 25°$, $B\hat{A}D = \theta$.



(a) Use the cosine rule to show that BD = $4\sqrt{5-4\cos\theta}$. [2]

Let $\theta = 40°$.

(b) (i) Find the value of $\sin C\hat{B}D$.

(ii) Find the two possible values for the size of $C\hat{B}D$.

(iii) Given that $C\hat{B}D$ is an acute angle, find the perimeter of ABCD. [12]

(c) Find the area of triangle ABD. [2]

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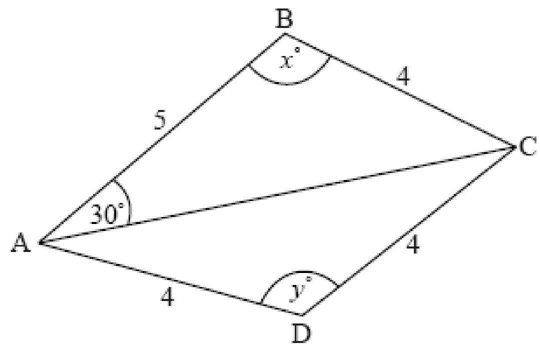
47. [Maximum mark: 14] ***[with GDC]***

The diagram below shows a quadrilateral ABCD with obtuse angles $\mathrm{A\hat{B}C}$ and $\mathrm{A\hat{D}C}$.



$\mathrm{AB} = 5$ cm, $\mathrm{BC} = 4$ cm, $\mathrm{CD} = 4$ cm, $\mathrm{AD} = 4$ cm, $\mathrm{B\hat{A}C} = 30°$, $\mathrm{A\hat{B}C} = x°$, $\mathrm{A\hat{D}C} = y°$.

(a) Use the cosine rule to show that $\mathrm{AC} = \sqrt{41 - 40\cos x}$. [1]

(b) Use the sine rule in triangle ABC to find another expression for AC. [2]

(c) (i) Hence, find x, giving your answer to two decimal places.

(ii) Find AC. [6]

(d) (i) Find y.

(ii) Hence, or otherwise, find the area of triangle ACD. [5]

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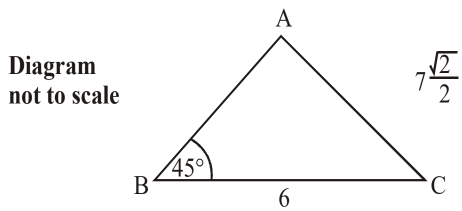
48. [Maximum mark: 10] ***[with GDC]***

The diagram shows a triangle ABC in which $\mathrm{AC} = 7\dfrac{\sqrt{2}}{2}$, $\mathrm{BC} = 6$, $\mathrm{A\hat{B}C} = 45°$.



(a) Use the fact that $\sin 45° = \dfrac{\sqrt{2}}{2}$ to show that $\sin\,\mathrm{B\hat{A}C} = \dfrac{6}{7}$. [2]

The point D is on (AB), between A and B, such that $\sin\,\mathrm{B\hat{D}C} = \dfrac{6}{7}$.

(b) (i) Write down the value of $\mathrm{B\hat{D}C} + \mathrm{B\hat{A}C}$.

(ii) Calculate the angle BCD.

(iii) Find the length of [BD]. [6]

(c) Show that $\dfrac{\text{Area of } \Delta\mathrm{BDC}}{\text{Area of } \Delta\mathrm{BAC}} = \dfrac{\mathrm{BD}}{\mathrm{BA}}$. [2]

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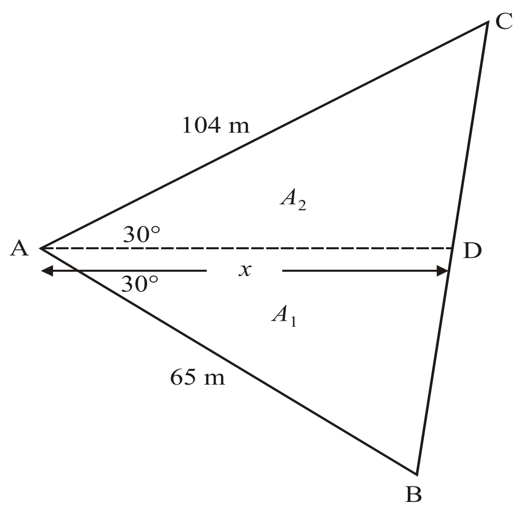
49. [Maximum mark: 18] ***[with GDC]***

A farmer owns a triangular field ABC. One side of the triangle, [AC], is 104 m, a second side, [AB], is 65 m and the angle between these two sides is 60°.

(a) Use the cosine rule to calculate the length of the third side of the field. [3]

(b) Given that $\sin 60° = \frac{\sqrt{3}}{2}$, find the area of the field in the form $p\sqrt{3}$ where $p \in Z$. [3]

Let D be a point on [BC] such that [AD] bisects the 60° angle. The farmer divides the field into two parts A_1 and A_2 by constructing a straight fence [AD] of length x metres, as shown on the diagram below.



(c) (i) Show that the area of A_1 is given by $\frac{65x}{4}$.

(ii) Find a similar expression for the area of A_2.

(iii) **Hence**, find the value of x in the form $q\sqrt{3}$, where $q \in Z$. [7]

(d) (i) Explain why $\sin A\hat{D}C = \sin A\hat{D}B$.

(ii) Use the result of part (i) and the sine rule to show that $\frac{BD}{DC} = \frac{5}{8}$. [5]

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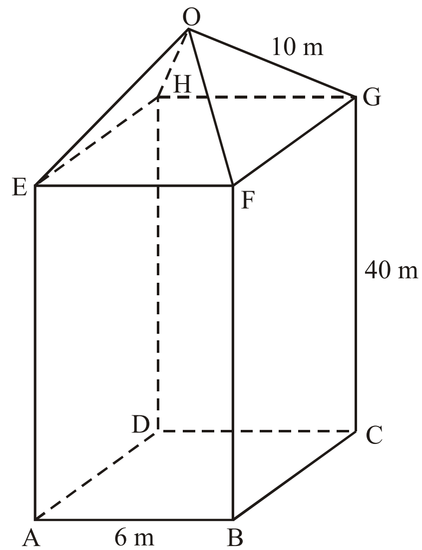
50. [Maximum mark: 14] ***[with GDC]***

An office tower is in the shape of a cuboid with a square base. The roof of the tower is in the shape of a square based right pyramid.

The diagram shows the tower and its roof with dimensions indicated. The diagram is **not** drawn to scale.



(a) Calculate, correct to three significant figures,

(i) the size of the angle between OF and FG; [3]

(ii) the shortest distance from O to FG; [2]

(iii) the total surface area of the four triangular sections of the roof; [3]

(iv) the size of the angle between the slant height of the roof and the plane EFGH; [2]

(v) the height of the tower from the base to O. [2]

A parrot's nest is perched at a point, P, on the edge, BF, of the tower. A person at the point A, outside the building, measures the angle of elevation to point P to be 79°.

(b) Find, correct to three significant figures, the height of the nest from the base of the tower. [2]

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51*. [Maximum mark: 16] ***[with GDC]***

In the diagram below, the points O(0, 0) and A(8, 6) are fixed. The angle $\mathrm{O\hat{P}A}$ varies as the point P(x ,10) moves along the horizontal line $y = 10$.

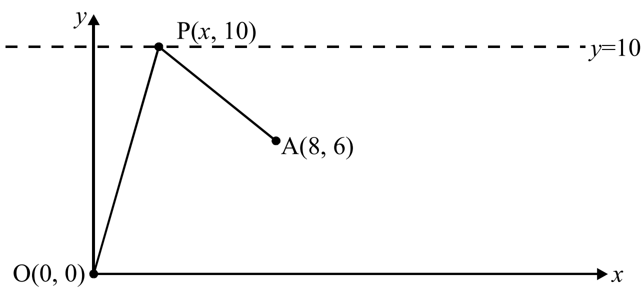

Diagram to scale

(a) (i) Show that $\mathrm{AP} = \sqrt{x^2 - 16x + 80}.$

(ii) Write down a similar expression for OP in terms of x. [2]

(b) Hence, show that $\cos\,\mathrm{O\hat{P}A} = \dfrac{x^2 - 8x + 40}{\sqrt{\{(x^2 - 16x + 80)(x^2 + 100)\}}}$, [3]

(c) Find, in degrees, the angle $\mathrm{O\hat{P}A}$ when $x = 8$. [2]

(d) Find the positive value of x such that $\mathrm{O\hat{P}A} = 60°$. [4]

Let the function f be defined by

$$f(x) = \cos\,\mathrm{O\hat{P}A} = \frac{x^2 - 8x + 40}{\sqrt{\{(x^2 - 16x + 80)(x^2 + 100)\}}},\quad 0 \le x \le 15.$$ [4]

(e) Consider the equation $f(x) = 1$.

(i) Explain, in terms of the position of the points O, A, and P, why this equation has a solution.

(ii) Find the **exact** solution to the equation. [5]

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52*. [Maximum mark: 16] ***[with GDC]***

The points P, Q, R are three markers on level ground, joined by straight paths PQ, QR, PR as shown in the diagram. QR = 9 km, $P\hat{Q}R = 35°$, $P\hat{R}Q = 25°$.

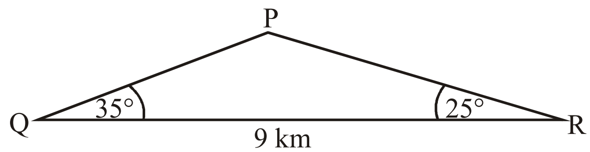
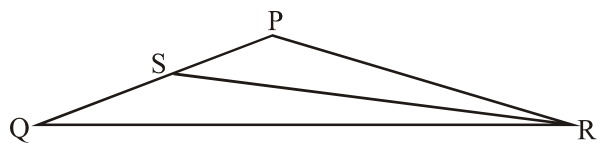


Diagram not to scale

(a) Find the length PR. [3]

(b) Tom sets out to walk from Q to P at a steady speed of 8 km h^{-1}. At the same time, Alan sets out to jog from R to P at a steady speed of a km h^{-1}. They reach P at the same time. Calculate the value of a. [7]

(c) The point S is on [PQ], such that RS = 2QS, as shown in the diagram.



Find the length QS. [6]

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