Question

Let $f(x) = \sin(x) + \frac{x}{2},\ 0 < x < 2\pi$.

(a) Find $f'(x)$.

$f'(x) = \cos(x) + \left[\frac{2(1) - x(0)}{4}\right] = \cos(x) + \frac{1}{2}$

[2 marks]

(b) Find $f''(x)$.

$f''(x) = -\sin(x)$

[2 marks]

(c) Identify the interval(s) on which $f(x)$ is decreasing.

$0 = \cos(x) + \frac{1}{2}$ $f'(\frac{2\pi}{3}) = -\sin(\frac{2\pi}{3}) = -\frac{\sqrt{3}}{3}$ $I = (\frac{2\pi}{3}, \frac{4\pi}{3})$

$\cos(x) = -\frac{1}{2}$, $x = \frac{2\pi}{3}, \frac{4\pi}{3}$ $f'(\frac{4\pi}{3}) = -\sin(\frac{4\pi}{3}) = \frac{\sqrt{3}}{3}$

[4 marks]

(d) Identify the interval on which $f(x)$ is concave upwards.

$\sin x = 0$ if $0 < x < \pi$: if $\pi < x < 2\pi$:

$x = \cancel{0}, \pi, \cancel{2\pi}$ $f''(\frac{\pi}{2}) = -\sin(\frac{\pi}{2}) = -1$ $f''(\frac{3\pi}{2}) = -\sin(\frac{3\pi}{2}) = 1$

$x = \pi$

[3 marks]

$f(x)$ is concave upwards when $x \in (\pi, 2\pi)$

(e) Write the equation of the tangent line to $y = f(x)$ at the inflection point. $f(\pi) = \sin\pi + \frac{\pi}{2} = \frac{\pi}{2}$, $P = (\pi, \frac{\pi}{2})$

$f'(\pi) = \cos(\pi) + \frac{1}{2} = -\frac{1}{2}$

[5 marks]

$y - \frac{\pi}{2} = -\frac{1}{2}(x - \pi)$

$y - \frac{\pi}{2} = -\frac{x}{2} + \frac{\pi}{2}$

$y = -\frac{1}{2}x + \pi$

Answers

Question	Example answer	Marks	Commentary
(a)	$f'(x) = \cos(x) + \frac{1}{2}$ **[2]**	2	Derivative of $\sin x$ and $\frac{x}{2}$ added together.
(b)	$f''(x) = (f')'(x)$ $= -\sin(x)$ **[2]**	2	Second derivative is the derivative of the first derivative. The derivative of $\cos x$.
(c)	$f'(x) < 0$ **[1]** $f'(x) = 0$ when $\cos x = -\frac{1}{2}$ $x = \frac{2\pi}{3}, \frac{4\pi}{3}$ Interval $\left]\frac{2\pi}{3}, \frac{4\pi}{3}\right[$ **[3]**	4	Base angle is $\frac{\pi}{3}$. Marks awarded for endpoints and interval notation (or equivalent e.g. $\frac{2\pi}{3} < x < \frac{4\pi}{3}$).
(d)	$f''(x) > 0$ **[1]** $\sin x < 0$ when $x \in]\pi, 2\pi[$ **[2]**	3	Endpoints and interval.

(e)	Inflection point: $(\pi, \mathrm{f}(\pi))$ **[1]** $\mathrm{f}(\pi) = \sin \pi + \frac{\pi}{2}$ $\mathrm{f}(\pi) = \frac{\pi}{2}$ **[1]** $\{\mathrm{f}\}'(\pi) = \cos \pi + \frac{1}{2}$ $\{\mathrm{f}\}'(\pi) = -\frac{1}{2}$ **[1]** Tangent line Point: $\left(\pi, \frac{\pi}{2}\right)$ gradient: $-\frac{1}{2}$ Point-gradient form of line $y - \frac{\pi}{2} = -\frac{1}{2}(x - \pi)$ **[2]** $y = -\frac{1}{2}(x - \pi) + \frac{\pi}{2}$ or $y = \pi - \frac{x}{2}$	5	**M1** mark for equation of a line. **A1** mark for correct equation.

Paper: 2 Marks: 10

Question

The position of a particle at any time, t ,is given by
$s\,(t) = 2t^4 - 8t^3 - 16t^2 - 8$, for all $t \geq 0$.

(a) Find the velocity of the particle at any time t.

$v(t) = s'(t) = 8t^3 - 24t^2 - 32t$

[3]

(b) Find the time(s) when the particle stops moving.

$0 = 8t^3 - 24t^2 - 32t$

$0 = (t^2 - 3t - 4)8t$

$0 = (t-4)(t+1)8t$

$t = 0,\ t = 4,\ t = -1$

[3]

(c) Find the time intervals when the particle is moving to the right and when the particle is moving to the left.

if $0 < t < 4$:

$v(1) = 8 - 24 - 32 = -48$

if $t > 4$:

$v(5) = 8(5)^3 - 24(5)^2 - 32(5) = 240$

Therefore, particle moves to left when $t \in (0, 4)$ and right when $t > 4$.

[4]

[Maximum mark: 10]

Answers

	Answer	Mark	Guidance
(a)	$v(t) = s'(t)$ $= 8t^3 - 24t^2 - 32t$	***M1*** ***A1A1***	The velocity function is the derivative of the position function. Award A1 for the first two correct terms and A1 for the second two correct terms.
(b)	$v(t) = 0$ $0 = 8t^3 - 24t^2 - 32t$ $0 = 8t(t^2 - 3t - 4)$ $0 = 8t(t - 4)(t + 1)$ $t = 0,\ -1,\ 4$ Since $t \geq 0$ $t = 0$ or $t = 4$	***M1*** ***M1*** ***A1***	The particle will stop moving when the velocity is equal to zero.
(c)	If $s'(t) > 0$ the particle moves to the right and if $s'(t) < 0$ the particle moves to the left. Possible method: number line test Moves left: $0 < t < 4$ Moves right: $4 < t < \infty$	***M1*** ***M1*** ***A1*** ***A1***	Knowing that when the position function is increasing the particle is moving to the right and when it is decreasing the particle is moving to the left. Accept other forms of these inequalities, such as interval notation.

Paper: 2 Marks: 7

Question

A model for the population growth of China is

$$P(t) = -0.000788t^3 + 0.0043t^2 + 7.24623t + 1286.78$$

in year t, where t is measured in years with $t = 0$ corresponding to the year 2000 and

$P(t)$ is measured in millions.

(a) Show that the population of China is forecast to peak during 2057.

$P'(t) = -0.002364t^2 + 0.0086t + 7.24623$

$0 = P'(t)$, By GDC, $t = 57.21$, which corresponds to during 2057.

[5]

(b) Find what the population of China will be when it peaks.

$P(57.21) = -0.000788(57.21)^3 + 0.0043(57.21)^2 + 7.24623(57.21) + 1286.78$

[2]

$P(57.21) = 1567.86$

Peak population will be 1,567,860,000 people.

[Maximum mark: 7]

Answer	Mark	Guidance
$P'(t) = 0$ $0 = -0.002364t^2 + 0.0086t + 7.24623$ $t = 57.2134\ldots$ $t \approx 57$ Therefore, $2000 + 57 = 2057$.	***M1*** ***A1A1*** ***M1*** ***A1*** ***AG***	Set the derivative equal to zero. Award A1 for the correct t^2 and t terms and A1 for the correct constant and zero terms. M1 for solving the quadratic and A1 for rounding up to a whole number.
$P(57) = -0.000788(57.2134)^3 + 0.0043(57.2134)^2$ $+ 7.24623(57.2134) + 1286.78$ ≈ 1567.86 Since $P(t)$ is measured in millions, the population in 2057 is approximately $1\,567\,860\,000$	***M1*** ***A1***	

Paper: 2 Marks: 8

Question

A student drops a water balloon from the top of the school science building, which is 80 feet in height. The velocity of the water balloon at time t seconds is $v(t) = -32t$ feet per second.

(a) Find $s(t)$, the height of the water balloon above the ground at time t.

$s(t) = \int -32t\, dt = -16t^2 + C$

$80 = -16(0)^2 + C$, $C = 80$

$s(t) = 80 - 16t^2$

[4]

(b) Find the number of seconds it will take the water balloon to hit the ground.

$0 = 80 - 16t^2$

$16t^2 = 80$

$t^2 = 5$

$t = \sqrt{5}$

* $-\sqrt{5}$ means nothing here.

It will take $\sqrt{5}$ seconds for the water balloon to hit the ground.

[2]

(c) Find the velocity of the water balloon when it hits the ground.

$v(\sqrt{5}) = -32(\sqrt{5}) \approx -71.6$ ft s^{-1}

[2]

The velocity of the water balloon just before it hits the ground is -71.6 ft s^{-1}.

[Maximum mark: 8]

	Answer	Mark	Guidance
(a)	$s(t) = \int v(t)dt$ $\int v(t)dt = \int(-32t)dt$ $= -16t^2 + c$ Solve for c: $80 = -16(0)^2 + c$ $c = 80$ $s(t) = -16t^2 + 80$	***M1*** ***A1*** ***M1*** ***A1***	The integral of the velocity function is equal to the position function. To find the complete position function you must solve for the constant c using the height of 80 ft at time 0, $(0, 80)$.
(b)	$s(t) = 0$ $0 = -16t^2 + 80$ $-80 = -16t^2$ $t^2 = \sqrt{5}$ $t = \pm\sqrt{5}$ $= \sqrt{5}$ ≈ 2.24 (s)	***M1*** ***A1***	To find when the water balloon hits the ground, the position will be at a height of zero. Recognise that the time will not be a negative value.
(c)	$v(t) = -32t$ $= -32(\sqrt{5})$ ≈ -71.6 The velocity of the water balloon when it hits the ground is -71.6 ft s^{-1}	***M1*** ***A1***	

Paper: 1 Marks: 6

Question

A function is defined by $g(x) = 5x - \dfrac{6}{x^2}, x \neq 0$.

(a) Find $g'(x)$.

$g'(x) = 5 + \dfrac{12}{x^3}$

[3]

The gradient of the graph of the function is 6.5 at the point P.

(b) Find the x-coordinate of P.

$6.5 = 5 + \dfrac{12}{x^3}$

[3]

$6.5x^3 = 5x^3 + 12$

[Maximum mark: 6]

$1.5x^3 = 12$

$x^3 = 8$

$x = 2$

$P_x = 2$

Answers

	Answer	Mark	Guidance
(a)	$g(x) = 5x - 6x^{-2}$ $g'(x) = 5 + 12x^{-3}$ or $g'(x) = 5 + \dfrac{12}{x^3}$	*A1* *A1* *A1*	For 5 For $+12$ For x^{-3} or $\dfrac{1}{x^3}$
(b)	$5 + \dfrac{12}{x^3} = 6.5$ $\dfrac{12}{x^3} = 1.5$ $x^3 = \dfrac{12}{1.5} = 8$ $x = 2$	*M1* *M1* *A1*	Set the gradient, 6.5, equal to $g'(x)$ and solve for x.

Paper: 1 Marks: 6

Question

A function $f(x)$ has a derivative function $f'(x)$ with the following properties in the domain $-5 < x < 3$:

x	$f'(x)$
$-5 < x < -1$	> 0
-1	0
$-1 < x < 0$	< 0
0	0
$0 < x < 3$	> 0

(a) Explain whether $f(-4)$ is greater than, less than or equal to $f(-2)$.

Less than, because the function is increasing in the interval $-4<x<-2$.

[2]

The point A(0, 5) lies on the graph of the function $f(x)$.

(b) Write down the equation of the tangent to the graph at $x = 0$.

$f'(0)=0$, so $m_0=0$

Tangent: $y=5$

[2]

(c) Explain whether A is a local minimum, local maximum or neither.

A is a local minimum, because the graph is decreasing before A and increasing immediately after it.

[2]

[Maximum mark: 6]

Answers

	Answer	Mark	Guidance
(a)	$f(-4) < f(-2)$ Because the function is increasing in the domain $-5 < x < -1$.	***A1*** ***R1***	A function is increasing when it has a positive gradient, $f'(x) > 0$, and decreasing when it has a negative gradient, $f'(x) < 0$.
(b)	At $x = 0$, the gradient of the tangent to the curve is zero. $y = 5$	***M1*** ***A1***	
(c)	*A* is a local minimum This is because it is the point where the gradient of the curve of the function changes from negative to positive.	***A1*** ***R1***	

Paper: 1 Marks: 6

Question

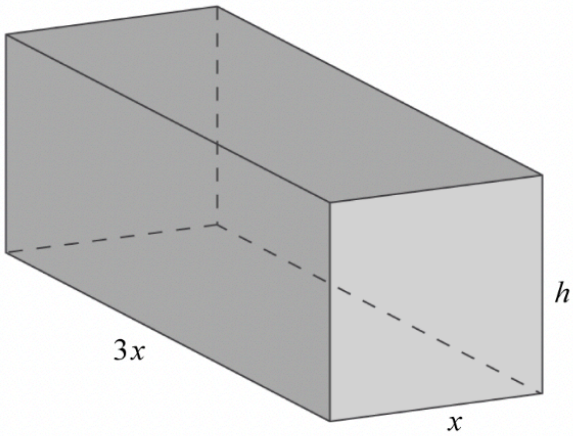
A closed box is cuboid in shape. The base has length $3x$ and width x, and the height is h.

$V = 3x(x)(h)$

$V = 3x^2h$

$100 = 3x(4) + 4h + 4x$

$100 = 16x + 4h$



The total length of all the edges of the box is 100 cm.

(a) The volume, V, of the box is given by the function $V(x) = 75x^2 - ax^3$.

Show that $a = 12$.

$3x^2h = 75x^2 - ax^3$

$100 - 16x = 4h$

$h = 25 - 4x$

$3x^2(25-4x) = 75x^2 - ax^3$

$\cancel{75x^2} - 12\cancel{x^3} = \cancel{75x^2} - a\cancel{x^3}$

$12 = a$

[2]

(b) Find $\dfrac{dV}{dx}$.

$\dfrac{dV}{dx} = 150x - 36x^2$

[2]

(c) Hence find the value of x which gives the maximum volume of the box.

$0 = 150x - 36x^2$

$0 = (150 - 36x)x$

$x = 0 \quad | \quad 36x = 150$

$x = \frac{25}{6}$

$x = \frac{25}{6}$ gives the maximum volume of the box.

[2]

[Maximum mark: 6]

Answers

	Answer	Mark	Guidance
(a)	$100 = 4(x) + 4(3x) + 4(h)$ $100 = 16x + 4h$ $V = 3x^2(25 - 4x)$ $V = 75x^2 - 12x^3$ $a = 12$	*M1* *A1* *AG*	Correct use of the sum of all the sides of the box to solve for h. Substitute for h.
(b)	$\frac{\mathrm{d}V}{\mathrm{d}x} = 150x - 36x^2$	*A1A1*	Award *A1* for each correct term.
(c)	$150x - 36x^2 = 0$ $x(150 - 36x) = 0$ $36x = 150$ $x = \frac{25}{6}$	*M1* *A1*	Setting the derivative equal to zero.

Paper: 1 Marks: 5

Question

A function is defined as $f(x) = 3x^2 + \frac{p}{x} + 8, x \neq 0$

(a) Find $f'(x)$

$f'(x) = 6x - \frac{p}{x^2}$

[2]

(b) The graph of the function has a local minimum when $x = 2$. Find the value of p.

$0 = 6(2) - \frac{p}{2^2}$

[3]

$0 = 12 - \frac{p}{4}$

[Maximum mark: 5]

$\frac{p}{4} = 12$

$p = 48$

Answers

	Answer	Mark	Guidance
(a)	$f'(x) = 6x - \frac{p}{x^2}$	*A1* *A1*	$6x$ $-\frac{p}{x^2}$ A1A0 if extra terms are present.
(b)	$f'(x) = 0$ $6(2) - \frac{p}{2^2} = 0$ $12 - \frac{p}{4} = 0$ $\frac{p}{4} = 12$ $p = 48$	*M1* *M1* *A1*	Setting the derivative equal to zero for a local minimum. Correctly substituting $x = 2$ into the derivative.

Paper: 1 Marks: 6

Question

A function is defined as $y = 2x^3 + px$

(a) Write down $\dfrac{dy}{dx}$

$\dfrac{dy}{dx} = 6x^2 + p$

[2]

The curve of the function has a local maximum at point P$(-2, q)$.

(b) Find the value of p

$0 = 6(-2)^2 + p$

$p = -24$

[2]

(c) Find the value of q

$q = 2(-2)^3 - 24(-2)$

$q = -16 + 48$

$q = 32$

[2]

[Maximum mark: 6]

Answers

	Answer	Mark	Guidance
(a)	$6x^2 + p$	***A1A1***	A1 for each correct term.
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ so $6(-2)^2 + p = 0$ $p = -24$	***M1*** ***A1***	Since this occurs at a local maximum, M1 for setting the derivative equal to 0.
(c)	$q = 2(-2)^3 - (24 \times -2)$ $q = 32$	***M1*** ***A1***	M1 for substituting for x in the function $y = 2x^3 - 24x$.

Paper: 2 Marks: 6

Question

Let $f(x) = x^3 - 4x^2 + x + 1$

(a) Find $f'(x)$

$f'(x) = 3x^2 - 8x + 1$

[2]

(b) Find the values of x when $f'(x) = 0$

By GDC, $x = 2.54, 0.132$ when $f'(x) = 0$

[2]

(c) The point P is the local maximum of the graph of $f(x)$ in the domain $-5 \leq x \leq 5$.

Write down the equation of the tangent to the curve at point P.

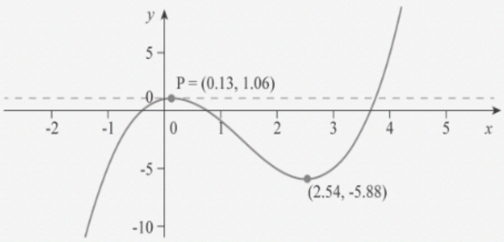
$f'(0.1315) = 0$

[2]

Tangent $= P_y = 1.065 \approx 1.07$

[Maximum mark: 6]

Answers

	Answer	Mark	Guidance
(a)	$f'(x) = 3x^2 - 8x + 1$	***A1A1***	(A1) for $3x^2$, (A1) for $(-8x+1)$ (A1) (A0) if additional terms are present.
(b)	$3x^2 - 8x + 1 = 0$ $x = 2.54, x = 0.131$	***A1A1***	Examples for solving: Quadratic Formula, Graphing Calculator, etc.
(c)	 (Click on the image to enlarge it) The tangent through P is horizontal. It has an equation in the form of $y = c$ rather than $y = mx + c$ The equation of this tangent is $y = 1.06$	***A1A1***	Notice the two turning points, the local maximum occurs at the value $x = 0.131$. (A1) for $y =$, (A1) for 1.06

Paper: 1 Marks: 6

Question

A straight line, L, has equation $3x - 2y - 12 = 0$.

(a) Find the gradient of the line.

$y = \frac{3}{2}x - 6$

$m = \frac{3}{2}$

[2]

A function is defined by $f(x) = 2x^2 - 2x + 10$.

(b) (i) Find $f'(x)$.

$f'(x) = 4x - 2$

[2]

(ii) A tangent is drawn to the graph of $f(x)$ such that it is parallel to the line L.

Find the value of x where the tangent touches $f(x)$.

$\frac{3}{2} = 4x - 2$

[2]

$\frac{7}{2} = 4x$

[Maximum mark: 6]

$x = \frac{7}{8}$ or 0.875

Answers

	Answer	Mark	Guidance
(a)	$2y = 3x - 12$ $y = \frac{3}{2}x - 6$ The gradient is 1.5	***M1*** ***A1***	Make y the subject of the equation to change the equation into the slope-intercept form of a line.
(b) (i)	$f'(x) = 4x - 2$	***A1A1***	One mark for each correct term.
(ii)	$1.5 = 4x - 2$ $4x = 3.5$ $x = 0.875$	***M1*** ***A1***	Set the gradient from part **(a)** equal to the derivative in part **b (i)** and solve for x.

Paper: 2 Marks: 6

Question

The function $f(x)$is defined by: $f(x) = x^3$

(a) Write down $f'(x)$.

$f'(x) = 3x^2$

[1]

The graph of the function $y = f(x)$ passes through the point A(3, 27).

(b) Find the gradient of the tangent to the curve at A.

$f'(3) = 3(3)^2 = 27$

[2]

(c) Find the equation of the normal to the curve at A.

Write your answer in the form $ax + by + d = 0$.

$n = -\frac{1}{27}$

[3]

$y - 27 = -\frac{1}{27}(x-3)$

[Maximum mark: 6]

$y - 27 = \frac{-x}{27} + \frac{1}{9}$

$y = -\frac{1}{27}x + \frac{244}{9}$

$27y = -x + 732$

$x + 27y - 732 = 0$

Answers

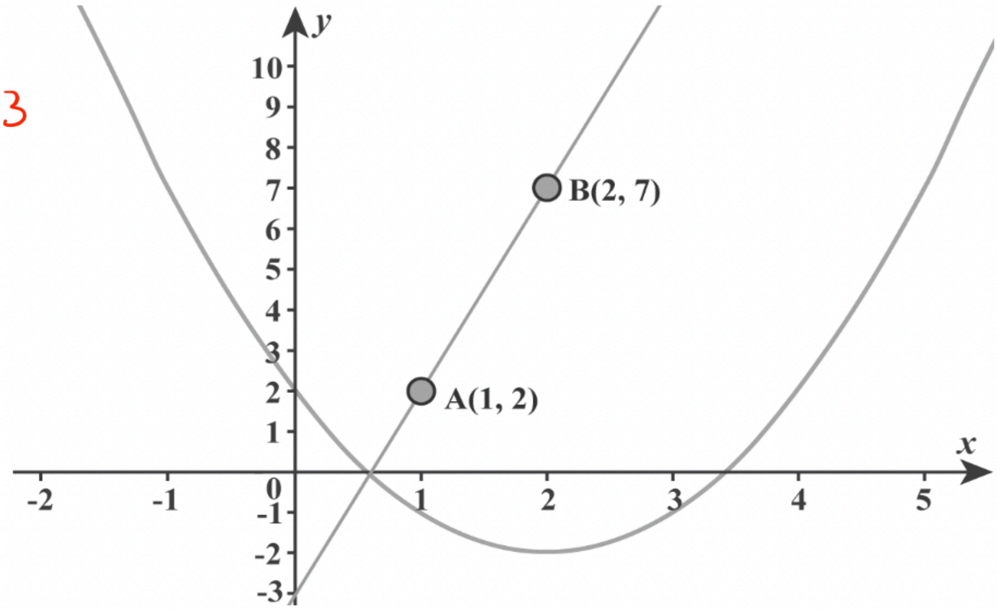
	Answer	Mark	Guidance
(a)	$f'(x) = 3x^2$	*A1*	
(b)	$f'(3) = 3 \times 3^2$ $f'(3) = 27$	*M1* *A1*	Substitute $x = 3$ into the answer to part **(a)**.
(c)	The normal gradient = $-\frac{1}{27}$ $y = -\frac{1}{27}x + c$ $27 = -\frac{1}{27} \times 3 + c$ $c = \frac{732}{27}$ or $\frac{244}{9}$ The equation of the normal is $y = -\frac{1}{27}x + \frac{732}{27}$ $27y = -x + 732$ $x + 27y - 732 = 0$	*M1* *M1* *A1*	Find the gradient of the normal which is the negative reciprocal of the gradient of the tangent, then substitute the gradient of the normal into the slope-intercept form of a line. Substitute the coordinates for A into this equation to find the value of c. Rearranging to $ax + by + d = 0$ form.

Paper: 2 Marks: 15

Question

The diagram below shows the line L_1 which passes through points $(1, 2)$ and $(2, 7)$ and the graph of the function $f(x) = x^2 - 4x + 2$.

$L_1 = 5x - 3$



(a) Write down the gradient of the line L_1.

$m = \frac{7-2}{2-1} = 5$

[1]

(b) Find $f'(x)$.

$f'(x) = 2x - 4$

[2]

(c) Find the coordinates of the point P where the tangent to the graph is parallel to L_1.

$f(4.5) = (4.5)^2 - 4(4.5) + 2$

$f(4.5) = 4.25$

$5 = 2x - 4$

$9 = 2x$

$x = 4.5$

$P = (4.5, 4.25)$

[3]

L_2 is the normal to the graph at point P.

(d) Find the equation of L_2 in the form $y = mx + c$.

$n_{f \text{ at } P} = -\frac{1}{5}$

$y - 4.25 = -\frac{1}{5}(x - 4.5)$

$y - 4.25 = -\frac{1}{5}x + \frac{4.5}{5}$

$y = -\frac{1}{5}x + 5.15$

[3]

The line L_2 also meets the graph of the function $f(x)$ at point Q.

(e) Determine the coordinates of Q.

$-\frac{1}{5}x + 5.15 = x^2 - 4x + 2$

$0 = x^2 - \frac{19}{5}x - 3.15$

By GDC, $x = -0.7, 4.5$

$f(-0.7) = (-0.7)^2 - 4(-0.7) + 2 = 5.29$

$f(4.5) = (4.5)^2 - 4(4.5) + 2 = 4.25$

$Q = (-0.7, 5.29)$ or $(4.5, 4.25)$

[2]

(f) Write down the equation of the axis of symmetry of the graph.

$x = \frac{4}{2} = 2$

[2]

(g) Determine for which values of x the function $f(x)$ is decreasing.

$f(x)$ is decreasing while $x \in (-\infty, 2)$

[2]

[Maximum mark: 15]

	Answer	Mark	Guidance
(a)	5	***A1***	
(b)	$f'(x) = 2x - 4$	***A1A1***	Award one mark for each correct term.
(c)	$2x - 4 = 5$ $x = 4.5$ $f(4.5) = (4.5)^2 - 4(4.5) + 2 = 4.25$ $(4.5, 4.25)$	***M1*** ***A1A1***	
(d)	Gradient of $L_2 = -0.2$ $y = -0.2x + c$ $4.25 = -0.2(4.5) + c$ $c = 5.15$ $y = -0.2x + 5.15$	***M1*** ***M1*** ***A1***	
(e)	$(-0.7, 5.29)$	***M1*** ***A1***	Attempting to find the intersection of the two curves or solving simultaneous equations. Correct coordinate pair.
(f)	$x = 2$	***A1*** ***A1***	For $x =$ a constant For 2
(g)	$x < 2$	***A1*** ***A1***	For $x <$ For 2

Question

Consider a function $f(x) = 2x^2 + \frac{4}{x}, x \neq 0$.

(a) Find $f(-1)$

$f(-1) = 2 - 4 = -2$

[1]

(b) Differentiate $f(x)$

$f'(x) = 4x - \frac{4}{x^2}$

[3]

(c) Find $f'(-1)$

$f'(-1) = -4 - 4 = -8$

[2]

L is the normal to the graph of the function $f(x)$ at $x = -1$

(d) Find the equation of L.

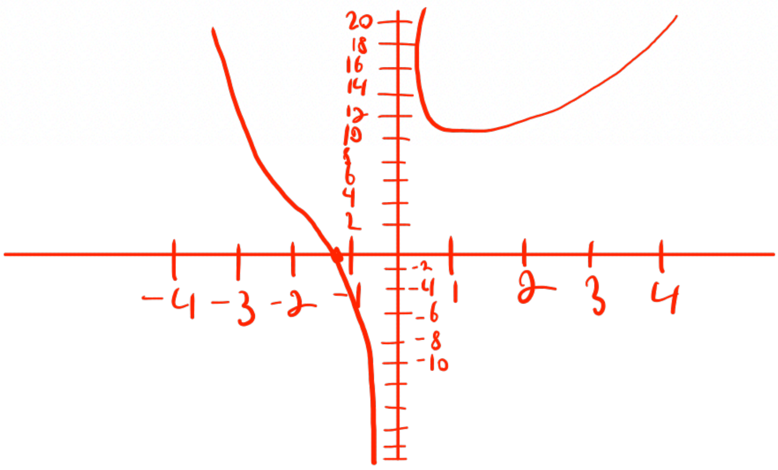
L = reciprocal of $-8 = \frac{1}{8}$

$y + 2 = \frac{1}{8}(x+1)$

$y + 2 = \frac{1}{8}x + \frac{1}{8} \rightarrow y = \frac{1}{8}x - \frac{18}{8}$

[3]

(e) Sketch the graph of $y = f(x)$ for $-4 \leq x \leq 4$ and $-10 \leq y \leq 20$.



[4]

The graph of the function $f(x)$ has a local minimum at point P.

(f) Using your answer to **(b)**, find the coordinates of P.

$0 = 4x - \frac{4}{x^2}$ $\quad f(1) = 2(1)^2 + \frac{4}{1} = 6$

[3]

$\frac{4}{x^2} = 4x$ $\quad P = (1, 6)$

[Maximum mark: 16]

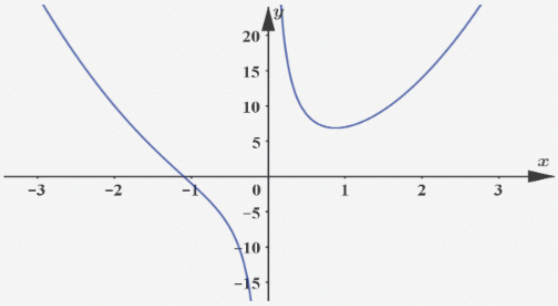
$4 = 4x^3$

$1 = x^3$

$x = 1$

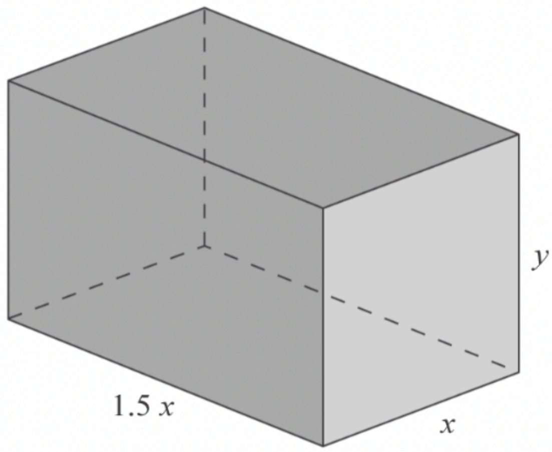
Answers

	Answer	Mark	Guidance
(a)	$f(-1) = 2(-1)^2 + \dfrac{4}{(-1)}$ $f(-1) = -2$	*A1*	
(b)	$f(x) = 2x^2 + 4x^{-1}$ $f'(x) = 4x - 4x^{-2}\left(= 4x - \dfrac{4}{x^2}\right)$	*A1A1A1*	Award A1 for $4x$, A1 for -4, and A1 for x^{-2}.
(c)	$f'(-1) = 4(-1) - \dfrac{4}{(-1)^2}$ $f'(-1) = -8$	*M1* *A1(ft)*	Correct substitution of -1 into the derivative of their f.
(d)	$y = \dfrac{1}{8}x + c$ $-2 = \dfrac{1}{8}(-1) + c$ $c = -\dfrac{15}{8}$ $y = \dfrac{1}{8}x - \dfrac{15}{8}$	*M1* *M1* *A1(ft)*	Using the opposite reciprocal slope of their part **(c)** answer. Correctly substituting their slope and $(-1, -2)$ into the equation of a line.

(e)	 (Click on the image to enlarge it)	***A1*** ***A1*** ***A1*** ***A1***	For axes, labelled with some indication of scale. For general shape of the curve. For x-intercept in correct position by eye. For local minimum in correct position by eye.
(f)	$f'(x) = 4x - \frac{4}{x^2} = 0$ $4x = \frac{4}{x^2}$ $x^3 = 1$ $x = 1$ $f(1) = 2(1)^2 + \frac{4}{1} = 6$ Point P has coordinates $(1, 6)$	***M1*** ***A1(ft)*** ***A1(ft)***	Setting their derivative of f equal to zero.

Question

A cardboard box is in the shape of a rectangular prism of length x cm, width $\frac{3}{2}x$ cm and height y cm.



(a) Write down an expression for the volume of the box.

$V = \frac{3}{2}x^2 y$

[1]

The total volume of the box is 1000 cm^3.

(b) Find an expression for y in terms of x

$1000 = \frac{3}{2}x^2 y$

$y = \frac{2000}{3x^2}$

[1]

(c) Show that the surface area of the box is given by $A(x) = 3x^2 + \dfrac{10\,000}{3x}$

$A = \frac{3}{2}x\left(\frac{2000}{3x^2}\right)(2) + \frac{3}{2}x^2(2) + x\left(\frac{2000}{3x^2}\right)(2)$

$A = \frac{12000x}{6x^2} + 3x^2 + \frac{8000x}{6x^2}$

[2]

$A = 3x^2 + \frac{20000}{6x^2}$, $A = 3x^2 + \frac{10000x}{3x^2}$, $A = 3x^2 + \frac{10000}{3x}$

(d) Find $A'(x)$

$A = 3x^2 + \frac{10000}{3}(x^{-1})$

[3]

$A' = 6x - \frac{10000}{3x^2}$

(e) Find the value of x which makes $A(x)$ a minimum.

$0 = 6x - \frac{10000}{3x^2}$

$10000 = 18x^3$

[2]

$\frac{10000}{3x^2} = 6x$

$x = 8.22$

(f) Find the minimum surface area of the box.

$A_{min} = 3(8.22)^2 + \frac{10000}{3(8.22)}$

[2]

$A_{min} = 202.74 + \frac{10000}{24.66}$

[Maximum mark: 11]

$A_{min} \cong 608 \text{ cm}^2$

Answers

	Answer	Mark	Guidance
(a)	$V = \frac{3}{2}x^2 y$	*A1*	
(b)	$1000 = \frac{3}{2}x^2 y$ $y = \frac{2000}{3x^2}$	*A1*	
(c)	$A(x) = 2(1.5x \times x) + 2(y \times 1.5x) + 2(x \times y)$ $A(x) = 3x^2 + 3xy + 2xy$ $A(x) = 3x^2 + 5x \times \frac{2000}{3x^2}$ $A(x) = 3x^2 + \frac{10\,000}{3x}$	*M1* *M1* *AG*	Correct surface area formula and substitution. Correctly substituting their y from part (**b**).
(d)	$A(x) = 3x^2 + \frac{10\,000}{3}x^{-1}$ $A'(x) = 6x - \frac{10\,000}{3}x^{-2} = 6x - \frac{10\,000}{3x^2}$	*A1* *A1* *A1*	For $6x$ For $-\frac{10\,000}{3}$ For x^{-2}

(e)	$$6x - \frac{10\,000}{3x^2} = 0$$ $$x^3 = \frac{10\,000}{18}$$ $$x = 8.22 \text{ (cm)}$$	***M1*** ***A1***	For setting their derivative equal to zero.
(f)	$$A(8.22) = 3(8.22)^2 + \frac{10\,000}{3(8.22)}$$ $$A(8.22) = 608 \text{ (cm}^2\text{)}$$	***M1*** ***A1***	Correct substituting of their x from part **(e)**.

Paper: 1 Marks: 6

Question

A particle moves along a straight line, starting at the origin.

After t seconds its position, s (measured in metres), satisfies the equation
$s^3 + e^s = t + 1$

(a) Express the velocity of the particle in terms of its position.

$S = (t + 1 - e^S)^{1/3}$

$V = S' = \frac{1}{3}(t + 1 - e^S)^{-2/3} \cdot (-e^S) =$

[3]

(b) Express the acceleration of the particle in terms of its position.

[3]

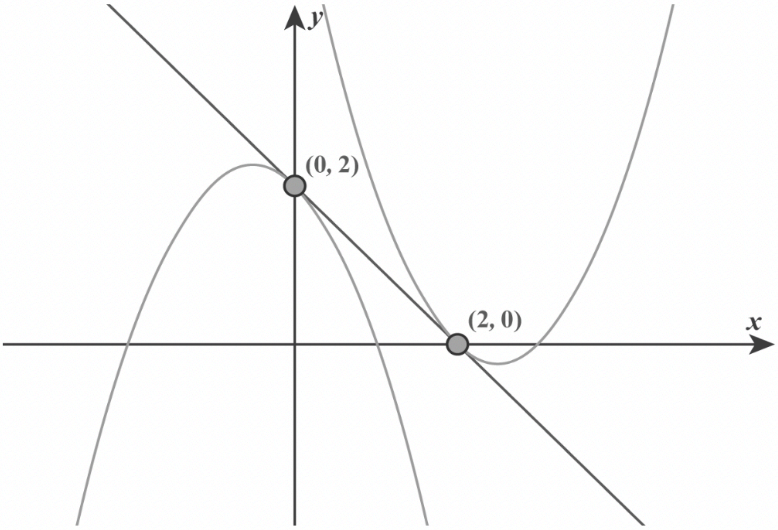
[Maximum mark: 6]

	Answer	Mark	Guidance
(a)	$3s^2\dfrac{\mathrm{d}s}{\mathrm{d}t}+\mathrm{e}^s\dfrac{\mathrm{d}s}{\mathrm{d}t}=1$ $\dfrac{\mathrm{d}s}{\mathrm{d}t}(3s^2+\mathrm{e}^s)=1$ $\dfrac{\mathrm{d}s}{\mathrm{d}t}=\dfrac{1}{3s^2+\mathrm{e}^s}$ $v=\dfrac{1}{3s^2+\mathrm{e}^s}$	***M1*** ***A1*** ***A1***	The velocity is the derivative of the position with respect to time. Differentiate the relationship given in the question with respect to time. M1 for correct implicit differentiation method attempted. A1 for the correct derivative. A1 for the correct velocity.
(b)	$0=\left(6s\dfrac{\mathrm{d}s}{\mathrm{d}t}\dfrac{\mathrm{d}s}{\mathrm{d}t}+3s^2\dfrac{\mathrm{d}^2s}{\mathrm{d}t^2}\right)+\left(\mathrm{e}^s\dfrac{\mathrm{d}s}{\mathrm{d}t}\dfrac{\mathrm{d}s}{\mathrm{d}t}+\mathrm{e}^s\dfrac{\mathrm{d}^2s}{\mathrm{d}t^2}\right)$ $0=6sv^2+3s^2a+\mathrm{e}^sv^2+\mathrm{e}^sa$ $a(3s^2+\mathrm{e}^s)=-v^2(6s+\mathrm{e}^s)$ $a=-\dfrac{v^2(6s+\mathrm{e}^s)}{3s^2+\mathrm{e}^s}$ $a=-\dfrac{6s+\mathrm{e}^s}{(3s^2+\mathrm{e}^s)^3}$	***M1*** ***A1*** ***A1***	The acceleration is the second derivative of the position with respect to time. Differentiate the first line of the previous calculation once more with respect to time.

Question

The diagram below shows part of the graphs of the functions

$$f(x) = -x^2 + px + q \text{ and } g(x) = x^2 + sx + t$$



The diagram also shows a line which is tangent to the graph of f at $(0, 2)$ and also tangent to the graph of g at $(2, 0)$.

(a) Find the value of p and of q.

[3]

(b) Find the value of s and of t.

[2]

(c) There is a second line that is also tangent to both parabolas. Find the equation of this line.

[9]

[Maximum mark: 14]

	Answer	Mark	Guidance
(a)	$2 = f(0) = -0^2 + p \times 0 + q$, and hence, $q = 2$ The gradient of the line is $\dfrac{0-2}{2-0} = -1$ $-1 = f'(0) = -2 \times 0 + p$, and hence $p = -1$	A1 M1 A1	The graph of f passes through the point $(0, 2)$. This line is tangent to the graph of f at $(0, 2)$. Putting all this together, gives $f(x) = -x^2 - x + 2$
(b)	$-1 = g'(2) = 2 \times 2 + s$, and hence $s = -5$ $0 = g(2) = 2^2 + s \times 2 + t$, and hence, $t = 6$	A1 A1	The line is also tangent to the graph of g at point $(2, 0)$. The graph of g passes through the point $(2, 0)$. Putting all this together, gives $g(x) = x^2 - 5x + 6$
(c)	Sample Method: Let $(u, f(u)) = (u, -u^2 - u + 2)$ be the point where this second line touches the graph of f and let $(v, g(v)) = (v, v^2 - 5v + 6)$ be the point where it touches the graph of g. Since it is a tangent of the graph of f at $(u, -u^2 - u + 2)$, $m = f'(u) = -2u - 1$ Since it is a tangent of the graph of g at $(v, v^2 - 5v + 6)$, $m = g'(v) = 2v - 5$ Since the line is passing through $(u, -u^2 - u + 2)$ and $(v, v^2 - 5v + 6)$,	M1 A1 A1 A1 M1 A1 A1	You now express the gradient, m, of this line in three different ways. The first two ways of expressing the gradient. Substituting this in the third expression and setting it equal to the first.
	$m = \dfrac{(v^2 - 5v + 6) - (-u^2 - u + 2)}{v - u}$ $m = \dfrac{v^2 + u^2 - 5v + u + 4}{v - u}$ $-2u - 1 = 2v - 5$ $4 = 2u + 2v$ $v = 2 - u$	M1 A1	The two solutions of this quadratic equation. For eliminating this equation.

$$
\begin{aligned}
-2u - 1 &= \frac{(2-u)^2 + u^2 - 5(2-u) + u + 4}{(2-u) - u} \\
-2u - 1 &= \frac{2u^2 + 2u - 2}{2 - 2u} \\
(2-2u)(-2u-1) &= 2u^2 + 2u - 2 \\
4u^2 - 2u - 2 &= 2u^2 + 2u - 2 \\
2u^2 - 4u &= 0 \\
u(u-2) &= 0
\end{aligned}
$$

$u = 0$ and $u = 2$

If $u = 0$, then $v = 2 - 0 = 2$, and the two points of tangency are $(0, 2)$ and $(2, 0)$. This gives the equation of the line in the previous parts, so this is not the line we are looking for.

If $u = 2$,

$$(2, f(2)) = (2, -2^2 - 2 + 2) = (2, -4)$$

and also gives

$v = 2 - 2 = 0$, so

$$(0, g(0)) = (0, 0^2 - 5 \times 0 + 6) = (0, 6)$$

One example of the equation of the line through these two points is

$$y = -5x + 6$$

(Accept equivalent equations in point-slope or standard forms)

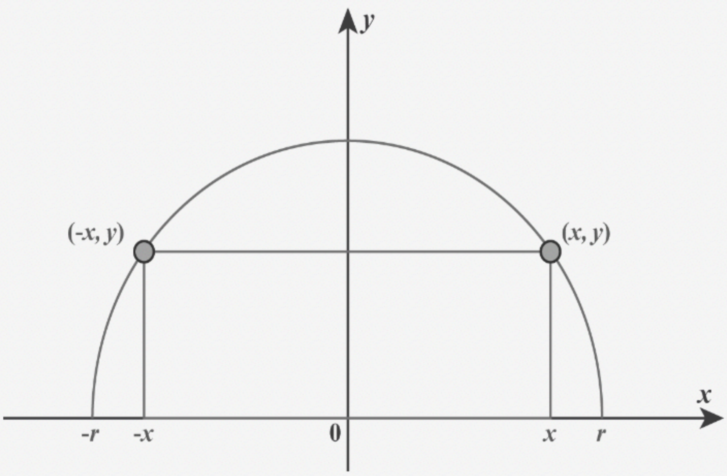
Question

A rectangle is positioned so that two of its vertices are on the x-axis and two of its vertices are on the semicircle of radius r with equation $x^2 + y^2 = r^2$, where $y > 0$

Find the largest possible area of such a rectangle in terms of r.

[Maximum mark: 6]

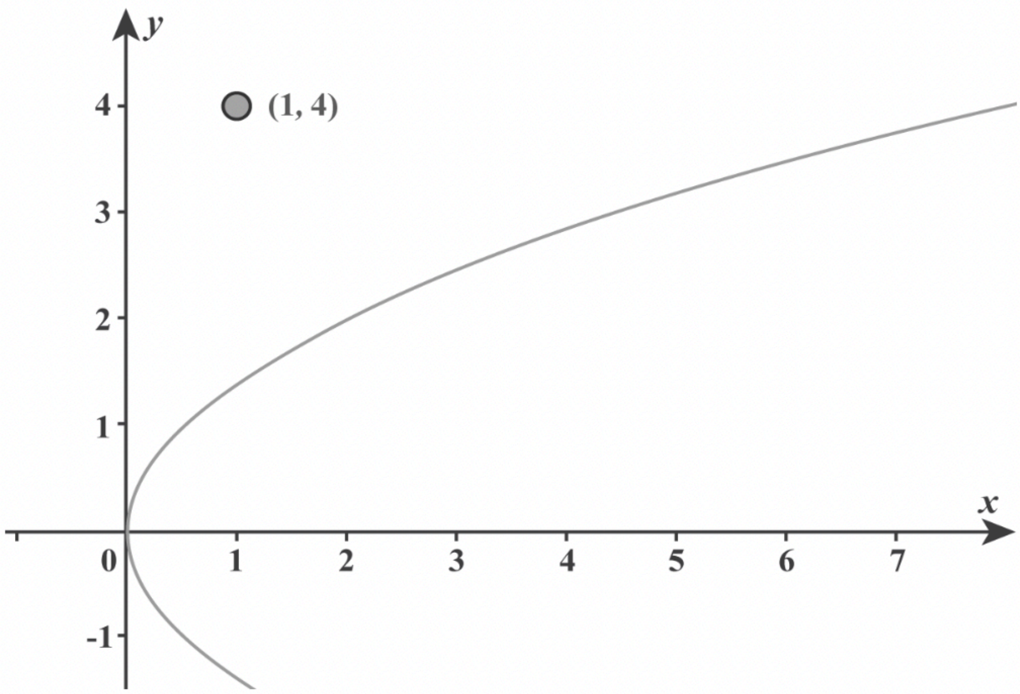
Answers

Answer	Mark	Guidance
	***M1*** ***M1*** ***M1*** ***M1*** ***A1***	For correct position of the semicircle and a rectangle. Let (x, y) be the vertex that lies in the first quadrant. Then, the rectangle has side lengths $2x$ and y, to find its area.
$A = 2xy$ $A = 2x\sqrt{r^2 - x^2}$ $A' = 2\sqrt{r^2 - x^2} - \frac{2x^2}{\sqrt{r^2 - x^2}} = \frac{2\left(r^2 - 2x^2\right)}{\sqrt{r^2 - x^2}}$ The area of the largest inscribed rectangle is $A\left(\frac{r}{\sqrt{2}}\right) = 2\frac{r}{\sqrt{2}}\sqrt{r^2 - \frac{r^2}{2}} = r^2$	***A1***	Correct area of the rectangle. To eliminate y use the fact that (x, y) lies on the circle $x^2 + y^2 = r^2$ which gives $y = \sqrt{r^2 - x^2}$. Correct substitution of y into the area formula.

		Remembering that the domain of this function is $0 \leq x \leq r$. Correct use of the product rule. Correct derivative. This derivative is 0 when $2x^2 = r^2$, that is $x = \frac{r}{\sqrt{2}}$ since $x \geq 0$
		This value of x gives a maximum value of Asince $A(0) = 0$ and $A(r) = 0$.

Question

Find the point on the parabola $y^2 = 2x$ that is closest to the point $(1, 4)$.



[Maximum mark: 6]

Answers

Answer	Mark	Guidance
$d = \sqrt{(x-1)^2 + (y-4)^2}$ $d = \sqrt{\left(\frac{y^2}{2} - 1\right)^2 + (y-4)^2}$ $d^2 = f(y) = \left(\frac{y^2}{2} - 1\right)^2 + (y-4)^2$	***M1*** ***M1*** ***M1***	The distance between the point $(1, 4)$ and the point (x, y). As the point (x, y) lies on the parabola we have $x = \frac{y^2}{2}$.
$f'(y) = 2\left(\frac{y^2}{2} - 1\right)y + 2(y-4) = y^3 - 8$ So $f'(y) = 0$ when $y = 2$ The corresponding value of x is $x = \frac{2^2}{2} = 2$ thus the point on $y^2 = 2x$ closest to the point $(1, 4)$ is $(2, 2)$.	***A1*** ***A1A1***	The minimum of d occurs at the same point as the minimum of d^2 so instead of minimising d we can minimise , as it is easier to work with. Differentiating $f(y)$. Award *A1* for correct x and *A1* for correct y.

Question

A particle moves along a straight line.

The velocity, v ms^{-1} t seconds after we start observing the movement is given by $v(t) = 1 - 2\sin 3t$.

(a) Find an expression for the acceleration of the particle.

[2]

(b) Find an expression for the displacement of the particle from its original position.

[6]

(c) Find the displacement of the particle from its original position $\frac{\pi}{6}$ seconds after we start observing the movement.

[3]

(d) Find the total distance travelled by the particle in the first $\frac{\pi}{6}$ seconds after we start observing the movement.

[10]

[Maximum mark: 21]

Answer	Mark	Guidance
$a(t) = v'(t) = -2 \times 3\cos 3t = -6\cos 3t$	*M1A1*	
$s(t) = \int 1 - 2\sin 3t \, \mathrm{d}t$ $= t - 2 \times \frac{1}{3}(-\cos 3t) + c$ $= t + \frac{2}{3}\cos 3t + c$	*M1A1* *A1* *A1*	Since $s'(t) = v(t)$, you integrate the velocity function to get the displacement.
$0 = 0 + \frac{2}{3}\cos(3 \times 0) + c$ $0 = 0 + \frac{2}{3} \times 1 + c$ $c = -\frac{2}{3}$ Hence, $s(t) = t + \frac{2}{3}\cos 3t - \frac{2}{3}$		Since you are looking for the displacement from the position at $t = 0$ you know that $s(0) = 0$.
$s\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + \frac{2}{3}\cos\frac{3\pi}{6} - \frac{2}{3}$ $= \frac{\pi}{6} - \frac{2}{3}$	*M1A1* *A1*	
$1 - 2\sin 3t = 0$ $\sin 3t = \frac{1}{2}$ If $0 < t < \frac{\pi}{6}$, then $0 < 3t < \frac{3\pi}{6} = \frac{\pi}{2}$. $3t = \frac{\pi}{6}$ $t = \frac{\pi}{18}$ $v(0) = 1 - 2\sin 0 = 1 > 0$	*M1* *A1* *M1* *A1A1* *M1* *A1* *A1*	Find the distance travelled to the left and to the right separately. First you find the time between 0 and $\frac{\pi}{6}$ when the velocity is 0.

$$\int_0^{\frac{\pi}{18}} 1 - 2\sin 3t\,\mathrm{d}t = \left[t + \frac{2}{3}\cos 3t - \frac{2}{3}\right]_0^{\frac{\pi}{18}}$$

$$= \left(\frac{\pi}{18} + \frac{2}{3}\cos\frac{3\pi}{18} - \frac{2}{3}\right) - \left(0 + \frac{2}{3}\cos(3\times 0) - \frac{2}{3}\right)$$

$$= \frac{\pi}{18} + \frac{2\sqrt{3}}{3\times 2} - \frac{2}{3}$$

$$= \frac{\pi}{18} + \frac{\sqrt{3}}{3} - \frac{2}{3}$$

A1

A1

A1

The only angle between 0 and $\frac{\pi}{2}$ for which $\sin\theta = \frac{1}{2}$ is $\theta = \frac{\pi}{6}$.

To find the distance travelled, we integrate the velocity function between 0 and $\frac{\pi}{18}$ and between $\frac{\pi}{18}$ and $\frac{\pi}{6}$ separately.

$$\int_{\frac{\pi}{18}}^{\frac{\pi}{6}} 1 - 2\sin 3t\,\mathrm{d}t = \left[t + \frac{2}{3}\cos 3t - \frac{2}{3}\right]_{\frac{\pi}{18}}^{\frac{\pi}{6}}$$

$$= \left(\frac{\pi}{6} + \frac{2}{3}\cos\frac{3\pi}{6} - \frac{2}{3}\right) - \left(\frac{\pi}{18} + \frac{2}{3}\cos\frac{3\pi}{18} - \frac{2}{3}\right)$$

$$= \left(\frac{\pi}{6} + 0 - \frac{2}{3}\right) - \left(\frac{\pi}{18} + \frac{2\sqrt{3}}{3\times 2} - \frac{2}{3}\right)$$

$$= \frac{2\pi}{18} - \frac{\sqrt{3}}{3}$$

The first integral gives the amount of movement to the right.

Hence the movement to the left is

$$-\left(\frac{2\pi}{18} - \frac{\sqrt{3}}{3}\right) = \frac{\sqrt{3}}{3} - \frac{2\pi}{18}$$

The total distance travelled is the sum of these two values:

$$\left(\frac{\pi}{18} + \frac{\sqrt{3}}{3} - \frac{2}{3}\right) + \left(\frac{\sqrt{3}}{3} - \frac{2\pi}{18}\right) = \frac{2\sqrt{3} - 2}{3} - \frac{\pi}{18}$$

The second integral gives the amount of the movement to the left.

Since this is a movement to the left, this integral is negative.

Question

The second derivative of function f is given by $f''(x) = 4e^{2x} + \dfrac{2}{e^x}$.

The point $(0, 4)$ is the minimum point of the graph of f.

(a) Find $f'(x)$.

[4]

(b) Find $f(x)$.

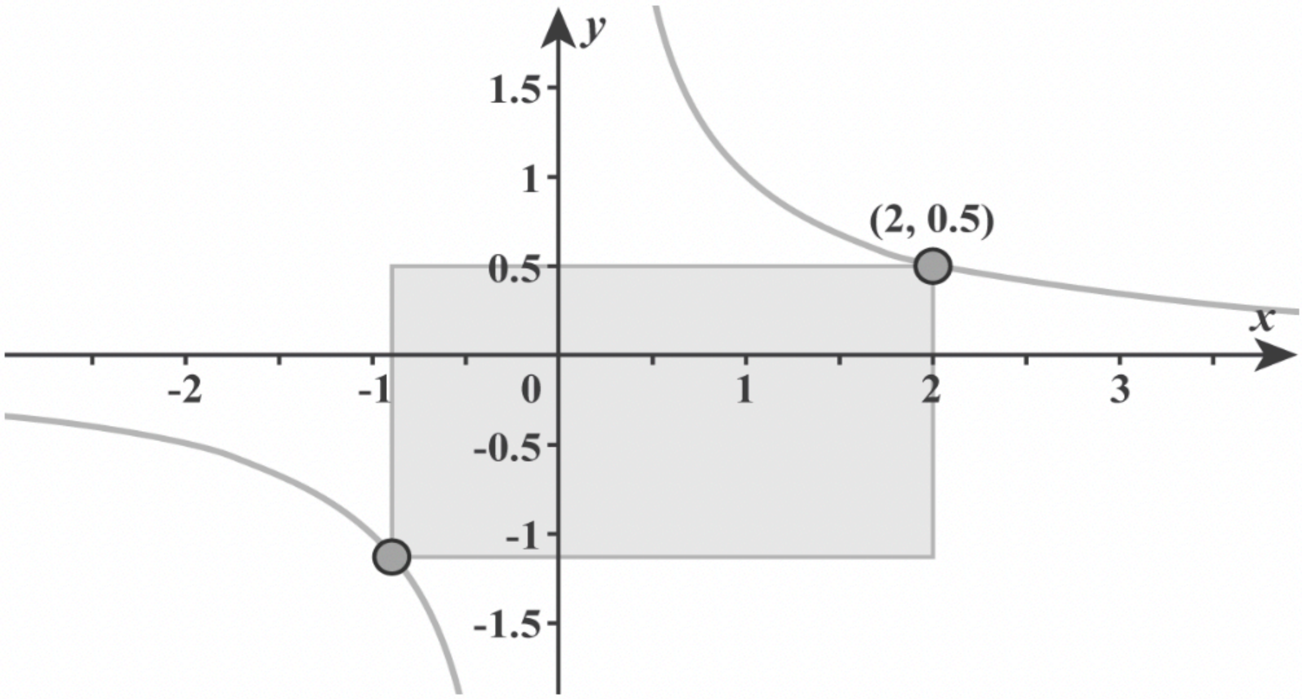
[4]

[Maximum mark: 8]

	Answer	Mark	Guidance
(a)	$f''(x) = 4e^{2x} + 2e^{-x}$ $f'(x) = \int 4e^{2x} + 2e^{-x}dx$ $= 4 \times \frac{1}{2}e^{2x} + 2 \times \frac{1}{-1}e^{-x} + c$ $= 2e^{2x} - 2e^{-x} + c$ $f'(0) = 2e^{2\times 0} - 2e^{-0} + c$ $0 = 2 \times 1 - 2 \times 1 + c$ $c = 0$ Hence, $f'(x) = 2e^{2x} - 2e^{-x}$	***M1*** ***A1*** ***M1*** ***A1***	You will use $\int e^{ax}dx = \frac{1}{a}e^{ax} + c$ several times. To find $f'(x)$ you must integrate $f''(x)$. To find the integration constant, use that $(0, 4)$ is a minimum point, so $f'(0) = 0$.
(b)	$f(x) = \int 2e^{2x} - 2e^{-x}dx$ $= 2 \times \frac{1}{2}e^{2x} - 2 \times \frac{1}{-1}e^{-x} + c$ $= e^{2x} + 2e^{-x} + c$ $f(0) = e^{2\times 0} + 2e^{-0} + c$ $4 = 1 + 2 \times 1 + c$ $c = 1$ Hence, $f(x) = e^{2x} + 2e^{-x} + 1 = e^{2x} + \frac{2}{e^x} + 1$	***M1*** ***A1*** ***M1*** ***A1***	To find $f(x)$ we integrate $f'(x)$. To find the integration constant, we use that $(0, 4)$ is a on the graph, so $f(0) = 4$.

Question

The diagram below shows part of the graph of $y = \dfrac{1}{x}$ along with the point $(2, 0.5)$ on the graph.



The diagram also shows a rectangle with sides parallel to the axes, one vertex is the given point and the opposite vertex on the branch of the graph in the third quadrant.

(a) Find the position of the vertex in the third quadrant that gives the rectangle with the smallest possible area.

[5]

(b) Calculate this smallest possible area.

[2]

(c) Suppose that a similar rectangle with opposite vertices $\left(a, \frac{1}{a}\right)$ and $\left(b, \frac{1}{b}\right)$ (where $a > 0$ and $b < 0$) has the same area. Find a relationship between a and b.

[4]

[Maximum mark: 11]

	Answer	Mark	Guidance
(a)	The horizontal side of the rectangle has length $w = 2 - x$. The vertical side of the rectangle has length $h = 0.5 - \frac{1}{x}$. $A(x) = wh$ $= (2 - x)\left(0.5 - \frac{1}{x}\right)$ $= 2 - 0.5x - \frac{2}{x}$ $A(x) = 2 - 0.5x - 2x^{-1}$ $A'(x) = -0.5 - 2 \times (-1)x^{-2} = -0.5 + \frac{2}{x^2}$ $0 = -0.5 + \frac{2}{x^2}$ $0.5 = \frac{2}{x^2}$ $x^2 = 4$ $x = \pm 2$ Since$x < 0$, this means that the position of the vertex when the area is minimal is $(-2, -0.5)$.	***M1*** ***A1*** ***M1*** ***A1*** ***A1***	Let the coordinates of the vertex in the third quadrant be $\left(x, \frac{1}{x}\right)$ for some $x < 0$. If the area is minimal, then $A'(x) = 0$.
(b)	For the vertex in part (**a**) the length of the sides of the rectangle are: $w = 2 - (-2) = 4$ and $h = 0.5 - (-0.5) = 1$, so the area is $4 \times 1 = 4$.	***M1*** ***A1***	

(c)	If $\left(a, \dfrac{1}{a}\right)$ and $\left(b, \dfrac{1}{b}\right)$ are the opposite vertices with $a > 0$ and $b < 0$ then the area is $(a-b)\left(\dfrac{1}{a} - \dfrac{1}{b}\right) = 2 - \dfrac{a}{b} - \dfrac{b}{a}.$ If this is equal to the area found in part (**b**), then $2 - \dfrac{a}{b} - \dfrac{b}{a} = 4$ $\dfrac{a}{b} + \dfrac{b}{a} = -2$ $\dfrac{a^2+b^2}{ab} = -2$	***M1*** ***A1*** ***M1*** ***A1***	
	$\dfrac{a^2+b^2}{ab} = -2$ $a^2 + b^2 = -2ab$ $a^2 + 2ab + b^2 = 0$ $(a+b)^2 = 0$ $a + b = 0$		

Paper: 1 Marks: 18

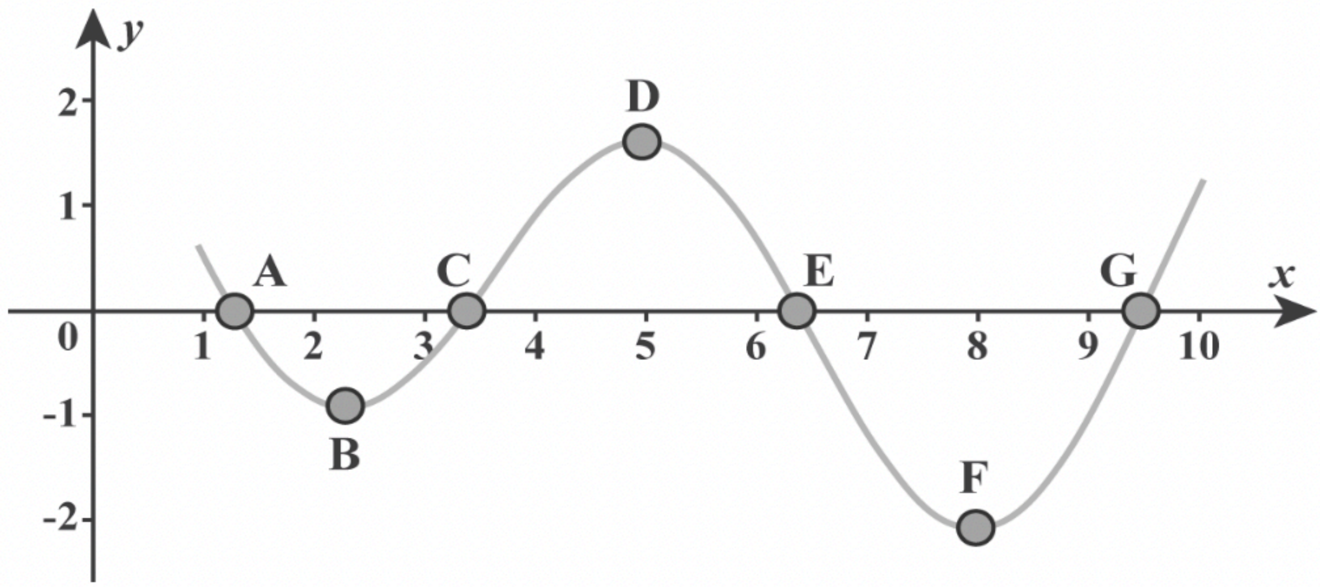
Question

Consider the function defined by $g(x) = \ln x \cos x$ for $1 \leq x \leq 10$.

(a) Find the x-intercepts of the graph of g.

[4]

The diagram below shows the graph of g', the derivative of g.



The local minimum, maximum points and the x-intercepts of the graph of g', are also indicated on the diagram. These points correspond to some points on the graph of g. (By 'corresponding point' we mean that they have the same x-coordinate.)

(b) State the points that correspond to local minimum points on the function $g(x)$.

[2]

(c) State the points that correspond to local maximum points on the function $g(x)$.

[2]

(d) State the points that correspond to points of inflexion on the function $g(x)$.

[2]

(e) Sketch the graph of g.

[4]

(f) Write down the number of x-intercepts the graph of g'' has.

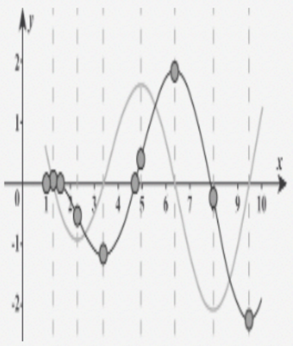
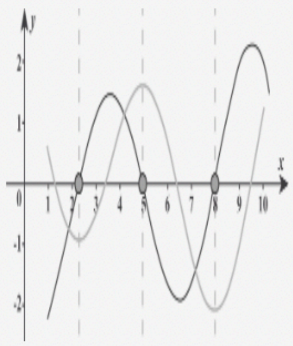
[1]

(g) Sketch the graph of g''.

[3]

[Maximum mark: 18]

	Answer	Mark	Guidance
(a)	Correct method for solving $\ln x \cos x = 0$	***M1***	Setting $\ln x = 0$ and $\cos x = 0$.
	$(1, 0)$	***A1***	If $\ln x = 0$, then $x = 1$.
	$\left(\frac{\pi}{2}, 0\right), \left(\frac{3\pi}{2}, 0\right)$ and $\left(\frac{5\pi}{2}, 0\right)$	***A1A1***	If $\cos x = 0$ and $1 \leq x \leq 10$, then $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$ Award (A1) for one or two correct x-intercepts and (A1) (A1) for all three correct x-intercepts for $\cos x = 0$.
(b)	Points C and G.	***A1A1***	At local minimum point, the graph of g is changing from decreasing to increasing, so the derivative is 0 and changing from negative to positive. Award (A1) for each correct point.
(c)	Points A and E.	***A1A1***	At local maximum point, the graph of g is changing from increasing to decreasing, so the derivative is 0 and changing from positive to negative. Award (A1) for each correct point.
(d)	Points B, D, and F.	***A1A1***	At a point of inflexion, the graph of g is changing concavity, so the second derivative is changing sign. This means that the first derivative is changing direction (from increasing to decreasing or from decreasing to increasing). Award (A1) for one or two correct points and (A1) (A1) for all three correct points.

(e)		***A1*** ***A1*** ***A1*** ***A1***	The diagram shows the graph of g (red) and g' (blue) together. The diagram indicates the intercepts identified in part (**a**) and the vertical lines show the places corresponding to the points identified in parts (**b**), (**c**) and (**d**). Correct x-intercepts. Correct position of local maximum/minimum points. Correct vertical position of the points of inflexion. Correct Shape.
(f)	There are three 3-intercepts of the graph of g''.	***A1***	The graph of g'' has an x-intercept, where the graph of g' has an extremum point. There are three such points.
(g)		***A1*** ***A1*** ***A1***	The diagram shows the graph of g'' (red) and g' (blue) together. The diagram indicates the intercepts mentioned in part (**f**) along with the vertical lines that help us find their place. Correct x-intercepts. Correct increasing/decreasing sections. Correct shape.

Paper: 1 Marks: 11

Question

Consider the function defined by $f(x) = \dfrac{\ln^2 x}{x}$ for $x > 0$.

(a) Find $f'(x)$.

[3]

(b) There is a local maximum and minimum point on the graph of f.

Find both of these points.

[5]

(c) Using the substitution $u = \ln x$ (or otherwise) find $\int f(x)\mathrm{d}x$.

[3]

[Maximum mark: 11]

Answer	Mark	Guidance
With $u(x) = \ln^2 x$ and $v(x) = x$, $f(x) = \frac{u(x)}{v(x)}$. $f'(x) = \frac{u'(x)v(x) - v'(x)u(x)}{v^2(x)}$ $= \frac{\frac{2\ln x}{x}x - \ln^2 x}{x^2} = \frac{2\ln x - \ln^2 x}{x^2}$	*M1* *A1A1*	We use the quotient rule to find the derivative. For $u(x) = \ln^2 x = (\ln x)^2$ the chain rule gives $u'(x) = 2\ln x \times \frac{1}{x} = \frac{2\ln x}{x}$. $v'(x) = 1$
$\frac{2\ln x - \ln^2 x}{x^2} = 0$ $\ln x(2 - \ln x) = 0$ The two solutions of this equation are: $f(1) = \frac{\ln^2 1}{1} = \frac{0^2}{1} = 0,$ Therefore, $(1, 0)$. $f(e^2) = \frac{\ln^2 e^2}{e^2} = \frac{(\ln e^2)^2}{e^2} = \frac{2^2}{e^2} = \frac{4}{e^2},$ Therefore, $\left(e^2, \frac{4}{e^2}\right)$.	*M1A1* *A1* *M1* *A1*	To find the x-coordinates of the extremum points you solve $f'(x) = 0$. $\ln x = 0$, which implies $x = 1$. For all other $x > 0$ values $f(x) > 0$ (since the numerator is a square, hence positive), so this is the local minimum point. $2 - \ln x = 0$, which implies $\ln x = 2$ and hence $x = e^2$. Since you have already found the local minimum point, this is the local maximum point.

For $u = \ln x$, $\dfrac{du}{dx} = \dfrac{1}{x}$. Hence, the integral is $\begin{aligned}\int f(x)dx &= \int \frac{\ln^2 x}{x}dx \\ &= \int \ln^2 x \times \frac{1}{x}dx \\ &= \int u^2 du \\ &= \frac{u^3}{3} + c \\ &= \frac{\ln^3 x}{3} + c\end{aligned}$	***M1*** ***A1*** ***A1***	This means that in the integral you can replace $\dfrac{1}{x}dx$ with du.

Paper: 1 Marks: 7

Question

Consider $f(x) = x\mathrm{e}^{2x}$

(a) Find $f'(x)$, $f''(x)$ and $f^{(3)}(x)$.

[4]

(b) Write down an expression for $f^{(n)}(x)$.

[3]

[Maximum mark: 7]

	Answer	Mark	Guidance
(a)	Use of the product rule: $f'(x) = e^{2x} + 2xe^{2x} = e^{2x}(2x+1)$ $f''(x) = 2e^{2x} + 2(2x+1)e^{2x} = e^{2x}(4x+4)$ $f^{(3)}(x) = 4e^{2x} + 2(4x+4)e^{2x} = e^{2x}(8x+12)$	***M1*** ***A1A1A1***	
(b)	These linear expressions can be rewritten as: $2x+1 = 1(2x+1)$ $4x+4 = 2(2x+2)$ $8x+12 = 4(2x+3)$ Recognising the pattern, you get: $f^{(n)}(x) = 2^{n-1}e^{2x}(2x+n)$	***M1*** ***A1A1***	In all three derivatives from part **(a)** we have e^{2x} multiplied by a linear expression in x. Looking for pattern in the form $e^{2x}(ax+b)$. Award *A1* for 2^{n-1} and *A1* for $(2x+n)$.

Paper: 1 Marks: 6

Question

Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

[Maximum mark: 6]

Answer	Mark	Guidance
$x^2 = 2x - x^2$ or $2x^2 - 2x = 0$	*M1*	For finding the points of intersection of the parabolas by solving their equations simultaneously.
$2x(x-1) = 0$	*A1*	
$x = 1$ or $x = 0$	*A1*	
$A = \int_0^1 (2x - x^2 - x^2)\mathrm{d}x = 2\int_0^1 (x - x^2)\mathrm{d}x$	*M1*	The top boundary is $y = 2x - x^2$ and the bottom boundary is $y = x^2$.
$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$	*M1*	Use of the area between two curves formula.
$= 2\left(\frac{1}{2} - \frac{1}{3}\right)$	*A1*	
$= \frac{1}{3}$		

Paper: 1 Marks: 6

Question

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6 \text{ m s}^{-1}$.

(a) Find the displacement of the particle during the time period $1 \leq t \leq 4$ s.

[3]

(b) Find the distance travelled during this time period.

[3]

[Maximum mark: 6]

Answer	Mark	Guidance
The displacement is		
$s(4) - s(1) = \int_1^4 v(t)\,\mathrm{d}t$	M1	Use of integration.
$= \int_1^4 t^2 - t - 6\mathrm{d}t$	M1	Correct work finding the definite integral.
$= \left[\frac{t^3}{3} - \frac{t^2}{2} - 6t\right]_1^4$	A1	
$= -\frac{9}{2}$		
The distance travelled is	M1	Note that $v(t) = t^2 - t - 6 = (t-3)(t+2)$ and so $v(t) \leq 0$ on the interval $[1, 3]$ and $v(t) \geq 0$ on $[3, 4]$.
$\int_1^4 \lvert v(t)\rvert \mathrm{d}t = \int_1^3 \lvert -v(t)\rvert \mathrm{d}t + \int_3^4 v(t)\,\mathrm{d}t$	M1	Use of integration.
$= \int_1^3 \left(-t^2 + t + 6\right)\mathrm{d}t + \int_3^4 \left(t^2 - t - 6\right)\mathrm{d}t$	A1	
$= \left[-\frac{t^3}{3} + \frac{t^2}{2} + 6t\right]_1^3 + \left[\frac{t^3}{3} - \frac{t^2}{2} - 6t\right]_3^4$		Correct work finding the definite integral.
$= \frac{61}{6}$ m		

Paper: 1 Marks: 6

Question

A particle moves in a straight line and has acceleration given by $a(t) = 6t + 4$

Its initial velocity is $v(0) = -6$ cm s^{-1} and its initial displacement is $s(0) = 9$ cm.

Find its position function, $s(t)$.

[Maximum mark: 6]

Answer	Mark	Guidance
$v'(t) = a(t) = 6t + 4$		Use of antidifferentiation.
Therefore,	*M1*	
$v(t) = \int a(t) = \int 6t + 4\ dt$	*A1*	Use of antidifferentiation again.
$v(t) = 6\frac{t^2}{2} + 4t + C = 3t^2 + 4t + C$		
Since $v(0) = C$ and we are given that $v(0) = -6$	*A1*	
so $C = -6$, which gives us the velocity function		
$v(t) = 3t^2 + 4t - 6$		
$s'(t) = v(t) = 3t^2 + 4t - 6$	*M1*	
Therefore,	*A1*	
$s(t) = \int v(t) = \int 3t^2 + 4t - 6\ dt$		
$s(t) = 3\frac{t^3}{3} + 4\frac{t^2}{2} - 6t + D = t^3 + 2t^2 - 6t + D$	*A1*	
Since $s(0) = D$ and we are given that $s(0) = 9$		
so $D = 9$, which gives us the position function		
$s(t) = t^3 + 2t^2 - 6t + 9$		

Paper: 1 Marks: 8

Question

Find f given that $f''(x) = 12x^2 + 6x - 4$, $f(0) = 4$ and $f(1) = 1$.

[Maximum mark: 8]

Answer	Mark	Guidance
$f'(x) = \int 12x^2 + 6x - 4\, dx$ $= 12\frac{x^3}{3} + 6\frac{x^2}{2} - 4x + C$ $= 4x^3 + 3x^2 - 4x + C$ $f(x) = \int 4x^3 + 3x^2 - 4x + C\, dx$ $= 4\frac{x^4}{4} + 3\frac{x^3}{3} - 4\frac{x^2}{2} + Cx + D$ $= x^4 + x^3 - 2x^2 + Cx + D$	*M1* *A1* *M1* *A1A1* *A1* *A1*	Attempting to find $f'(x)$ by using integration. Attempting to find $f(x)$ by using integration once again. A1 for $x^4 + x^3 - 2x^2$ A1 for $(Cx + D)$ A1 for finding D.
Using $f(0) = 4$ and $f(1) = 1$: $f(0) = 0 + D = 4$ $D = 4$ $f(1) = 1 + 1 - 2 + C + 4 = 1$ $C = -3$ $f(x) = x^4 + x^3 - 2x^2 - 3x + 4$	*A1*	A1 for finding C. A1 for the final $f(x)$.

Paper: 1 Marks: 7

Question

Find the coordinates of the points on the curve $y = x^4 - 6x^2 + 4$ where the tangent is horizontal.

[Maximum mark: 7]

Answer	Mark	Guidance
$\frac{dy}{dx} = \frac{d}{dx}(x^4 - 6x^2 + 4) = 4x^3 - 12x$ $4x^3 - 12x = 4x(x^2 - 3)$ Thus, $\frac{dy}{dx} = 0$ if $x = 0$ or $x^2 - 3 = 0$, that is $x = \sqrt{3}$ or $x = -\sqrt{3}$ So the given curve has horizontal tangents when $x = 0$, $x = \sqrt{3}$ and $x = -\sqrt{3}$. The corresponding points are $(0, 4)$, $(\sqrt{3}, -5)$and $(-\sqrt{3}, -5)$	*A1A1* *M1* *A1* *A1A1* *A1*	For each correct term of the derivative. Horizontal tangents occur when the derivative is zero. Award one mark for the three correct y-coordinates.

Paper: 2 Marks: 10

Question

Consider the function defined by $f(x) = x^2 + 2x + 1$.

(a) Find the equation of the tangent at $x = 3$.

[7]

(b) Find the equation of the normal at $x = 3$.

[3]

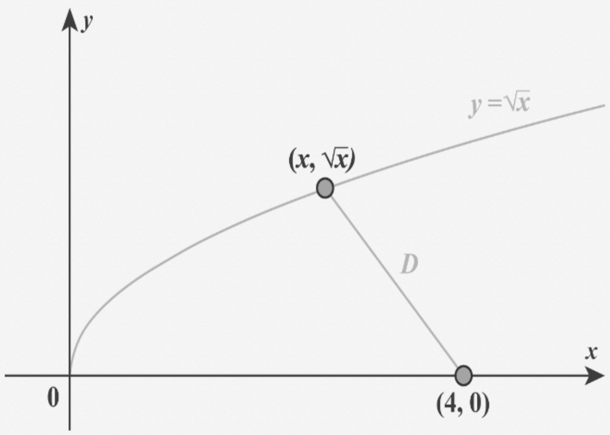
[Maximum mark: 10]

	Answer	Mark	Guidance
(a)	$f(3) = 3^2 + 2(3) + 1$ $f(3) = 9 + 6 + 1$ $f(3) = 16$ Therefore the coordinates are $(3, 16)$. $f'(x) = 2x + 2$ $f'(3) = 2(3) + 2$ $f'(3) = 6 + 2$ $f'(3) = 8$ So the gradient at $x = 3$ is $m = 8$ $16 = 8(3) + c$ $c = -8$ $y = 8x - 8$	*M1* *A1* *A1* *M1* *A1* *M1* *A1*	First, find the coordinates of the point by finding $f(3)$. Second, find the gradient of the curve at the point $(3, 16)$. Award one mark for the correct $f'(x)$. Award a method mark for substituting $x = 3$ into $f'(x)$. Third, find the equation of the tangent line at the point $(3, 16)$. One possible method is to substitute $(3, 16)$ and $m = 8$ into the gradient-intercept form of a line and solving for c.
(b)	If the gradient of the tanget at $x = 3$ is 8, then the gradient of the normal at $x = 3$ is $-\frac{1}{8}$. $16 = -\frac{1}{8}(3) + c$ $c = \frac{131}{8}$ Therefore, $y = -\frac{1}{8}x + \frac{131}{8}$	*A1* *M1* *A1*	First, find the gradient of the normal line at $x = 3$. Second, find the equation of the normal at $x = 3$ by substituting the point $(3, 16)$and the normal gradient of $m = -\frac{1}{8}$ into the gradient-intercept form of a line and solving for c.

Question

Find the exact coordinates of the point on the curve $y = \sqrt{x}$ that is closest to the point $(4,\ 0)$.

[Maximum mark: 8]

Answer	Mark	Guidance
	***A1*** ***M1A1*** ***A1*** ***A1*** ***M1***	Start with a sketch and represent the information.
$D^2 = (x-4)^2 + (\sqrt{x})^2$ or $D^2 = (4-x)^2 + (\sqrt{x})^2$ $D^2 = x^2 - 8x + 16 + x$ $D = (x^2 - 7x + 16)^{\frac{1}{2}}$ **Method 1** (calculus) $f'(x) = 2x - 7$ Let $f'(x) = 2x - 7 = 0$ Hence $(2x-7) = 0 \Rightarrow x = \dfrac{7}{2}$ Thus the y-coordinate becomes $y = \sqrt{\dfrac{7}{2}}$.	***A1A1*** ***M1A1*** ***A1A1***	For an accurate sketch. Use of the distance formula. Correct expression. To find the minimum of D is the same as finding the minimum of D^2. Now you can differentiate $D^2 = f(x)$. Correct derivative. Setting the derivative equal to zero.
Thus the coordinates on $y = \sqrt{x}$ that is $\left(\dfrac{7}{2}, \sqrt{\dfrac{7}{2}}\right)$. **Method 2** (vertex of a quadratic) $x = \dfrac{-b}{2a} = \dfrac{-(-7)}{2 \times 1} = \dfrac{7}{2}$ Since $y = \sqrt{x}$ $y = \sqrt{\dfrac{7}{2}}$ Thus the coordinates are $\left(\dfrac{7}{2}, \sqrt{\dfrac{7}{2}}\right)$.		As in the previous method, we find the value of x for which $D^2 = x^2 - 7x + 16$ is minimal. This value is the first coordinate of the vertex of the parabola $y = x^2 - 7x + 16$. Use of the formula $x = \dfrac{-b}{2a}$, with correct b and a.

Paper: 1 Marks: 6

Question

The graph of of a function f passes through the point $\left(\frac{\pi}{6}, 5\right)$.

Given that $f'(x) = -4\sin(2x)$, find $f(x)$.

[Maximum mark: 6]

Answer	Mark	Guidance
$f(x) = \int f'(x)\mathrm{d}x$	*M1*	The function of $f(x)$ is the antiderivative of $f'(x)$.
$\int f'(x)\mathrm{d}x = \int -4\sin(2x)\,\mathrm{d}x$	*A1*	The graph of f passes through the point $\left(\frac{\pi}{6}, 5\right)$.
$= \frac{4\cos(2x)}{2} + c$	*M1*	
$= 2\cos(2x) + c$	*M1*	Correct substitution.
$f\left(\frac{\pi}{6}\right) = 5$	*A1*	Correct simplification.
$5 = 2\cos(2 \times \frac{\pi}{6}) + c$	*A1*	
$5 = 2\cos\frac{\pi}{3} + c$		Correct value for c
$5 = 2 \cdot \frac{1}{2} + c$		
$5 = 1 + c$		
$c = 4$		
$f(x) = 2\cos(2x) + 4$		

Question

Let $f(x) = \dfrac{2x}{x^2 - 3}$.

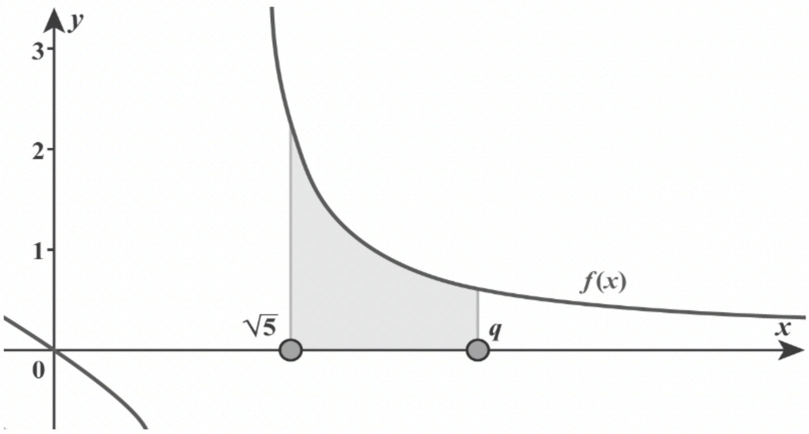
(a) Show that $f'(x) = -\dfrac{2x^2 + 6}{(x^2 - 3)^2}$

[4]

(b) Find $\displaystyle\int \frac{2x}{x^2 - 3} dx$

[4]

The following diagram shows part of the graph of f .



(c) The shaded region is enclosed by the graph of f, the x-axis, and the lines $x = \sqrt{5}$ and $x = q$. The region has an area of $\ln 5$. Find the value of q.

[6]

[Maximum mark: 14]

	Answer	Mark	Guidance
(a)	$f(x)' = \dfrac{(2x)'(x^2-3)-(2x)(x^2-3)'}{(x^2-3)^2}$ $= \dfrac{2(x^2-3)-(2x)(2x)}{(x^2-3)^2}$ $= \dfrac{2x^2-6-4x^2}{(x^2-3)^2}$ $= \dfrac{-2x^2-6}{(x^2-3)^2}$ $= -\dfrac{2x^2+6}{(x^2-3)^2}$	***M1*** ***A1*** ***A1*** ***A1***	Use of the quotient rule. Correct numerator. Correct Denominator. Correctly simplifying.
(b)	Let $u = x^2 - 3$ and $\dfrac{\mathrm{d}u}{\mathrm{d}x} = 2x$ $\displaystyle\int \frac{2x}{x^2-3}\mathrm{d}x = \int \frac{1}{x^2-3}\cdot 2x\mathrm{d}x$ $\displaystyle = \int \frac{1}{u}\frac{\mathrm{d}u}{\mathrm{d}x}\mathrm{d}x$ $\displaystyle = \int \frac{1}{u}\mathrm{d}u$ $= \ln\lvert u\rvert + c$ $= \ln\lvert x^2-3\rvert + c$	***M1*** ***M1*** ***A1*** ***A1***	Use of substitution. Correct substitution.

(c)	$\int_{\sqrt{5}}^{q} \frac{2x}{x^2-3}\mathrm{d}x = \left[\ln(x^2-3)\right]_{\sqrt{5}}^{q}$ $= \ln(q^2-3) - \ln((\sqrt{5})^2-3)$ $= \ln(q^2-3) - \ln(5-3)$ $= \ln(q^2-3) - \ln 2$ $= \ln \frac{q^2-3}{2}$	***M1*** ***M1*** ***A1*** ***M1*** ***M1*** ***A1***	The area of the shaded region. Use the answer from (**b**). $\ln \frac{q^2-3}{2} = \ln 5$ Dropping the like base logs from both sides of the equations.
	$\frac{q^2-3}{2} = 5$ $q^2 - 3 = 10$ $q^2 = 13$ $q = \sqrt{13}$		Remember q is greater than 0.