A.1 Kinematics SL

What you need to understand

- that the motion of bodies through space and time can be described and analysed in terms of position, velocity, and acceleration
- velocity is the rate of change of position, and acceleration is the rate of change of velocity
- the change in position is the displacement

A.1 Kinematics SL

What you need to understand

- the difference between distance and displacement
- the difference between instantaneous and average values of velocity, speed and acceleration, and how to determine them

A.1 Kinematics SL

What you need to understand

- the equations of motion for solving problems with uniformly accelerated motion as given by

$$s = \frac{u+v}{2}t$$
$$v = u + at$$
$$s = ut + \frac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$

A.1 Kinematics SL

What you need to understand

- motion with uniform and non-uniform acceleration
- the behaviour of projectiles in the absence of fluid resistance, and the application of the equations of motion resolved into vertical and horizontal components
- the qualitative effect of fluid resistance on projectiles, including time of flight, trajectory, velocity, acceleration, range and terminal speed.

A.1 Kinematics SL

Distance and displacement

·**Mechanics** is the branch of physics which concerns itself with forces, and how they affect a body's motion.

·**Kinematics** is the sub-branch of mechanics which studies only a body's motion without regard to causes.

·**Dynamics** is the sub-branch of mechanics which studies the forces which *cause* a body's motion.

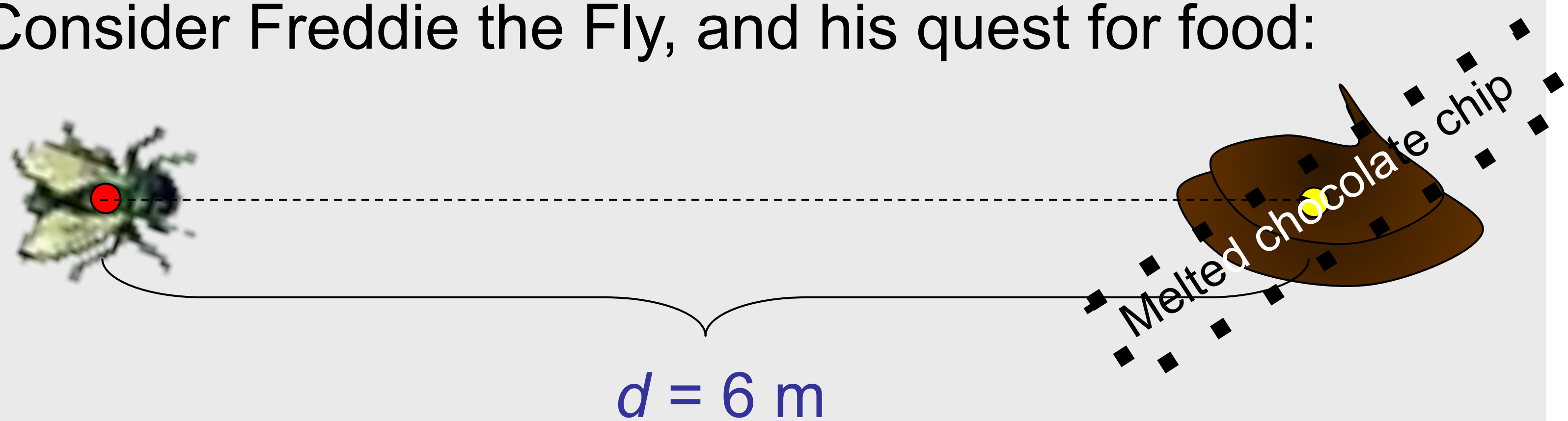
A.1 Kinematics SL

Distance and displacement

·Kinematics is the study of **displacement**, **velocity** and **acceleration**, or in short, a study of motion.

·A study of motion begins with **position** and change in **position**.

·Consider Freddie the Fly, and his quest for food:



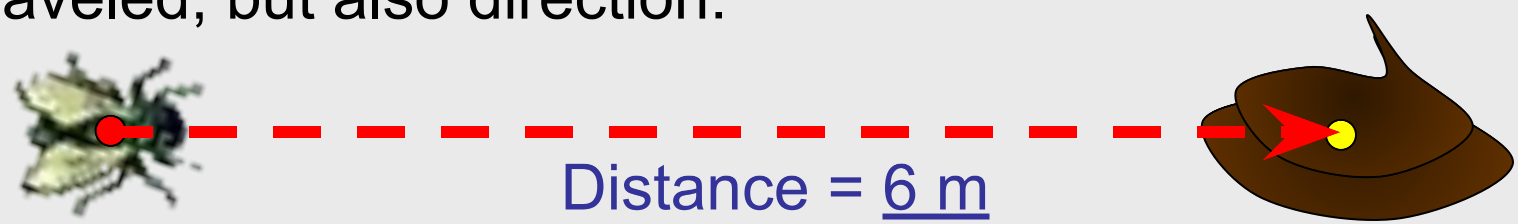
·The distance Freddie travels is simply how far he has flown, without regard to direction. Freddie's distance is 6 meters.

A.1 Kinematics SL

Distance and displacement

·**Distance** is simply how far something has traveled without regard to direction. Freddy has gone 6 m.

·**Displacement**, on the other hand, is not only distance traveled, but also direction.



Distance = 6 m

Displacement = 6 m

in the positive x-direction

·This makes displacement a vector. It has a *magnitude* (6 m) and a *direction* (+ x-direction).

·We say Freddie travels through a displacement of 6 m in the positive x-direction.

A.1 Kinematics SL

Distance and displacement

·Let's revisit some previous examples of a ball moving through some displacements…

Displacement A

Displacement B

·Displacement A is just 15 m to the right (or +15 m for short).

Vector

·Displacement B is just 20 m to the left (or -20 m for short).

FYI

Scalar

·Distance A is 15 m, and Distance B is 20 m. There is no regard for direction in distance.

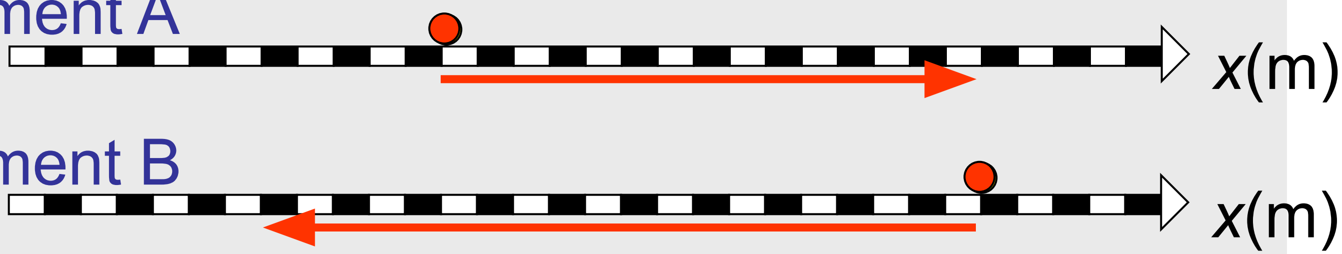
A.1 Kinematics SL

Distance and displacement

·Now for some detailed analysis of these two motions…

Displacement A

Displacement B



·Displacement Δx (or s) has the following formulas:

displacement

$$\Delta x = x_2 - x_1$$

$$s = x_2 - x_1$$

where x_2 is the final position and x_1 is the initial position

FYI

·Many textbooks use Δx for displacement, and IB uses s. Don't confuse the "change in Δ" with the "uncertainty Δ" symbol. And don't confuse s with seconds!

A.1 Kinematics SL

Distance and displacement

$$\Delta x = x_2 - x_1$$
$$s = x_2 - x_1$$

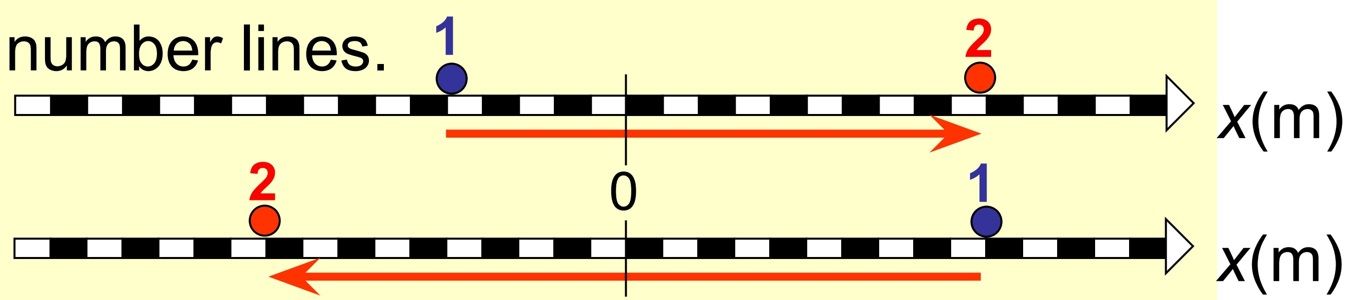
displacement

where x_2 is the final position and x_1 is the initial position

EXAMPLE: Use the displacement formula to find each displacement. Note that the x = 0 coordinate has been placed on the number lines.

Displacement A

Displacement B



SOLUTION:

- For A: s = (+10) – (-5) = +15 m.
- For B: s = (-10) – (+10) = -20 m.

FYI

- The correct direction (sign) is automatic!

A.1 Kinematics SL

Speed and velocity

·**Velocity** v is a measure of how fast an object moves through a displacement.

·Thus, velocity is displacement divided by time, and is measured in meters per second ($m\ s^{-1}$).

$$v = \Delta x / \Delta t$$

$$v = s / t$$

velocity

EXAMPLE: Find the velocity of the second ball (Ball B) if it takes 4 seconds to complete its displacement.

SOLUTION:

- For B: $s = (-10) - (+10) = -20$ m.
- But $t = 4$ s. Therefore $v = -20\ m / 4\ s = -5\ m\ s^{-1}$.
- Note that v "inherits" its direction from s.

A.1 Kinematics SL

Speed and velocity

·From the previous example we calculated the velocity of the ball to be -5 m s^{-1}.

·Thus, the ball is moving 5 m s^{-1} to the left.

·With disregard to the direction, we can say that the ball's speed is 5 m s^{-1}.

·We define **speed** as distance divided by time, with disregard to direction.

PRACTICE: A runner travels 64.5 meters in the negative x-direction in 31.75 seconds. Find her velocity, and her speed.

SOLUTION: ·Her velocity is -64.5 / 31.75 = -2.03 m s^{-1}.

·Her speed is 64.5 / 31.75 = +2.03 m s^{-1}.

A.1 Kinematics SL

Acceleration

·**Acceleration** is the change in velocity over time.

acceleration

$$a = \Delta v / \Delta t$$

$$a = (v - u) / t$$

where v *is the final velocity and* u *is the initial velocity*

·Since u and v are measured in m/s and since t is measured in s, a is measured in m/s^2, or in IB format, a is measured in m s^{-2}.

FYI

·Many textbooks use $\Delta v = v_f - v_i$ for change in velocity, v_f for final velocity and v_i initial velocity. IB gets away from the subscripting mess by choosing v for final velocity and u for initial velocity.

A.1 Kinematics SL

Acceleration

acceleration

$a = \Delta v / \Delta t$

$a = (v - u) / t$

where v *is the final velocity and* u *is the initial velocity*

EXAMPLE: A driver sees his speed is 5.0 m s^{-1}. He then simultaneously accelerates and starts a stopwatch. At the end of 10. s he observes his speed to be 35 m s^{-1}. What is his acceleration?

SOLUTION: Label each number with a letter:

- v = 35 m s^{-1}, u = 5.0 m s^{-1}, and t = 10. s.
- Next, choose the formula: $a = (v - u) / t$.
- Now substitute and calculate:
- a = (35 – 5) / 10 = 3.0 m s^{-2}.

A.1 Kinematics SL

Acceleration

acceleration

$a = \Delta v / \Delta t$

$a = (v - u) / t$ *where* v *is the final velocity and* u *is the initial velocity*

PRACTICE:

(a) Why is velocity a vector?

(b) Why is acceleration a vector?

SOLUTION:

(a) Velocity is a displacement over time. Since displacement is a vector, so is velocity.

(b) Acceleration is a change in velocity over time. Since velocity is a vector, so is acceleration.

A.1 Kinematics SL

Solving problems using equations of motion for uniform acceleration

·Back in the 1950s, military aeronautical engineers thought that humans could not withstand much of an acceleration, and therefore put little effort into pilot safety belts and ejection seats.

·An Air Force physician by the name of Colonel Stapp, however, thought humans could withstand higher accelerations.

·He designed a rocket sled to accelerate at up to 40*g* (at which acceleration you would feel like you weighed 40 times your normal weight!).

A.1 Kinematics SL

Solving problems using equations of motion for uniform acceleration

·The human to be tested would be Stapp himself.

·An accelerometer and a video camera were attached to the sled. Here are the results:

A.1 Kinematics SL

Solving problems using equations of motion for uniform acceleration

·Here are the data.

·In 1954, America's original Rocketman, Col. John Paul Stapp, attained a then-world record land speed of **1000 kmh^{-1}**, going from a standstill to a speed faster than a .45 bullet in **5.0 seconds** on an especially-designed rocket sled, and then screeched to a dead stop in **1.4 seconds**, sustaining more than 40g's of force, all in the interest of safety.

·There are TWO accelerations in this problem:

(a) He speeds up from 0 to 1000 kmh^{-1} in 5.0 s.

(b) He slows down from 1000 kmh^{-1} to 0 in 1.4 s.

A.1 Kinematics SL

Solving problems using equations of motion for uniform acceleration

·There are TWO accelerations in this problem:

(a) He speeds up from 0 to 1000 kmh^{-1} in 5.0 s.

(b) He slows down from 1000 kmh^{-1} to 0 in 1.4 s.

EXAMPLE: Was Stapp more uncomfortable while he was speeding up, or while he was slowing down?

SOLUTION: While slowing down. Why?

Convert from kmh^{-1} to ms-1

$$1000\ kmh^{-1} * \frac{1000\ m}{3600\ s} = 280\ (277)\ ms^{-1}$$

A.1 Kinematics SL

Solving problems using equations of motion for uniform acceleration

·There are TWO accelerations in this problem:

(a) He speeds up from 0 to 280 ms^{-1} in 5.0 s.

(b) He slows down from 280 ms^{-1} to 0 in 1.4 s.

EXAMPLE: Find Stapp's acceleration during the speeding up phase.

SOLUTION:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{280\text{ m/s} - \mathbf{0\text{ m/s}}}{5\text{ s}} = 60\text{ m/s}^2$$

EXAMPLE: Find Stapp's acceleration during the slowing down phase.

$$\bar{a} = \frac{v - u}{t} = \frac{0\text{ m/s} - 280\text{ m/s}}{1.4\text{ s}} = {}^{-}200\text{ m s}^{-2}$$

A.1 Kinematics SL

Determining instantaneous and average values for velocity, speed and acceleration

·Consider a car whose position is changing.

·A police officer is checking its speed with a radar gun as shown.

·The radar gun measures the position of the car during each successive snapshot, shown in yellow.

·How can you tell that the car is speeding up?

·What are you assuming about the radar gun time?

A.1 Kinematics SL

Determining instantaneous and average values for velocity, speed and acceleration

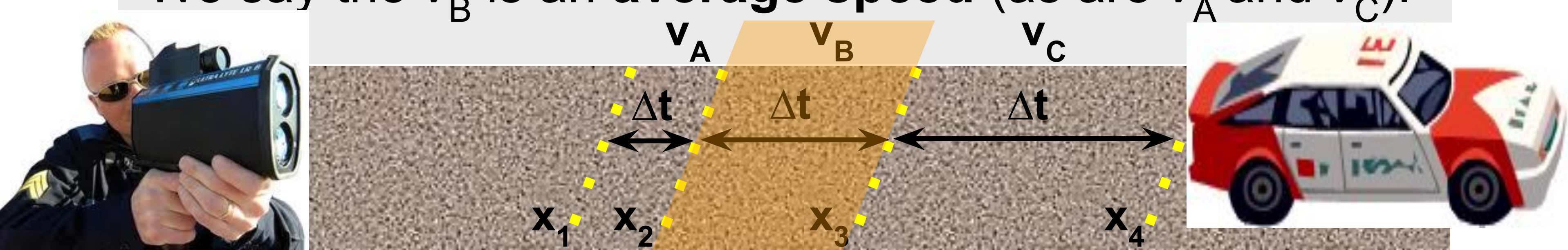
·We can label each position with an x and the time interval between each x with a Δt.

·Then $v_A = (x_2 - x_1)/\Delta t$, $v_B = (x_3 - x_2)/\Delta t$, and finally $v_C = (x_4 - x_3)/\Delta t$.

·Focus on the interval from x_2 to x_3.

·Note that the speed changed from x_2 to x_3, and so v_B is NOT really the speed for that whole interval.

·We say the v_B is an **average speed** (as are v_A and v_C).



A.1 Kinematics SL

Determining instantaneous and average values for velocity, speed and acceleration

·If we increase the sample rate of the radar gun (make the Δt smaller) the positions will get closer together.

·Thus the velocity calculation is more exact.

·We call the limit as Δt approaches zero in the equation $v = \Delta x / \Delta t$ the **instantaneous velocity**.

·For this level of physics we will just be content with the average velocity. Limits are beyond the scope of this course. You can use the Wiki extensions to explore limits and derivatives, if interested.

A.1 Kinematics SL

Determining instantaneous and average values for velocity, speed and acceleration

·By the same reasoning, if Δt gets smaller in the acceleration equation, our acceleration calculation becomes more precise.

·We call the limit as Δt approaches zero of the equation $a = \Delta v / \Delta t$ the **instantaneous acceleration**.

·For this level of physics we will be content with the average acceleration. See the Wiki for extensions if you are interested!

A.1 Kinematics SL

Equations of motion for uniform acceleration

·The equations for uniformly accelerated motion are also known as the **kinematic equations**. They are listed here

$s = ut + (1/2)at^2$	*Displacement*
$v = u + at$	*Velocity*
$v^2 = u^2 + 2as$	*Timeless*
$s = (u + v)t / 2$	*Average displacement*

·They can only be used if the acceleration *a* is CONSTANT (uniform).

·They are used so commonly throughout the physics course that we will name them.

A.1 Kinematics SL

Equations of motion for uniform acceleration

·From $a = (v - u)/t$ we get

$$at = v - u.$$

·Rearrangement leads to $v = u + at$, the <u>velocity equation</u>.

·Now, if it is the case that the acceleration is constant, then the average velocity can be found by taking the sum of the initial and final velocities and dividing by 2 (just like test grades). Thus

$$\textit{average velocity} = (u + v) / 2.$$

·But the displacement is the average velocity times the time, so that $s = (u + v)t / 2$, the <u>average displacement equation</u>.

A.1 Kinematics SL

Equations of motion for uniform acceleration

·We have derived $v = u + at$ and $s = (u + v)t / 2$.

·Let's tackle the first of the two harder ones.

$s = (u + v)t / 2$	*Given*
$s = (u + u + at)t / 2$	$v = u + at$
$s = (2u + at)t / 2$	*Like terms*
$s = 2ut/2 + at^2 / 2$	*Distribute t/2*
$s = ut + (1/2)at^2$	*Cancel 2*

·which is the <u>displacement equation</u>.

·Since the equation $s = (u + v)t/2$ only works if the acceleration is constant, $s = ut + (1/2)at^2$ also <u>works only if the acceleration is constant</u>.

A.1 Kinematics SL

Equations of motion for uniform acceleration

·We now have derived $v = u + at$, $s = (u + v)t / 2$ and $s = ut + (1/2)at^2$. Let's tackle the timeless equation.

·From $v = u + at$ we can isolate the t.

$$v - u = at$$

$$t = (v - u)/a$$

·From $s = (u + v)t / 2$ we get:

$$2s = (u + v)t$$

$$2s = (u + v)(v - u) / a$$

$$2as = (u + v)(v - u)$$

$$2as = \cancel{uv} - u^2 + v^2 - \cancel{vu}$$

$$v^2 = u^2 + 2as$$

Multiply by 2

$t = (v - u)/a$

Multiply by a

F O I L

Cancel (uv = vu)

A.1 Kinematics SL

Equations of motion for uniform acceleration

·Just in case you haven't written these down, here they are again.

$s = ut + (1/2)at^2$	*Displacement*	kinematic equations
$v = u + at$	*Velocity*	
$v^2 = u^2 + 2as$	*Timeless*	a is constant
$s = (u + v)t/2$	*Average displacement*	

·We will practice using these equations soon. They are extremely important.

·Before we do, though, we want to talk about freefall and its special acceleration g.

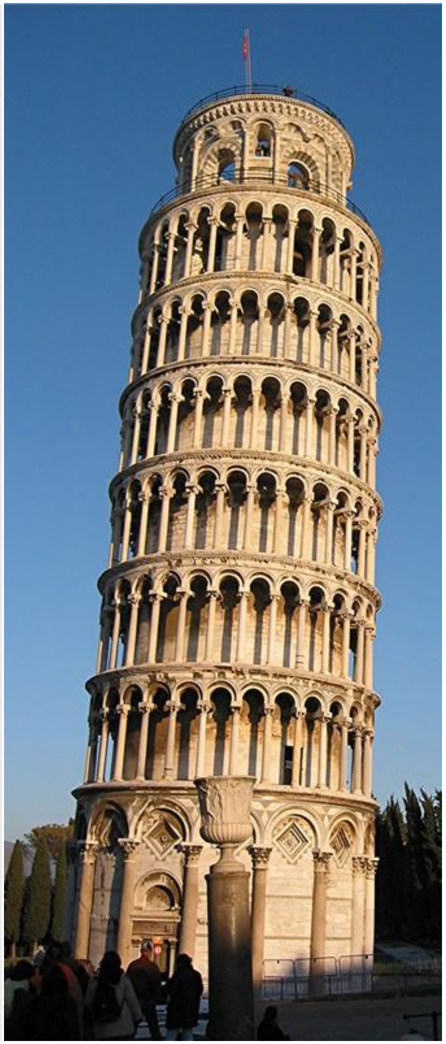
A.1 Kinematics SL

Determining the acceleration of free-fall experimentally

·Everyone knows that when you drop an object, it picks up speed when it falls.

·Galileo did his famous freefall experiments on the tower of Pisa long ago, and determined that all objects fall at the same acceleration in the absence of air resistance.

·Thus, as the next slide will show, an apple and a feather will fall side by side!

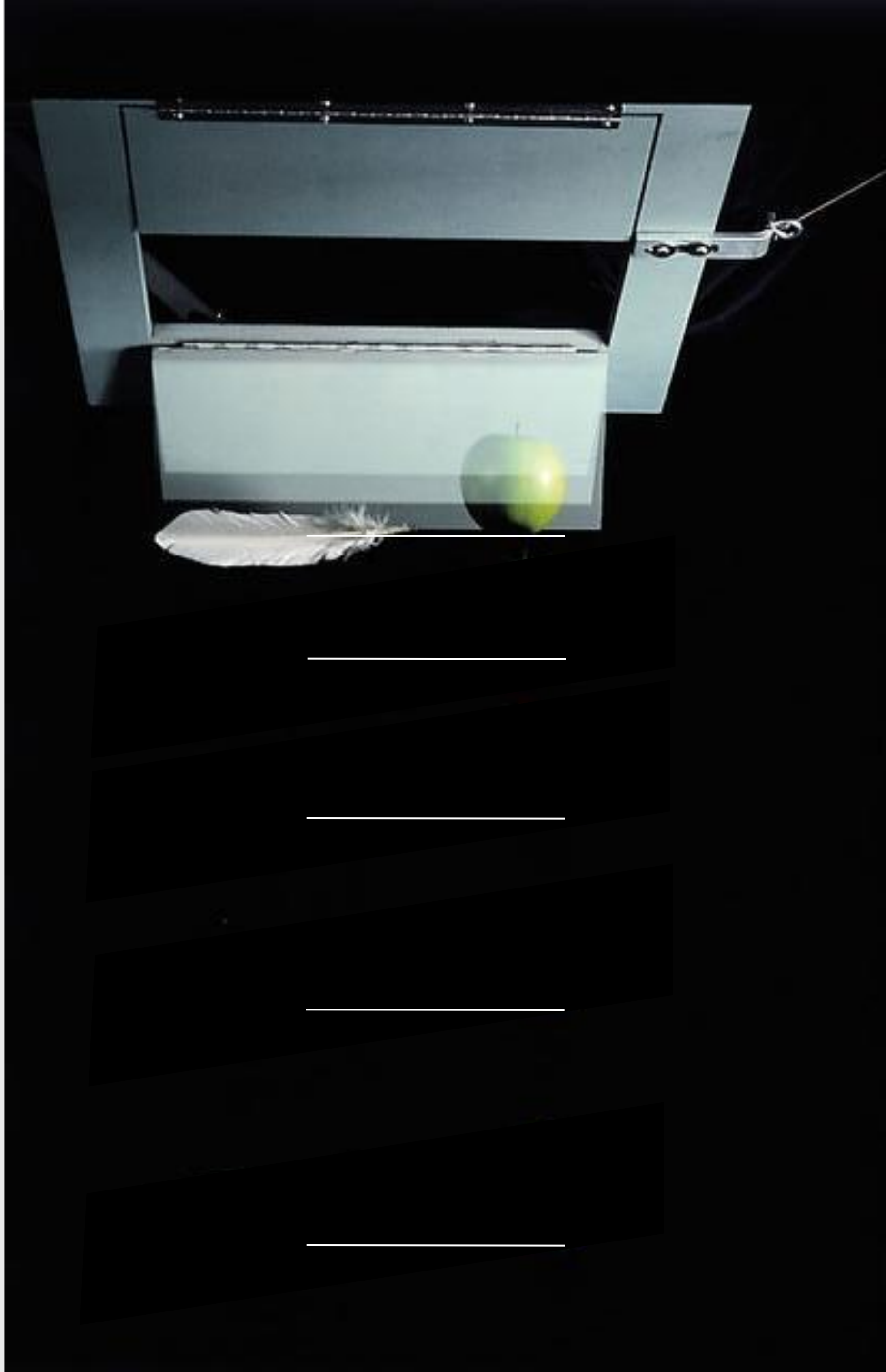


A.1 Kinematics SL

Determining the acceleration of free-fall experimentally

·Consider the multiflash image of an apple and a feather falling in a partial vacuum:

·If we choose a convenient spot on the apple, and mark its position, we get a series of marks like so:

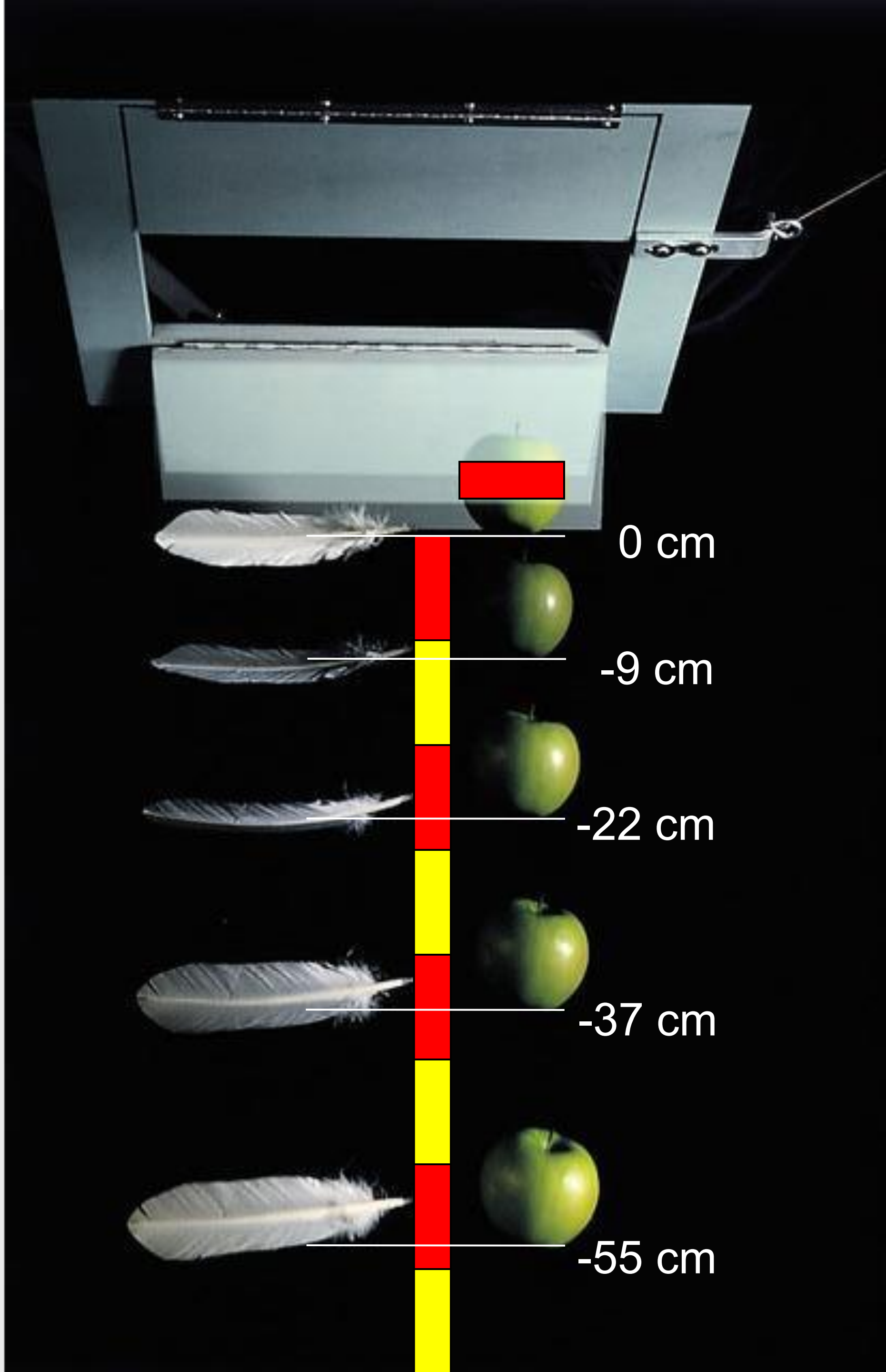


A.1 Kinematics SL

Determining the acceleration of free-fall experimentally

·Now we SCALE our data. Given that the apple is 8 cm in horizontal diameter we can superimpose this scale on our photograph.

·Then we can estimate the position in cm of each image.



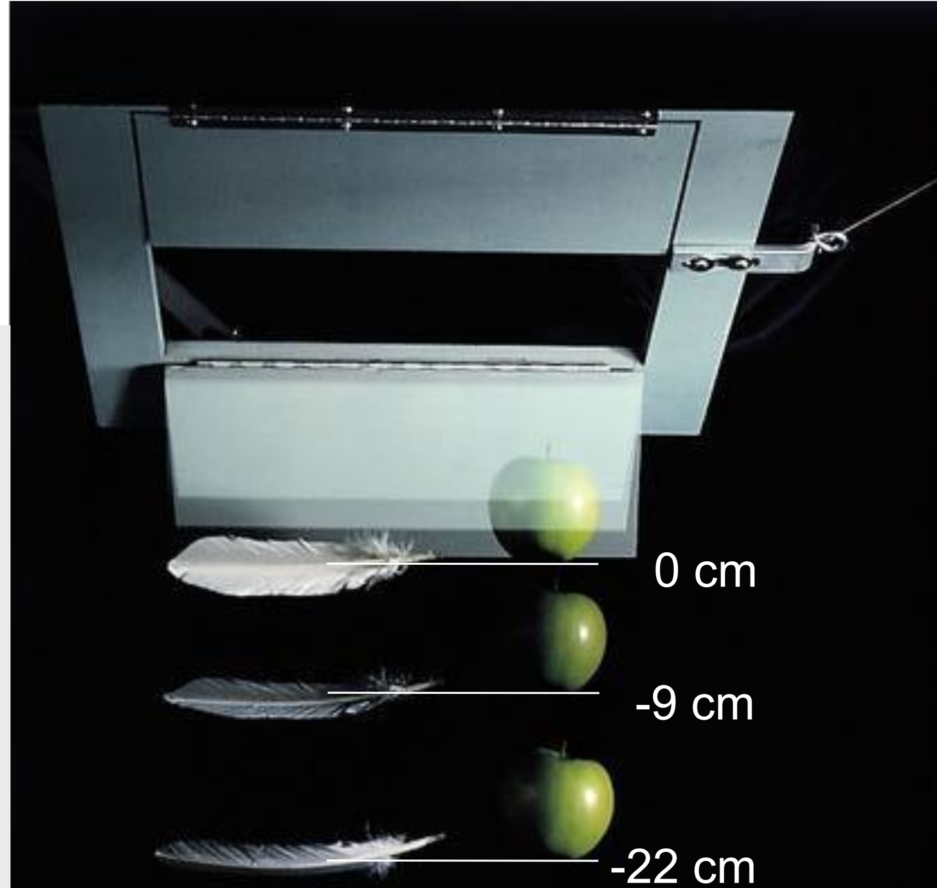
A.1 Kinematics SL

Determining the acceleration of free-fall experimentally

·Suppose we know that the time between images is 0.056 s.

·We make a table starting with the **raw data** columns of *t* and *y.*

·We then make **calculations** columns in Δt, Δy and *v*.



t (s)	y (cm)	Δt	Δy	v
.000	0			
.056	-9	.056	-9	-161
.112	-22	.056	-13	-232
.168	-37	.056	-15	-268
.224	-55	.056	-18	-321

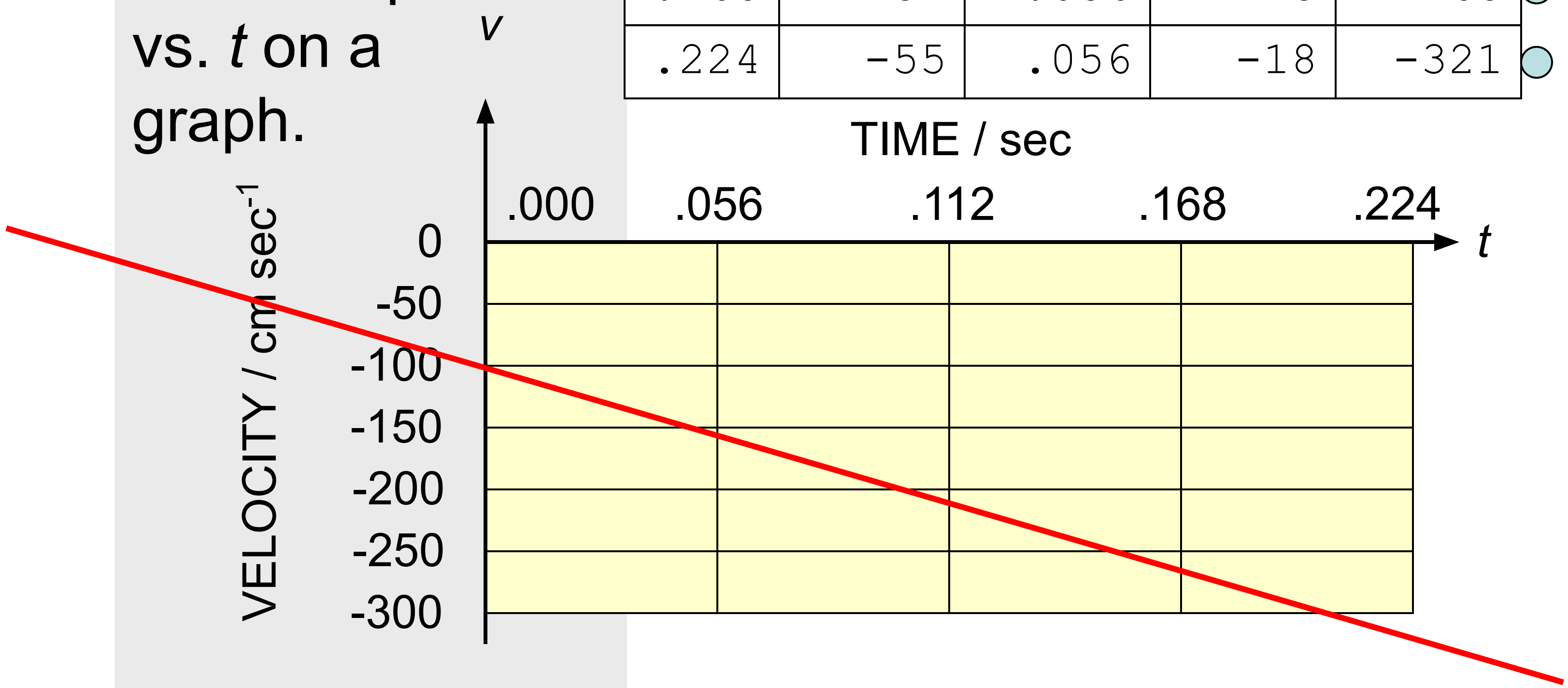
FYI: Since $v = \Delta y / \Delta t$, the first v entry is also BLANK.

A.1 Kinematics SL

Determining the acceleration of free-fall experimentally

·Now we plot *v* vs. *t* on a graph.

t(s)	y(cm)	Δt	Δy	v
.000	0			
.056	-9	.056	-9	-161
.112	-22	.056	-13	-232
.168	-37	.056	-15	-268
.224	-55	.056	-18	-321



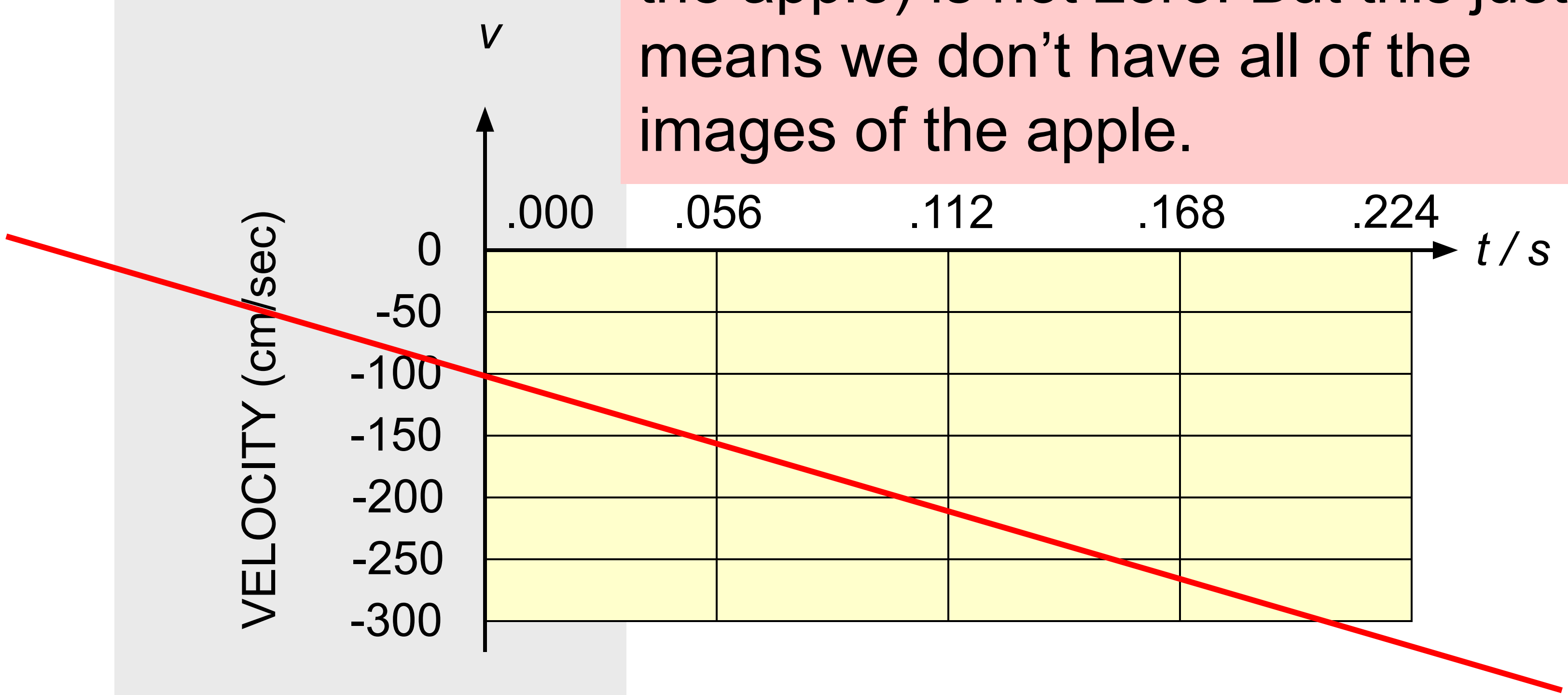
A.1 Kinematics SL

Determining the acceleration of free-fall experimentally

FYI

·The graph *v* vs. *t* is linear. Thus *a* is constant.

·The *y*-intercept (the initial velocity of the apple) is not zero. But this just means we don't have all of the images of the apple.



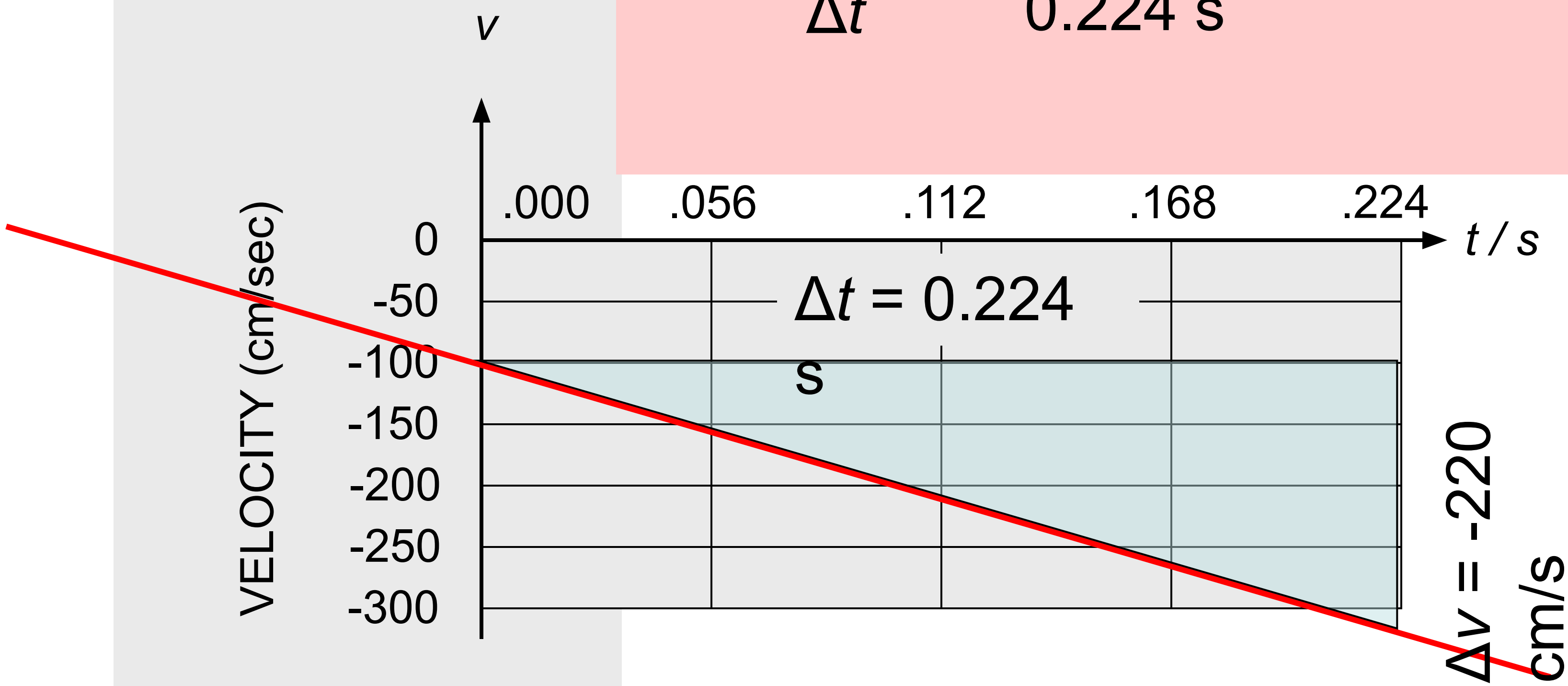
A.1 Kinematics SL

Determining the acceleration of free-fall experimentally

FYI

·Finally, the acceleration is the **slope** of the *v* vs. *t* graph:

$$\bar{\mathbf{a}} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{-220 \text{ cm/s}}{0.224 \text{ s}} = -982 \text{ cm/s}^2$$



A.1 Kinematics SL

Determining the acceleration of free-fall experimentally

·Since this acceleration due to gravity is so important we give it the name g.

·ALL objects accelerate at $-g$, where

$$g = 980 \text{ cm s}^{-2}$$

in the absence of air resistance.

·We can list the values for g in three ways:

$g = 980 \text{ cm s}^{-2}$

$g = 9.80 \text{ m s}^{-2}$

$g = 32 \text{ ft s}^{-2}$

We usually round the metric value to 10: $g = 10. \text{ m s}^{-2}$

magnitude of the freefall acceleration

·Hammer and feather drop Apollo 15

A.1 Kinematics SL

Solving problems using equations of motion for uniform acceleration

-General:

$s = ut + (1/2)at^2$, and

$v = u + at$, and

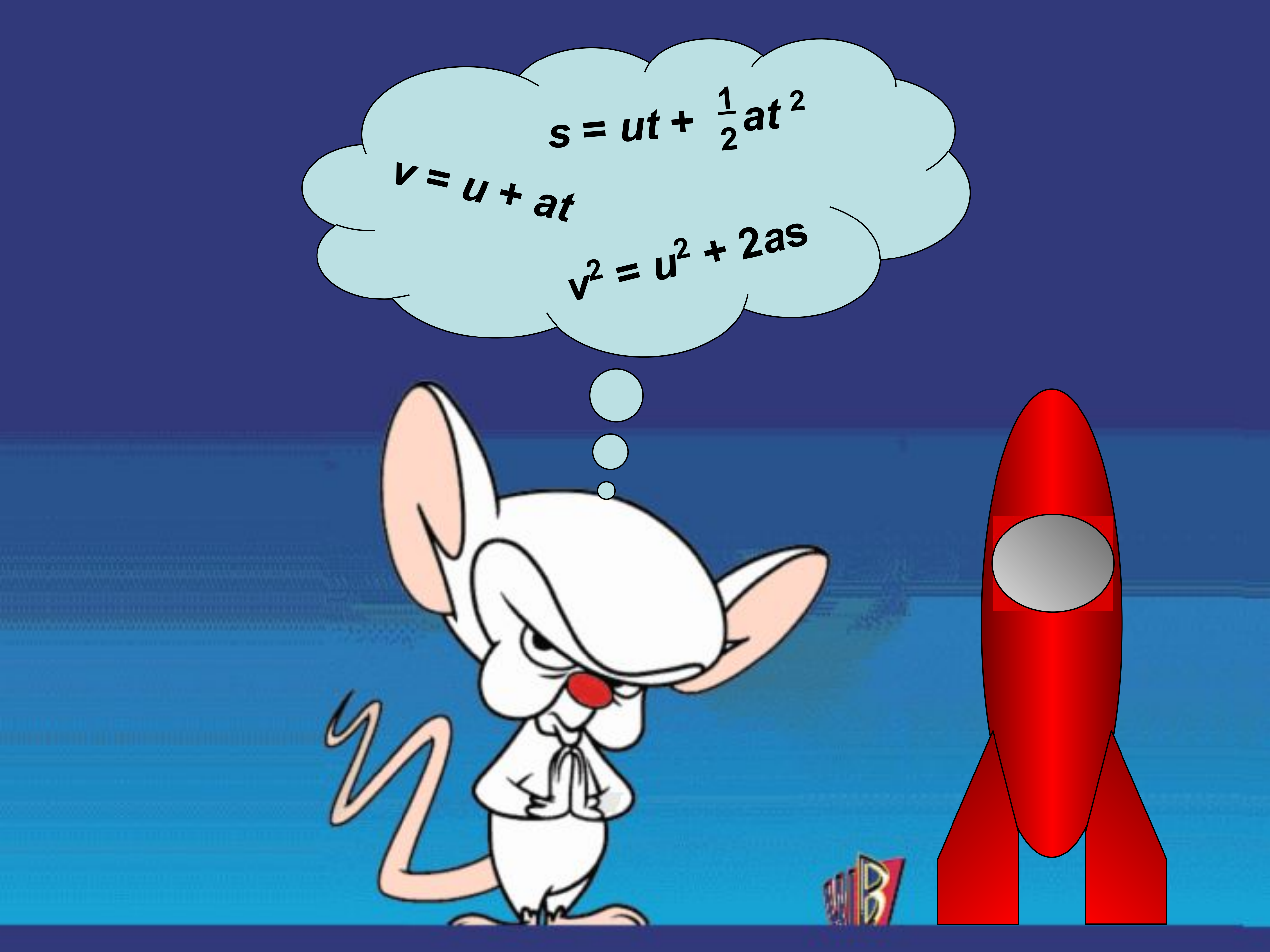
$v^2 = u^2 + 2as$, and

$s = (u + v)t / 2$;

-Freefall: Substitute ‘$-g$’ for ‘a’ in all of the above equations.

FYI

·The kinematic equations will be used throughout the year. We must master them NOW!


$s = ut + \frac{1}{2}at^2$
$v = u + at$
$v^2 = u^2 + 2as$

A.1 Kinematics SL

Solving problems using equations of motion for uniform acceleration

EXAMPLE: How far will Pinky and the Brain go in 30.0 seconds if their acceleration is 20.0 m s $^{-2}$?

KNOWN

$a = 20\ \text{m/s}^2$	Given
$t = 30\ \text{s}$	Given
$u = 0\ \text{m/s}$	Implicit

FORMULAS

$$s = ut + \tfrac{1}{2}at^2$$
$$\cancel{v = u + at}$$
$$\cancel{v^2 = u^2 + 2as}$$

WANTED $s = ?$

- t is known - drop the timeless eq'n.
- Since v is not wanted, drop the velocity eq'n:

SOLUTION

$$s = ut + \tfrac{1}{2}at^2$$
$$s = 0(30) + \tfrac{1}{2}20(30)^2$$
$$s = 9000\ \text{m}$$

A.1 Kinematics SL

Solving problems using equations of motion for uniform acceleration

EXAMPLE: How fast will Pinky and the Brain be going at this instant?

KNOWN

a = 20 m/s^2 Given
t = 30 s Given
u = 0 m/s Implicit

WANTED v = ?

·t is known - drop the timeless eq'n.

·Since v is wanted, drop the displacement eq'n:

FORMULAS

$\cancel{s = ut + \frac{1}{2}at^2}$

$v = u + at$

$\cancel{v^2 = u^2 + 2as}$

SOLUTION

$v = u + at$

$v = 0 + 20(30)$

$v = 600\ \text{m s}^{-1}$

A.1 Kinematics SL

Solving problems using equations of motion for uniform acceleration

EXAMPLE: How fast will Pinky and the Brain be going when they have traveled a total of 18000 m?

KNOWN

$a = 20\text{ m/s}^2$ Given

$s = 18000\text{ m}$ Given

$u = 0\text{ m/s}$ Implicit

FORMULAS

$\cancel{s = ut + \frac{1}{2}at^2}$

$\cancel{v = u + at}$

$v^2 = u^2 + 2as$

WANTED $v = ?$

·Since t is not known - drop the two eq'ns which have time in them.

SOLUTION

$v^2 = u^2 + 2as$

$v^2 = 0^2 + 2(20)(18000)$

$v = 850\text{ m s}^{-1}$

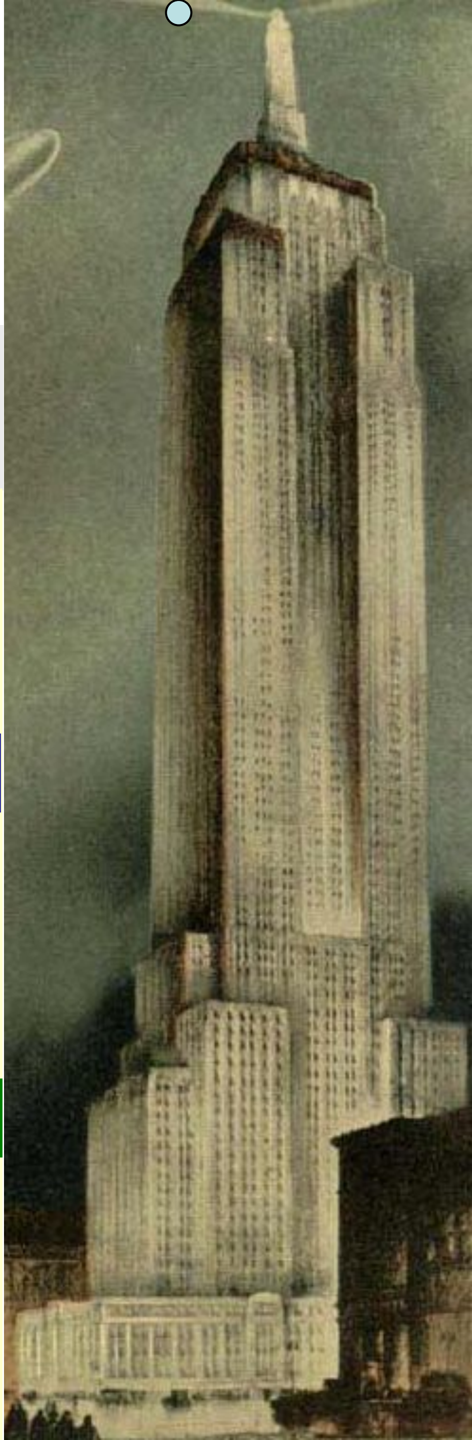
A.1 Kinematics SL

Solving problems using equations of motion for uniform acceleration

EXAMPLE: A ball is dropped off of the Empire State Building (381 m tall). How fast is it going when it hits ground?

KNOWN		*FORMULAS*
$a = -10\ \text{m/s}^2$	Implicit	$\cancel{s = ut + \frac{1}{2}at^2}$
$s = -381\ \text{m}$	Given	$\cancel{v = u + at}$
$u = 0\ \text{m/s}$	Implicit	$v^2 = u^2 + 2as$

WANTED $v = ?$	*SOLUTION*
·Since t is not known - drop the two eq'ns which have time in them.	$v^2 = u^2 + 2as$ $v^2 = 0^2 + 2(-10)(-381)$ $v = -87\ \text{m s}^{-1}$



A.1 Kinematics SL

Solving problems using equations of motion for uniform acceleration

EXAMPLE: A ball is dropped off of the Empire State Building (381 m tall). How long does it take to reach the ground?

KNOWN

$a = -10\ \text{m/s}^2$ Implicit

$s = -381\ \text{m}$ Given

$u = 0\ \text{m/s}$ Implicit

FORMULAS

$$s = ut + \tfrac{1}{2}at^2$$

$$\cancel{v = u + at}$$

$$\cancel{v^2 = u^2 + 2as}$$

WANTED $t = ?$

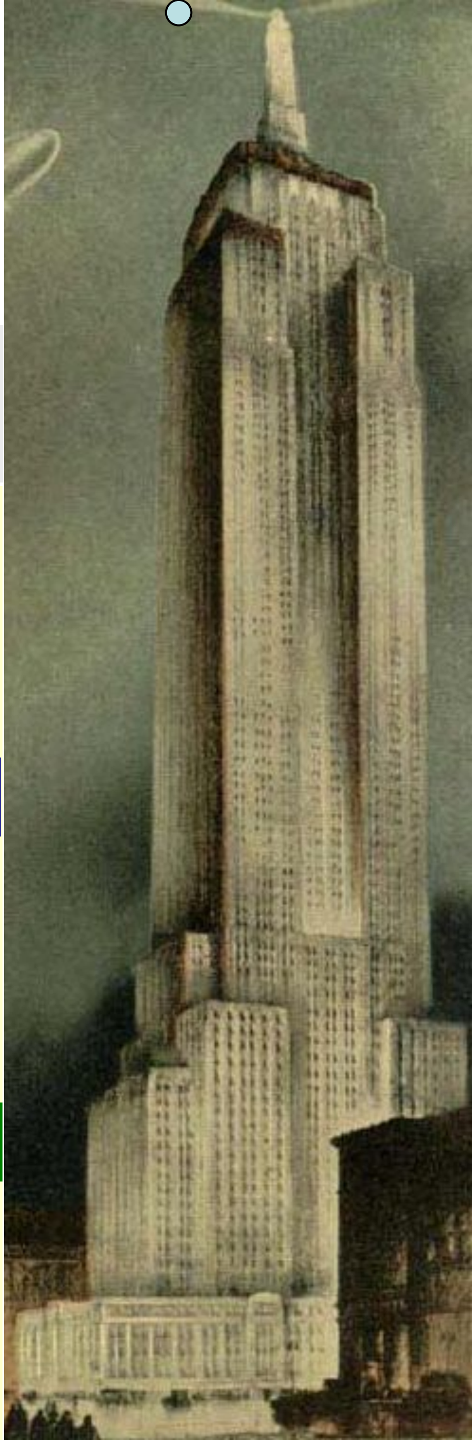
- Since t is desired and we have s drop the last two eq'ns.

SOLUTION

$$s = ut + \tfrac{1}{2}at^2$$

$$-381 = 0t + \tfrac{1}{2}(-10)t^2$$

$$t = 8.7\ \text{s}$$



A.1 Kinematics SL

Solving problems using equations of motion for uniform acceleration

EXAMPLE: A cheer leader is thrown up with an initial speed of 7 m s^{-1}. How high does she go?

KNOWN

a = -10 m/s^2 Implicit

u = 7 m s^{-1} Given

v = 0 m/s Implicit

FORMULAS

$\cancel{s = ut + \frac{1}{2}at^2}$

$\cancel{v = u + at}$

$v^2 = u^2 + 2as$

WANTED s = ?

·Since t is not known - drop the two eq'ns which have time in them.

SOLUTION

$v^2 = u^2 + 2as$

$0^2 = 7^2 + 2(-10)s$

$s = 2.45$ m

A.1 Kinematics SL

Solving problems using equations of motion for uniform acceleration

EXAMPLE: A ball is thrown upward at 50 m s^{-1} from the top of the 300-m Millau Viaduct, the highest bridge in the world. How fast does it hit ground?

KNOWN

a = -10 m/s^2 Implicit
u = 50 m s^{-1} Given
s = -300 m Implicit

FORMULAS

$\cancel{s = ut + \frac{1}{2}at^2}$
$\cancel{v = u + at}$
$v^2 = u^2 + 2as$

WANTED v = ?

·Since t is not known - drop the two eq'ns which have time in them.

SOLUTION

$v^2 = u^2 + 2as$
$v^2 = 50^2 + 2(-10)(-300)$
v = -90 m s^{-1}

A.1 Kinematics SL

Solving problems using equations of motion for uniform acceleration

EXAMPLE: A ball is thrown upward at 50 m s^{-1} from the top of the 300-m Millau Viaduct, the highest bridge in the world. How long is it in flight?

KNOWN		*FORMULAS*
a = -10 m/s^2	Implicit	$\cancel{s = ut + \frac{1}{2}at^2}$
u = 50 m s^{-1}	Given	$v = u + at$
v = -90 m s^{-1}	Calculated	$\cancel{v^2 = u^2 + 2as}$

WANTED t = ?

·Use the simplest t equation.

SOLUTION

$$v = u + at$$

$$-90 = 50 + (^{-}10)t$$

$$t = 14 \text{ s}$$

A.1 Kinematics SL

Sketching and interpreting motion graphs

·The **slope** of a displacement-time graph is the velocity.

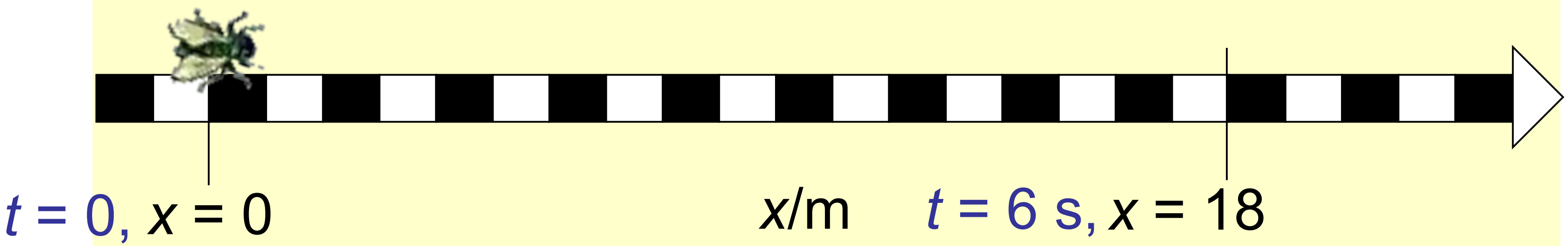
·The **slope** of the velocity-time graph is the acceleration. We already did this example with the falling feather/apple presentation.

·You will have ample opportunity to find the slopes of distance-time, displacement-time and velocity-time graphs in your labs.

A.1 Kinematics SL

Sketching and interpreting motion graphs

EXAMPLE: Suppose Freddie the Fly begins at $x = 0$ m, and travels at a constant velocity for 6 seconds as shown. Find two points, sketch a displacement vs. time graph, and then find and interpret the slope and the area of your graph.



SOLUTION:

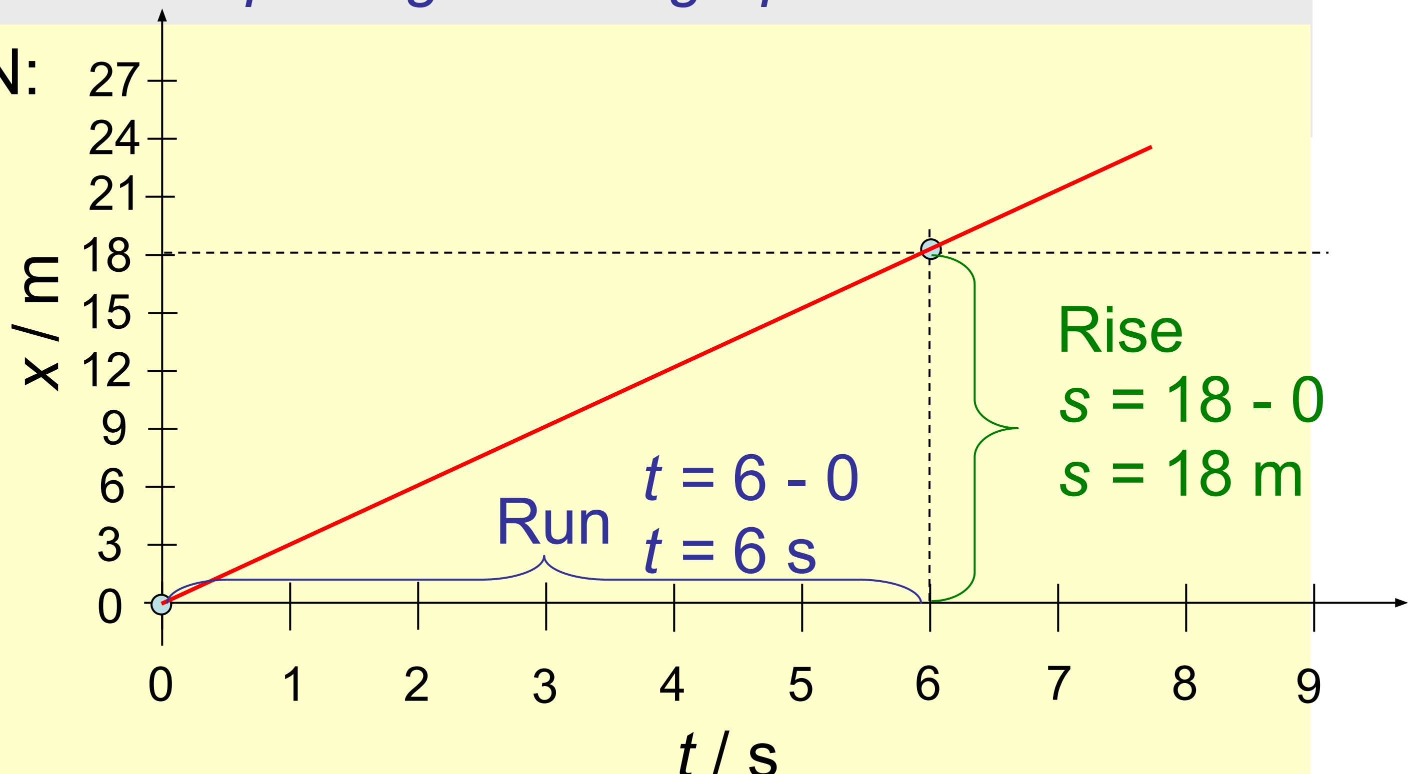
·The two points are (0 s, 0 m) and (6 s, 18 m).

·The sketch is on the next slide.

A.1 Kinematics SL

Sketching and interpreting motion graphs

SOLUTION:



·The slope is rise over run or 18 m / 6 s

·Thus the slope is 3 m s^{-1}, which is interpreted as Freddie's velocity.

A.1 Kinematics SL

Sketching and interpreting motion graphs

·The **area** under a velocity-time graph is the displacement.

·The **area** under an acceleration-time graph is the change in velocity.

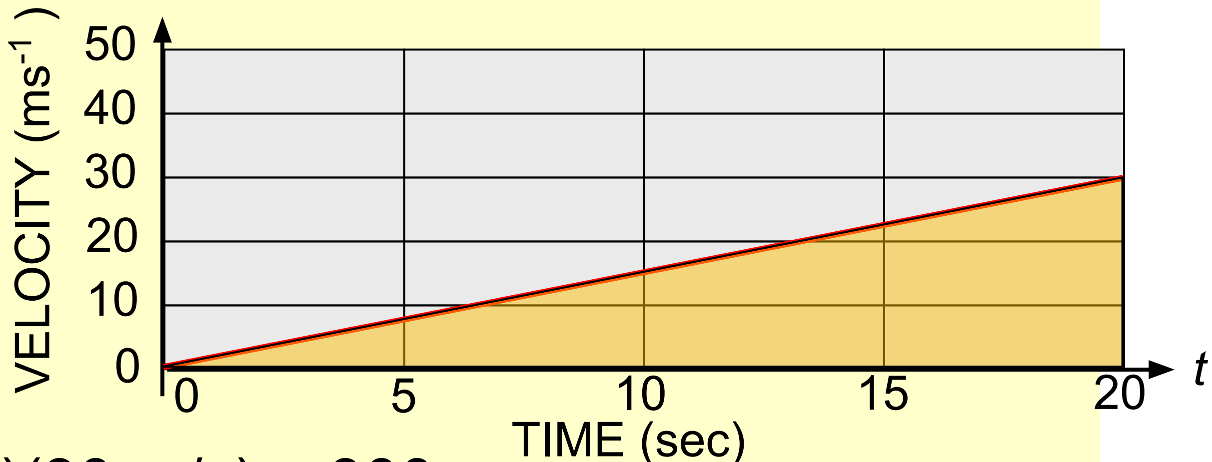
·You will have ample opportunity to draw distance-time, displacement-time and velocity-time graphs in your labs.

A.1 Kinematics SL

Sketching and interpreting motion graphs

EXAMPLE: Calculate and interpret the area under the given v vs. t graph. Find and interpret the slope.

SOLUTION:



·The area of a triangle is $A = (1/2)bh$.

·Thus

$$A = (1/2)(20\text{ s})(30\text{ m/s}) = 300\text{ m}.$$

·This is the displacement of the object in 20 s.

·The slope is (30 m/s) / 20 s = 1.5 m s^{-2}.

·This is the acceleration of the object.

A.1 Kinematics SL

Qualitatively describing the effect of fluid resistance on falling objects or projectiles, including reaching terminal speed

-Students should know what is meant by terminal speed.

-This is when the drag force exactly balances the weight.

A.1 Kinematics SL

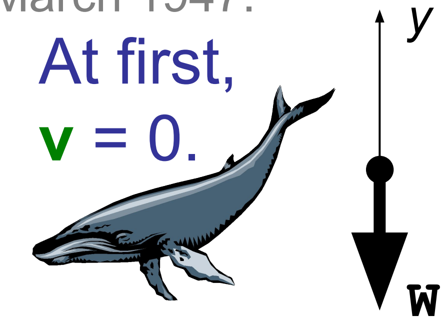
"A female Blue Whale weighing 190 metric tonnes (418,877lb) and measuring 27.6m (90ft 5in) in length suddenly materialized above the Southern Ocean on 20 March 1947."

Guinness World Records. *Falkland Islands Philatelic Bureau. 2 March 2002.*

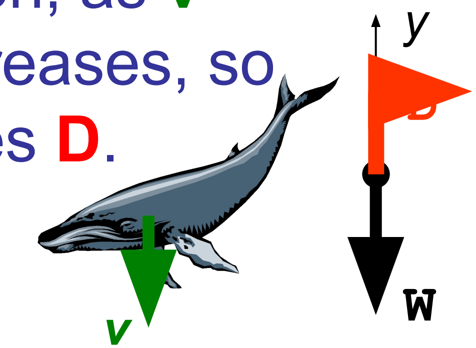
Qualitatively describing the effect of fluid resistance on falling objects or projectiles, including reaching terminal speed

- Suppose a blue whale suddenly materializes high above the ground.
- The drag force *D* is proportional to the speed squared.
- Thus, as the whale picks up speed, the drag force increases.
- Once the drag force equals the whale's weight, the whale will stop accelerating.
- It has reached **terminal speed**.

At first, **v** = 0.

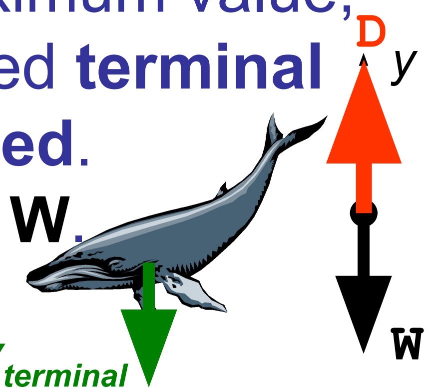


Then, as **v** increases, so does **D**.



v reaches a maximum value, called **terminal speed**.

D = **W**.



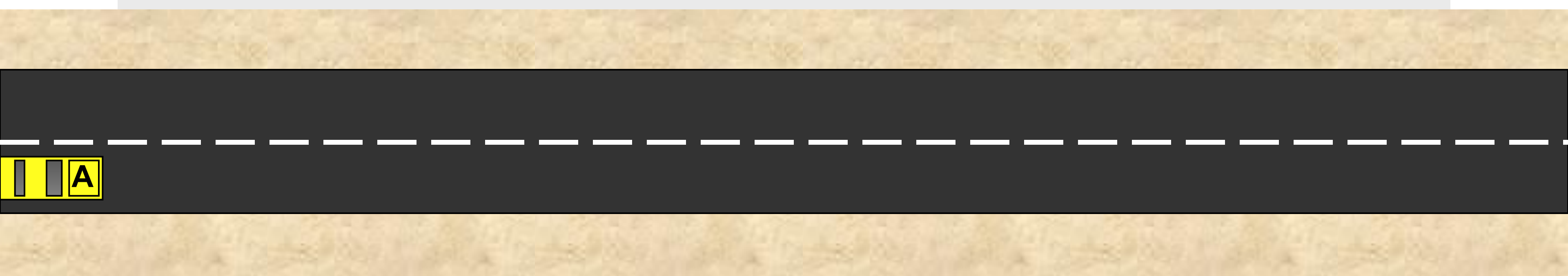
A.1 Kinematics SL

Determine relative velocity in one and two dimensions

·Suppose you are a passenger in a car on a perfectly level and straight road, moving at a constant velocity. Your velocity relative to the pavement might be 60 kph.

·Your velocity relative to the driver of your car is zero. Whereas your velocity relative to an oncoming car might be 120 kph.

·Your velocity can be measured relative to **any** reference frame.



A.1 Kinematics SL

Determine relative velocity in one and two dimensions

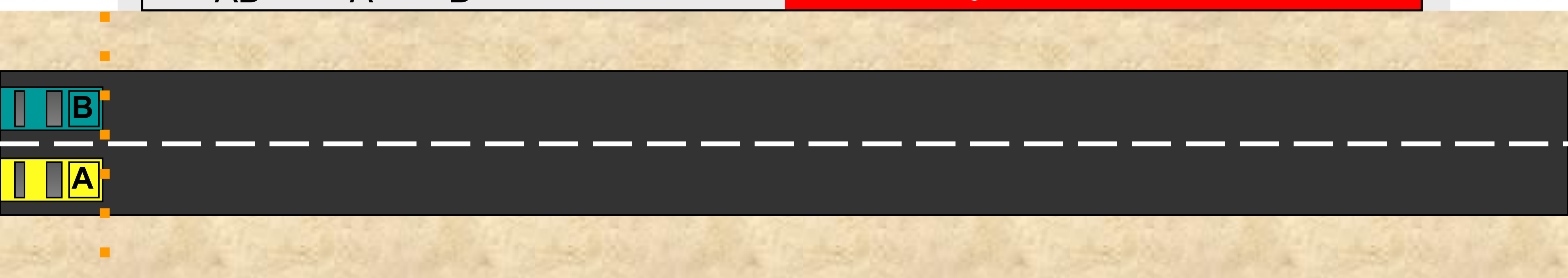
·Consider two cars, A and B, shown below.

·Suppose you are in car A which is moving at v_A = +20 m s^{-1} and next to you is car B, moving at v_B = +40 m s^{-1}.

·As far as you are concerned your velocity v_{AB} relative to car B is -20 m s^{-1} because you seem to be moving backwards relative to B's coordinate system.

·We write

$\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B$	velocity of A relative to B



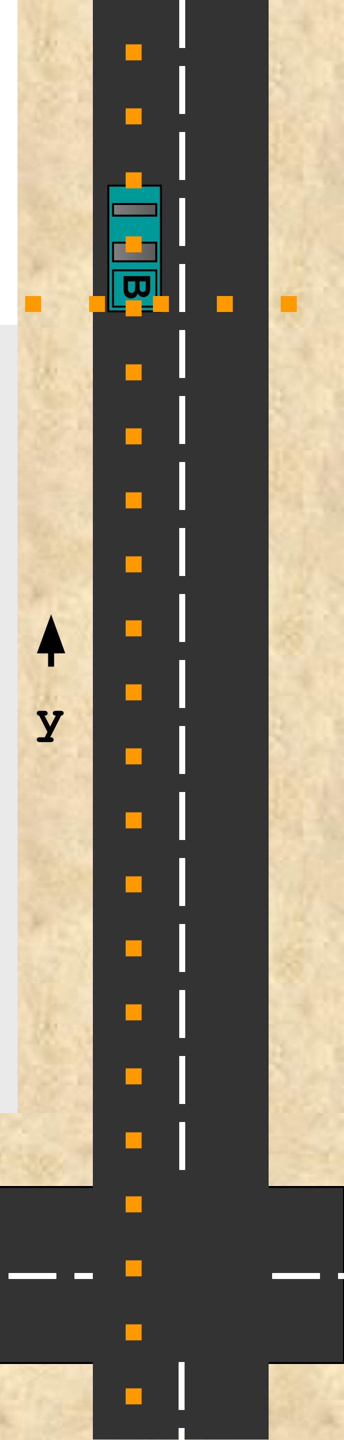
A.1 Kinematics SL

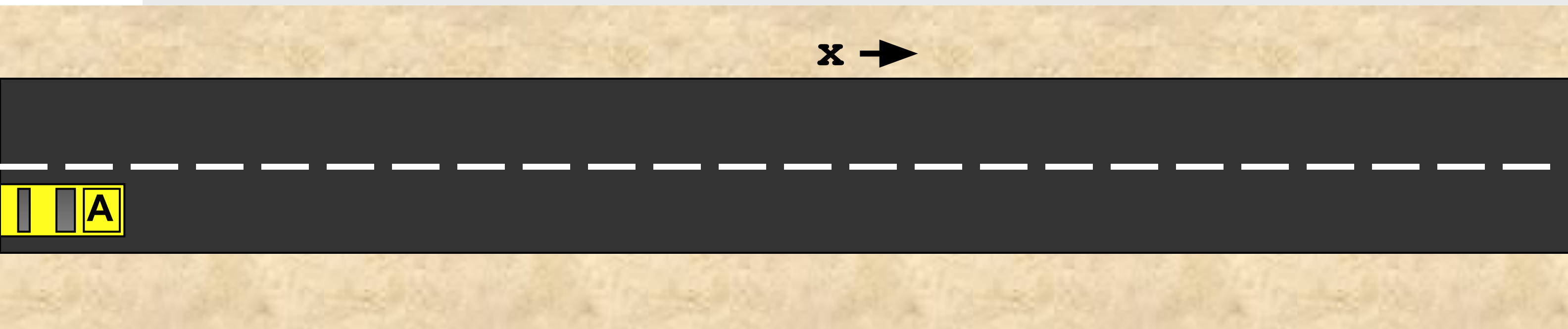
Determine relative velocity in one and two dimensions

·The equation works even in two dimensions.

·Suppose you are in car A which is moving at v_A = +40 m s^{-1} and approaching you at right angles is a car B is moving at v_B = -20 m s^{-1} as shown.

·Since A and B are moving perpendicular to one another, use a vector diagram to find v_{AB}. The solution is on the next slide.

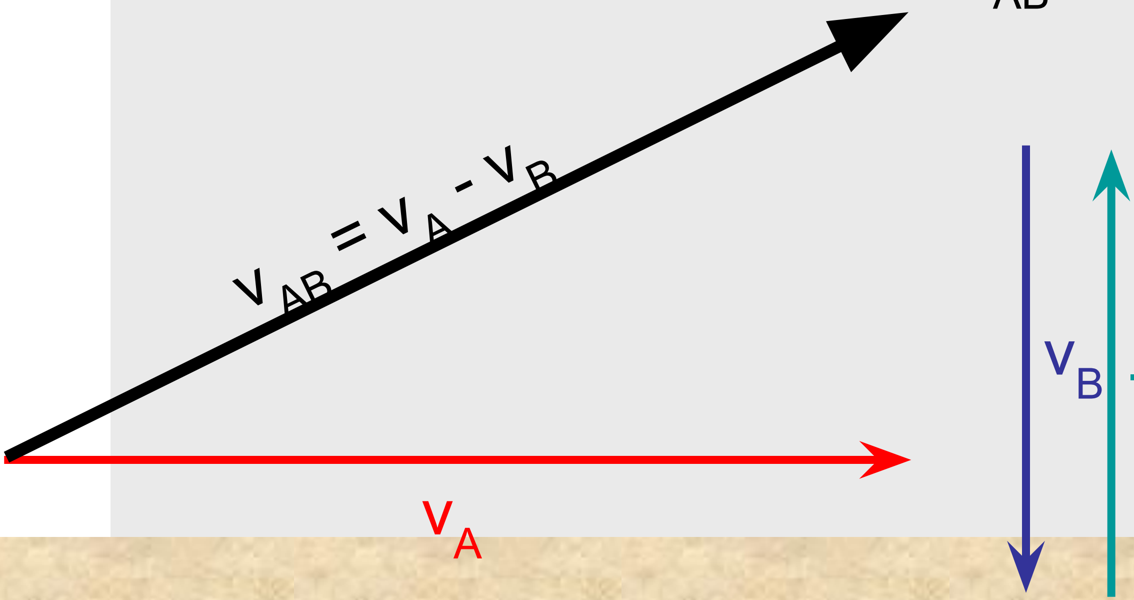




A.1 Kinematics SL

Determine relative velocity in one and two dimensions

·Draw in the vectors and use $\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B$.



$$v_{AB}^{\ 2} = v_A^{\ 2} + v_B^{\ 2}$$

$$v_{AB}^{\ 2} = 40^2 + 20^2$$

$$v_{AB} = 45 \text{ m s}^{-1}$$

A.1 Kinematics SL

Projectile motion

·A **projectile** is an object that has been given an initial velocity by some sort of short-lived force, and then moves through the air under the influence of gravity.

·Baseballs, stones, or bullets are all examples of projectiles executing **projectile motion**.

·You know that all objects moving through air feel an a resistance (recall sticking your hand out of the window of a moving car).

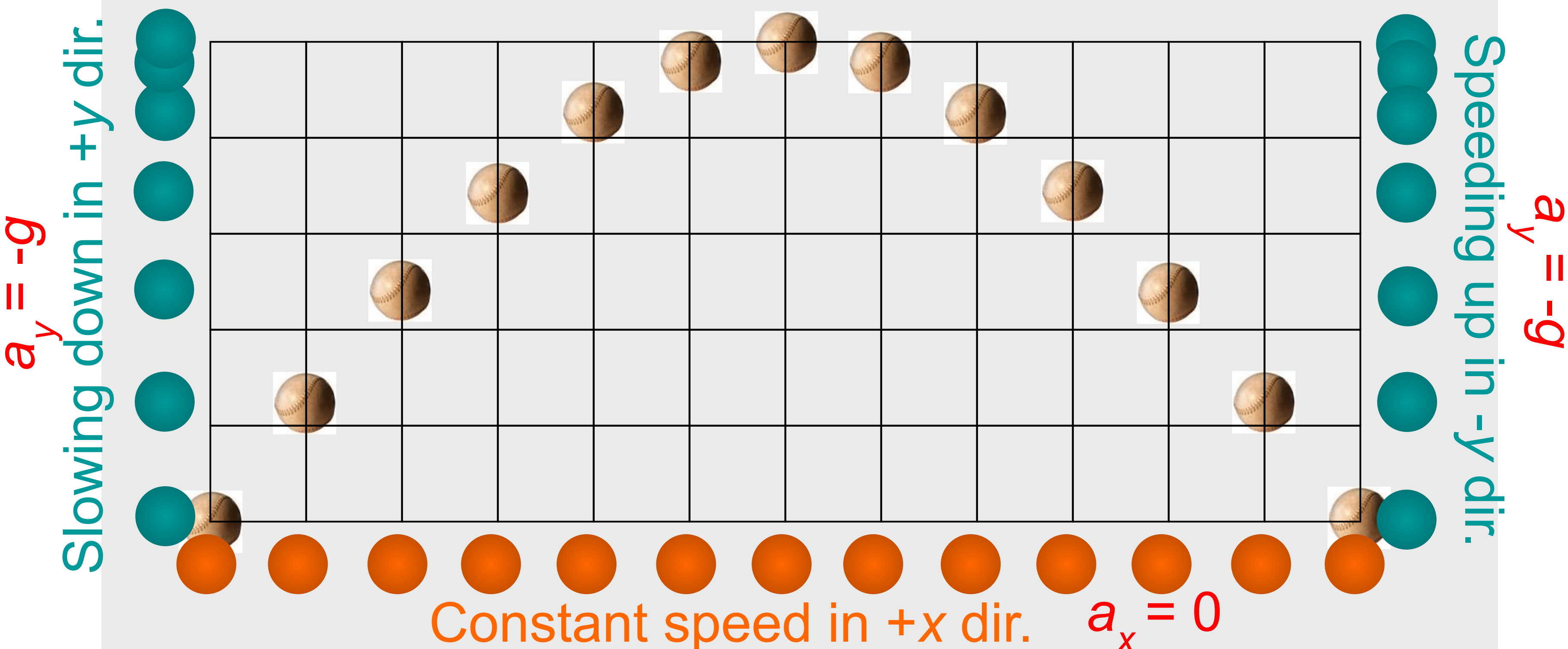
FYI

·We will ignore air resistance in the discussion that follows…

A.1 Kinematics SL

Analysing projectile motion

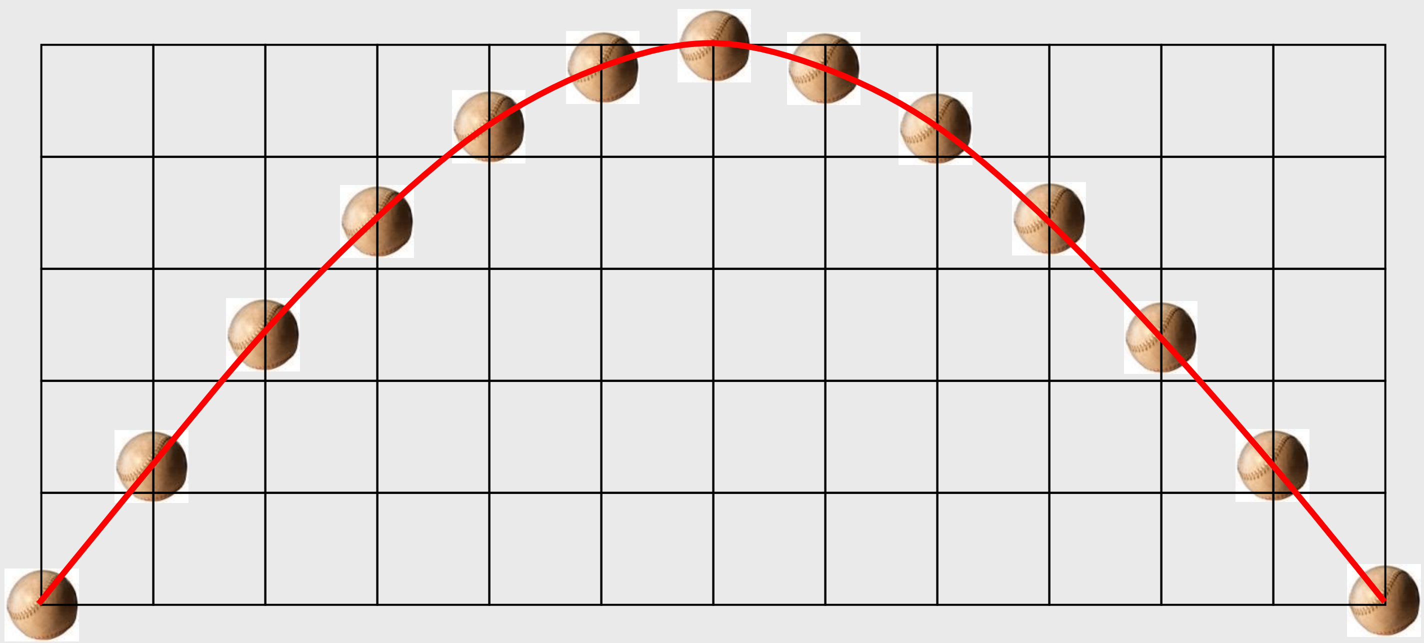
·Regardless of the air resistance, the vertical and the horizontal components of velocity of an object in projectile motion are independent.



A.1 Kinematics SL

Analysing projectile motion

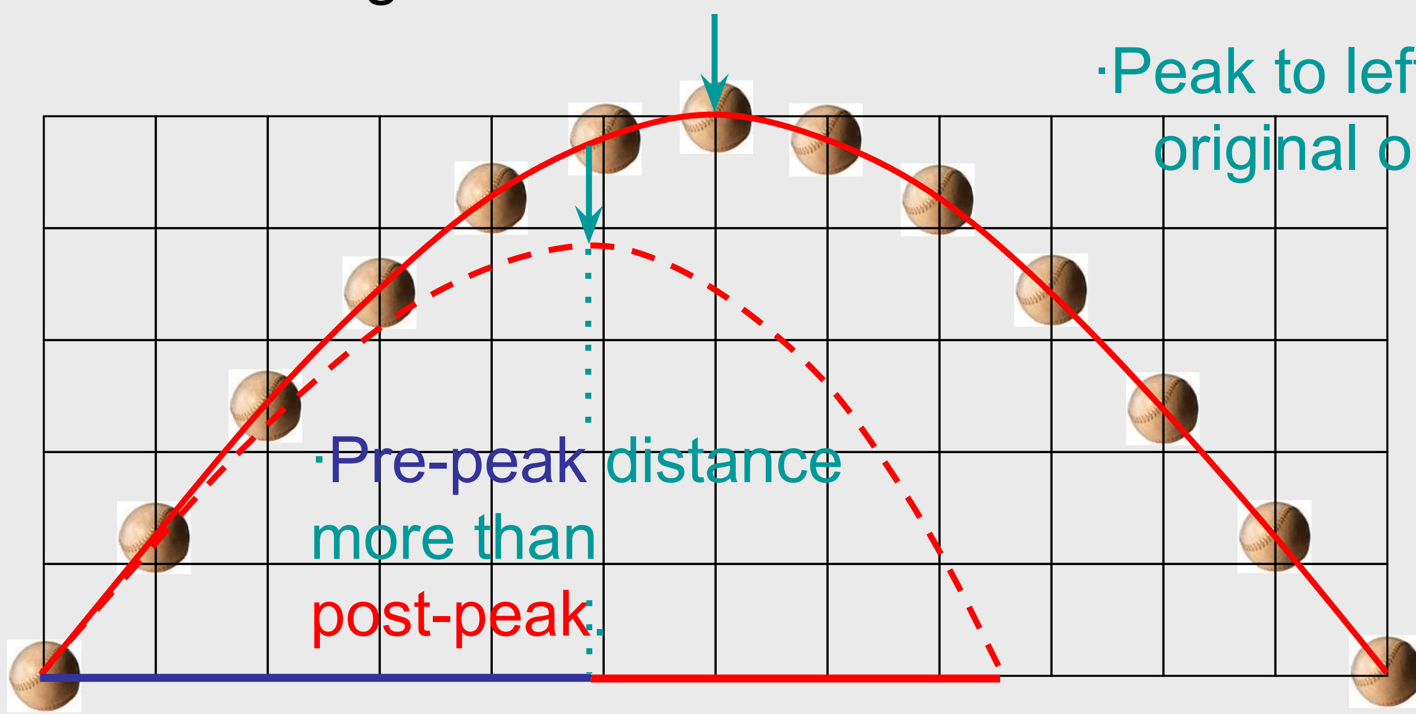
·The trajectory of a projectile in the absence of air is **parabolic**. Know this!



A.1 Kinematics SL

Analysing projectile motion with fluid resistance

·If there is air resistance, it is proportional to the square of the velocity. Thus, when the ball moves fast its deceleration is greater than when it moves slow.



SKETCH POINTS

A.1 Kinematics SL

Analysing projectile motion

·Recall the kinematic equations:

$$s = ut + (1/2)at^2 \qquad \textit{Displacement}$$
$$v = u + at \qquad \textit{Velocity}$$

kinematic equations 1D

a is constant

·Since we worked only in 1D at the time, we didn't have to distinguish between *x* and *y* in these equations.

·Now we appropriately modify the above to meet our new requirements of simultaneous equations:

$$\Delta x = u_x t + (1/2)a_x t^2$$
$$v_x = u_x + a_x t$$
$$\Delta y = u_y t + (1/2)a_y t^2$$
$$v_y = u_y + a_y t$$

kinematic equations 2D

a_x *and* a_y are constant

A.1 Kinematics SL

Analysing projectile motion

$$\Delta x = u_x t + (1/2)\cancelto{0}{a_x} t^2$$
$$v_x = u_x + \cancelto{0}{a_x} t$$
$$\Delta y = u_y t + \boxed{(1/2)a_y t^2}$$
$$v_y = u_y + \boxed{a_y t}$$

kinematic equations 2D

a_x *and* a_y are constant

PRACTICE: Show that the reduced equations for projectile motion are

$$\Delta x = u_x t \qquad \Delta y = u_y t - 5t^2$$
$$v_x = u_x \qquad v_y = u_y - 10t$$

reduced equations of projectile motion

SOLUTION:

·a_x = 0 in the absence of air resistance.

·a_y = -10 in the absence of air resistance.

A.1 Kinematics SL

Analysing projectile motion

$\Delta x = u_x t$	$\Delta y = u_y t - 5t^2$	reduced equations of projectile motion
$v_x = u_x$	$v_y = u_y - 10t$	

EXAMPLE: Use the reduced equations above to prove that projectile motion is parabolic in nature.

SOLUTION: Just solve for t in the first equation and substitute it into the second equation.

$\Delta x = u_x t$ becomes $t = x / u_x$ so that $t^2 = x^2 / u_x^2$.

Then since $y = u_y t - 5t^2$, we have

$$y = (u_y / u_x)x - (5 / u_x^2)x^2.$$

FYI

- The equation of a parabola is $y = Ax + Bx^2$.
- In this case, $A = u_y / u_x$ and $B = -5 / u_x^2$.

A.1 Kinematics SL

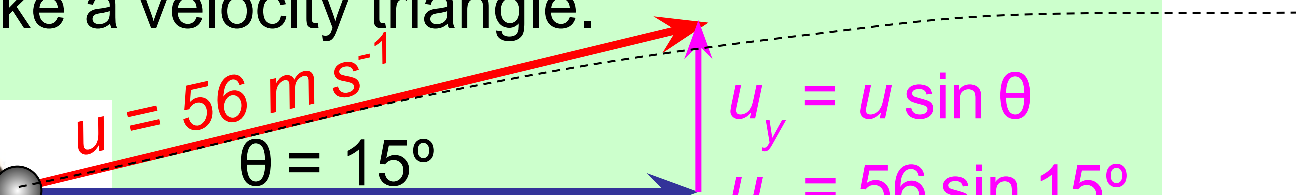
Analysing projectile motion

$\Delta x = u_x t$	$\Delta y = u_y t - 5t^2$	reduced equations of projectile motion
$v_x = u_x$	$v_y = u_y - 10t$	

PRACTICE: A cannon fires a projectile with a muzzle velocity of 56 ms^{-1} at an angle of inclination of 15º.

(a) What are u_x and u_y?

SOLUTION: Make a velocity triangle.



$u_x = u \cos\theta$

$u_x = 56 \cos 15º$

$u_x = 54 \text{ m s}^{-1}$

$u_y = u \sin\theta$

$u_y = 56 \sin 15º$

$u_y = 15 \text{ m s}^{-1}$.

A.1 Kinematics SL

Analysing projectile motion

$$\Delta x = u_x t \qquad \Delta y = u_y t - 5t^2$$
$$v_x = u_x \qquad v_y = u_y - 10t$$

projectile motion

PRACTICE: A cannon fires a projectile with a muzzle velocity of 56 ms^{-1} at an angle of inclination of 15º.

(b) What are the tailored equations of motion?

(c) When will the ball reach its maximum height?

SOLUTION: (b) Just substitute $u_x = 54$ and $u_y = 15$:

$$\Delta x = 54t \qquad \Delta y = 15t - 5t^2$$
$$v_x = 54 \qquad v_y = 15 - 10t$$

tailored equations for this particular projectile

(c) At the maximum height, $v_y = 0$. Why? Thus $v_y = 15 - 10t$ becomes $0 = 15 - 10t$ so that

$$10t = 15 \text{ or } t = 1.5 \text{ s.}$$

A.1 Kinematics SL

Analysing projectile motion

$\Delta x = 54t$	$\Delta y = 15t - 5t^2$
$v_x = 54$	$v_y = 15 - 10t$

ta
this particular projectile

PRACTICE: A cannon fires a projectile with a muzzle velocity of 56 ms^{-1} at an angle of inclination of 15º.

(d) How far from the muzzle will the ball be when it reaches the height of the muzzle at the end of its trajectory?

SOLUTION:

From symmetry $t_{up} = t_{down} = 1.5$ s so $t = 3.0$ s.

Thus

$$\Delta x = 54t$$

$$\Delta x = 54(3.0)$$

$$\Delta x = 160 \text{ m.}$$

A.1 Kinematics SL

Analysing projectile motion

$\Delta x = 54t$	$\Delta y = 15t - 5t^2$
$v_x = 54$	$v_y = 15 - 10t$

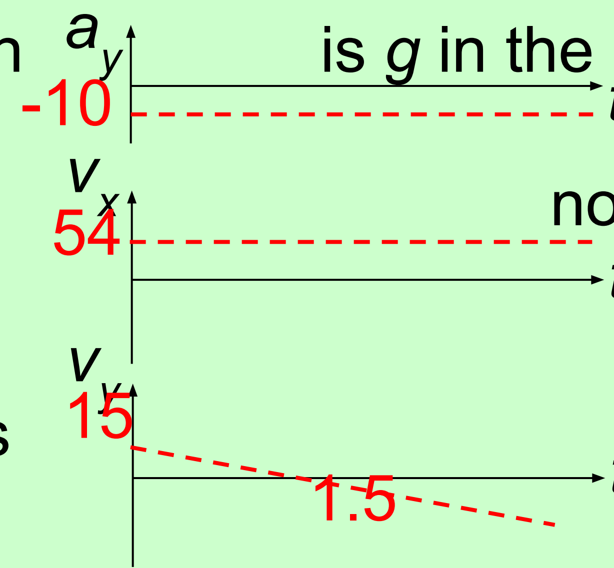
ta… this particular projectile

PRACTICE: A cannon fires a projectile with a muzzle velocity of 56 ms^{-1} at an angle of inclination of 15º.

(e) Sketch the following graphs:

a vs. t, v_x vs. t, v_y vs. t:

SOLUTION: The only acceleration is g in the ^-y-direction.



·v_x = 54, a constant. Thus it does not change over time.

·$v_y = 15 - 10t$ Thus it is linear with a negative gradient and it crosses the time axis at 1.5 s.

A.1 Kinematics SL

Analysing projectile motion

Which **one** of the following is a true statement concerning the vertical component of the velocity and the acceleration of a projectile when it is at its maximum height? *(The acceleration of free fall is g.)*

	Vertical component of velocity	Acceleration
A.	~~maximum~~	~~zero~~
B.	~~maximum~~	g
C.	zero	~~zero~~
D.	zero	g

·The acceleration is ALWAYS *g* for projectile motion-since it is caused by Earth and its field.

·At the maximum height the projectile switches from upward to downward motion. $v_y = 0$ at switch.

A.1 Kinematics SL

Analysing projectile motion

A stone is thrown at an angle to the horizontal. Ignoring air resistance, the horizontal component of the initial velocity of the stone determines the value of

A. range only.

~~B. maximum height only.~~

~~C. range and maximum height.~~

~~D. range and time of flight.~~

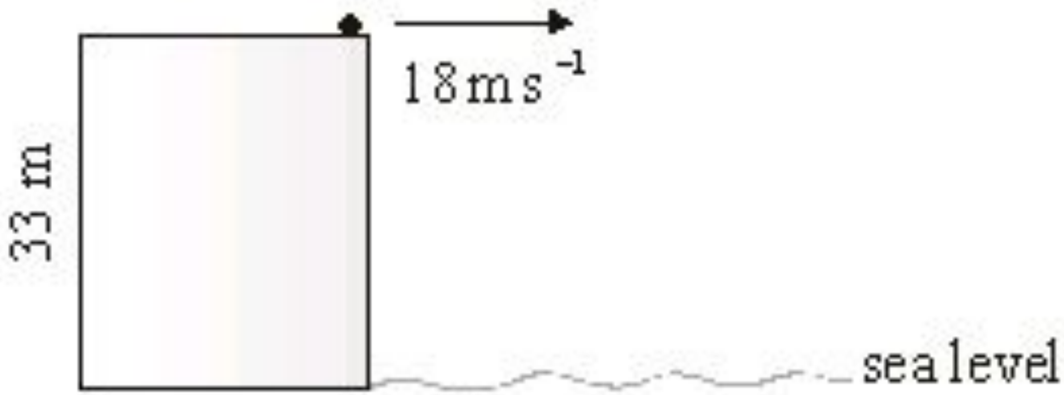
·The flight time is limited by the y motion.

·The maximum height is limited by the y motion.

A.1 Kinematics SL

Analysing projectile motion

A stone is thrown horizontally from the top of a vertical cliff of height 33 m as shown below.



The initial horizontal velocity of the stone is 18 m s^{-1} and air resistance may be assumed to be negligible.

(a) State values for the horizontal and for the vertical acceleration of the stone.

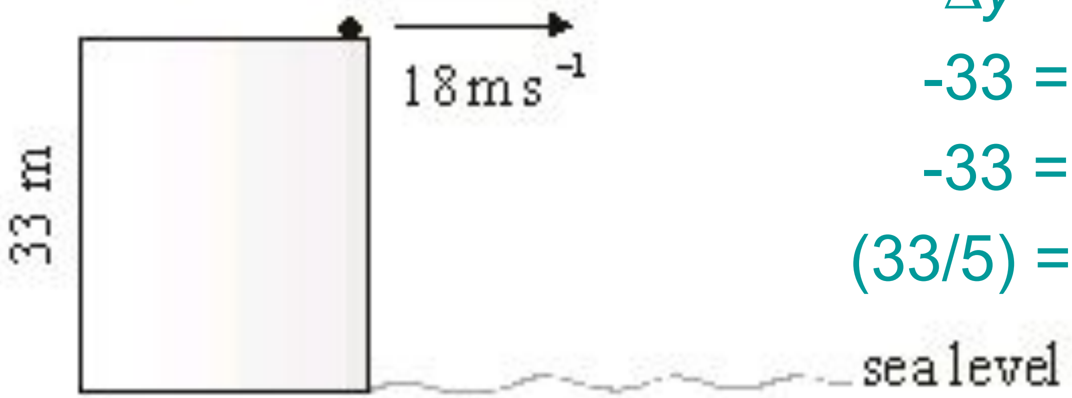
Horizontal acceleration: $a_x = 0$. ....

Vertical acceleration: $a_y = -10\ ms^{-2}$. ....

A.1 Kinematics SL

Analysing projectile motion

A stone is thrown horizontally from the top of a vertical cliff of height 33 m as shown below.



$\Delta y = u_y t - 5t^2$

$-33 = 0t - 5t^2$

$-33 = -5t^2$

$(33/5) = t^2$

The initial horizontal velocity of the stone is 18 m s^{-1} and air resistance may be assumed to be negligible.

(b) Determine the time taken for the stone to reach sea level.

·Fall time limited by y-equations:

·$t = 2.6$ s.

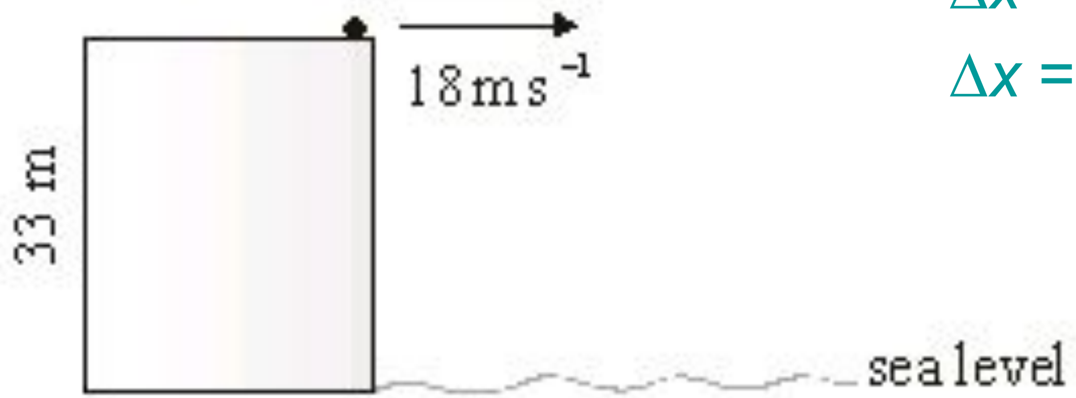
A.1 Kinematics SL

Analysing projectile motion

A stone is thrown horizontally from the top of a vertical cliff of height 33 m as shown below.

$\Delta x = u_x t$

$\Delta x = 18(2.6)$



The initial horizontal velocity of the stone is 18 m s^{-1} and air resistance may be assumed to be negligible.

(c) Calculate the distance of the stone from the base of the cliff when it reaches sea level.

·Use x-equations and t = 2.6 s:

....

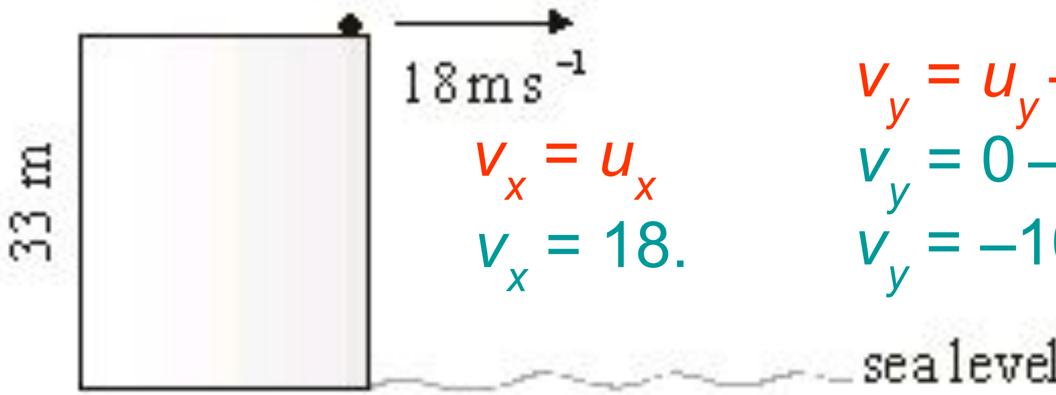
·Δx = 47 m.

....

A.1 Kinematics SL

Analysing projectile motion

A stone is thrown horizontally from the top of a vertical cliff of height 33 m as shown below.



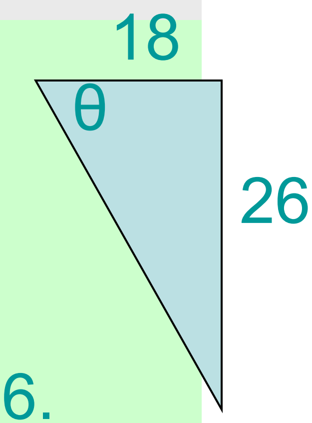
$v_x = u_x$

$v_x = 18.$

$v_y = u_y - 10t$

$v_y = 0 - 10t$

$v_y = -10(2.6) = -26.$



The initial horizontal velocity of the stone is 18 m s^{-1} and air resistance may be assumed to be negligible.

(d) Calculate the angle that the velocity makes with the surface of the sea.

$\cdot\tan\theta = 26/18$

....

$\theta = \tan^{-1}(26/18) = 55°.$

....

A.1 Kinematics SL

Analysing projectile motion

A stone is projected horizontally from the top of a cliff. Neglecting air resistance, which *one* of the following correctly describes what happens to the horizontal component of velocity and to the vertical component of velocity?

	Horizontal component of velocity	Vertical component of velocity
A.	~~Decreases~~	Increases
B.	~~Decreases~~	Constant
C.	Constant	~~Constant~~
D.	Constant	Increases

·The horizontal component of velocity is $v_x = u_x$ which is CONSTANT.

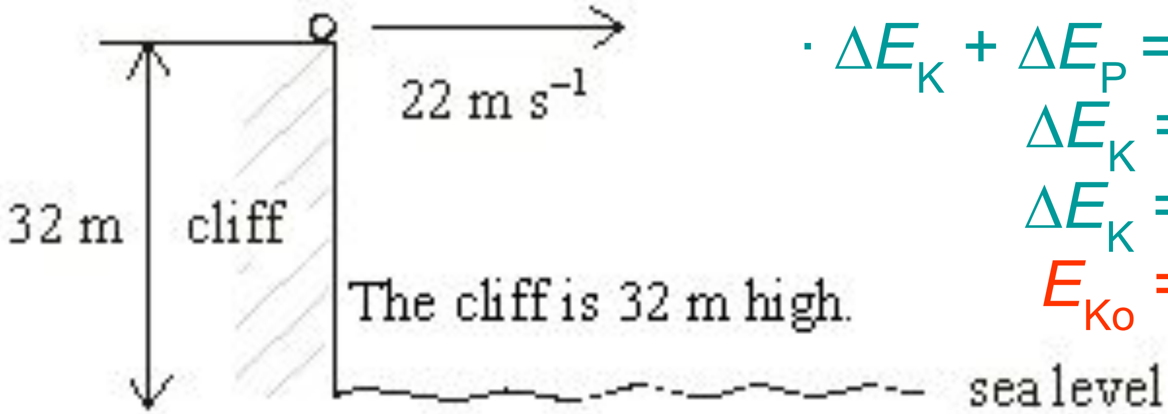
·The vertical component of velocity is $v_y = u_y - 10t$, which is INCREASING (negatively).

A.1 Kinematics SL

Analysing projectile motion

This question is about projectile motion.

A stone of mass 0.44 kg is thrown horizontally from the top of a cliff with a speed of 22 m s^{-1} as shown below.



· $\Delta E_K + \Delta E_P = 0$

$\Delta E_K = -\Delta E_P$

$\Delta E_K = -mg\Delta h$

$E_{Ko} = (1/2)mu^2$

(a) Calculate the total kinetic energy of the stone at sea level assuming air resistance is negligible.

$\Delta E_K = -(0.44)(9.8)(-32) = {}^{+}138 \text{ J} = E_K - E_{Ko}$

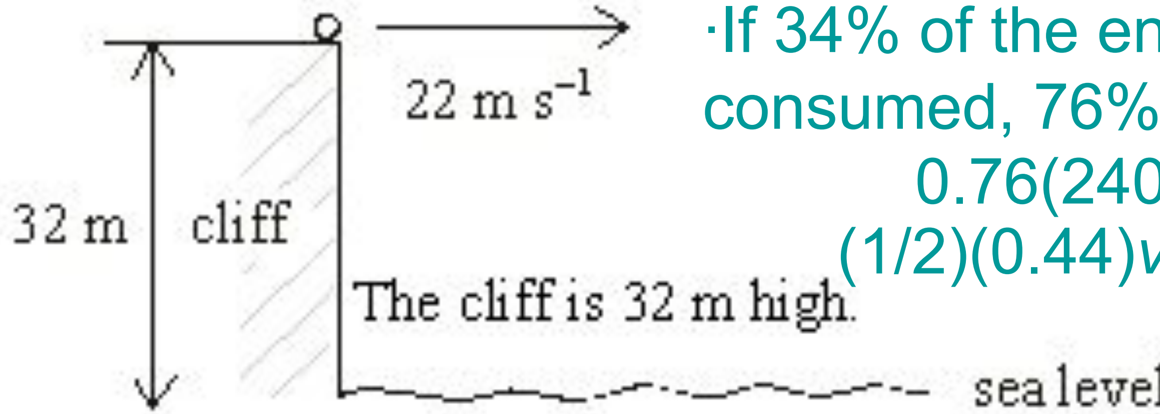
$E_K = {}^{+}138 + (1/2)(0.44)(22^2) = 240 \text{ J}.$

A.1 Kinematics SL

Analysing projectile motion

This question is about projectile motion.

A stone of mass 0.44 kg is thrown horizontally from the top of a cliff with a speed of 22 m s^{-1} as shown below.



The cliff is 32 m high.

·If 34% of the energy is consumed, 76% remains.

$0.76(240) = 180$ J

$(1/2)(0.44)v^2 = 180$ J

(b) In practice, air resistance is not negligible. During the motion of the stone from the top of the cliff to sea level, 34% of the total energy of the stone is transferred due to air resistance. Determine the speed at which the stone reaches sea level.

·$v = 29$ ms^{-1}.

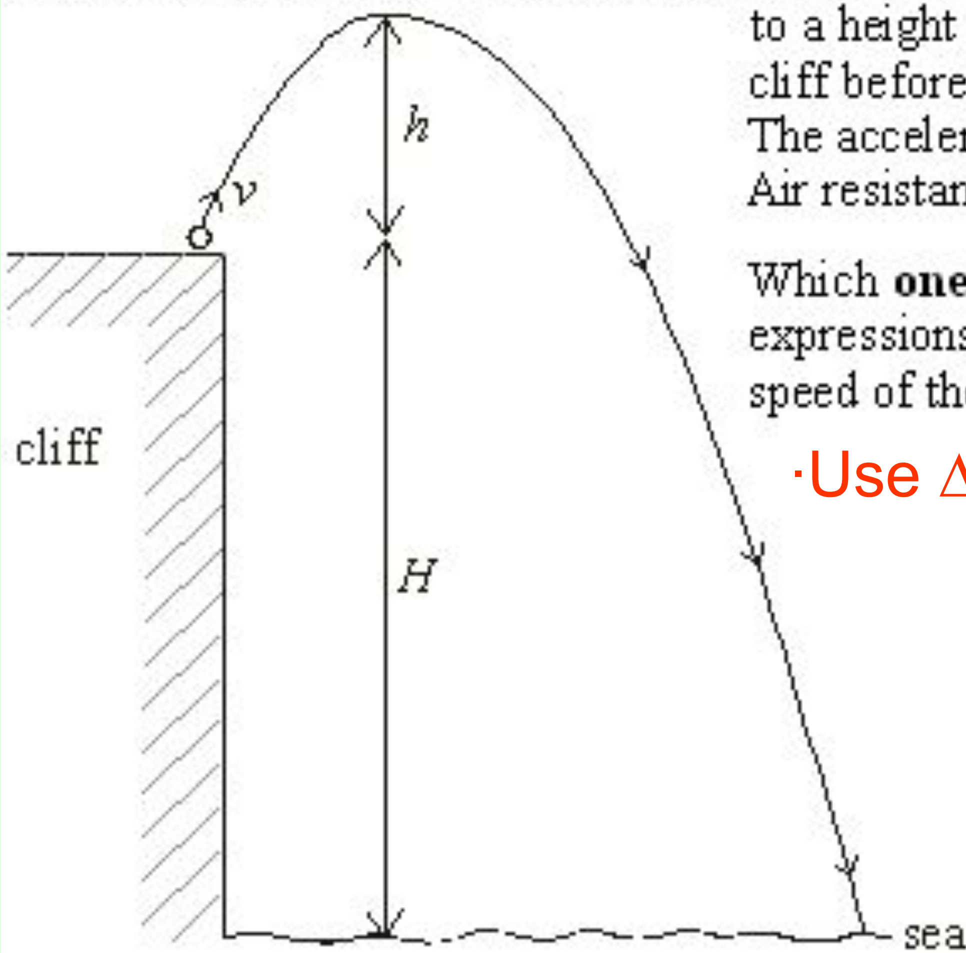
....

$$(1/2)mv_f^2 - (1/2)mv^2 = -\Delta E_P$$
$$\cancel{m}v_f^2 = \cancel{m}v^2 + -2\cancel{m}g(0-H)$$
$$v_f^2 = v^2 + 2gH$$

A.1 Kinematics SL

Analysing projectile motion

A stone is thrown with speed v from the top of a cliff of height H, as shown. The stone is thrown at an angle to the horizontal so that it rises to a height h above the top of the cliff before falling into the sea. The acceleration of free fall is g. Air resistance is negligible.



Which **one** of the following expressions gives correctly the speed of the stone as it hits the sea?

·Use $\Delta E_K + \Delta E_P = 0$.

A. $v + \sqrt{(2gh)}$

B. $v + \sqrt{(2gH)}$

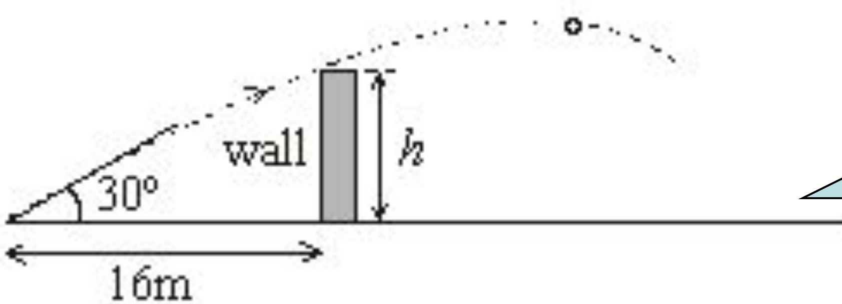
C. $\sqrt{(2g(h + H))}$

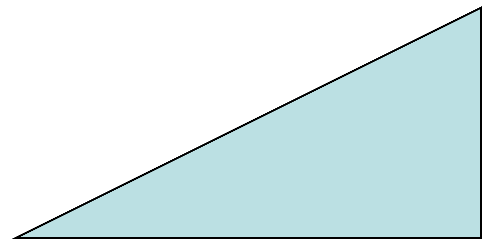
D. $\sqrt{(v^2 + 2gH)}$

A.1 Kinematics SL

Analysing projectile motion

A ball is projected from ground level with a speed of 28 m s^{-1} at an angle of 30° to the horizontal as shown below.





$u_y = u \sin \theta$

$u_y = 28 \sin 30°$

$u_x = u \cos \theta$

$u_x = 28 \cos 30°$

There is a wall of height h at a distance of 16 m from the point of projection of the ball. Air resistance is negligible.

(a) Calculate the initial magnitudes of

(i) the horizontal velocity of the ball;

$u_x = 24$ m s^{-1}. ....

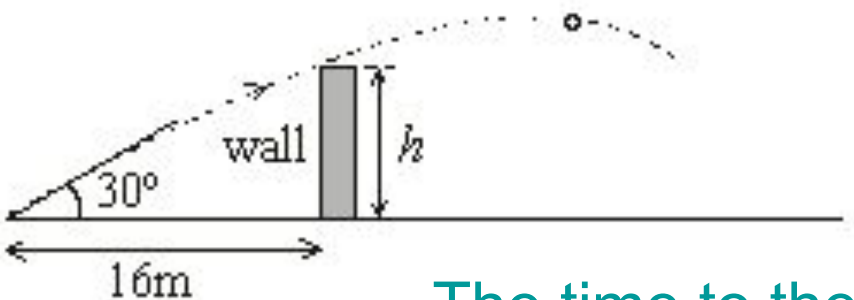
(ii) the vertical velocity of the ball.

$u_y = 14$ m s^{-1}. ....

A.1 Kinematics SL

Analysing projectile motion

A ball is projected from ground level with a speed of 28 m s^{-1} at an angle of 30° to the horizontal as shown below.



$\Delta x = u_x t$

$16 = 24t$

$t = 16 / 24 = 0.67$

- The time to the wall is found from Δx…

There is a wall of height h at a distance of 16 m from the point of projection of the ball. Air resistance is negligible.

(b) The ball just passes over the wall. Determine the maximum height of the wall.

$\Delta y = u_y t - 5t^2$

....

$\Delta y = 14t - 5t^2$

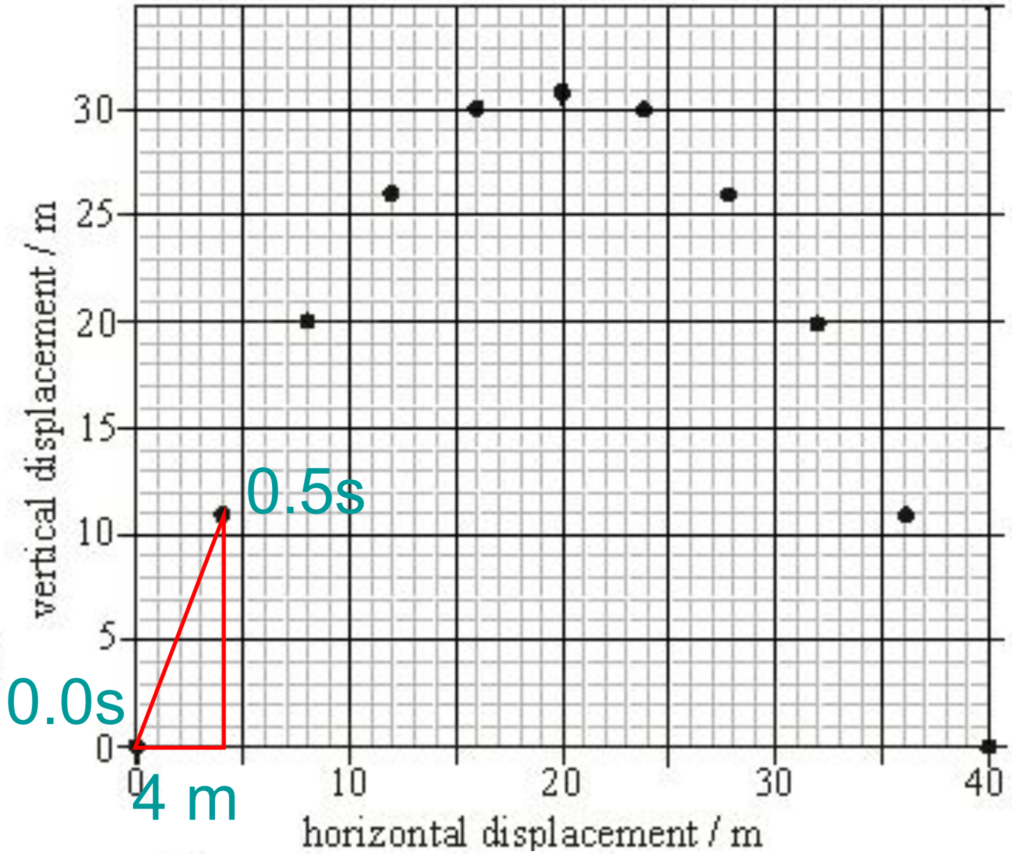
....

$\Delta y = 14(0.67) - 5(0.67)^2 = 7.1$ m.

A.1 Kinematics SL

Analysing projectile motion

A ball is kicked at an angle to the horizontal. The diagram shows the position of the ball every 0.50 s.



The acceleration of free fall is $g = 10$ m s^{-2}. Air resistance may be neglected.

(a) Using the diagram determine, for the ball

(i) the horizontal component of the initial velocity.

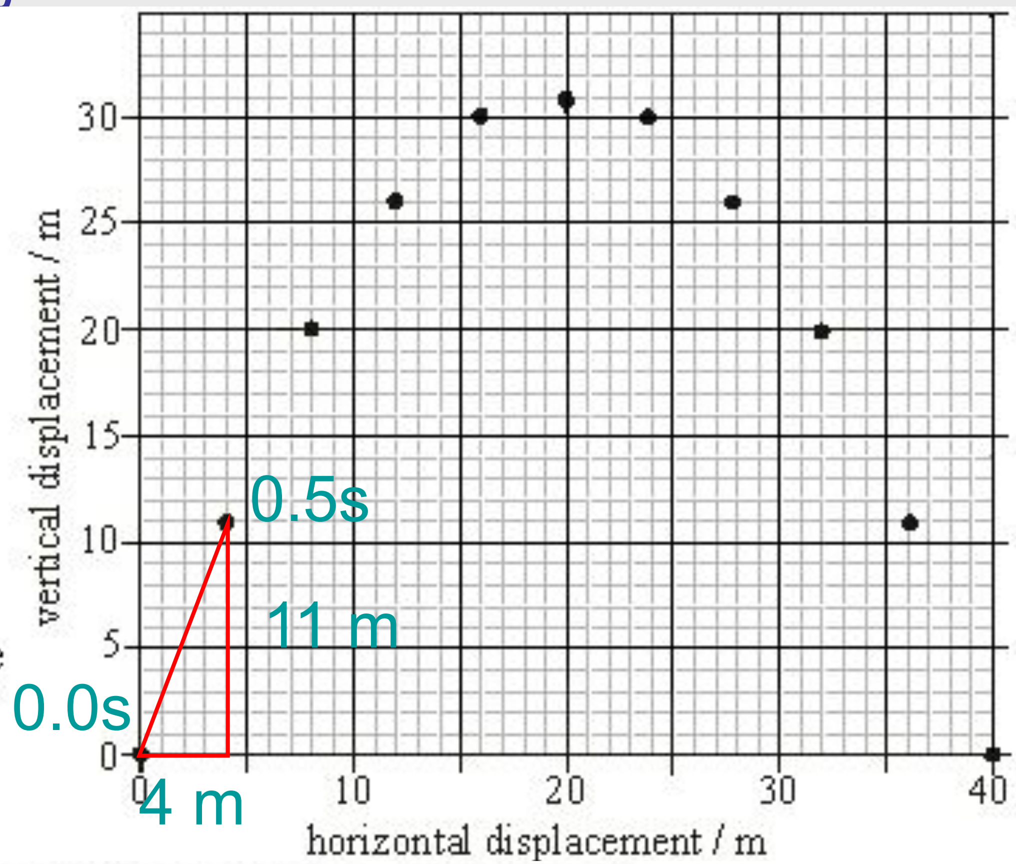
$u_x = \Delta x / \Delta t = (4 - 0) / (0.5 - 0.0) = 8$ ms^{-1}.

A.1 Kinematics SL

Analysing projectile motion

A ball is kicked at an angle to the horizontal. The diagram shows the position of the ball every 0.50 s.

The acceleration of free fall is $g = 10\ \text{m s}^{-2}$. Air resistance may be neglected.



(a) Using the diagram determine, for the ball

(ii) the vertical component of the initial velocity.

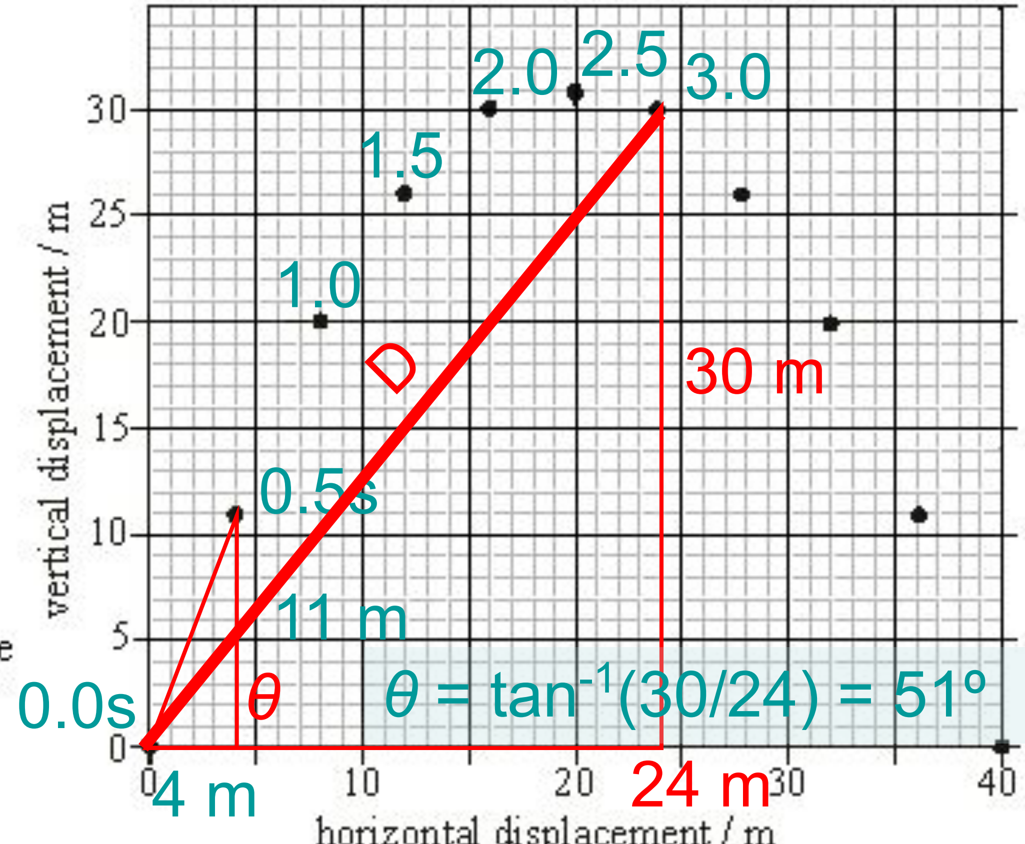
$u_y = \Delta y / \Delta t = (11 - 0) / (0.5 - 0.0) = 22\ \text{ms}^{-1}$.

A.1 Kinematics SL

Analysing projectile motion

A ball is kicked at an angle to the horizontal. The diagram shows the position of the ball every 0.50 s.

The acceleration of free fall is $g = 10\ \mathrm{m\ s^{-2}}$. Air resistance may be neglected.



(a) Using the diagram determine, for the ball

(iii) the magnitude of the displacement after 3.0 s.

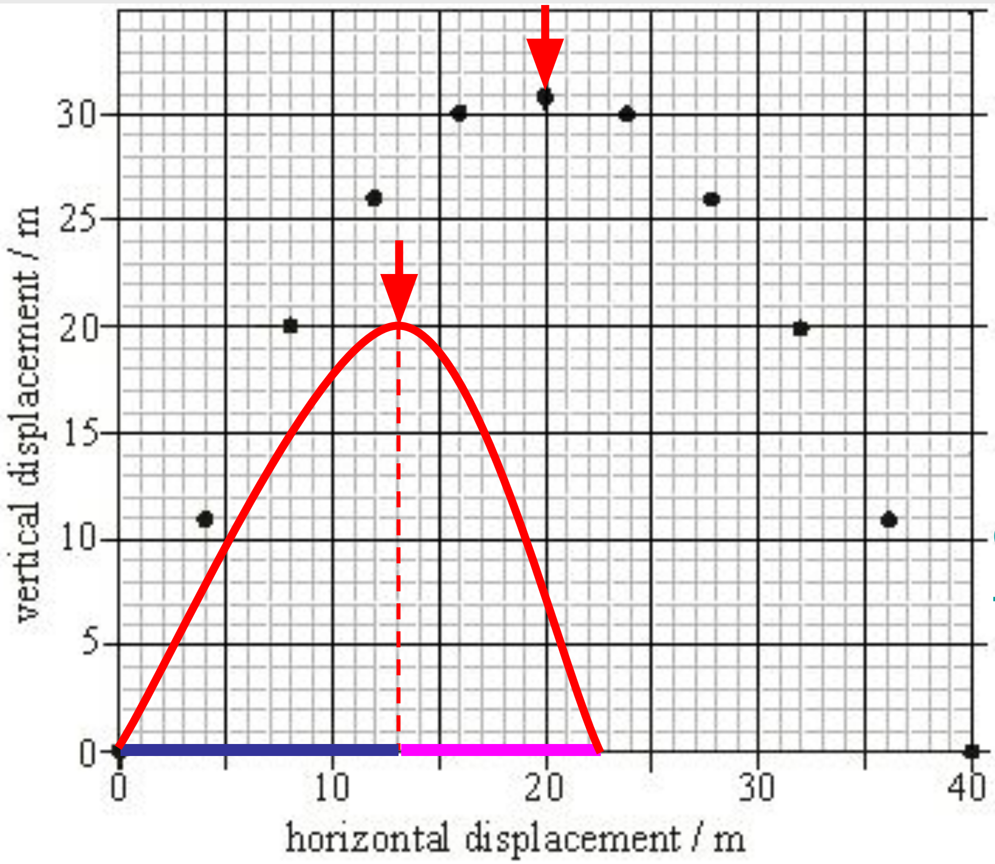
$D^2 = 24^2 + 30^2$ so that D = 38 m ,@ θ = 51°.

A.1 Kinematics SL

Analysing projectile motion

A ball is kicked at an angle to the horizontal. The diagram shows the position of the ball every 0.50 s.

The acceleration of free fall is $g = 10\ \mathrm{m\ s^{-2}}$. Air resistance may be neglected.



(b) On the diagram above draw a line to indicate a possible path for the ball if air resistance were not negligible.

·New peak below and left.

·Pre-peak greater than post-peak.