

INTERNATIONAL BACCALAUREATE  
*Mathematics: analysis and approaches*  
**MAA**

**EXERCISES [MAA 4.9]**  
**DISCRETE DISTRIBUTIONS IN GENERAL**  
*Compiled by Christos Nikolaidis*

**O. Practice questions**

1. [Maximum mark: 6] **[with / without GDC]**

The probability distribution of the discrete random variable  $X$  is given by the table

$x$	1	2	3	4	5
$P(X = x)$	0.4	0.2	0.15	0.15	0.1

Find

- (a) the expected value  $E(X)$  of  $X$ . [2]
- (b) the mode of  $X$ . [1]
- (c) the median of  $X$ . [1]
- (d) the lower quartile  $Q_1$  and the upper quartile  $Q_3$ . [2]

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2. [Maximum mark: 6] **[without GDC]**

The probability distribution of the discrete random variable  $X$  is given by the table

$x$	1	2	3
$P(X = x)$	$a$	$2a$	$b$

Find the values of  $a$  and  $b$  given that  $E(X) = 2.2$

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3. [Maximum mark: 6] **[without GDC]**

The probability distribution of the discrete random variable  $X$  is given by the table

$x$	1	2	3
$P(X = x)$	0.2	0.4	0.4

Nikos selects a number at random.

If he selects 1 he earns 10 €. If he selects 2 he earns 5 €. If he selects 3 he loses 4 €

- (a) Find the expected value of  $X$ . [3]
- (b) Find the expected value of the profit for Nikos. Is the game fair? [3]

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4. [Maximum mark: 5] **[without GDC]**

A discrete random variable  $X$  has its probability distribution given by

$$P(X = x) = \frac{x}{6}, \text{ where } x \text{ is } 1, 2, 3.$$

- (a) Complete the following table showing the probability distribution for  $X$ . [2]

$x$	1	2	3
$P(X = x)$			

- (b) Find  $E(X)$ . [3]

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5. [Maximum mark: 5] **[without GDC]**

Each of the following 10 words is placed on a **card** and put in a hat.

**ONE, TWO, THREE, FOUR, FIVE, SIX, SEVEN, EIGHT, NINE, TEN**

We pick a card at random. Let  $X$  be the size (number of letters) of the corresponding word.

- (a) Give the probability distribution for  $X$  (i.e. the table of probabilities). [3]

- (b) Find the expected number of  $X$ . [2]

(a)

	ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE	TEN
$X$	3	3	5	4	4	3	5	5	4	3
$P(X=x)$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

(b)  $E(X) = \frac{3+3+5+4+4+3+5+5+4+3}{10} = \frac{39}{10} = 3.9$

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6. [Maximum mark: 18] **[with / without GDC] mainly for HL**

The table below shows the probability distribution of a discrete random variable  $X$ .

$x$	0	1	2
$P(X = x)$	0.3	0.2	0.5

Complete the following table.

$E(X)$	
$E(X^2)$	
$E(X^3)$	
$\text{Var}(X)$ using $E(X^2) - E(X)^2$	
$\text{Var}(X)$ using $E(X - \mu)^2$	
MODE	
MEDIAN	
$Q_1$	
$Q_3$	
$E(2X + 1)$	
$\text{Var}(2X + 1)$	

7\*. [Maximum mark: 8] **[with / without GDC]** **only for HL**

A discrete random variable  $X$  has its probability distribution given by

$$P(X = x) = \frac{x^2}{14}, \text{ where } x \text{ is } 1, 2, 3, \dots, n.$$

(a) Find the value of  $n$ . [2]

(b) Find (i)  $E(X)$  (ii)  $E(X^2)$  (iii)  $\text{Var}(X)$ . [6]

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8. [Maximum mark: 5] **[with GDC]** **only for HL**

The table below shows the probability distribution of a discrete random variable  $X$ .

$x$	0	1	2
$P(X = x)$	$a$	$b$	$c$

Given that  $E(X) = 1$  and  $\text{Var}(X) = 0.8$  find the values of  $a$ ,  $b$  and  $c$ .

$$1 = b + 2c$$

$$0.8 = (0-1)^2 a + (1-1)^2 b + (2-1)^2 c$$

$$0.8 = a + c$$

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**A. Exam style questions (SHORT)**

9. [Maximum mark: 6] **[with GDC]**

The probability distribution of the discrete random variable  $X$  is given by the following table.

$x$	1	2	3	4	5
$P(X = x)$	0.4	$p$	0.2	0.07	0.02

- (a) Find the value of  $p$ . [3]  
 (b) Calculate the expected value of  $X$ . [3]

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10. [Maximum mark: 6] **[without GDC]**

A discrete random variable  $X$  has a probability distribution as shown in the table below.

$x$	0	1	2	3
$P(X = x)$	0.1	$a$	0.3	$b$

- (a) Find the value of  $a + b$ . [2]  
 (b) Given that  $E(X) = 1.5$ , find the value of  $a$  and of  $b$ . [4]

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11. [Maximum mark: 5] **[with GDC]**

The table below shows the probability distribution of a discrete random variable  $X$ .

$x$	0	1	2	3
$P(X = x)$	0.2	$a$	$b$	0.25

Given that  $E(X) = 1.55$ , find the value of  $a$  and of  $b$ .

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12\*. [Maximum mark: 6] **[with GDC]**

In a game a player rolls a biased tetrahedral (four-faced) die. The probability of each possible score is shown below.

Score	1	2	3	4
Probability	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{10}$	$x$

Find the probability of a total score of six after two rolls.

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13. [Maximum mark: 6] **[with GDC]**

The following table shows the probability distribution of a discrete random variable  $X$ .

$x$	-1	0	2	3
$P(X = x)$	0.2	$10k^2$	0.4	$3k$

- (a) Find the value of  $k$ . [3]  
 (b) Find the expected value of  $X$ . [3]

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14. [Maximum mark: 6] **[without GDC]**

A discrete random variable  $X$  has its probability distribution given by

$$P(X = x) = k(x + 1), \text{ where } x \text{ is } 0, 1, 2, 3, 4.$$

- (a) Complete the following table showing the probability distribution for  $X$  (in terms of  $k$ ) [2]

$x$	0	1	2	3	4
$P(X = x)$					

- (b) Show that  $k = \frac{1}{15}$ . [1]  
 (c) Find  $E(X)$ . [3]

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15. [Maximum mark: 6] **[without GDC]**

The probability distribution of a discrete random variable  $X$  is defined by

$$P(X = x) = cx(5 - x), \quad x = 1, 2, 3, 4.$$

(a) Find the value of  $c$  [3]

(b) Find  $E(X)$ . [3]

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16. [Maximum mark: 5]

The probability distribution of a discrete random variable  $X$  is given by

$$P(X = x) = k \left( \frac{2}{3} \right)^x \quad \text{for } x = 0, 1, 2, \dots$$

Find the value of  $k$ .

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(a) Kim states that the probability distribution for her pack of cards is as follows.

$x$	0	3	4	9
$P(X = x)$	0.3	0.45	0.2	0.35

[2]

- |            |     |     |      |     |
|------------|-----|-----|------|-----|
| $x$        | 0   | 3   | 4    | 9   |
| $P(X = x)$ | 0.4 | $k$ | $2k$ | 0.3 |

[2]

- the probability that the number on the card drawn

- [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

18. [Maximum mark: 8]

A biased die with four faces is used in a game. A player pays 10 counters to roll the die. The table below shows the possible scores on the die, the probability of each score and the number of counters the player receives in return for each score.

Score	1	2	3	4
Probability	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{10}$
Number of counters player receives	4	5	15	$n$

(a) The player throws the die twice. Find the probability that

(i) he has a total score of 3.

(ii) he has a total score of 4.

[4]

(b) Find the value of  $n$  in order for the player to get an expected return of 9 counters per roll.

$$(a)(i) \frac{1}{2} \cdot \frac{1}{5} = \frac{1}{10} = 0.1 \quad (ii) \left(\frac{1}{5}\right)^2 + \left(\frac{1}{5} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{5}\right) = 0.1$$

$$(b) 9 = \frac{1}{2}(4) + \frac{1}{5}(5) + \frac{1}{5}(15) + \frac{1}{10}(n)$$

$$9 = 2 + 1 + 3 + \frac{n}{10}$$

$$3 = \frac{n}{10}$$

$$n = 30$$

**B. Exam style questions (LONG)**

**19.** [Maximum mark: 15] *[without GDC]*

Two fair **four**-sided dice, one red and one green, are thrown. For each die, the faces are labelled 1, 2, 3, 4. The score for each die is the number which lands face down.

The sample space is shown below:

	1	2	3	4
1	•	•	•	•
2	•	•	•	•
3	•	•	•	•
4	•	•	•	•

- (a) Write down the probability that two scores of 4 are obtained. [1]

Let  $X$  be the number of 4s that land face down.

- (b) Complete the following probability distribution for  $X$ . [3]

$x$	0	1	2
$P(X = x)$			

- (c) Find  $E(X)$ . [3]

Chris plays a game where he rolls the dice above.

If two 4s are obtained he wins 20€.

If only one 4 is obtained he wins 5€.

If no 4 is obtained he loses 2€

- (d) Find the expected amount earned in one game. [3]

- (e) If Chris plays this game 100 times find the amount he is expected to win. [2]

- (f) If Chris plays this game twice find the probability that he earns 18€. [3]

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[illegible]



[illegible]

21. [Maximum mark: 16] **[with GDC]**

A **four-sided** die has three blue faces and one red face. The die is rolled.

Let  $B$  be the event a blue face lands down, and  $R$  be the event a red face lands down.

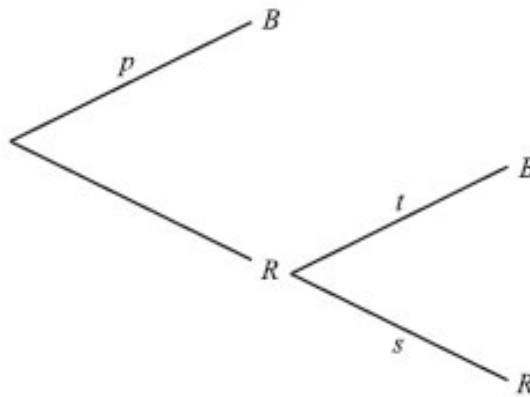
(a) Write down the values of

(i)  $P(B)$

(ii)  $P(R)$

[2]

(b) If the blue face lands down, the die is not rolled again. If the red face lands down, the die is rolled once again. This is represented by the following tree diagram, where  $p, s, t$  are probabilities.



Find the value of  $p$ , of  $s$  and of  $t$ .

[2]

Guiseppe plays a game where he rolls the die. If a blue face lands down, he scores 2 and is finished. If the red face lands down, he scores 1 and rolls one more time. Let  $X$  be the total score obtained.

(c) (i) Show that  $P(X = 3) = \frac{3}{16}$ .

(ii) Find  $P(X = 2)$

[3]

(d) (i) Construct a probability distribution table for  $X$ .

(ii) Calculate the expected value of  $X$ .

[5]

(e) If the total score is 3, Guiseppe wins \$10. If the total score is 2, Guiseppe gets nothing. He plays the game twice. Find the probability that he wins exactly \$10.

[4]

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This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

22\*. [Maximum mark: 18] **[with GDC]**

A box contains 2 red and 8 blue balls. Find

- (a) the expected number of **red balls** if we select two balls with replacement. [4]
- (b) the expected number of **red balls** if we select two balls without replacement. [4]
- (c) the expected number of **red balls** if we select three balls without replacement. [5]
- (d) the expected number of **trials** if we select a ball without replacement until we find a blue one. [5]

$$(a) E(x) = \frac{2}{10} \cdot \frac{1}{9} = \frac{2}{90} = \frac{1}{45}$$



**24\*\*.** [Maximum mark: 15] **[with GDC]**

Two children, Alan and Belle, each throw two fair cubical dice simultaneously. The score for each child is the sum of the two numbers shown on their respective dice.

- (a) (i) Calculate the probability that Alan obtains a score of 9. [3]  
 (ii) Calculate the probability that Alan and Belle both obtain a score of 9. [3]
- (b) (i) Calculate the probability that Alan and Belle obtain the same score, [5]  
 (ii) Deduce the probability that Alan's score exceeds Belle's score. [5]
- (c) Let  $X$  denote the largest number shown on the four dice.
- (i) Show that for  $P(X \leq x) = \left(\frac{x}{6}\right)^4$ , for  $x = 1, 2, 3, 4, 5, 6$
- (ii) Complete the following probability distribution table.

$x$	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{1296}$	$\frac{15}{1296}$				$\frac{671}{1296}$

- (iii) Calculate  $E(X)$ . [7]

(a)(i)