

INTERNATIONAL BACCALAUREATE
Mathematics: analysis and approaches
MAA

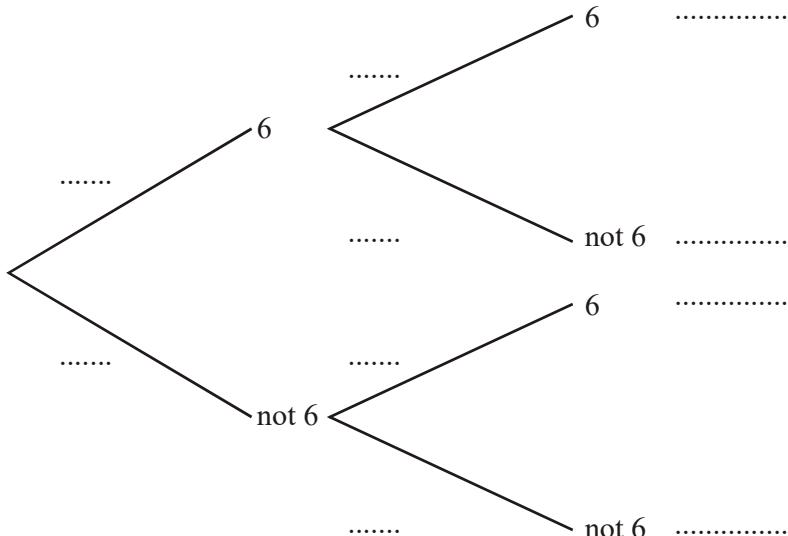
EXERCISES [MAA 4.8]
PROBABILITY II (TREE DIAGRAMS)
Compiled by Christos Nikolaidis

O. Practice questions

1. [Maximum mark: 7] **[without GDC]**

Two ordinary, 6-sided dice are rolled and the total score is noted.

- (a) Complete the tree diagram by entering probabilities and listing outcomes. [2]

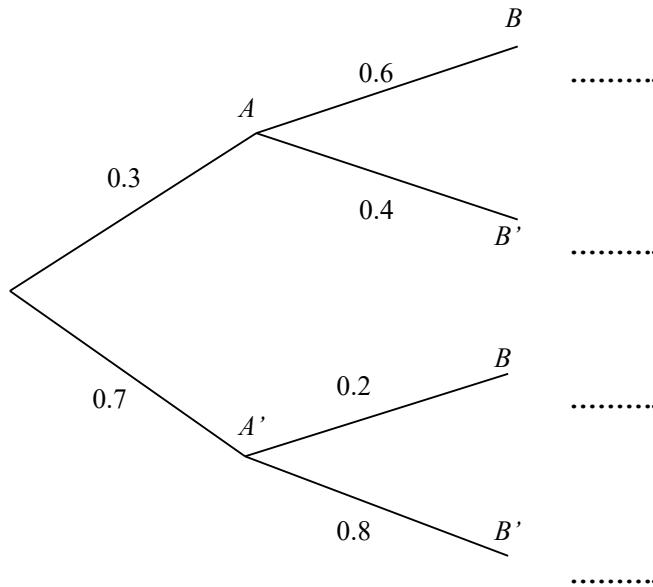


- (b) Find the probability of getting two sixes. [1]
(c) Find the probability of getting one six only. [2]
(d) Find the probability of getting one or more sixes. [2]

.....
.....
.....
.....
.....
.....
.....
.....

2. [Maximum mark: 10] **[without GDC]**

In the following diagram complete the probabilities of the four different scenarios



- (a) Write down the following probabilities

[3]

| | | | | | |
|----------------|--|----------------|--|-----------------|--|
| $P(A)$ | | $P(A')$ | | $P(A \cap B)$ | |
| $P(A \cap B')$ | | $P(A' \cap B)$ | | $P(A' \cap B')$ | |

- (b) Calculate the following probabilities

[3]

| | |
|---------------|--|
| $P(B)$ | |
| $P(B')$ | |
| $P(A \cup B)$ | |

- (c) Write down the following conditional probabilities

[2]

| | | | | | | | |
|------------|--|-------------|--|-------------|--|--------------|--|
| $P(B A)$ | | $P(B' A)$ | | $P(B A')$ | | $P(B' A')$ | |
|------------|--|-------------|--|-------------|--|--------------|--|

- (d) Calculate the following conditional probability

[2]

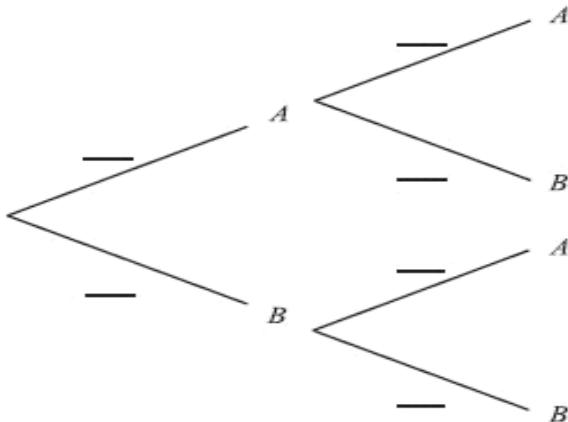
| | |
|------------|--|
| $P(A B)$ | |
|------------|--|

A. Exam style questions (SHORT)

3. [Maximum mark: 6] **[with GDC]**

A bag contains four apples (A) and six bananas (B). A fruit is taken from the bag and eaten. Then a second fruit is taken and eaten.

- (a) Complete the tree diagram below by writing probabilities in the spaces provided. [3]



- (b) Find the probability that one of each type of fruit was eaten. [3]
-
.....
.....

4. [Maximum mark: 5] **[without GDC]**

A girl walks to school every day. If it is not raining, the probability that she is late is $\frac{1}{5}$.

If it is raining, the probability that she is late is $\frac{2}{3}$. The probability that it rains on a particular day is $\frac{1}{4}$. On one particular day the girl is late. Find the probability that it was raining on that day.

.....
.....
.....
.....
.....
.....
.....
.....
.....

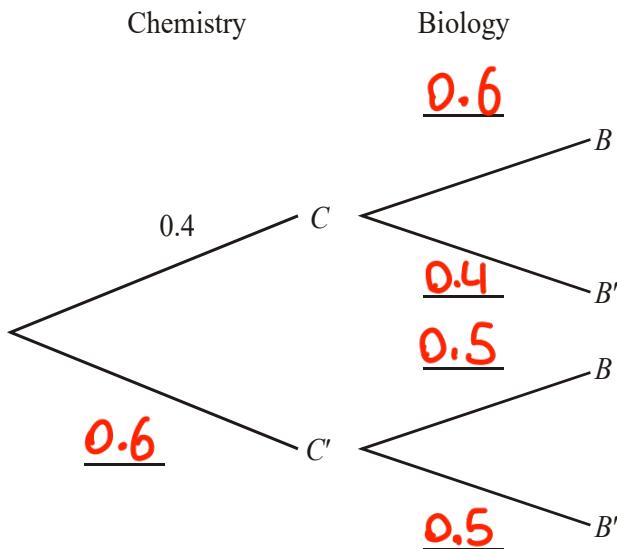
5. [Maximum mark: 5] **[with GDC]**

The events B and C are dependent, where C is the event “a student takes Chemistry”, and B is the event “a student takes Biology”. It is known that

$$P(C) = 0.4, \quad P(B | C) = 0.6, \quad P(B | C') = 0.5.$$

- (a) Complete the following tree diagram.

[1]



- (b) Calculate the probability that a student takes Biology. [2]

- (c) Given that a student takes Biology, what is the probability that the student takes Chemistry? [2]

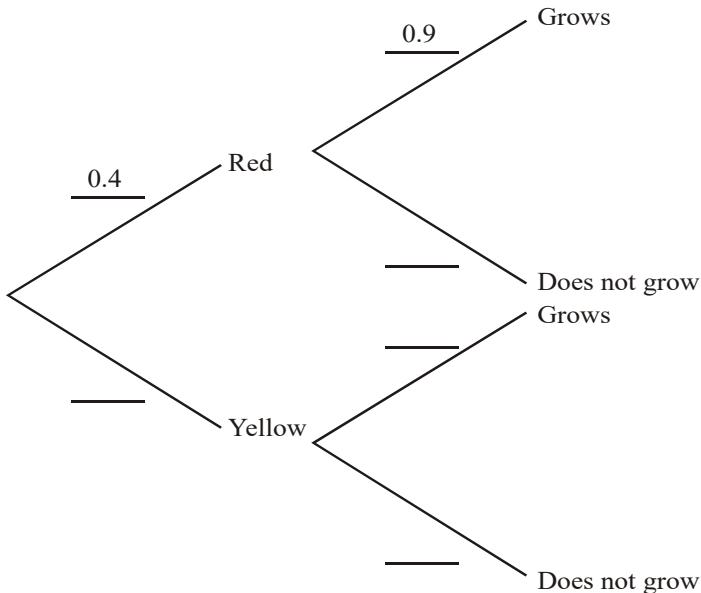
(b) $0.4(0.6) + 0.6(0.5) = 0.54$

(c) 0.4

- 6.** [Maximum mark: 8] **[with GDC]**

A packet of seeds contains 40% red seeds and 60% yellow seeds. The probability that a red seed grows is 0.9, and that a yellow seed grows is 0.8. A seed is chosen at random from the packet.

- (a) Complete the probability tree diagram below.



[3]

- (b) (i) Calculate the probability that the chosen seed is red and grows.
(ii) Calculate the probability that the chosen seed grows.
(iii) Given that the seed grows, calculate the probability that it is red.

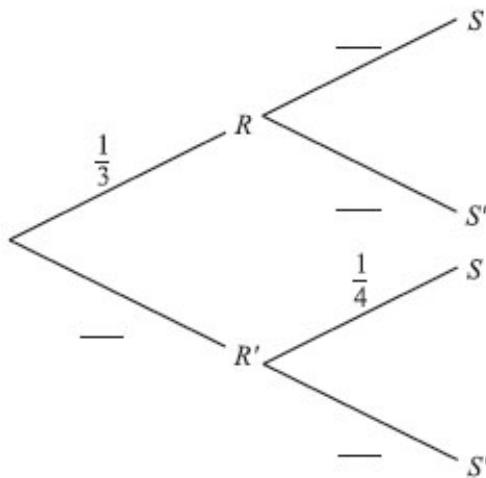
[5]

7. [Maximum mark: 8] *[without GDC]*

The following probabilities were found for two events R and S .

$$P(R) = \frac{1}{3}, P(S | R) = \frac{4}{5}, P(S | R') = \frac{1}{4}.$$

- (a) Complete the tree diagram.



[3]

- (b) Find the following probabilities.

(i) $P(R \cap S)$.

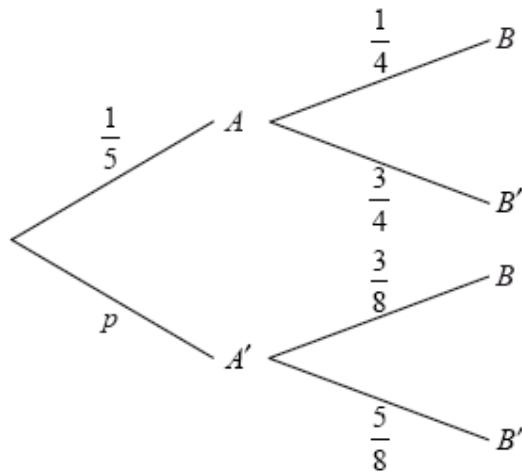
(ii) $P(S)$.

$$\text{(iii)} \quad P(R \mid S).$$

[5]

8. [Maximum mark: 7] ***[without GDC]***

The diagram below shows the probabilities for events A and B , with $P(A') = p$.



- (a) Write down the value of p . [1]
(b) Find $P(B)$. [3]
(c) Find $P(A' | B)$. [3]

9. [Maximum mark: 7] **[with GDC]**

A company uses two machines, A and B, to make boxes. Machine A makes 60 % of the boxes.

80 % of the boxes made by machine A pass inspection.

90 % of the boxes made by machine B pass inspection.

A box is selected at random.

- (a) Find the probability that it passes inspection. [3]

(b) The company would like the probability that a box passes inspection to be 0.87.
Find the percentage of boxes that should be made by machine B to achieve this. [4]

- 10.** [Maximum mark: 8] ***[with GDC]***

In any given season, a soccer team plays 65 % of their games at home.

When the team plays at home, they win 83 % of their games.

When they play away from home, they win 26 % of their games.

The team plays one game.

- (a) Find the probability that the team wins the game. [4]

(b) If the team does not win the game, find the probability that the game was played at home. [4]

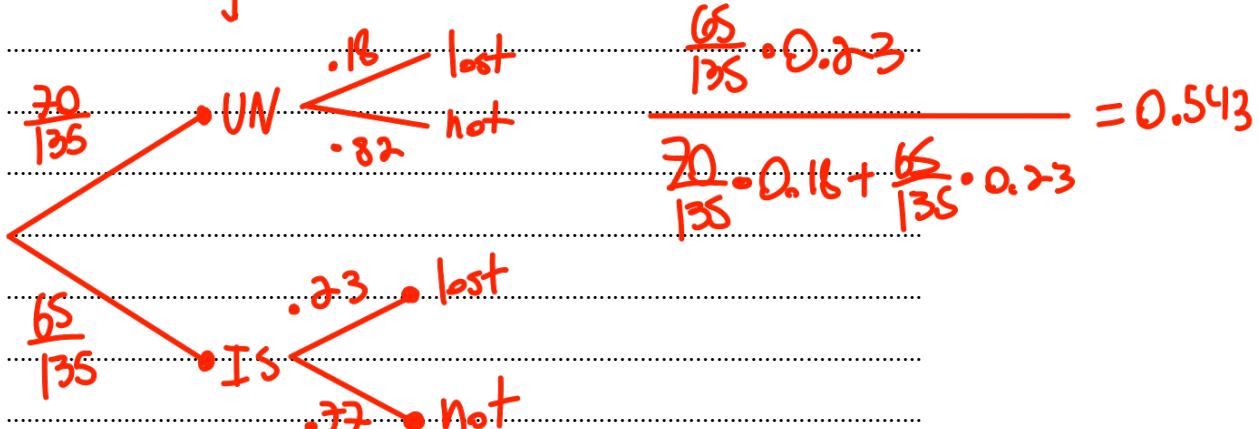
- 11.** [Maximum mark: 6] ***[with GDC]***

The probability that it rains during a summer's day in a certain town is 0.2. In this town, the probability that the daily maximum temperature exceeds 25°C is 0.3 when it rains and 0.6 when it does not rain. Given that the maximum daily temperature exceeded 25°C on a particular summer's day, find the probability that it rained on that day.

- 12.** [Maximum mark: 6] **[with GDC]**

Only two international airlines fly daily into an airport. UN Air has 70 flights a day and IS Air has 65 flights a day. Passengers flying with UN Air have an 18% probability of losing their luggage and passengers flying with IS Air have a 23% probability of losing their luggage. You overhear someone in the airport complain about her luggage being lost. Find the probability that she travelled with IS Air.

Total flights = 135



13. [Maximum mark: 6] **[with GDC]**

Robert travels to work by train every weekday from Monday to Friday. The probability that he catches the 08.00 train on Monday is 0.66. The probability that he catches the 08.00 train on any other weekday is 0.75. A weekday is chosen at random.

- (a) Find the probability that he catches the train on that day. [3]
(b) Given that he catches the 08.00 train on that day, find the probability that the chosen day is Monday. [3]

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

14. [Maximum mark: 6] **[with GDC]**

The local Football Association consists of ten teams. Team A has a 40 % chance of winning any game against a higher-ranked team, and a 75 % chance of winning any game against a lower-ranked team. If A is currently in fourth position, find the probability that A wins its next game.

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

- 15.** [Maximum mark: 6] **[with GDC]**

Box A contains 6 red balls and 2 green balls. Box B contains 4 red balls and 3 green balls. A fair cubical die with faces numbers 1, 2, 3, 4, 5, 6 is thrown. If an even number is obtained, a ball is selected from box A; if an odd number is obtained, a ball is selected from box B.

- (a) Calculate the probability that the ball selected was red.
 - (b) Given that the ball selected was red, calculate the probability that it came from Box B.

- 16.** [Maximum mark: 6] *[without GDC]*

Bag 1 contains 4 red cubes and 5 blue cubes. Bag 2 contains 7 red cubes and 2 blue cubes. Two cubes are drawn at random, the first from Bag 1 and the second from Bag 2.

- (a) Find the probability that the cubes are of the same colour.
(b) Given that the cubes selected are of different colours, find the probability that the red cube was selected from Bag 1.

17. [Maximum mark: 7] *[without GDC]*

The probability that a man leaves his umbrella in any shop he visits is $\frac{1}{3}$.

- (a) After visiting two shops in succession, find the probability
(i) that he didn't leave the umbrella in any of them.
(ii) that he left the umbrella in one of them. [4]

(b) After visiting two shops in succession, he finds he has left his umbrella in one of them. What is the probability that he left his umbrella in the second shop? [3]

18**. [Maximum mark: 7] **[with GDC]**

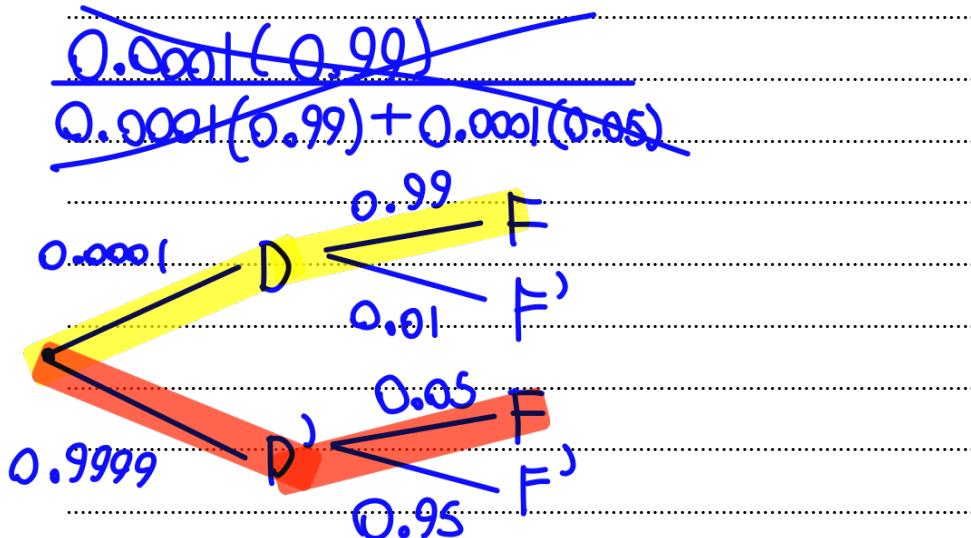
A new blood test has been shown to be effective in the early detection of a disease.

The probability that the blood test correctly identifies someone with this disease is 0.99, and the probability that the blood test correctly identifies someone without that disease is 0.95. The incidence of this disease in the general population is 0.0001.

A doctor administered the blood test to a patient and the test result indicated that this patient had the disease. What is the probability that the patient has the disease?

$$\underline{0.0001 \cdot 0.99}$$

$$\begin{aligned} & \cancel{(0.9999)(0.95)} + \cancel{(0.9999)(0.05)} + (0.0001)(0.99) + 0.0001(0.01) \\ & = 0.000099 \end{aligned}$$



$$P(D|F) \rightarrow 0.0001(0.99) = 0.00098$$

$$P(D|F') \rightarrow 0.0001(0.99) + 0.9999(0.05)$$

19. [Maximum mark: 5] **[with GDC]**

A class contains 13 girls and 11 boys. The teacher randomly selects four students.

Determine the probability that all four students selected are girls.

$$\frac{13}{24} \cdot \frac{12}{23} \cdot \frac{11}{22} \cdot \frac{10}{21} = 0.0673$$

20. [Maximum mark: 6] **[with GDC]**

A painter has 12 tins of paint. Seven tins are red and five tins are yellow. Two tins are chosen at random. Calculate the probability that both tins are the same colour.

- 21.** [Maximum mark: 6] ***[with GDC]***

A box contains 22 red apples and 3 green apples. Three apples are selected at random, one after the other, without replacement.

- (a) The first two apples are green. What is the probability that the third apple is red? [2]
(b) What is the probability that exactly two of the three apples are red? [4]

- 22***. [Maximum mark: 4] *[with GDC]*

A bag contains 10 red balls, 10 green balls and 6 white balls. Two balls are drawn at random from the bag without replacement. What is the probability that they are of different colours?

$$\{R, G\}, \{R, W\}, \{G, W\}, \{G, R\}, \{W, R\}, \{W, G\}$$

$$\frac{10}{86} \cdot \frac{10}{85} + \frac{10}{26} \cdot \frac{6}{25} + \frac{10}{86} \cdot \frac{6}{25} = 0.338$$

$$\frac{10}{20} \cdot \frac{10}{25} + \frac{10}{20} \cdot \frac{6}{25} + \frac{10}{20} \cdot \frac{6}{25} + \frac{10}{20} \cdot \frac{10}{25} + \frac{10}{20} \cdot \frac{6}{25} + \frac{10}{20} \cdot \frac{6}{25} = 0.677$$

23*. [Maximum mark: 7] **[without GDC]**

A bag contains 2 red balls, 3 blue balls and 4 green balls. A ball is chosen at random from the bag and is not replaced. A second ball is chosen.

- (a) Find the probability of choosing two green balls. [2]
- (b) Find the probability of choosing two balls of the same colour. [2]
- (c) Find the probability of choosing one green ball and one blue ball in any order. [3]

Total balls = 9

$$(a) \frac{4}{9} \cdot \frac{3}{8} = \frac{12}{72} = \frac{1}{6}$$

$$(b) \frac{4}{9} \cdot \frac{3}{8} + \frac{3}{9} \cdot \frac{2}{8} + \frac{2}{9} \cdot \frac{1}{8} = \frac{12}{72} + \frac{6}{72} + \frac{2}{72} = \frac{20}{72} = \frac{5}{18}$$

$$(c) \frac{4}{9} \cdot \frac{3}{8} + \frac{3}{9} \cdot \frac{4}{8} = \frac{12}{72} + \frac{12}{72} = \frac{24}{72} = \frac{1}{3}$$

$$\frac{\binom{4}{2}}{\binom{9}{2}} = \frac{6}{36} = \frac{1}{6}$$

1,2,3,4

1,2 | 1,3 | 1,4 | 2,3 | 3,4 | 2,4 |

→ This is all options of two green balls over all options of two balls in general.

Option A

[MAA 4.8] PROBABILITY II (TREE DIAGRAMS)

24**. [Maximum mark: 6] **[with GDC]**

There are 25 disks in a bag. Some of them are black and the rest are white. Two are simultaneously selected at random. Given that the probability of selecting two disks of the same colour is equal to the probability of selecting two disks of different colour, how many black disks are there in the bag?

Let $x = \text{No. of black disks}$

Let $25-x = \text{No. of white disks}$

A n choose r means how many different combinations of r numbers you can make out of n options.

$$50\% = \frac{\binom{x}{2} + \binom{25-x}{2}}{\binom{25}{2}}$$

$$\frac{1}{2} \left(\frac{25!}{2!(23!)^2} \right) = \frac{x!}{2!(x-2)!} + \frac{(25-x)!}{2!(23-x)!}$$

$$\frac{1}{2} \left(\frac{25 \cdot 24}{4} \right) = \frac{x(x-1)}{4} + \frac{(25-x)(24-x)}{4}$$

$$\frac{300}{2} = \frac{x^2 - x}{4} + \frac{600 - 49x + x^2}{4}$$

$$300 = 2x^2 - 50x + 600$$

$$0 = 2x^2 - 50x + 300$$

$$0 = x^2 - 25x + 150$$

$$0 = (x^2 - 15x)(-10x + 150)$$

$$0 = x(x-15) - 10(x-15)$$

$$0 = (x-10)(x-15)$$

$$x = 10, 15$$

Option B

[MAA 4.8] PROBABILITY II (TREE DIAGRAMS)

24**. [Maximum mark: 6] **[with GDC]**

There are 25 disks in a bag. Some of them are black and the rest are white. Two are simultaneously selected at random. Given that the probability of selecting two disks of the same colour is equal to the probability of selecting two disks of different colour, how many black disks are there in the bag?

Let $x = \text{No. of black disks}$

Let $25-x = \text{No. of white disks}$

$$P(\text{Same}) = \frac{x(x-1)}{25 \cdot 24} + \frac{(25-x)(24-x)}{25 \cdot 24}$$

$$P(\text{Different}) = \frac{x(25-(x-1))}{25 \cdot 24} + \frac{(25-x)(x-1)}{25 \cdot 24}$$

$$\frac{x^2 - x}{600} + \frac{x^2 - 49x + 600}{600} = \frac{-x^2 + 26x}{600} + \frac{-x^2 + 26x - 25}{600}$$

$$x^2 - x + x^2 - 49x + 600 = -x^2 + 26x - x^2 + 26x - 25$$

$$2x^2 - 50x + 600 = -2x^2 + 52x - 25$$

$$4x^2 - 2x + 625 = 0$$

25*. [Maximum mark: 8] **[without GDC]**

Two women, Ann and Bridget, play a game in which they take it in turns to throw an unbiased six-sided die. The first woman to throw a '6' wins the game. Ann is the first to throw.

- (a) Find the probability that
- (i) Ann wins on her first throw;
 - (ii) Bridget wins on her first throw;
 - (iii) Ann wins on her second throw; [4]
- (b) Find the probability that Ann wins the game. [3]
- (c) Find the probability that Bridget wins the game. [1]

$$(a.i) \frac{1}{6}$$

$$(a.ii) \frac{5}{6} \cdot \frac{1}{6} = \frac{5}{36}$$

$$(a.iii) \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6}$$

$$(b) r = \left(\frac{5}{6}\right)^2$$

$$P(A \text{ wins}) = \frac{\frac{1}{6}}{1 - \left(\frac{5}{6}\right)^2} = \frac{1}{6} \cdot \frac{36}{11} = \frac{6}{11}$$

$$(c) P(B) = \frac{\frac{5}{36}}{1 - \left(\frac{5}{6}\right)^2} = \frac{\frac{1}{6} \cdot \frac{36}{11}}{\frac{36}{11} - \frac{25}{36}} = \frac{1}{5}$$

Si la probabilidad que Ann gana es $\frac{6}{11}$ en $\frac{11}{11}$, que sea 1 o 100%, entonces la que Bridget gana es el resto:

$$1 - \frac{6}{11} = \frac{11}{11} - \frac{6}{11} = \frac{5}{11} = P(B \text{ Wins})$$

26.** [Maximum mark: 6] **[without GDC]**

Jack and Jill play a game, by throwing a die in turn. If the die shows a 1, 2, 3 or 4, the player who threw the die wins the game. If the die shows a 5 or 6, the other player has the next throw. Jack plays first and the game continues until there is a winner.

- (a) Write down the probability that Jack wins on his first throw. [1]
- (b) Calculate the probability that Jill wins on her first throw. [2]
- (c) Calculate the probability that Jack wins the game. [3]

$$(a) \frac{4}{6} = \frac{2}{3}$$

$$(b) \frac{2}{6} \cdot \frac{4}{6} = \frac{8}{36} = \frac{2}{9}$$

$$(c) \frac{4}{6} \quad \frac{2}{6} \cdot \frac{2}{6} \cdot \frac{4}{6}$$

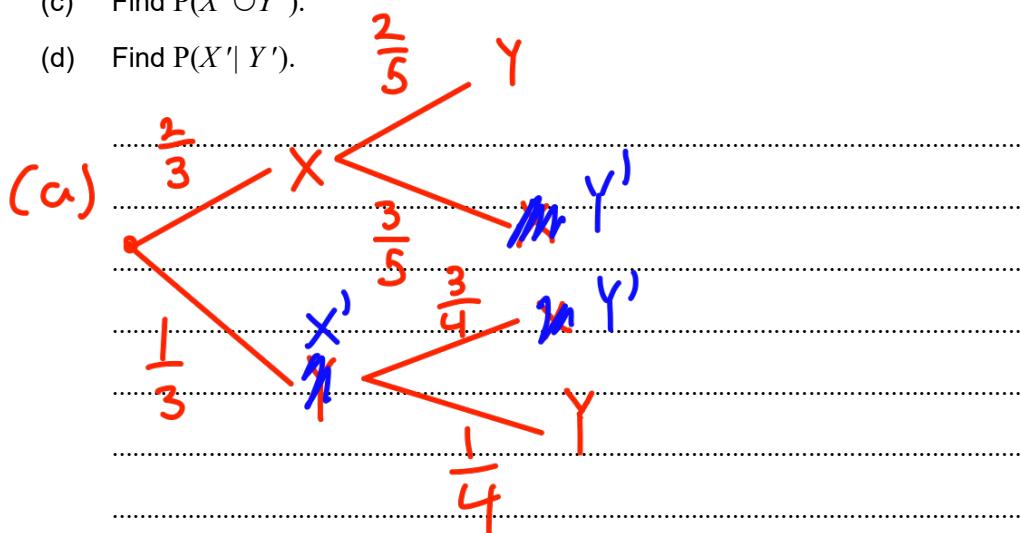
$$\begin{aligned} & \left(1 - \left(\frac{2}{6} \right)^2 \right) \quad \frac{2}{6} \cdot \frac{2}{6} \cdot \frac{2}{6} \cdot \frac{2}{6} \cdot \frac{4}{6} \\ & = \frac{1}{6} \cdot \frac{32}{36} = \frac{6}{36} = \frac{1}{6} \end{aligned}$$

B. Exam style questions (LONG)

27. [Maximum mark: 10] **[with GDC]**

It is given that $P(X) = 2/3$, $P(Y|X) = 2/5$ and $P(Y|X') = 1/4$.

- (a) Draw and complete a tree diagram to represent this information. [3]
 (b) Find $P(Y')$; [2]
 (c) Find $P(X' \cup Y')$. [2]
 (d) Find $P(X'|Y')$. [3]



$$(b) P(Y') = \frac{2}{3} \cdot \frac{3}{5} = \frac{2}{5} \quad \frac{2}{3} \cdot \frac{3}{5} + \frac{1}{3} \cdot \frac{3}{4} = \frac{8}{20} + \frac{5}{20} = \frac{13}{20}$$

$$(c) \frac{1}{3} \cdot \frac{1}{4} + \frac{2}{3} = \frac{1}{12} + \frac{8}{12} = \frac{9}{12} = \frac{3}{4} \quad P(X' \cup Y') = P(X \cap Y')$$

$$(d) \frac{2}{5} \cdot \frac{1}{3} \cdot \frac{3}{4} = \frac{3}{12} = \frac{1}{4}$$

$$= 1 - \left(\frac{2}{3} \cdot \frac{2}{5} \right) = \frac{11}{15}$$

28. [Maximum mark: 10] **[with GDC]**

Dumisani is a student at IB World College. The probability that he will be woken by his alarm clock is $\frac{7}{8}$. If he is woken by his alarm clock the probability he will be late for school is $\frac{1}{4}$.

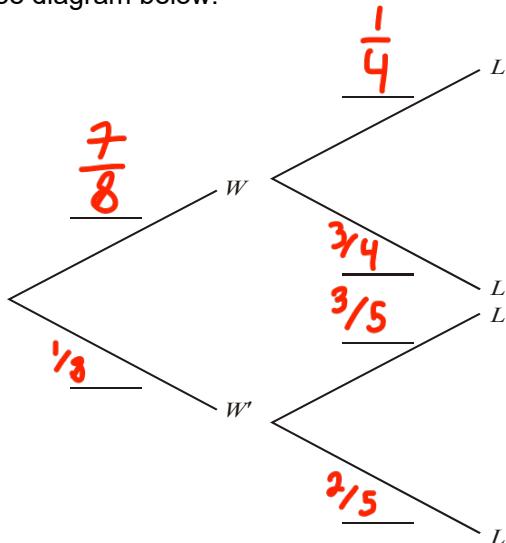
school is $\frac{1}{4}$.

If he is not woken by his alarm clock the probability he will be late for school is $\frac{3}{5}$.

Let W be the event "Dumisani is woken by his alarm clock".

Let L be the event "Dumisani is late for school".

- (a) Complete the tree diagram below.



[3]

- (b) Calculate the probability that Dumisani will be late for school. [3]

- (c) Given that Dumisani is late for school what is the probability that he was woken by his alarm clock? [4]

$$(b) \frac{7}{8} \cdot \frac{1}{4} + \frac{1}{8} \cdot \frac{3}{5} = \frac{7}{32} + \frac{3}{40} = 0.294$$

$$(c) \frac{\frac{7}{8} \cdot \frac{1}{4}}{\frac{1}{8} \cdot \frac{3}{5} + \frac{7}{8} \cdot \frac{1}{4}} = 0.797$$

29. [Maximum mark: 13] **[with GDC]**

Bag A contains 2 red and 3 green balls.

- (a) Two balls are chosen at random from the bag without replacement. Find the probability that 2 red balls are chosen. [2]

Bag B contains 4 red and n green balls.

- (b) Two balls are chosen without replacement from this bag. If the probability that two

$$\text{red balls are chosen is } \frac{2}{15}, \text{ show that } n = 6. \quad [4]$$

A standard die with six faces is rolled. If a 1 or 6 is obtained, two balls are chosen from bag A, otherwise two balls are chosen from bag B.

- (c) Calculate the probability that two red balls are chosen. [4]

- (d) Given that two red balls are chosen, find the probability that a 1 or a 6 was obtained on the die. [3]

$$(a) \frac{2}{5} \cdot \frac{1}{4} = \frac{2}{20} = \frac{1}{10}$$

$$(b) \frac{4}{n-4} \cdot \frac{3}{n-3} = \frac{2}{15}$$

$$\frac{12}{n^2+7n+12} = \frac{2}{15}$$

$$180 = 2n^2 + 14n + 24$$

$$0 = 2n^2 + 14n - 156$$

By GDC: $n = 6$

$$(c) \frac{2}{6} \cdot \frac{1}{10} + \frac{4}{6} \cdot \frac{2}{15} = 0.122$$

$$(d) \frac{\frac{2}{6} \cdot \frac{1}{10}}{\frac{2}{6} \cdot \frac{1}{10} + \frac{4}{6} \cdot \frac{2}{15}} = 0.273$$