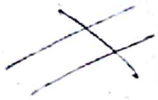
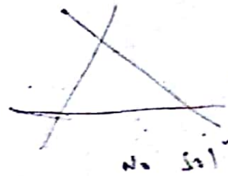
Singular cases :

Shubham Pharts

i) 

no sol^n.

ii) 

No sol^n

iii) 

infinite sol^n

iv) 

fig ii) , sum of two eq^n $\neq$ third eq^n it is inconsistence.

(iii) $0 = 0$ " $0 = 0$, infinitely many sol^n.

$$u\begin{bmatrix}1\\2\\3\end{bmatrix} + v\begin{bmatrix}1\\0\\1\end{bmatrix} + w\begin{bmatrix}1\\3\\4\end{bmatrix} = b \quad , \quad b = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

co-planar.

Q) $x + y = 4.$

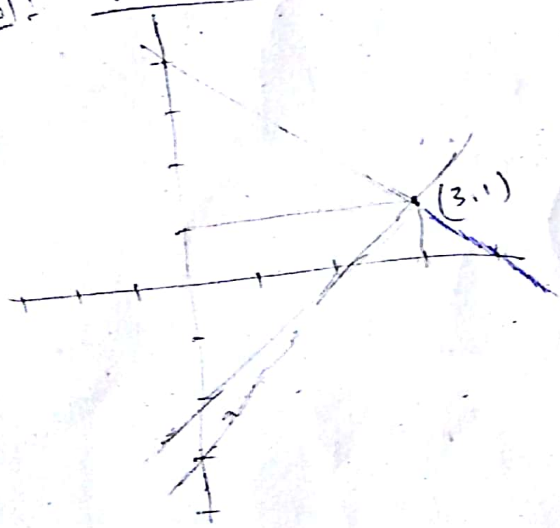
$2x - 2y = 4.$

Draw row & column picture and hence find sol^n if any.

Sol: row pic.

$\frac{x}{4} + \frac{y}{4} = 1$ —①

$\frac{x}{2} + \frac{y}{-2} = 1$ —②.

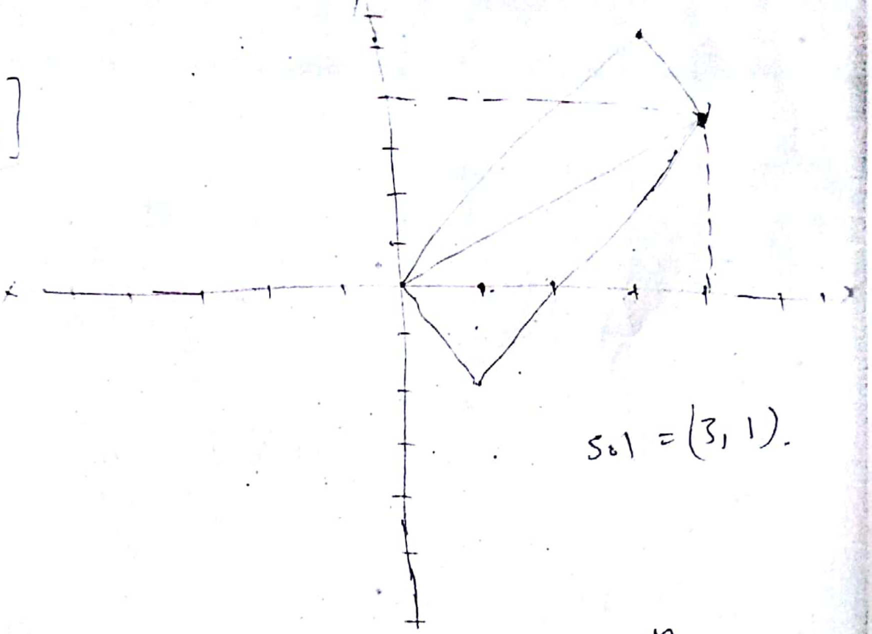


Column pic

$$x\begin{bmatrix}1\\2\end{bmatrix}+y\begin{bmatrix}1\\-2\end{bmatrix}=\begin{bmatrix}4\\4\end{bmatrix}$$

$$\Rightarrow 3\begin{bmatrix}1\\2\end{bmatrix}+1\begin{bmatrix}1\\-2\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}3\\6\end{bmatrix}+\begin{bmatrix}1\\-2\end{bmatrix}$$



Sol = (3, 1).

Q) Draw two pic in 2 planes for the eqn

$x-2y=0$ & $x+y=6$.

Gaussion Elimination method:

$$\text{I}\begin{cases}2u+v+w=5 & -① \\ 4u-6v=-2 & -② \\ -2u+7v+2w=9 & -③\end{cases}$$

first pivot element.

$$\text{II}\begin{cases}2u+v+w=5 & -④ \\ -8v-2w=-12 & -⑤\ [②-①\times 2] \\ 8v+3w=14 & -⑥\ [①+③].\end{cases}$$

$$\text{III}\begin{cases}2u+v+w=5 \\ -8v-2w=-12 \\ w=2. & [5+6]\end{cases}$$

$-8v=-12+2(2)=-8$

Soln of $(u, v, w)=(1, 1, 2)$

Solve: ① $u+v+w=6$

$u+2v+2w=11$

$2u+3v-4w=3$

② $\begin{bmatrix} & 11 & \end{bmatrix} = \begin{matrix}7\\10\\3\end{matrix}$

Gaussian Elimination method:

$2u + v + w = 5$

$4u - 6v = -2$

$-2u + 7v + 2w = 9$

$Ax = b.$

$$\begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 9 \end{bmatrix} \quad -①$$

step 1: subtract 2 times of 1st row from 2nd row.

step 2: subtract (−1) " 1st row " 3rd row.

step 3: subtract (−1) " 2nd row " 3rd row.

Using step 1:

$$\left.\begin{aligned} 2u + v + w &= 5 \\ -8v - 2w &= -12 \\ -2u + 7v + 2w &= 9 \end{aligned}\right\} -②$$

$$Eb = \begin{bmatrix} 5 \\ -12 \\ 9 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ -2 \\ 9 \end{bmatrix} = \begin{bmatrix} 5 \\ -12 \\ 9 \end{bmatrix}$$

$EAx = Eb = c.$

where $E = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$FEAx = Fc$

Step 2: $\left.\begin{aligned} 2u + v + w &= 5 \\ -8v - 2w &= -12 \\ 8v + 3w &= 14 \end{aligned}\right\}$ — (3)

$$\begin{array}{c} F \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array} \begin{bmatrix} 5 \\ -12 \\ 9 \end{bmatrix} = \begin{bmatrix} 5 \\ -12 \\ 14 \end{bmatrix} \quad \text{where } F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Step 3: $\left.\begin{aligned} 2u + v + w &= 5 \\ -8v - 2w &= -12 \\ w &= 2 \end{aligned}\right\}$ — (4)

$GFEAx = Gd$

$$\begin{array}{c} G \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \end{array} \begin{bmatrix} 5 \\ -12 \\ 14 \end{bmatrix} = \begin{bmatrix} 5 \\ -12 \\ 2 \end{bmatrix}$$

$GFEA = U$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 0 & -8 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$GFEA = U.$

$G^{-1}(GFEA) = G^{-1}U.$

$(G^{-1}G)FEA = G^{-1}U$

$I\,FEA = G^{-1}U$

$FEA = G^{-1}U$

$\vdots$

$A = E^{-1}F^{-1}G^{-1}U.$

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$$\underset{E^{-1}}{\begin{bmatrix}1&0&0\\2&1&0\\0&0&1\end{bmatrix}}\underset{E}{\begin{bmatrix}1&0&0\\-2&1&0\\0&0&1\end{bmatrix}}=\underset{I}{\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}}$$

$$\underset{F^{-1}}{\begin{bmatrix}1&0&0\\0&1&0\\-1&0&1\end{bmatrix}}\underset{F}{\begin{bmatrix}1&0&0\\0&1&0\\1&0&1\end{bmatrix}}$$

$$\underset{G^{-1}}{\begin{bmatrix}1&0&0\\0&1&0\\0&-1&1\end{bmatrix}}\underset{G}{\begin{bmatrix}1&0&0\\0&1&0\\0&1&1\end{bmatrix}}$$

$A = E^{-1}F^{-1}G^{-1}U$

$A = LU \qquad (L = E^{-1}F^{-1}G^{-1})$

$$L = \begin{bmatrix}1&0&0\\2&1&0\\-1&-1&1\end{bmatrix} \qquad \boxed{\therefore A = LU}$$

Q) $A = \begin{bmatrix}2&3&3\\0&5&7\\6&9&8\end{bmatrix}$

let $A = LU$

let $L = \begin{bmatrix}1&0&0\\l_1&1&0\\l_2&l_3&1\end{bmatrix}$ & $U = \begin{bmatrix}u_1&u_2&u_3\\0&u_4&u_5\\0&0&u_6\end{bmatrix}$

$LU = A$.

$$\begin{bmatrix}1&0&0\\l_1&1&0\\l_2&l_3&1\end{bmatrix}\begin{bmatrix}u_1&u_2&u_3\\0&u_4&u_5\\0&0&u_6\end{bmatrix}=\begin{bmatrix}2&3&3\\0&5&7\\6&9&8\end{bmatrix}$$

finding values of u_1, u_2, u_7 — u_8, $l_1 l_2 l_3$.

1) $a_{ij} = i + j$

& $b_{ij} = (-1)^{i+j}$; $i, j = 1, 2$.

AB & BA.

2) find two inner products of

$$\begin{bmatrix} 1 & -2 & 7 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 7 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -2 \\ 7 \end{bmatrix} \begin{bmatrix} 3 & 5 & 2 \end{bmatrix}$$

3) multiply Ax to find sol^n.

Ax = zero vector. can you find more sol^n.

to Ax = zero.

$$Ax = \begin{bmatrix} 3 & -6 & 0 \\ 0 & 2 & -2 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

Triangular Factorization:

$A = L\,U$ → upper triangular

L ↓ lower triangular

LU Decomposition method:

$$2u + v + w = 5$$
$$4u - 6v = -2$$
$$-2u + 7v + 2w = 9$$

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$$A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix} \times \begin{bmatrix} u \\ v \\ w \end{bmatrix} ; \quad b = \begin{bmatrix} 5 \\ -2 \\ 9 \end{bmatrix}$$

$A = LU$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 2 & 1 & 1 \\ 0 & -8 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

Sol^n.

Given system : $Ax = b$ — ①

$A = LU$

$(LU)x = b$

$L(Ux) = b$; ② associativity.

Let $Ux = D$ — ③

② ⇒ $L(Ux) = b$

⇒ $LD = b$

$$\Rightarrow \underset{3\times 3}{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}} = \underset{3\times 1}{\begin{bmatrix} 5 \\ -2 \\ 9 \end{bmatrix}}$$

⇒ $d_1 = 5$

$2d_1 + d_2 = -2 \Rightarrow d_2 = -12$

$-d_1 - d_2 + d_3 = 9 \Rightarrow d_3 = 2$

$$\therefore D = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -12 \\ 2 \end{bmatrix}$$

③ ⇒ $Ux = D$

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & -8 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 5 \\ -12 \\ 2 \end{bmatrix}$$

$\Rightarrow 2u + v + w = 5$ $\Rightarrow$ $\boxed{u = 1}$.

$\Rightarrow -8v - 2w = -12$

$w = 2$

$\therefore -8v - 2w = -12$

$\boxed{v = 1}$

$\therefore x = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

$$\boxed{A^{-1} = \frac{1}{|A|} \text{Adj}[A]}$$

(ii) If the inverse of matrix exists, then it cannot act as have two different inverses.

If possible let B & C ... inverses of A.

$BA = I$ - (1)

$AC = I$ - (2)

(1) $\times C$ = $BAC = IC = C$ - (3)

(2) $\times B$ = $BAC = IB = B$ - (4)

(3) & (4) $\Rightarrow C = B$.

ii) If A is invertible, then one and only one sol^n to $Ax = b$.

$Ax = b$

$(A^{-1}A)x = A^{-1}b$

$A x = A^{-1}b$.

iii) Suppose, there is a non-zero vector 'x' such that $Ax = 0$, then A cannot have an inverse.

$Ax = 0$

$\times$ (If A^{-1} exists), then.

$(A^{-1}A)x = A^{-1}0$

$x = 0$

iv) ~~A~~ If A be 2x2 matrix, ~~then~~ of the form

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ Then the inverse of this matrix exists

if and only if $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$

v) The diagonal matrix has an inverse provided no diagonal ~~matrix~~ entries are zero.

Determination of inverse by Gauss-Jordan method:

$$\begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$d_1 u = b_1 \Rightarrow u = b_1/d_1$

$d_2 v = b_2 \Rightarrow v = b_2/d_2$

$d_3 w = b_3 \Rightarrow w = b_3/d_3$

$Ax = b$, $A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix}$ $x = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$, $b = \begin{bmatrix} 5 \\ -2 \\ 9 \end{bmatrix}$

find inverse of A:

let B be inverse of A.

$\therefore AB = I$.

To find inverse, consider. $[A \mid I]$

$$= \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 4 & -6 & 0 & 0 & 1 & 0 \\ -2 & 7 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\approx \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & -8 & -2 & -2 & 1 & 0 \\ 0 & 8 & 3 & 1 & 0 & 1 \end{array}\right]$$

$R_2 = R_2 + (-2)R_1$

$R_3 = R_3 + R_1$

$$\approx \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1/4 & \frac{1}{4} & -1/8 & 0 \\ 0 & 8/3 & 1 & 1/3 & 0 & 1/3 \end{array}\right]$$

$R_2 = R_2 \times -\frac{1}{8}$

$R_3 = R_3 \times \frac{1}{3}$.

$$\approx \left[\begin{array}{ccc|ccc} 2 & 0 & 3/4 & \frac{3}{4} & 1/8 & 0 \\ 0 & 1 & 1/4 & 1/4 & -1/8 & 0 \\ 0 & 8/3 & 1 & 1/3 & 0 & 1/3 \end{array}\right]$$

As: reduces to $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/4 & -5/16 & \\ 0 & 1 & 0 & 1/2 & -3/8 & -1/4 \\ 0 & 0 & 1 & -1 & 1 & \end{array}\right]$

$A = LU$ — ①

$A = LDU'$ — ②

$L \rightarrow$ Lower triangular matrix with diagonal element as unity.

$U \rightarrow$ U.T.M with non-zero diagonal element

$U' \rightarrow$ U.T.M with all diagonal elements to be unity.

$D \rightarrow$ diagonal matrix with all pivot elements as diagonal elements.

Transpose of a matrix:

$A \quad A^T$

$A = (a_{ij})_{m \times n}$

$A^T = (a_{ji})_{n \times m}$

i) $(AB)^T = B^T A^T$

ii) $(A^{-1})^T = (A^T)^{-1}$

$AA^{-1} = I$

$(AA^{-1})^T = (I)^T = I$

$\Rightarrow (A^{-1})^T A^T = I$

$(A^{-1})^T A^T (A^T)^{-1} = I (A^T)^{-1}$

$(A^{-1})^T = (A^T)^{-1}$

Symmetric matrix:

$A = A^T$

$a_{ij} = a_{ji}$, $\forall i, j$

$a_{ij} = -a_{ji}$, $i=j$

$2a_{ij} = 0$

$\Rightarrow a_{ij} = 0$

A symmetric matrix need not be invertable, but if the inverse exists, it is also symmetric.

let $A \to$ symmetric

$A = A^T$.

let A^{-1} exist.

prove that A^{-1} is symmetric.

$(A^{-1})^T = (A^T)^{-1}$.

$(A^{-1})^T = A^{-1}$

$B^T = B$ if $A^{-1} = B$

$\Rightarrow B$ is symmetric

$\Rightarrow A^{-1}$ is "

→ If A is symetric then AA^T and A^TA will be also symmetric but not necessarily to be equal.

$A = [1 \ 2]$ $A^T = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $AA^T = [1 \ 2] \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = [5]_{1\times1}$

$A^TA = \begin{bmatrix} 1 \\ 2 \end{bmatrix} [1 \ 2] = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}_{2\times2}$

$A = LU$, $\boxed{A = LDU}$ — ①.

A is symmetric

$A = A^T$.

$A^T = (LDU)^T$.

$A = A^T = U^T D^T L^T$ — ②.

① & ②, $A = LDU = U^T D^T L^T$

$L = U^T$ & $U = L^T$

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1) use Gauss-Jordan method to find inverse of

i) $A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ ii) $A_2 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ iii) $A_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

2) if $A = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, find A^TB, B^TA, AB^T, BA^T.

3) using G.J Method.

$[A : I]$ $AB^{-1} = I$

$$\begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4) change I into inverse as you reduce A from following.

$I \to A^{-1}$ $[A \mid I] = \begin{bmatrix} 1 & 3 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{bmatrix}$ & $[A \mid I] = \begin{bmatrix} 1 & 4 & 1 & 0 \\ 3 & 9 & 0 & 1 \end{bmatrix}$

5) A^T, A^{-1} & $(A^{-1})^T$ & $(A^T)^{-1}$

for $A_1 = \begin{bmatrix} 1 & 0 \\ 9 & 3 \end{bmatrix}$ & $A_2 = \begin{bmatrix} 1 & c \\ c & 0 \end{bmatrix}$

6) $(AB)^T = B^TA^T$ A^TB^T

$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$, $AB = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

7) factorize the following symmetric matrices into

$A = LDL^T$, D = diagonal matrix $U = DU'$

$A_1 = \begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix}$, $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ eg: $U = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$

$= \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

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Vector space :

Group $G = \{a, \overset{+}{*}\}$.

i) if $a, b \in G \Rightarrow a+b \in G$ (closure property)

ii) $a, b, c \in G$

$a+(b+c) = (a+b)+c$ (Associative)

iii) $e \in G$ s.t $a+e = a \;\forall\; a \in G$.

$5+0 = 5$ (identity)

iv) $a \in G$, $\exists$ an element $b \in G$

s.t $a+b = 0$.

Ring: $R = \{a, *, \cdot\}$

v) any $a, b \in R$

$a \cdot b \in R$ (closure)

vi) $a, b, c \in R$

$a \cdot (b \cdot c) = (a \cdot b) \cdot c$ (associative)

$a \cdot (b+c) = a \cdot b + a \cdot c$ (distributive).

vii) any $a \in R$, $\exists$ an element $e \in R$

s.t $ae = e \cdot a = a$.

viii) For any $a \in R$ $\exists$ $b \in R$.

s.t $a \cdot b = e = b \cdot a$

Vector space (V)

A vector space 'V' over a field 'F' consists of a set on which two operations namely addition & scalar multiplication are defined so that for any two elements x & y belonging to $\in V$, there exists a unique element $x+y \in V$ and for any scalar $a \in F$ there must exist

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a unique element $ax \in V$ such that the following conditions are satisfied.

Field:

VS1 : For any $x, y \in V$, $x+y = y+x$ (commutative)

VS2 : For any $x, y, z \in V$, $(x+y)+z = x+(y+z)$ (Associative)

VS3 : There exists an element in V which denoted by 0 such that $x+0 = x = 0+x$.

VS4 : For each element $x \in V$, there exists an element $y \in V$ s.t. $x+y=0$.

VS5 : For each element $x \in V$, $1 \cdot x = x$.

VS6 : For each pair of element $a, b \in F$, & each element $x \in V$, we have $(ab)x = a(bx)$

VS7 : For each element $a \in F$, & $x, y \in V$ we have

$a(x+y) = ax + ay$

VS8 : For each pair element $a, b \in V$ & $x \in V$, we have

$(a+b)x = ax + bx$

$u = (a_1, a_2, \ldots a_n)$ $\quad a_i \in F$

$v = (b_1, b_2 \ldots b_n)$ $\quad b_i \in F$

$u, v \in V(F^n)$ $\quad u+v = (a_1+b_1, a_2+b_2 + \ldots a_n+b_n)$

$eu = e(a_1, a_2 \ldots a_n) = (ea_1, ea_2, \ldots ea_n)$

$u = (3, -2, 0) \in V(F3)$

$v = (-1, 1, 4) \in V(F3)$

$e = -5$.

$u+v = (3, -2, 0) + (-1, 1, 4)$

$= (2, -1, 4) \in V(F3)$

$eu = -5(3, -2, 0)$

$= (-15, 10, 0) \in V(F^3)$

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F → complex field.

V(F)

$u = (1+i, 2i)$

$v = (2-3i, 4i)$

$u+v = (1+i, 2i) + (2-3i, 4i)$

$= (3-2i, 6i)$

$e = (1+i, 3i+4)$

$\therefore eu = 5i(1+i, 2i)$

$= (5i(1+i), 5i(2i))$

$= (5i+5i^2, 10i^2)$

$= (-5+5i, -10) \in V(F)$.

1) $S = \{(a_1, a_2) : a_1, a_2 \in R\}$

$(a_1, a_2), (b_1, b_2) \in S$.

$e \in R$.

$(a_1, a_2) + (b_1, b_2) = \{a_1+b_1, a_2-b_2\}$ —(i)

$(b_1, b_2) - (a_1, a_2) = (b_1+a_1, b_2-a_2)$ —(ii)

VS1 is not satisfied.

∵ Right side of (i) & (ii) are not equal.

L.S of (i) & (ii) are also not equal.

which contradiction (VS1)

∴ S → not a vector space over R.

2) $S = \{(a_1, a_2) : a_1, a_2 \in R\}$

$(a_1, a_2), (b_1, b_2) \in S$, $e \in R$.

$(a_1, a_2) + (b_1, b_2) = (a_1+b_1, 0)$.

$(b_1, b_2) + (a_1, a_2) = (b_1+a_1, 0)$

LS of VS3, $\{(a_1, a_2) + (b_1, b_2)\} + (c_1, c_2) = (a_1+b_1, 0) + (c_1, c_2)$

$= (a_1+b_1+c_1, 0)$

R.S of VS3 $= (a_1, a_2) + \{(b_1, b_2) + (c_1, c_2)\}$

$= (a_1, a_2) + \{b_1 + c_1, 0\}$

$= (a_1 + b_1 + c_1, 0)$

$1(a_1, a_2) = (a_1, 0) \neq (a_1, a_2)$

VS5 is not satisfied.

$u = (a_1, a_2 \, -- \, a_n) \rightarrow$ n-tuple

$n = 3, \quad a_i \in F.$

$u = (a_1, a_2, a_3) \quad u \in F^3, R^3.$

$x, y \in V, \; x+y \in V$

$a \in F, \; ax \in V$

$$A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \\ 6 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 7 & 5 \\ 3 & 3 \\ 8 & 2 \end{bmatrix}.$$

$$A + B = \begin{bmatrix} 9 & 5 \\ 4 & 8 \\ 14 & 5 \end{bmatrix} \in V.$$

$$c = -5, \quad cA = (-5)\begin{bmatrix} 2 & 0 \\ 1 & 5 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} -10 & 0 \\ -5 & -25 \\ -30 & -15 \end{bmatrix} \in V.$$

Theorem: Cancellation Law of vector Addition:

statement: If x, y and z are vectors s.t $x + z = y + z$. — (1)

then $x = y$.

proof: $x \in V; y \in V$ s.t $x + y \in V$.

By (VS4) we have for any $z \in V, \exists \, y \in V$

s.t $z + y = 0$.

L.S, $x = x+0 = x+y+z+y$ $[\because y+z=0]$

$= y+z+y$.

$= y+z+y$ by (i)

$= 0+y$

$= y+0$.

$\therefore x=y$.

Theorem :2:

In any vector space 'v' the following statement are true.

i) $0x = 0$ for each $x \in V$

ii) $(-a)x = -(ax) = a(-x)$. for each $x \in V$ & $a \in F$.

iii) $0a = 0$ for each $a \in F$.

Q) At the end of may month, for a function store head the following inventory.

	American	spanish	mediterranian.	Danish.
Living Room suits	4	2	1	3
Bedroom suits	5	1	1	4
Dinning suit.	3	1	2	6

Record this data as $M_{3\times4}$.

To prepare for June sale to double its inventory on each of the item listed in the preciding table. Assuming that none of the present stoke is sold until the additional furniture arrives verify the inventory on hand after the order is filled which described by $2M$. Also if the inventory at the end of June is described by the matrix A

$$A = \begin{bmatrix} 5 & 3 & 1 & 2 \\ 6 & 2 & 1 & 5 \\ 1 & 0 & 3 & 3 \end{bmatrix}, \quad 2M - A$$

how many suits were sold for june sale.

Sol:- $M = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 5 & 1 & 1 & 4 \\ 3 & 1 & 2 & 6 \end{bmatrix}$.

$$2M - A = \begin{bmatrix} 8 & 4 & 2 & 6 \\ 10 & 2 & 2 & 8 \\ 6 & 2 & 4 & 12 \end{bmatrix} - \begin{bmatrix} 5 & 3 & 1 & 2 \\ 6 & 2 & 1 & 5 \\ 1 & 0 & 3 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 & 1 & 4 \\ 4 & 0 & 1 & 3 \\ 5 & 2 & 1 & 9 \end{bmatrix}$$

Subspace: subspace of a vector space is a non empty subspace that satisfies all the requirements of a vectorspace. & linear combination rule must be satisfied. (ie) (i) If we at any vector x, y in sub space, then $x+y \in$ subspace.

(ii) If we multiply any vector x in subspace by any scalar 'c', then $cx \in$ subspace.

$0 \in S \in V$ — smallest subspace $= 0$

$0+0 = 0 \in S$ — largest subspace is vector space itself.

$c0 = 0 \in S$.

Field: Any Ring containing atleast two elements is called a field, if it is commutative, has a unity element & is such that all non-zero elements have its ~~co~~multiplicative inverse.

unity element eq. $u \cdot u_0 = u$ $u \in R$.

Subspaces:

let $S \subset V(F)$, then S will form a subspace of V iff and only the following conditions satisfied.

i) If any $x, y \in S$, $x+y \in S$. [closed under addition]

ii) for any $x \in S$ ~~scalar~~ & any scalar $c \in F$, $cx \in S$. [closed under multiplication].

iii) S will contain zero vector.

iv) Each vector in S will have its additive inverse in S.

Theorem: let V be a vector space & W is subset of V, then W is subspace of V iff and only if the following three conditions are satisfied.

i) $0 \in W$

ii) $x+y \in W$ for any $x \in W$, $y \in W$.

iii) $cx \in W$ for any $x \in W$ & any scalar $c \in F$.

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Proof: Let ~~V~~ W be a subspace of the vector space V. W is assumed to be a vector space.

let $0'$ be a zero vector $\in W$.

s.t $x + 0' = x$ (i) for any $x \in W$.

$x \in V$ $[W \subset V]$

$x + 0 = x$ — (ii)

(i) & (ii), $\Rightarrow x + 0' = x = x + 0$.

$\Rightarrow 0' = 0$ [by left cancellation property]

let conditions (i), (ii) & (iii) hold in W.

R.T.P. that W is a vector space

let $x \in W$.

then by property (iii) $\Rightarrow (-1)x \in W$, $(-1) \in F$.

$\Rightarrow 1(-x) \in W$.

$\Rightarrow -x \in W$.

$\Rightarrow$ Additive inverse belongs to W.

$\Rightarrow W$ is a vector space.

Ex (1) let 'n' be a non-negative integer.

let $P_n(F)$ having ~~$P_c(F_c)$~~ all polynomials in $P(F)$ having degree equal to n.

$$P_n(F) = a_n x^n + a_{n-1} x^{n-1} + \cdots a_1 x + a_0$$

$P_1(F) = a_1 x + a_0 \rightarrow$ deg 1

if, $a_1 = 0$, $P(F) = a_0 \rightarrow$ deg 0.

$P(F) = 0. \rightarrow$ deg (-1). (i) is satisfied (ii) & (iii) also satisfied.

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ie. Proof ② S.T the set of diagonal matrices is a subspace of an $M_{n\times n}(F)$.

$M'_{n\times n} \rightarrow a_{ij} = 0$ for $i \neq j$.

Sol/ (i) dig matrix $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow$ diagonal matrix.

(i) condition satisfied.

(ii) $A_{n\times n}$, $B_{n\times n}$ → diagonal matrices, $\in M'_{n\times n}$.

for $i \neq j$.

$(A+B)_{ij} = A_{ij} + B_{ij} = 0 + 0 = 0$.

$\Rightarrow (A+B) \in M'_{n\times n}$. (ii) is satisfied.

(iii) $(cA)_{ij} = c(A_{ij}) = c0 = 0$. (iii) is satisfied.

③ set of matrices $M_{m\times n}(R)$ having non negative entries is not a subspace of $M_{m\times n}(R)$.

Sol/ (iii) condition is not satisfied, $M_{m\times n}(R)$ is not a subspace.

④ <u>Theorem:</u> Any intersection of subspaces of vector space V is a subspace of V.

<u>Linear combination of vectors.</u>

Let V be a vector space and S is a non-empty subset of V. A vector $v \in V$ is called linear combination of vectors S. is S. ∃ a finite no. of vectors $u_1, u_2 \ldots u_n$ is S. and some scalars $a_1, a_2 \ldots a_n$ in F.

S.T: $v = a_1u_1 + a_2u_2 + \cdots a_nu_n$.

1.) $v = (2,6,8)$ how to determine these vector as linear combination of $u_1 = (1,2,1)$, $u_2 = (-2,-4,-2)$, $u_3 (0,2,3)$, $u_4 = (2,0,-3)$, $u_5 = (-3,8,16)$.

S1 If possible let

$$v(2,6,8) = a_1(1,2,1) + a_2(-2,-4,-2) + a_3(0,2,3) + a_4(2,0,-3) + a_5(-3,8,16)$$

$$(2,6,8) = (a_1 - 2a_2 + 2a_4 - 3a_5,\ 2a_1 - 4a_2 + 2a_3 + 8a_5,\ a_1 - 2a_2 + 3a_3 - 3a_4 + 16a_5)$$

comparing,

$$a_1 - 2a_2 + 2a_4 - 3a_5 = 2 \quad -① $$
$$2a_1 - 4a_2 + 2a_3 + 8a_5 = 6 \quad -② $$
$$a_1 - 2a_2 + 3a_3 - 3a_4 + 16a_5 = 8 \quad -③ $$

Eliminating a_1 from ② & ③ using ①.

$$\left.\begin{aligned} a_1 - 2a_2 + 2a_4 - 3a_5 &= 2 \\ 2a_3 - 4a_4 + 14a_5 &= 2 \\ a_3 - 2a_4 + 7a_5 &= 1 \\ 3a_3 - 5a_4 + 19a_5 &= 6 \end{aligned}\right\} ④ \quad \begin{matrix} [①\times(-2) + ②] \\ [①\times(-1) + ③] \end{matrix}$$

Eliminating a_3 from last eqn of ④ using 2nd.

$$\left.\begin{aligned} a_1 - 2a_2 + 2a_4 - 3a_5 &= 2 \\ a_3 - 2a_4 + 7a_5 &= 1 \\ a_4 - 2a_5 &= 3 \end{aligned}\right\} ⑤$$

$$\left.\begin{aligned} a_1 + 2a_4 &= 2 + 2a_2 + 3a_5 \\ a_3 - 2a_4 &= 1 - 2a_5 \\ a_4 &= 3 + 2a_5 \end{aligned}\right\} \quad (6)$$

If $a_2 = 0$, $a_5 = 0$.

$a_1 + 2a_4 = 2$

$a_3 - 2a_4 = 1$

$a_1 + a_3 = 3$

$\boxed{a_4 = 3}$

$a_3 = 1 + 2(3) = 7.$

$\boxed{a_3 = 7}$

$\therefore \boxed{a_1 = -4}$

$a_1 = -4$

$a_2 = 0$

$a_3 = 7$

$a_4 = 3$

$a_5 = 0$

Show that $2x^3 - 2x^2 + 12x - 6$ is a linear combination of $(x^3 - 2x^2 - 5x - 3)$ and $(3x^3 - 5x^2 - 4x - 9)$ in $P_3(R)$.

but $3x^3 - 2x^2 + 7x + 8$.

Sol: If $(2x^3 - 2x^2 + 12x - 6)$ is a Linear comb of $(x^3 - 2x^2 - 5x - 3)$ & $(3x^3 - 5x^2 - 4x - 9)$ such that two scalars a, b s.t $(2x^3 - 2x^2 + 12x - 6)$

$= a(x^3 - 2x^2 - 5x - 3) + b(3x^3 - 5x^2 - 4x - 9).$

$\Rightarrow (a + 3b)x^3 + (-2a - 5b)x^2 + (-5a - 4b)x + (-3a - 9b)$

comparing both sides, we get.

$a + 3b = 2$

$-2a - 5b = -2$

$-5a - 4b = 12$

$-3a - 9b = -6$

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Eliminating a from last 3 eqⁿ using first eqⁿ

$a+3b=2$

$\left.\begin{array}{l} b=2 \\ 11b=22 \\ 0.b=0 \end{array}\right\} b=2$

$a+3(2)=2$

$a=-4$

$\therefore \boxed{a=-4, \quad b=2}$

$\therefore 2x^3-2x^2+12x-6 = -4(x^3-2x^2-5x-3)+2(3x^3-5x^2-4x-9).$

If possible let.

$3x^3-2x^2+7x+8 = a(\quad)+b(\quad)$

$\therefore$ $a+3b=3$

$-2a-5b=-2$

$-5a-4b=7$

$-3a-9b=8$

$\Rightarrow$ $a+3b=3$

$b=4$

$11b=22$

$0=17!$

<u>Linearly dependent vectors:</u>

let V be vector space over a field F.

a finite subspace $S=\{\alpha_1, \alpha_2 - \alpha_n\}$ of the vectors of V is said to be linearly dependent if their exists a set of scalars $a_1 a_2 -- a_n \in F$ not all zero such that this L.C

$a_1\alpha_1 + a_2\alpha_2 + \ldots a_n\alpha_n = 0.$

<u>Linearly Independent vectors:</u>

Linearly Independent vectors:

let V be a vector space over a field F, a finite subspace. $S = \{\alpha_1, \alpha_2 -- \alpha_n\}$ of vectors V is said to be linearly independent if there exists a set of scalars $a_1, a_2 -- a_n \in F$, s.t

$a_1\alpha_1 + a_2\alpha_2 + -- a_n\alpha_n = 0 \Rightarrow a_i = 0 \; \forall \; i = 1, 2, --- n$

Eg: null set is linearly independent

Deductions:

1) If 2 vectors be Linearly dependent then one of them is a scalar multiple of another.

let α, β: $\quad a, b \in F$.

$a\alpha + b\beta = 0$

let $a \neq 0$.

$a\alpha = -b\beta$

$\alpha = \left(\frac{-b}{a}\right)\beta \quad a \neq 0.$

2) A system consisting of a single non-zero vector is always linearly Independent.

$S = \{\alpha\}, \quad \alpha \neq 0.$

$a\alpha = 0$ where $a \in F$.

$\Rightarrow a = 0 \quad [\because \alpha \neq 0]$

$\Rightarrow S = \{\alpha\}$ is L.I.

3) Every superset of a L.D of vectors will L.D.

$S = \{\alpha_1, \alpha_2 \ldots \alpha_n\}$.

where $\alpha_1, \alpha_2 \ldots \alpha_n$ is L.D.

$\therefore \exists$ a set of scalars $a_1, a_2 \ldots a_n \in F$.

not all zero : s.t.

$a_1\alpha_1 + a_2\alpha_2 \ldots + a_k\alpha_k + \ldots a_n\alpha_n = 0$.

let $a_k \neq 0$.

consider the super set $S' = \{\alpha_1, \alpha_2 \ldots \alpha_n, \beta_1, \beta_2 \ldots \beta_n\}$

$\Rightarrow S \subset S'$.

consider. $a_1\alpha_1 + a_2\alpha_2 + \ldots a_k\alpha_k + \ldots a_n\alpha_n + 0.\beta_1 + 0.\beta_2 + \ldots 0.\beta_n = 0$.

Hence again $a_k \neq 0$.

$\Rightarrow S'$ is L.D.

4). Any subset of L.I set of vectors is L.I.

$S = \{\alpha_1, \alpha_2 \ldots \alpha_n\}$ L.I

$\exists\ a_1, a_2 \ldots a_n \in F$ s.t.

$a_1\alpha_1 + a_2\alpha_2 + \ldots + a_n\alpha_n = 0$.

$\Rightarrow a_i = 0\ \forall i$

consider a subset $S' = \{\alpha_1, \alpha_2 \ldots \alpha_k\} \subset S$. $k \leq n$.

$a_1\alpha_1 + a_2\alpha_2 + \ldots + a_k\alpha_k = 0$.

$\Rightarrow a_i = 0\ \forall i = 1, 2, \ldots k$.

$\Rightarrow \{\alpha_1, \alpha_2 \ldots \alpha_n\}$ is L.I.

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Theorem !

let 'V' be a vector space over the field F. then the set 'S' of a non-zero vectors $\{\alpha_1, \alpha_2 - \alpha_n\} \in V$ is L.D. if and only if some element of S be a linear combination of others.

proof:

let $S = \{\alpha_1, \alpha_2 -- \alpha_n\}$ be L.D.
$\alpha_i \neq 0 \ \forall i = 1, 2 -- n$.

$\exists \ a_1, a_2 -- a_n \in F$ not all zero st.

$\therefore \ a_1\alpha_1 + a_2\alpha_2 + a_k\alpha_k + -- + a_n\alpha_n = 0$.

let $a_k \neq 0$.

$$\therefore \ a_k\alpha_k = -\sum_{\substack{i=1 \\ i \neq k}}^{n} a_i\alpha_i$$

$$\Rightarrow \quad \alpha_k = -\sum_{\substack{i=1 \\ i \neq k}}^{n} \frac{a_i}{a_k}\alpha_i$$

Now let one of the elements of S, say α_j be represented as Linear combination of the other elements of S.

$$\text{i.e.} \quad \alpha_j = \sum_{\substack{i=1 \\ i \neq j}}^{n} b_i\alpha_i$$

$$\Rightarrow \quad \sum_{\substack{i=1 \\ i \neq j}}^{n} b_i\alpha_i + (-1)\alpha_j = 0.$$

$\therefore$ coeff of α_j is non-zero.

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$\therefore$ we can $\{\alpha_1, \alpha_2 - - \alpha_j - - \alpha_n\}$.

is Linearly dependent.

<u>Linear span L(S):</u> Let 'V' be a vector space over a field 'F' and 'S' be any non empty subset of V. then the linear span of S is defined as the set of all linear combinations of finite set of elements of S is called Linear span denoted by L(S).

$$L(S) = \left\{ a_1\alpha_1 + a_2\alpha_2 + - - a_n\alpha_n : \begin{array}{l} a_i \in F \; \& \\ \alpha_i \in S \; \forall i \end{array} \right\}$$

$L(S) \subset V$

L(S) is generated a span on S, S is called generator ~~by~~ of L(S).

$$L(S) = \left\{ \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \right\}$$

$$S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

$$= \lambda_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \lambda_2 \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$S \subset L(S)$.

Theorem :

The linear span of L(S) of any subset of S of V is a subspace of V generated by S.

Ex ① S.T. the vectors $(-1,2,1)$, $(3,0,-1)$ $(-5,4,3)$ are linearly dependent in $V_3(R)$.

$a(-1,2,1) + b(3,0,-1) + c(-5,4,3) = 0.$ $(0,0,0)$

where $a,b,c \in R$.

$\Rightarrow -a+3b-5c = 0$

$2a + 0\cdot b + 4c = 0$

$a - b + 3c = 0.$

$b = c$

$a = -2c$

let $c = 1$, $b = 1$

$a = -2$.

if rank of coeff matrix is < than no. of unknowns.

$$\begin{vmatrix} -1 & 3 & -5 \\ 2 & 0 & 4 \\ 1 & -1 & 3 \end{vmatrix} = 0 \therefore \text{rank} < 3.$$

② Let x, y, z be the elements of a vectorspace V over F and let $a, b \in F$. S.T. x, y, z are L.D if $(x + ay + bz)$, y, z be L.D.

Sol:- $k_1(x + ay + bz) + k_2 y + k_3 z = 0.$

$k_1 x + (k_1 a + k_2) y + (k_1 b + k_3) z = 0 (0,0,0)$

$k_1 \neq 0$, $k_2, k_3 = 0$.

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Basis:

Let V be a vectorspace over field F and S is the subset of vector space $S \subset V(F)$ such that (i) S is a set of linearly independent vectors

(ii) Each vector in V can be represented as a linear combination of finite number of elements of S, (i.e) S generates V. such that $L(S) = V$ then S is called a "basis of the vector space V".

$B = \{(1,0,0), (0,1,0), (0,0,1)\}$ we have to prove

(i) Linearly independent:

$(a_1, a_2, a_3) \in R$ s.t.

$a_1(1,0,0) + a_2(0,1,0) + a_3(0,0,1) = (0,0,0)$

$1.a_1 + 0.a_2 + 0.a_3 = 0 \Rightarrow a_1 = 0$

$0.a_1 + 1.a_2 + 0.a_3 = 0 \Rightarrow a_2 = 0$

$0.a_1 + 0.a_2 + 1.a_3 = 0 \Rightarrow a_3 = 0.$

$\therefore$ B is an independent set of vectors. — (A)

ii) Linear combination:

Let $\alpha = (\alpha_1, \alpha_2, \alpha_3) \in V(R)$

$(\alpha_1, \alpha_2, \alpha_3) = \alpha_1(1,0,0) + \alpha_2(0,1,0) + \alpha_3(0,0,1)$ — (B)

$\therefore$ (A) & (B) $\Rightarrow$ B is basis of vector space V (R)

$B' = \{(1,1,0), (1,0,1), (0,1,1)\}$

(i) let $a, b, c \in R$ s.t.

$a(1,1,0) + b(1,0,1) + c(0,1,1) = (0,0,0)$

$a + b + 0 = 0$ — (i)

$a + 0 + c = 0$ — (ii)

$0 + b + c = 0$ — (iii)

$2(a+b+c) = 0$

$\Rightarrow a+b+c = 0$ — (iv)

(iv) − (i) $\Rightarrow$ $c = 0$.

(iii) $\Rightarrow$ $b = 0$.

(ii) $\Rightarrow$ $a = 0$.

B' is linearly independent.

ii) $(\alpha_1, \alpha_2, \alpha_3) = \frac{1}{2}\underbrace{(\alpha_1+\alpha_2-\alpha_3)}_{a_1}(1,1,0) + \frac{1}{2}\underbrace{(\alpha_1+\alpha_3-\alpha_2)}_{a_2}(1,0,1)$

$+ \frac{1}{2}\underbrace{(\alpha_2+\alpha_3-\alpha_1)}_{a_3}(0,1,1)$

$\therefore$ B' is basis of vector space $V(R)$.

Dimension :

The vector space V is said to be finite dimensional or finitely generated if there exists a finite subset of V such that Linear span $L(S) = V$, otherwise V is said to be infinite dimension.

The no. of elements in a basis of a finite dimensional vector space $V(F)$ is called dimension of vector space "$\dim V$".

$V_n(F) \rightarrow$ 'n' dimension of vectorspace.

Theorem-1: In a finite dimensional vector space V over the field 'F' with basis set $B = \{x_1, x_2 \ldots x_n\}$ every vector x belonging to V $x \in V$ is uniquely expressable as a linear combination of vectors 'V'.

Theorem-2:

There exists a basis for each finite dimensional vector space.

Th-3:

A set of vectors 'S' consisting of the 'n' vectors
$e_1 = (1, 0, 0 \ldots 0)$, $e_2 = (0, 1, 0 \ldots 0)$
$e_n = (0, 0 \ldots 1)$ → is a basis of $V_n(F)$.

Theorem-4:

If $V(F)$ be a finite dimensional vector space then any two bases of 'V' have same no. of elements.

Theorem-5:

If 'W' be a proper subspace of a finite dimensional vector space 'V' then 'W' is also a finite dimensional and $\dim W \le \dim V$.

Theorem-6:

If 'W_1 & W_2' be two subspaces of a finite dimensional vector space $V(F)$ then

$\dim(W_1 \cup W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$

Q) Show that the vectors $(1,2,1), (2,1,0), (1,-1,2)$ form a basis of the vector space V_3 over the field of Real numbers.

* We know that if $V(F)$ be a finite dimensional vector space of dimension n, then any set of 'n' linearly independent vectors of 'V' will form a basis of 'V'. *

Sol: $S' = \{(1,0,0), (0,1,0), (0,0,1)\}$

$S = \{(1,2,1), (2,1,0), (1,-1,2)\}$

let $a, b, c \in F$ s.t.

$a(1,2,1) + b(2,1,0) + c(1,-1,2) = 0$

$$\Rightarrow \left.\begin{aligned} a+2b+c &= 0 \\ 2a+b-c &= 0 \\ a+0\cdot b+2c &= 0 \end{aligned}\right\}$$

homogeneous system of equations so, these system has trivial sol^n.

$$\begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & -1 \\ 1 & 0 & 2 \end{vmatrix} \Rightarrow 1(2) - 2(4+1) + 1(-1)$$

$\Rightarrow 2 - 10 - 1 \neq 0 \cdot \Rightarrow a = b = c = 0.$

$\therefore (1,2,1), (2,1,0), (1,-1,2)$ are linearly independent.

Q) Show that the set $S = \{(1,0,0), (1,1,0), (1,1,1), (0,1,0)\}$ spans the vector space R^3 but is not a basis set.

Sol: $\alpha \in V$.

$\alpha = (\alpha_1, \alpha_2, \alpha_3) \in V$ & $a, b, c, d \in R$.

If possible, let.

$(\alpha_1, \alpha_2, \alpha_3) = a(1,0,0) + b(1,1,0) + c(1,1,1) + d(0,1,0)$

$\Rightarrow \alpha_1 = a+b+c$

$\alpha_2 = b+c+d$

$\alpha_3 = c$

$\Rightarrow c = \alpha_3$.

$b = \alpha_2 - c - d = \alpha_2 - \alpha_3 - d$.

$a = \alpha_1 - b - c \Rightarrow \alpha_1 - (\alpha_2 - \alpha_3 - d) - \alpha_3$

$\Rightarrow \alpha_1 - \alpha_2 + \alpha_3 + d - \alpha_3$

$a \Rightarrow \alpha_1 - \alpha_2 + d$.

If $d = 0$,

$$\left.\begin{aligned} a &= \alpha_1 - \alpha_2 \\ b &= \alpha_2 - \alpha_3 \\ c &= \alpha_3 \end{aligned}\right\} \text{at least one of the scalars is different from zeros.}$$

Q) find R basis of vector containing $\{(1,1,0), (1,1,1)\}$

Row space, column space & Rank of a Matrix.

$$[A]_{m \times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}_{m \times n}.$$

$$\left.\begin{aligned} R_1 &= (a_{11}, a_{12} \cdots a_{1n}) \\ R_2 &= (a_{21}, a_{22} \cdots a_{2n}) \\ R_i &= (a_{i1}, a_{i2}, \cdots a_{in}) \\ &\cdots \\ R_m &= (a_{m1}, a_{m2} \cdots a_{mn}) \end{aligned}\right\} \text{each in an } n\text{-triple vector.}$$

$$c_1 = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} \quad c_2 = \begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{bmatrix} \quad c_3 = \begin{bmatrix} a_{13} \\ a_{23} \\ \vdots \\ a_{m3} \end{bmatrix} \quad c_n = \begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix}$$

each n vector is an ~~multiple~~ m-tuple vector.

⇒ The row vectors which generate a ~~the~~ vector space is called row space of matrix A 'R(A)'.

⇒ The column vector generate a vector space which is called a column space of A 'C(A)'.

⇒ The row space ~~are~~ R(A) is a sub space of V.S of $V_n(F)$ while C(A) is the subspace of a vector space of $V_m(F)$.

⇒ Dimension of vector space R(A) is called Row Rank of A ($\leq n$) and dimension of V.S C(A) is called column Rank of A. ($\leq m$).

Theorem : 1 :- premultiplication by a non-singular matrix doesnot change the row rank of a matrix.

Theorem - 2 : The Row rank of a matrix is same as its rank

Th - 3 : The row Rank & column Rank of a matrix are equal.