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$x+2y=3$ — (1)
$4x+5y=6$ — (2)
} linear equⁿ

If equⁿ has a solution – Consistent System.

No. of equⁿ = No. of unknown → Unique Solution.

Method of Solution :–

i) Elemination
ii) Determinant

(1) (2) − (1) × (−4)

$-3y=-6$
$\Rightarrow y=2$

From equⁿ (1) →

$x+(2\times 2)=3$
$\Rightarrow x=-1$

Determinant method :

$$\begin{bmatrix}1 & 2\\4 & 5\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}3\\6\end{bmatrix}$$

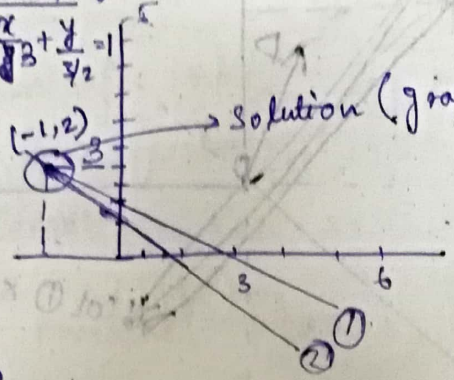
$$x=\frac{\begin{vmatrix}3 & 2\\6 & 5\end{vmatrix}}{\begin{vmatrix}1 & 2\\4 & 5\end{vmatrix}},\quad y=\frac{\begin{vmatrix}1 & 3\\4 & 6\end{vmatrix}}{\begin{vmatrix}1 & 2\\4 & 5\end{vmatrix}}$$

Row Representation :

$x+2y=3 \rightarrow \frac{x}{3}+\frac{y}{3/2}=1$

$4x+5y=6$

$\frac{x}{3/2}+\frac{y}{6/5}=1$



→ solution (graphically)

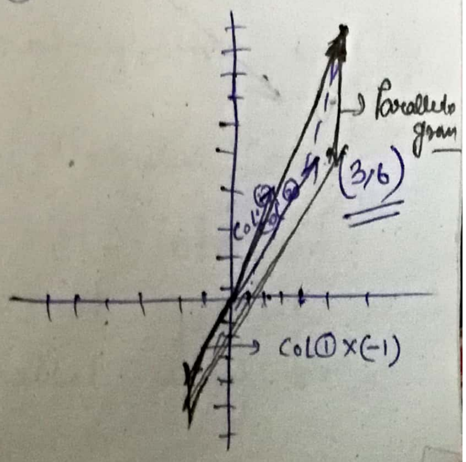
Column form :

$$x\begin{bmatrix}1\\4\end{bmatrix}+y\begin{bmatrix}2\\5\end{bmatrix}=\begin{bmatrix}3\\6\end{bmatrix}$$

↓ column vector.

(−1) col. (1) + 2. col (2)

$$\begin{bmatrix}-1\\-4\end{bmatrix}+\begin{bmatrix}4\\10\end{bmatrix}$$



Diagonal will give the solution

Three Equations with three unknowns

→ Each of them represent planes. For line requirement two equ's are needed.

$$2u + v + w = 5$$

$$4u - 6v = -2$$

$$-2u + 7v + 2w = 9$$

$4u = -2 + 6v$

$\Rightarrow 2u = -1 + 3v$

$-1 + 3v + 7v + 2w = 9$

$\Rightarrow 10v + 2w = 10$

$\Rightarrow 5v + w = 5$

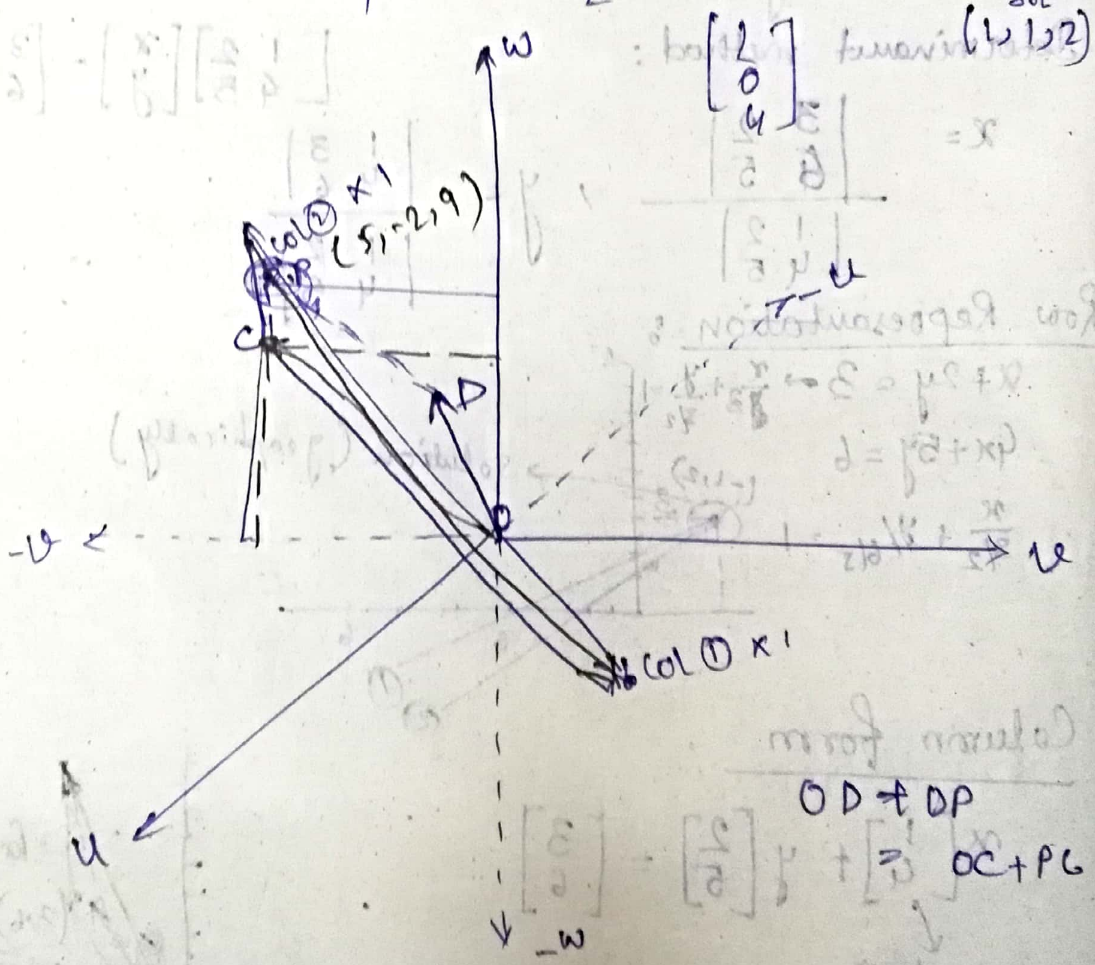
Row form

Ans:- $(1, 1, 2)$

Column form

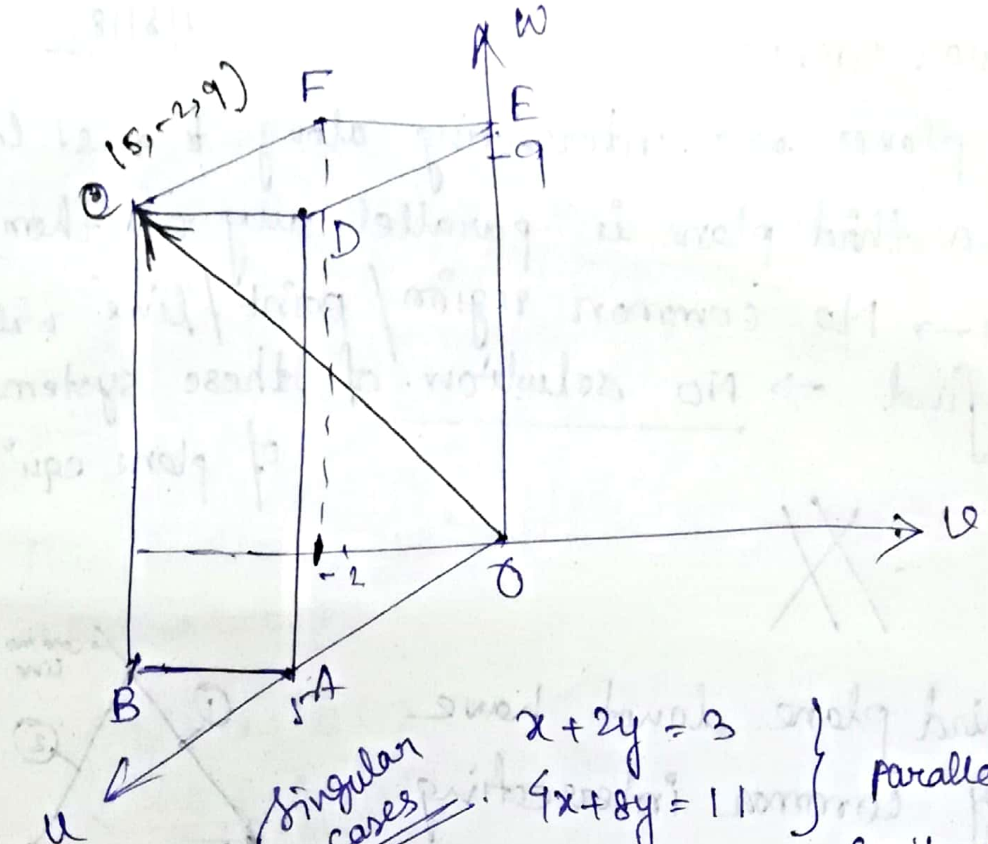
$$\underset{1}{u}\begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix} + \underset{1}{v}\begin{bmatrix} 1 \\ -6 \\ 7 \end{bmatrix} + \underset{2}{w}\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 9 \end{bmatrix}$$

Sol^n: $(1, 1, 2)$



OD + OP

OC + PC



Singular cases:

$$\left.\begin{aligned} x+2y&=3 \\ 4x+8y&=11 \end{aligned}\right\}$$ parallel lines. Can't get solution.

Inconsistance system.

$u+v+w=2$ — (i)

$2u \quad +3w=5$ — (ii)

$3u+v+4w=6$ — (iii)

LHS

(i)+(ii) → (iii)

Now, (iii) − ((i)+(ii))

$\boxed{0=-1}$ ✗

Implies System of three equations are inconsistant parallel systems.

If not the parallel planes, then also there is posibility of not getting any solution → ~~Singular~~ (Singular case)

there, three st. lines do not intersect each other. So, we are not getting solution. they are parallel may be.

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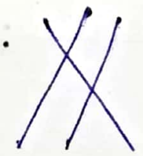
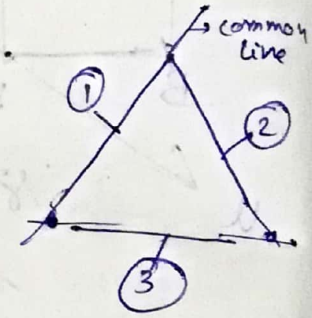
Singular Cases:

1) Two planes are intersecting along ~~to~~ a st. line. If a third plane is parallel any of them, then → No common region / point / line will be find. ⇒ No solution of these system of plane equ^ns.

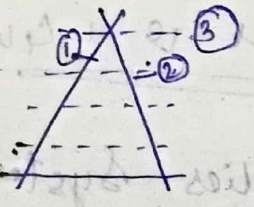


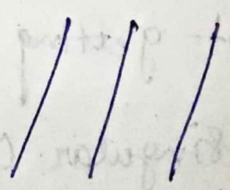
2) Third plane donot have any common intersecting line with the other two planes ⇒ No solution.



Exceptions

3) modified point ②
In this case, the third plane is lifting up gradually. so finally they have common intercepting St. line
⇒ Infinite Solution.



4)  all parallel planes
⇓
No Solution.

$$\left.\begin{aligned} u+v+w&=2 \\ 2u\quad+3w&=5 \\ 3u+v+4w&=6 \end{aligned}\right\} \begin{aligned} &— 1 \\ &— 2 \\ &— 3 \end{aligned}$$

Figure ii

L.H.S equ^n 1 + 2 = equ^n 3

R.H.S $\neq$

This implies, system of equ^{n}s is inconsistent.

$\Rightarrow$ No solution.

$$\left\{\begin{aligned} u+v+w&=2 \quad — \text{i} \\ 2u+\quad 3w&=5 \quad — \text{ii} \\ \hline 3u+v+4w&=7 \\ 3u+v+4w&=7 \quad — \text{iii} \\ \hline 0&=0 \end{aligned}\right.$$

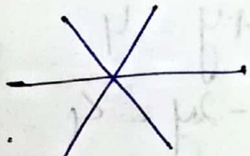
equ^n i + ii = equ^n iii

System of equ^{n}s is consistent

$u, v, w \rightarrow$ can have many values.

for $0=0$. $\Rightarrow$ Infinitely many sol^n.

Row form $\rightarrow$ Each rows represents the each plane.



Column form:

$u+v+w=2$

$2u+3w=5$

$3u+v+4w=6$

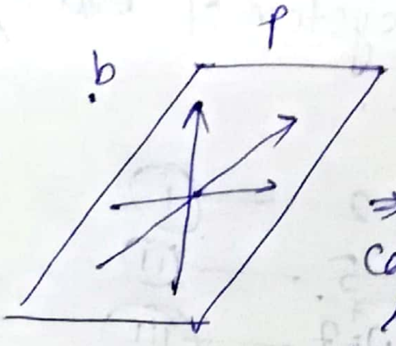
$$u\begin{bmatrix}1\\2\\3\end{bmatrix}+v\begin{bmatrix}1\\0\\1\end{bmatrix}+w\begin{bmatrix}1\\3\\4\end{bmatrix}=\cancel{\begin{bmatrix}2\\5\\6\end{bmatrix}}=b$$

L.H.S of Column form $\rightarrow$

These column are lying on a same plane provided any suitable cond^n, sum of these 3

$\begin{bmatrix}0\\0\\0\end{bmatrix}$. Then three planes are lying on the same planes.

These three columns along with the R.H.S 'b' column are lying on a same plane, then only → Consistent.



⇒ In this case 'b' vector lies outside the common plane ⇒ No solⁿ is found.

(Inconsistent).

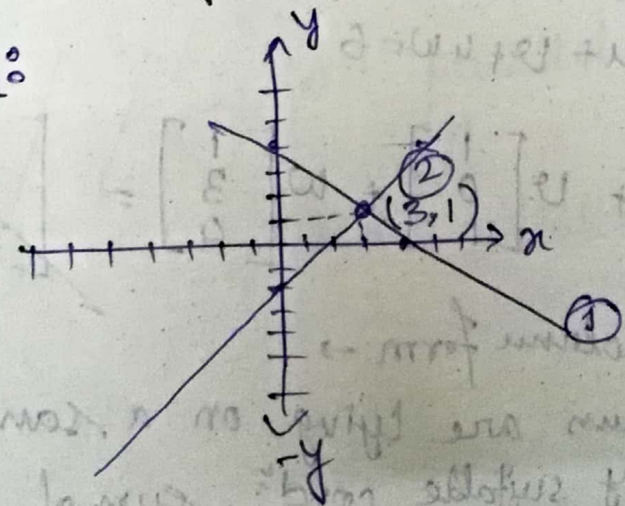
Q.1) For the equⁿs

$x+y=4$

$2x-2y=4 \qquad \frac{x}{2}+\frac{y}{-2}=1$

Draw the row picture & column pic and find the solution for this system if any.

Row form :

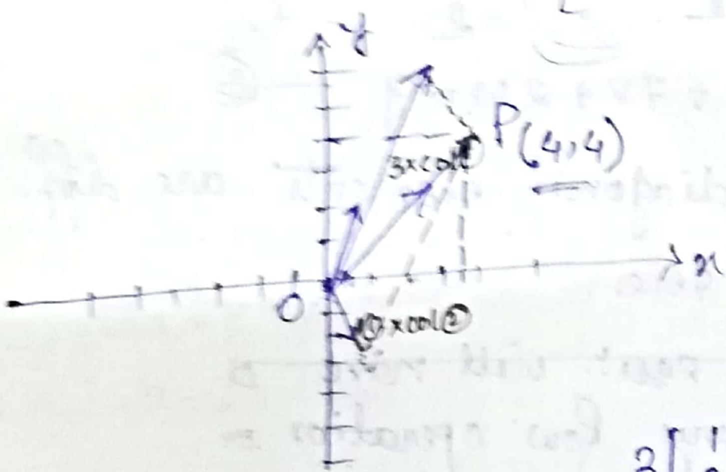


Solⁿ → (3,1)

Column form :

$$x\begin{bmatrix}1\\2\end{bmatrix} + y\begin{bmatrix}1\\-2\end{bmatrix} = \begin{bmatrix}4\\4\end{bmatrix}$$

let, $x=3$, $y=1$ } Solution



$$3\begin{bmatrix}1\\2\end{bmatrix} + 1\begin{bmatrix}1\\-2\end{bmatrix}$$

$$= \begin{bmatrix}3\\6\end{bmatrix} + \begin{bmatrix}1\\-2\end{bmatrix}$$

$\vec{OP}$ ⇒ Sum of L.H.S

H/W

2) Draw two pictures in two planes for the equ^ns → $x - 2y = 0$
$x + y = 6$

Find the solution.

3) For 4 linear equ^ns with ② unknowns (x,y) the row pictures shows that 4 st. lines. the column pic is in 4 dimensional space. The equ^ns have no solution unless the vector of R.H.S is a combination of proper linear combination of L.H.S ② column vectors.

Gaussian Elimination Method:-

$2u+v+w=5$ —— (1)

$4u-6v=-2$ —— (2)

$-2u+7v+2w=9$ —— (3)

All diagonal elements are diff. from zero.

~~# last eqn. will have 0.~~

elimentary Row operation =

(2) − (1)×2

$\cancel{0\cdot u+4v-2w}=$

Case-I

$2u+v+w=5$ —— (4)

$-8v-2w=-12$ —— (5)

(iii) + (i)

$8v+3w=14$ —— (6)

Case-II

(5) + (6)

$2u+v+w=5$

$0-8v-2w=-12$

$0 \quad 0 \quad \boxed{w=2}$

$-8v=-8$

$v=1$

$u=1$

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Solve by Gaussian Elemination method.

Q) ①

$$u + v + w = 6$$
$$u + 2v + 2w = 11$$
$$2u + 3v - 4w = 3$$

Q) ②

$$u + v + w = 7$$
$$u + 2v + 2w = 10$$
$$2u + 3v - 4w = 3$$

1)

$$\begin{pmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 2 & 11 \\ 2 & 3 & -4 & 3 \end{pmatrix} \xrightarrow[R_3 \to R_3 - 2R_1]{R_2 \to R_2 - R_1} \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & 5 \\ 0 & 1 & -6 & -9 \end{pmatrix}$$

$$\downarrow R_3 \to R_3 - R_2$$

$$\begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & -7 & -14 \end{pmatrix}$$

$\therefore$ The third row says $\rightarrow$ $-7w = -14$

$\therefore$ $w = 2$.

$\therefore$ $v + w = 5$

$\Rightarrow v = 5 - w = 5 - 2 = 3$

$\therefore$ $u + v + w = 6$

$\Rightarrow u = 6 - v - w = 6 - 3 - 2 = 1$

Ans
$u = 1$
$v = 3$
$w = 2$

2)

$$\begin{pmatrix} 1 & 1 & 1 & 7 \\ 1 & 2 & 2 & 10 \\ 2 & 3 & -4 & 3 \end{pmatrix} \xrightarrow[R_3 \to R_3 - 2R_1]{R_2 \to R_2 - R_1} \begin{pmatrix} 1 & 1 & 1 & 7 \\ 0 & 1 & 1 & 3 \\ 0 & 1 & -6 & 4 \end{pmatrix}$$

$$\downarrow R_3 \to R_3 - R_2$$

$$\begin{pmatrix} 1 & 1 & 1 & 7 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & -7 & 1 \end{pmatrix}$$

$\therefore$ $-7w = 1$

$\Rightarrow w = -1/7$

$\therefore$ $v + w = 3$

$\therefore$ $v = 3 + 1/7 = 22/7$

$u + v + w = 7$

$\therefore u = 7 - \frac{22}{7} + \frac{1}{7} = \frac{49 - 22 + 1}{7} = \frac{28}{7} = 4$

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Gaussian Elimination Method :

$2u + v + w = 5$

$4u - 6v = -2$

$-2u + 7v + 2w = 9$

Matrix :- vector representation

$Ax = b$

$$\begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 9 \end{bmatrix} \quad \text{--- (1)}$$

Step-1 Subtract 2 times of 1st row from the 2nd row

Step-2 " (-1) " " 1st row from 3rd row

Step-3 Subtract (-1) times of 2nd row from 3rd.

using step-1

$$\left.\begin{aligned} 2u + v + w &= 5 \\ -8v - 2w &= -12 \\ -2u + 7v + 2w &= 9 \end{aligned}\right\} \quad (2)$$

$$Eb = \begin{bmatrix} 5 \\ -12 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ -2 \\ 9 \end{bmatrix}$$

$$\boxed{EAx = Eb} \; c$$

Where, $E = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Using step 2

$$\left.\begin{aligned} 2u+v+w &= 5 \\ -8v-2w &= -12 \\ 8v+3w &= 14 \end{aligned}\right\} \rightarrow (3)$$

$FEAx = Fc = d$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \boxed{1} & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ -12 \\ 9 \end{bmatrix} = \begin{bmatrix} 5 \\ -12 \\ 14 \end{bmatrix}$$

where, $F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ (3)

Using step 3

$$\left.\begin{aligned} 2u+v+w &= 5 \\ -8v-2w &= -12 \\ w &= 2 \end{aligned}\right\} \rightarrow (4)$$ (upper triangular Matrix)

$GFEAx = Gd$

$$G \begin{bmatrix} 5 \\ -12 \\ 14 \end{bmatrix} = \begin{bmatrix} 5 \\ -12 \\ 2 \end{bmatrix}$$

$G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \boxed{1} & 1 \end{bmatrix}$, $E = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$$GFEA = U = \begin{bmatrix} 2 & 1 & 1 \\ 0 & -8 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$ → Pivot Elements

Inverse of Subtraction is Addition

$G^{-1}(GFEA) = G^{-1}U$

$(G^{-1}G)FEA = G^{-1}U$

$\Rightarrow IFEA = G^{-1}U$

$\Rightarrow FEA = G^{-1}U$

$F^{-1}(FEA) = F^{-1}G^{-1}U$

$\Rightarrow EA = F^{-1}G^{-1}U$

$\Rightarrow (E^{-1}E)A = E^{-1}F^{-1}G^{-1}U$

$\Rightarrow IA = E^{-1}F^{-1}G^{-1}U$

$\Rightarrow \boxed{A = E^{-1}F^{-1}G^{-1}U} \rightarrow A = LU$

$E^{-1}E = I$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \boxed{1} & 0 & 1 \end{bmatrix}$$

$$F^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{cc} G^{-1} & G \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$L = E^{-1}F^{-1}G^{-1}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} \rightarrow \text{unity}$$

A = Lower triangular matrix × Upper triangular matrix

Q) Represent a following matrix A as a product of lower & upper triangular matrix

(i) $A = \begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{bmatrix}$

Let, $A = LU$

where, $L = \begin{bmatrix} 1 & 0 & 0 \\ l_1 & 1 & 0 \\ l_2 & l_3 & 1 \end{bmatrix}$ & $U = \begin{bmatrix} u_1 & u_2 & u_3 \\ 0 & u_4 & u_5 \\ 0 & 0 & u_6 \end{bmatrix}$

$LU = A$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_1 & 1 & 0 \\ l_2 & l_3 & 1 \end{bmatrix} \begin{bmatrix} u_1 & u_2 & u_3 \\ 0 & u_4 & u_5 \\ 0 & 0 & u_6 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{bmatrix}$$

$$\begin{bmatrix} u_1 & u_2 & u_3 \\ l_1u_1 & l_1u_2+u_4 & l_1u_3+u_5 \\ l_2u_1 & l_2u_2+l_3u_4 & u_3l_2+u_5l_3+u_6 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{bmatrix}$$

$u_1 = 2$, $u_2 = 3$, $u_3 = 3$

$l_1 = 0$, $l_2 = 3$,

1) Write down the 2x2 matrices A & B that have entries $a_{ij} = i+j$

& $b_{ij} = (-1)^{i+j}$

and then find AB & BA.

2) Find two inner product of

$$\begin{bmatrix} 1 & -2 & 7 \end{bmatrix}_{1\times 3} \begin{bmatrix} 1 \\ -2 \\ 7 \end{bmatrix}_{3\times 1} = \begin{bmatrix} 1+4+49 \end{bmatrix} = \begin{bmatrix} 54 \end{bmatrix}_{1\times 1}$$

$$\begin{bmatrix} 1 & -2 & 7 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 3-10+7 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -2 \\ 7 \end{bmatrix}_{3\times 1} \begin{bmatrix} 3 & 5 & 1 \end{bmatrix}_{1\times 3} = \begin{bmatrix} 3 & 5 & 1 \\ -6 & -10 & -2 \\ 21 & 35 & 7 \end{bmatrix}_{3\times 3}$$

3) Multiply AX to ~~multiply~~ find a solution AX = zero vector. Can u find more solutions to AX = zero where

$$A = \begin{bmatrix} 3 & -6 & 0 \\ 0 & 2 & -2 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

Formula

$$(A+B)^2 = A(A+B) + B(A+B)$$

$$= (A+B)(B+A)$$

$$= (A^2 + AB + BA + B^2)$$

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Triangluar Factorization:

$$A = L\,U$$

L: lower triangular; U: upper triangular

LU decomposition Method.

$2u+v+w = 5$

$4u - 6v = -2$

$-2u + 7v + 2w = 9$

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix} \quad x = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad b = \begin{bmatrix} 5 \\ -2 \\ 9 \end{bmatrix}$$

$A = LU$

$$\Rightarrow L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} ; \quad U = \begin{bmatrix} 2 & 1 & 1 \\ 0 & -8 & -2 \\ 0 & 0 & 1 \end{bmatrix}.$$

Given system: $AX = b$ — (1)

$\because A = LU$

$\therefore (LU)X = b$

$\Rightarrow L(UX) = b$ — (2) [By associativity

let, $(UX) = D$ — (3)

Then equⁿ (2) $\rightarrow$

$LD = b$ (L, b are known)

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}_{3\times3} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}_{3\times1} = \begin{bmatrix} 5 \\ -2 \\ 9 \end{bmatrix}_{3\times1}$$

$\therefore d_1 = 5$

$2d_1 + d_2 = -2 \quad \Rightarrow d_2 = -12$

$-d_1 - d_2 + d_3 = 9 \quad \Rightarrow d_3 = 9 + d_1 + d_2$

$= \cancel{8}\ 2$

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -12 \\ 2 \end{bmatrix}$$

From (3) $\rightarrow$

$$UX = D$$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 1 \\ 0 & -8 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 5 \\ -12 \\ 2 \end{bmatrix}$$

$2u + v + w = 5$ $\quad \therefore \boxed{u = 1}$

$-8v - 2w = -12$ $\quad \therefore \boxed{v = 1}$

$\boxed{w = 2}$

$$\therefore X = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$ Solution for linear system of equ^n.

$$Ax = b$$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 9 \end{bmatrix}$$

$$Ax = b$$

$$(A^{-1}A)x = A^{-1}b$$

$$\boxed{x = A^{-1}b}$$

Rule

i) $A^{-1} = \dfrac{1}{|A|} \mathrm{Adj}[A]$

ii) Gauss-Jordan Rule

Inverse of Matrix

i) If the inverse of a matrix exists then it can't have two different inverses.
[unique inverse]

Explain :- If B & C are the inverses of A.

$\therefore BA = I \quad \text{--- (1)}$

$AC = I \quad \text{--- (2)}$

postmultiplying relation (1) by C matrix.

$BAC = IC = C \longrightarrow (3)$

premultiplying relation (2) by B matrix

$BAC = BI = B \longrightarrow (4)$

from (3) & (4), implies that $B = C$
shows uniqueness of inverse matrix.

ii) If A is invertible, then one & only one solution to $AX = b$ will be $A^{-1}b$.

$$AX = b$$

$$(A^{-1}(A)X) = A^{-1}b$$

$$\Rightarrow \boxed{X = A^{-1}b}$$

iii) Suppose, there is a non-zero vector X, such that $AX = 0$, then A can't have an inverse.

$$A^{-1}AX = A^{-1}0$$

$$\Rightarrow X = 0$$ (not possible because X is non zero vector.)

iv) If A, B are 2x2 matrices ~~then~~ of the form, then the inverse of this matrix exists if and only if —

$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb \neq 0$ $\quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times2}$

v) A diagonal matrix has an inverse provided no diagonal entries are zero.

Gauss-Jordan Inverse Method :

Coefficient matrix → $\begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$

$$\begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$d_1 u = b_1$

$d_2 V = b_2$

$d_3 \omega = b_3$

Let, B be the inverse of A.

$\therefore$ $\boxed{AB = I}$ — (1) (from definition)

We shall apply no. of suitable elementary row operations on the both sides from left.

We should choose the row operations so that A matrix will reduce to identity matrix.

$$\left\{E_n \cdots E_2(E_1 A)\right\}B = E_n \cdots E_2 E_1 I$$

$$\Downarrow$$

$$I \quad B = E_m E_{m-1} \cdots E_2 E_1 I = D \text{ (let)}$$

(non identity matrix)

$\therefore IB = D$

$\therefore B = D$

Determine

$AX = b$ $\quad A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix}$ $\quad x = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$

$b = \begin{bmatrix} 5 \\ -2 \\ 9 \end{bmatrix}$

Find Inverse of A :-

Let, B be the inverse of A.

$\therefore AB = I$

Applying Gauss Jordan Method

To find inverse, consider

$$[A \mid I]$$

$$= \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 4 & -6 & 0 & 0 & 1 & 0 \\ -2 & 7 & 2 & 0 & 0 & 1 \end{array}\right]$$

$\downarrow R_2 \to R_2 - 2R_1, \quad R_3 \to R_3 + R_1$

$$\approx \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & -8 & -2 & -2 & 1 & 0 \\ 0 & 8 & 3 & 1 & 0 & 1 \end{array}\right]$$

$$\Big\downarrow \begin{array}{l} R_2 \to -\frac{1}{8}R_2 \\ R_3 \to R_3 \times \frac{1}{3} \end{array}$$

$$\simeq \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{4} & \frac{1}{4} & -\frac{1}{8} & 0 \\ 0 & \frac{8}{3} & 1 & \frac{1}{3} & 0 & \frac{1}{3} \end{array}\right]$$

$$\Big\downarrow R_1 \to R_1 - R_2$$

$$\simeq \left[\begin{array}{ccc|ccc} 2 & 0 & \frac{3}{4} & \frac{3}{4} & \frac{1}{8} & 0 \\ 0 & 1 & \frac{1}{4} & \frac{1}{4} & -\frac{1}{8} & 0 \\ 0 & \frac{8}{3} & 1 & \frac{1}{3} & 0 & \frac{1}{3} \end{array}\right]$$

$$\Big\downarrow R_3 \to R_3 - \frac{8}{3}R_2$$

$$\left[\begin{array}{ccc} 2 & 0 & \frac{3}{4} \\ 0 & 1 & \frac{1}{4} \\ 0 & 0 & \frac{1}{3} \end{array}\right.$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{4} & -\frac{5}{16} & -\frac{3}{8} \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{3}{8} & -\frac{1}{4} \\ 0 & 0 & 1 & -1 & 1 & -1 \end{array}\right]$$

Inverse of A

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$A = LU$ —— (1)

$A = LDU'$ —— (2)

L: Lower triangular matrix with diagonal matrix is unity.

U: Upper triangular matrix, with non-zero diagonal elements.

U': Upper triangular matrix with all the diagonal elements to be unity.

D: Diagonal matrix with ~~one~~ all pivot elements as diagonal elements.

Transpose of a Matrix: (A^T)

Properties

$A = A(ij)_{m \times n}$

$A^T = (a_{ji})_{n \times m}$

(i)

considering AB.

$(AB)^T = B^T A^T$

(ii) $(A^{-1})^T = (A^T)^{-1}$

~~$A_1 = B^{-1}$~~

Proof $AC = I \quad [\Rightarrow C = A^{-1}]$

$A C C^{-1} = I C^{-1}$

$\therefore \boxed{A = C^{-1}}$

$AA^{-1} = I$

$(A A^{-1})^T = I^T = I$

Now using property (i)

$$(A^{-1})^T A^T = I$$

$$\Rightarrow (A^{-1})^T (A^T \cdot (A^T)^{-1}) = I \cdot (A^T)^{-1}$$

$$\Rightarrow (A^{-1})^T \cdot I = (A^T)^{-1}$$

$$\Rightarrow \boxed{(A^{-1})^T = (A^T)^{-1}}$$ proved

Symmetric Matrix :

$$A = A^T$$

$$a_{ij} = a_{ji} \quad \forall i, j$$

Skew symmetric Matrix

$$a_{ij} = -a_{ji} \quad , i = j$$

$$2a_{ij} = 0$$

$$a_{ij} = 0$$

Statement

A symmetric matrix need not be invertible, but if the inverse exists then it is also symmetric

let, $A \rightarrow$ Symmetric Matrix

$$\therefore A = A^T$$

let, A^{-1} exists.

Prove that A^{-1} is symmetric.

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$$(A^{-1})^T = (A^T)^{-1}$$

$$\Rightarrow (A^{-1})^T = A^{-1}$$

$B^T = B$ if $A^{-1} = B$.

that means, A^{-1} is symmetric because B is also symmetric.

Resulting matrix of AA^T or A^TA will be symmetric matrix but not necessarily they are similar to each other.

Ex- $A = [1\ 2]$ $A^T = \begin{bmatrix}1\\2\end{bmatrix}$

$$\underset{1\times2\ 2\times1}{AA^T} = [1\ 2]\begin{bmatrix}1\\2\end{bmatrix} = [5]_{1\times1}$$

Now, $A^TA = \begin{bmatrix}1\\2\end{bmatrix}[1\ 2] = \begin{bmatrix}1 & 2\\2 & 4\end{bmatrix}_{2\times2}$

$A = LDU$ — (1)

A: Symmetric Matrix.

$\therefore A = A^T$

$$A^T = (LDU)^T$$

$$A^T = U^T D^T L^T$$

$$\Rightarrow A = U^T D^T L^T \text{ — (2)}$$

$$\therefore LDU = U^T D^T L^T$$

Comparing —

$D = D^T$

$L = U^T$

$U = L^T$

1) Use Gauss-Jordan Method to find inverse of

i) $A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ii) $A_2 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

iii) $A_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

2. If $A = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ find A^TB, B^TA, AB^T, BA^T.

3) Use gauss-Jordan elimination on $[A \cdot I]$ to solve $AA^{-1} = I$ given that $\begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} [x_1 \; x_2 \; x_3] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4. change I ~~A~~ into A^{-1} as you reduce A to I from the following

$[A \vdots I] = \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array}\right]$

&

$[A \vdots I] = \left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 3 & 9 & 0 & 1 \end{array}\right]$

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5. Find A^T, A^{-1} & $(A^{-1})^T$ & $(A^T)^{-1}$ for

$$A = \begin{bmatrix} 1 & 0 \\ 9 & 3 \end{bmatrix} \quad \& \quad A = \begin{bmatrix} 1 & c \\ c & 0 \end{bmatrix}$$

6. Verify that $(AB)^T = B^T A^T$ but those are different from $A^T B^T$ where

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$$

7. Factorize the following symmetric matrices into $A = LDL^T$ $D \to$ Diagonal Matrix

$$A = \begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix}, \quad A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

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Vector Space

Elements of vector space are vectors.

Group: Contains elements are defined in a set, only one single operation

$G = \{a, * \text{operation}\}$

4 Rules for Group :- If Operation is Addition, then —

(i) if $a, b \in G \Rightarrow a+b \in G$ [Closure Property]

(ii) [Associative property]

$a, b, c \in G$

$a+(b+c) = (a+b)+c$

(iii) $\exists\ e \in G$

such $a+e = a \ \forall\ a \in G$

$\Downarrow$

[$=0$ (identity element)]

(iv) Existance of Inverse Element

$a \in G\ \exists$ an element $b \in G$

s.t. $a+b=0$

$5+(-5)=0$

Commutative Group :- If $(a+b) \in G$

$\therefore (b+a) \in G$.

Ring: $R = \{ a, \underset{\text{operation}}{*}, \underset{\text{other operation}}{**} \}$

under addition previous all rules are satisfied.

under multiplication ——

(v) any $a, b \in R$
$\Rightarrow a \cdot b \in R$ [closure]

(vi) $a, b, c \in R$
$a \cdot (b \cdot c) = (a \cdot b) \cdot c \in R$ [associativity]

(vii) Existance of identity

any $a \in R$, $\exists$ an element $e \in R$
s.t. $a \cdot e = e \cdot a = a$

(viii) any $a \in R$, $\exists$ an element $b \in R$
s.t. $a \cdot b = e = b \cdot a$ [Identity element]

$a+b = b+a$

Distributing property
$a \cdot (b+c) = a \cdot b + a \cdot c$

Field: An a algebraic structure:
$F = \{ a, *, ** \}$

i) Every non-zero elements must have inverse in this a structure.

ii) All other properties are satisfield.

Definition:- Any Ring containing atleast two elements is called a field, if it is commutative, has a unity element & is such that, all non zero elements have it's multiplicative inverse.

Vector Space (V) / Linear Space

A vector space (V) over a field 'F' consists of a set on which two operations namely addition & scalar multiplication are defined so that for any two elements x & y belonging to V there must exist an unique element $(x+y) \in V$ and for any scalar $a \in$ 'F' there must exist an unique element $ax \in V$ such that the following conditions are satisfied :→

VS 1) For $x, y \in V$, $(x+y) \in V$
[Commutative]

VS 2) For $x, y, z \in V$, $(x+y)+z = x+(y+z)$
[Associativity]

VS 3) There exists an element in V denoted by '0' such that
$x+0 = x = 0+x$

VS 4) For each element $x \in V$ there exists an element $y \in V$

Such that $x+y=0$

(VS 5) for each element $x \in V$, $1 \cdot x = x$

(VS 6) For each pair of element $a, b \in$ 'F' and each element '$x \in V$ we have

$(ab)x = a(bx)$ [associativity under multiplication]

(VS 7) for each element $a \in F$ & each pair of $x, y \in V$, we have $a(x+y) = ax+ay$

[Distributive Property]

(VS 8) For each pair of element $a, b \in F$ and for each $x \in V$, we have

$(a+b) \cdot x = ax + bx$

Ex-1) $u = (a_1, a_2, \ldots, a_n)$ $a_i \in F$

$v = (b_1, b_2, \ldots, b_n)$ $b_i \in F$

u & v in n-dimensions $\in F^n$

or

$u, v \in V(F^n)$

$\rightarrow$ $u + v = (a_1+b_1, a_2+b_2, \ldots, a_n+b_n) \in V(F^n)$

$c \in F^n$

$c.u = c(a_1, a_2, \ldots, a_n) \in V(F^n)$

$= (ca_1, ca_2, ca_n)$

Real Field

Q) $u = (3, -2, 0) \in V(F^3)$

$v = (-1, 1, 4) \in V(F^3)$

c (scalar) $= -5$

$\Rightarrow u+v = (2, -1, 4) \in V(F^3)$

$c \cdot u = (-5) \cdot (3, -2, 0)$

$= (-15, 10, 0) \in V(F^3)$

Such all other properties are satisfied.

Q) ~~a~~ $F \to$ complex field $(a+ib)$

$V(F)$

let, $u = (1+i, 2i)$

$v = (2-3i, 4i)$

$u+v = (1+i, 2i) + (2-3i, 4i)$

$= (3-2i, 6i)$

$c = \cancel{(1+i, 3i+4), (6i)}\ -5i$

$c \cdot u = \cancel{(1+i, 3i+4)}\ 5i(1+i, 2i)$

$= (5i(1+i), 5i \cdot 2i)$

$= (5i-5, -10) \in V(F)$

Q) $S = \{(a_1, a_2) : a_1, a_2 \in R\}$ Real field

$(a_1, a_2), (b_1, b_2) \in S$ such that

$c \in R$ defined by $(a_1, a_2) + (b_1, b_2) = \{a_1 + b_1, a_2 - b_2\}$

Whether the 'S' is a vector space or not?

$\Rightarrow$ $(b_1, b_2) + (a_1, a_2) = (b_1 + a_1, b_2 - a_2)$

So, $(a_1, a_2) + (b_1, b_2) \neq (b_1, b_2) + (a_1, a_2)$

Not following ~~Associativity~~. Commutative
Which contradicts VS1.

$S \rightarrow$ not a vector space over R.

Q) $S = \{(a_1, a_2) : a_1, a_2 \in R\}$

$(a_1, a_2), (b_1, b_2) \in S$

$c \in R$ $c(a_1, a_2) = (ca_1, 0) \in R$

defined by —

$(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, 0)$

$\Rightarrow$ For Scalar Multiplication $\rightarrow$

$c(a_1, a_2)$

$= (ca_1, \cancel{ca_2}\,0) \neq (a_1, a_2)$ $1x = x$ VS5 Not satisfied.

~~Here~~ (Consider $c = 1$)

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$u = (a_1, a_2, \ldots, a_n) \longrightarrow$ n tuple

If $n = 3$, $\qquad a_i \in F$

$u = (a_1, a_2, a_3)$

$\therefore u \in F^3$

$$\underset{3\times3}{(A)} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}$$ → can be treated as a tuple → row vector

Column vector

$\left[x, y \in V \Rightarrow x+y \in V \right.$
$\left. a \in F, \quad ax \in V \right]$ condition of vector space.

Ex ① Vector space consists the elements that are matrices of order $(m \times n)$.

$M_{m\times n} \in V(F)$

Now, Check the above conditions are satisfying or not?

⇒ let, $\underset{3\times2}{A_{m\times n}}, \underset{3\times2}{B_{m\times n}} \in V(F)$

let,

$$\underset{3\times2}{A} = \begin{bmatrix} 2 & 0 \\ 1 & 5 \\ 6 & 3 \end{bmatrix} \qquad \underset{3\times2}{B} = \begin{bmatrix} 7 & 5 \\ 3 & 3 \\ 8 & 2 \end{bmatrix}$$

Now, $A + B = \begin{bmatrix} 9 & 5 \\ 4 & 8 \\ 14 & 5 \end{bmatrix}_{3\times2}$, every element $\in F$

So, $(A+B)_{3\times 2} \in V(F)$

$\Rightarrow (A+B)_{m\times n} \in V(F)$ —— (i) cond^n proved

Now, let, scalar quantity $(c) = -5$

$\therefore c(A)_{3\times 2} = -5\begin{bmatrix} 2 & 0 \\ 1 & 5 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} -10 & 0 \\ -5 & -25 \\ -30 & -15 \end{bmatrix}_{3\times 2} \in V(F)$

So $c(A)_{m\times n} \in V(F)$ —— (ii) cond^n proved

$\therefore M_{m\times n} \in V(F)$ it will form a vector space

Theorem : Cancellation law of vector addition $\longrightarrow$

If x, y & z are vectors in a vector space V, such that $x+z = y+z$, then $x = y$.

Proof := Property of V.S-4

$x \in V$; $y \in V$ st $x+y=0$

By V.S-4, for any set $z \in V$,

$\exists\ v \in V$ st. $z+v = 0$

(y is not necessarily inverse of z)

L.H.S $x = x + 0$ (VS-3)

$= x + (z + y)$ [$\because y + z = 0$]

$= \cancel{x+y}\ \cancel{(x+z)+y}\ y + (z+x)$

$= \cancel{(y+z)} + y + 0$ [where $x+z = y+z$, given in problem]

$= \cancel{0+y}$

$= y + \cancel{0}$

$\therefore \boxed{x = y}$ Proved

Theorem-2

In any vector space V, the following statements are true —

(i) $0x = 0$ for each $x \in V$

(ii) $(-a)x = -(a x) = a(-x)$ for each $x \in V$ & $a \in F$.

(iii) $0 \cdot a = 0$ for each $a \in F$.

8) At the end of may month a furniture store had the following inventory ___

M matrix

	A	S	M	D
Living room suts	4	2	1	3
Bedroom "	5	1	1	4
Dinning "	3	1	2	6

Record this data in (3x4) matrix M to prepare it's June sale, the store to double it's inventory of ~~all~~ each of the items listed in the preceeding table. Assuming that, none of the present stock is sold untill the additional furniture arrise. verify that the inventer on an after the order is filled is described by the matrix 2M. also if the inventory at the end of June is described by the matrix A.

$$A = \begin{bmatrix} 5 & 3 & 1 & 2 \\ 6 & 2 & 1 & 5 \\ 1 & 0 & 3 & 3 \end{bmatrix}_{3\times4}$$

then interpretation (2M-A).

How many suits were sold during the June Sale?

$(2M)_{3\times 4}$:

$$M = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 5 & 1 & 1 & 4 \\ 3 & 1 & 2 & 6 \end{bmatrix}$$

$$2M = \begin{bmatrix} 8 & 4 & 2 & 6 \\ 10 & 2 & 2 & 8 \\ 6 & 2 & 4 & 12 \end{bmatrix}_{3\times 4}$$ at the end of June

at the end of June month if the inventory matrix →

$$A = \begin{bmatrix} 5 & 3 & 1 & 2 \\ 6 & 2 & 1 & 5 \\ 1 & 0 & 3 & 3 \end{bmatrix}$$

No. of furnitures sold ⇒

$$2M - A = \begin{bmatrix} 8 & 4 & 2 & 6 \\ 10 & 2 & 2 & 8 \\ 6 & 2 & 4 & 12 \end{bmatrix} - \begin{bmatrix} 5 & 3 & 1 & 2 \\ 6 & 2 & 1 & 5 \\ 1 & 0 & 3 & 3 \end{bmatrix}$$

8)

for L Room ⟵ $= \begin{bmatrix} 3 & 1 & 1 & 4 \\ 4 & 0 & 1 & 3 \\ 5 & 2 & 1 & 9 \end{bmatrix}$

for Bed Room ⟵

for Dinning Room Suits ⟵

Objective Type Question

1) Every v.s contain a zero vector. → True

2) A v.s may have more than one zero vector → False

3) Any v.s $a.x = bx$ implies $a=b$ → False [unless $x=0$]

4) Any v.s, $ax = by$ implies $x=y$ → False [unless $x=0$]

5) A vector of F^n, may be regarded as a matrix $(M)_{n\times 1}(F)$ → True

$$M_{n\times 1} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}_{n\times 1} \in F^n$$

6) An $M_{m\times n}$ matrix has m colums & n rows — False

7) If f is a polynominal of degree n, and c is a non zero scalar then $c.f$ is a polynominal of degree n → True.

8) A non zero scalar of F, may be considered to be a polynominal of degree zero – True.

$$f = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n.$$

$a_1 = a_2 = \ldots a_n = 0$

$$\boxed{f = a_0 x^0}$$

Sub-Space :

A subspace of a v.s is a non-empty subset that satisfies all the requirements of a v.s..

[vital criteria are :-

$x \in v.s$

$y \in v.s$

$\therefore x+y \in v.s$

&

and linear combination rule must be satisfied.

(a) If we add any vector x & y in the subspace, then $(x+y)$ also belong to the sub space

(b) If we multiply any vector 'x' in the subspace by any scalar 'c' then $c.x \in$ Subspace.

Smallest subspace ~~of a~~, a set contain zero vector $\rightarrow$ $0 \in S \subseteq V$

$0+0=0 \in S$
$c.0=0 \in S$
} Both conditions are satisfied.

Largest sub-space — The vector space itself.

zero must be included in the sub-space & v-s.

Q. ~~V.S.~~ $M_{3\times3} \in$ V.S.
~~Ra~~ Take Lower triangular matrices from v-s. Will these matrices form a ~~vector sp~~ subspace?

$\Rightarrow$ $M_{3\times3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3} \in V$

$L_1 = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$

Now, those two main properties should be satisfied.

$$L_2 = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \in S$$ (Because L is a subset of V)

$$L_1 + L_2 = \begin{bmatrix} 2 & 0 & 0 \\ l_1'' & 2 & 0 \\ l_2'' & l_3'' & 2 \end{bmatrix} \in L$$ (Since S is a subspace of V)

[Closure property holds]

$$c \cdot L_1 = \begin{bmatrix} c & 0 & 0 \\ l_1''' & c & 0 \\ l_2''' & l_3''' & c \end{bmatrix} \in S$$

[Scalar multiplication property Holds]

$\therefore$ So we can 's' is a subspace of V.

Subspaces:

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necessary of ~~being~~ subspace: When any subset follows all the '8' conditions (1, 2, 5, 6, 7, 8) (3, 4) also must be satisfied → That subset will be a subspace of the V.S.

Let, S be a subset of V.S 'V' defined over the field 'F' then 'S' will form a subspace 'V' iff the following conditions are satisfied-

Conditions

① for any $x, y \in S$, $(x+y) \in S$
[closed under Addition operation]

② for any $x \in S$ & any scalar $c \in F$,
$cx \in S$ [closed under scalar multiplication]

③ V.S. 3
S will contain 'zero' ~~elemer~~ vector.

④ V.S. 4
each vector in S will have its additive inverse in S.

Theorem

Let, V be a V.S. and W is a subset of 'V' then 'W' is a subspace of 'V' iff the following three conditions are satisfied. (Necessary conditions)

① $0 \in W$

② $x+y \in W$ $\forall x \in W, y \in W$

(iii) $cx \in W$ $\forall x \in W$, & $c \in F$

Proof

Let, W be a subspace of V.S. 'V'

$\therefore$ from the definition of a V.S, we can say, condition (ii) & (iii) can hold.

$\because$ W is assumed to be a V.S.

Let, $0'$ be a zero vector $\in W$

s.t. $x+0' = x$ $\forall x \in W$ — (1)

$\therefore x \in V$ [Since $W \subset V$]

$\therefore x+0 = x$. — (2)

from (1) & (2)

$\therefore x+0' = x = x+0$

$\Rightarrow 0' = 0$ [by 'x' cancellation property]

Sufficients conditions :

Conditions (i), (ii) & (iii) holds the W.

R.T.P that W is a vector space.

Let, $x \in W$.

$c = -1 \in F$

Then by property (iii) $(-1)x \in W$.

$\therefore \; 1(-x) \in W$

$\Rightarrow \; -x \in W$

$\therefore$ Additive Inverse also belongs to W.

So W is a Vector space. Proved

Example

Let, n be a non-negative integer and let $P_n(F)$ consists of all polynomials in $P(F)$ having degree less than or equals to n, then S.T. $P_n(F)$ is a Vector space.

$P_n(F) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

$P_1(F) = a_1 x + a_0 \rightarrow$ (of Degree $= 1$)

If $a_1 = 0$, then polynomial will be of degree '0'. $P(F) = a_0$

Now

$P(F) = 0 \rightarrow$ It has degree (-1).

Now, by condition (i) $\rightarrow$ satisfied.

$0 \in P_n(F)$

Addition of polynomial, never exceeds n 'degr

$\Downarrow$ $(<= n)$

So, condition (ii) $\rightarrow$ Satisfied.

Addition of two polynomial having degree $<= n$ $\in P_n(F)$

Taking a polynomial and multiplying scalar, that will also belong to $P_n(F)$
[Con^n: (iii) Satisfied]

$\therefore$ $P_n(F)$ forms a V.S. Proved

Example

Show that the set of diagonal matrices is a subspace of $M_{n\times n}(F)$

$\rightarrow$ Let, for a set of all diagonal matrices

$M'_{n\times n} \rightarrow a_{ij} = 0$ for $i \neq j$

$$\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \rightarrow \text{If diagonal elements are zeros.}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \text{dig. matrix (zero matrix)}$$

$\in M'_{n\times n}$

Cond^n: (i) Satisfied.

$(A)_{n\times n}$, $(B)_{n\times n}$ $\rightarrow$ two diagonal matrices

For $i \neq j$

$(A+B)_{ij} = A_{ij} + B_{ij} = 0 + 0 = 0$

$\therefore$ $(A+B) \in M'_{n\times n}$

Cond^n: (ii) is satisfied.

$(cA)_{ij} = cA_{ij} = c \cdot 0 = 0$

$\therefore (cA)_{ij} \in M'_{n \times n}$

Con (iii) satisfied.

$\therefore M'_{n\times n}$ subspace.

Ex-2 The set of matrices $M_{m\times n}(R)$ having non-negative entries is not a subspace of $M_{m\times n}(R)$.

→ Cond (iii) will not be satisfied. as we take c = negative value. which allow not be taken.

So, $M'_{m\times n}(R)$ is not Sub-space.

Theorem

Any intersection of Subspaces of a v.s. 'V' is a subspace of 'V'.

Linear Combination of Vectors

Let, V be a v.s. and 's' is a non-empty subset of 'V'. A vector 'v' $\in V$ is called a linear combination of vectors of 's' if there exists a finite no. vectors $\{u_1, u_2, \dots, u_n\}$ in 's' and some scalars $\{a_1, a_2, \dots, a_n\}$ in the field 'F' such that —

$$v = a_1u_1 + a_2u_2 + \dots + a_nu_n$$

Ex $v = (2,6,8)$

How to determine this vector as a linear combination of

$u_1 = (1,2,1)$, $u_2 = (-2,-4,-2)$

$u_3 = (0,2,3)$, $u_4 = (2,0,-3)$

$u_5 = (-3,8,16)$

If possible, let

$$v = a_1(1,2,1) + a_2(-2,-4,-2) + a_3(0,2,3)$$
$$\Rightarrow (2,6,8) + a_4(2,0,-3) + a_5(-3,8,16)$$

$\therefore$

$$(2,6,8) = (a_1 - 2a_2 + 2a_4 - 3a_5,\ 2a_1 - 4a_2 + 2a_3 + 8a_5,$$
$$a_1 - 2a_2 + 3a_3 - 3a_4 + 16a_5)$$

Comparing →

$a_1 - 2a_2 + 2a_4 - 3a_5 = 2$ —— (1)

$2a_1 - 4a_2 + 2a_3 + 8a_5 = 6$ —— (2)

$a_1 - 2a_2 + 3a_3 - 3a_4 + 16a_5 = 8$ —— (3)

Infinitely many Solutions.

Use, gauss Elemination Technique.

Eliminating a_1 from (2) & (3) using (1)

$$\left.\begin{aligned} a_1 - 2a_2 + 2a_4 - 3a_5 &= 2 \\ 2a_3 - 4a_4 + 14a_5 &= 2 \\ 3a_3 - 5a_4 + 13a_5 &= 6 \end{aligned}\right\} (4)$$

$a_3 - 2a_4 + 7a_5 = 1$

Eliminating a_3 from ~~1st~~ last equ^n. of (4) using 2nd,

$$\left.\begin{aligned} a_1 - 2a_2 + 2a_4 - 3a_5 &= 2 \\ a_3 - 2a_4 + 7a_5 &= 1 \\ a_4 - 8a_5 &= 3 \end{aligned}\right\} (5)$$

From System (5) →

$$\left.\begin{aligned} a_1 + 2a_4 &= 2 + 2a_2 + 3a_5 \\ a_3 - 2a_4 &= 1 - 7a_5 \\ a_4 &= 3 + 8a_5 \end{aligned}\right\} (6)$$

(Canonical Form)

For any value of a_2 & a_5, system can be solved.

If $\boxed{a_2 = 0, \; a_5 = 0}$

$a_1 + 2a_4 = 2$ $\therefore \boxed{a_1 = -4}$

$a_3 - 2a_4 = 1$ $\therefore \boxed{a_3 = 7}$

$\boxed{a_4 = 3}$

$\therefore \boxed{(2,6,8) = -4u_1 + 0.u_2 + 7u_3 + 3.u_4 + 0.u_5}$

Ans

8) S.T. $2x^3 - 2x^2 + 12x - 6$ is a linear combination of $(x^3 - 2x^2 - 5x - 3)$ and $(3x^3 - 5x^2 - 4x - 9)$ in $P_3(R)$ but $3x^3 - 2x^2 + 7x + 8$ is not. 26/9/18

Sol^n $\rightarrow$ If $(2x^3 - 2x^2 + 12x - 6)$ be represented as linear combination of $(x^3 - 2x^2 - 5x - 3)$ and $(3x^3 - 5x^2 - 4x - 9)$ then there must exists two scalars say a, b, such that

$$2x^3 - 2x^2 + 12x - 6 = a(x^3 - 2x^2 - 5x - 3) + b(3x^3 - 5x^2 - 4x - 9)$$

$$= (a + 3b)x^3 + (-2a - 5b)x^2 + (-5a - 4b)x - 3a - 9b$$

$\therefore$ $a + 3b = 2$

$2a + 5b = 2$

$5a + 4b = -12$

$3a + 9b = 6$

Eliminating from last 3 equations using 1st eqⁿ

$a+3b=2 \Rightarrow a=-4$

$\left.\begin{array}{l} -b=\cancel{}-2 \\ 11b=22 \end{array}\right\} \Rightarrow b=2$

$0 \times b=0$

So, $\boxed{a=-4,\ b=2}$ (possible solution)

So, our assumption is correct.

$\therefore 2x^3-2x^2+12x-6 = -4(x^3-2x^2-5x-3)$
$+2(3x^3-5x^2-4x-9)$

Let, if possible,

$3x^3-2x^2+7x+8 = a(x^3-2x^2-5x-3)+$
$b(3x^3-5x^2-4x-9)$

$\therefore a+3b=3$

$2a+5b=2$

$5a+4b=-7$

$3a+9b=-8$

So, our assumption is wrong.

This can't be represented as linear combination.

$a+3b=3$

$\left.\begin{array}{l} -b=-4 \\ -11b=-12 \\ 0\cdot b=17 \end{array}\right\}$ Not consistance

Linear Dependency & Independency :-

Vector space over field $F \to V(F)$

let, ~~subspace~~ set $\Rightarrow S = \{\alpha_1, \alpha_2, \dots \alpha_n\} \in V$

If $v \in V$

$v = a_1\alpha_1 + a_2\alpha_2 + \dots + a_n\alpha_n$

where, $a_1, a_2 \dots a_n \in F$

Linearly Dependent Vectors :-

let, V be a v.s. over the field F, a finite subset $S = \{\alpha_1, \alpha_2 \dots \alpha_n\}$ of the vectors of V is said to be linearly dependent if there exists a set of scalars $(a_1, a_2, \dots a_n) \in F$ not all zeros, such that

$a_1\alpha_1 + a_2\alpha_2 + \dots + a_n\alpha_n = 0$

Linearly Independent Vector

is said to be linearly independent if there exists a set of scalars

s.t. $a_1\alpha_1 + a_2\alpha_2 + \dots + a_n\alpha_n = 0$

$\Rightarrow a_i = 0 \;\forall\; i = 1, 2, 3 \dots$

// Null set is assumed to be linearly independent

P.1 If two vectors be linearly dependent, then one of them is a scalar multiple of other.

α, β $\quad a, b \in F$

$a\alpha + b\beta = 0$

let, $a \neq 0$

$a\alpha = -b\beta$

$\Rightarrow \boxed{\alpha = -b/a\ \beta}$ $\quad$ α is represented as a scalar multiple of β.

P.2 A system consisting of a single non-zero vector is always linearly independent.

$S = \{\alpha\}$, $\alpha \neq 0$

$a \in F$

$a\alpha = 0$

$\Rightarrow a = 0 \quad \because \alpha \neq 0$

So, the set is linearly independent.

P.3 Every super set of linearly dependent set of vectors is linearly dependent.

$S = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$

Where $\alpha_1, \alpha_2, \ldots, \alpha_n$ all are linearly dependent.

$\exists$ set of scalars $a_1, a_2, \ldots, a_n \in F$ not all zeros.

s.t. $a_1\alpha_1 + a_2\alpha_2 + \ldots + a_n\alpha_n = 0$.

let, $a_k \neq 0$

Consider the superset $S' = \{\alpha_1, \alpha_2 \ldots \alpha_n, \beta_1, \beta_2 \ldots \beta_m\}$

$\therefore a_1\alpha_1 + a_2\alpha_2 + \ldots + a_k\alpha_k + \ldots + 0\cdot\beta_1 + 0\cdot\beta_2 + \ldots 0\cdot\beta_m = 0$

Here again $a_k \neq 0$

$\Rightarrow S'$ is linearly dependent. Proved

D-4

Any subset of linearly independent set of vector is linearly independent.

$S = \{\alpha_1, \alpha_2 \ldots \alpha_n\}$

$\Downarrow$

linearly independent

$\exists\ a_1, a_2 \ldots, a_n \in F$ s.t

$a_1\alpha_1 + a_2\alpha_2 + \ldots + a_n\alpha_n = 0$

$\Rightarrow a_i = 0 \quad \forall\ i = 1, 2 \ldots n$

Consider a subset $S' = \{\alpha_1, \alpha_2 \ldots \alpha_k\} \subseteq S$

$(k \leq n)$

$a_1\alpha_1 + a_2\alpha_2 + \ldots + a_k\alpha_k = 0$

$\Rightarrow a_i = 0 \quad \forall\ i = 1, 2, \ldots k < n$

$\therefore \{\alpha_1, \alpha_2 \ldots \alpha_k\}$ linearly independent

Theorem

Let, 'V' be a V.S. over the field F, then the set 'S' of non-zero vectors $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ belonging to 'V' is linearly dependent, iff some element of 'S' be a linear combination of others.

Let, $S = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ be linearly dependent vectors $\exists$ $a_1, a_2, \ldots, a_n \in F$, not all zeros, s.t. $a_1\alpha_1 + a_2\alpha_2 + \ldots + a_n\alpha_n = 0$

$\alpha_i \neq 0$, $\forall$ $i = 1, 2, 3, \ldots, n$.

Let, $a_k \neq 0$

$\therefore$ $a_k\alpha_k = -\sum\limits_{\substack{i=1 \\ i\neq k}}^{n} a_i\alpha_i$

$\therefore$ $\boxed{\alpha_k = -\sum\limits_{\substack{i=1 \\ i\neq k}}^{n} \frac{a_i}{a_k}\alpha_i}$ So, our assumption is right.

Now, let one of the elements of 'S' α_j be represented as linear combination of the other elements of 'S', i.e.

$$\alpha_j = \sum_{\substack{i=1 \\ i\neq j}}^{n} b_i\alpha_i$$

$\Rightarrow \sum\limits_{\substack{i=1 \\ i\neq j}}^{n} b_i\alpha_i + (-1)\alpha_j = 0$

$\longrightarrow$ Atleast one scalar, is different from zero.

∴ All vectors are linearly dependent.

Linear Span : L(S)

Let, V be a v.s. over field 'F' and 'S' be any non-empty subset of ~~field~~ 'V' then the linear span of S is defined as the set of all linear combination of finite set of 'S'.

$$L(S) = \{a_1\alpha_1 + a_2\alpha_2 + \dots + a_n\alpha_n : a_i \in F \ \& \ \alpha_i \in S \ \forall i\}$$

$L(S) \subseteq V$

L(S) is generated or spanned by 'S'.

$S \Rightarrow$ Generator of L(S).

$$L(S) = \left\{ \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \right\}$$

$$S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

$$\lambda_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \lambda_2 \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$\therefore S \subseteq L(S)$

Statement

The linear span L(S) of any subset S of V is a subspace of V generated by 'S'.

Show that, the vectors $(-1,2,1)$, $(3,0,-1)$ & $(-5,4,3)$ are linearly dependent in $V_3(R)$.

$$a(-1,2,1)+b(3,0,-1)+c(-5,4,3)=0$$

where, $a,b,c \in R$

$$\left.\begin{aligned} -a+3b-5c&=0 \\ 2a+0.b+4c&=0 \\ a-b+3c&=0 \end{aligned}\right\} \text{Homogeneous Equations.}$$

$2a=-4c$

$\Rightarrow a=-2c$

$b=a+3c=-2c+3c=c$

If Rank of Matrix < No. of unknown

$\rightarrow$ Homogeneous system of equations have non zero solution.

$$\Delta = \begin{vmatrix} -1 & 3 & -5 \\ 2 & 0 & 4 \\ 1 & -1 & 3 \end{vmatrix} = 0 \quad [\Rightarrow \text{Rank} < 3]$$

Let, $\left.\begin{aligned} c&=1 \\ b&=1 \\ a&=-2 \end{aligned}\right\}$ Non zero values.

$\therefore$ linearly dependent

Proved

Ex Let, x, y, z be the elements of vs V (F) and let $a, b \in F$ show that x, y, z are linearly dependent if ~~$x+y$~~ $(x+ay+bz)$, $(y, z$ this be linearly dependent.

$P, Q, R \in F$

$$P(x+ay+bz) + Qy + Rz = 0$$

$px = 0$

$pa + Q = 0$

$pb + R = 0$

$\therefore$ at least one of this coefficient is different from zero.

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Basis & Dimension 4/10/18

Basis :-

Let, V be a vector space over the field "F" and S is a subset of V(F). such that —

(i) S is a set of linearly independent vectors in V.

(ii) Each vector in V can be represented as a linear combination of finite no.s of elements of 'S' that is 'S' generates V, such that $L(S) = V$.

Then 'S' is called the "**Basis**" of the Vector space 'V'.

Example :- $B \subset V(R)$

$$B = \{(1,0,0),(0,1,0),(0,0,1)\}$$

↳ These should be L.I.

(i) Linearly Independence

$(a_1, a_2, a_3) \in R$ s.t

$$a_1(1,0,0) + a_2(0,1,0) + a_3(0,0,1) = (0,0,0).$$

$\therefore$
$$1 \cdot a_1 + 0 \cdot a_2 + 0 \cdot a_3 = 0 \Rightarrow a_1 = 0$$
$$0 \cdot a_1 + 1 \cdot a_2 + 0 \cdot a_3 = 0 \Rightarrow a_2 = 0$$
$$0 \cdot a_1 + 0 \cdot a_2 + 1 \cdot a_3 = 0 \Rightarrow a_3 = 0$$

These, the elements of B ~~is~~ are linearly independent.

$(\alpha_1, \alpha_2, \alpha_3) = \alpha_1(1,0,0) + \alpha_2(0,1,0) + \alpha_3(0,0,1)$

$\therefore$ Both the conditions are satisfied, so B is a basis of $V(R)$. Proved.

Basis is not necessarily be unique.

$B' = \{(1,1,0), (1,0,1), (0,1,1)\}$

Let, $a, b, c \in R$

s.t

$a + b + 0 \cdot c = 0$ —(i) $\Rightarrow$ $a + b = 0$

$a + 0 \cdot b + c = 0$ —(ii) $\Rightarrow$ $a + c = 0 \rightarrow a = -$

$0 \cdot a + b + c = 0$ —(iii) $\Rightarrow$ $b + c = 0$

$\therefore b = -c$

$2(a+b+c) = 0$ —(iv)

(iv) − (i) $\Rightarrow c = 0$

$\therefore a = 0$
$b = 0$

$\therefore$ The elements of B' are linearly independent

Now,

let, $\alpha = (\alpha_1, \alpha_2, \alpha_3) \in V(R)$

$$(\alpha_1, \alpha_2, \alpha_3) = \frac{1}{2}(\alpha_1 + \alpha_2 - \alpha_3)(1,1,0) + \frac{1}{2}(\alpha_1 + \alpha_3 - \alpha_2)(1,0,1) + \frac{1}{2}(\alpha_2 + \alpha_3 - \alpha_1)(0,1,1)$$

$$= a_1(1,1,0) + a_2(1,0,1) + a_3(0,1,1)$$

$\therefore$ It is linear combination.

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$\therefore B'$ is also a basis of $V(R)$. Proved.

Dimension :

Finite Dimensional Vector Space :-

The vector space 'V' is said to be finite dimensional or finitely generated, if there exists a finite subset 'S' of $V(F)$ such that $L(S) = V$, otherwise V is said to be infinite Dimensional.

dim (Null space) = 0

Dimension :

The no. of elements in a basis of a finite dimensional vector space $V(F)$ is called the dimension of the vector space, denoted as $\dim(V)$

$V_n(F) \rightarrow$ Basis contains n elements.

Th. 1

In a finite dimensional vector space V over the field F, with basis $B = \{\alpha_1, \alpha_2 \cdots \alpha_n\}$ every vector $\alpha \in V$ is uniquely expressible as a linear combination of vectors of 'B'

Th. 2

There exists a basis for each finite dimensional vector space.

Th. 3

A set of vectors 'S' consisting of the

n vectors $\rightarrow$ $e_1 = (1, 0, 0 \dots 0)$
$e_2 = (0, 1, 0 0 \dots 0)$
$\dots$
$e_n (0, 0 \dots 0, 1)$
is a basis of $V_n(F)$.

Th. 4

If $V(F)$ be a finite dimensional vector space, then any two bases of V have same no. of elements.

Th. 5

If W be a proper subspace of a finite dimensional vector space V, the W is also finite dimensional and $\dim(W) < \dim(V)$

Th. 6

If W_1 & W_2 be two subspaces of a finite dimensional vector space $V(F)$, then $\dim(W_1 \cup W_2) = \dim(W_1) + \dim(W_2) - \dim(W_1 \cap W_2)$

Ex Show that the vectors $(1, 2, 1)$, $(2, 1, 0)$, $(1, -1, 2)$ form a basis of $V_3(R)$.

→ We know that, if $V(F)$ be a finite dimensional vector space of dim n

then any set n L Independent vectors of V will form a basis of V.

Let, $S = \{(1,2,1), (2,1,0), (1,-1,2)\}$

We have to show that the elements of S are linearly independent, so that S can be a basis of V.

Let, $a, b, c \in F$ s.t.

$a(1,2,1) + b(2,1,0) + c(1,-1,2) = 0$

$\therefore \left.\begin{aligned} a + 2b + c &= 0 \\ 2a + b - c &= 0 \\ a + 0 + 2c &= 0 \end{aligned}\right\}$ Homogeneous System of equations.

$\therefore$ Coefficient Matrix

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -1 \\ 1 & 0 & 2 \end{bmatrix}$$

$2 - 2(4+1) + (1)$

$= \cancel{2-8}\ 3 - 10 = -7 \neq 0$

$\therefore$ Rank is $= 3$

$\Rightarrow$ each ~~of~~ $a = b = c = 0$

$\therefore$ The set of vectors are linearly independent.

$\therefore$ These can form basis of $V_3(R)$.

Ex. Show that the set

$S = \{(1,0,0), (1,1,0), (1,1,1)\}, (0,1,0)\}$

Spans the vector space R^3 but is not a basis set.

$\rightarrow \alpha \in V$

$\alpha = \{\alpha_1, \alpha_2, \alpha_3) \in V$, let, $a, b, c, d \in R$

If possible let,

$(\alpha_1, \alpha_2, \alpha_3) = a(1,0,0) + b(1,1,0) + c(1,1,1)$
$+ d(0,1,0)$

$\Rightarrow$

$\Rightarrow \alpha_1 = a+b+c$

$\alpha_2 = b+c+d$

$\alpha_3 = c$

$\Rightarrow c = \alpha_3$

$\Rightarrow b = \alpha_2 - \alpha_3 - d$

$\Rightarrow a = \alpha_1 - \alpha_2 + \alpha_3 + d - \alpha_3$

$= \alpha_1 - \alpha_2 + d$

If $d = 0$

$\therefore a = \alpha_1 - \alpha_2$

$b = \alpha_2 - \alpha_3$

$c = \alpha_3$

$\therefore$ So, atleast we can find one of them is different from zero.

$\Rightarrow$ Implies that the given set is not linearly independent.

Ex Find the basis of R^3 containing the vectors $\{(1,1,0), (1,1,13)\}$.

~~find the~~

31/10/18

Row Space, Column Space & Rank of matrix

$$[A]_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ - & - & - & - \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$$

$$\left.\begin{aligned} R_1 &= (a_{11}, a_{12}, \ldots, a_{1n}) \\ R_2 &= (a_{21}, a_{22}, \ldots, a_{2n}) \\ &\cdots \\ R_i &= (a_{i1}, a_{i2}, \ldots a_{in}) \\ &\cdots \\ R_n &= (a_{n1}, a_{n2}, \ldots a_{mn}) \end{aligned}\right\} \text{each is an } n\text{-tupple vector.}$$

$\Rightarrow m$ such "n-tupple" vectors

$$c_1 = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} \qquad c_2 = \begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{bmatrix}$$

$\vdots$

$$c_n = \begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix}$$

each n vector is a m-tuple vector

The row vectors which generate a vector space, is called the row-space of a matrix 'A'.

$R_A(A)$ $\qquad$ $\dim(R(A)) =$ Row-ranked of A

$c_1R_1 + c_2R_2 + \dots + c_mR_m \Rightarrow$ Row-space

Column-space

The column-vectors generate a vector space which is called 'column-space of A'

$C(A)$.

$R(A) \subset V_n(F)$ while $C(A) \subset V_m($
subspace of $\qquad$ subspace of

$\dim(C(A)) \rightarrow$ Column-ranked of matrix A

* Row rank of matrix A $<= n$
Col. " " " $<= m$

Th. 1

Pre-multiplication by a non-singular matrix, doesn't change the row-rank of a matrix.

Th. 2

The row-rank of a matrix is same as it's rank.

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Th. 3
The row rank and column rank of a matrix are equal

Q. Find the row-space and row-rank of the following matrix :-

$$M = \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ -1 & 2 & 1 & 1 \end{bmatrix}$$

Soln. :-

Row-space : of M is linear combination of row vectors R_1, R_2, R_3, R_4 and is given by —

$R_1 = (1,0,0,1)$
$R_2 = (-1, 1, 0, 0)$
$R_3 = (0, 2, 1, 0)$
$R_4 = (-1, 2, 1, 1)$

Ans —

$c_1R_1 + c_2R_2 + c_3R_3 + c_4R_4$
$= c_1(1,0,0,1) + c_2(-1,1,0,0) + c_3(0,2,1,0) + c_4(-1,2,1,1)$
$c_i \in R$

Row rank → No. of Independent rows.

$$M = \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ -1 & 2 & 1 & 1 \end{bmatrix} \longrightarrow \text{echelon form}$$

$$\simeq \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 2 & 1 & 2 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 + R_1 \\ R_4 \to R_4 + R_1 \end{matrix}$$

$$\simeq \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \begin{matrix} R_3 \to R_3 - 2R_2 \\ R_4 \to R_4 - 2R_2 \end{matrix}$$

$$\simeq \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 2 \end{bmatrix} \quad R_4 \to R_4 - R_3$$

$$\simeq \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_4 \to \tfrac{1}{2} R_4$$

$$\simeq \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{matrix} R_1 \to R_1 - R_4 \\ R_2 \to R_2 - R_4 \end{matrix}$$

$$\simeq \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = N \quad R_3 \to R_3 + 2R_4$$

All Rows are linearly independent in this identity matrix ($\cancel{A}N$)

So, Rank of $\cancel{M}N = 4$

Since, $M \simeq N$

$\therefore$ Rank of $(M) = 4$

Examine the linear dependence of the set of vectors $\alpha = (1, 2, -3)$

$\beta = (2, -3, 1)$

$\gamma = (-3, 1, 1)$

Sol^n : A = [illegible] ~ [illegible]

(A, linearly independent & [illegible])

3) S.T. the column rank of the matrix

$$\begin{bmatrix} 2 & 4 & 4 \\ 4 & 6 & 8 \\ 3 & 3 & 1 \end{bmatrix}$$ is 3.

4) S.T. the row rank of A = [illegible] is '3'

5) If the row rank of matrix

$$\begin{bmatrix} 3 & 4 & -3 & 5 \\ 1 & 2 & -1 & 7 \\ 1 & 1 & 2 & 9 \\ 2 & -1 & 1 & K \end{bmatrix}$$ is be '3'

Then S.T. K = 7

Characteristic Equations

Matrix polynomial $\rightarrow$ of degree 'n'

$$F = A_0 + A_1 x + A_2 x^2 + \dots + A_n x^n$$

$A_i \rightarrow$ Matrices of same order

A_n is non-singular

Eg. $\rightarrow$ $F = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 7 \\ 8 & 2 \end{bmatrix} x + \begin{bmatrix} 5 & 11 \\ 3 & 9 \end{bmatrix} x^2 + \dots$

$$+ \dots + \begin{bmatrix} 7 & 33 \\ 2 & 9 \end{bmatrix} x^5$$

Degree = 5

Consider

$$A = \begin{bmatrix} 1+3x+2x^2 & x^2 & 6-4x \\ 1+x^3 & 4+3x^2 & 1-2x+3x^3 \\ 3-2x+2x^3 & 7 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3x+2x^2+0\cdot x^3 & 0+0\cdot x+x^2+0\cdot x^3 & 6-4x+0 \dots \\ 1+0\cdot x+0\cdot x^2+1\cdot x^3 & 4+0\cdot x+3x^2+0\cdot x^3 & 1-2x+0 \dots \\ 3-2x+0\cdot x^2+2x^3 & 7+0\cdot x+0\cdot x^2+0\cdot x^3 & 5+0x \dots \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 6 \\ 1 & 4 & 1 \\ 3 & 7 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 & -4 \\ 0 & 0 & -2 \\ -2 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 2 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix} x^2$$

$$+ \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 3 \\ 2 & 0 & 0 \end{bmatrix} x^3$$

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Characteristic Eqn

$A \rightarrow$ Square matrix of order 'n'

$(A - \lambda I)$

$I \rightarrow$ Identity matrix of same order as 'A'!

$\lambda \rightarrow$ Any scalar

$\boxed{|A - \lambda I| = 0} \rightarrow$ characteristic Equation.
[roots can be imaginary]

Expand $|A - \lambda I| \rightarrow$ polynomial of λ in degree 'n'

$$(-1)^n \left[\lambda^n + P_1 \lambda^{n-1} + P_2 \lambda^{n-2} + \dots P_{n-1} \lambda^1 + P_n\right] = 0$$

where, $P_1, P_2 \dots P_n \Rightarrow$ coefficients whose ~~of~~ elements are the elements of given matrix.

$|A - \lambda I| \Rightarrow$ characteristic polynomial.

Roots of characteristic equation $\rightarrow$ __Eigen values__

or __Latent Roots__

or __Characteristics Roots__

Set of All the ~~roots~~ eigen value of the given matrix $(A) \rightarrow$ is called __Spectrum of 'A'__.

Ch:1 Deductions corresponds to Eigen Value

Characteristic roots of Diagonal matrix are the elements of it's leading diagonal.

2. Zero is an eigen value of matrix 'A' if and only if, 'A' is ~~non~~ singular.