

國立臺灣大學電機資訊學院資訊工程研究所

碩士論文

Department of Computer Science and Information Engineering

College of Electrical Engineering and Computer Science

National Taiwan University

Master Thesis

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MsWave

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國立臺灣大學  
資訊工程研究所

碩士論文

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撰

國立臺灣大學（碩）博士學位論文  
口試委員會審定書

論文中文題目

論文英文題目

本論文係○○○君（○學號○）在國立臺灣大學○○學系、所完成之碩（博）士學位論文，於民國○○年○○月○○日承下列考試委員審查通過及口試及格，特此證明

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系主任、所長

（簽名）

（是否須簽章依各院系所規定）

# 致謝

這裡將簡單介紹如何利用  $\text{\LaTeX}$  來編輯你的畢業論文，若不知道  $\text{\LaTeX}$  是什麼或是沒有概念的話，建議你可以簡單看過放在此資料夾裡的[李果正 - 大家來學  \$\text{\LaTeX}\$](#) 前四章內容，在下載適合的  $\text{\LaTeX}$  整合發行套件之後（請看第 III.項），可以嘗試用剛安裝好的  $\text{\LaTeX}$  編輯器來編譯[thesis.tex](#)這份文件，編譯的方法可以看下面第 V.項的介紹，若編譯成功，所編譯出來的 thesis.pdf 文件的應該會跟此 demo.pdf 文件一模一樣，而且沒有任何問號符號，走到這一步的話，就差不多可以開始邊學習  $\text{\LaTeX}$  邊編輯你的畢業論文了！基本上會使用到的指令都包含在論文的各章節裡，怎麼在論文裡寫公式或是放圖之類的就自行看 tex 檔學吧。如果有任何問題或建議可以來信與我討論，我的信箱是[dran31545@gmail.com](mailto:dran31545@gmail.com)，或是到此範本[Google Project](#)裡面的[Issues](#)貼上你的問題與建議，我會盡我所能更新此範本，也歡迎大家自行重製、改良此範本並散布給他人，祝大家順利畢業！

要編輯致謝請打開[acknowledgementsCH.tex](#)

## I. 此範本參考並修改自下列網站的資料：

- [如何用  \$\text{\LaTeX}\$  排版臺灣大學碩士論文](#)  
—台灣大學論文  $\text{\LaTeX}$  樣版原創者[黃子桓](#)的教學網頁
- [\$\text{\LaTeX}\$  常用語法及論文範本](#)  
—[Hitripod](#)所修改的範本，這裡參考了許多他所寫的格式和內容
- [使用  \$\text{\LaTeX}\$  做出精美的論文](#)
- [XeTeX：解決  \$\text{\LaTeX}\$  惱人的中文字型問題](#)
- [台灣大學碩士、博士論文的  \$\text{\LaTeX}\$  模板](#)

## II. 幾個有用的參考資料及網路資源：

- [李果正 - 大家來學  \$\text{\LaTeX}\$](#) —建議先看完前四章
- [WIKIBOOKS- \$\text{\LaTeX}\$](#) —好用的線上工具書
- [Working with a .bib file using JabRef](#)
- [Using BibDesk - A short tutorial](#)
- [\$\text{\LaTeX}\$  for Physicists](#)

### III. 下載 L<sup>A</sup>T<sub>E</sub>X 整合發行套件，可參考 [TeX Collection](#)：

1. [MacTeX](#): For **MacOSX**，下載 [MacTeX.pkg](#)
2. [ProTeXt](#): For **Windows**，下載 [ISO file](#)
3. [TeX Live](#): For **GNU/Linux** and **MacOSX**, and **Windows**，下載 [ISO file](#)
4. [CTAN](#): The Comprehensive TeX Archive Network.

### IV. 好用的程式：

- 文獻管理系統：
  1. [JabRef](#)  
可參考 [Working with a .bib file using JabRef](#) 或是 [Google](#) 及 [YouTube](#)
  2. [BibDesk](#) (For Mac)  
可參考 [Using BibDesk - A short tutorial](#) 或是 [Google](#) 及 [YouTube](#)
- 方程式編輯器：[Daum Equation Editor](#) (Chrome App，必須使用 Google 瀏覽器)

### V. 編譯流程：

1. `xelatex thesis`  
對 `thesis.tex` 進行第一次 XeLaTeX 編譯，產生 `thesis.pdf` 以及其他檔案
2. `bibtex thesis`  
對 `thesis.tex` 進行 BibTeX 編譯，產生 `bbl` 檔以及 `blg` 檔
3. `xelatex thesis`  
對 `thesis.tex` 進行第二次 XeLaTeX 編譯，產生目錄、圖表連結及參考文獻
4. `xelatex thesis`  
對 `thesis.tex` 進行第三次 XeLaTeX 編譯，產生參考文獻連結，完成編譯

**注意！**此範本使用 `cite` 套件，可依據你利用文獻管理系統所整理好的 [thesisbib.bib](#) 檔在論文最後產生參考文獻頁面，若你的系所規定要在每個章節的後面產生參考文獻，則可以用 `chapterbib` 套件，來對每個有附參考文獻的章節 `tex` 檔進行一次 BibTeX 編譯產生 `bbl` 檔，如範例的 [introduction.tex](#)、[THM.tex](#) 和 [EXP.tex](#)，如果有這需要請把 [thesis.tex](#) 檔裡使用 `cite` 套件的指令利用註解符號 `%` 來取消使用 `cite` 套件，並刪去出現在使用 `chapterbib` 套件指令前面的註解符號 `%` 來啟動使用 `chapterbib` 套件

```
\usepackage{cite}
%\usepackage{chapterbib}
改成
```

```
%\usepackage{cite}
\usepackage{chapterbib}
```

再來利用註解符號% 取消會把參考文獻放在論文最後的指令

```
\bibliographystyle{unsrt}
\addcontentsline{toc}{chapter}{\bibname}
\bibliography{thesisbib}
```

改成

```
%\bibliographystyle{unsrt}
%\addcontentsline{toc}{chapter}{\bibname}
%\bibliography{thesisbib}
```

再把用來輸入章節檔案的 \input 指令改成 \include 指令

```
\input{introduction}  =>  \include{introduction}
\input{THM}            =>  \include{THM}
\input{EXP}            =>  \include{EXP}
```

最後記得在每個有附參考文獻的章節加上產生參考文獻的指令，即在 [introduction.tex](#)、[THM.tex](#) 和 [EXP.tex](#) 三個檔案裡最後啟動下面兩行指令

```
%\bibliographystyle{unsrt} => \bibliographystyle{unsrt}
%\bibliography{thesisbib}  => \bibliography{thesisbib}
```

而編譯時則需要對有附參考文獻的 [introduction.tex](#)、[THM.tex](#) 和 [EXP.tex](#) 各做一次 BibTeX 編譯，編譯流程如下

1. xelatex thesis  
對 thesis.tex 進行第一次 XeLaTeX 編譯，產生 thesis.pdf 及其他檔案
2. bibtex introduction  
對 introduction.tex 進行 BibTeX 編譯，產生 bbl 檔以及 blg 檔
3. bibtex THM  
對 THM.tex 進行 BibTeX 編譯，產生 bbl 檔以及 blg 檔
4. bibtex EXP  
對 EXP.tex 進行 BibTeX 編譯，產生 bbl 檔以及 blg 檔
5. xelatex thesis  
對 thesis.tex 進行第二次 XeLaTeX 編譯，產生目錄、圖表連結及參考文獻

## 6. xelatex thesis

對 thesis.tex 進行第三次 XeLaTeX 編譯，產生參考文獻連結，完成編譯

## VI. 補充說明與注意事項：

- 口試委員會審定書：

請到台大圖書館網頁的[電子論文服務](#)下載論文格式範本，並修改成正確的格式，也可到此範本所在資料夾的[cert.doc](#)修改。當然你也可以利用 LaTeX 來編輯，你只要填好[ntuvars.tex](#)檔的資料，並去除在 thesis.tex 裡下面這行的註解符號%

```
%\makecertification
```

編譯完後就可以產生審定書格式。口試通過後，請把已經簽名的審定書掃描成 pdf 檔，再取代原本的[cert.pdf](#)，即可放上已簽名的審定書。處理審定書出現的指令在 thesis.tex 裡

```
%----- generate the certification ...
%\makecertification
%----- includepdf by using package ...
\addcontentsline{toc}{chapter}{口試委員會審定書}
\includepdf[pages={1}]{cert.pdf}
```

- 浮水印：

資料夾已經附上浮水印檔案了，若學校有更改，到請到台大圖書館網頁的[電子論文服務](#)下載pdf 格式的浮水印到此範本所在資料夾。若要開啟關閉浮水印功能，即自行刪去或加上下面位於[thesis.tex](#)指令的註解符號%

```
%\CenterWallPaper{0.174}{watermark.pdf}
%\setlength{\wpXoffset}{6.1725cm}
%\setlength{\wpYoffset}{10.5225cm}
```

- 單面印刷與雙面印刷：

此範本為單面印刷，若論文頁數超過 80 頁，依規定需要用雙面印刷，此時只需把 thesis.tex 裡的

```
\documentclass[a4paper, 12pt, oneside]{book}
改成
\documentclass[a4paper, 12pt, twoside]{book}
```

- 如何加入附錄？

在thesis.tex裡，依需求選擇 input 或 include，刪去% 符號來輸入附錄章節

```
%----- Input your appendix here -----  
%\input{AppendixA}  
%or %chapter cite == \include  
%\include{AppendixA}
```

在章節檔 AppendixA.tex 裡，開頭打

```
\chapter{First appendix title}
```

即可，以此類推。

- 系上規定論文圖表須全部放到最後獨立出來的章節，且章節不出現在目錄中：

在thesis.tex裡，依需求選擇 input 或 include，刪去% 符號來輸入圖表章節

```
%----- Input your Figure chapter here -----  
%\input{EndFigTab}  
%chapter cite == \include  
%\include{EndFigTab}
```

在章節檔EndFigTab.tex裡有範例和說明可供參考，要注意正文的圖表和附錄的圖表要分清楚，即在EndFigTab.tex內

```
\renewcommand{\thefigure}{\arabic{chapter}.  
\arabic{figure}}  
\renewcommand{\thetable}{\arabic{chapter}.  
\arabic{table}}  
%--- Input your main figures and tables here ---
```

這幾行之後章節計數器格式已切換為 1...9，放正文的圖表，

```
\renewcommand{\thefigure}{\Alph{chapter}.  
\arabic{figure}}  
\renewcommand{\thetable}{\Alph{chapter}.  
\arabic{table}}  
%--- Input your appendix figures and tables here ---
```

這幾行之後章節計數器格式已切換為 A...Z，放附錄的圖表。另外要取消圖表的浮動功能，才能讓圖表按照指令出現順序排好，即把平常使用的圖表指令



```
\begin{figure}[htb]
...
\begin{table}[htb]
```

改成

```
\begin{figure}[!]
...
\begin{table}[!]
```

剩下的只要注意章節圖表的計數器設定即可。`\ref` 和 `\label` 指令可以在此圖表章節與正文章節使用。

- 如果我想要修改 `margin`(文字邊界) 的話，可以從哪裡下手呢？請打開[ntu.sty](http://ntu.sty)修改下面這行的上下左右參數即可：

```
\RequirePackage[top=3cm,left=3cm,bottom=2cm,right=3cm]
{geometry}
```

- 我想引用 Twomey (1974): Pollution and planetary albedo 這篇論文，如何用 `\cite` 引用它的時候在內文顯示 Twomey (1974) [編號]？建議使用 `natbib` 套件，參考資料如下：

[LaTeX/Bibliography Management](#)

[Overview of Bibtex-Styles](#)

[Reference sheet for natbib usage](#)

- `XYTeX`：  
此範本中文字體使用 `XYTeX` 轉換，細節請參考[Hitripod](#)寫的[XeTeX：解決 LaTeX 惱人的中文字型問題](#)。
- 如何輸入英文‘單引號’和“雙引號”以及不同長度的破折號？  
可以參考[李果正-大家來學 L<sup>A</sup>T<sub>E</sub>X](#)第 17 頁針對標點符號的遊戲規則，範例如下，輸入以下指令：

```
\單引號'\
``雙引號''\
-hyphen\
--en-dash\
---em-dash\
```

則顯示：

```
‘單引號’
“雙引號”
```

-hyphen  
—en-dash  
—em-dash

# 中文摘要

請打開並編輯[abstractCH.tex](#)

關鍵字：壹、貳、參、肆、伍、陸、柒

# Abstract

Open and edit [abstractEN.tex](#)

Key words:A, B, C, D, E, F, G

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# Chapter 1

## Get started with L<sup>A</sup>T<sub>E</sub>X

Three common font styles in this text:

- **Item1:** *Italic* 中文 123
- **Item2:** **Bold** 中文 123
- **Item3:** *slant* 中文 123

About the advance latex grammer see the next section 1.1.

### 1.1 L<sup>A</sup>T<sub>E</sub>X Adadvanced Features

The following features would be introduced in the coming subsections:

- SubSection 1.1.1: **Figure**
- SubSection 1.1.3: **Verb**
- SubSection 1.1.3: **Verb**
- SubSection 1.1.4: **Enumeration**
- SubSection 1.1.2: **Table**
- SubSection 1.1.5: **Code Display**
- SubSection 1.1.6: **Math**

- SubSection 1.1.7: **Algorithms**

### 1.1.1 Figure



Figure 1.1: A picture of a tiger.

Figure 1.1 is a picture of a tiger.

### 1.1.2 Table

[Table examples on WIKIBOOKS.](#)

Table 1.1: Table Example 1		
Start	End	Character Block Name
3400	4DB5	CJK Unified Ideographs Extension A
4E00	9FFF	CJK Unified Ideographs

Table 1.2: Table Example 2		
Item		
Animal	Description	Price (\$)
Gnat	per gram	13.65
	each	0.01
Gnu	stuffed	92.50
Emu	stuffed	33.33
Armadillo	frozen	8.99

Table 1.3: Table Example 3

Allocation	Allocation, Element, Type, Script
<b>Data Types</b>	Byte2, Byte3, and Byte4 Float2, Float3, Float4 Int2, Int3, Int4 Long2, Long3, Long4 Matrix2f, Matrix3f, Matrix4f Short2, Short3, Short4
<b>Graphics</b>	Mesh ProgramFragment, ProgramRaster ProgramStore, ProgramVertex RSSurfaceView

Table 1.4: Table Example 4

Team sheet		
Goalkeeper	GK	Paul Robinson
Defenders	LB	Lucas Radebe
	DC	Michael Duberry
	DC	Dominic Matteo
	RB	Didier Domi
Midfielders	MC	David Batty
	MC	Eirik Bakke
	MC	Jody Morris
Forward	FW	Jamie McMaster
Strikers	ST	Alan Smith
	ST	Mark Viduka

Table 1.5: Table Example 5

Team	P	W	D	L	F	A	Pts
Manchester United	6	4	0	2	10	5	12
Celtic	6	3	0	3	8	9	9
Benfica	6	2	1	3	7	8	7
FC Copenhagen	6	2	1	2	5	8	7

### 1.1.3 Verb

Let's take a overview on how to type special characters:

<FRAMEWORKS\_BASE>/graphics/java/android/renderscript

<sup>1</sup> You could also go back to the beginning of the chapter by the **hyperref**.

### 1.1.4 Enumeration

1. Enumerated Item1
2. Enumerated Item2
3. Enumerated Item3

### 1.1.5 Code Display

Here is a "Hello, DanDing." example:

```
void main(int argc, char **argv)
{
    printf(" ' _> ` ");
}
```

Another example with line numbers:

```
1 void main(int argc, char **argv)
2 {
3     printf(" ' _> ` ");
4 }
```

Matlab example:

```
1 function y = demo(x) % This is a comment.
2     str = 'hello there';
3     y = x + 1;
4 end
```

---

<sup>1</sup>Path of <APP\_intermediates>: <ANDROID\_ROOT>/ out/ target/ common/ obj/ APPS/ APP-NAME\_intermediates/

### 1.1.6 Math

- Inline mode:

The solution to  $\sqrt{x} = 5$  is  $x = 25$ .

- Display mode:

The solution to

$$\sqrt{x} = 5$$

is

$$x = 25.$$

- Numbered mode:

$$2 + 2 = 4 \tag{1.1}$$

- Non-numbered:

$$2 + 2 = 4$$

- Aligning:

$$\begin{aligned} 2x^2 + 3(x-1)(x-2) &= 2x^2 + 3(x^2 - 3x + 2) \\ &= 2x^2 + 3x^2 - 9x + 6 \\ &= 5x^2 - 9x + 6 \end{aligned}$$

- Fractions:

$$\frac{n!}{k!(n-k)!} = \binom{n}{k}$$

- Matrix:

$$A_{m,n} = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{pmatrix}$$

[More examples on WIKIBOOKS.](#)

### 1.1.7 Algorithms

---

**Algorithm 1** Calculate  $y = x^n$

---

**Require:**  $n \geq 0 \vee x \neq 0$

**Ensure:**  $y = x^n$

$y \leftarrow 1$

**if**  $n < 0$  **then**

$X \leftarrow 1/x$

$N \leftarrow -n$

**else**

$X \leftarrow x$

$N \leftarrow n$

**end if**

**while**  $N \neq 0$  **do**

**if**  $N$  is even **then**

$X \leftarrow X \times X$

$N \leftarrow N/2$

**else** [ $N$  is odd]

$y \leftarrow y \times X$

$N \leftarrow N - 1$

**end if**

**end while**

---

[More examples on WIKIBOOKS.](#)

# Chapter 2

## Introduction

HiHi Iam r44 . The organization of this thesis is as follows. In chapte ??, the theoretical background and definition of surface plasmon will be included [1]. Chapte ?? contains description of experiment methods such as atomic force microscopy and scanning electron microscopy.

### 2.1 Thesis Overview

In this section, we describe the overview of this thesis.

## Chapter 3

### Related Works

HiHi Iam r44 . The organization of this thesis is as follows. In chapte ??, the theoretical background and definition of surface plasmon will be included [1]. Chapte ?? contains description of experiment methods such as atomic force microscopy and scanning electron microscopy.



# Chapter 4

## Methodology

HiHi Iam r44 . The organization of this thesis is as follows. In chapte ??, the theoretical background and definition of surface plasmon will be included [1]. Chapte ?? contains description of experiment methods such as atomic force microscopy and scanning electron microscopy.

### 4.1 Problem Setup

There are a query set  $Q = \{q_1, q_2, \dots, q_T\} \subset \mathbb{R}^D$  at the server  $P$  and a dataset  $X_i \subset \mathbb{R}^D$  on each local machine  $M_i$ . For each coming query  $q_t$ , we want to find its  $k_{th}$  nearest neighborhood among these distributed datasets while reducing the transmission cost between  $P$  and each  $M_i$ .

### 4.2 Overall Framework

In this section, we describe the overall framework of our work. Then, we will give the details about the framework in the following sections.

There are two main phases in our framework. For each  $X_i$ , the first phase only needs to be done for once. On the other hand, we need to run the second phase for each new query  $q_t$ .

The first phase is an preprocessing procedure for the second phase. Its goal is to im-

prove the performance of pruning in the second phase. We will prove in the Sec 4.5.2 that this pruning power is highly correlated to the distribution of the norm of the feature vectors. As a result, each  $M_i$  would learn an orthogonal matrix  $W_i$  for its  $X_i$  to fit our desired distribution and then send each  $W_i$  back to  $P$ . We can notice that this phase is only dependent on  $X_i$  and independent of  $q_t$ . Therefore, we only need to do the first phase for once. we give the details about how to learn  $W_i$ , how to send it back to  $P$  in the Sec 4.3.

The second phase is the main procedure of our framework. Note that  $P$  have already got  $W_i$  for each  $X_i$  in the beginning of the second phase. For each coming query  $q_t$ , we iteratively prune some candidates which are impossible to be the  $k$ NN of  $q_t$  to reduce the search space until there are only  $k$  candidates left.

To prune candidates iteratively, we divide the second phase into several rounds. For each round  $j$ , we use a *Select* function  $S_i(q_t, j; \theta_t)$  to generate the values for trasmitting from  $P$  to  $M_i$ , where  $S_i$  is the importance-selecting function of  $M_i$  and  $\theta_t$  is its parameters for  $q_t$ . (We put the details of  $S_i$  at the Sec ??.) By these values, each  $M_i$  could calculate the bounds between each candidate  $x_l$  and  $q_t$ . With these bounds,  $P$  would be able to determine which candidates are definitely not our answer and then disregards them in the following rounds. By these pruning, we could achieve the goal of saving transmission cost from avoiding to consider the unnecessary candidates.

Note that we could use the square of the Euclidean distance instead of the origin Euclidean distance to find  $k$ NN as it is non-negative. So we will use the former one in our framework.

### 4.3 Orthogonal Transformation

Since the In this section, we describe the overview of this thesis.

### 4.3.1 Definition of Orthogonal Transformation

**Definition 1.** A matrix  $W \in \mathbb{R}^{D \times D}$  is orthogonal if whose columns and rows are orthogonal vectors, i.e.

$$W^T W = W W^T = I$$

where  $I$  is the identity matrix.

### 4.3.2 Property of Orthogonal Transformation

**Property 1.** Let  $x, y \in \mathbb{R}^{D \times 1}$ , and  $W \in \mathbb{R}^{D \times D}$  be an orthogonal matrix. Then,

$$Dist(x, y) = \sum_{d=1}^D (x[d] - y[d])^2 = \sum_{d=1}^D (W[d, :]x - W[d, :]y)^2 = Dist(Wx, Wy)$$

where  $W[d, :]$  is the  $d_{th}$  row of  $W$ .

We will use this important property in the Sec ??.

## 4.4 Enhance the Bounds by the Orthogonal Transformation

In the Sec 4.2, we mentioned that the first phase is an auxiliary step for the second phase. After the introduction of the orthogonal transformation, we introduce this powerful tool into the first phase in our framework.

### 4.4.1 The Goal of the First Phase

Our goal in the first phase is to reduce the ranges of the bounds used in the second phase. Since we will use a threshold to prune the impossible candidates according to their bounds in the The pruning procedure, the ranges of the bounds would be one of the most influential factor of the pruning power.

Suppose that we want to prune all candidates whose lower bounds are upper than a threshold  $thr$ . In these figures, we could see that when the ranges of their bounds is

short, more candidates would be pruned than those with long ranges of the bounds. In other words, the shorter the range of the bound, the higher chance this candidate would be pruned if it is not our final answer of  $k$ NN.

#### 4.4.2 Definition of the Bounds

First, we need to define the bounds for the pruning. Recall that given a query  $q_t$ , our goal is to find its  $k$ NN in these distributed datasets  $X_i$ . Intuitively, we need to calculate the square of the Euclidean distance  $Dist(q_t, x), \forall x \in \cup_i X_i$ . However, to calculate  $Dist(q_t, x)$ , we need to send the whole  $q_t$  to the local machines or send the whole  $x$  to  $P$ , which causes a huge transmission cost. Therefore, instead of the exact value of Euclidean distances, our proposed framework uses bounds to find the  $k$ NN.

**Definition 1.**  $\forall x, y \in \mathbb{R}^D$ , a lower bound  $LB(x, y)$  and a upper bound  $UB(x, y)$  must satisfy the following inequation:

$$LB(x, y) \leq Dist(x, y) \leq UB(x, y)$$

#### 4.4.3 Relation Between the Norms and the Bounds

To achieve the goal of reducing the length of the ranges of the bounds, we could look into the derivation of the bounds. Suppose there are two vectors  $x, y \in \mathbb{R}^D$ , but we could only observe the first  $s$  dimensions of  $x$  and  $\sum_{d=s+1}^D x[d]^2$ , which is the square of two norm of the unobserved part  $x[s+1 : D]$ . In the Sec 4.5.2, we give the bounds as

$$\begin{aligned} LB(x, y) &= \sum_{d=1}^s (x[d] - y[d])^2. \\ UB(x, y) &= \sum_{d=1}^s (x[d] - y[d])^2 \\ &\quad + \sum_{d=s+1}^D x[d]^2 + \sum_{d=s+1}^D y[d]^2 \\ &\quad + 2 \times \sqrt{\sum_{d=s+1}^D x[d]^2 \times \sum_{d=s+1}^D y[d]^2}. \end{aligned}$$

Therefore, we could get the length of the range by the subtraction.

$$Len = UB(x, y) - LB(x, y) = \sum_{d=s+1}^D x[d]^2 + \sum_{d=s+1}^D y[d]^2 + 2 \times \sqrt{\sum_{d=s+1}^D x[d]^2 \times \sum_{d=s+1}^D y[d]^2}.$$

Since the term  $\sum_{d=s+1}^D x[d]^2$  is given, all we could do is to reduce the length with the help of the  $\sum_{d=s+1}^D y[d]^2$  term. If we could reduce  $\sum_{d=s+1}^D y[d]^2$ , the term  $\sum_{d=s+1}^D x[d]^2 \times \sum_{d=s+1}^D y[d]^2$  would also decrease and then make  $Len$  smaller. Therefore, our goal now becomes to make the term  $\sum_{d=s+1}^D y[d]^2$  as small as possible, which is the square norm of the vector  $y[s+1 : D]$ .

Note that the lower (upper) bounds is non-decreasing (non-increasing) as  $s$  becomes larger and  $LB(x, y) = UB(x, y)$  when  $s = D$ . This means that  $LB$  and  $UB$  would be exactly equal to  $\sum_{d=1}^D (x[d] - y[d])^2$  eventually.

To reduce the length of the ranges  $Len$  for each  $s$ , we hope to make  $\sum_{d=s+1}^D y[d]^2$  as small as possible. However, since the feature vector  $y$  is given from datasets, the value of  $\sum_{d=s+1}^D y[d]^2$  is already determined when  $s$  is given. As a result, we introduce the orthogonal transformation to achieve this goal.

#### 4.4.4 Equivalent Bounds After Transformation

From the Sec 4.3.2, we know the distance of two vectors won't be changed after an orthogonal transformation. Now we use this property to achieve our goal to reduce the length of the ranges  $Len$  given  $s$ .

Given an orthogonal transformation  $W \in \mathbb{R}^{D \times D}$ , we have  $Dist(x, y) = Dist(Wx, Wy)$ . This means that the bounds we derivated before could also be the bounds for  $Dist(Wx, Wy)$ . That is,

$$LB(x, y) \leq Dist(x, y) = Dist(\hat{x}, \hat{y}) \leq UB(x, y)$$

where  $\hat{x} = Wx$ .

This also means that we could use the same way to derivate the lower bounds and upper bounds for  $Dist(Wx, Wy)$  and these bounds are also the bounds for  $Dist(x, y)$ .

That is,

$$LB(\hat{x}, \hat{y}) \leq Dist(x, y) = Dist(\hat{x}, \hat{y}) \leq UB(\hat{x}, \hat{y})$$

So, we could use the bounds  $LB(\hat{x}, \hat{y}), UB(\hat{x}, \hat{y})$  for  $Dist(x, y)$  and the length of range we want to reduce becomes

$$\hat{Len}(W) = UB(\hat{x}, \hat{y}) - LB(\hat{x}, \hat{y}) = \sum_{d=s+1}^D \hat{x}[d]^2 + \sum_{d=s+1}^D \hat{y}[d]^2 + 2 \times \sqrt{\sum_{d=s+1}^D \hat{x}[d]^2 \times \sum_{d=s+1}^D \hat{y}[d]^2}.$$

which becomes a function of  $W$ .

As a result, instead of trying to reduce  $\sum_{d=s+1}^D y[d]^2$  which is impossible as we mentioned in the Sec 4.4.3, our goal becomes to reduce  $\sum_{d=s+1}^D \hat{y}[d]^2$  with the help of  $W$ .

#### 4.4.5 Reduce the Norm with Orthogonal Transformation

For  $y \in \mathbb{R}^{D \times 1}$  and  $W \in \mathbb{R}^{D \times D}$ , given  $s$ , we want to reduce  $\sum_{d=s+1}^D \hat{y}[d]^2$  as much as possible. However, since  $s$  is unknown while deciding  $W$  in the first phase of our framework, we have to handle all possible values which  $s$  could be. Moreover, since  $\sum_{d=1}^D \hat{y}[d]^2$  is equal to  $\sum_{d=s+1}^D y[d]^2$ , which is independent with  $W$ , if the  $\sum_{d=s+1}^D \hat{y}[d]^2$  decreases with some  $W$ , the term  $\sum_{d=1}^s \hat{y}[d]^2$  must increase. Here, we use a more general strategy to deal with these problems.

We could look the term  $\sum_{d=s+1}^D \hat{y}[d]^2$  from a different angle. Actually, this term is the square norm of the latter part of the vector  $\hat{y}$ . Therefore, although  $\sum_{d=1}^D \hat{y}[d]^2$  is a constant for  $W$ , we could reduce the square norm of the latter part by increasing the forward part of it. In other words, we move the norm of the latter part of  $y$  to its forward part. To accomplish it, we design an objective function and then optimize this function to find our ideal  $W$ .

$$f(W; y) = \sum_{d=1}^D w_d \times \hat{y}[d]^2 = \sum_{d=1}^D w_d \times (W[d, :]y[d])^2 \quad (4.1)$$

where  $w_d = d, \forall d = 1 : D$ .

Because  $w_d$  would give the larger penalty as  $d$  increases, the elements in the latter part

of  $\hat{y}$  would be forced to become small while minimizing this objective function. This is exactly our goal to reduce  $\sum_{d=s+1}^D \hat{y}[d]^2$ . Therefore, our question becomes how to optimize this objective function with the constraints that  $W$  must be an orthogonal matrix.

#### 4.4.6 Optimize with Orthogonal Constraints

haha

$$F_i(W) = \sum_{x \in X_i} f(W; x) = \sum_{x \in X_i} \sum_{d=1}^D w_d \times \hat{x}[d]^2 = \sum_{x \in X_i} \sum_{d=1}^D w_d \times (W[d, :]x[d])^2 \quad (4.2)$$

haha

### 4.5 Prune by the Bounds

In this section, we describe the method to prune the candidates by bounds.

#### 4.5.1 Prune the Candidates with the Bounds

For the query  $q_t$ , if we already know  $LB(q_t, x)$  and  $UB(q_t, x) \forall x \in \cup_i X_i$ , we could use the  $k_{th}$  smallest upper bounds and directly prune those  $x$  whose lower bounds are higher than this value  $thr$ . I.e., we want to prune

$$\{x | LB(q_t, x) > thr, \forall x \in \cup_i X_i\}$$

where  $thr$  is the  $k_{th}$  largest  $UB(q_t, x) \forall x \in \cup_i X_i$ .

Here is an example of the pruning procedure.

TODO: Draw the figure about pruning.

We will talk about the details to generate these bounds and find the threshold in the Sec haha.

### 4.5.2 Derivation of the Bounds

Suppose there are two vectors  $x, y \in \mathbb{R}^D$ , we know the square of their Euclidean distance is

$$Dist(x, y) = \sum_{d=1}^D (x[d] - y[d])^2 \quad (4.3)$$

However, if we could only observe the first  $s$  dimensions of  $x$ , we could decompose their distance as

$$Dist(x, y) = \sum_{d=1}^D (x[d] - y[d])^2 = \sum_{d=1}^s (x[d] - y[d])^2 + \sum_{d=s+1}^D (x[d] - y[d])^2. \quad (4.4)$$

Since the first component of (4.4) is already known, all we need to do is to deal with the second term. Therefore, we further expand the second term as following:

$$\sum_{d=s+1}^D (x[d] - y[d])^2 = \sum_{d=s+1}^D x[d]^2 + \sum_{d=s+1}^D y[d]^2 - \sum_{d=s+1}^D 2 \times x[d] \times y[d].$$

By this analysis, we find the final term is the inner product between two partial vector  $x[s+1 : D]$  and  $y[s+1 : D]$ , which could be approximated by Cauchy–Schwarz inequality

$$\sum_{d=s+1}^D x[d] \times y[d] \leq \sqrt{\sum_{d=s+1}^D x[d]^2 \times \sum_{d=s+1}^D y[d]^2}. \quad (4.5)$$

After combing with (4.4) and (4.5), we derive the bounds as

$$LB(x, y) = \sum_{d=1}^s (x[d] - y[d])^2. \quad (4.6)$$

$$\begin{aligned} UB(x, y) &= \sum_{d=1}^s (x[d] - y[d])^2 \\ &+ \sum_{d=s+1}^D x[d]^2 + \sum_{d=s+1}^D y[d]^2 \\ &+ 2 \times \sqrt{\sum_{d=s+1}^D x[d]^2 \times \sum_{d=s+1}^D y[d]^2}. \end{aligned} \quad (4.7)$$

We could notice that the calculation of the bounds only needs the first  $s$  dimensions of  $x$  and  $\sum_{d=s+1}^D x[d]^2$ . Therefore, we only need one more number to get the bounds for the



unobserved part  $x[s + 1 : D]$ .

### 4.5.3 Calculation of the Bounds

After derivation of the bounds, we describe the procedure of calculating them in our framework.

For the query  $q_t$ , at the first round (i.e.  $j = 1$ ),  $P$  sends the first  $s_1$  dimensions of  $q_t$  and  $\sum_{d=s_1+1}^D q_t[d]^2$  to each  $M_i$ . With these values, each  $M_i$  would be able to calculate the lower bounds  $LB(q_t, x)$  and upper bounds  $UB(q_t, x)$  for each  $x \in X_i$ . Then, after  $P$  getting the  $k_{th}$  smallest upper bounds as  $thr$ , we could run the pruning procedure.

In each following round (i.e.  $j > 1$ ),  $P$  sends the next  $s_j$  dimensions of  $q_t$  to each  $M_i$  whose instances were not pruned completely. These  $M_i$  will update their bounds as following

$$\begin{aligned}
 LB_j(x, y) &= LB_{j-1}(x, y) + \sum_{d=p_{j-1}+1}^{p_j} (x[d] - y[d])^2. \quad (4.8) \\
 UB_j(x, y) &= UB_{j-1}(x, y) + \sum_{d=1}^s (x[d] - y[d])^2 \\
 &\quad + \sum_{d=s+1}^D x[d]^2 + \sum_{d=s+1}^D y[d]^2 \\
 &\quad + 2 \times \sqrt{\sum_{d=s+1}^D x[d]^2 \times \sum_{d=s+1}^D y[d]^2}. \quad (4.9)
 \end{aligned}$$

where  $p_j = \sum_{i=1}^j s_i$ ,  $LB_j$  and  $UB_j$  indicate the lower bounds and upper bounds at the iteration  $j$  respectively.

### 4.5.4 Find the Threshold in Distributed Machines

The question now is to find the threshold  $thr$  for pruning. Since TODO: Put this section here or after derived the bounds.

## 4.6 Coordinate Descent to Decide the Pivots

## 4.7 Importance-Selecting Function and Overall Framework

At each round  $j$  for  $q_t$ , we need to decide what to send from  $P$  to each  $M_i$  and then calculate the bounds at  $M_i$ .

In this section, we describe the overview of our framework.

# Bibliography

- [1] S.A. Maier. *Plasmonics: fundamentals and applications*. Springer Verlag, 2007.