Assignment 6

Chayapon Thunsetkul 6280742

October 28, 2022

1. Assume that $o(g) = n$ then $g^n = e_G$ where e_G is the identity in G. Then, by a lemma, we have $\theta(g^n) = \theta(e_G) = e_H$ where e_H is the identity in H. Also, by the definition of an isomorphism, $\theta(g^n) = (\theta(g))^n = e_H^n = e_H$. It follows from this that $o(\theta(g))|n = o(g)$.

   Next, suppose that $o(\theta(g)) = m$ and $\theta(g)^m = e_H$. Then, since θ is an isomorphism, θ^{-1} is also an isomorphism, and $g^m = \theta^{-1}(\theta(g)^m) = \theta^{-1}(e_H) = e_G$ It follows from this that $o(g)|m = o(\theta(g))$.

   Thus, $o(g) = o(\theta(g))$

2. Given any $x \in \mathbb{Z}_{50}$ we can find an integer a such that $7a = x \mod 50$. This follows from the fact that 7 and 50 are coprime and 7 can generate $\mathbb{Z}_{50}$. Now, $\theta(x) = \theta(7a) = a\theta(7) = 13a$.

3. We have $\lambda_d(g) = dg$ for $g \in G$. Reading off the **d** in the Cayley table gives the map (a d f g)(b h e c). Representing this into numbers; (1 4 6 7)(2 8 5 3).

4. By a lemma, we have $o(6) = 5$ in $\mathbb{Z}_{30}$, $o(15) = 3$ in $\mathbb{Z}_{45}$, and $o(4) = 25$ in $\mathbb{Z}_{25}$. By another lemma, the order of $(6, 15, 4)$ in $\mathbb{Z}_{30} \times \mathbb{Z}_{45} \times \mathbb{Z}_{25}$ is the $\text{lcm}(5, 3, 25) = 25$.

5. Let $a \in \mathbb{Z}_{25}$ then $b \in \mathbb{Z}_5$ and $o((a, b)) = 5$ in $\mathbb{Z}_{25} \times \mathbb{Z}_5 \iff \text{lcm}(o(a), o(b)) = 5$. Since 5 is a prime $o(a), o(b)$ must either be 1 or 5 but not both 1. Suppose $o(a) = 5$ then $25/\gcd(a, 25) = 5 \iff \gcd(a, 25) = 5$ which leaves 4 possible values of a. Suppose $o(b) = 5$ then $5/\gcd(b, 5) = 5 \iff \gcd(b, 5) = 1$. This leaves 4 possible values of b. Adding all the possible values when $o(a) = 5$ or $o(b) = 5$ or $o(a) = 1$ or $o(b) = 1$ and subtracting the case where $o(a) = o(b) = 1$ gives $1 + 1 + 4 + 4 - 1 = 9$.