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Assignment 5

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1. $U(30) = \{1, 7, 11, 13, 17, 19, 23, 29\}$

left cosets of H in $U(30)$:

$\{1, 11\}, \{7, 17\}, \{13, 23\}, \{19, 29\}$

2. let $|H| = n$ then by Lagrange's theorem,

$42 \mid n$ and $n \mid 420$

since $K < H < G$

n is either $42 \times 5 = 210$

or $n = 42 \times 2 = 84$

3. Since $H \cap K$ is a subgroup of both H and K, it follows that $|H \cap K|$ divides $|H|$ and $|H \cap K|$ divides $|K|$, but since $|H|$ and $|K|$ are relativly prime, the only common divider is 1, hence, $|H \cap K| = 1$. Thus, $H \cap K = \{1\}$

4. The order of $\tau = 5$ and $o(\sigma) = 7$. By a lemma $5 \mid |G|$ and $7 \mid |G|$. It follows that $|G|$ is atleast 35.

5. By Lagrange's theorem,

$|G| = |G:H| \cdot |H|$

$|G| = |G:H| \cdot |H:K| \cdot |K|$

and since $K \leq G$

$|G| = |G:K| \cdot |K|$

Hence, $|G:H| \cdot |H:K| = |G:K|$

6. 1) $\forall h \in H$ we have $\det(h) = 1$ and $\det(Ah) = \det(A) \cdot \det(h) = 2$. By a lemma, $\forall B \in G$ such that $\det(B) = 2$ implies that $BH = AH$. Also, $B \in BH = AH$. Thus, AH contains all elements in G with determinant $= 2$.

2) Let $A \in G$, then the left coset AH contains only elements with determinants $= \det(A)$

3) $G = \bigcup_{r \in \mathbb{R}/\{0\}} A_r H$

where $A_r = \sqrt{r} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

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7. By a lemma, $g \in G$ then $o(g) \mid 8$.

The dividers of 8 are, 1, 2, 4, 8.

since the idutity is unique, there must exists an element with order 2, 4, or 8.

if $o(g) = 4$ then $o(g^2) = 2$

if $o(g) = 8$ then $o(g^4) = 2$

both $g^2, g^4 \in G \Leftarrow g \in G$.

8. Case $a = 1$: Trivially $aH = H = Ha$

case $a \neq 1$: if $a \in H$ then, by a lemma, $aH = H = Ha$.
Else if $a \notin H$ then $aH \cap H = \emptyset$ and $Ha \cap H = \emptyset$.

since $|G : H| = 2$ implies $|H| = \frac{|G|}{2}$
it follows that $|Ha| = |aH| = \frac{|G|}{2}$.

This means that $Ha = \{g \in G, g \notin H\}$
$= aH$.