Using QAOA for Routing and Wavelength Assignment Optimisation in Optical Networks

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Routing and Wavelength Assignment (RWA)

- In wavelength division multiplexing (WDM) network, a connection between a source and a destination node is established by a lightpath.
- A lightpath is a set of links associated with a unique wavelength.
- A lightpath must
  1. have all of its link using the same wavelength (wavelength continuity), and
  2. not use the same wavelength as another lightpath sharing same link(s).
- The task of assigning routes and wavelengths optimally is called the Routing and wavelength assignment (RWA) problem[1].

Motivation

- The problem is NP-complete and exact solutions are intractable at large scales[1–3].
- Classical heuristics (e.g. shortest-path first-fit) provide good but not necessarily optimal solutions[1–7].
- Quantum and quantum-inspired methods, have been investigated as potential alternatives[8, 9].
- One of the main limitation of these quantum approaches is that hard constraint must be modelled as penalties in the cost function, yielding infeasible solution.

QAOA

Quantum Approximate Optimisation Algorithm

- Given a cost function $f(x)$, define the *problem Hamiltonian*:

$$\hat{H}_P|\boldsymbol{x}\rangle = f(\boldsymbol{x})|\boldsymbol{x}\rangle. \tag{1}$$

  and another Hamiltonian called the *mixer Hamiltonian* usually taken to be the transverse-field Hamiltonian:

$$\hat{H}_m = -\sum_j \hat{X}_j, \tag{2}$$

- Define two parameterised unitaries: the *phase operator*

$$U_P(\gamma) = e^{-i\gamma \hat{H}_P} \tag{3}$$

  and the *Mixing operator:*

$$U_M(\beta) = e^{-i\beta \hat{H}_M} \tag{4}$$

- Initial state: equal superposition $|\psi_i\rangle = \frac{1}{\sqrt{2^n}} \sum_{\boldsymbol{x}} |\boldsymbol{x}\rangle$

QAOA

circuit

After P alternating layers, the final state is

$$|\boldsymbol{\beta}, \boldsymbol{\gamma}\rangle = \hat{U}_M(\beta_{P-1})\hat{U}_P(\gamma_{P-1}) \cdots \hat{U}_M(\beta_0)\hat{U}_P(\gamma_0)|\psi_i\rangle, \tag{5}$$

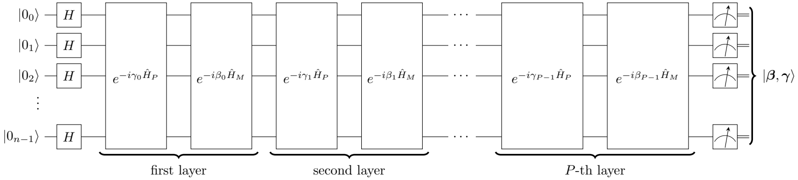


Figure: A P-layer QAOA circuit on n qubits. Each layer applies a problem unitary

Objective: find parameters $(\boldsymbol{\beta}, \boldsymbol{\gamma})$ minimising

$$\langle f \rangle = \langle \boldsymbol{\beta}, \boldsymbol{\gamma}|H_P|\boldsymbol{\beta}, \boldsymbol{\gamma}\rangle.$$

QAOA

Quantum Alternating Operator Ansatz

- Consider a problem where we only want to find a solution that belongs to a set of *feasible* solution which is a (proper) subset of the set of all solutions.
- Using the tranverse-field Hamiltonian does not preserve the feasible subspace.
- The extension of the original QAOA introduced in [10] allows the mixing Hamiltonian to be problem specific.
- Restricts dynamics to a feasible subspace $\Rightarrow$ guarantees valid solutions.
- Ensures all sampled solutions respect hard constraints.

Formulation: Cost Function

- Binary decision variable:

$$x_{r,\lambda} = \begin{cases} 1 & \text{if route } r \text{ is assigned with the wavelength } \lambda, \\ 0 & \text{otherwise} \end{cases}$$

- Cost function to minimise:

$$f(\boldsymbol{x}) = f_{\text{collision}}(\boldsymbol{x}) + f_{\text{wavelength}}(\boldsymbol{x})$$

where
  - $f_{\text{collision}}(\boldsymbol{x}) := \sum_{r,r' \in S} \sum_{\lambda \in \Lambda} x_{r,\lambda} x_{r',\lambda}$: penalises lightpath collisions.
  - $f_{\text{wavelength}}(\boldsymbol{x}) := \sum_{r \in R} \sum_{\lambda \in \Lambda} h(\lambda) x_{r,\lambda}$: penalises higher-index wavelengths.

The problem Hamiltonian corresponding to the cost function is

$$\hat{H}_p = \frac{1}{4} \sum_{r,r' \in S} \sum_{\lambda \in \Lambda} (\mathbb{1} - \hat{Z}_{r,\lambda})(\mathbb{1} - \hat{Z}_{r',\lambda}) + \frac{1}{2} \sum_{r \in R} \sum_{\lambda \in \Lambda} h(\lambda)(\mathbb{1} - \hat{Z}_{r,\lambda}). \tag{6}$$

Formulation: Qubit encoding

- The solution, $\boldsymbol{x}$, is encoded as bitstring
- One-hot encoding: A valid bitstring will have a constraint that each sub-bitstring will have a Hamming weight of one.

Pair 1			Pair 2			Pair 3		
Route 1	Route 2	Route 3	Route 1	Route 2	Route 3	Route 1	Route 2	Route 3
10	00	00	00	10	00	01	00	00
λ_1 λ_2	λ_1 λ_2	λ_1 λ_2	λ_1 λ_2	λ_1 λ_2	λ_1 λ_2	λ_1 λ_2	λ_1 λ_2	λ_1 λ_2

Figure: The bitstring representing a valid solution for an RWA instance where $P = 3, N = 3, \Lambda = 2$.

The mixer Hamiltonian that preserves the feasible solution subspace is

$$\hat{H}_m = \sum_{i=1}^{P} \sum_{j=1}^{N\times\Lambda} (\hat{X}_{i,j}\hat{X}_{i,j+1} + \hat{Y}_{i,j}\hat{Y}_{i,j+1}). \tag{7}$$

Formulation

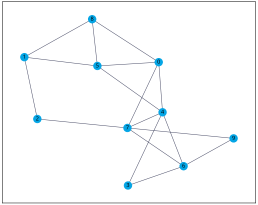
Two-steps scheme

- This amount of qubit may be unpractical when the scheme is applied on a utility scale.
- The two-steps scheme is as follows:
  1. Reduce the RWA problem into RA problem by letting the number of available wavelength, $\Lambda = 1$.
  2. Use the formulation to obtain the route for each source-destination node pair.
  3. Reintroduce the original number of available wavelength and let the route option be the ones obtained from the WA step.
- The two-step scheme reduces the number of qubits usage from $P \times N \times \Lambda$ to $\max(P \times N, P \times \Lambda)$.

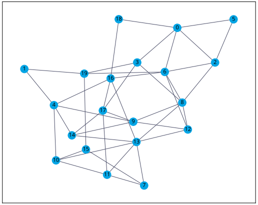
Numerical Results

Classical simulation setup

- Number of requests: 2 to 10
- Number of alternate routes: 1 to 10
- Number of wavelengths: 1 to $|P|$
- Circuit depths 1 to 4
- Two network topologies
- Result compared to the k shortest paths first-fit



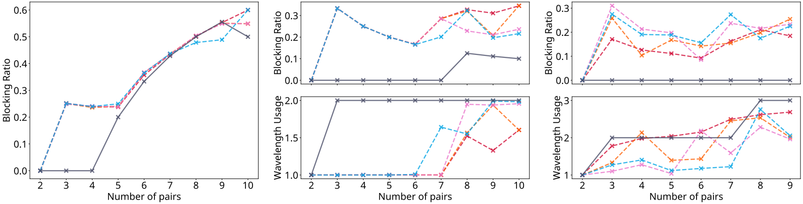
(a) $|V| = 10$ nodes and $|E| = 22$ links. (Small)



(b) $|V| = 20$ nodes and $|E| = 42$ links. (Large)

Numerical Results: Performance

- Blocking ratio:
  - At circuit depth $= 1$: 20.7% vs. 6% (k-SPFF)
  - At circuit depth $= 4$: reduced to 18.0%
- Wavelength usage:
  - QAOA (depth 1): 82.4% vs. 86.9% (k-SPFF)
  - At depth 4: 79.6%



(a) $N = 2, \Lambda = 1$, small topology **(b)** $N = 1, \Lambda = 3$, large topology **(c)** $N = 1, \Lambda = 3$, large topology

Figure: The blocking ratio and number of wavelength used. -×- is circuit depth 1, -×- is 2, -×- is 3, -×- is 4, and —×— is k-SPFF.

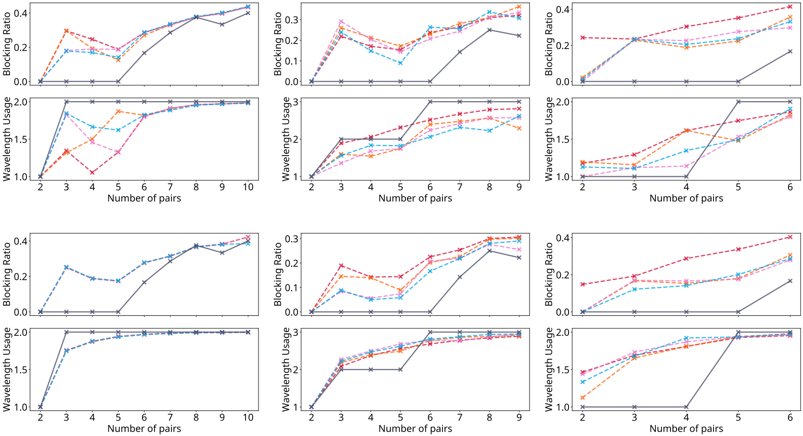
Numerical Results

Effect of the modified cost function

Modification: set $f_{\text{wavelength}} = 0$.

Blocking ratio: $\downarrow$ by $\sim$3.25%, **Wavelength usage:** $\uparrow$ by $\sim$17%

Top rows: before cost function modification, **bottom row:** after



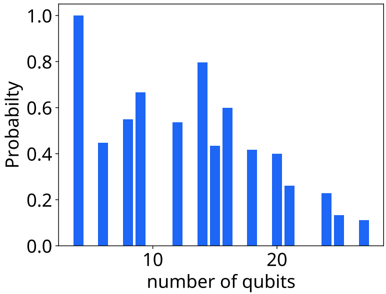
(a) $N = 1, |\Lambda| = 2$, small topology

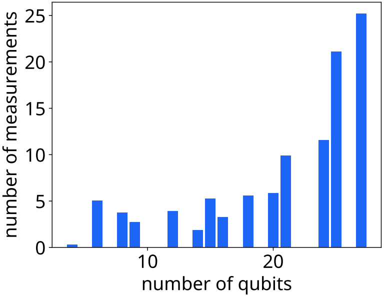
(b) $N = 1, |\Lambda| = 3$, small topology

(c) $N = 2, |\Lambda| = 2$, small topology

Quality of Solutions: Distribution Analysis

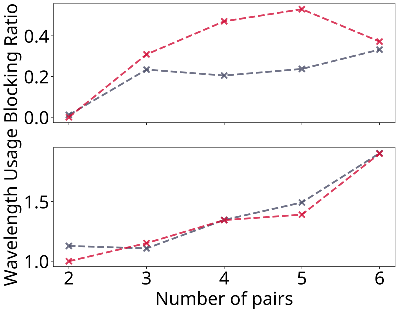
- Many sampled solutions are low-quality $\Rightarrow$ expected performance pulled down.
- Occasionally samples high-quality solutions (better than heuristic).
- For small instances ($<$ 20 qubits), $\sim$10 measurement shots are sufficient to reach $>$ 95% confidence of finding a better solution.



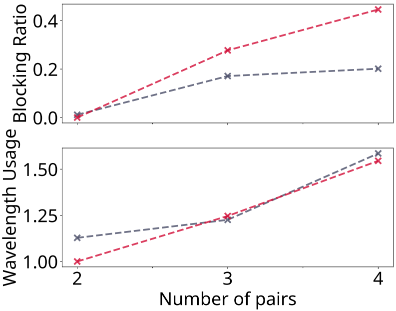


Numerical Results: Two-steps scheme

- Joint formulation ⇒ up to 20% lower blocking
- Also uses fewer wavelengths than two-step scheme
- Confirms benefit of joint optimisation



(a) $|P| = 2, |\Lambda| = 2$, small topology



(b) $|P| = 2, |\Lambda| = 3$, small topology

Figure: -×-is the joint RWA formulation, and -×-is the two-steps formulation.

Conclusion

- I proposed QAOA formulation for the joint RWA problem.
- The encoding uses one-hot constraints and a tailored mixer to guarantee feasibility.
- The joint formulation outperforms the two-step scheme.
- At small scales (≤ 27 qubits), QAOA underperforms classical heuristics in blocking.
- Smarter cost-function shaping and adaptive mixers may improve performance.
- Larger quantum hardware (> 27 qubits) will be needed to test the formulation at meaningful scales.

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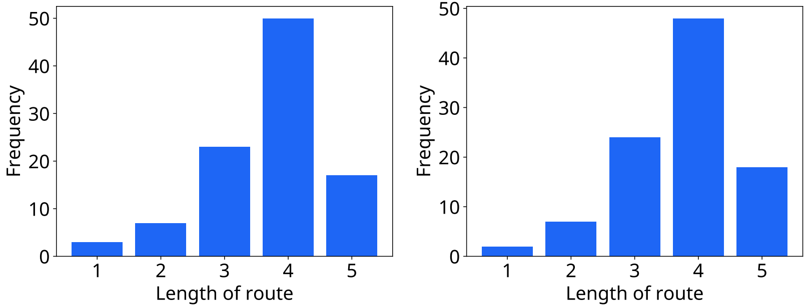
Appendix

Length of routes selected as alternate routes

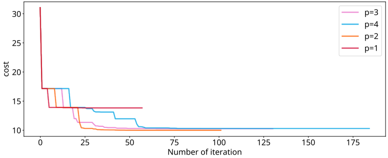


(a) Small **(b)** Large

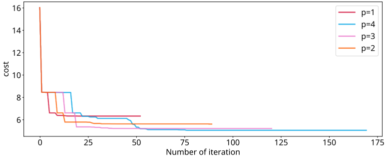
Figure: The frequency of the length of route (number of links) in each network topology

Appendix

Expected cost at each iteration of the optimisation process



(a) $|P| = 9, N = 1, |\Lambda| = 3$, small topology



(b) $|P| = 6, N = 2, |\Lambda| = 2$, small topology

Figure: The average cost of the solution at each iteration of QAOA.

Appendix

Runtime

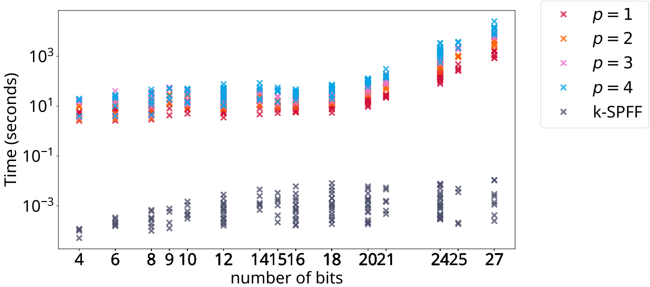


Figure: Runtime of the simulation at different problem size (quantified by the number of bits needed for the formulation).