1.

a. Função de Transferência

$$V_{in}(t) = V_R(t) + V_L(t)$$

$$V_{in}(t) = R \times i_L(t) + L \times \frac{di_L(t)}{dt}$$

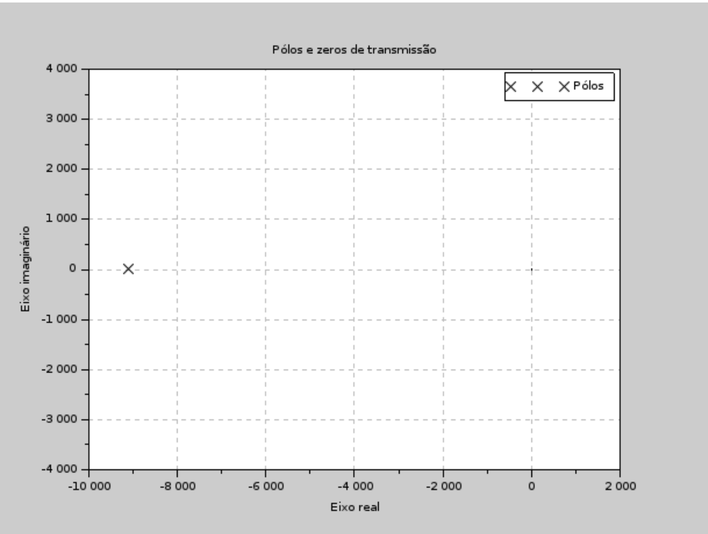
$$V_{in}(s) = R \times I_L(s) + L \times sI_L(s)$$

$$V_{in}(s) = (R + sL) \times I_L(s)$$

$$\frac{I_L(s)}{V_{in}(s)} = \frac{1}{R + sL} = \frac{1/L}{s + (R/L)} = \frac{1}{R} \times \left(\frac{R/L}{s + (R/L)}\right) = H(s)$$

b. Entrada pulso

$$I_L(s) = 1 \times H(s) = \frac{1/1{,}1 \times 10^{-3}}{s + \left(\frac{10}{1{,}1 \times 10^{-3}}\right)} = \frac{9{,}1 \times 10^2}{s + 9{,}1 \times 10^3}$$



O sistema é estável, pois gera saídas limitadas para entradas limitadas. A estabilidade pode ser observada através da posição dos polos, que se encontram ao lado esquerdo do plano s, isto é, possui parte real negativa, o que sinaliza que a exponencial irá convergir ao valor de regime permanente.

c. Características

$$ganho = k = 1/L = 9{,}1 \times 10^2$$

$$polo = p = R/L = \frac{10}{1{,}1 \times 10^{-3}} = 9{,}1 \times 10^3$$

$$constante\ de\ tempo = \tau = \frac{1}{p} = \frac{1{,}1 \times 10^{-3}}{10} = 1{,}1 \times 10^{-4}$$

d. Entrada degrau

$$I_L(s) = V_{in}(s) \times H(s)$$

$$I_L(s) = \left(V_{in} \times \frac{1}{s}\right) \times \frac{1}{R} \times \left(\frac{p}{s+p}\right)$$

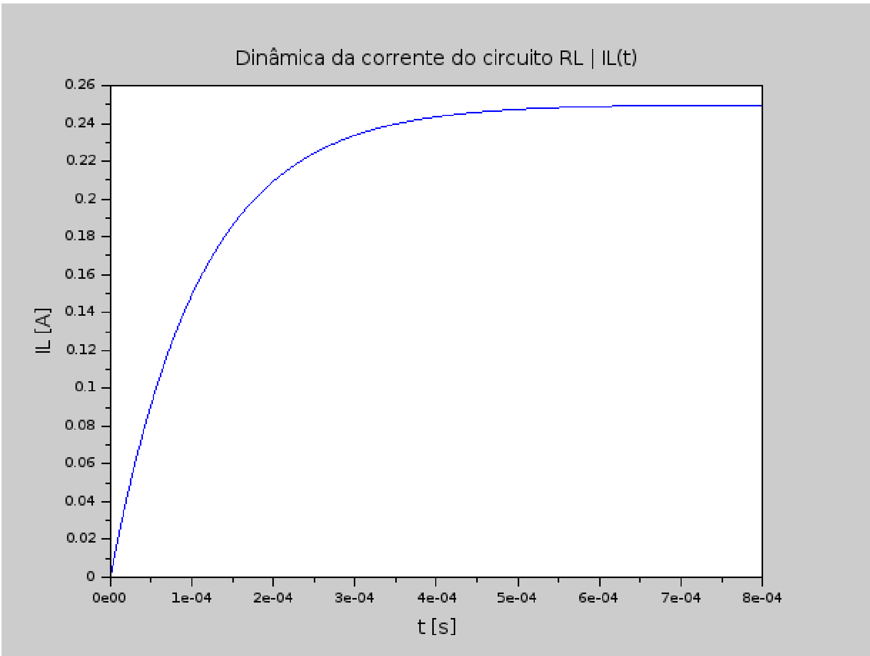
$$I_L(s) = \left(\frac{V_{in}}{R}\right) \times \left(\frac{1}{s} \times \frac{p}{s+p}\right)$$

$$I_L(t) = \left(\frac{V_{in}}{R}\right) \times (1 - e^{-p \times t})$$

$$I_L(t) = \left(\frac{V_{in}}{R}\right) \times \left(1 - e^{-t/\tau}\right)$$

$$I_L(t) = \frac{2{,}5}{10} \times \left(1 - e^{-\frac{t}{1{,}1 \times 10^{-4}}}\right)$$

$$I_L(t) = 0{,}25 \times \left(1 - e^{-\frac{t}{1{,}1 \times 10^{-4}}}\right)$$



2.

$constante\ de\ tempo$

$$T'(t = 45seg) = 0{,}95 \times T' = T' \times \left(1 - e^{-45/\tau}\right)$$

$$0{,}95 = 1 - e^{-45/\tau}$$

$$e^{-45/\tau} = 1 - 0{,}95$$

$$e^{-45/\tau} = 0{,}05$$

$$\ln\left(e^{-45/\tau}\right) = \ln 0{,}05$$

$$-45/\tau = \ln 0{,}05$$

$$\tau = -45/\ln 0{,}05$$

$$\tau = 15{,}02\ seg$$

$polo$

$$p = 1/\tau = 6{,}66 \times 10^{-2}$$

$ganho$

$$T = T' = k/p = k \times \tau$$

$$k = T'/\tau = T'/0{,}63 \times T' = 1{,}58$$

Função de transferência

$$H(s) = \frac{k}{s + p} = \frac{1{,}58}{s + 6{,}66 \times 10^{-2}}$$

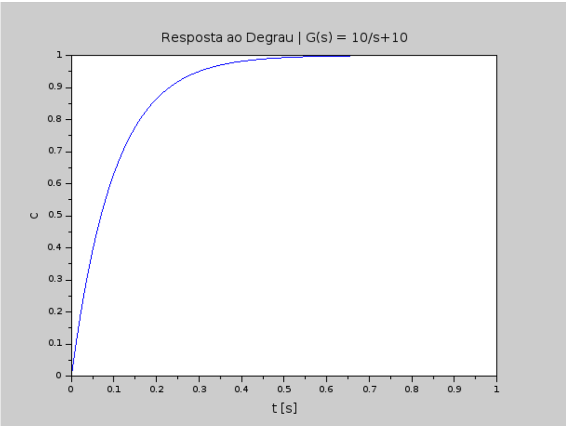
3.

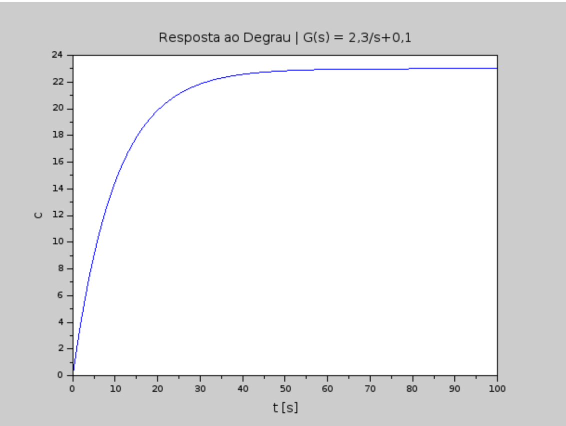
$$C(s) = R(s) \times G(s) = \left(\frac{1}{s}\right) \times \left(\frac{k}{s+p}\right)$$

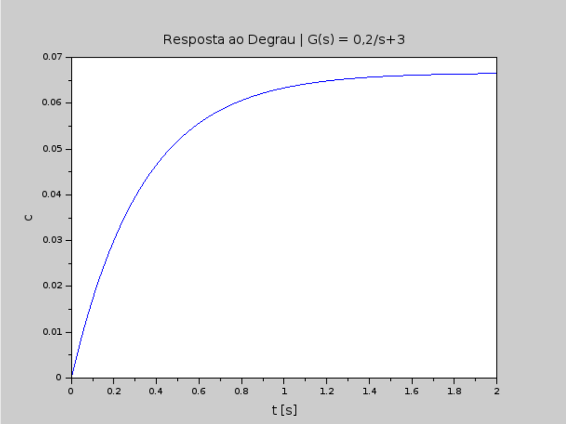
$$C(s) = \left(\frac{1}{s}\right) \times \frac{1}{p/k} \times \left(\frac{p}{s+p}\right)$$

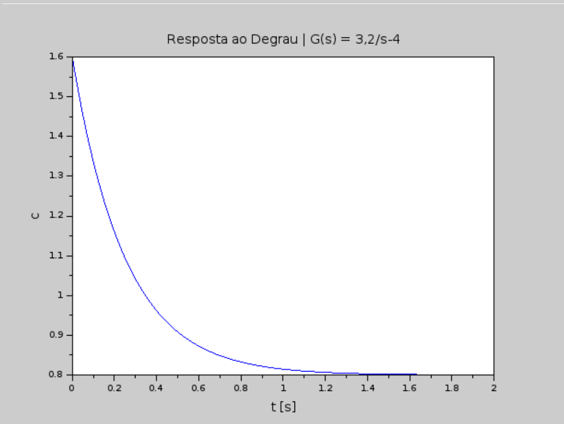
$$C(s) = \left({}^{k}/_{p}\right) \times \left(\frac{1}{s}\right) \times \left(\frac{p}{s+p}\right)$$

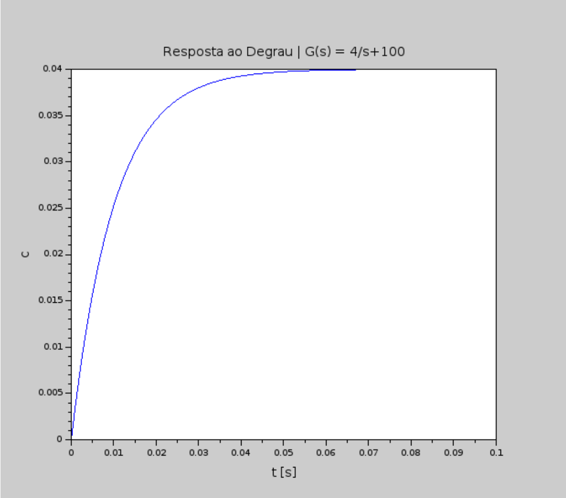
$$C(t) = \left({}^{k}/_{p}\right) \times \left(1 - e^{-p \times t}\right)$$

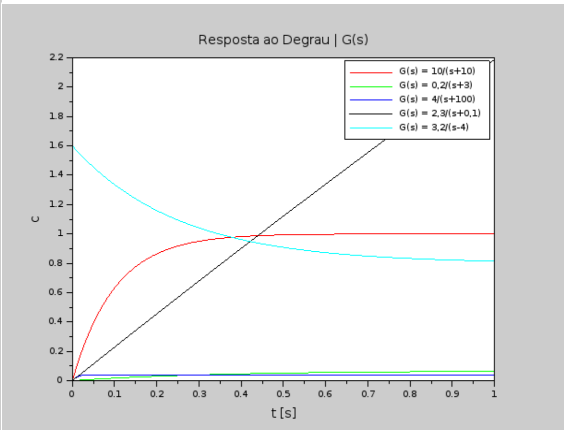












4.

$$Y(s) = R(s) \times G_{MF}(s)$$

$$\frac{Y(s)}{R(s)} = G_{MF}(s)$$

$$= \frac{G(s)}{1 + G(s) \times H(s)} \qquad H(s) = 1$$

$$= \frac{G(s)}{1 + G(s)}$$

$$= \frac{G(s)}{1 + G(s)}$$

$$= \frac{1}{{}^{1}/_{G(s)} + 1} \qquad G(s) = \frac{b}{s + a}$$

$$= \frac{b}{s + (a + b)} = \frac{k}{s + p}$$

$$R(S) = 1\ (degrau\ uniátio)$$

$$\begin{cases} t_{s,2\%} = {}^{4}/_{p} = 1s \Longrightarrow p = 4 \\ Y = {}^{k}/_{p} = 10 \Longrightarrow k = 10 \times p = 40 \end{cases}$$

$$\begin{cases} k = b = 40 \\ p = a + b \Longrightarrow a = p - b = -36 \end{cases}$$

$$G(s) = \frac{40}{s - 36} \qquad G_{MF}(s) = -\frac{36}{s + 4}$$

5.

a.

$$I_L(t) = I_{in}(t) = I_C(t) + I_R(t)$$

$$\frac{V_{in}(s)}{Z_{in}} = \frac{V_C(s)}{Z_C} + \frac{V_R(s)}{R}$$

$$\frac{V_{in}(s)}{Z_{in}} = \frac{V_O(s)}{Z_C} + \frac{V_O(s)}{R}$$

$$\frac{V_{in}(s)}{Z_{in}} = \left(\frac{1}{Z_C} + \frac{1}{R}\right) \times V_O(s)$$

$$\frac{V_{in}(s)}{Z_{in}} = \left(\frac{R + Z_C}{Z_C \times R}\right) \times V_O(s)$$

$$\frac{V_O(s)}{V_{in}(s)} = \frac{1}{Z_{in}} \times \left(\frac{Z_C \times R}{R + Z_C}\right)$$

$$\frac{V_O(s)}{V_{in}(s)} = \frac{1}{Z_L + Z_{RC}} \times Z_{RC} \qquad \frac{1}{Z_{RC}} = \frac{1}{Z_C} + \frac{1}{R} = \frac{R + Z_C}{Z_C \times R} \Longrightarrow Z_{RC} = \frac{Z_C \times R}{R + Z_C}$$

$$\frac{V_O(s)}{V_{in}(s)} = \frac{1}{Z_L/_{Z_{RC}} + 1}$$

$$\frac{V_O(s)}{V_{in}(s)} = \frac{1}{Z_L \times (R + Z_C)/_{Z_C \times R} + 1}$$

$$\frac{V_O(s)}{V_{in}(s)} = \frac{Z_C \times R}{Z_L \times (R + Z_C) + Z_C \times R}$$

$$\frac{V_O(s)}{V_{in}(s)} = \frac{Z_C \times R}{Z_L \times Z_C + (Z_L + Z_C) \times R}$$

$$\frac{V_O(s)}{V_{in}(s)} = \frac{Z_C}{Z_L \times Z_C/_R + Z_L + Z_C}$$

$$\frac{V_O(s)}{V_{in}(s)} = \frac{1}{Z_L/_R + Z_L/Z_C + 1}$$

$$\frac{V_O(s)}{V_{in}(s)} = \frac{1}{LC \times s^2 + L/_R \times s + 1}$$

$$\frac{V_O(s)}{V_{in}(s)} = \frac{1/_{LC}}{s^2 + 1/_{CR} \times s + 1/_{LC}} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n \times s + \omega_n^2}$$

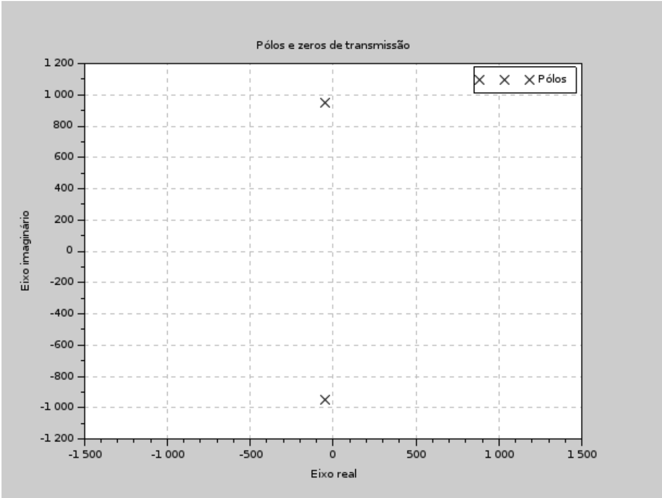
b.

$$\omega_n^2 = {}^{1}/_{LC} \Rightarrow \omega_n = \sqrt{{}^{1}/_{LC}} = \sqrt{{}^{1}/_{(1{,}1 \times 10^{-3})(10^{-3})}} = 9{,}53 \times 10^{2}$$

$$2\xi\omega_n = {}^{1}/_{CR} \Rightarrow \xi = \frac{1}{2CR\omega_n} = \frac{1}{2 \times (10^{-3}) \times 10 \times 9{,}53 \times 10^{-1}} = 5{,}24 \times 10^{-2}$$

c.

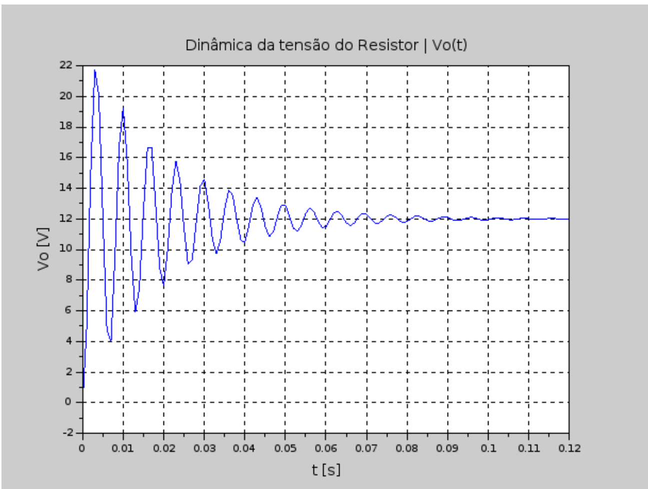
$$s_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1} = -50.0 \pm j$$



d. O sistema é estável, pois gera saídas limitadas para entradas limitadas. A estabilidade pode ser observada através da posição dos polos, que se encontram ao lado esquerdo do plano s, isto é, possui parte real negativa, o que sinaliza que a exponencial irá convergir ao valor de regime permanente.

e. $0 < \xi < 1$ os polos são complexos conjugados e o sistema é dito subamortecido.

f.



6.

a.

$$\frac{Y(s)}{R(s)} = G_{MF}(s)$$

$$= \frac{G(s)}{1 + G(s) \times H(s)} \qquad H(s) = 1$$

$$= \frac{G(s)}{1 + G(s)}$$

$$= \frac{G(s)}{1 + G(s)}$$

$$= \frac{1}{{}^{1}/_{G(s)} + 1} \qquad G(s) = \frac{16}{s(s+4)}$$

$$= \frac{16}{s(s+4) + 16}$$

$$= \frac{16}{s^2 + 4s + 16} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n \times s + \omega_n^2}$$

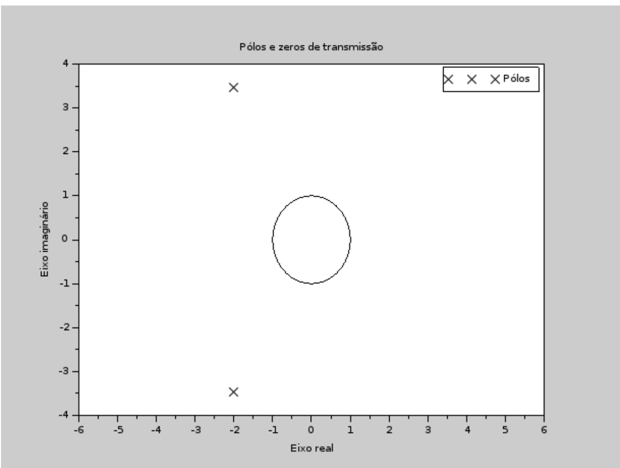
b. O sistema é estável, pois gera saídas limitadas para entradas limitadas. A estabilidade pode ser observada através da posição dos polos, que se encontram ao lado esquerdo do plano s, isto é, possui parte real negativa, o que sinaliza que a exponencial irá convergir ao valor de regime permanente.

c.

$$\omega_n^2 = 16 \Longrightarrow \omega_n = 4$$

$$2\xi\omega_n = 4 \Longrightarrow \xi = \frac{4}{2\omega_n} = \frac{1}{2} = 0{,}5$$

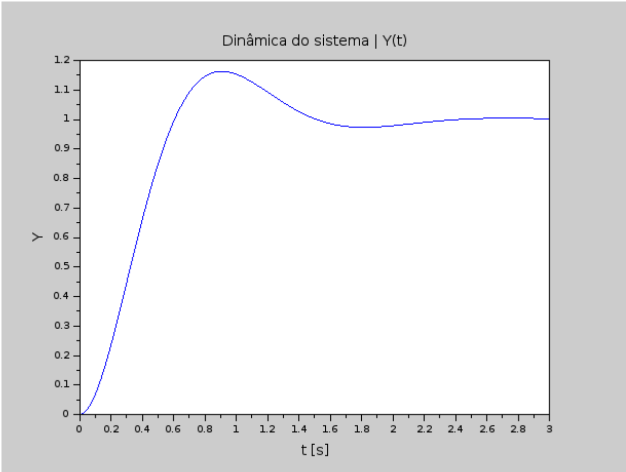
d.



e. $0 < \xi < 1$ os polos são complexos conjugados e o sistema é dito subamortecido.

f.

$$t_{s,5\%} = \frac{3}{\xi\omega_n} = 1{,}5seg \quad t_{s,2\%} = \frac{4}{\xi\omega_n} = 2seg \quad M_P(\%) = e^{-\frac{\xi\times\pi}{\sqrt{1-\xi^2}}} \times 100 = 16{,}3\%$$



7.

$$\frac{Y(s)}{R(s)} = G_{MF}(s)$$

$$= \frac{G(s)}{1 + G(s) \times H(s)} \qquad H(s) = k_p$$

a. $G(s) = \frac{5}{s^2+2{,}2s+1} = \frac{5}{s^2+2{,}2s-4+5}$