1.

a. Função de Transferência

$$V_{in}(t) = V_R(t) + V_L(t)$$

$$V_{in}(t) = R \times i_L(t) + L \times \frac{di_L(t)}{dt}$$

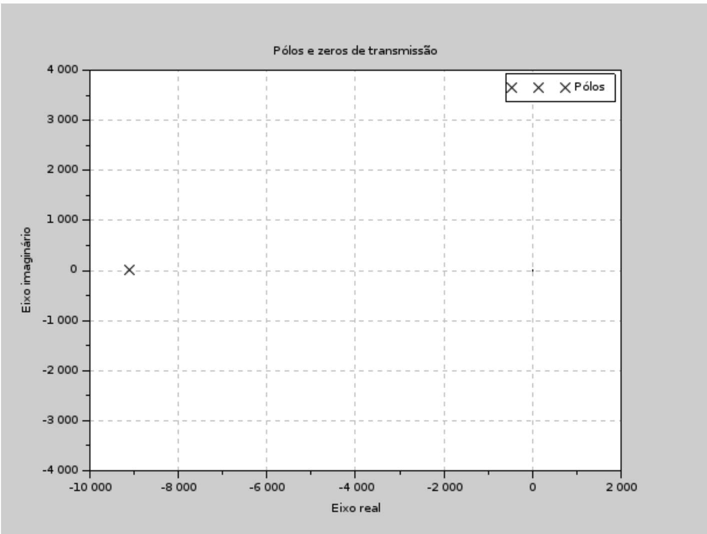
$$V_{in}(s) = R \times I_L(s) + L \times sI_L(s)$$

$$V_{in}(s) = (R + sL) \times I_L(s)$$

$$\frac{I_L(s)}{V_{in}(s)} = \frac{1}{R + sL} = \frac{1/L}{s + (R/L)} = \frac{1}{R} \times \left(\frac{R/L}{s + (R/L)}\right) = H(s)$$

b. Entrada pulso

$$I_L(s) = 1 \times H(s) = \frac{1/1{,}1 \times 10^{-3}}{s + \left(\frac{10}{1{,}1 \times 10^{-3}}\right)} = \frac{9{,}1 \times 10^{2}}{s + 9{,}1 \times 10^{3}}$$



O sistema é estável, pois gera saídas limitadas para entradas limitadas. A estabilidade pode ser observada através da posição dos polos, que se encontram ao lado esquerdo do plano s, isto é, possui parte real negativa, o que sinaliza que a exponencial irá convergir ao valor de regime permanente.

c. Características

$$ganho = k = 1/L = 9{,}1 \times 10^{2}$$

$$polo = p = R/L = \frac{10}{1{,}1 \times 10^{-3}} = 9{,}1 \times 10^{3}$$

$$constante\ de\ tempo = \tau = \frac{1}{p} = \frac{1{,}1 \times 10^{-3}}{10} = 1{,}1 \times 10^{-4}$$

d. Entrada degrau

$$I_L(s) = V_{in}(s) \times H(s)$$

$$I_L(s) = \left(V_{in} \times \frac{1}{s}\right) \times \frac{1}{R} \times \left(\frac{p}{s+p}\right)$$

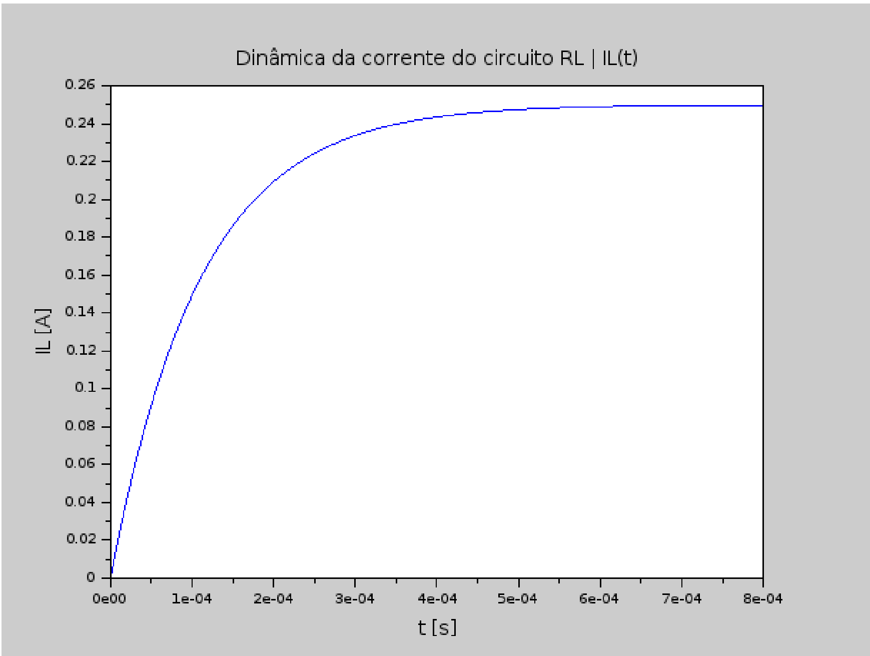
$$I_L(s) = \left(\frac{V_{in}}{R}\right) \times \left(\frac{1}{s} \times \frac{p}{s+p}\right)$$

$$I_L(t) = \left(\frac{V_{in}}{R}\right) \times (1 - e^{-p \times t})$$

$$I_L(t) = \left(\frac{V_{in}}{R}\right) \times \left(1 - e^{-t/\tau}\right)$$

$$I_L(t) = \frac{2,5}{10} \times \left(1 - e^{-\frac{t}{1,1 \times 10^{-4}}}\right)$$

$$I_L(t) = 0,25 \times \left(1 - e^{-\frac{t}{1,1 \times 10^{-4}}}\right)$$



2.

$$constante\ de\ tempo$$

$$T'(t = 45seg) = 0{,}95 \times T' = T' \times \left(1 - e^{-45/\tau}\right)$$

$$0{,}95 = 1 - e^{-45/\tau}$$

$$e^{-45/\tau} = 1 - 0{,}95$$

$$e^{-45/\tau} = 0{,}05$$

$$\ln\left(e^{-45/\tau}\right) = \ln 0{,}05$$

$$-45/\tau = \ln 0{,}05$$

$$\tau = -45/\ln 0{,}05$$

$$\tau = 15{,}02\ seg$$

$$polo$$

$$p = 1/\tau = 6{,}66 \times 10^{-2}$$

$$ganho$$

$$T = T' = k/p = k \times \tau$$

$$k = T'/\tau = T'/0{,}63 \times T' = 1{,}58$$

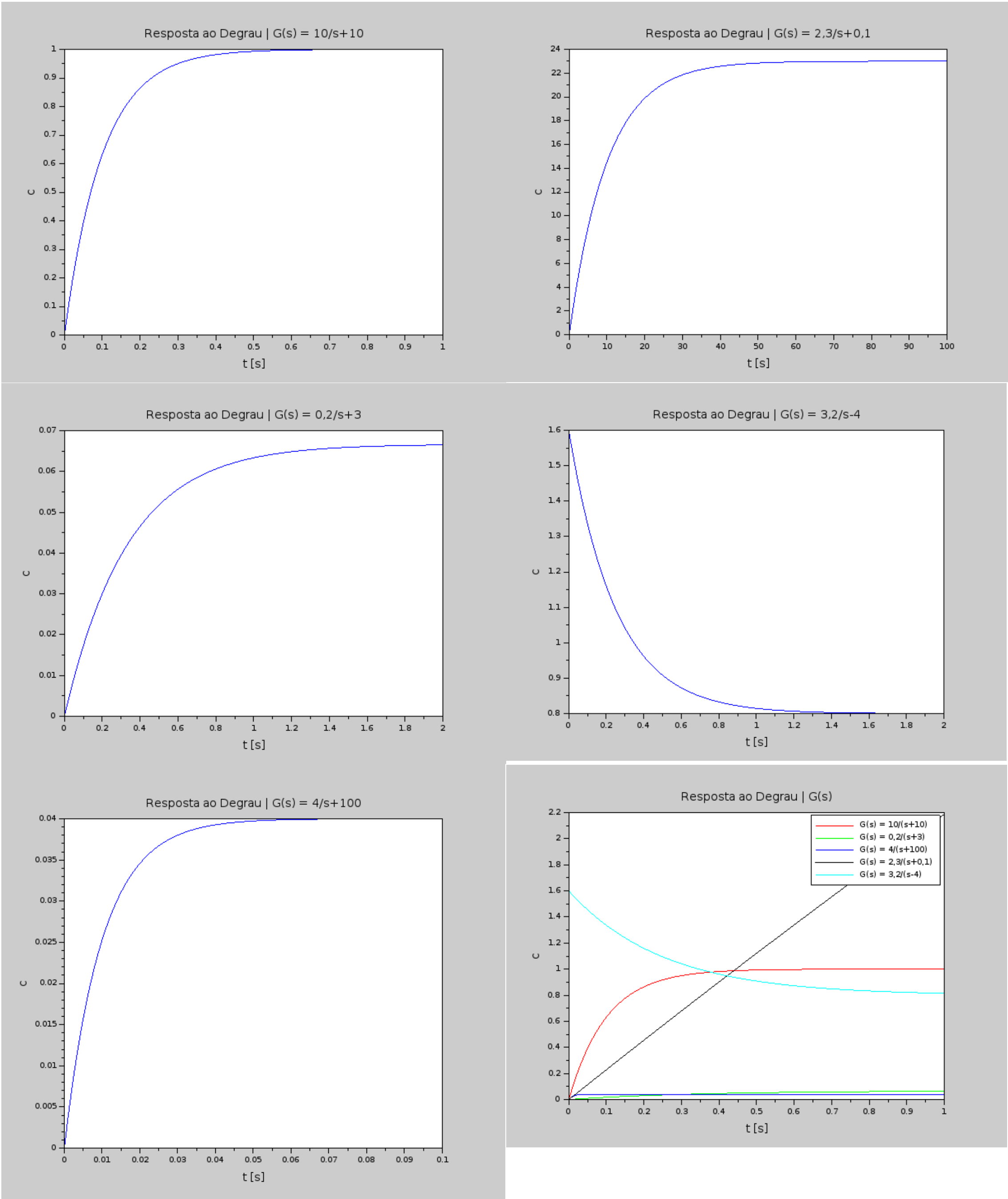
Função de transferência

$$H(s) = \frac{k}{s + p} = \frac{1{,}58}{s + 6{,}66 \times 10^{-2}}$$

3.

$$C(s) = R(s) \times G(s) = \left(\frac{1}{s}\right) \times \left(\frac{k}{s+p}\right) = \left({}^{k}/_{p}\right) \times \left(\frac{1}{s}\right) \times \left(\frac{p}{s+p}\right)$$

$$C(t) = L^{-1}[C(s)] = R(t) * G(t) = \left({}^{k}/_{p}\right) \times \left(1 - e^{-p \times t}\right)$$



4.

$$Y(s) = R(s) \times G_{MF}(s)$$

$$\frac{Y(s)}{R(s)} = G_{MF}(s)$$

$$= \frac{G(s)}{1 + G(s) \times H(s)} \qquad H(s) = 1$$

$$= \frac{G(s)}{1 + G(s)}$$

$$= \frac{G(s)}{1 + G(s)}$$

$$= \frac{1}{{}^{1}/_{G(s)} + 1} \qquad G(s) = \frac{b}{s + a}$$

$$= \frac{b}{s + (a + b)} = \frac{k}{s + p}$$

$$R(S) = 1\ (degrau\ uniátio)$$

$$\begin{cases} t_{s,2\%} = {}^{4}/_{p} = 1s \Longrightarrow p = 4 \\ Y = {}^{k}/_{p} = 10 \Longrightarrow k = 10 \times p = 40 \end{cases}$$

$$\begin{cases} k = b = 40 \\ p = a + b \Longrightarrow a = p - b = -36 \end{cases}$$

$$G(s) = \frac{40}{s - 36} \qquad G_{MF}(s) = -\frac{36}{s + 4}$$

5.

a.

$$I_L(t) = I_{in}(t) = I_C(t) + I_R(t)$$

$$\frac{V_{in}(s)}{Z_{in}} = \frac{V_C(s)}{Z_C} + \frac{V_R(s)}{R}$$

$$\frac{V_{in}(s)}{Z_{in}} = \frac{V_O(s)}{Z_C} + \frac{V_O(s)}{R}$$

$$\frac{V_{in}(s)}{Z_{in}} = \left(\frac{1}{Z_C} + \frac{1}{R}\right) \times V_O(s)$$

$$\frac{V_{in}(s)}{Z_{in}} = \left(\frac{R + Z_C}{Z_C \times R}\right) \times V_O(s)$$

$$\frac{V_O(s)}{V_{in}(s)} = \frac{1}{Z_{in}} \times \left(\frac{Z_C \times R}{R + Z_C}\right)$$

$$\frac{V_O(s)}{V_{in}(s)} = \frac{1}{Z_L + Z_{RC}} \times Z_{RC} \qquad \frac{1}{Z_{RC}} = \frac{1}{Z_C} + \frac{1}{R} = \frac{R + Z_C}{Z_C \times R} \Longrightarrow Z_{RC} = \frac{Z_C \times R}{R + Z_C}$$

$$\frac{V_O(s)}{V_{in}(s)} = \frac{1}{Z_L/_{Z_{RC}} + 1}$$

$$\frac{V_O(s)}{V_{in}(s)} = \frac{1}{Z_L \times (R + Z_C)/_{Z_C \times R} + 1}$$

$$\frac{V_O(s)}{V_{in}(s)} = \frac{1}{Z_L/_R + Z_L/Z_C + 1}$$

$$\frac{V_O(s)}{V_{in}(s)} = \frac{1}{LC \times s^2 + L/_R \times s + 1}$$

$$\frac{V_O(s)}{V_{in}(s)} = \frac{1/_{LC}}{s^2 + 1/_{CR} \times s + 1/_{LC}}$$

b.

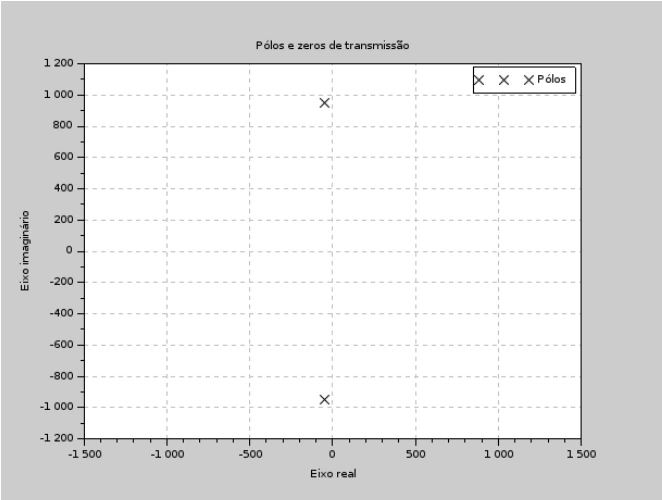
$$\frac{V_O(s)}{V_{in}(s)} = \frac{1/_{LC}}{s^2 + 1/_{CR} \times s + 1/_{LC}} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n \times s + \omega_n^2}$$

$$\omega_n^2 = 1/_{LC} \Longrightarrow \omega_n = \sqrt{1/_{LC}} = \sqrt{1/_{(1{,}1 \times 10^{-3})(10^{-3})}} = 9{,}53 \times 10^2$$

$$2\xi\omega_n = 1/_{CR} \Longrightarrow \quad \xi = \frac{1}{2CR\omega_n} = \frac{1}{2 \times (10^{-3}) \times 10 \times 9{,}53 \times 10^{-1}} = 5{,}24 \times 10^{-2}$$

c.

$$s_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1} = -50.0 \pm j$$

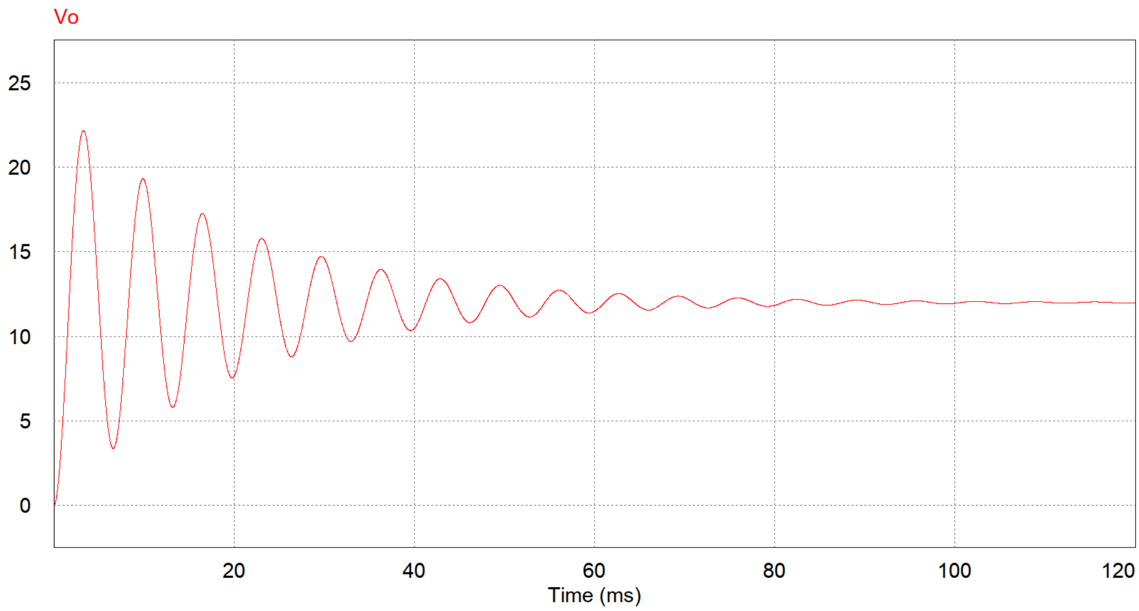


d. O sistema é estável, pois gera saídas limitadas para entradas limitadas. A estabilidade pode ser observada através da posição dos polos, que se encontram ao lado esquerdo do plano s, isto é, possui parte real negativa, o que sinaliza que a exponencial irá convergir ao valor de regime permanente.

e. $0 < \xi < 1$ os polos são complexos conjugados e o sistema é dito subamortecido.

f.

$$\frac{V_O(s)}{V_{in}(s)} = \frac{{}^{1}/_{LC}}{s^2 + {}^{1}/_{CR} \times s + {}^{1}/_{LC}} = \frac{9{,}09 \times 10^5}{s^2 + 100 \times s + 9{,}09 \times 10^5}$$



6.

a.

$$\frac{Y(s)}{R(s)} = G_{MF}(s)$$

$$= \frac{G(s)}{1 + G(s) \times H(s)} \qquad H(s) = 1$$

$$= \frac{G(s)}{1 + G(s)}$$

$$= \frac{G(s)}{1 + G(s)}$$

$$= \frac{1}{{}^{1}/_{G(s)} + 1} \qquad G(s) = \frac{16}{s(s+4)}$$

$$= \frac{16}{s(s+4) + 16}$$

$$= \frac{16}{s^2 + 4s + 16} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n \times s + \omega_n^2}$$

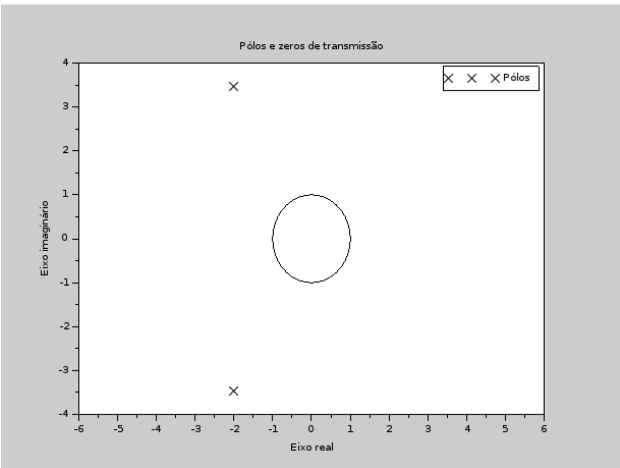
b. O sistema é estável, pois gera saídas limitadas para entradas limitadas. A estabilidade pode ser observada através da posição dos polos, que se encontram ao lado esquerdo do plano s, isto é, possui parte real negativa, o que sinaliza que a exponencial irá convergir ao valor de regime permanente.

c.

$$\omega_n^2 = 16 \Longrightarrow \omega_n = 4$$

$$2\xi\omega_n = 4 \quad \Longrightarrow \xi = \frac{4}{2\omega_n} = \frac{1}{2} = 0{,}5$$

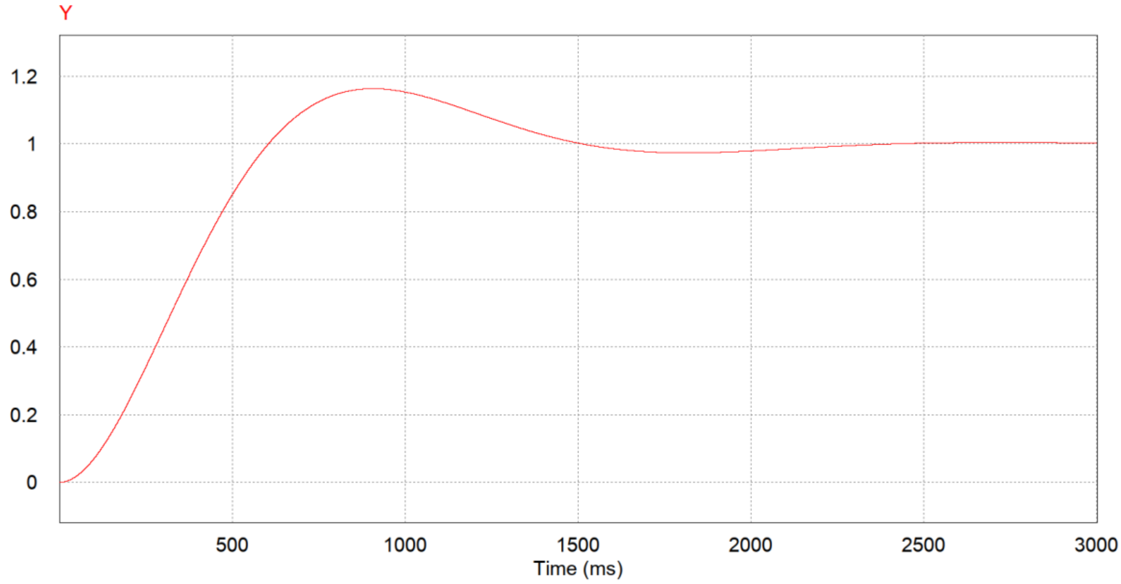
d.



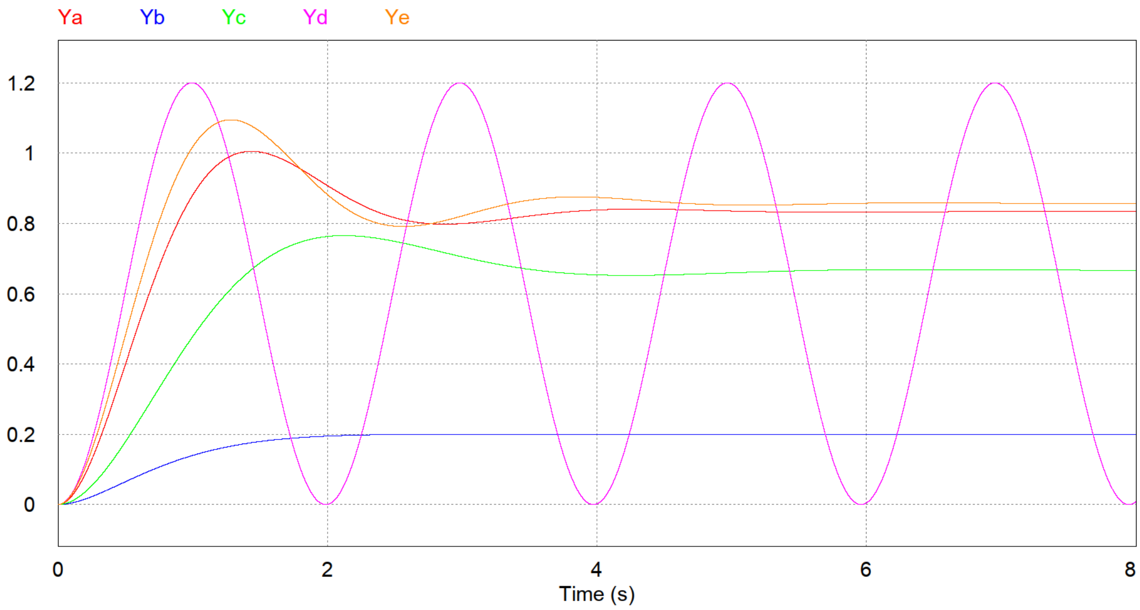
e. $0 < \xi < 1$ os polos são complexos conjugados e o sistema é dito subamortecido.

f.

$$t_{s,5\%} = \frac{3}{\xi\omega_n} = 1{,}5seg \quad t_{s,2\%} = \frac{4}{\xi\omega_n} = 2seg \quad M_P(\%) = e^{-\frac{\xi\times\pi}{\sqrt{1-\xi^2}}} \times 100 = 16{,}3\%$$



7.



a. $G(s) = \frac{5}{s^2+2{,}2s+1}$

b. $G(s) = \frac{1}{s^2+4s+4}$

c. $G(s) = \frac{2}{s^2+1{,}8s+1}$

d. $G(s) = \frac{6}{s^2+4}$

f. $G(s) = \frac{6}{s^2-2s+1}$

8.

$$G(s)=\frac{\omega_n^2}{s\times(s+2\xi\omega_n)+\omega_n^2}=\frac{\omega_n^2}{s^2+2\xi\omega_n\times s+\omega_n^2}$$

$$M_P(\%)\leq 16{,}3\%$$

$$e^{-\frac{\xi\times\pi}{\sqrt{1-\xi^2}}}\times 100\leq 16{,}3\%$$

$$-\frac{\xi\times\pi}{\sqrt{1-\xi^2}}\leq \ln 0{,}163$$

$$-\frac{\pi}{\ln 0{,}163}\leq\frac{\sqrt{1-\xi^2}}{\xi}$$

$$\left(-\frac{\pi}{\ln 0{,}163}\right)^2\leq\frac{1-\xi^2}{\xi^2}$$

$$\xi\leq\sqrt{\frac{1}{1+\left(-\frac{\pi}{\ln 0{,}163}\right)^2}}\leq 0{,}5$$

$$\xi\leq-\frac{\ln 0{,}163}{\sqrt{\pi^2+\ln^2 0{,}163}}\leq 0{,}5$$

$$t_{s,5\%}=\frac{3}{\xi\omega_n}\leq\ 3seg\Longrightarrow\omega_n\geq\frac{1}{\xi}\Longrightarrow\omega_n\geq 2$$

9.

$$M_p(\%) = \frac{Y_m - Y_f}{Y_f} \times 100 \qquad \xi = -\frac{\ln[M_p(\%) \times 100]}{\sqrt{\pi^2 + \ln^2[M_p(\%) \times 100]}} \qquad \omega_n = \frac{4}{t_{s,2\%} \times \xi}$$

a.

$$\begin{cases} Y_m = 145 \\ Y_f = 100 \\ t_{s,2\%} = 15ms \end{cases} \Rightarrow \begin{cases} M_p(\%) = 45\% \\ \xi = 0{,}246 \\ \omega_n = 1{,}08 \times 10^3 \end{cases}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n \times s + \omega_n^2} = \frac{1{,}17 \times 10^6}{s^2 + 533{,}3 \times s + 1{,}17 \times 10^6}$$

b.

$$\begin{cases} Y_m = 116{,}3 \\ Y_f = 100 \\ t_{s,2\%} = 16ms \end{cases} \Rightarrow \begin{cases} M_p(\%) = 16{,}3\% \\ \xi = 0{,}5 \\ \omega_n = 5{,}0 \times 10^2 \end{cases}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n \times s + \omega_n^2} = \frac{2{,}5 \times 10^5}{s^2 + 2{,}5 \times 10^5 \times s + 2{,}5 \times 10^5}$$

c.

$$\begin{cases} Y_m = 105 \\ Y_f = 100 \\ t_{s,2\%} = 20ms \end{cases} \Rightarrow \begin{cases} M_p(\%) = 5\% \\ \xi = 0{,}69 \\ \omega_n = 2{,}90 \times 10^2 \end{cases}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n \times s + \omega_n^2} = \frac{8{,}40 \times 10^4}{s^2 + 400 \times s + 8{,}40 \times 10^4}$$

d.

$$\begin{cases} Y_m = 100 \\ Y_f = 100 \\ t_{s,2\%} = 25ms \end{cases} \Rightarrow \begin{cases} M_p(\%) = 0 \\ \xi = 1 \\ \omega_n = 160 \end{cases}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n \times s + \omega_n^2} = \frac{2{,}5 \times 10^4}{s^2 + 320 \times s + 2{,}5 \times 10^4}$$

10.

a.

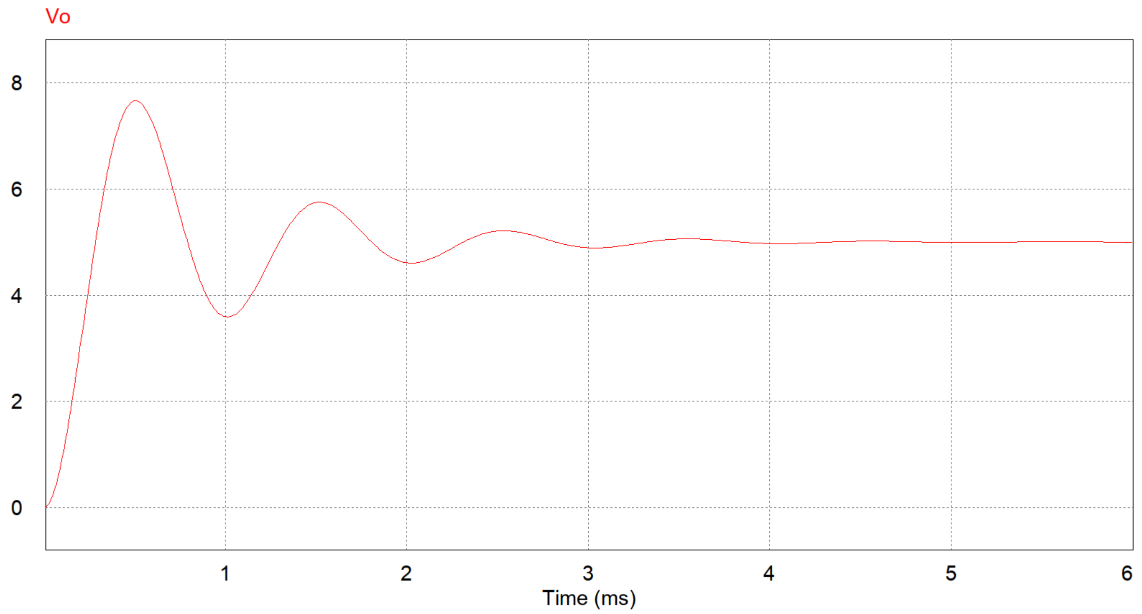
$$\frac{V_O(s)}{V_{in}(s)} = \frac{{}^{1}/_{LC}}{s^2 + {}^{1}/_{CR} \times s + {}^{1}/_{LC}} = \frac{4{,}0 \times 10^7}{s^2 + 2{,}5 \times 10^3 \times s + 4{,}0 \times 10^7} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n \times s + \omega_n^2}$$

$$\omega_n^2 = {}^{1}/_{LC} \Longrightarrow \omega_n = \sqrt{{}^{1}/_{LC}} = \sqrt{{}^{1}/_{(2{,}5 \times 10^{-4})(10^{-4})}} = 6{,}3 \times 10^3$$

$$2\xi\omega_n = {}^{1}/_{CR} \Longrightarrow \ \xi \ = \frac{1}{2CR\omega_n} = \frac{1}{2 \times (10^{-4}) \times 4 \times 6{,}3 \times 10^3} = 0{,}198$$

b. $0 < \xi < 1$ os polos são complexos conjugados e o sistema é dito subamortecido.

c.



11.

$$\begin{cases} \xi > 1\ (sem\ oscila\c{c}\tilde{a}o) \\ e(\infty) = 0 \\ t_{s,2\%} = 1ms \quad \Longrightarrow \omega_n = \dfrac{4}{t_{s,2\%} \times \xi} = \dfrac{4}{10^{-3} \times 1} = 4 \times 10^3 \end{cases}$$

$$\frac{Y(s)}{R(s)} = \frac{G_{PID}(s) \times G(s)}{1 + G_{PID}(s) \times G(s)}$$

Equação característica,

$$\Delta(s) = 1 + \frac{100}{(s + 2000) \times (s + 4000)} k_p + \frac{k_i}{s} + k_d \times s = 0$$

$$\Delta(s) = 1 + \frac{100 \times \left(k_d \times s^2 + k_p \times s + k_i\right)}{(s + 2000) \times (s + 4000) \times s} = 0$$

$$\Delta(s) = \frac{(s + 2000) \times (s + 4000) \times s + 100 \times \left(k_d \times s^2 + k_p \times s + k_i\right)}{(s + 2000) \times (s + 4000) \times s} = 0$$

$$\Delta(s) = (s^2 + 6000 \times s + 8000000) \times s + 100 \times k_d \times s^2 + 100 \times k_p \times s + 100 \times k_i = 0$$

$$\Delta(s) = s^3 + 6000 \times s^2 + 8000000 \times s + 100 \times k_d \times s^2 + 100 \times k_p \times s + 100 \times k_i = 0$$

$$\Delta(s) = s^3 + (6000 + 100 \times k_d) \times s^2 + \left(8000000 + 100 \times k_p\right) \times s + 100 \times k_i = 0$$

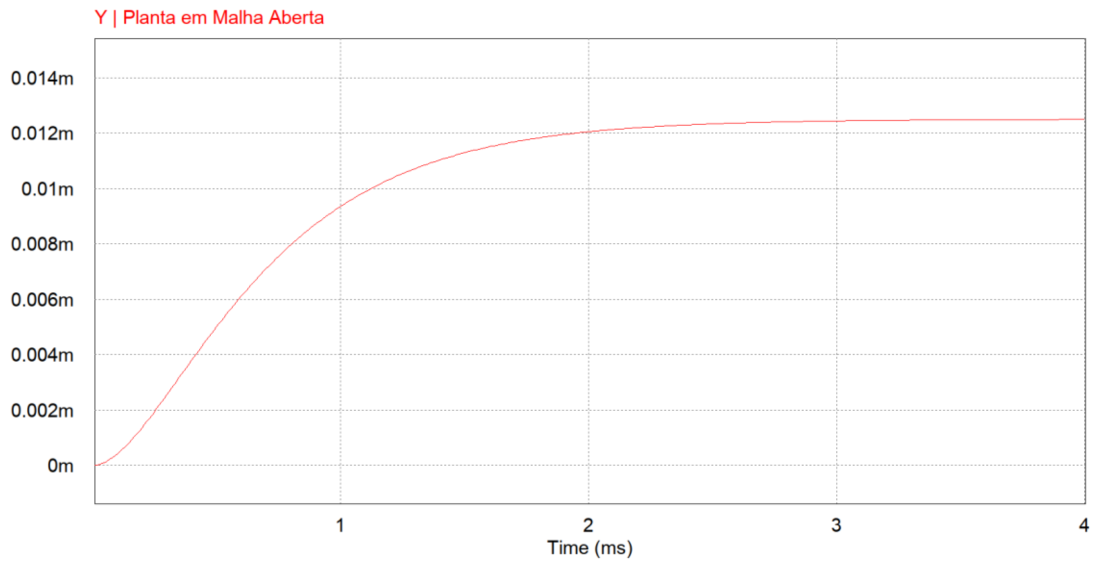
$$\Delta(s) = (s + \alpha\omega_n) \times (s^2 + 2\xi\omega_n \times s + {\omega_n}^2) = 0$$

Para dois polos dominantes: $\alpha\omega_n = 4 \times 10^3 \Longrightarrow \alpha = {}^{4 \times 10^3}/_{4 \times 10^3} = 1$

$$\Delta(s) = s^3 + (2\xi\omega_n + \omega_n) \times s^2 + ({\omega_n}^2 + 2\xi{\omega_n}^2) \times s + {\omega_n}^3 = 0$$

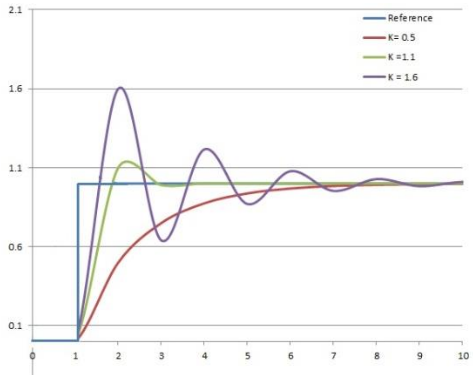
Os parâmetros do controlador PID são,

$$\begin{cases} k_p = {}^{{\omega_n}^2 + 2\xi{\omega_n}^2 - 8000000}/_{100} = 4 \times 10^5 \\ k_d = {}^{2\xi\omega_n + \omega_n - 6000}/_{100} = 60 \\ k_i = {}^{{\omega_n}^3}/_{100} = 6{,}4 \times 10^8 \end{cases}$$

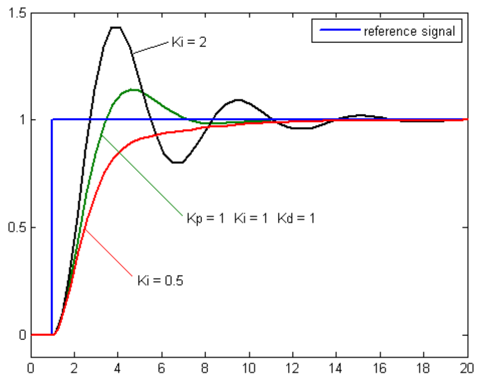

Y | Planta em Malha Aberta
0.014m
0.012m
0.01m
0.008m
0.006m
0.004m
0.002m
0m
1
2
3
4
Time (ms)

12.

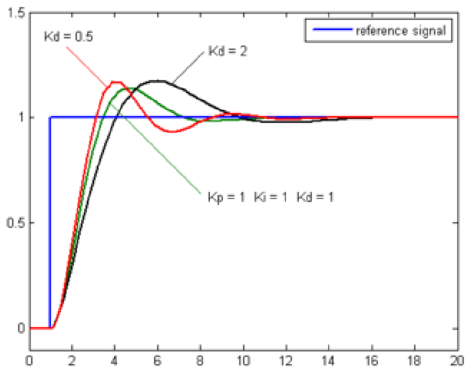
Atenuação da ação proporcional, reduzindo o ganho (k_p) entre o sinal de saída e à amplitude do erro $e(t)$ (tratamento para o overshoot).



Atenuação da ação integral, reduzindo o ganho (k_i) entre o sinal de saída e à magnitude e duração do erro $e(t)$, corrigindo o erro de offset gerado pela ação proporcional, acelerando a resposta do sistema, permitindo-o chegar ao valor de regime mais rapidamente (tratamento para o tempo de estabilização).



Manutenção da ação derivativa, para que o sistema permaneça a responder rapidamente a uma perturbação.



14.

$$M_p(\%) = \frac{Y_m - Y_f}{Y_f} \times 100 \qquad \xi = -\frac{\ln[M_p(\%) \times 100]}{\sqrt{\pi^2 + \ln^2[M_p(\%) \times 100]}} \qquad \omega_n = \frac{4}{t_{s,2\%} \times \xi}$$

$$\begin{cases} Y_m = 1{,}163 \\ Y_f = 1 \\ t_{s,2\%} = 8ms \end{cases} \Longrightarrow \begin{cases} M_p(\%) = 16{,}3\% \\ \xi = 0{,}5 \\ \omega_n = 10^3 \end{cases}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n \times s + \omega_n^2} = \frac{10^6}{s^2 + 10^3 \times s + 10^6}$$

$$\frac{Y(s)}{R(s)} = \frac{G_c(s) \times G(s)}{1 + G_c(s) \times G(s) \times H(s)} = \frac{1}{{}^{1}/_{G_c(s) \times G(s)} + H(s)}$$

$$Sem\ controlador: \quad G_c(s) = 1 \quad H(s) = 1 \quad G(s) = \frac{x}{s^2 + 2\xi\omega_n \times s + \omega_n^2}$$

$$\frac{Y(s)}{R(s)} = \frac{x}{s^2 + 2\xi\omega_n \times s + \omega_n^2 + x}$$

$$Controlador\ P: \quad G_c(s) = k_p \quad H(s) = 1 \quad G(s) = \frac{x}{s^2 + 2\xi\omega_n \times s + \omega_n^2}$$

$$\frac{Y(s)}{R(s)} = \frac{x \times k_p}{s^2 + 2\xi\omega_n \times s + \omega_n^2 + x \times k_p}$$