

1.

a. Função de Transferência

$$V_{in}(t) = V_R(t) + V_L(t)$$

$$V_{in}(t) = R \times i_L(t) + L \times \frac{di_L(t)}{dt}$$

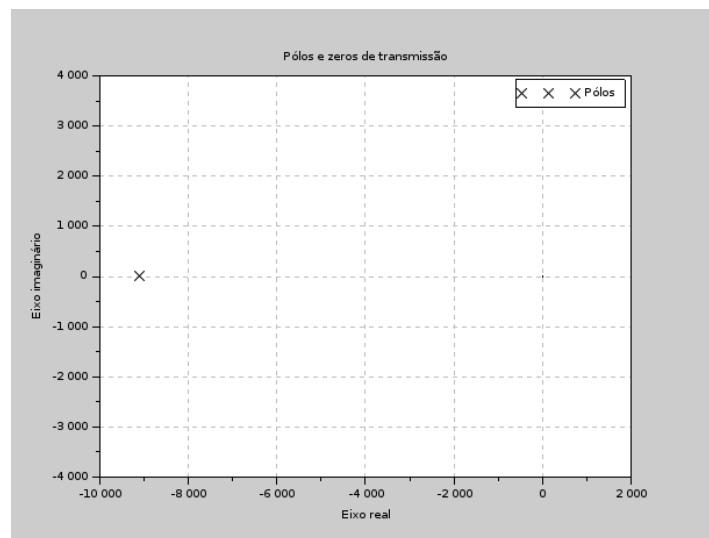
$$V_{in}(s) = R \times I_L(s) + L \times sI_L(s)$$

$$V_{in}(s) = (R + sL) \times I_L(s)$$

$$\frac{I_L(s)}{V_{in}(s)} = \frac{1}{R + sL} = \frac{1/L}{s + (R/L)} = \frac{1}{R} \times \left(\frac{R/L}{s + (R/L)} \right) = H(s)$$

b. Entrada pulso

$$I_L(s) = 1 \times H(s) = \frac{1/1,1 \times 10^{-3}}{s + \left(\frac{10}{1,1 \times 10^{-3}} \right)} = \frac{9,1 \times 10^2}{s + 9,1 \times 10^3}$$



O sistema é estável, pois gera saídas limitadas para entradas limitadas. A estabilidade pode ser observada através da posição dos polos, que se encontram ao lado esquerdo do plano s, isto é, possui parte real negativa, o que sinaliza que a exponencial irá convergir ao valor de regime permanente.

c. Características

$$ganho = k = 1/L = 9,1 \times 10^2$$

$$polo = p = R/L = \frac{10}{1,1 \times 10^{-3}} = 9,1 \times 10^3$$

$$constante\ de\ tempo = \tau = \frac{1}{p} = \frac{1,1 \times 10^{-3}}{10} = 1,1 \times 10^{-4}$$

d. Entrada degrau

$$I_L(s) = V_{in}(s) \times H(s)$$

$$I_L(s) = \left(V_{in} \times \frac{1}{s}\right) \times \frac{1}{R} \times \left(\frac{p}{s+p}\right)$$

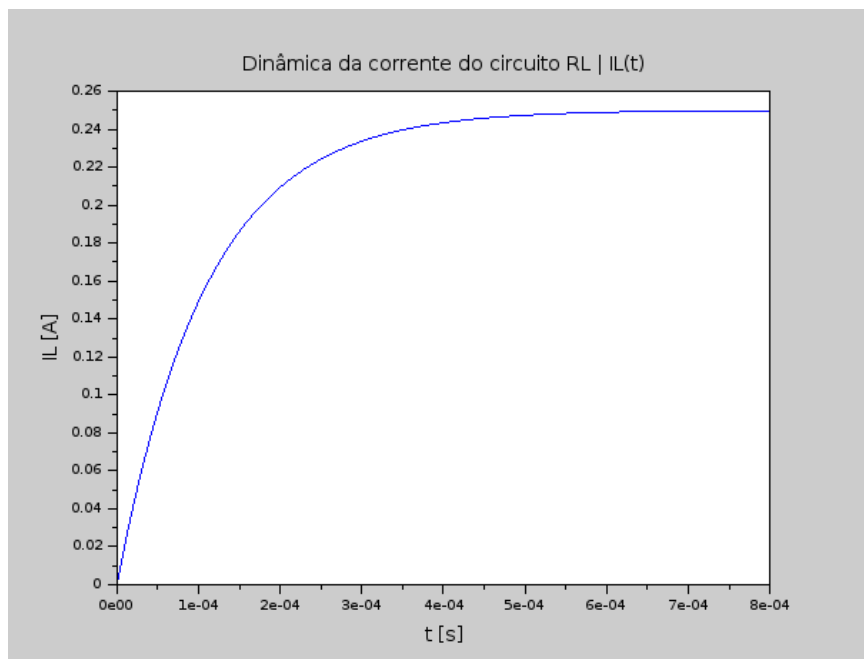
$$I_L(s) = \left(\frac{V_{in}}{R}\right) \times \left(\frac{1}{s} \times \frac{p}{s+p}\right)$$

$$I_L(t) = \left(\frac{V_{in}}{R}\right) \times (1 - e^{-p \times t})$$

$$I_L(t) = \left(\frac{V_{in}}{R}\right) \times (1 - e^{-t/\tau})$$

$$I_L(t) = \frac{2,5}{10} \times \left(1 - e^{-\frac{t}{1,1 \times 10^{-4}}}\right)$$

$$I_L(t) = 0,25 \times \left(1 - e^{-\frac{t}{1,1 \times 10^{-4}}}\right)$$



2.

constante de tempo

$$T'(t = 45\text{seg}) = 0,95 \times T' = T' \times (1 - e^{-45/\tau})$$

$$0,95 = 1 - e^{-45/\tau}$$

$$e^{-45/\tau} = 1 - 0,95$$

$$e^{-45/\tau} = 0,05$$

$$\ln(e^{-45/\tau}) = \ln 0,05$$

$$-45/\tau = \ln 0,05$$

$$\tau = -45/\ln 0,05$$

$$\tau = 15,02 \text{ seg}$$

polo

$$p = 1/\tau = 6,66 \times 10^{-2}$$

ganho

$$T = T' = k/p = k \times \tau$$

$$k = T'/\tau = T'/0,63 \times T' = 1,58$$

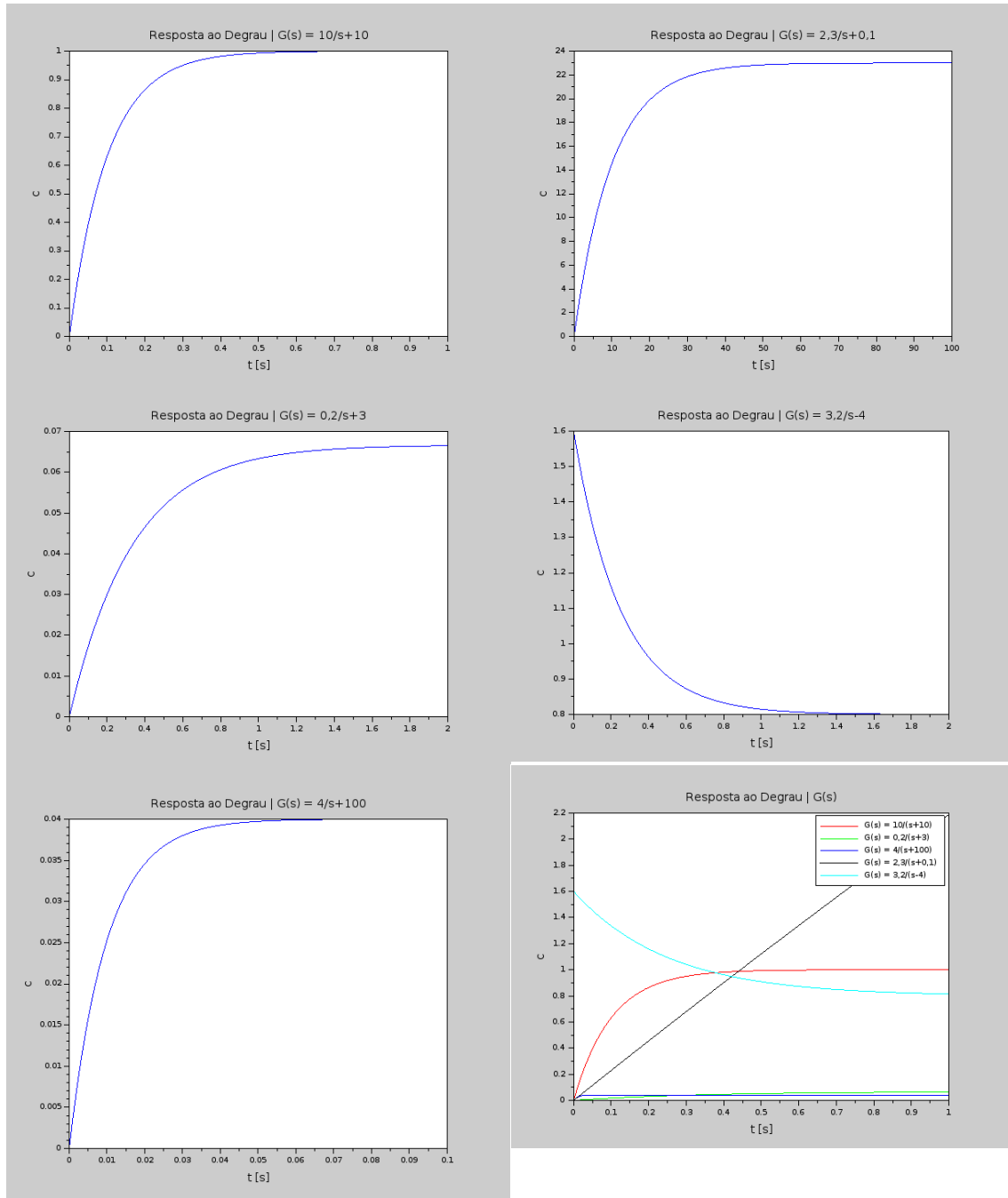
Função de transferência

$$H(s) = \frac{k}{s + p} = \frac{1,58}{s + 6,66 \times 10^{-2}}$$

3.

$$C(s) = R(s) \times G(s) = \left(\frac{1}{s}\right) \times \left(\frac{k}{s+p}\right) = \left(\frac{k}{p}\right) \times \left(\frac{1}{s}\right) \times \left(\frac{p}{s+p}\right)$$

$$C(t) = L^{-1}[C(s)] = R(t) * G(t) = \left(\frac{k}{p}\right) \times (1 - e^{-p \times t})$$



4.

$$Y(s) = R(s) \times G_{MF}(s)$$

$$\frac{Y(s)}{R(s)} = G_{MF}(s)$$

$$= \frac{G(s)}{1 + G(s) \times H(s)} \quad H(s) = 1$$

$$= \frac{G(s)}{1 + G(s)}$$

$$= \frac{G(s)}{1 + G(s)}$$

$$= \frac{1}{1/G(s) + 1} \quad G(s) = \frac{b}{s + a}$$

$$= \frac{b}{s + (a + b)} = \frac{k}{s + p}$$

$$R(s) = 1 \text{ (degrau uni tio)}$$

$$\begin{cases} t_{s,2\%} = 4/p = 1s \Rightarrow p = 4 \\ Y = k/p = 10 \Rightarrow k = 10 \times p = 40 \end{cases}$$

$$\begin{cases} k = b = 40 \\ p = a + b \Rightarrow a = p - b = -36 \end{cases}$$

$$G(s) = \frac{40}{s - 36} \quad G_{MF}(s) = -\frac{36}{s + 4}$$

5.

a.

$$I_L(t) = I_{in}(t) = I_C(t) + I_R(t)$$

$$\frac{V_{in}(s)}{Z_{in}} = \frac{V_C(s)}{Z_C} + \frac{V_R(s)}{R}$$

$$\frac{V_{in}(s)}{Z_{in}} = \frac{V_O(s)}{Z_C} + \frac{V_O(s)}{R}$$

$$\frac{V_{in}(s)}{Z_{in}} = \left(\frac{1}{Z_C} + \frac{1}{R} \right) \times V_O(s)$$

$$\frac{V_{in}(s)}{Z_{in}} = \left(\frac{R + Z_C}{Z_C \times R} \right) \times V_O(s)$$

$$\frac{V_O(s)}{V_{in}(s)} = \frac{1}{Z_{in}} \times \left(\frac{Z_C \times R}{R + Z_C} \right)$$

$$\frac{V_O(s)}{V_{in}(s)} = \frac{1}{Z_L + Z_{RC}} \times Z_{RC}$$

$$\frac{1}{Z_{RC}} = \frac{1}{Z_C} + \frac{1}{R} = \frac{R + Z_C}{Z_C \times R} \Rightarrow Z_{RC} = \frac{Z_C \times R}{R + Z_C}$$

$$\frac{V_O(s)}{V_{in}(s)} = \frac{1}{Z_L/Z_{RC} + 1}$$

$$\frac{V_O(s)}{V_{in}(s)} = \frac{1}{Z_L \times (R + Z_C)/Z_C \times R + 1}$$

$$\frac{V_O(s)}{V_{in}(s)} = \frac{1}{Z_L/R + Z_L/Z_C + 1}$$

$$\frac{V_O(s)}{V_{in}(s)} = \frac{1}{LC \times s^2 + L/R \times s + 1}$$

$$\frac{V_O(s)}{V_{in}(s)} = \frac{1/LC}{s^2 + 1/CR \times s + 1/LC}$$

b.

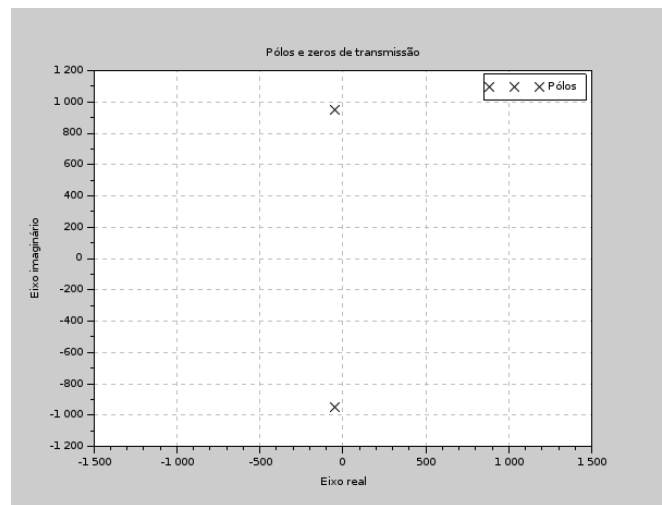
$$\frac{V_O(s)}{V_{in}(s)} = \frac{1/LC}{s^2 + 1/CR \times s + 1/LC} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n \times s + \omega_n^2}$$

$$\omega_n^2 = 1/LC \Rightarrow \omega_n = \sqrt{1/LC} = \sqrt{1/(1,1 \times 10^{-3})(10^{-3})} = 9,53 \times 10^2$$

$$2\xi\omega_n = 1/CR \Rightarrow \xi = \frac{1}{2CR\omega_n} = \frac{1}{2 \times (10^{-3}) \times 10 \times 9,53 \times 10^{-1}} = 5,24 \times 10^{-2}$$

c.

$$s_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1} = -50.0 \pm j$$

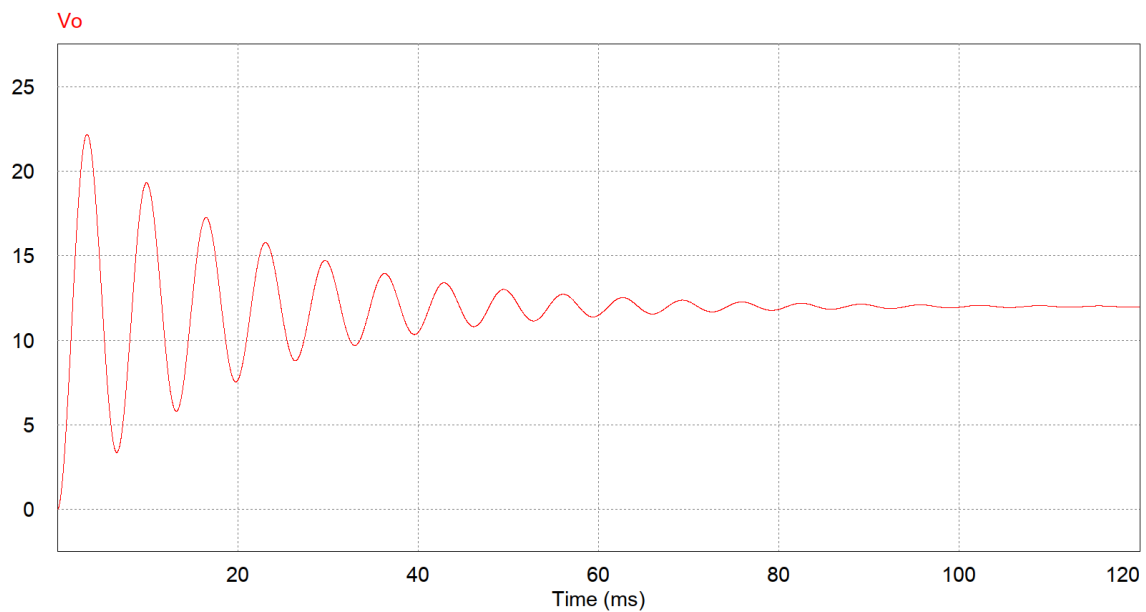


d. O sistema é estável, pois gera saídas limitadas para entradas limitadas. A estabilidade pode ser observada através da posição dos polos, que se encontram ao lado esquerdo do plano s, isto é, possui parte real negativa, o que sinaliza que a exponencial irá convergir ao valor de regime permanente.

e. $0 < \xi < 1$ os polos são complexos conjugados e o sistema é dito subamortecido.

f.

$$\frac{V_o(s)}{V_{in}(s)} = \frac{1/LC}{s^2 + 1/CR \times s + 1/LC} = \frac{9,09 \times 10^5}{s^2 + 100 \times s + 9,09 \times 10^5}$$



6.

a.

$$\begin{aligned}\frac{Y(s)}{R(s)} &= G_{MF}(s) \\ &= \frac{G(s)}{1 + G(s) \times H(s)} \quad H(s) = 1 \\ &= \frac{G(s)}{1 + G(s)} \\ &= \frac{G(s)}{1 + G(s)} \\ &= \frac{1}{1/G(s) + 1} \quad G(s) = \frac{16}{s(s+4)} \\ &= \frac{16}{s(s+4) + 16} \\ &= \frac{16}{s^2 + 4s + 16} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}\end{aligned}$$

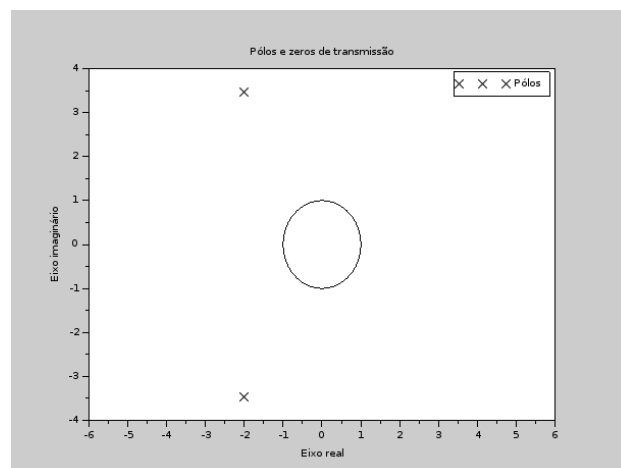
b. O sistema é estável, pois gera saídas limitadas para entradas limitadas. A estabilidade pode ser observada através da posição dos polos, que se encontram ao lado esquerdo do plano s , isto é, possui parte real negativa, o que sinaliza que a exponencial irá convergir ao valor de regime permanente.

c.

$$\omega_n^2 = 16 \Rightarrow \omega_n = 4$$

$$2\xi\omega_n = 4 \Rightarrow \xi = \frac{4}{2\omega_n} = \frac{1}{2} = 0,5$$

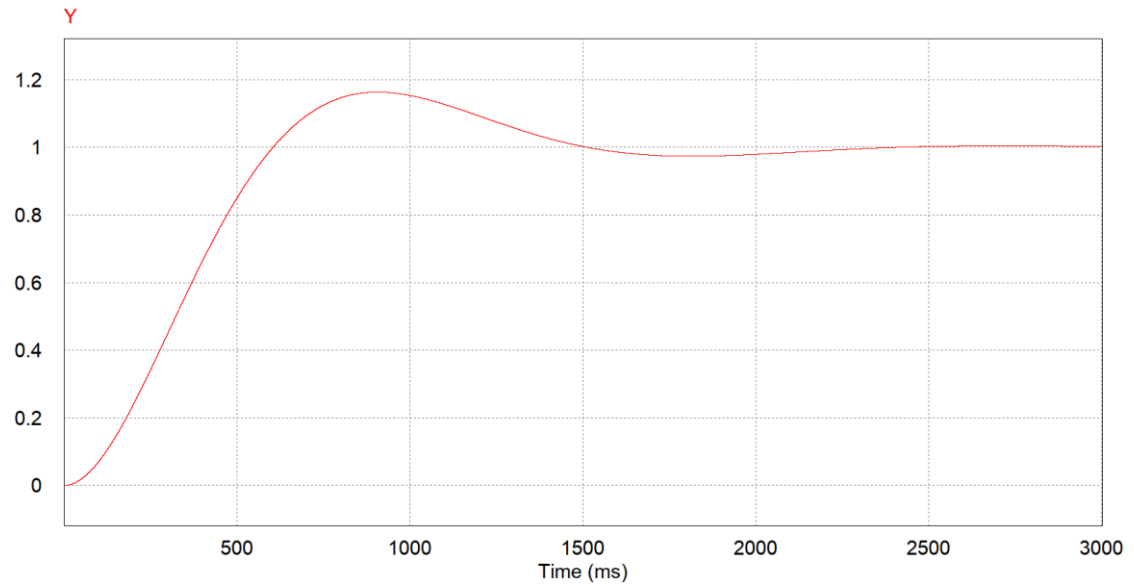
d.



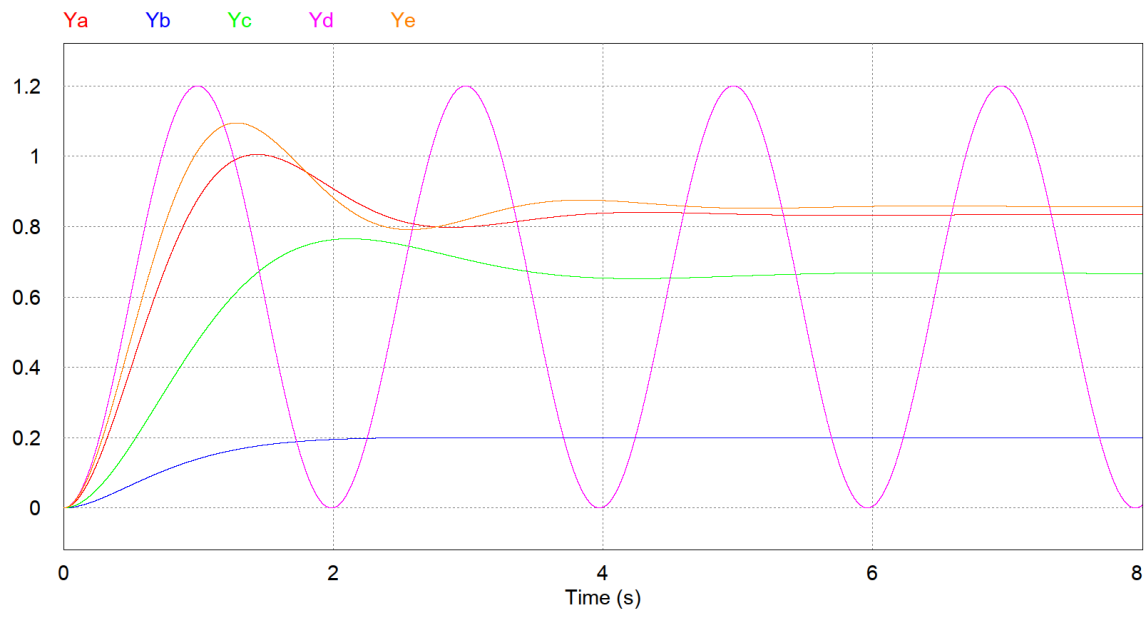
e. $0 < \xi < 1$ os polos são complexos conjugados e o sistema é dito subamortecido.

f.

$$t_{s,5\%} = \frac{3}{\xi\omega_n} = 1,5\text{seg} \quad t_{s,2\%} = \frac{4}{\xi\omega_n} = 2\text{seg} \quad M_p(\%) = e^{-\frac{\xi \times \pi}{\sqrt{1-\xi^2}}} \times 100 = 16,3\%$$



7.



a. $G(s) = \frac{5}{s^2 + 2,2s + 1}$

b. $G(s) = \frac{1}{s^2 + 4s + 4}$

c. $G(s) = \frac{2}{s^2 + 1,8s + 1}$

d. $G(s) = \frac{6}{s^2 + 4}$

f. $G(s) = \frac{6}{s^2 - 2s + 1}$

8.

$$G(s) = \frac{\omega_n^2}{s \times (s + 2\xi\omega_n) + \omega_n^2} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n \times s + \omega_n^2}$$

$$M_P(\%) \leq 16,3\%$$

$$e^{-\frac{\xi \times \pi}{\sqrt{1-\xi^2}}} \times 100 \leq 16,3\%$$

$$-\frac{\xi \times \pi}{\sqrt{1-\xi^2}} \leq \ln 0,163$$

$$-\frac{\pi}{\ln 0,163} \leq \frac{\sqrt{1-\xi^2}}{\xi}$$

$$\left(-\frac{\pi}{\ln 0,163}\right)^2 \leq \frac{1-\xi^2}{\xi^2}$$

$$\xi \leq \sqrt{\frac{1}{1 + \left(-\frac{\pi}{\ln 0,163}\right)^2}} \leq 0,5$$

$$\xi \leq -\frac{\ln 0,163}{\sqrt{\pi^2 + \ln^2 0,163}} \leq 0,5$$

$$t_{s,5\%} = \frac{3}{\xi\omega_n} \leq 3\text{seg} \Rightarrow \omega_n \geq \frac{1}{\xi} \Rightarrow \omega_n \geq 2$$

9.

$$M_p(\%) = \frac{Y_m - Y_f}{Y_f} \times 100$$

$$\xi = -\frac{\ln[M_p(\%) \times 100]}{\sqrt{\pi^2 + \ln^2[M_p(\%) \times 100]}}$$

$$\omega_n = \frac{4}{t_{s,2\%} \times \xi}$$

a.

$$\begin{cases} Y_m = 145 \\ Y_f = 100 \\ t_{s,2\%} = 15ms \end{cases} \Rightarrow \begin{cases} M_p(\%) = 45\% \\ \xi = 0,246 \\ \omega_n = 1,08 \times 10^3 \end{cases}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n \times s + \omega_n^2} = \frac{1,17 \times 10^6}{s^2 + 533,3 \times s + 1,17 \times 10^6}$$

b.

$$\begin{cases} Y_m = 116,3 \\ Y_f = 100 \\ t_{s,2\%} = 16ms \end{cases} \Rightarrow \begin{cases} M_p(\%) = 16,3\% \\ \xi = 0,5 \\ \omega_n = 5,0 \times 10^2 \end{cases}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n \times s + \omega_n^2} = \frac{2,5 \times 10^5}{s^2 + 2,5 \times 10^5 \times s + 2,5 \times 10^5}$$

c.

$$\begin{cases} Y_m = 105 \\ Y_f = 100 \\ t_{s,2\%} = 20ms \end{cases} \Rightarrow \begin{cases} M_p(\%) = 5\% \\ \xi = 0,69 \\ \omega_n = 2,90 \times 10^2 \end{cases}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n \times s + \omega_n^2} = \frac{8,40 \times 10^4}{s^2 + 400 \times s + 8,40 \times 10^4}$$

d.

$$\begin{cases} Y_m = 100 \\ Y_f = 100 \\ t_{s,2\%} = 25ms \end{cases} \Rightarrow \begin{cases} M_p(\%) = 0 \\ \xi = 1 \\ \omega_n = 160 \end{cases}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n \times s + \omega_n^2} = \frac{2,5 \times 10^4}{s^2 + 320 \times s + 2,5 \times 10^4}$$

10.

a.

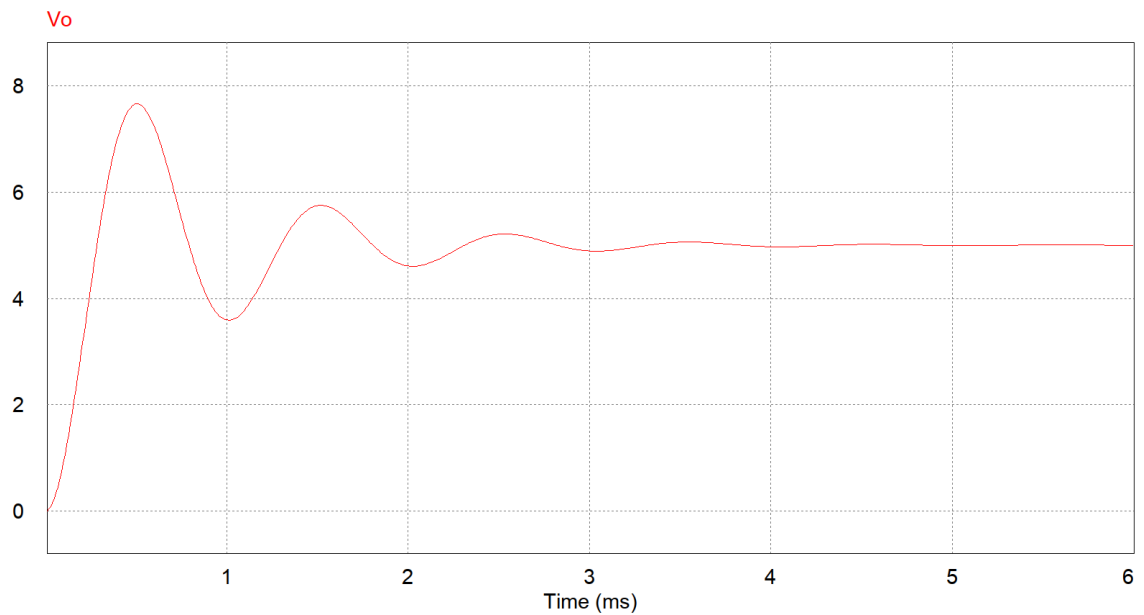
$$\frac{V_o(s)}{V_{in}(s)} = \frac{1/LC}{s^2 + 1/CR \times s + 1/LC} = \frac{4,0 \times 10^7}{s^2 + 2,5 \times 10^3 \times s + 4,0 \times 10^7} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n \times s + \omega_n^2}$$

$$\omega_n^2 = 1/LC \Rightarrow \omega_n = \sqrt{1/LC} = \sqrt{1/(2,5 \times 10^{-4})(10^{-4})} = 6,3 \times 10^3$$

$$2\xi\omega_n = 1/CR \Rightarrow \xi = \frac{1}{2CR\omega_n} = \frac{1}{2 \times (10^{-4}) \times 4 \times 6,3 \times 10^3} = 0,198$$

b. $0 < \xi < 1$ os polos são complexos conjugados e o sistema é dito subamortecido.

c.



11.

$$\begin{cases} \xi > 1 \text{ (sem oscilação)} \\ e(\infty) = 0 \\ t_{s,2\%} = 1ms \Rightarrow \omega_n = \frac{4}{t_{s,2\%} \times \xi} = \frac{4}{10^{-3} \times 1} = 4 \times 10^3 \end{cases}$$

$$\frac{Y(s)}{R(s)} = \frac{G_{PID}(s) \times G(s)}{1 + G_{PID}(s) \times G(s)}$$

Equação característica,

$$\Delta(s) = 1 + \frac{100}{(s + 2000) \times (s + 4000)} k_p + \frac{k_i}{s} + k_d \times s = 0$$

$$\Delta(s) = 1 + \frac{100 \times (k_d \times s^2 + k_p \times s + k_i)}{(s + 2000) \times (s + 4000) \times s} = 0$$

$$\Delta(s) = \frac{(s + 2000) \times (s + 4000) \times s + 100 \times (k_d \times s^2 + k_p \times s + k_i)}{(s + 2000) \times (s + 4000) \times s} = 0$$

$$\Delta(s) = (s^2 + 6000 \times s + 8000000) \times s + 100 \times k_d \times s^2 + 100 \times k_p \times s + 100 \times k_i = 0$$

$$\Delta(s) = s^3 + 6000 \times s^2 + 8000000 \times s + 100 \times k_d \times s^2 + 100 \times k_p \times s + 100 \times k_i = 0$$

$$\Delta(s) = s^3 + (6000 + 100 \times k_d) \times s^2 + (8000000 + 100 \times k_p) \times s + 100 \times k_i = 0$$

$$\Delta(s) = (s + \alpha\omega_n) \times (s^2 + 2\xi\omega_n \times s + \omega_n^2) = 0$$

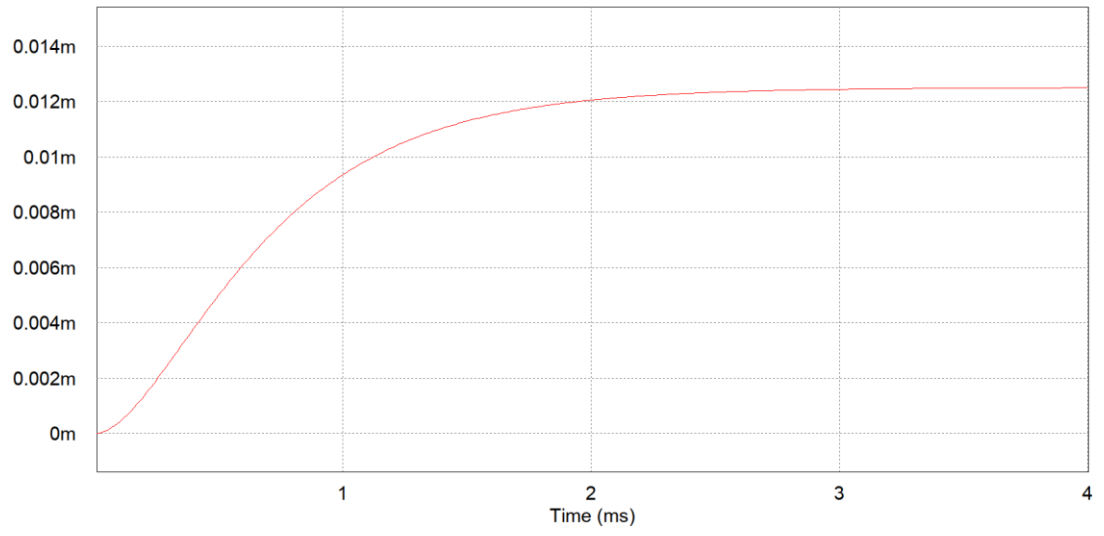
$$\text{Para dois polos dominantes: } \alpha\omega_n = 4 \times 10^3 \Rightarrow \alpha = 4 \times 10^3 / 4 \times 10^3 = 1$$

$$\Delta(s) = s^3 + (2\xi\omega_n + \omega_n) \times s^2 + (\omega_n^2 + 2\xi\omega_n^2) \times s + \omega_n^3 = 0$$

Os parâmetros do controlador PID são,

$$\begin{cases} k_p = \omega_n^2 + 2\xi\omega_n^2 - 8000000 / 100 = 4 \times 10^5 \\ k_d = 2\xi\omega_n + \omega_n - 6000 / 100 = 60 \\ k_i = \omega_n^3 / 100 = 6,4 \times 10^8 \end{cases}$$

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14.

$$M_p(\%) = \frac{Y_m - Y_f}{Y_f} \times 100 \quad \xi = -\frac{\ln[M_p(\%) \times 100]}{\sqrt{\pi^2 + \ln^2[M_p(\%) \times 100]}} \quad \omega_n = \frac{4}{t_{s,2\%} \times \xi}$$

$$\begin{cases} Y_m = 1,163 \\ Y_f = 1 \\ t_{s,2\%} = 8ms \end{cases} \Rightarrow \begin{cases} M_p(\%) = 16,3\% \\ \xi = 0,5 \\ \omega_n = 10^3 \end{cases}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n \times s + \omega_n^2} = \frac{10^6}{s^2 + 10^3 \times s + 10^6}$$

$$\frac{Y(s)}{R(s)} = \frac{G_c(s) \times G(s)}{1 + G_c(s) \times G(s) \times H(s)} = \frac{1}{1/G_c(s) \times G(s) + H(s)}$$

Sem controlador: $G_c(s) = 1$ $H(s) = 1$ $G(s) = \frac{x}{s^2 + 2\xi\omega_n \times s + \omega_n^2}$

$$\frac{Y(s)}{R(s)} = \frac{x}{s^2 + 2\xi\omega_n \times s + \omega_n^2 + x}$$

Controlador P: $G_c(s) = k_p$ $H(s) = 1$ $G(s) = \frac{x}{s^2 + 2\xi\omega_n \times s + \omega_n^2}$

$$\frac{Y(s)}{R(s)} = \frac{x \times k_p}{s^2 + 2\xi\omega_n \times s + \omega_n^2 + x \times k_p}$$