MATH 131: Numerical Methods for scientists and engineers – Discussion 2: Paper

The goals of this discussion section are:

- Get a deeper understanding of Taylor's theorem.

- Get a deeper understanding of root-finding algorithms.

1. Consider the function $f(x) = 2x\cos(2x) - (x-2)^2$ and $x_0 = 0$.

   (a) Find the third Taylor polynomial of f and use it to approximate $f(0.4)$.
   (b) Give the remainder error term.
   (c) Using the second Taylor polynomial at $h = 0$, show that $2h\cos(2h) - (h-2)^2 + 2h + h^2 = 4 + O(h^3)$.

2. Consider the function $f(x) = \sqrt{x} - \cos(x)$ for $x \in [0, 1]$.

   (a) Using the bisection method, provide x_0, x_1, x_2 and evaluate $f(x_0)$, $f(x_1)$, $f(x_2)$.
   (b) Sketch the problem and place x_0, x_1, x_2, $f(x_0)$, $f(x_1)$, $f(x_2)$.
   (c) What is the expected rate of convergence ?

3. Consider the function $f(x) = \cos(2x) - x$ for $x \in [0, 1]$.

   (a) Rewrite the problem $f(x) = 0$ into a fixed-point problem. Define the associated function g.
   (b) Explain how to use the fixed-point iteration method to solve $x = g(x)$ for $x \in [0, 1]$. State your initial guess, and provide x_1 and x_2.
   (c) Sketch the problem and place x_0, x_1, x_2.
   (d) Find a bound for $|g'(x)|$ for $x \in [0, 1]$.
   (e) If the method converging ? Justify your answer.

4. Consider the function $f(x) = x^2 - 6$.

   (a) Find the zeros of f.
   (b) Considering $x_0 = 3$, use Newton's method to find x_1 and x_2.
   (c) Considering $x_0 = 3$ and $x_1 = 2$, use the Secant method to find x_2.
   (d) Which one between the 2 previous methods is closer to the exact solution ?
   (e) How would you guarantee convergence of Newton's method ?