MATH 131: Numerical Methods for scientists and engineers
Midterm 3–VERSION A–Fall 2017
November 20, 2017
Instructions

Read the following instructions carefully:

- Write your name on the front page of your green/blue book.
- Write the VERSION (VERSION A or VERSION B) of your exam on the front page of your green/blue book.
- Make a grading table on the back of the first cover of your green/blue book.
- Write each problem on a separate page. Please make sure you clearly mark the problem you are working on (e.g., 1a)).
- No calculator or electronic devices allowed.
- It is important to show your work for each problem. Write full sentences. Credit will NOT be given for correct answers without justification. Also, partial credit will be given for incorrect answers if some of the work is correct.
- Clearly mark out (cross out) any work that you are not including in your answer and you do not want graded.
- Good luck!

1. (20 pts, 4 each) Consider the Initial Value Problem (IVP)

$$y'(t) = -20y, \quad y(0) = 5, \quad t \in [0, 2]$$

   (a) What do you need to ensure well-posedness of the IVP ?
   (b) Find a Lipschitz constant, L, for this IVP.
   (c) Solve this IVP.
   (d) Is the equation stiff ? Justify (no more than two lines) your answer.
   (e) Given a general N and $\Delta t = \frac{2}{N}$, write Euler's method to approximate this IVP.

2. (20 pts, 4 each)

   (a) Give the local truncation error $\tau_j(\Delta t)$ for fourth order Runge-Kutta method.
   (b) Define the stability of a numerical method.
   (c) Define the consistency of a numerical method.
   (d) Define the convergence of a numerical method.
   (e) Given a general Δt, give a 3-step explicit method (you can even create one).

3. (12 pts, 4 each) Consider the differential equation

$$y^{(3)}(t) = e^t + y, \quad y(0) = 5, \quad t \in [0, 2]$$

   (a) Rewrite this problem into a system of first-order ODEs.
   (b) Given a general N and $\Delta t = \frac{2}{N}$, write Euler's method to approximate the obtained system.
   (c) Given a general N and $\Delta t = \frac{2}{N}$, write the second order Taylor's method to approximate the obtained system.

WRITE YOUR NAME AND DON'T FORGET TO PUT THIS PAGE BACK IN YOUR EXAM AT THE END

$Name$: –

1. (28 pts, 4 each) Identify all the following numerical methods:

   (a) $w_{j+1} = w_j + \Delta t f(t_j, w_j)$

   (b) $w_{j+1} = w_j + \dfrac{\Delta t}{2}\left(f(t_j, w_j) + f(t_{j+1}, w_j + \Delta t f(t_j, w_j))\right)$

   (c) $w_{j+1} = w_j + \Delta t\, f\left(t_j + \dfrac{\Delta t}{2}, w_j + \dfrac{\Delta t}{2} f(t_j, w_j)\right)$

   (d) $w_{j+1} = w_j + \dfrac{\Delta t}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

   $$k_1 = f(t_j, w_j)$$
   $$k_2 = f\left(t_j + \frac{\Delta t}{2}, w_j + \frac{\Delta t}{2} k_1\right)$$
   $$k_3 = f\left(t_j + \frac{\Delta t}{2}, w_j + \frac{\Delta t}{2} k_2\right)$$
   $$k_4 = f(t_{j+1}, w_j + \Delta t k_3)$$

   (e) $w_{j+1} = w_j + \dfrac{\Delta t}{2}\left(3f(t_j, w_j) - f(t_{j-1}, w_{j-1})\right)$

   (f) $w_{j+1} = w_j + \Delta t\left(f(t_j, w_j) + \dfrac{\Delta t}{2} f'(t_j, w_j)\right)$

   (g) $w_{j+1} = w_j + \dfrac{h}{12}\left(5f(t_{j+1}, w_{j+1}) + 8f(t_j, w_j) - f(t_{j-1}, w_{j-1})\right)$

   i. Euler's method:——————-
   ii. 2-step Adams-Moulton method:——————-
   iii. second order Taylor's method:——————-
   iv. fourth Runge-Kutta method:——————-
   v. Modified Euler's method:——————-
   vi. second Runge-Kutta method:——————-
   vii. 2-step Adams-Bashforth method:——————-

2. (20 pts, 4 each) Place the following numerical methods in the table below (several can belong to the same category).

   (a) Euler's method
   (b) Taylor's method
   (c) Runge-Kutta of order n
   (d) Adams-Bashforth methods
   (e) Adams-Moulton methods

	1-step method	multi-step method
Explicit method	____ ____ ____ ____	____ ____ ____ ____
Implicit method	____ ____ ____ ____	____ ____ ____ ____