MATH 131: Numerical Methods for scientists and engineers
Midterm 2–VERSION A–Fall 2017
October 23, 2017
Instructions

Read the following instructions carefully:

- Write your name on the front page of your green/blue book.
- Write the VERSION (VERSION A or VERSION B) of your exam on the front page of your green/blue book.
- Make a grading table on the back of the first cover of your green/blue book.
- Write each problem on a separate page. Please make sure you clearly mark the problem you are working on (e.g., 1a)).
- No calculator or electronic devices allowed.
- It is important to show your work for each problem. Write full sentences. Credit will NOT be given for correct answers without justification. Also, partial credit will be given for incorrect answers if some of the work is correct.
- Clearly mark out (cross out) any work that you are not including in your answer and you do not want graded.
- Good luck!

WRITE YOUR NAME AND DON'T FORGET TO PUT THIS PAGE BACK IN YOUR EXAM AT THE END

Name : –

1. (20 pts, 10 each)
   (a) Given $x_0 = 2$, $x_1 = 3$, $f(x_0) = 2$, $f(x_1) = 3$, construct the linear Lagrange Polynomial interpolant of f that passes through the points $(x_0, f(x_0))$, $(x_1, f(x_1))$. To get full credit, you must give the Lagrange polynomials you need.
   (b) Use Newton's divided difference formula to construct an interpolating polynomial of degree two of f for the following data: $(x_0, f(x_0)) = (-1, 1)$, $(x_1, f(x_1)) = (0, 0.5)$, $(x_2, f(x_2)) = (1, 3)$.
2. (20 pts, 10 each)
   (a) Given a function f defined on $[a, b]$ and a set of nodes $a = x_0 < x_1, \cdots < x_n = b$. Define a cubic spline interpolant $S(x)$ of $f(x)$ (*Hint: List the conditions* $s(x)$ *has to satisfy.)*
   (b) Construct the natural cubic spline interpolant for the following data:

x	f(x)
0	1
0.5	2.72

3. (20 pts, 5, 15)

   (a) What is the order of the error bound while using an $(n+1)$-point formula approximating $f'(x_0)$?

   (b) Use the most accurate 3 points formula to determine each missing entry in the table below:

x	f(x)	f'(x)
7.1	1	
7.2	3	
7.3	2	

   We recall some formulas

$$f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h} - \frac{h}{2} f''(\xi)$$

$$f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h} - \frac{h^2}{6} f'''(\xi)$$

$$f'(x_0) = \frac{3f(x_0) - 4f(x_0-h) + f(x_0-2h)}{2h} + \frac{h^2}{3} f'''(\xi)$$

$$f'(x_0) = \frac{-3f(x_0) + 4f(x_0+h) - f(x_0+2h)}{2h} + \frac{h^2}{3} f'''(\xi)$$

4. (20 pts, 5 each)

   (a) Give the general expression of the Lagrange polynomials $L_{n,k}(x)$.

   (b) What is the name of the polynomial interpolant $P(x)$ of f such that $P(x_i) = f(x_i)$, $i = 0, \ldots, n$ and $P'(x_i) = f'(x_i)$, $i = 0, \ldots, n$?

   (c) What kind of polynomials are used for Gaussian quadratures ?

   (d) Explain adaptive quadrature methods in no more than two lines.

5. (20 pts, 16, 4) Below is a MATLAB implementation of the trapezoid rule with some code missing.

   (a) Fill the blanks below to correctly execute trapezoid rule. READ THE CODE CAREFULLY!

```
function [I] = trapezoid_rule(f,a,b,N)

% function to approximate $\int_a^b f(x) dx$ using the Trapezoid rule
% INPUTS:
% f is the function at hand
% a is the lower bound of the interval
% b is the upper bound of the interval
% N is the number of panels used
% OUTPUTS:
% I is the approximate integral

    I = 0;
    h = ___________________________________;

    for j = 0: N-1
        x_j   = ___________________________________;
        x_jp1 = ___________________________________;
        I     = I + _______________________________;
    end

end
```

   (b) Explain the differences between the Trapezoid rule and the Simpson's rule (number of points needed, order of the error bound, etc.).