MATH 131: Numerical Methods for scientists and engineers – Discussion 8: Paper

The goals of this discussion section are:

- Get a deeper understanding of numerical methods for IVPs.

1. (This exercise has been given on a midterm)
   Match the numerical methods (Euler's method, 2-step Adams-Moulton method, second order Taylor's method, fourth Runge-Kutta method, Modified Euler's method, second Runge-Kutta method, 2-step Adams-Bashforth method) to their expression:

   (a) $w_{j+1} = w_j + \Delta t f(t_j, w_j)$

   (b) $w_{j+1} = w_j + \dfrac{\Delta t}{2} \left(f(t_j, w_j) + f(t_{j+1}, w_j + \Delta t f(t_j, w_j)) \right)$

   (c) $w_{j+1} = w_j + \Delta t\, f\left(t_j + \dfrac{\Delta t}{2}, w_j + \dfrac{\Delta t}{2} f(t_j, w_j) \right)$

   (d) $w_{j+1} = w_j + \dfrac{\Delta t}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

   $$k_1 = f(t_j, w_j)$$
   $$k_2 = f\left(t_j + \frac{\Delta t}{2}, w_j + \frac{\Delta t}{2} k_1 \right)$$
   $$k_3 = f\left(t_j + \frac{\Delta t}{2}, w_j + \frac{\Delta t}{2} k_2 \right)$$
   $$k_4 = f(t_{j+1}, w_j + \Delta t k_3)$$

   (e) $w_{j+1} = w_j + \dfrac{\Delta t}{2} \left(3f(t_j, w_j) - f(t_{j-1}, w_{j-1}) \right)$

   (f) $w_{j+1} = w_j + \Delta t \left(f(t_j, w_j) + \dfrac{\Delta t}{2} f'(t_j, w_j) \right)$

   (g) $w_{j+1} = w_j + \dfrac{h}{12} \left(5f(t_{j+1}, w_{j+1}) + 8f(t_j, w_j) - f(t_{j-1}, w_{j-1}) \right)$

   Place those numerical methods in the table below (several can belong to the same category).

	1-step method	multi-step method
Explicit method		
Implicit method		

2. Similar to what has been done in class for the 2-step Adams-Bashforth method, derive the 2-step Adams-Moulton method.