Topics For Midterm

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Section 3.1: You should be able to:

- Construct Lagrange polynomial interpolants using $Eq.3.1$ in the book.
  Example problems § 3.1 #1, 2, 5, 6

$$P(x) = f(x_0)L_{n,0}(x) + \cdots + f(x_n)L_{n,n}(x) = \sum_{k=0}^{n} f(x_k)L_{n,k}(x) = \Pi_{i=0,i\neq k}^{n} \frac{(x-x_i)}{(x_k-x_i)} (Eq3.1)$$

1 For the given functions $f(x)$, let $x_0 = 0$, $x_1 = .6$, and $x_2 = .9$. Construct interpolation polynomials of degree at most one and at most two to approximate $f(.45)$ and find the absolute error.

i. $f(x) = \cos(x)$.

Degree 1 We want to construct an interpolation polynomial of degree one, that is $n = 1$, therefore we need $n+1$ points.
Consider the following. For second degree, we need 2 nodal points that is x_0, x_1. Not including x_2

$$y_0 = f(x_0), y_1 = f(x_1)$$

$$L_{1,0} = \frac{(x-x_1)}{(x_0-x_1)}, L_{1,1} = \frac{(x-x_0)}{(x_1-x_0)},$$

$$P_1(x) = y_0(L_{1,0}) + y_1(L_{1,1})$$

Degree 2 We want to construct an interpolation polynomial of degree two, that is $n = 2$, therefore we need $n+1$ points.
Consider the following. For second degree, we need 3 nodal points that is x_0, x_1, x_2

$$y_0 = f(x_0), y_1 = f(x_1), y_2 = f(x_2)$$

$$L_{2,0} = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}, L_{2,1} = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}, L_{2,2} = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$P_2(x) = y_0(L_{2,0}) + y_1(L_{2,1}) + y_2(L_{2,2})$$

Absolute Error We want to find the absolute error given $f(.45)$, we have previously found P_1 and P_2. First we evalute the value of $x = .45$ at out interpolant polynomials.

$$P_1(.45), P_2(.45)$$

Absolute is as follows

$$AbsErr1 = |P_1(.45) - f(.45)|$$

$$AbsErr2 = |P_2(.45) - f(.45)|$$

ii. $f(x) = \sqrt{1+x}$

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$$y_0 = f(x_0), y_1 = f(x_1)$$

$$L_{1,0} = \frac{(x-x_1)}{(x_0-x_1)}, L_{1,1} = \frac{(x-x_0)}{(x_1-x_0)},$$

$$P_1(x) = y_0(L_{1,0}) + y_1(L_{1,1})$$

Degree 2 We want to construct an interpolation polynomial of degree two, that is $n = 2$, therefore we need $n+1$ points.
Consider the following. For second degree, we need 3 nodal points that is x_0, x_1, x_2

$$y_0 = f(x_0), y_1 = f(x_1), y_2 = f(x_2)$$

$$L_{2,0} = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}, L_{2,1} = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}, L_{2,2} = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$P_2(x) = y_0(L_{2,0}) + y_1(L_{2,1}) + y_2(L_{2,2})$$

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$$P_1(.45), P_2(.45)$$

Absolute is as follows

$$AbsErr1 = |P_1(.45) - f(.45)|$$

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iii. $f(x) = \ln(x+1)$

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$$P_1(x) = y_0(L_{1,0}) + y_1(L_{1,1})$$

Degree 2 We want to construct an interpolation polynomial of degree two, that is $n = 2$, therefore we need $n+1$ points.
Consider the following. For second degree, we need 3 nodal points that is x_0, x_1, x_2

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$$P_2(x) = y_0(L_{2,0}) + y_1(L_{2,1}) + y_2(L_{2,2})$$

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$$P_1(.45), P_2(.45)$$

Absolute is as follows

$$AbsErr1 = |P_1(.45) - f(.45)|$$

$$AbsErr2 = |P_2(.45) - f(.45)|$$

iv. $f(x) = \tan(x)$

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Consider the following. For second degree, we need 2 nodal points that is x_0, x_1. Not including x_2

$$y_0 = f(x_0), y_1 = f(x_1)$$

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$$y_0 = f(x_0), y_1 = f(x_1), y_2 = f(x_2)$$

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$$P_2(x) = y_0(L_{2,0}) + y_1(L_{2,1}) + y_2(L_{2,2})$$

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$$P_1(.45), P_2(.45)$$

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$$AbsErr1 = |P_1(.45) - f(.45)|$$

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