MATH 131: Numerical Methods for scientists and engineers – Discussion 4: Paper

The goals of this discussion section are:

- Get a deeper understanding of Lagrange interpolant, Newton's divided differences and cubic splines.

1. Use Lagrange interpolating polynomials of degree one, two and three to approximate $f(8.4)$, knowing that $f(8.1) = 16.9$, $f(8.3) = 17.6$, $f(8.6) = 18.5$ and $f(8.7) = 18.8$.
   (a) Sketch the situation.
   (b) Give the general expression of the Lagrange polynomials $L_{n,k}$ and the Lagrange interpolant.
2. Repeat the previous exercise using Newton's divided differences. Comment on the result.
   (a) Give the general expression of the coefficients involved in the Newton's divided differences.
   (b) Make a table and store all coefficients.
3. Construct the natural cubic spline for the following data:

x	0.1	0.2	0.4	0.5	0.8
f(x)	1.2	-1.1	2	2.3	0.5

   (a) List all conditions you need to build this cubic spline.
4. BONUS: Download on Catcourses `cubic_spline.mlx`. Complete the `MATLAB` function, called `cubic_spline` that inputs a set of data points (x, y) =`(datx,daty)`, x the numbers at which to interpolate, and outputs the cubic spline interpolant, S, evaluated at x using natural cubic spline interpolant:

   ```
   function S = cubic_spline(x,datx,daty)
   ```

   Use your textbook to complete the code. Use the code to interpolate a the data points:

x	0.9	1.3	1.9	2.1	2.6	3.0	3.9	4.4	4.7	5.0	6.0	7.0	8.0	9.2	10.5	11.3	11.6	12.0	12.6	13.0	13.3
f(x)	1.3	1.5	1.85	2.1	2.6	2.7	2.4	2.15	2.05	2.1	2.25	2.3	2.25	1.95	1.4	0.9	0.7	0.6	0.5	0.4	0.25

   Interpolate using `x=1.0:0.1:14`.
   Hint: your results should be the same as the ones obtained in Table 3.19 of your textbook.