MATH 131: Numerical Methods for scientists and engineers
Midterm 1–VERSION B–Fall 2017
September 22, 2017
Instructions

Read the following instructions carefully:

- Write your name on the front page of your green/blue book.
- Write the VERSION (VERSION A or VERSION B) of your exam on the front page of your green/blue book.
- Make a grading table on the back of the first cover of your green/blue book.
- Write each problem on a separate page. Please make sure you clearly mark the problem you are working on (e.g., 1a)).
- No calculator or electronic devices allowed.
- It is important to show your work for each problem. Write full sentences. Credit will NOT be given for correct answers without justification. Also, partial credit will be given for incorrect answers if some of the work is correct.
- Clearly mark out (cross out) any work that you are not including in your answer and you do not want graded.
- Good luck!

WRITE YOUR NAME AND DON'T FORGET TO PUT THIS PAGE BACK IN YOUR EXAM AT THE END

$Name$: –

1. (20pts, 10 each) Let $f(x) = -3x^3 + 4x - 2$.
   (a) Use Newton's method to find x_1 if $x_0 = 1$.
   (b) Use the Secant method to find x_2 if $x_0 = 2$ and $x_1 = 1$.

2. (30pts, 10 each) Let $g(x) = \sqrt{x+6}$.

   (a) Show that $c = 3$ is a fixed point of the function $g(x)$.

   (b) Find a bound for $|g'(x)|$ for $x \in [0, 4]$.

   (c) Is the fixed-point method converging to the fixed point $c = 3$ in the interval $[0, 4]$? If so, what is the rate of convergence ?

3. (20pts, 5 each)

   (a) Give Taylor's theorem for a function f at a point x_0.

   (b) Using Newton's method, what is the rate of convergence for the sequence of error ?

   (c) Give the largest interval of possible approximations p^* of a number p up to 10^{-5} relative error.

   (d) Suppose p^* approximates the number p. What are the different types of error that you can use to quantify the accuracy of the approximation ?

4. (30pts, 25, 5) Below is a MATLAB implementation of the bisection method with some code missing.

   (a) Fill the blanks below to correctly execute the bisection method. READ CAREFULLY THE CODE !

```
function [c,err,n] = bisection(f,a,b,tol,N)

% function to solve f(x) = 0 using the bisection method over [a,b]
% ASSUMPTIONS: we assume that f(a)*f(b)<0
% INPUTS:
% f is the function at hand
% a is the lower bound of the tested interval
% b is the upper bound of the tested interval
% tol is the error tolerance
% N is the maximum number of iterations
% OUTPUTS:
% c is the computed root
% err is the error bound at the end
% n is the last iteration before breaking

    n = 0;
    a_n = a;
    b_n = b;
    err = b_n-a_n;

    while err > tol && n < N
        x_n = _____________;
        if f(x_n)*f(a_n) > 0
            ______ = ______;
            ______ = ______;
        elseif f(x_n)*f(b_n) >= 0
            ______ = ______;
            ______ = ______;
        end
        err = _____________;
        n = n+1;
    end
    c = (b_n + a_n) / 2.0;
end
```

   (b) Is the code working if you consider $f(x) = x^4$, $a = -1$ and $b = 2$? Explain why.