MATH 131: Numerical Methods for scientists and engineers – Discussion 7: Coding

The goals of this discussion section are:

- Get a deeper understanding of IVPs, Euler's method, Taylor's method.
  Use the codes you developed for Assignment 4.

1. Consider the IVP
$$\frac{dy}{dt} = \frac{\sin(2t) - 2ty}{t^2}, \quad y(1) = 2, \quad t \in [1, 5]$$
   (a) Show that the problem is well-posed.
   (b) Find the exact solution (by hand, not with a computer).

2. **Pair-up with a classmate and** write a Matlab function called `taylor_method_three` that solve the IVP
$$\frac{dy}{dt} = f(t, y), \quad a \leq t \leq b, \quad y(0) = \alpha \tag{1}$$
   using third order Taylor method. The header should look like

```
function w = taylor_method_three(f, fp, fpp, a,b,alpha,N)
```

   where N is the number of intervals used, so that $\Delta t = \frac{b-a}{N}$, `fp` and `fpp` being the first and second derivatives of f. Use this method to solve the IVP
$$\frac{dy}{dt} = \frac{\sin(2t) - 2ty}{t^2}, \quad y(1) = 2, \quad t \in [1, 5]$$
   with $N = 10, 10^2, 10^3$. Plot the three solutions with respect to t. On another figure, make a `loglog` plot of absolute error at $t = 5$ versus the number of intervals for all three methods on the same plot. To compute the exact solution use the matlab function `dsolve`.

3. Use your code `euler_timestep` from Assignment 4, and compare the results you obtained with the ones obtained with `taylor_method_three`. Plot on the same graph the error and comment. Do you recover the theoretical orders of error bounds ?