

MATH 131: Numerical Methods for scientists and engineers – Discussion 10: Paper

The goals of this discussion section are:

- Get a deeper understanding of direct solvers for linear systems.

1. Consider the system:

$$\begin{array}{ccccrccl} x_1 & +2x_2 & +3x_3 & +4x_4 & = & 2 \\ 2x_1 & & -3x_3 & +4x_4 & = & 1 \\ 4x_1 & -2x_2 & -3x_3 & +x_4 & = & 0 \\ 3x_1 & -x_2 & -4x_3 & +2x_4 & = & -3 \end{array}$$

- (a) Rewrite this system into the form $Ax = b$ where A and b are matrices to determine.
 - (b) Check if the system has a unique solution.
 - (c) Is the matrix symmetric?
 - (d) Use Gaussian Elimination and Backward substitution to solve this system.
 - (e) Perform the LU factorization to solve the system. Provide L, U .
2. Create a 3×3 matrix A such that:
- (a) The matrix is symmetric, with non zeros entries outside of the diagonal.
 - (b) The determinant is equal to 3.

Perform LU factorization on that matrix and solve the system $Ax = b$ for $b = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$.