MATH 131: Numerical Methods for scientists and engineers – Discussion 6: Paper

The goals of this discussion section are:

- Get a deeper understanding of numerical differentiation and integration.

1. (Similar exercise has been given on a midterm)
   (a) What is the order of the error bound while using an $(n+1)$-point formula approximating $f'(x_0)$?
   (b) Explain how to derive an $(n+1)$-point formula to approximate derivatives. Be precise and concise.
   (c) Use the most accurate 3 points formula to determine each missing entry in the table below:

x	f(x)	f'(x)
7.1	1.2	
7.2	3.1	
7.3	2.4	

   We recall some formulas

$$f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h} - \frac{h}{2}f''(\xi)$$

$$f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h} - \frac{h^2}{6}f'''(\xi)$$

$$f'(x_0) = \frac{3f(x_0) - 4f(x_0-h) + f(x_0-2h)}{2h} + \frac{h^2}{3}f'''(\xi)$$

$$f'(x_0) = \frac{-3f(x_0) + 4f(x_0+h) - f(x_0+2h)}{2h} + \frac{h^2}{3}f'''(\xi)$$

2. Approximate $\int_0^2 x^2 \ln(x^2+1)\,dx$ using $h = 0.25$:
   (a) use Composite Trapezoid rule.
   (b) use Composite Simpson's rule.
   (c) use Composite Midpoint rule.

   For the Composite Trapezoid rule and Composite Simpson's rule, recall the order of the error with respect to h. Discuss which one will give a more accurate result.