On Possible Worlds

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Introduction

In "Possible Worlds", R.C. Stalnaker (cf. [1]) responds to the metaphysical treatment of foundational aspects of the possible world semantics of modal logic as established in David Lewis's "Counterfactuals" (cf. [2]). Stalnaker identifies Lewis's position on the theory of possible worlds as that of an extreme realist, a position with which he is not comfortable. Consequently, he commits himself to a rational interpretation of the four theses put forward by Lewis which claim that the actual world is one among many possible 'concrete' worlds whose actuality is indexical in nature and that the notion of a possible world is conceptually irreducible. In the process, Stalnaker succesfully establishes the independence of a more plausible subset of Lewis's theses, motivated in part by incompatibilities that emerge in co-existing with the remainder. In what follows, he identifies his position on the matter as that of a moderate realist and defends the same against R.M. Adam's nominal realism of possible worlds (cf. [3]). In conclusion, he remarks on the scope of his position in saying that the notion of a possible world is a basic concept in a true account of the way in which we represent our world in propositional acts and attitudes, the development and defense of which, remains undone.

I argue that all three sets of theses corresponding to the three positions encountered in "Possible Worlds" are descriptively insufficient. I agree that Stalnaker's investigation is illuminating in nature, especially in contrasting Lewis' and Adams' theses about possible worlds. In particular I discuss how category theory is a ideal framework for coming up with a descriptive, insightful, intuitive and pragmatically useful theory for "possible worlds".

Summary

Stalnaker motivates a need for a serious metaphysical treatment of the notion of a possible world that has emerged in constructing the relational semantics of modal algebras. For this notion to serve a purpose more than that of a heuristic device, such a treatment is deemed necessary. Therefore, he critiques a relevant treatment appearing in Lewis's "Counterfactuals". Lewis claims that he believes in the existence of possible worlds similar to the one we inhabit. He justifies his belief by pointing out that ordinary language permits talking about the ways in which things comprising this world could have been and that taking this at face value permits belief in existence of entities which might be called 'ways things could have been', which he prefers to call 'possible worlds'. In response, Stalnaker points out that the rhetorical force of Lewis's argument lies in substituting 'possible worlds' for 'ways things could have been'. This substitution is evidently ambiguous. A critical look at the four theses put forward by Lewis is therefore in order. At the outset, Stalnaker clarifies that he does not believe in the extreme realism of possible worlds put forward by Lewis and that he aims to establish the independence of a more plausible subset of the same and thereby defend the coherence of a manifestly moderate realism of possible worlds. Lewis proposes:

- T1 Possible worlds exist.
- T2 Other possible worlds are things of the same sort as the actual world "I and all my surroundings".
- T3 The indexical analysis of the adjective 'actual' is the correct analysis.
- **T4** Possible worlds are conceptually irreducible.

T1 - **T4** are aimed towards answering: "What are possible worlds?" "Do they exist?" "How are possible worlds different from the actual world?" "What is the nature of possible worlds?".

Firstly, Stalnaker agrees with Lewis's arguments in support of **T1**: Possible worlds do exist in the sense implicit in "ways things could have been". **T2** however, Stalnaker argues, is incompatible with **T1**. **T2** informally characterises the actual world by the paraphrase "I and my surroundings", which is comprised of concrete entities. According to Lewis, a possible worlds, in

being similar to the same, is therefore, concrete or is at least comprised of concrete entities. Here, Stalnaker points out that "the way things are" is a state of the world and not the world itself. Thus, **T2** is rejected as a candidate for the subset that he aims to construct.

Though **T2** is found to be incompatible with **T1**, the way in which the actuality of the actual world is likened to the "way things are" and thereby transmitted to the possible worlds, is rooted in **T3**. Indeed, **T3** seems to imply that the actuality of the actual world is simply a world-relative attribute, so that, the concept of actuality does not distinguish, from an absolute standpoint, the actual world from the others. Hence, this might lead one to infer that the other possible worlds are just as real as the actual world. Stalnaker argues that this reasoning is evidently faulty and claims that the semantical thesis that the indexical analysis of "actual" is correct can be separated from the metaphysical thesis that the actuality of the actual world is determined by the relationship between the world and its elements. On the other hand, **T3** is compatible with **T1** since it does not imply the non-existence of possible worlds. After all, the belief that the "actual world" is the only one that is real is content-less as there is no way of justifying the same by arguing what is not real.

Stalnaker agrees with **T4** and defends it against R.M. Adam's response to Lewis's extreme realism. Adams argues that in order to avoid extreme realism one must find an eliminative reduction of possible worlds. Contrary to **T4**, Adams begins wiith the notion of a proposition and defines a world-story as a maximal consistent set of such propositions. Hence, the notion of a possible world can be given a contextual analysis in terms of world-stories. However, what is it that this formulation answers? Stalnaker talks about two problems, which also appear in Adams' analysis:

- P1: How can explanations given in terms of possible worlds help us to understand anything unless we are told what possible worlds are, in terms of notions that are independent of those that possible worlds themselves are intended to explain?
- **P2**: Believing in **T1** and **T2** necessarily commits one to extreme realism.

Stalnaker points out that key to understanding P2 is a cognizance of the dis-

tinction between a way that our world might have been and the world which is a way that our world might have been. This distinction, in fact resolves $\mathbf{P2}$. In this light, it is surprising that Adams develops his world-story theory inspired by $\mathbf{P2}$. It is important to understand what is meant by possibility in relation to sets of propositions in Adams' treatment. Intuitively, a set of propositions is possible if all its members can be true together. Moreover, this notion cannot be defined in terms of possible worlds or world-stories, without circularity. It should however be a consequence of the theory that a set of propositions is possible if and only if its members are are simultaneously true in some possible world. Adams' also assumes implicitly that every proposition has a contradictory or converse, where it is understood that propositions denoted by symbols A and B are contradictory if and only if $\{A, B\}$ is not possible and for every set of possible propositions Γ , either $\Gamma \cup \{A\}$ or $\Gamma \cup \{B\}$ is possible.

It is now pointed out that there is a reduction of propositions to possible worlds, competing with the reduction of possible worlds to propositions. Stalnaker claims that Adams is in fact aware of this and discusses the same, concluding that the two proposals are *equivalent*. This is false, Stalnaker claims, and shows that the former among the two is stronger. Subsequently, he asserts two consequences of the possible worlds analysis which do not follow from the world-story theory, namely:

- I: Identity condition. Necessarily equivalent truths are identical.
- C: Closure condition. For ever set of propositions, there is a proposition, which, necessarily, is true if and only if every member of the set is true.

I and C are transported to Adams' world-story theory, which, Stalnaker argues, establishes an equivalence. This follows from realising that, in the world-story framework, it can now be deduced that there exists a set of propositions of which all others are truth-functions. This set is that of the strongest contingent propositions. Calling these propositions as basic ones (or alternatively propositional elements), it follows that, propositions, in general, can be defined as sets of these propositional elements. Moreover, a non-basic proposition will be true if one of its members is true. This reformulation, admits the obvious defintion for the previously primitive notion of possibility. The said equivalence has thus been derived.

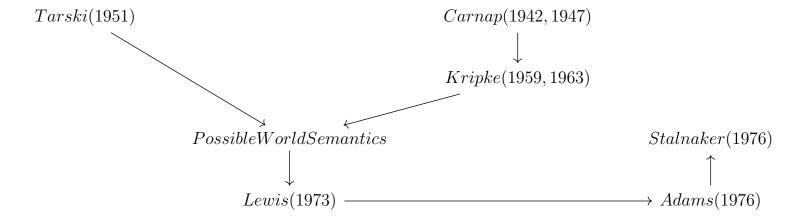
Hence, Stalnaker extracts **T1**,**T3** and **T4** and puts them together, calling the resulting theory, a moderate realism of possible worlds. It is noted that though he begins with the world-story framework given by Adams, which is minimal with respect to the inherent structure, the subsequent metamorphosis of the same into the possible worlds analysis is solely motivated by independently plausible assumptions and about *propositions* and theory-neutral considerations of economy of formulation.

The realism that Stalnaker defends lies in his claim that the concept of a possible world is a basic concept in a true account of the way in which we represent the world in our propositional acts and attitudes.

Critique

It is evident, that all three sets of theses put forward by Lewis, Stalnaker and Adams (hereby **TL**, **TS**, **TA** respectively) are insufficient in defining "possible worlds" and the structure of the class of all "possible worlds". Are they aware of it? One can surely say that Stalnaker is. Indeed, none among **TL**, **TS** and **TA** answer **P1** satisfactorily. In what follows, I point out a natural direction that might be taken so as to come up with a richer and descriptive theory of possible worlds.

Firstly, it is useful to understand why and how a metaphysical treatment for "possible worlds" becomes necessary. This has been done partly in the very first paragraph of the preceding summary. I summarise the same along with some other background in the following diagram (meant to describe chronological data and flow of idea only) and proceed.



The provability interpretation of the necessity operator □ and its relation to intuitionism gave a strong impetus to mathematical studies in modal logic, which resulted, in particular, in establishing connections with algebra and topology by McKinsey and Tarski (1944, 1946, 1948), and finally led to the discovery of relational representations of modal algebras by Jonsson and Tarski (1951). This relational semantics was also invented by philosophers independently: Carnap (1942, 1947), Kripke (1959, 1963a,b) who apparently were not aware of (Jonsson and Tarski 1951) (cf. [7]). David Lewis used the same in order to explain semantic properties of counterfactual conditionals. On a different note, it is certainly surprising, even absurd, that Lewis ended up demanding the existence of 'concrete' possible worlds in explaining counterfactuals.

A primary reason for the lack of descriptiveness in **TL** and **TS** is, in my opinion, because of an incomplete interpretation of what one means by 'ways things might have been'. Intuitively, one realises that the idea of existence of possibilities is central to human experience, and as a first step, this idea is extracted out of statements in a natural language. In doing so, it is evident that one is considering different modalities associated with propositional attitudes. Moreover, one is aware of different phenomena and possibilities associated with them. In particular, one is able to select a "possibility" for each phenomena and subsequently fit all of them together so as picture a possible world. It is therefore, clear that depending on the class of phenomena one chooses to consider, one pictures "possible worlds" of different kinds. How-

ever, it is now necessary to remember that Lewis, Stalnaker and Adams are interested in considering total possible worlds. So, they attempt to discuss all class of phenomena. As a side note, here by phenomena, I am not referring to its particular instances, rather it's essence and equivalence classes of such essences. We do have with us, some constructions which we have called "possible worlds" - strictly mutually compatible instances of phenomena belonging to some class(es). Thus intuitively, our constructions are no different from those made by Adams in his world-story theory. In fact, since **TA** along with **I** and **C** is equivalent to Stalnakers possible world analysis, the former is certainly more intuitive and therefore preferable. The lack of descriptiveness is primarily due to the choice of considering total possible worlds. Hence, it is necessary to restrict oneself to different classes of phenomena and investigate the constructions one calls "possible worlds".

Therefore, in this manner, the nominal realism in $\mathbf{TA} \cup \{\mathbf{I}\} \cup \{\mathbf{C}\}$ emerges when one asks, "Are these constructions real?". Adams chooses to reject reality of these constructions. The question in itself is familiar to us. It is the same one that one asks for mathematical abstractions. Therefore, Extreme Realism of possible worlds put forward by Lewis enjoys a similar status as Platonic Realism in Mathematics. The similarity in itself is not very surprising as we in fact investigating "reality" and "actuality" in different contexts. It is however worthwhile to note how the difference between the two has been used by Stalnaker to point out incompatibilities in \mathbf{TL} and faults in interpreting $\mathbf{T3}$. However, if the reality of possible worlds is of no consequence to the actual world (after all they as causally disconnected), it is certainly plausible that finding better theories explaining phenomena and propositional attitudes with respect to the actual world is pragmatically more useful.

It is also important to note that a descriptive theory of possible worlds would also have strong relationships with theories of space-time, matter and phenomena. Such relationships are however lost when in Adams reductive methods. A possible direction for work regarding possible worlds is therefore clear. One would prefer a bottom-up approach like Adams rather than a top-down one like Lewis, Stalnaker. In doing so, one would inevitably end up asking oneself:

- What does one mean by a thing?
- How do such things interact?

• How do *things* make up our world or rather a (consciousness-free) world?

Of course, it is worthwhile to note that thing might be replaced by other objects primitive to a perspective that one seeks to use. The first among the above set of questions is certainly familiar to us. It is the title of Martin Heidegger's illuminating work, "What is a thing?" (cf. [4]). Recently, similar ideas are also considered in "What is a thing?: Topos Theory in the Foundations of Physics" written by Andreas Doring and Chris Isham aiming to use a topos-theoretic formulation for general theories for physics. In particular, category theory has gradually come up in various sciences, as the ideal framework for investigating music (cf. [5]), anthropology and the foundations of mathematics. Yet another recent work by Healy and Caudell (cf. [6]) proposes a categorical framework as a vehicle for defining ontologies in an unambiguous language with analytical and constructive features. It is also interesting to note that the internal logic of topos is intuionistic. Not surprisingly, topos theory is very constructive in nature and the intuionistic ideas inherent in it are reminiscent of the mathematical foundations of modal logic. (cf. [7]).

References

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