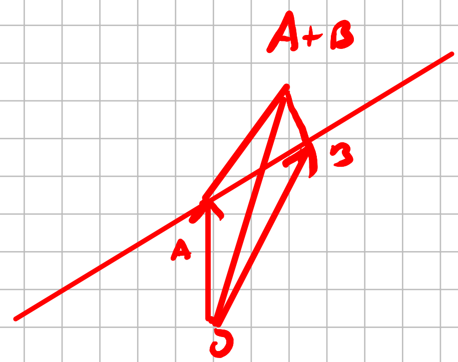
$A = (a_{ij})$ $\quad \sum_{i=1}^{n} a_{ij} = 1 \quad \forall\, j = \overline{1, n}$

$A^* \Rightarrow \sum_{j=1}^{n} a^*_{ij} = 1$ $\quad A^* = \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{pmatrix}$ $\quad A^* \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$(1, 1, 1)$ vect. propriu pt A^* coresp. lui $\lambda = 1$ $\Rightarrow 1 \in \sigma(A^*) = \sigma(A)$

$\Rightarrow 1 \in \sigma(A)$

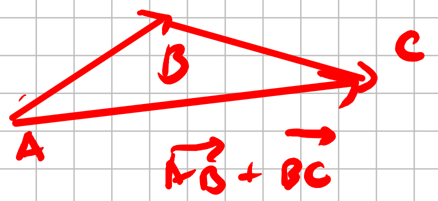
dreapta în plan $d: ax+by+c=0$ $\quad (a,b)\neq(0,0)$

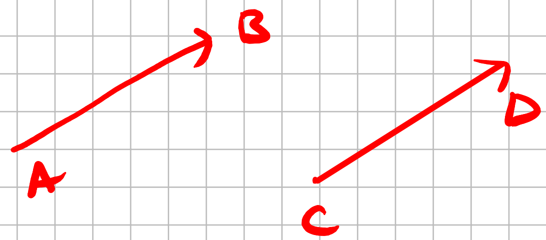


$(x_1,y_1),\ (x_2,y_2)\in d \quad a(x_1+x_2)+b(y_1+y_2)+c=0\ ?$

$ax_1+by_1+c+ax_2+by_2 = 0-c\neq 0$

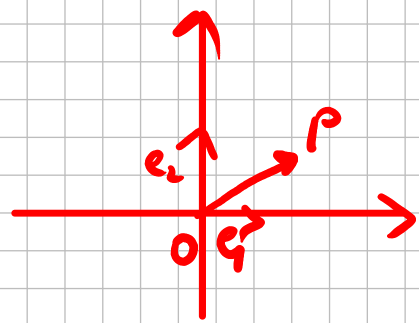
$d\subset\mathbb{R}^2$ subsp. vectorial $\Leftrightarrow c=0$.





$\varphi_O : V \to L \qquad \varphi_O(A) = \overrightarrow{OA}$

Planul geometric V, $L = \mathbb{R}^2$ $\qquad P \mapsto \overrightarrow{OP}$



$A^2(\mathbb{R})$: $\mathbb{R}^2$ cu structura de sp. afin

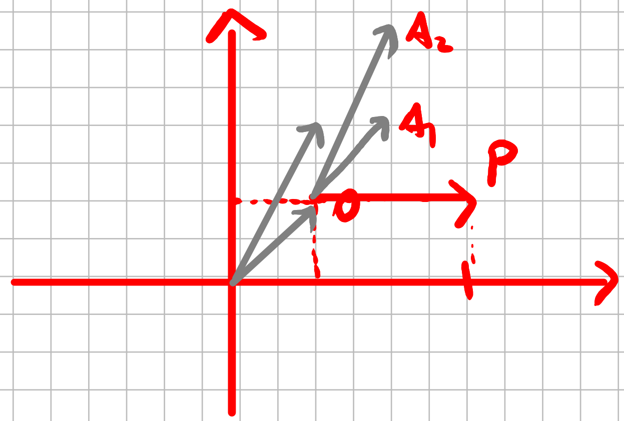
Ex. $O = (1,1)$ $\quad B = \{f_1, f_2\} = \{(1,1), (1,2)\}$

Să se descompună $P = (3,1)$ în raport cu reperul $\{O; f_1, f_2\}$

$$\overrightarrow{OP} = \text{"}P - O\text{"} = (3,1) - (1,1) = (2,0)$$

$$\left(\begin{array}{cc|c} 1 & 1 & 2 \\ 1 & 2 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & -2 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -2 \end{array}\right)$$

Cond. lui P sînt $(4,-2)$



$f_1 := \overrightarrow{OA_1}$, $f_2 := \overrightarrow{OA_2}$

$(1,1) = (x,y) - (1,1) \Rightarrow (x,y) = (2,2)$

$(1,2) = (x,y) - (1,2) \Rightarrow (x,y) = (2,4)$

$\{A\}$ afin-indep. $\forall A \in V$

$\{A, B\}$ afin-indep $\Leftrightarrow A \neq B$ $(\Leftrightarrow \overrightarrow{AB} \neq 0)$

$\{A, B, C\}$ afin indep $\Leftrightarrow \{\overrightarrow{AB}, \overrightarrow{AC}\}$ liniar-indep $\Leftrightarrow$ $\{A, B, C\}$ nu sunt coliniare

$\{A, B, C, D\}$ afin-indep $\Leftrightarrow \{\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}\}$ liniar-indep $\Leftrightarrow$ $\{A, B, C, D\}$ formează tetraedru

Ex. 2 $d : ax + by = c$ este subsp. afin in $A^2(\mathbb{R})$

$O = (x_0, y_0) \in d$

$L' = \{ \overrightarrow{OP} \mid P \in d \} = \{ (x - x_0, y - y_0) \mid (x, y) \in d \}$ subsp. vectorial ?

$P_1 = (x_1, y_1)$, $P_2 = (x_2, y_2)$ $\alpha, \beta \in \mathbb{R}$

$\alpha \overrightarrow{OP_1} + \beta \overrightarrow{OP_2} \in L'$? $\Leftrightarrow$ $\alpha(x_1 - x, y_1 - y) + \beta(x_2 - x, y_2 - y) \in L'$ $\Leftrightarrow$

$\Leftrightarrow$ $(\underbrace{\alpha x_1 - \alpha x + \beta x_2 - \beta x, \alpha y_1 - \alpha y + \beta y_2}_{\parallel} - \beta y) \in L'$

$(x_3 - x, y_3 - y)$ $(x_3, y_3) \in d$

$\in d \Rightarrow \alpha \overrightarrow{OP_1} + \beta \overrightarrow{OP_2} \in L'$

$(\alpha x_1 + \beta x_2 - (\alpha + \beta - 1)x, \alpha y_1 + \beta y_2 - (\alpha + \beta - 1)y) - (x, y) \in L'$?

$a(\alpha x_1 + \beta x_2 - (\alpha + \beta - 1)x) + b(\alpha y_1 + \beta y_2 - (\alpha + \beta - 1)y) =$

$$= \alpha(ax_1 + by_1) + \beta(ax_2 + by_2) - (\alpha+\beta-1)(ax+by) =$$

$$= \alpha c + \beta c - (\alpha+\beta-1)c = c$$

$d \subset \mathbb{R}^2 \qquad d: ax+by=c \qquad \mathrm{Dir}(d) = \{\overrightarrow{OP} \mid P\in d\}$, $O \in d$ fixat

$O' \in d \qquad \mathrm{Dir}'(d) = \{\overrightarrow{O'P} \mid P \in d\} = \mathrm{Dir}(d)$

$$\overrightarrow{O'P} = \underbrace{\overrightarrow{O'O}}_{\in\,\mathrm{Dir}(d)} + \underbrace{\overrightarrow{OP}}_{\in\,\mathrm{Dir}(d)} \in \mathrm{Dir}(d)$$

$(x_0,y_0)\in d \qquad \mathrm{Dir}(d) = \{(x-x_0,\, y-y_0) \mid (x,y)\in d\}$

$$a(x-x_0)+b(y-y_0) = ax+by-(ax_0+by_0) = c-c=0$$

$\mathrm{Dir}(d) = \{(x,y)\in\mathbb{R}^2 \mid ax+by=0\}$

$V' = \{x\in K^n \mid Ax=b\} \Rightarrow \mathrm{Dir}(V')\ (= L') = \{x \in K^n \mid Ax = 0\}$

$V' = \{ x + x_0 \mid x \in L' \}$ pt. un $x_0 \in V'$ fixat

Dim $\mathrm{Dir}(d) = 1 \Rightarrow \mathrm{Dir}(d) = \langle (b, -a) \rangle = \{ (bt, -at) \mid t \in \mathbb{R} \}$

$d = \{ (x_0 + bt, y_0 - at) \mid t \in \mathbb{R} \}$

$d: \begin{cases} x = bt + x_0 \\ y = -at + y_0 \end{cases}$ ec. parametrică a dreptei

$d_1, d_2 \quad d_1: a_1x + b_1y = c_1, \quad d_2: a_2x + b_2y = c_2$

$d_1 \parallel d_2 \Leftrightarrow \mathrm{Dir}(d_1) = \mathrm{Dir}(d_2) \Leftrightarrow \exists\, t$ a.î. $(b_2, -a_2) = (b_1t, -a_1t) \Leftrightarrow$

$\exists\, t$ a.î. $a_2 = a_1t,\ b_2 = b_1t$

$d_1: x + y = 2,\ d_2: x - y = 3 \qquad d_1 \nparallel d_2 \qquad$ pt că $\{(1,1), (1,-1)\}$ liniar indep.

$d_1: x + 2y = 3,\ d_2: 2x + 4y = 5 \Rightarrow d_1 \parallel d_2$ şi $d_1 \neq d_2$

Dacă $V_1 \parallel V_2$, $\dim V_1 = \dim V_2$ și $V_1 \neq V_2 \Rightarrow V_1 \cap V_2 = \emptyset$

În spațiu

$d \subset \mathbb{R}^3$ $\quad d : \begin{cases} x_1 = a_1 t + b_1 \\ x_2 = a_2 t + b_2 \\ x_3 = a_3 t + b_3 \end{cases}$ $\quad \langle (a_1, a_2, a_3) \rangle = \mathrm{Dir}(d)$

$\mathrm{dir}(d)$ - vectorul directo.

$d_1 \parallel d_2 \Leftrightarrow$ au vectorii directori proporționali

$\Pi \subset \mathbb{R}^3$ este plan $\Leftrightarrow$ $\Pi = \{ x \in \mathbb{R}^3 \mid a_1 x_1 + a_2 x_2 + a_3 x_3 = b \}$ $(a_1, a_2, a_3) \neq (0,0,0)$

$\mathrm{Dir}(\Pi) = \{ x \in \mathbb{R}^3 \mid a_1 x_1 + a_2 x_2 + a_3 x_3 = 0 \} = (a_1, a_2, a_3)^\perp$

$\Pi_1 : a_1 x_1 + b_1 x_2 + c_1 x_3 = d_1$ $\qquad \Pi_1 \parallel \Pi_2$?

$\Pi_2 : a_2 x_1 + b_2 x_2 + c_2 x_3 = d_2$

$\mathrm{Dir}(\pi_1) = \mathrm{Dir}(\pi_2) \Leftrightarrow \mathrm{Dim}(\mathrm{Dir}(\pi_1) \cap \mathrm{Dir}(\pi_2)) = 2$

$\mathrm{Dir}(\pi_1) \cap \mathrm{Dir}(\pi_2) : \begin{cases} a_1 x_1 + b_1 x_2 + c_1 x_3 = 0 \\ a_2 x_1 + b_2 x_2 + c_2 x_3 = 0 \end{cases}$

$\dim(\ ) = 3 - \mathrm{rg}\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{pmatrix} \overset{!}{=} 2 \Rightarrow \mathrm{Rg}\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{pmatrix} = 1$

$\Leftrightarrow (a_1, b_1, c_1) = \lambda (a_2, b_2, c_2)$ cu $\lambda \neq 0$

$\pi_1 \parallel \pi_2 \Leftrightarrow (a_1, b_1, c_1) = \lambda (a_2, b_2, c_2)$

$\pi_1 \neq \pi_2 \Leftrightarrow d_1 \neq \lambda d_2$

Fie $d_1 : \begin{cases} x_1 = m_1 t + n_1 \\ x_2 = m_2 t + n_2 \\ x_3 = m_3 t + n_3 \end{cases}$ $\qquad \pi : a x_1 + b x_2 + c x_3 = d$

$d_1 \parallel \Pi \Leftrightarrow \mathrm{Dir}(d_1) \subset \mathrm{Dir}(\Pi) \Leftrightarrow (m_1, m_2, m_3) \in \mathrm{Dir}(\Pi)$

$\Leftrightarrow \boxed{a m_1 + b m_2 + c m_3 = 0}$

$d: \begin{cases} 2x + 3y - z = 1 \\ x + y + z = 3 \end{cases}$ $\quad \dim(d) = 3 - \mathrm{rg}\begin{pmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \end{pmatrix} = 3 - 2 = 1$

a dreaptă

Să se determine ecuația parametrică a lui d

$\begin{cases} 2x + 3y = 1 + z \\ x + y = 3 - z \end{cases} \Big| \cdot(-2) \Rightarrow \begin{cases} 2x + 3y = 1 + z \\ -2x - 2y = -6 + 2z \end{cases}$

$\qquad \qquad y = -5 + 3z$

$x = 3 - z + 5 - 3z = 8 - 4z$

$d: \begin{cases} x = -4t + 8 \\ y = 3t - 5 \\ z = t \end{cases}$

$$d: \frac{x-8}{-4} = \frac{y+5}{3} = \frac{z}{1}$$

In general

$$d: \frac{x-x_0}{m_1} = \frac{y-y_0}{m_2} = \frac{z-z_0}{m_3}$$

d trece prin (x_0, y_0, z_0) și are direcția (m_1, m_2, m_3)

$A, B \in V$ $\quad O \in AB \quad L = \langle \vec{AB} \rangle = \langle \vec{OA} \rangle$

$$\vec{OB} = \vec{OA} + \vec{AB} \Rightarrow \vec{AB} = \vec{OB} - \vec{OA}$$

Mijlocul lui $[AB]$ $\quad C \in (AB)$ a.î. $\vec{AC} = \frac{\vec{AB}}{2} = \frac{\vec{AC} + \vec{CB}}{2} \Rightarrow \vec{AC} = \vec{CB}$

$$\vec{OC} = \vec{OA} + \vec{AC} = \vec{OA} + \frac{\vec{AB}}{2} = \frac{2\vec{OA} + \vec{AB}}{2} = \frac{2\vec{OA} + \vec{OB} - \vec{OA}}{2} = \frac{\vec{OA} + \vec{OB}}{2}$$

$\triangle ABC$ $\quad A_1 = \text{mij}(BC)$, $B_1 = \text{mij}(AC)$, $C_1 = \text{mij}(AB)$

Dacă $\text{char } K \neq 2, 3$ $\quad AA_1 \cap BB_1 \cap CC_1 = \{G\}$ c.d.g. al $\triangle$.

$\text{char } K = 3$ $\quad AA_1 \parallel BB_1 \parallel CC_1$

$\{\overrightarrow{AB}, \overrightarrow{AC}\}$ liniar-indep, bază în $L(ABC)$

(ABC) planul gen. de $\{A, B, C\}$

$\overrightarrow{BA_1} = \overrightarrow{A_1C}$ $\quad \overrightarrow{BA_1} + \overrightarrow{A_1C} = \overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{AC} - \overrightarrow{AB}$

$$\overrightarrow{AA_1} = \overrightarrow{AB} + \overrightarrow{BA_1} = \overrightarrow{AB} + \frac{\overrightarrow{BC}}{2} = \overrightarrow{AB} + \frac{\overrightarrow{AC} - \overrightarrow{AB}}{2} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2}$$

$$\overrightarrow{AB_1} = \frac{\overrightarrow{AC}}{2}, \quad \overrightarrow{AC_1} = \frac{\overrightarrow{AB}}{2}$$

$$\overrightarrow{BB_1} = \overrightarrow{BA} + \overrightarrow{AB_1} = -\overrightarrow{AB} + \frac{\overrightarrow{AC}}{2} = \frac{\overrightarrow{AC} - 2\overrightarrow{AB}}{2}$$

$$\overrightarrow{CC_1} = \overrightarrow{CA} + \overrightarrow{AC_1} = -\overrightarrow{AC} + \frac{\overrightarrow{AB}}{2} = \frac{\overrightarrow{AB} - 2\overrightarrow{AC}}{2}$$

Cho $k = 3$ $\quad -2 = 1$, $\frac{1}{2} = 1$ $\Rightarrow$

$$\overrightarrow{AA_1} = \overrightarrow{AB} + \overrightarrow{AC}$$

$$\overrightarrow{BB_1} = \overrightarrow{AB} + \overrightarrow{AC}$$

$$\overrightarrow{CC_1} = \overrightarrow{AB} + \overrightarrow{AC}$$

$\Rightarrow AA_1 \parallel BB_1 \parallel CC_1$.

$f: A^3(\mathbb{R}) \to A^3(\mathbb{R})$ apl. afină $\mathcal{F}(\overrightarrow{AB}) = \mathcal{F}(B-A) = \mathcal{F}(B) - \mathcal{F}(A)$

$$\overrightarrow{f(A)f(B)} = f(B) - f(A)$$

$f: A^n(K) \to A^m(K)$ $f(x) = Ax + b$. $A \in M_{m,n}(K),\ b \in K^m$

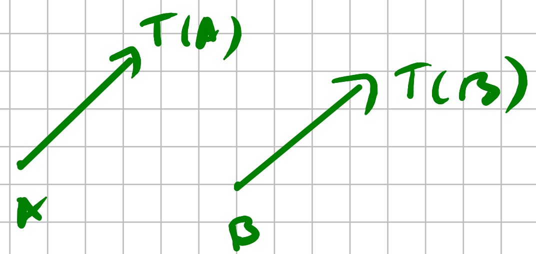
forma generală a unei aplicații afine.

$\mathcal{F}(X) = A \cdot X$

$b \neq 0 \Rightarrow f(x) = x + b$ este apl. afină cu $\Lambda(f) = Id_{K^n}$

translație de vector b.

$T_v : V \to V$ $\forall A, B$ $\overrightarrow{AT_v(A)} = \overrightarrow{BT_v(B)} = v$



$$\overrightarrow{AB} = \overrightarrow{T_v(A)T_v(B)}$$

$$\overrightarrow{T_v(A)T_v(B)} = \overrightarrow{T_v(A)A} + \overrightarrow{AB} + \overrightarrow{BT_v(B)} = -v + \overrightarrow{AB} + v = \overrightarrow{AB}$$

$T_v(AB) \parallel AB$ dacă $v \notin \langle \overrightarrow{AB} \rangle \Rightarrow T_v(AB) \overset{\parallel}{\neq} AB$

<u>Omotetii</u> $\quad \mathcal{H}_O^\lambda : V \to V \quad \mathcal{H}_O^\lambda(O) = O \quad A \neq O, \lambda \in K \setminus \{0\}$

$$\overrightarrow{O\mathcal{H}_O^\lambda(A)} = \lambda \cdot \overrightarrow{OA}$$

$\mathcal{H}_O(AB) \parallel AB$ dacă $O \notin AB$.

$O \in AB \Rightarrow \mathcal{H}_O(AB) = AB$. $\quad$ Caz $\mathcal{H}_O^\lambda : A^2(\mathbb{R}) \to A^2(\mathbb{R})$

$O = (x_0, y_0) \quad \mathcal{H}_O^\lambda(x, y) = (x', y')$

$$(x' - x_0, y' - y_0) = \lambda(x - x_0, y - y_0)$$

$$\Rightarrow \quad (x', y') = (\lambda x + (1-\lambda)x_0, \lambda y + (1-\lambda) y_0) \Rightarrow$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} (1-\lambda)x_0 \\ (1-\lambda)y_0 \end{pmatrix}$$

$$\Lambda(\mathcal{H}_0^{\lambda}) = \lambda I_n$$

$T_v \circ T_w = T_{v+w}$ $\qquad T_w(A) = B \quad \Rightarrow \overrightarrow{AB} = w$

$(T_v \circ T_w)(A) = C$

$$(T_v \circ T_w)(A) = T_v(T_w(A))$$

$$\overrightarrow{AC} = \overrightarrow{A\,T_v(B)} = \overrightarrow{AB} + \overrightarrow{BT_v(B)} = w + v$$

$$\mathcal{H}_0^{\lambda_1} \circ \mathcal{H}_0^{\lambda_2} = \mathcal{H}_0^{\lambda_1 \lambda_2}$$

Algebric $(\{T_v \mid v \in L\}, \circ)$ grup abelian $\simeq (L, +)$

$\{\mathcal{H}_0^{\lambda} \mid \lambda \in K \setminus \{0\}, \cdot\} \simeq (K^{\times}, \cdot)$

Grupul generat de translații și omotetii s.n. grupul dilatărilor și este caracterizat de: $f \in G \Leftrightarrow f(AB) \parallel AB \quad \forall A, B \in V$.

Spații afine euclidiene.

$A=(1,3)$, $B=(2,7)$ $\quad d(A,B) = |\overrightarrow{AB}| = \sqrt{(2-1)^2+(7-3)^2} = \sqrt{1+16} = \sqrt{17}$

$A=(x_1, \dots, x_n)$, $B=(y_1, \dots, y_n)$ $\quad d_2(A,B) = \sqrt{(y_1-x_1)^2 + \dots + (y_n-x_n)^2}$

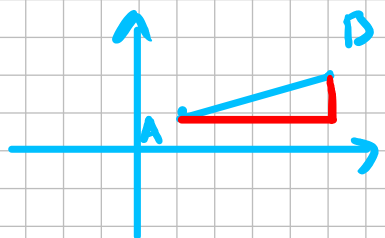
distanța euclidiană

1) $d(A,A)=0$

2) $d(A,B)=d(B,A)$

3) $d(A,C) \le d(A,B)+d(B,C)$ ineg. triunghiului

Distanța Manhattan $\quad d_1(A,B) = |y_1-x_1| + \dots + |y_n-x_n|$



— dist. euclidiană

— dist. Manhattan

$d_i : a_i x + b_i y = c_i \quad i = \overline{1,2}$

$Dir(d_i) = \langle (-b_i, a_i) \rangle = (a_i, b_i)^{\perp} \qquad L_1 \perp L_2 \Rightarrow L_1^{\perp} \perp L_2^{\perp}$

$d_1 \perp d_2 \Leftrightarrow a_1 a_2 + b_1 b_2 = 0$

$d : ax + by = c$ perp. din P pe d este $\dfrac{x - x_0}{a} = \dfrac{y - y_0}{b}$

$P = (x_0, y_0) \in A^2(\mathbb{R})$

$\hat{I}n$ $A^3(\mathbb{R})$ $\quad E^3(\mathbb{R})$ sp. afin euclidian pe $\mathbb{R}^3$

$d_i : \begin{cases} x = a_i t + b_i \\ y = c_i t + d_i \\ z = e_i \cdot t + f_i \end{cases} \quad i = \overline{1,2} \quad \Rightarrow d_1 \perp d_2 \Leftrightarrow a_1 a_2 + c_1 c_2 + e_1 e_2 = 0$

$\Pi_i : a_i x + b_i y + c_i z = d_i \qquad i = \overline{1,2}$

$\mathrm{Dir}(\Pi_1)^{\perp} = \langle (a_1, b_1, c_1) \rangle \subset \mathrm{Dir}(\Pi_2) \Leftrightarrow a_2 \cdot a_1 + b_2 \cdot b_1 + c_2 \cdot c_1 = 0$

$(a_1, b_1, c_1) =: \vec{n}_{\Pi_1}$ vectorul normal la Π_1

$$d : \frac{x - x_0}{m} = \frac{y - y_0}{n} = \frac{z - z_0}{l}$$

$\Pi : a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

$\mathrm{Dir}(d) \subset \mathrm{Dir}(\Pi)^{\perp} \Leftrightarrow (m, n, l) \in \mathrm{Dir}(\Pi)^{\perp} = \langle (a, b, c) \rangle$

$\Leftrightarrow \exists \lambda \neq 0$ a.î. $(m, n, l) = \lambda (a, b, c)$.

Fie $P = (x_0, y_0, z_0)$ $\quad \Pi : a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$

$$d(P, \Pi) = \min\{ |\overrightarrow{PQ}| \mid Q \in \Pi \}$$

Pasul 1: Construim $PQ \perp \Pi$, $Q \in \Pi$

$$PQ : \frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c} \iff PQ : \begin{cases} x = at + x_0 \\ y = bt + y_0 \\ z = ct + z_0 \end{cases}$$

Pasul 2:

$PQ \cap \Pi = \{Q\}$ $\quad a(at + x_0 - x_1) + b(bt + y_0 - y_1) + c(ct + z_0 - z_1) = 0$

$$(a^2+b^2+c^2)t = a(x_1-x_0) + b(y_1-y_0) + c(z_1-z_0)$$

$$t = \frac{a}{a^2+b^2+c^2}(x_1-x_0) + \frac{b}{a^2+b^2+c^2}(y_1-y_0) + \frac{c}{a^2+b^2+c^2}(z_1-z_0)$$

$$Q : \begin{cases} x = \dfrac{a^2(x_1-x_0) + ab(y_1-y_0) + ac(z_1-z_0)}{a^2+b^2+c^2} + x_0 \\[2ex] y = \dfrac{ab(x_1-x_0) + b^2(y_1-y_0) + bc(z_1-z_0)}{a^2+b^2+c^2} + y_0 \end{cases}$$

$$z = \frac{ac(x_1-x_0)+bc(y_1-y_0)+c^2(z_1-z_0)}{a^2+b^2+c^2}+z_0$$

Pas 3 $\quad |\vec{PQ}| = \frac{1}{a^2+b^2+c^2}\sqrt{\begin{aligned}&a^4(x_1-x_0)^2+a^2b^2(y_1-y_0)^2+a^2c^2(z_1-z_0)^2+\\&a^2b^2(x_1-x_0)^2+b^4(y_1-y_0)^2+b^2c^2(z_1-z_0)^2+\\&a^2c^2(x_1-x_0)^2+b^2c^2(y_1-y_0)^2+c^4(z_1-z_0)^2\end{aligned}}$

$$= \frac{1}{a^2+b^2+c^2}\sqrt{a^2(x_1-x_0)^2\cdot(a^2+b^2+c^2)+b^2(y_1-y_0)^2(a^2+b^2+c^2)+c^2(z_1-z_0)^2(a^2+b^2+c^2)}$$

$$= \sqrt{\frac{a^2(x_1-x_0)^2+b^2(y_1-y_0)^2+c^2(z_1-z_0)^2}{a^2+b^2+c^2}} = \frac{|a(x_1-x_0)+b(y_1-y_0)+c(z_1-z_0)|}{\sqrt{a^2+b^2+c^2}}$$

Ex. Fie $P=(1,3,1)$ și $d: \frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{0}$

Calculați $d(P,d)$.

Pasul 1: $\Pi \perp d$, $P \in \Pi$ $\quad \Pi: 2\cdot(x-1)+3(y-3)+0\cdot(z-1)=0$

$\Pi: 2x+3y = 11$

Pasul 2: $d \cap \Pi$

$$d: \begin{cases} x = 2t+1 \\ y = 3t-1 \\ z = 1 \end{cases}$$

$d \cap \Pi: \quad 4t+2+9t-3 = 11 \Rightarrow 13t = 12 \Rightarrow t = \frac{12}{13}$

$\{Q\} = d \cap \Pi \qquad Q = \left(\frac{24}{13}+1, \frac{36}{13}-1, 1\right) = \left(\frac{37}{13}, \frac{23}{13}, 1\right)$

$dist(P,d) = |\overrightarrow{PQ}| = \sqrt{\left(\frac{37}{13}-1\right)^2 + \left(\frac{23}{13}-3\right)^2 + 0} = \frac{1}{13}\sqrt{24^2+16^2}$

$= \frac{8}{\sqrt{13}}$

$d_i : \frac{x-x_i}{a_i} = \frac{y-y_i}{b_i} = \frac{z-z_i}{c_i}$ $\quad i=\overline{1,2}$

d_1, d_2 coplanare $\quad (\exists \pi$ a.î. $d_1 \subset \pi, d_2 \subset \pi)$

dacă $(a_1, b_1, c_1) \Rightarrow (a_2, b_2, c_2) \Rightarrow d_1 \parallel d_2$

$d_1 \cap d_2 = \{P\} \Rightarrow \{\mathrm{dir}(d_1), \mathrm{dir}(d_2)\}$ liniar indep.

$P = (x_0, y_0, z_0)$

$$\frac{x_0-x_1}{a_1} = \frac{y_0-y_1}{b_1} = \frac{z_0-z_1}{c_1} \quad , \quad \frac{x_0-x_2}{a_2} = \frac{y_0-y_2}{b_2} = \frac{z_0-z_2}{c_2}$$

$(\Leftarrow)$ $\quad (x_2-x_1, y_2-y_1, z_2-z_1) \in \langle \mathrm{dir}(d_1), \mathrm{dir}(d_2) \rangle$

d_1, d_2 coplane $\iff$ $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

d_1, d_2 necoplane $\Rightarrow$ $\exists! d$ a.î. $d \cap d_1 = \{P_1\}$, $d \cap d_2 = \{P_2\}$
$d \perp d_1$, $d \perp d_2$

<u>Ex. 3</u> $d_1: \frac{x-1}{1} = \frac{y}{0} = \frac{z+1}{1}$, $d_2: \frac{x-2}{0} = \frac{y+1}{1} = \frac{z}{1}$

Perp. comună a lui d_1 și d_2 și distanța între d_1 și d_2

$d_1: \begin{cases} x = s+1 \\ y = 0 \\ z = s-1 \end{cases}$ $d_2: \begin{cases} x = 2 \\ y = t-1 \\ z = t \end{cases}$

$$d(s,t) = P_1(s)P_2(t) : \begin{cases} x = 1-s \\ y = t-1 \\ z = t-s+1 \end{cases}$$

$$d(s,t) \perp d_1,\ d(s,t) \perp d_2 \iff \begin{matrix} \overrightarrow{P_1(s)P_2(t)} \cdot (1,0,1) = 0 \\ \overrightarrow{P_1(s)P_2(t)} \cdot (0,1,1) = 0 \end{matrix}$$

$$\iff \begin{cases} 1-s+t-s+1=0 \\ t-1+t-s+1=0 \end{cases} \iff \begin{cases} t-2s=-2 \\ 2t-s=0 \end{cases} \Rightarrow \begin{cases} s=2t \\ -3t=-2 \end{cases} \Rightarrow \begin{cases} t=\frac{2}{3} \\ s=\frac{4}{3} \end{cases}$$

$$P_1 = \left(\frac{7}{3}, 0, \frac{1}{3}\right),\quad P_2 = \left(2, -\frac{1}{3}, \frac{2}{3}\right)$$

$$P_1P_2 : \frac{x-\frac{7}{3}}{2-\frac{7}{3}} = \frac{y-0}{-\frac{1}{3}-0} = \frac{z-\frac{1}{3}}{\frac{2}{3}-\frac{1}{3}} \iff \frac{x-\frac{7}{3}}{-\frac{1}{3}} = \frac{y}{-\frac{1}{3}} = \frac{z-\frac{1}{3}}{\frac{1}{3}}$$

$P_1P_2: \quad \dfrac{x-\frac{2}{3}}{-1} = \dfrac{y}{-1} = \dfrac{z-\frac{1}{3}}{1}$

$$\operatorname{dist}(d_1, d_2) = |\overrightarrow{P_1P_2}| = \sqrt{\frac{1}{9}+\frac{1}{9}+\frac{1}{9}} = \frac{1}{\sqrt{3}}.$$