NUME ....

EXAMEN LA CALCUL DIFERENTIAL SI INTEGRAL

I. (1) [1 punct] Sa se arate ca seria $\sum\limits_{n=7}^{\infty} \frac{4}{n^2-7n+12}$ este convergenta si sa se gaseasca suma ei.

(2) [1 puncte] Studiati onvergenta seriei

$$\sum_{n=1}^{\infty} \frac{a(a+2)(a+4)(a+2n)}{1 \cdot 3 \cdot 5 \cdots (2n+1)}$$

in functie de valorile parametrului $a \in (0, \infty)$.

II. (1) [1 punct] Sa se studieze convergenta simpla si convergenta uniforma a sirului de functii $(f_n)_{n \geq 1}$.

$$f_n : [0,2] \to \mathbb{R}, \quad f_n(x) = \frac{1}{1+x^2n} + x.$$

(2) [1 punct] Calculati integrala improprie

$$\int_0^\infty x^2 e^{-2x} dx.$$

III. [3 puncte] Consideram functia $f : \mathbb{R}^3 \to \mathbb{R}$

$$f(x,y,z) = -z^8 + 8yz - 4y^2 - x^4 + 32x$$

1) Calculati $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}, \frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y^2}, \frac{\partial^2 f}{\partial z^2}, \frac{\partial^2 f}{\partial x \partial y}, \frac{\partial^2 f}{\partial x \partial z}, \frac{\partial^2 f}{\partial y \partial z}$.

2) Calculati $H_f(x,y,z)$.

3) Determinati punctele de minim si de maxim local ale functiei f.

4) Calculati $df(1,1,2)$

IV. 1) [1 punct] Calculati

$$\iint_D (x^3 + xy^2) dx dy$$

unde $D = \{(x,y) \in \mathbb{R}^2 | x^2 + y^2 \leq 1, \ x \leq 0\}$.

2) [1 punct] Calculati integrala

$$\iiint_V (xy - x^2 + z \sin z) \, dx dy dz, \text{ unde } V = [1,2] \times [0,1] \times [0,\pi].$$

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