MASINI TURING

1. Exemple
2. Definitia formala
3. Limbaje Turing-recunoscute si limbaje Turing-decidabile

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MT

> APD

> AFD

memorie infinita dar acces secvential

memorie finita

memorie infinita, fara restrictii de acces + un dispozitiv de citire/scriere

⇒ o MT poate calcula orice functie intuitiv calculabila

⇒ exista probleme pe care nicio MT nu le poate rezolva

⇒ o MT poate face tot ce poate face un calculator real.

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Alan Matison TURING, 1936-7, "On Computable Numbers, With an Application to the Entscheidungsproblem", *Proceedings of the London Mathematical Society*, (2) 42, pp 230-265; correction ibid. 43, pp 544-546 (1937).

..."an infinite memory capacity obtained in the form of an infinite tape marked out into squares on each of which a symbol could be printed. At any moment there is one symbol in the machine; it is called the scanned symbol. The machine can alter the scanned symbol and its behavior is in part determined by that symbol, but the symbols on the tape elsewhere do not affect the behavior of the machine. However, the tape can be moved back and forth through the machine, this being one of the elementary operations of the machine. Any symbol on the tape may therefore eventually have an innings.“

(vezi A. TURING: "Intelligent Machinery“, (1948) p. 61, reprinted in "*Cybernetics: Key Papers“,* eds. C.R. EVANS and A.D.J. ROBERTSON, University Park Press, Baltimore, 1968. p. 31)

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Cele mai importante contributii ale lui A. TURING la dezvoltarea calculatoarelor digitale sunt considerate urmatoarele:

- principiul programului memorat;
- demonstratia existentei calculatorului universal.

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- Numere calculabile
- Numere necalculabile
- Functii calculabile
- Functii necalculabile

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Variante de MT:

- banda infinita in ambele directii;
- banda infinita la dreapta si in jos;
- banda circulara
- cursor capabil sa examineze blocuri de celule;
- cursor capabil fie sa modifice simbolul citit fie sa se deplaseze;
- etc. (vom reveni)

=> Variate reprezentari grafice

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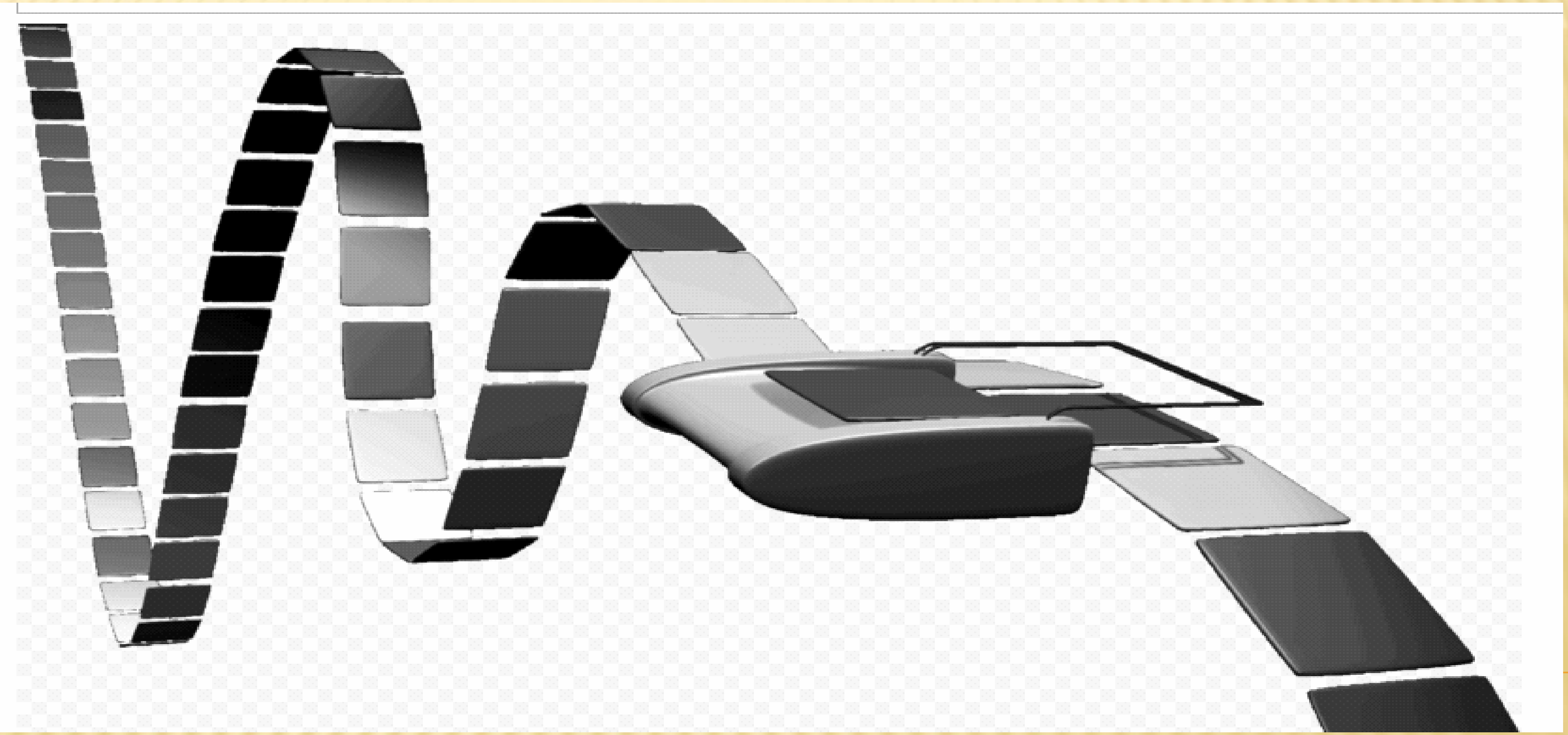

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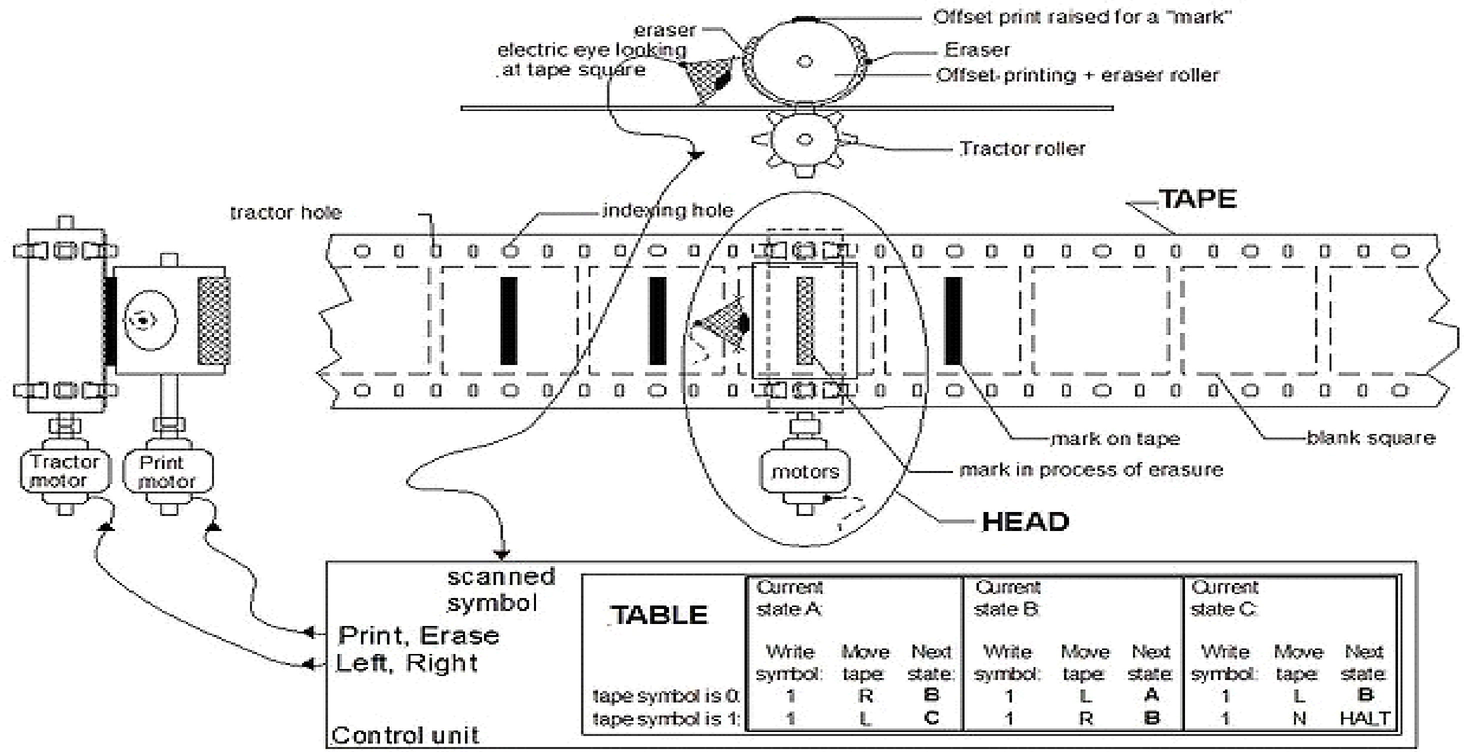

A fanciful mechanical Turing machine's TAPE and HEAD. The TABLE instructions might be on another "read only" tape, or perhaps on punch-cards. Usually a "finite state machine" is the model for the TABLE.

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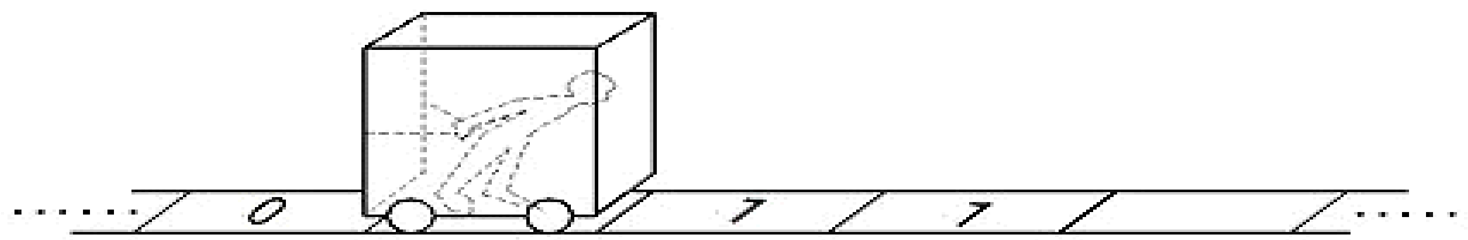

A poor mug in a box, reading, writing, erasing as per his list of instructions. After Boolos and Jeffrey figure 3-1, p. 21

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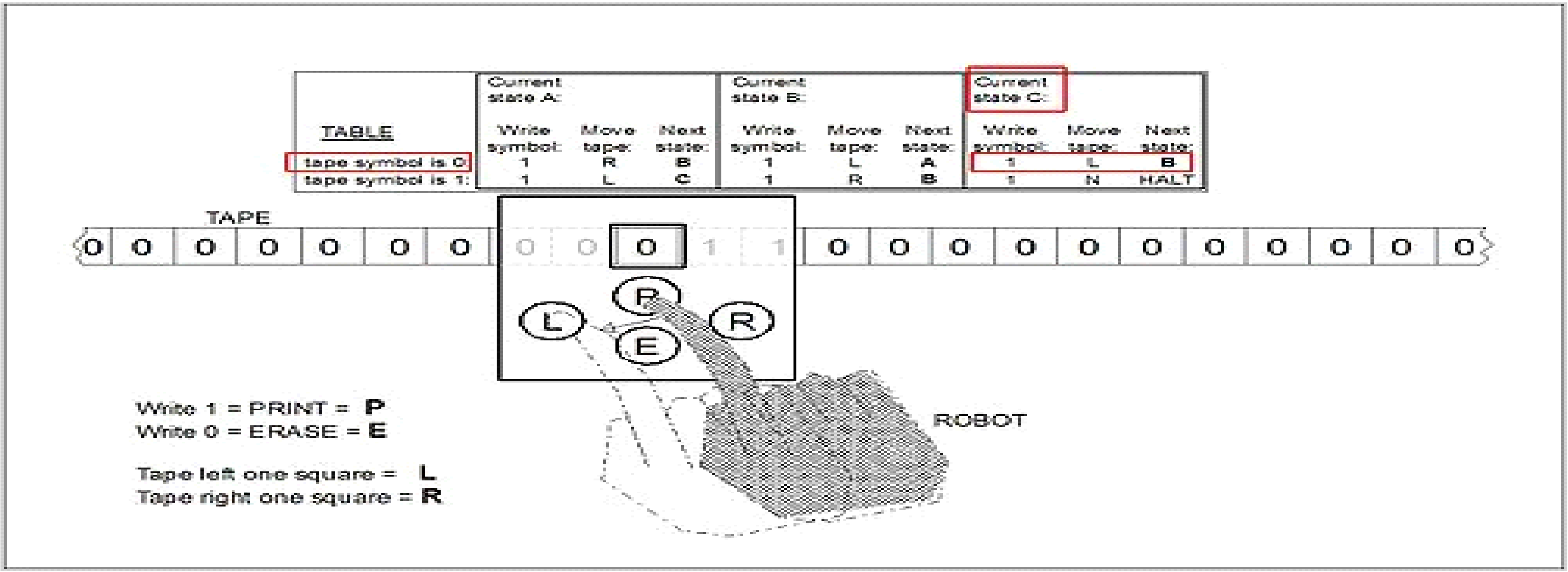

This is a robot with a console enlisted to work as a two-symbol three-state Busy Beaver. Robot is working on a tape initially printed with 0/blanks. Robot has looked at the symbol in the window (symbol 0), has read the instruction ("state") **C** and is about to PRINT a 1. Then it will push the tape-LEFT button. Lastly it will look toward instruction ("state") **B**. (The print/erase mechanism is out of sight, beneath the window. Maybe the tape is clear and the mechanism pulls off sticky 0's and sticks on 1's to PRINT and vice versa to ERASE.)

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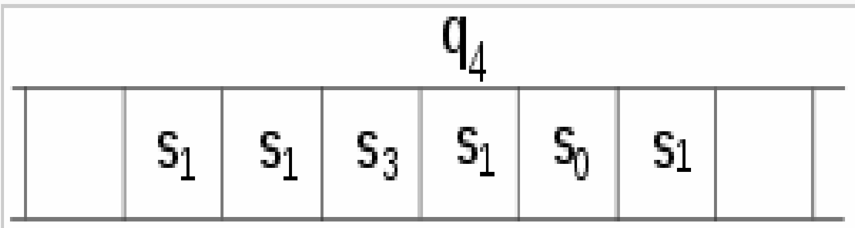
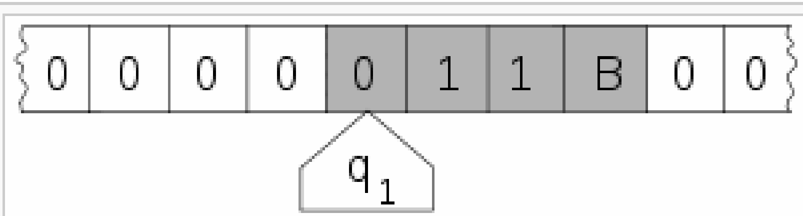


The head is always over a particular square of the tape; only a finite stretch of squares is given. The instruction to be performed (q_4) is shown over the scanned square. (Drawing after Kleene (1952) p.375.)



Here, the internal state (q_1) is shown inside the head, and the illustration describes the tape as being infinite and pre-filled with "0", the symbol serving as blank. The system's full state (its *configuration*) consists of the internal state, the contents of the shaded squares including the blank scanned by the head ("11B"), and the position of the head. (Drawing after Minsky (1967) p. 121).

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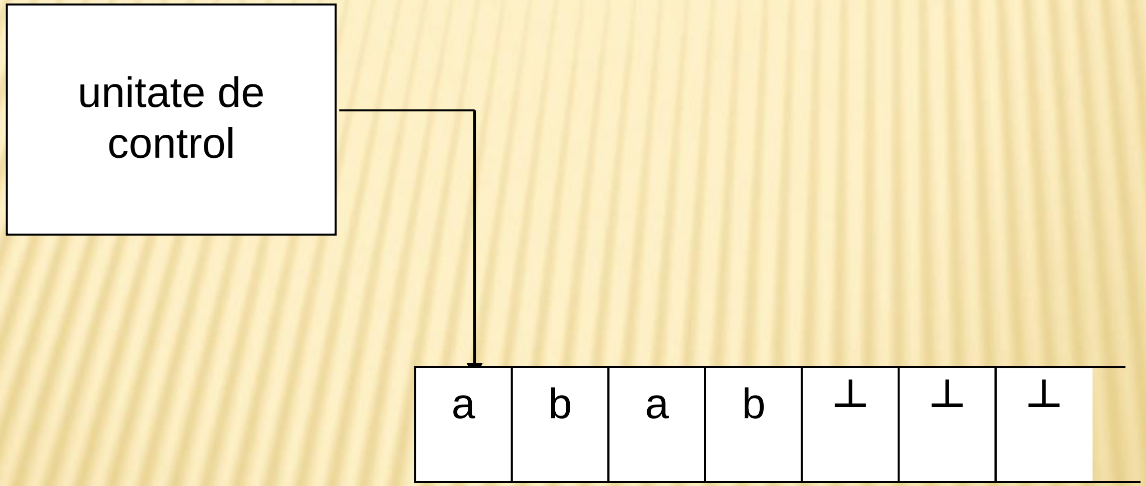
Descriere informala



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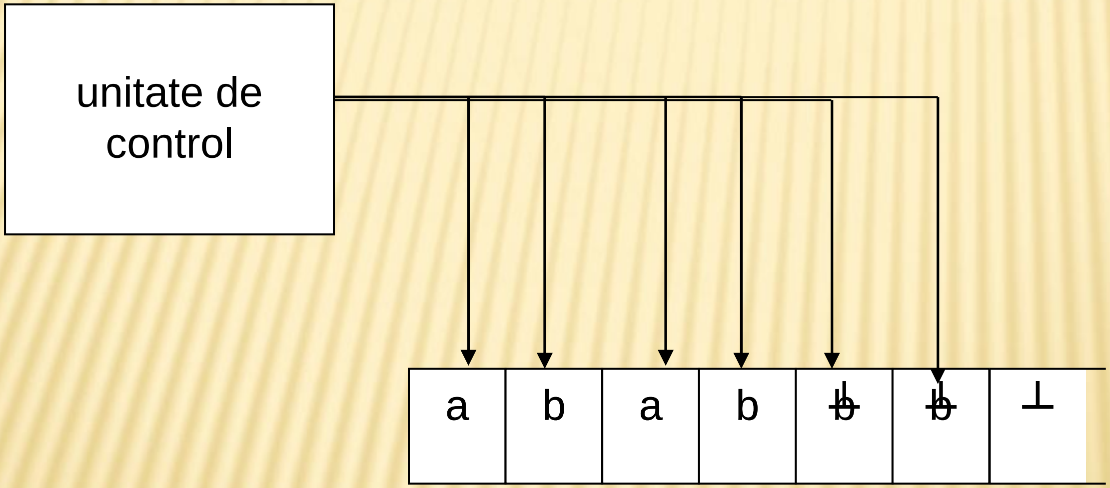

Descriere informala (cont.)

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MT poate sa simuleze

- orice calculator real
- orice limbaj de programare.

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<u>Exemplu</u>

Fie L = {w#w | w$\in${0,1}*}

Construim o MT care sa testeze apartenenta unei secvente ‘’binare” la L;

idee:

avem voie sa ne deplasam la stanga si la dreapta in secventa de intrare si putem “marca” un simbol, odata ce l-am examinat

Cursorul va scana in mod repetat secventa de intrare:

- la fiecare trecere va compara un simbol din stanga cu unul situat in dreapta lui # si, daca coincid, le inlocuieste cu x;
- daca toate simbolurile din secventa au fost inlocuite cu x, atunci MT trece in una dintre starile finale de acceptare; altfel trece in una dintre starile finale de respingere.

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$L = \{w\#w \mid w \in \{0,1\}^*\}$

=> *algoritmul:*

(P1) se scaneaza secventa de intrare **s** in cautarea unor simboluri $s \notin \{0,1\}$:

daca un astfel de simbol ESTE gasit, atunci secventa este respinsa;

(P2) se scaneaza secventa de intrare **s** in cautarea simbolului special # :

daca simbolul NU este gasit, atunci secventa este respinsa;

(P3) se scaneaza prima pereche de simboluri cele mai din stanga din cele 2 subsecvente;

daca NU coincid, atunci secventa este respinsa; altfel, se inlocuiesc cu x [si se trece la pasul 4];

(P4) se scaneaza urmatoarea pereche de simboluri, pana se epuizeaza simbolurile din stanga lui #;

daca la dreapta lui # mai raman simboluri binare, atunci secventa de intrare este respinsa; altfel, secventa de intrare este acceptata.

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1. Exemple
2. Definitia formala
3. Limbaje Turing-recunoscute si limbaje Turing-decidabile

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Definitia 1

MT = un sistem (Q, Σ, Γ, δ, q_0, q_a, q_r) unde:

- Q = mulţime finită, nevida: mulţimea stărilor;
- Σ = mulţime finită, nevida: alfabetul de intrare; $\Sigma \cap Q = \emptyset$;
- Γ = mulţime finită, nevida: alfabetul benzii; $\Sigma \subset \Gamma$, $␣ \in \Gamma \setminus \Sigma$;
- $\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{ L , R \}$: funcţia de tranziţie;
- $q_0 \in Q$: starea iniţială;
- $q_a \in Q$: starea finala de acceptare a secventei de intrare;
- $q_r \in Q$: starea finala de respingere a secventei de intrare

Notatie

MT = {M | M este o masina Turing}.

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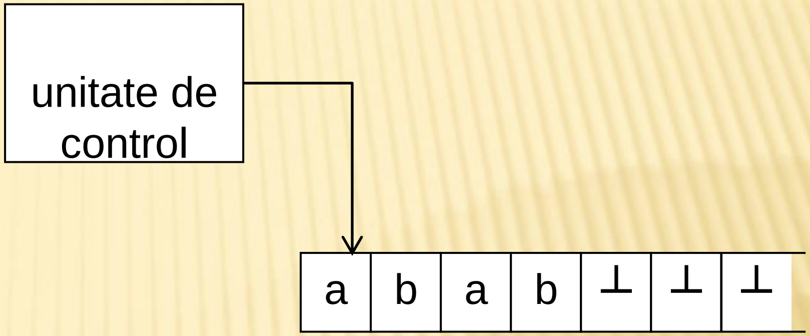

<u>Observatie</u>: Modul de calcul al MT:

1) Initial, MT se afla in starea q_0

si primeste pe banda, in primele n locatii din extr. stg., secventa de intrare $w=w_1w_2\ldots w_n\in\Sigma^*$.

Primul blank $\perp$ care apare pe banda marcheaza sfarsitul secv. de intrare.

2) Initial,cursorul se afla in extremitatea stanga a benzii (in prima locatie):

$\delta(q_0,w_1)= (p,b,R)$, unde $p=q_0$ sau $p=q_i$

$\Leftrightarrow$ MT, aflata in starea q_0, citeste pe banda de intrare simbolul w_1 =>

- MT trece in starea p,
- inlocuieste simbolul w_1 cu simbolul b in celula examinata si
- deplaseaza cursorul cu o celula la dreapta celulei examinate.

3) Daca MT incearca sa deplaseze cursorul <u>dincolo</u> de extremitatea stanga a benzii, acesta <u>ramane</u> in dreptul primei locatii din extremitatea stanga.

4) Calculul continua pana MT ajunge in q_a sau q_r si se opreste. Altfel, cicleaza nedefinit.

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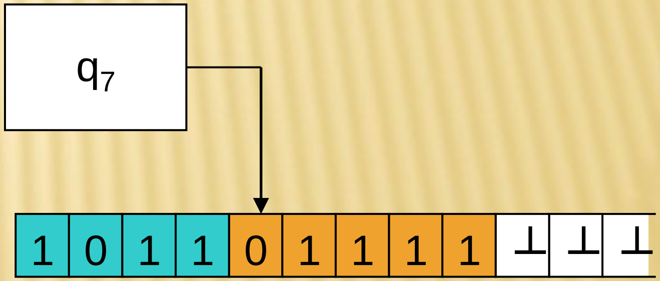
Definitia 2

Configuratie a unei $M \in MT$ = un triplet format din:

- starea curenta a M, q;
- continutul curent al benzii, v·w ;
- pozitia curenta a cursorului

Notatie

vqw, $v,w \in \Gamma^*$, $q \in Q$.



Exemplu

Configuratia $1011q_7 01111$ inseamna

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<u>Definitia 3</u>

Configuratia C_1 produce configuratia C_2 $\Leftrightarrow$

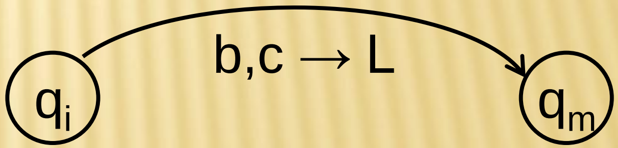
MT trece – corect – din C_1 in C_2 intr-un singur pas $\Leftrightarrow$

Fie $a,b,c \in \Gamma$,

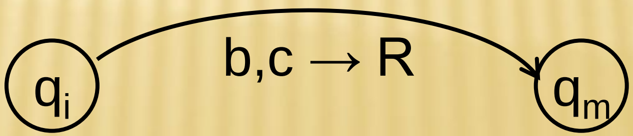
$v,w \in \Gamma^*$,

$q_i, q_m \in Q$

Spunem ca o configuratie vaq_ibw <u>produce</u> configuratia vq_macw daca $\delta(q_i,b)=(q_m,c,L)$;



Analog: o configuratie vaq_ibw <u>produce</u> configuratia $vacq_mw$ daca $\delta(q_i,b)=(q_m,c,R)$.



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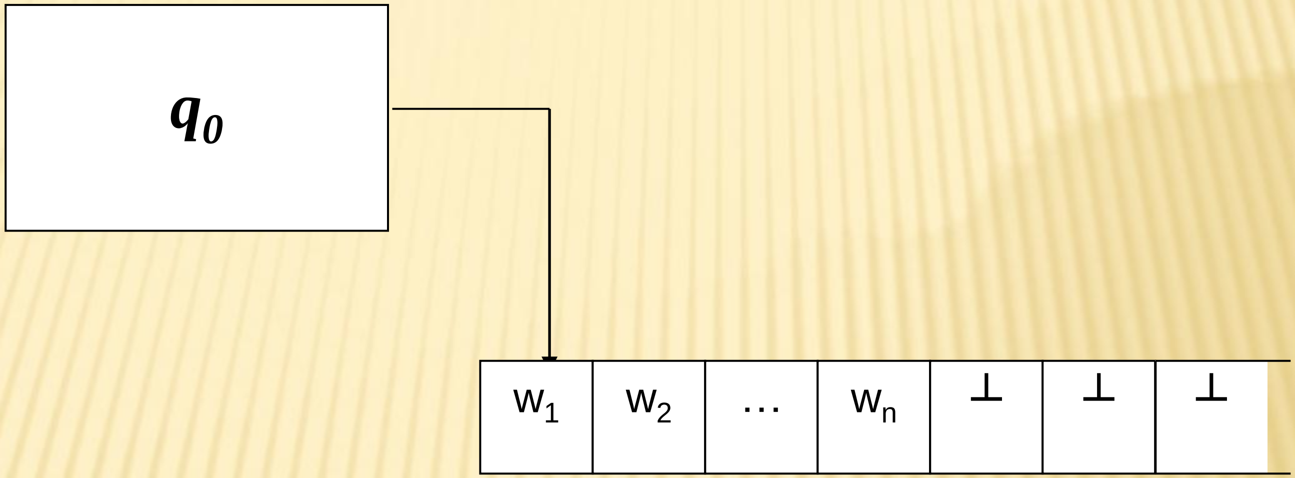

<u>Cazuri particulare de configuratii</u>:

1) Configuratia initiala: $q_0 w$;

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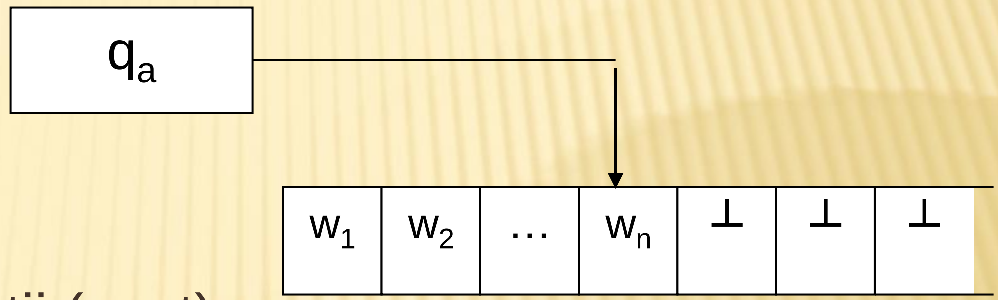
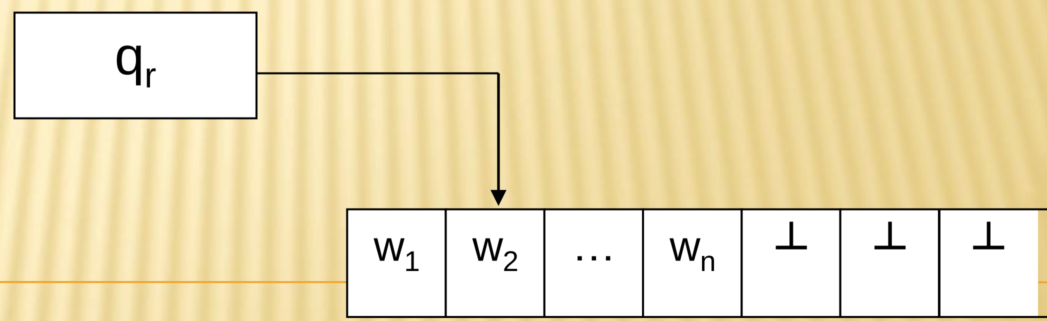


Cazuri particulare de configuratii (cont):

2) Configuratii de oprire:

- configuratia de acceptare : $q=q_a$,



- configuratia de respingere : $q=q_r$;

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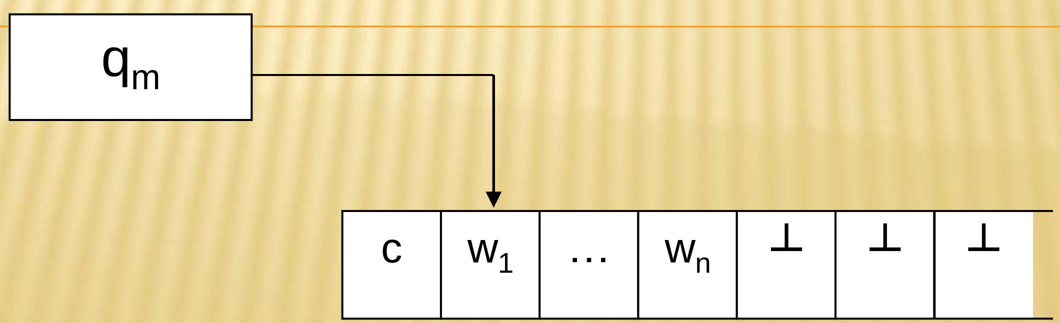
<u>Cazuri particulare de configuratii (cont)</u>:

3) Cursorul este in extr. stanga a benzii => config. curenta $q_i bw$ produce:

- $q_m cw$ iff cursorul ramane pe loc,

- $cq_m w$ iff cursorul se deplaseaza la dreapta;



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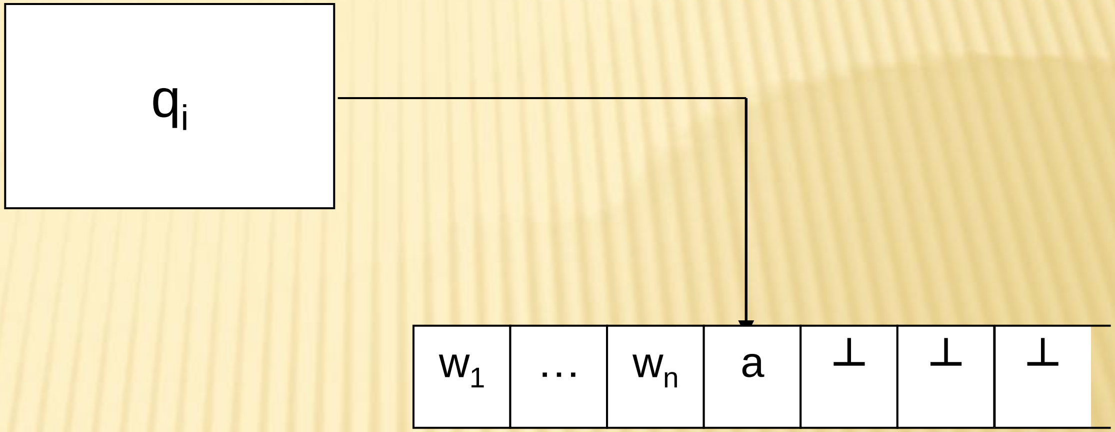

<u>Cazuri particulare de configuratii (cont.)</u>:

4) Cursorul este in extr. dreapta a benzii =>

config. curenta waq_i este echivalenta cu $waq_i\perp$.

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Definitia 4

$M \in MT$ M accepta secventa de intrare $w \in \Sigma^* \Leftrightarrow$

$\exists$ o secventa de configuratii $C_1, C_2, \ldots, C_s$ astfel incat:

1) C_1 este configuratia initiala a lui M pentru intrarea w,
2) $\forall\ 1 \leq i \leq s-1$: $C_i \rightarrow C_{i+1}$,
3) C_s este o configuratie de acceptare;

Definitia 5

Fie $M \in MT$:

$L(M)$ = limbajul masinii Turing M = $\{w \in \Sigma^* \mid M \text{ accepta } w\}$.

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Definitia 6

Limbajul $L \subseteq \Sigma^*$ se numeste Turing-acceptat (Turing-recunoscut) = recursiv enumerabil $\Leftrightarrow$

$\exists$ $M \in MT$: $L=L(M)$.

Definitia 7

1) $M \in MT$ se numeste decidenta $\Leftrightarrow$ M se opreste indiferent ce secventa primeste la intrare.

2) Fie $M \in MT$ si $L \subseteq \Sigma^*$; Spunem ca M decide asupra limbajului L $\Leftrightarrow$

(i) $L=L(M)$,

(ii) M este decidenta.

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Definitia 8

Limbajul $L \subseteq \Sigma^*$ se numeste [Turing-]decidabil = recursiv $\Leftrightarrow$

$\exists$ $M \in MT$ decidenta: $L=L(M)$.

Observatie

$\forall$ L Turing-decidabil => L este Turing-acceptat dar

(rec.) (r.e.)

<≠

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Exemplul 1

$L_1 = \{w \in \{0,1\}^* \mid \exists n \in N: |w|=2n+1\}$ este Turing-decidabil =>

M_1="Fie secventa de intrare $w \in \{0,1\}^*$:

1. Se scaneaza banda si se bareaza primul simbol intalnit (nemarcat).

Daca nu exista, atunci M_1 respinge.

2. Se scaneaza banda si se bareaza primul simbol intalnit (nemarcat).

Daca nu exista, atunci M_1 accepta; altfel, se aduce cursorul in extremitatea stanga si se reia de la Pasul 1."

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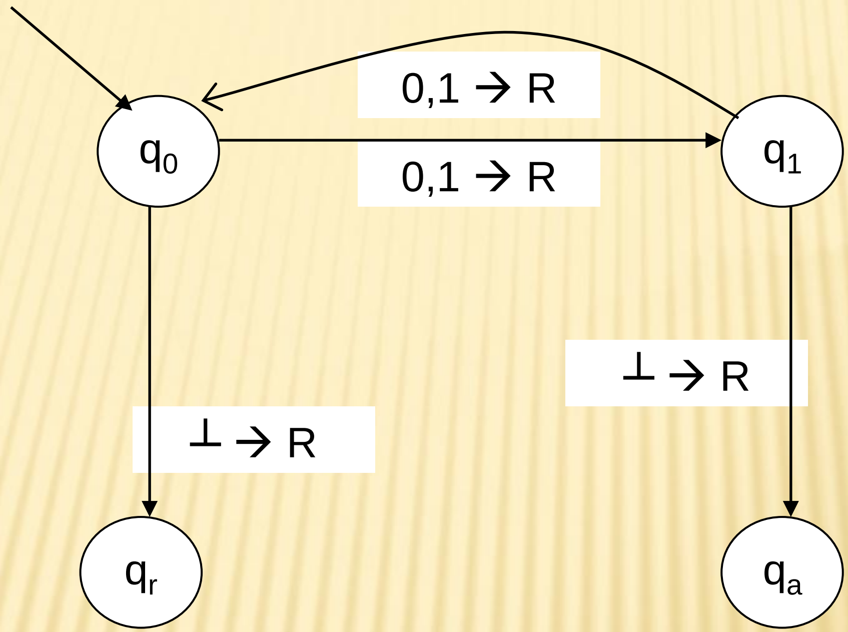

Definim formal $M_1=(\{q_0, q_a, q_r, q_1\}, \Sigma=\{0,1\}, \Gamma=\{0,1,\perp\}, \delta, q_0, q_a, q_r)$, unde δ:

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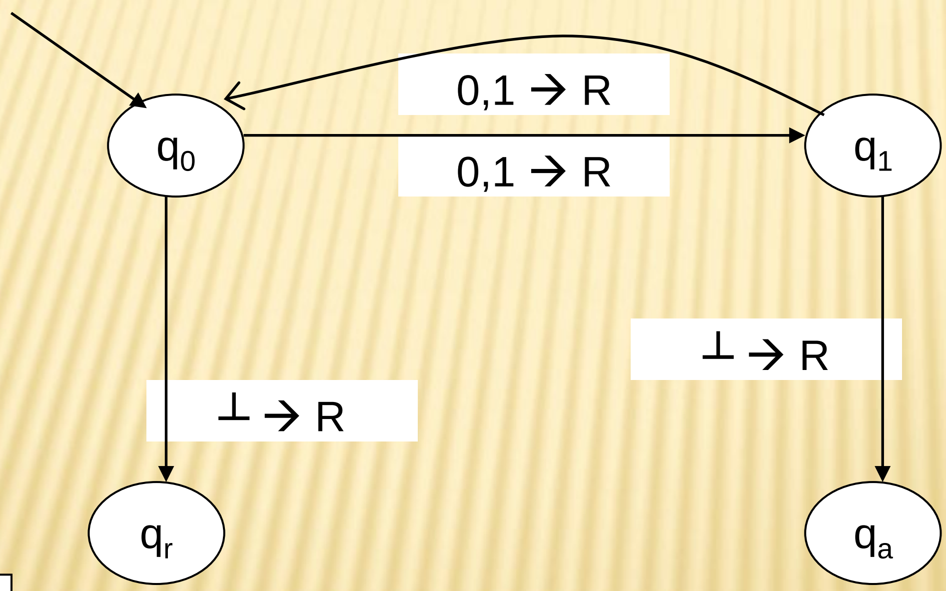
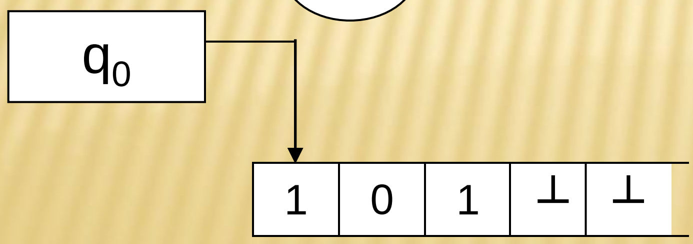
Exemplu de secventa acceptata de M_1:

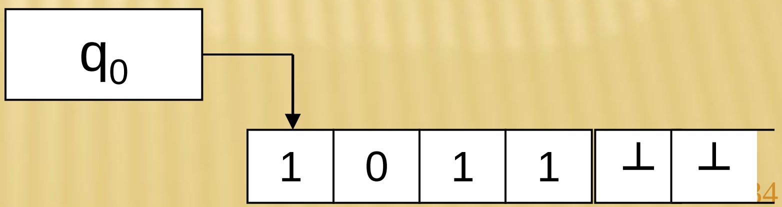
$q_0 101 =$
$q_0 101\perp \rightarrow$
$1 q_1 01\perp \rightarrow$
$10 q_0 1\perp \rightarrow$
$101 q_1 \perp \rightarrow$
$101 q_a$





Exemplu de secventa respinsa de M_1:

$q_0 1011 =$
$q_0 1011\perp \rightarrow$
$1 q_1 011\perp \rightarrow$
$10 q_0 11\perp \rightarrow$
$101 q_1 1\perp \rightarrow$
$1011 q_0 \perp \rightarrow$
$1011 q_r$.



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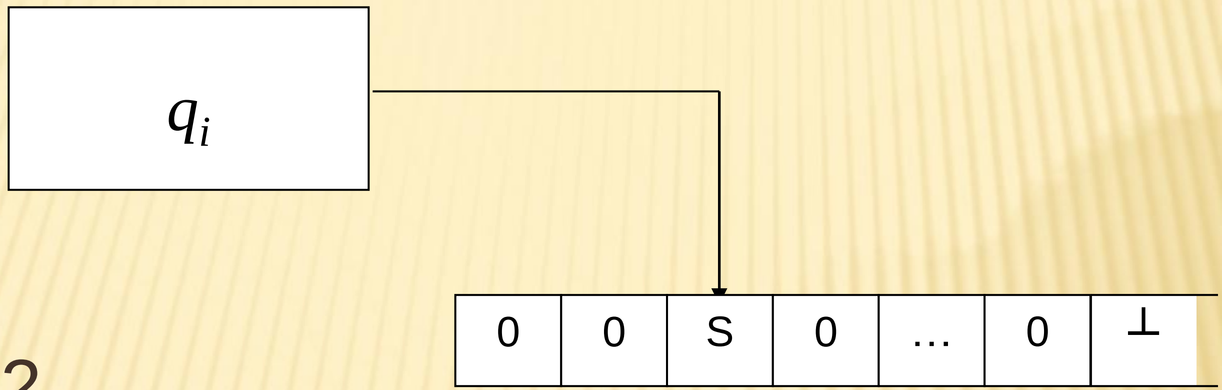

Exemplul 2

$L_2 = \{0^K \mid \exists n \in N: k=2^n\}$ este Turing-decidabil =>

M_2="Fie secventa de intrare $w \in \{0\}^*$:

1. Se scaneaza banda de la stanga la dreapta;

daca se intalneste un simbol diferit de 0, atunci M_2 respinge.

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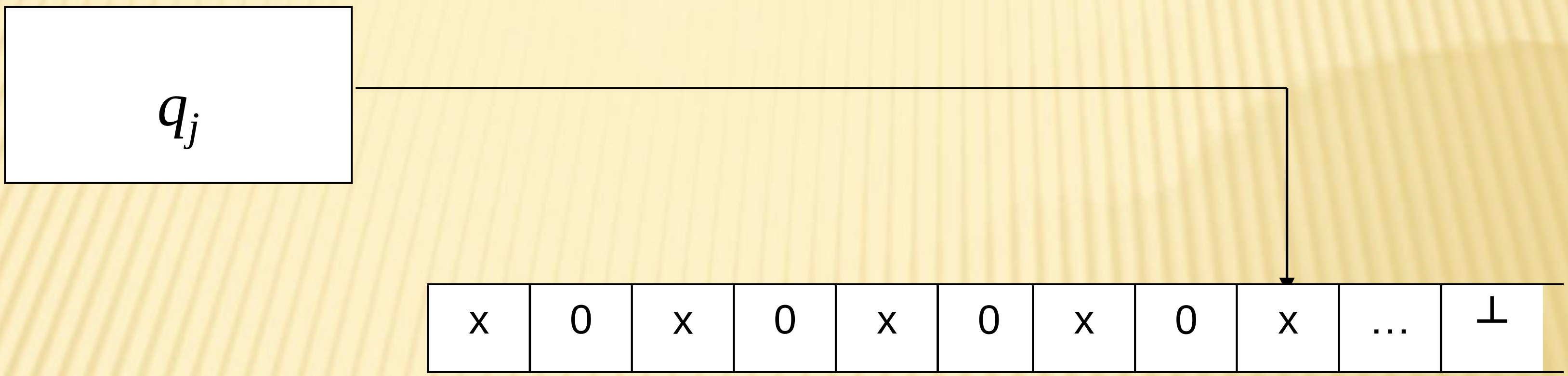

2. Se aduce cursorul in extremitatea stanga si se scaneaza banda, barandu-se simbolurile 0 **din 2 in 2**, incepand cu **primul** 0 nebarat intalnit.

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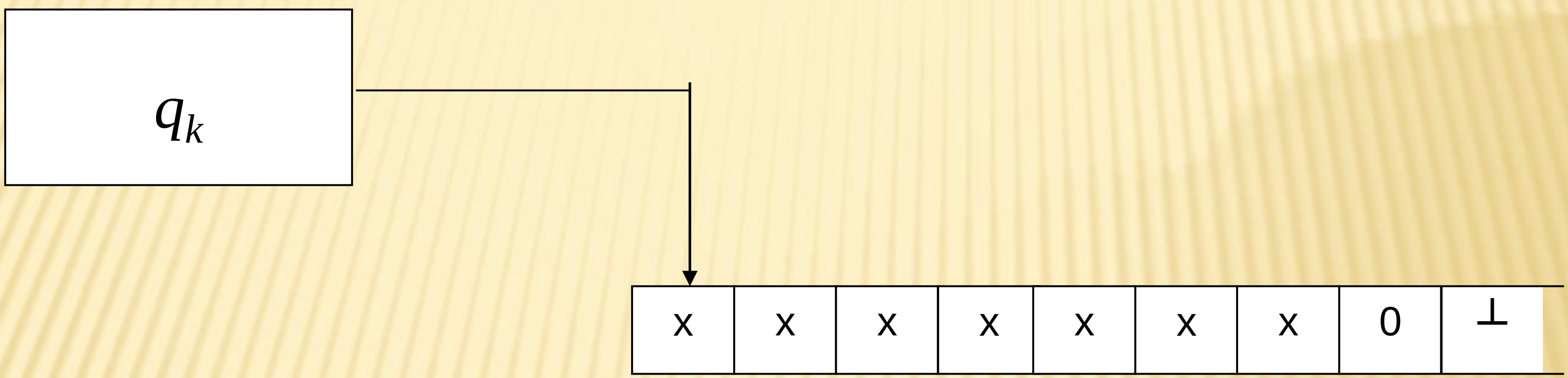

3. Daca la Pasul 2 banda continea un singur simbol 0, atunci M_2 accepta.

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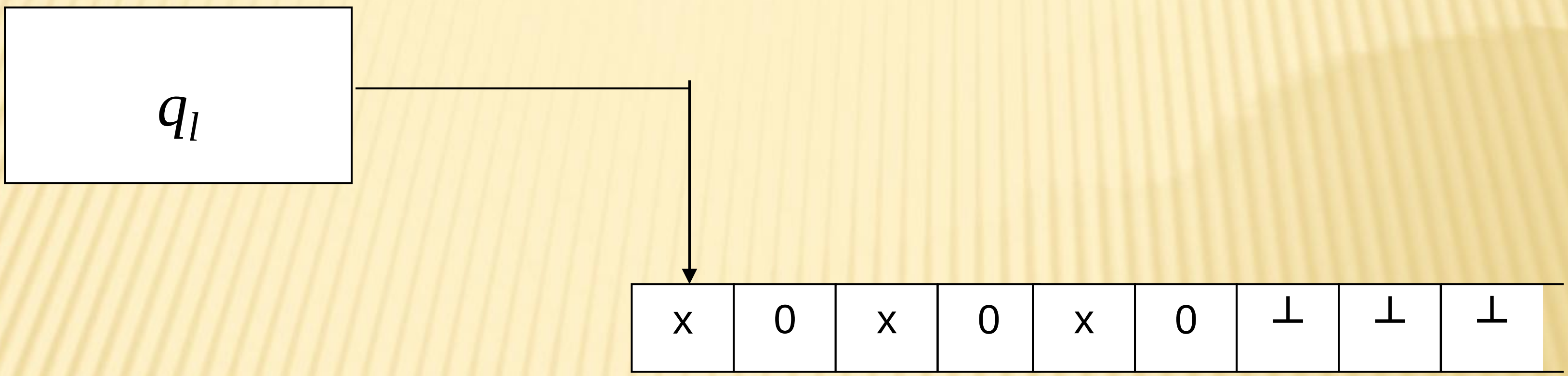

4. Daca la Pasul 2 banda continea mai mult de un simbol 0 si **numarul de simboluri 0 era impar**, atunci M_2 respinge.

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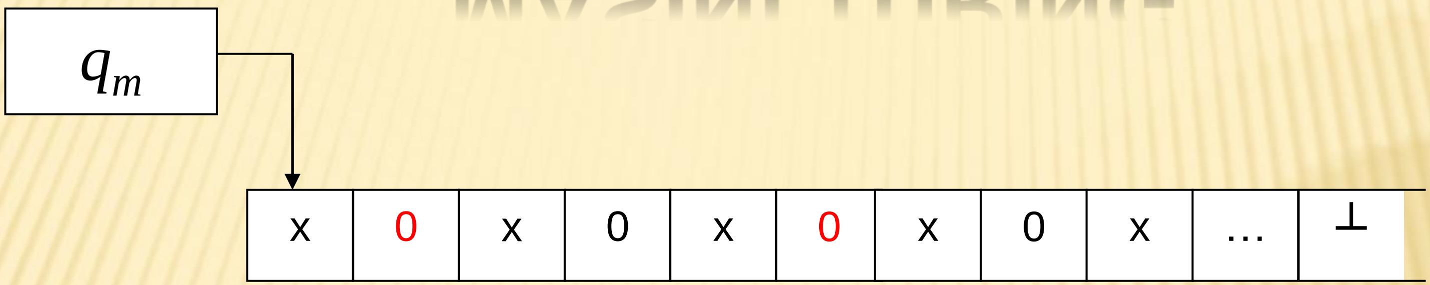
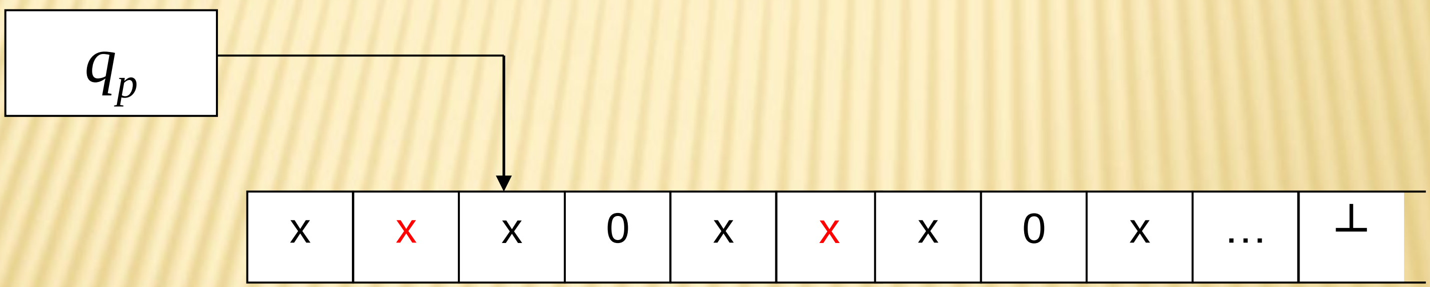
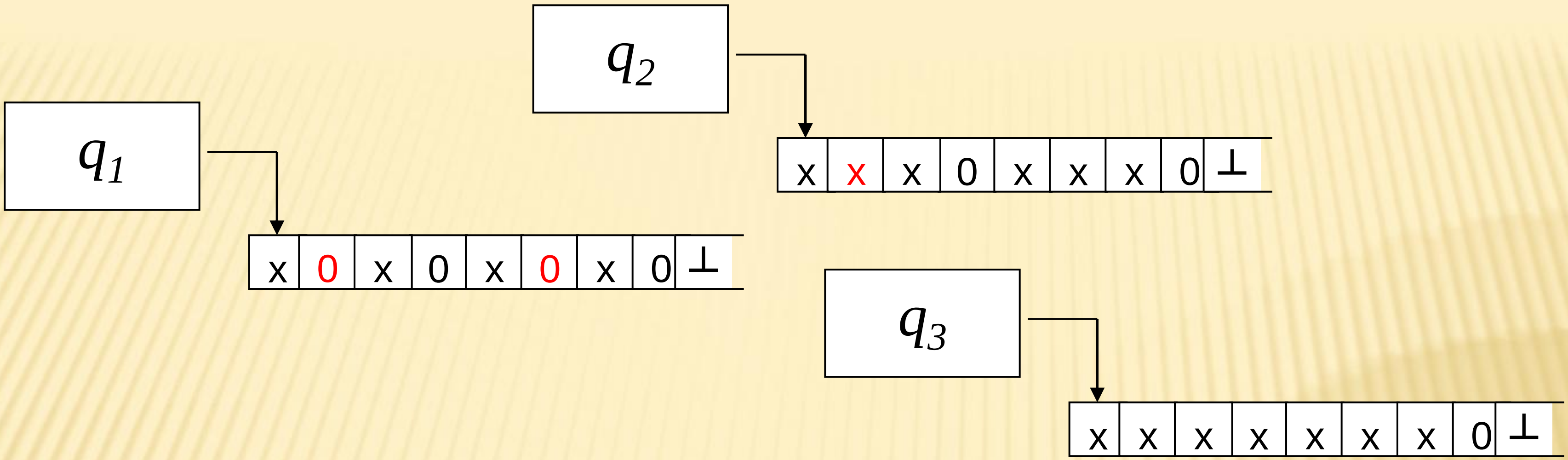


5. Se readuce cursorul in extremitatea stanga a benzii.



6. Se reia Pasul 2.”



Exemplul 2 (sintetic)

$L_2 = \{0^K \mid \exists n \in N: k=2^n\}$ este Turing-decidabil =>

M_2="Fie secventa de intrare $w \in \{0\}^*$:

1. Se scaneaza banda de la stanga la dreapta;

daca se intalneste un simbol diferit de 0, atunci M_2 respinge.

2. Se aduce cursorul in extremitatea stanga si se scaneaza banda, barandu-se simbolurile 0 din 2 in 2, incepand cu **primul** 0 nebarat intalnit.

3. Daca banda contine ACUM un singur simbol 0, atunci M_2 accepta.

Daca banda contine mai mult de un simbol 0 si numarul de simboluri 0 este impar, atunci M_2 respinge.

4. Se readuce cursorul in extremitatea stanga a benzii si se reia Pasul 2."

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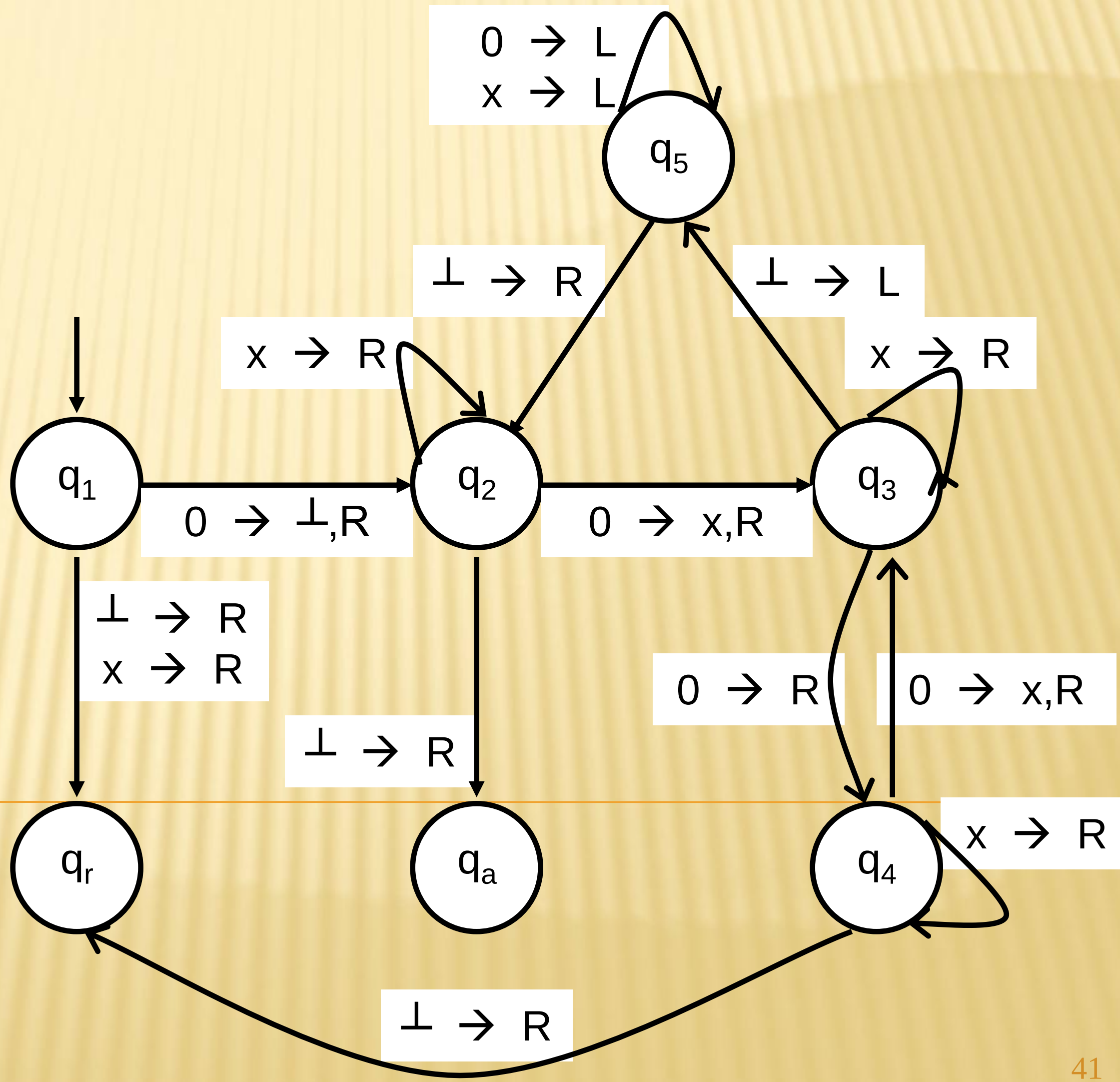

Definim formal

$M_2=(\{q_1,q_a,q_r,q_2,q_3,q_4,q_5\},$
$\Sigma=\{0\}, \Gamma=\{0,x,\perp\}, \delta, q_1, q_a, q_r),$

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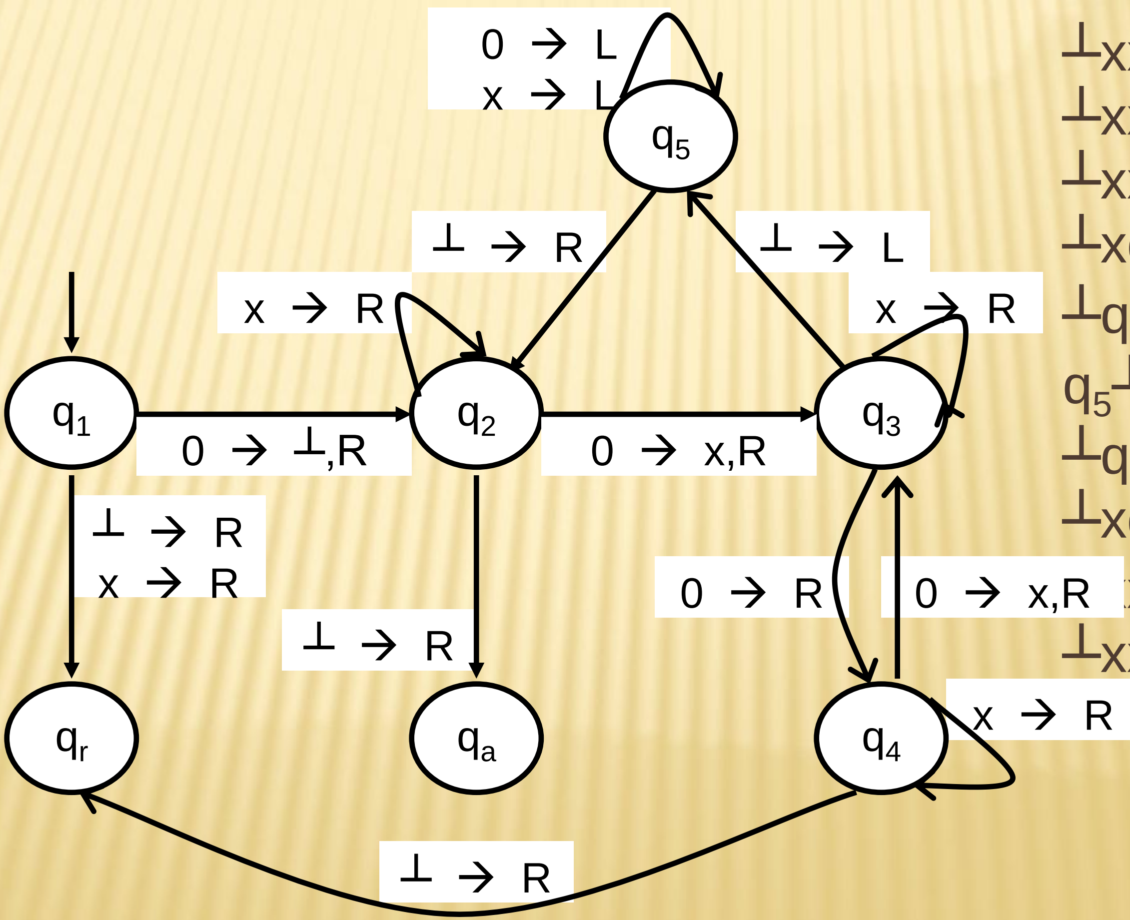
Exemplu de secventa acceptata de M_2: 0000

$q_1 0000 \rightarrow$
$\perp q_2 000 \rightarrow$
$\perp x q_3 00 \rightarrow$
$\perp x 0 q_4 0 \rightarrow$
$\perp x 0 x q_3 \perp \rightarrow$
$\perp x 0 q_5 x \perp \rightarrow$
$\perp x q_5 0 x \perp \rightarrow$
$\perp q_5 x 0 x \perp \rightarrow$
$q_5 \perp x 0 x \perp \rightarrow$
$\perp q_2 x 0 x \perp \rightarrow$
$\perp x q_2 0 x \perp \rightarrow$



$\perp x x q_3 x \perp \rightarrow$
$\perp x x x q_3 \perp \rightarrow$
$\perp x x q_5 x \perp \rightarrow$
$\perp x q_5 x x \perp \rightarrow$
$\perp q_5 x x x \perp \rightarrow$
$q_5 \perp x x x \perp \rightarrow$
$\perp q_2 x x x \perp \rightarrow$
$\perp x q_2 x x \perp \rightarrow$
$x q_2 x \perp \rightarrow$
$\perp x x x q_2 \perp \rightarrow$
$x x \perp q_a \rightarrow$

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<u>Observatie</u>

Testul din Pasul 4 al MT M_2 se poate efectua si prin apelarea MT M_1.

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Exercitii

MT pentru limbajele:

$L_3 = \{ \#_0 w = \#_1 w \mid w \in \{0,1\}^* \}$;

$L_4 = \{ 0^n 1^n \mid n \in N \}$;

$L_5 = \{ a^i b^j c^k \mid i \cdot j = k,\ \ i,j,k \in N^+ \}$;

$L_6 = \{ \#x_1\#x_2\#\ldots\#x_n \mid x_i \in \{0,1\}^* \text{ si } x_i \neq x_j,\ \forall 1 \leq i \neq j \leq n,\ n \in N \}$.

Simularile se pot efectua pe:

http://math.hws.edu/eck/js/turing-machine/TM.html

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1. Exemple
2. Definitia formala
3. Limbaje Turing-recunoscute si limbaje Turing-decidabile