Synoptic :→

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Make UP Examination
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Max. Marks: 100
Class: SE (Comp and IT)
Course Code: BS41
Name of the Course: Applied Mathematics-II

Duration: 3 Hours
Semester: IV
Branch: Comp and IT

Instructions:
(1) All questions are compulsory
(2) Assume suitable data if necessary

Q1 :→

Ans (a)

$$\begin{vmatrix} 5-\lambda & -6 & -6 \\ -1 & 4-\lambda & 2 \\ 3 & -6 & -4-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0$$

— (2) marks

$$\Rightarrow \lambda = 1, 2, 2$$

we know that each characteristic root of A is also a root of the minimal poly. of A. So if $f(x)$ is the minimal poly of A then $(x-1)(x-2)$ are the factors of $f(x)$.

— (1) mark

Let us see whether the poly $(x-2)(x-1) = x^2 - 3x + 2$ annihilates A or not

$$A^2 = \begin{bmatrix} 13 & -18 & -18 \\ -3 & 10 & 6 \\ 9 & -18 & -14 \end{bmatrix}$$

— (1) mark

$\therefore A^2 - 3A + 2I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$f(x) = x^2 - 3x + 2$ annihilates A

Thus $f(x)$ is a monic & minimal poly of A. Hence A is a delogatory.

② marks

Q1 :→

Ans b) $A'A = \begin{bmatrix} 4 & 6 \\ 6 & 13 \end{bmatrix}$

The charac eqn of $A'A$ is $\begin{vmatrix} 4-\lambda & 6 \\ 6 & 13-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda^2 - 17\lambda + 16 = 0 \Rightarrow \lambda = 16, 1$

$\sigma_1 = \sqrt{16}$, $\boxed{\sigma_1 = 4}$, $\boxed{\sigma_2 = 1}$

$\therefore D = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$

$v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\quad v_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

Since $(v_1 \cdot v_2) = (1 \ 2) \cdot (-2 \ 1) = 0$

② marks

$\therefore v_1, v_2$ are orthogonal

$\|v_1\| = \sqrt{5}$ $\quad \|v_2\| = \sqrt{5}$

$v_1 = \frac{v_1}{\|v_1\|} = \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$, $\quad v_2 = \frac{v_2}{\|v_2\|} = \begin{bmatrix} -2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$

$\therefore V = \begin{bmatrix} 1/\sqrt{5} & -2/\sqrt{5} \\ 2/\sqrt{5} & 1/\sqrt{5} \end{bmatrix}$

$u_1 = \frac{1}{\sigma_1} A v_1 = \frac{1}{4} \begin{bmatrix} 8/\sqrt{5} \\ 4/\sqrt{5} \end{bmatrix} = \begin{bmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$

② marks

$$v_2 = \begin{bmatrix} -1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$$

$$\therefore U = \begin{bmatrix} 2/\sqrt{5} & -1/\sqrt{5} \\ 1/\sqrt{5} & 2/\sqrt{5} \end{bmatrix}$$

— (2) marks

$$A = UDV^T = \begin{bmatrix} 2/\sqrt{5} & -1/\sqrt{5} \\ 1/\sqrt{5} & 2/\sqrt{5} \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{5} & 2/\sqrt{5} \\ -2/\sqrt{5} & 1/\sqrt{5} \end{bmatrix}$$

(3)

Q1:

Ans (c)

$$\begin{vmatrix} 10-\lambda & -2 & -5 \\ -2 & 2-\lambda & 3 \\ -5 & 3 & 5-\lambda \end{vmatrix} = 0$$

$\lambda^3 - 17\lambda^2 + 42\lambda = 0 \Rightarrow \lambda = 0, 3, 14$

Let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ such that $[A - \lambda I]X = 0$

— (2) marks

when $\lambda = 0$ $\therefore \begin{bmatrix} 1 \\ -5 \\ 4 \end{bmatrix}$ is the 1st Eigen vector

— (2) marks

$\lambda = 3$ $\therefore \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\lambda = 14$, $\begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}$

— (4) marks

or

Q1:

Ans (c) i)

$$M = \begin{array}{c} \begin{matrix} A & B & C & D \end{matrix} \\ \begin{bmatrix} 0 & 0 & 1/3 & 0 \\ 1/2 & 0 & 1/3 & 0 \\ 1/2 & 0 & 0 & 1 \\ 0 & 1 & 1/3 & 0 \end{bmatrix} \end{array}$$

$PR_{t+1} = M \cdot PR_t$

— (1) Mark

Let $PR = \begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix}$ —— (1) ma

$PR_1 = \begin{bmatrix} 1/12 \\ 2.5/12 \\ 6/12 \\ 4/12 \end{bmatrix}$ → After 1st iteration —— (1) mark

We can have as many iterations for accurate answer (equilibrium value)

After 2nd iteration

$PR_2 = \begin{bmatrix} 2/12 \\ 15/12 \\ 4.5/12 \\ 3.5/12 \end{bmatrix}$ $PR = \begin{bmatrix} 1 \\ 4 \\ 2 \\ 3 \end{bmatrix}$ —— (2) marks

Node B has the highest value that's why PR = 4 for B.

Ans
C ii) :→

Dangling Node :→ Nodes with no outgoing cycles.

problem : Algorithm discussed so far won't work

No solution —— (1) mar

Let $PR_0 = \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix}$, $M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}$ — (1 mark) (3)

Initial page rank Transition matrix

$v_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2/3 \end{pmatrix}$

$v_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 6 \\ 2/3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ — (1) mark

not possible

Q2:

Ans a) prepare the table — (4) marks

$N = 8$, $\Sigma x = 544$, $\Sigma x^2 = 36$, $\Sigma y = 552$,
$\Sigma y^2 = 44$, $\Sigma xy = 24$,

$r = \dfrac{\Sigma xy}{\sqrt{\Sigma x^2 \cdot \Sigma y^2}} = 0.6030$ — (2) marks

Ans (b) prepare the table — (4) marks

$N = 8$, $\Sigma D^2 = 4$

$R = 1 - \dfrac{6\,\Sigma D^2}{N^3 - N} = 0.952$ — (2) marks

Ans (c) prepare the table — (3) marks
$\Sigma x = 56$, $\Sigma y = 40$, $\Sigma xy = 364$, $\Sigma x^2 = 524$

Let $y = a + bx$ be the eqn of line of regression of y on x — 1 marks

$\Sigma y = na + b\Sigma x$

$40 = 8a + 56b$ — (1)

And $\Sigma xy = a\Sigma x + b\Sigma x^2$

$364 = 56a + 524b$ — (2)

— 2 marks

solving (1) & (2) $\therefore \boxed{a = \frac{6}{11}}$, $\boxed{b = \frac{7}{11}}$

$\therefore y = \frac{6}{11} + \frac{7}{11}x$

$\Rightarrow y = \frac{6}{11} + \frac{7}{11} \times 10 = \frac{76}{11}$

— 2 marks

or

Ans (c) Line of regression of x on y is

$2x - 9y + 6 = 0 \Rightarrow 2x = 9y - 6$

$\Rightarrow x = \frac{9y}{2} - 3 \Rightarrow \boxed{b_{xy} = \frac{9}{2}}$

Line of regression of y on x be

$x - 2y + 1 = 0 \Rightarrow \boxed{b_{yx} = \frac{1}{2}}$

$\therefore r = \sqrt{b_{xy} \cdot b_{yx}} = \sqrt{\frac{9}{2} \cdot \frac{1}{2}} = \frac{3}{2} > 1$

— 4 marks

which is not possible. So our choice of line of regression is incorrect.

Now, we can take the line of regression of x on y be $x - 2y + 1 = 0$

$\Rightarrow x = 2y - 1 \Rightarrow \boxed{b_{xy} = 2}$

— 2 marks

2nd Line of regression of y on x is
$2x - 9y + 6 = 0 \Rightarrow b_{yx} = \frac{2}{9}$
$r = \sqrt{b_{xy} \cdot b_{yx}} = \sqrt{2 \cdot \frac{2}{9}} = \frac{2}{3} < 1$
Hence the correlation coeffi betn
x & y is $\frac{2}{3}$. — 2 marks

Q3:→

Ans a) Let X_i denote the no. of pts. on the ith dice.
If S denotes the sum of the points of n dice Then $S = \sum_{i=1}^{n} E(X_i)$ — ① — 2 marks

$E(X_i) = \sum_{x} P_i x_i$
$= \frac{1}{6} \cdot 1 + \frac{1}{6} \cdot 2 + \frac{1}{6} \cdot 3 + \cdots \frac{1}{6} \cdot 6$
$= \frac{1}{6}(1 + 2 + \cdots 6) = \frac{7}{2}$ — 2 marks

① $\Rightarrow S = \frac{n7}{2}$

If π denotes the product of pts.
$E(\pi) = E(X_1) \cdot E(X_2) \cdots E(X_n)$
$= \frac{7}{2} \cdot \frac{7}{2} \cdots \cdots \frac{7}{2} = \left(\frac{7}{2}\right)^n$ — 2 marks

Ans b) :→ $\text{cov}(X, Y) = E(XY) - E(X) \cdot E(Y)$
$= 4$
$\rho = \frac{\text{Cov}(X, Y)}{\sigma_x \cdot \sigma_y}$ — 1 mark

$\sigma_x = \sqrt{\text{Var}(X)}$, $\sigma_y = \sqrt{\text{Var}(Y)}$ — 2 marks

$Var(X) = E(X^2) - [E(X)]^2$
$= 5$
$Var(Y) = E(Y^2) - [E(Y)]^2 = 7$ — 2 marks

$\sigma_x = \sqrt{5}$, $\sigma_y = \sqrt{7}$
$\therefore \rho = \frac{4}{\sqrt{35}}$ — 1 mark

Q3:-

Ans c): $P(x < \frac{1}{2}, y > \frac{1}{2})$
$= P(0 < x < \frac{1}{2}, \frac{1}{2} < y < 1)$
$= \int_{x=0}^{1/2} \int_{1/2}^{1} f(x,y)\, dx\, dy = \frac{11}{40}$ — 2 marks

$P(y < \frac{3}{4}) = P(0 < y < \frac{3}{4}) = \int_0^{3/4} f(y)\, dy$
$f(y) = \int_x f(x,y)\, dx = \int_0^1 \frac{2}{5}(2x+3y)\, dx$
$\boxed{f(y) = \frac{2}{5}(1+3y)}$ — 3 marks

$\therefore P(y < \frac{3}{4}) = \int_0^{3/4} \frac{2}{5}(1+3y)\, dy$
$= \frac{19}{40}$ — 2 marks

Now,
$f(x) = \int_y f(x,y)\, dy = \int_0^1 \frac{2}{5}(2x+3y)\, dy$
$= \frac{2}{5}\left(2x + \frac{3}{2}\right)$ — 1 mark

or

Ans (c) $E(X) = \int_x x f(x)\, dx = \int_0^4 x \frac{x}{8}\, dx$ — 1 mark

5

$\therefore f(x) = \int_y f(x,y)\,dy = \int_1^5 \frac{xy}{96}\,dy$

$= \frac{x}{8}$ — 2 marks

$E(X) = \int_0^4 \frac{x^2}{8}\,dx = \frac{8}{3}$

$E(Y) = \int_y y\, f(y)\,dy$, $f(y) = \int_x f(x,y)\,dx$

$= \frac{y}{12}$ — 1 mark

$\therefore E(Y) = \int_1^5 y\,\frac{y}{12}\,dy$, $E(XY) = \int_x\int_y xy\, f(x,y)\,dx\,dy$

$= \frac{31}{9}$ $= \frac{248}{27}$ — 3 marks

$E(2X+3Y) = \frac{141}{9}$ — 1 mark

Q4 :→

Ans a) $P(X=x) = \frac{e^{-m}\, m^x}{x!}$ — 1 mark

$\frac{\cancel{e^{-m}}\, m^2}{2!} = 9\,\frac{\cancel{e^{-m}}\, m^4}{4!} + 90\,\frac{\cancel{e^{-m}}\, m^6}{6!}$

$\Rightarrow \frac{1}{2} = \frac{9m^2}{4\times3\times2} + \frac{90m^4}{6\times5\times4\times3\times2}$ — 3 marks

$\therefore m^4 + 3m^2 - 4 = 0$

$\Rightarrow (m^2+4)(m^2-1) = 0$

$\Rightarrow m^2 = -4$ or 1

Discard the -ve value. The mean is 1

since $m > 0$ $\therefore$ The statement is correct. — 2 marks

Ans b) $z = \frac{x-m}{\sigma} = \frac{x-65}{5}$

$x = 75 \quad \therefore z = \frac{75-65}{5} = 2$ — (2) marks

$P(X > 75) = P(z > 2)$

$= 0.5 - (\text{area from } z=0 \text{ to } z=2)$

$= 0.5 - 0.4772 = 0.0228$ — (1) mark

P(a student has not scored more than 75)

$= 1 - 0.0228 = 0.9772$ — (1) mark

P(all 3 students have not scored more than 75 marks) $= 0.93$ — (1) mark

P(atleast one of 3 has scored more than 75 marks) $= 1 - 0.93 = 0.07$ — (1) mark

Ans c):→ $n = 10$, $P(x) = {}^{10}C_x\, p^x q^{10-x}$

$P(5) = 2 \cdot P(4)$ — (2) marks

$\therefore {}^{10}C_5\, p^5 q^{10-5} = 2\ {}^{10}C_4\, p^4 q^{10-4}$

$\therefore \frac{p}{5} = \frac{q}{3}$

$p = \frac{5}{8}, \quad q = 1 - p = \frac{3}{8}$ — (3) marks

$P(x) = {}^{10}C_x \left(\frac{5}{8}\right)^x \left(\frac{3}{8}\right)^{10-x}$

$P(x=0)$

$\therefore {}^{10}C_0 \left(\frac{5}{8}\right)^0 \left(\frac{3}{8}\right)^{10} = \left(\frac{3}{8}\right)^{10}$ — (3) marks

or

Ans c) i) $E(x) = np = 2$, $Var(x) = npq = \frac{4}{3}$ — (1) mark

$\therefore \frac{\cancel{np}q}{\cancel{np}} = \frac{4/3}{2} \Rightarrow q = \frac{2}{3}, \; p = \frac{1}{3}, \; n = 6$ — (c) 1 mark

$\therefore P(X=x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

$= {}^{6}C_{x} \left(\frac{1}{3}\right)^{x} \left(\frac{2}{3}\right)^{6-x}$ — 1 mark

$\therefore$ prob. distr of X

x:	0	1	2	3	4	5	6
$P(X)$:	$\frac{64}{729}$	$\frac{192}{729}$	$\frac{240}{729}$	$\frac{160}{729}$	$\frac{60}{729}$	$\frac{12}{729}$	$\frac{1}{729}$

— 1 mark

Ans c ii) $n=5$

The prob. for ship to be wrecked is
$q = \frac{1}{10} = 0.1 \quad \therefore p = 0.9$ — 2 marks

$P(x) = {}^{5}C_{x}(0.9)^{x}(0.1)^{5-x}$ — 1 mark

P(atleast 4 ships will arrive safely)
$= P(x \geq 4) = P(x=4) + P(x=5)$
$= 0.91854$ — 1 mark

Q5 :→

Ans a) Null Hyp (H_0): Accidents are equally distributed over all the days of a week
Alternative Hyp (H_a): Accidents do not occur equally. — 1 mark

Calculation of Test Statistic: If the accidents occur equally on all days of a week, there will be $\frac{84}{7} = 12$ accidents — 1 mark per day

$\therefore \chi^2 = \Sigma \frac{(O-E)^2}{E} = 2.33$ — ① mark

Level of significance : $\alpha = 0.05$

Degree of freedom $\nu = n-1 = 7-1 = 6$ — ① mark

Critical value : For 6 d.f. at 5% LOS the table value of χ^2 is 12.59 — ① mark

Decision : Since the calculated value is less than table value, The Hypo. is accepted.

$\therefore$ The accidents occur equally on all working days. — ① mark

Ans b) :→ Null Hypo (H_0) : $\mu = 5.4$

Alter. " (H_a) : $\mu \neq 5.4$ — ① mark

Test statistic : Here $n > 30$

$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{6.2 - 5.4}{10.24/\sqrt{50}} = 0.55$ — ② mark

LOS $= \alpha = 0.05$

Critical value : The value of z_α at 5% LOS is 1.96 — ① mark

Decision : Since the computed value of $|z| = 0.55$ is less than critical value $\therefore$ The Null Hypo. is accepted $\therefore$ The sample is drawn from the population with mean 5.4 — ② marks

5:

Ans c): Null Hypo. (H_0) : $\mu_1 = \mu_2$
Alternative " (H_a) : $\mu_1 \neq \mu_2$ — (1) mark

Calculation of Test statistic :→ unbiased Estimate of common population SD. is

$$S_p = \sqrt{\frac{\Sigma(x_i-\bar{x})^2 + (\Sigma y_i-\bar{y})^2}{n_1+n_2-2}} = 1.81$$

$$S.E. = S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = 0.91$$

$$t = \frac{\bar{x}_1-\bar{x}_2}{S.E.} = -2.64 \Rightarrow |t| = 2.64$$

— (3) marks

Level of significance : $\alpha = 0.05$
Critical value : $\nu = 9+7-2 = 14$ d.f.
The value is 2.145 — (02) marks

Decision : Since the computed value of $|t| = 2.64$ is greater than table value $t_\alpha = 2.145$, the Null Hypo. is rejected
∴ The samples cannot be considered to have been drawn from the same population.

(2) marks

or

Ans c):→ Null Hypo $H_0 : \mu_1 = \mu_2$
Alternative Hypo. $H_a : \mu_1 \neq \mu_2$ — (1) mark

Calculation of Test statistic :

$\bar{x}_1 - \bar{x}_2 = 72 - 70 = 2$

$S.E. = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{64}{32} + \frac{36}{36}} = \sqrt{3}$ (3) marks

$\therefore z = \frac{\bar{x}_1 - \bar{x}_2}{S.E} = \frac{2}{\sqrt{3}} = 1.15$

$|z| = 1.15$

Level of significance : $\alpha = 1\%$

Critical value : The value of z_α at 1% level of significance from the table is 2.58 (2) mark

Decision : → Since the computed value of $|z| = 1.15$ is less than the critical value $z_\alpha = 2.58$ the hypo. is accepted. (2) mar