Model-Based Trading: Theory and Practice

E4729 – Spring 2020

Lecture 1

COLUMBIA UNIVERSITY
IN THE CITY OF NEW YORK

Welcome to E4729

Agenda

- About Ken
- Class Organization
  - Syllabus
  - Assignments
  - Final Project
- Guest Lecturer(s)

Ken Gleason

- Automated Trading
- Technology Risk Management
- 20+ years in financial services / technology
- 15 years in electronic trading
  - Development of statistical arbitrage trading systems
  - Principal trading - market making, portfolio trading, statistical arbitrage
  - Institutional Agency execution algorithms
  - Trading Analytics
- Operational Risk in Electronic Trading
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Topics (*Order and Content Subject to Change*)

Nature of Markets and Prices
Sources for Value and Reasons for Trading
Mechanisms for trading
Liquidity and Transparency
Econometric Issues

Limit Order Markets
Limit Order Markets
Floor Markets
Dealer Markets
Limit Order Book Mechanics

Univariate Time Series Analysis
Stationarity and Ergodicity
Moving Average Models
Autoregressive Models
Forecasting, Filtering, and Model Estimation

Information-Based Models
Informed Traders and Uninformed Traders Roll Model of Trade Prices
Random Walk Model of Security Price
Roll Model of Bid, Ask and Transaction Prices

Sequential Trade Models
Trade Quantities and Price Behavior
Simple Sequential Trade Model
Market Dynamics: Bid and Ask Quotes Over Time
Order Flow and Probability of Informed Trading
Distributions of Buys and Sells
Event Uncertainty and Poisson Arrivals

Strategic Trade Models
Strategic Behavior of Informed Traders Strategic Behavior of Uninformed Traders Link between Strategic and Sequential Trade Models

Generalized Roll Model
Structural Model and Its Statistical Representation
Forecasting and Filtering
General Univariate Random Walk
Decompositions Identification in Random Walk Decompositions

Market Data Analysis
Working with bar data Working with tick data Issues with resampling Applied Time Series Analysis

Transaction Costs I: Components
Component Transaction Costs
Implementation Shortfall
Market Impact, Timing Risk and Opportunity Costs Optimal Trading Strategies

Trading Benchmarks
Benchmark Prices, BAM and VWAP
VWAP Trading Strategies
Models of Order Slicing and Timing Analyzing Algorithm Performance

Developing Execution Algorithms
Trading Objectives and Execution Strategy Approaches
Developing a framework for trade simulation using tick-level data
Target functions and tolerance functions Schedule Based Strategies, TWAP, VWAP, etc. Volume following strategies
Event Driven Strategies
Multi-factor strategies

Applied Stock Forecasting
A systematic analysis of technical indicators Lee-Ready Trade Direction Analysis Autoregressive and moving average models

Transaction Costs II: Performance Analysis
Trading Algorithm Execution and Performance
Error correction
Data enrichment: joining market data with transaction data to analyze market impact, reversion and adverse selection Interpretation of performance vs benchmarks Effective visualizations and representation

Statistical Arbitrage
Arbitrage Pricing Theory – a review Correlation, Cointegration and other methods Pairs Trading Strategies

Assignments and Final Project

- There will be three assignments

- Final Project
  - A project of your choice, relevant to the material covered in class
  - Exhibit understanding of theory and application
  - You will submit a proposal for your project (counts as 1 assignment, due date TBA)
  - The last three weeks of lecture will be reserved for group project presentations
  - More details in coming weeks

- First assignment: form groups of 5-6 and submit them on Courseworks. Due February 7th .

Grading

- Class Participation 10%
- 3 group homework assignments - 20% each
- Final Group Project 30%
  - 5% for proposal
  - 10% for presentation
  - 15% for written submission

Academic Standards Integrity

Consider the IEOR Policy on Academic Integrity

- Do not assume that past projects (found here or elsewhere) are original research.

- Beware of inadvertent "over-borrowing"
  - Use of source code without modification
  - Sequencing of final papers
  - Formatting (even if it's exactly the way you'd do it)

- Always Note Your Sources. Grades will be lowered for missing sources.

Any Questions Before We Get Started??

Lecture 1

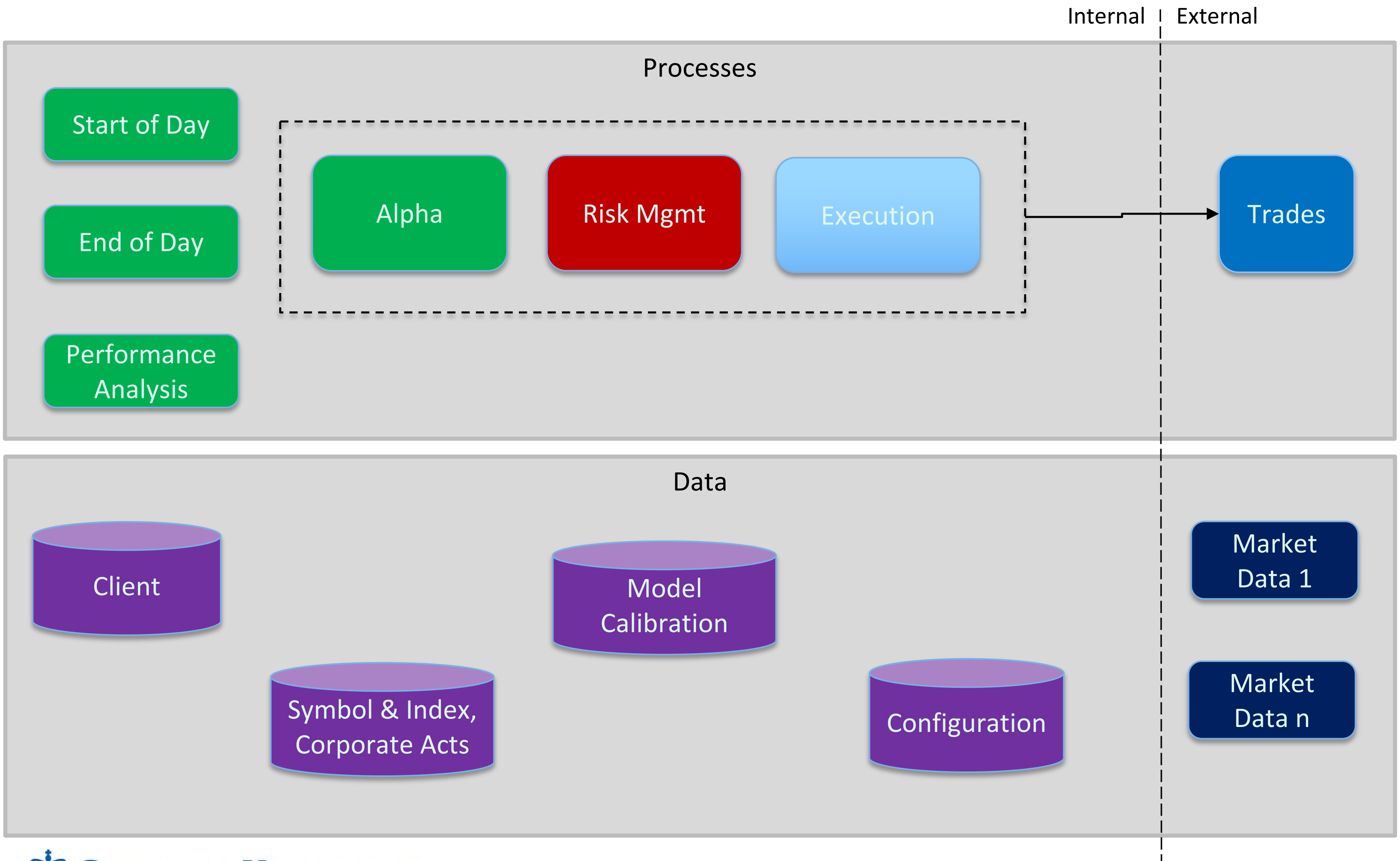
- The Model Based Trading Ecosystem
- Market Microstructure Fundamentals
- Generating Random Data

The Model-Based Trading Ecosystem

Model-Based Trading Ecosystem

- Motivation: What it means to be model-based and data-driven
  - Acquisition
  - Transformation
  - Validation
  - Maintenance
  - <u>Data is integral to every facet</u>

Trading Data Ecosystem



Market Microstructure Fundamentals

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Market Microstructure:

What is it??

Market Microstructure

- Market microstructure is the study of *processes and outcomes of exchanging assets* under explicit *trading rules/mechanisms*.
- Market microstructure research exploits the structure provided by specific trading mechanisms to model *how prices are formed* and how *price-setting rules evolve* in the markets.
- As microstructure research is set in the markets for financial assets, this enhances our ability to understand both the *returns* to financial assets and the processes by which markets become *efficient*.
- Aside from being valuable for illuminating prices and markets, microstructure research has immediate application in the *regulation of markets*, and in design and *formulation of new trading mechanisms*.

Prices and Markets

What determines a price?

In the standard economics paradigm, it is the *intersection of supply and demand curves* for a particular good:

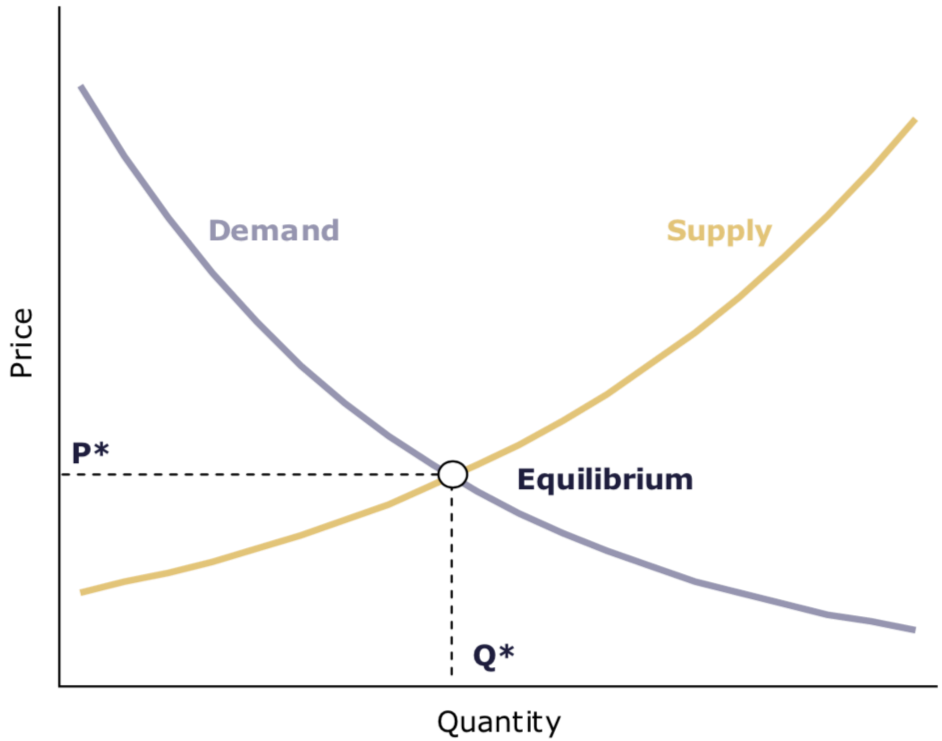
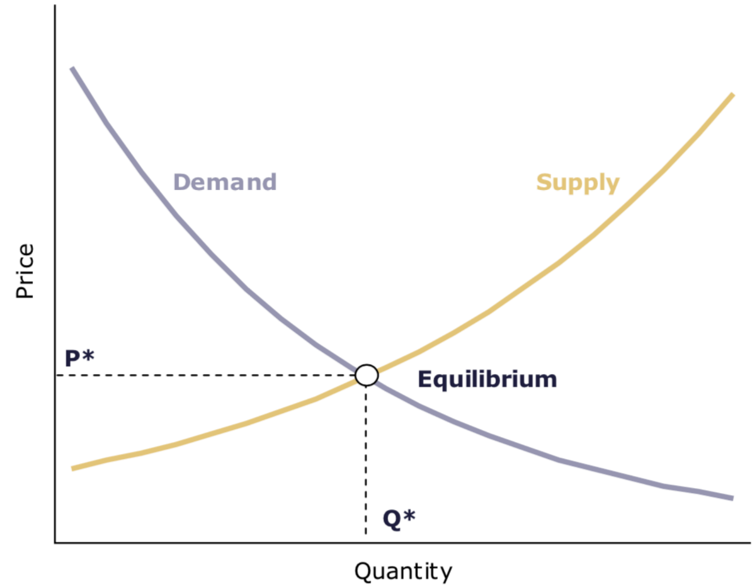


Figure 1: Supply and Demand Curves and Equilibrium Prices

Prices and Markets

- In equilibrium, this certainly must be the case.
- But *how*, exactly is this equilibrium actually attained?
- What is in the economy that coordinates the desires of demanders and suppliers so that a price emerges and trade occurs?
- This is study of price formation is a key element of microstructure research.



Price Formation

There are two traditional approaches to the mechanism for price formation:

1. The Agnostic (General Equilibrium) Approach

2. The Walrasian Auctioneer

The Agnostic (General Equilibrium) Approach

- The Trading Mechanism is irrelevant
- What matters are the properties of the *equilibrium prices*.
- These properties can be determined by simply solving for the *market-clearing price*; how exactly this market clearing was achieved was not of interest.
- Such an *agnostic* approach to price setting can be found for example in *rational expectation* literature where the questions of interest involve how traders use information in prices to determine their equilibrium demands, and where behavior out of equilibrium is *not* considered.
- Two advantages of this approach are its simplicity and its generality. Implicit in this approach, however, is the assumption that the trading mechanism plays *no role* in affecting the resulting equilibrium; that whatever the trading mechanism employed, the *same* equilibrium will arise.
- Though simplifying, this assumption is particularly *troubling* for markets in which traders have *differential (asymmetric) information*.

The Walrasian Auctioneer

The price formation process could be captured by the general representation of a *Walrasian auctioneer*. This auctioneer aggregates traders' demands and supplies to find a market-clearing price:

1. Each trader submits his or her demand, or even the demand schedule, to the auctioneer.
2. The auctioneer announces a *potential* trading price.
3. Traders then determine their optimal demands at that price.
4. No actual trading occurs until each trader has a chance to *revise* his order.
5. A new potential price is suggested, traders again revise any orders, and the process continues until there is no further revision.
6. Equilibrium prevails where each trader submits his optimal order at the *equilibrium price*, (the price at which quantity supplied *equals* the quantity demanded)

The Walrasian Auctioneer, cont'd.

- This representation views markets prices as arising from a process of <u>*tatonement*</u>, or a series of preliminary auctions.
- There is *no* trading allowed outside equilibrium and incentive issues are not considered.
- Because the price is adjusted until there is no excess demand, the Walrasian auctioneer does not take any trading position, but serves *only* to redirect quantities from sellers to buyers.
- Moreover, this auction activity is *costless*, so there are no frictions in the exchange process.
- The equilibrium price thus emerges as the natural outcome of an unseen trading game in which buyers and sellers costlessly exchange assets.

Microstructure's Beginnings

- The Walrasian auctioneer provides a simple and elegant way to envision the price-setting process. But does it, in fact, capture the actual process by which prices are formed?
- In the case of financial assets, there are markets (e.g. London gold fixing) that bear at least an approximate resemblance to the Walrasian framework.
- But there are many other markets that *differ dramatically*, with specific market participants playing roles *far removed* from the *passive* one of the auctioneer.
- Perhaps more important is the issue of *trader behavior*. If trading involves more than simply matching supplies and demands in equilibrium, then the trading mechanism may have an importance of its own.
- Such concerns raised by a number of economists led to the formal study of microstructure.
- The most critical analysis of trading was that of Demsetz [1968], who examined (along with the nature of transaction costs) how the time dimension of supply and demand affected market prices, setting the stage for formal study of market microstructure.

Microstructure's Beginnings, cont'd.

- Demsetz began with the simple observation that trade may involve some *cost*.
- This cost could be *explicit*, arising, for example, from the charges levied by a particular market venue (operations, equipment, etc.),
- Or it could be *implicit*, reflecting costs connected with the immediate execution of trades. These implicit costs, referred to as the *price of immediacy*, arose because, unlike in the Walrasian auction, trading had a *time dimension*.
- In particular, while over time the number of sellers might equal the number of buyers, at any particular point in time such an outcome was *not* guaranteed.
- If the number of traders wishing to sell immediately did not equal the number who wished to buy immediately, the imbalance of trade would make it impossible to find a market-clearing price at a give time *t*.

The Time Dimension of Trading

Let's look at the Widget Market

Scenario 1:

1. One market participant wishes to buy at the same time another wishes to sell
2. An orderly auction (whether Walrasian or otherwise) can proceed
3. An equilibrium price is established and a trade occurs

Scenario 2:

1. One market participant wishes to buy (or sell) but no other market
2. participants wish to sell (or buy) *at that time.*
3. How can an equilibrium price be achieved???

The Price of Immediacy

By paying a price! Demsetz argued that this lack of equilibrium could be *overcome* by paying a price for immediacy:

- Specifically, at any point in time there are *two sources* of supply and demand in the markets
- On the demand side, there is one demand arising from traders who want to buy <u>immediately</u>, and other coming from traders who want to buy but do not feel the need to do so at this particular time
- The same is true on the supply side
- If there is an imbalance of traders wanting to buy now, then either
  - Some buyers have to wait for sellers to arrive, or
  - Buyers can offer a *higher price* to induce waiting sellers to transact immediately
  - Similarly, if there is an imbalance of *sellers* wanting to trade now, a *lower price* must be offered to induce more demanders (buyers) to trade immediately
- This results in *two prices*, not one, characterizing the equilibrium

Departure from the Walrasian Framework

- This idea that the price could contain a cost of immediacy captured an aspect of price process not envisioned in Walrasian framework.

- Now there are actually *two supply curves* and *two demand curves*, reflecting the *two time frames* of the trading process.

- While a traders willing to wait might trade at the simple price envisioned in the Walrasian framework, trades occurring immediately would not share this outcome.

- This meant that even the notion of an equilibrium price was problematic – the price depended on whether one *wanted* to buy or to sell, and not simply on the *willingness* to trade

Departure from the Walrasian Framework

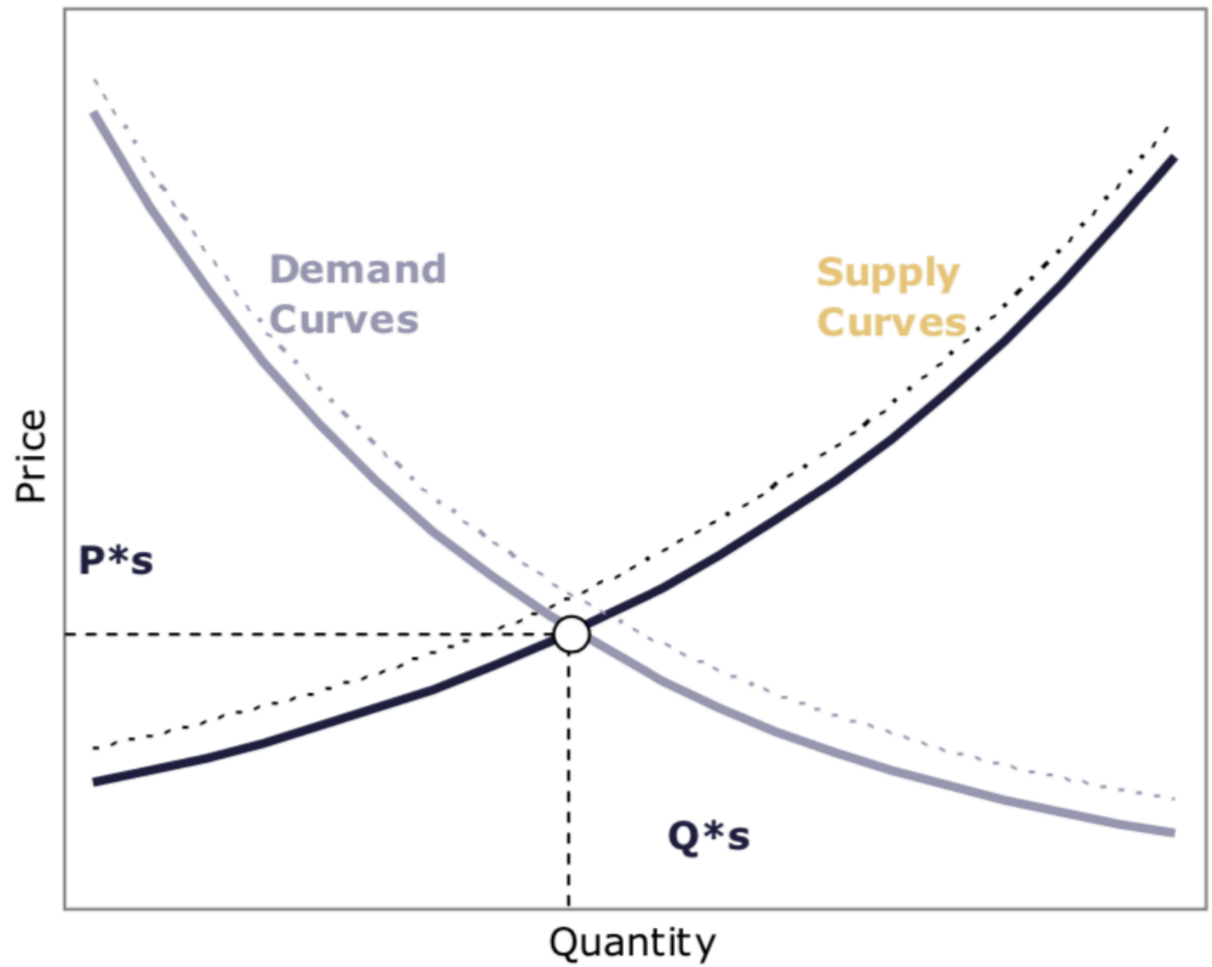


Figure 2: Multiple Supply and Demand Curves and Prices

Departure from the Walrasian Framework

- Of equal importance was the implication that the specific ***structure*** of the market mechanism could ***affect*** the trading price.

- Since the size of the price concession needed to trade immediately, i.e. the ***spread***, depended on the number of traders, factors such as volume could affect the cost of immediacy and thus the market price.

- Demsetz addressed these structural issues, and empirically investigated the relation of the size of the spread and the volume of trade on NYSE. His work suggested that the behavior of the markets, much like the behavior of the firms, cold *only* be understood by examining their structure and organization.

- If the actual mechanism used to set prices is *not merely a channel* to an inevitable outcome, but rather is an *input* into the equilibrium price itself, then how such mechanisms work cannot be ignored.

Departure from the Walrasian Framework

- Demsetz's model analyzes the behavior of one simple trading mechanism, and has limited scope in addressing the much more complex aspects observed in actual trading mechanisms.
- Equally important, however, are the *interactions* between the *market mechanism* and *trader behavior*.
- If the trading mechanism matters in setting prices, then so too will it matter in affecting trader's *order decisions*.
- Consequently, the *exogeneity* of the order process to the price-setting mechanism is *unlikely* to hold.
- The question of how prices are set, then, is a lot more complicated than the simple concept of the Walrasian auctioneer!

Departure from the Walrasian Framework

- Microstructure theories depart from the standard theory of exchange (in the sense of Walrasian tatonement) by

  - Assuming market agents' trading activities are *asynchronous*, *discrete*, and *temporary*
  - And by treating moment-to-moment *aggregate exchange behavior* on the part of market agents as essential to the price-formation process and an important descriptive aspects of the markets.

- Although the study of microstructure dates back to early history, with the rapid developments in financial markets, the advent of modern exchanges and alternative exchange mechanisms and venues, the field has acquired a distinct and important identity.

Sources of Value and Reasons for Trade

- Why do trades happen, and how are they valued? It is generally assumed that security value consists of *private* and *common* value components:
- Private values
  - *Idiosyncratic* to the agent and are usually known by the agent when the trading strategy is decided.
  - These components arise from the price difference in investment horizon, risk exposure, endowments, tax situations etc.
- Common values
  - The same for everyone in the market, are often realized only after trade has occurred.
  - Common value component reflects the cash flows from the security, as summarized in the present value of the flows or the security's resale value.
  - Common value effects generally dominate the private value effects in security markets.

Trade Mechanisms in Economic Settings

- Microstructure analyses are usually very specific about the mechanisms or *protocols* used to accomplish trade.
- One common and important mechanism is the *continuous limit order market* (LOM).
- Other specific mechanisms include
  - *bargaining*
  - *auctions*
  - *dealer markets*
  - a variety of *derivative markets*
- These mechanisms may operate in *parallel*, and hence many markets are *hybrids*.

Multiple Characterizations of Prices

- The *market-clearing price*, at least as it arises from Walrasian tatonement, *rarely* appears in microstructure analyses.

- At a single instant there may be *many prices* (See Figure 2 above), depending on
  - *direction* (buying or selling)
  - the *speed* with which the trade must be accomplished (immediacy)
  - the agent's *identity* or other attributes (transparency)
  - *trade size* (quantity)
  - and the agent's *relationship* to the counterparty

- Some prices (like bids and offers) may be hypothetical or conditional

Liquidity

“Liquidity” refers to the availability of participants and their willingness to trade in a market:

- The *more* participants there are in a market and/or the more participants are willing to trade, the more *liquid* a market is considered to be.

- The *less* participants there are in a market and/or the more participants are willing to trade, the more *illiquid* a market is considered to be.

Liquidity

Liquidity is sometimes characterized as “depth, breath and resiliency” of a market:

- In a *deep market*
  - If we look a little above the current market price, there is a *large incremental quantity* available for sale.
  - Similarly, below the current price, there is a large incremental quantity that is sought by one or more buyers.

- A *broad market* has *many participants,* none of whom is presumed to exert significant market power.

- In a *resilient market*, the *price effects* that are associated with the trading process (as opposed to fundamental valuations) are *small* and *die out quickly*.

Suppliers and Demanders

It is sometimes useful to characterize agents as *suppliers* or *demanders* of liquidity:

- Liquidity suppliers have traditionally been associated with the *financial services industry*, that is brokers, dealers and other intermediaries that are (sometimes called the *sell side*) of the market.

- Liquidity *demanders* in this view are the *customers*, the *individuals and institutional investors* characterized by trading needs (and sometimes called the *buy side*).

Suppliers and Demanders

From a narrower perspective, liquidity supply and demand differentiates agents who are available for trade or *offer the option to trade*, and those who spontaneously *decide to trade:*

- Thus, the liquidity *suppliers are passive*, and *demanders are active*. In any particular trade, the *active* side is the party who "seals the deal" by *accepting the terms offered* by the passive side. In other words, the passive side "makes" the markets and the active side "takes" the market.

- With the rise of markets that are widely, directly, and electronically accessible, the role of liquidity demander and supplier is a strategic choice that can be quickly *reversed*.

The alignment of liquidity demand and supply with a particular type of institution is therefore of diminished relevance in many (but not all!) modern markets.

Consolidation vs. Segmentation of Venues

- As we mentioned earlier, liquidity is generally *enhanced*, and individual agents can trade at lower cost, when the number of participants *increases*.

- This force favors market *consolidation* (concentration) of trading activity in a single mechanism or venue.

- Differences in market participants (e.g. retail versus institutional investors), and innovations by market participants, however, may drive market *segmentation* (fragmentation).

Consolidation vs. Segmentation of Venues

The *number of participants* in a security markets logically depends on attributes of the security, in addition to the trading mechanism.

There will tend to be greater interest (and liquidity) in a security if:

- The *aggregate value* of the underlying asset is high
- The value-relevant *information* is comprehensive, uniform, and credible
- The security is a component of an important *index*, there will be high interest in trading the security.

Transparency

Transparency is a *market attribute* that refers to how much *information* market participants (and potential participants) posses:

- *Electronic markets* that communicate the bids and offers of buyers and sellers and the process of executed trades in real time are considered *highly transparent*.
- *Dealer markets*, on the other hand, often have no publicly visible bids or offers, nor any trade reporting, and are therefore usually considered *opaque*.
- The line between these two is increasingly blurred, however, as innovation creates various types of hybrid markets.

Data Characteristics

Let's talk about four characteristics of microstructure data:

1. Microstructure data are *discrete.*

2. Microstructure data are often *well-ordered.*

3. Microstructure data samples are typically *large.*

4. Microstructure data can be considered to be *new* (and *old* as well)

Microstructure Data are Discrete

- Most microstructure series consist of *discrete* events *randomly* arrange in *continuous time*.
- Within the time series taxonomy, they are formally classified as point processes.
- Point process characterizations are becoming increasingly important, but for many purposes it suffices to treat observations as continuous variables realized at regular discrete times.

Microstructure Data are Well-Ordered

- The sequence of observations in the data set closely correspond to the sequence in which the economic events actually happened.
- In contrast, most *macroeconomic* data are *time-aggregated*. This gives rise to simultaneity and uncertainty about the direction of causality.
- The fine temporal resolution (sometimes described as “ultra-high frequency”) often supports stronger conclusions about causality.

Microstructure Data Samples are Large

- By most economic standards, observation counts are (very!) large.
- Tens of thousands of data points for a single asset on one day is not unusual.
- One would not ordinarily question the validity of asymptotic statistical approximations in the samples of this size.
- Furthermore, despite the number of observations, the data samples are often *small* in terms of *calendar span* (on the order of days or at best months).

Microstructure Data are New (and Old)

- Microstructure data samples are *new* - we don't have a long historical date for many markets, at least compared to the availability of non-microstructure economic data.

- Data samples may also be characterized as old, though, because market institutions are changing so rapidly that even samples a few years previous may be seriously out of date. It is essential to consider market structure regime changes and their potential effects on studies.

Important Microstructure Questions

Below are some significant outstanding questions (along with countless other ones) that market structure studies need to address:

- How exactly is information impounded in prices?
- How is (a given) market structure related to the valuation of
- securities?
- How do we enhance the information aggregation process?
- How do we avoid market failures?
- What type of trading arrangement maximizes efficiency?
- What is the most optimal reward process for liquidity provision?
- What is the trade-off between “fairness” and “efficiency”?
- What is the optimal balance between centralization and fragmentation?
- What is the significance of human interactions in trading process?

Transaction Costs

Money managers consistently *underperform* their paper portfolio benchmarks (some by as much as 2 to 3% annually).

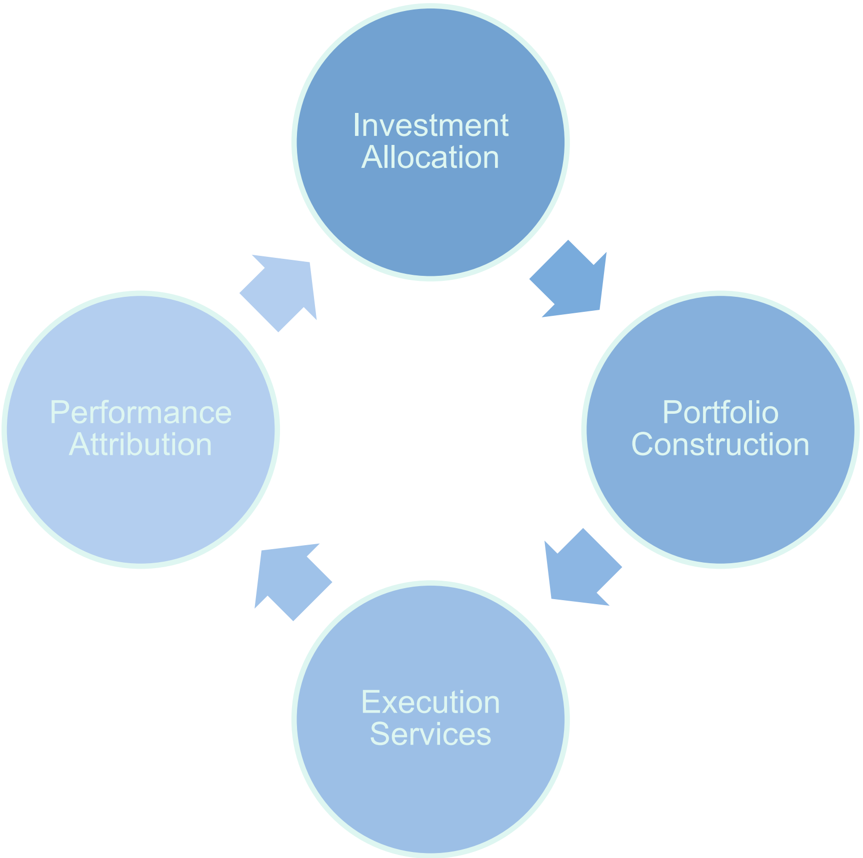
- One contributing factor to the underperformance is the *transaction costs* associated with implementing of investment decisions.
- The implementation of a financial decision is *not free*. It has an associated cost and usually results in reduced portfolio return.
- If managers do not properly manage these costs during all phases of investment cycle, many of the fund's superior investments will become unprofitable.

Investment Cycle

The financial investment cycle consists primarily of four stages.

Transaction costs affect the decisions in *each* and every phase of the investment cycle:

- Asset Allocation
- Portfolio Construction
- Execution Services
- Performance Attribution



Asset Allocation

- The process of distributing investment dollars across various investment classes as a means of *diversifying* risk and targeting a specific level of portfolio *return*.

- The more traditional asset classes consist of cash, bond and stocks.

- More recently other investment classes such as commodities, real estate, private equity, hedge funds, oil and gas and timber have become more conventional.

Regardless of investment class, transaction costs have a *significant* effect on overall portfolio returns.

Portfolio Construction

- The portfolio construction phase of the investment cycle consists primarily of specifying the *exact instruments* to purchase or sell in each asset class, for example bonds and stocks.
- In the construction of an equity portfolio, managers often need to decide upon different categories of stock such as large cap, mid cap, or small cap; different sectors; or even the decision of growth or value stock.
- The stocks are selected based on their expected return and associated risk.
- If transaction costs are not incorporated into the investment decision, the result could be an *inefficient* portfolio mix caused by an *inaccurate* assessment of risk.

Execution Services

- The execution services phase of the investment cycle consists of the actual implementation of the investment decision.
- This phase, which is also referred to as the *implementation phase* or trading phase, involves making decision regarding how, when and where to buy or sell stocks.
- Decision makers have the responsibility of evaluating all potential trading options to determine the best method of implementing the specific trading list. They need to decide on:
  - The appropriate *trading strategy* (aggressive or passive)
  - the appropriate *trading option* (principal or agency)
  - the appropriate *trading venue* (traditional broker, ECN, or crossing system)
  - the *broker(s)* who will execute the trade.
- Since this phase is the most involved with the actual trading of an order, decision makers need to exert careful cost control.

Performance Attribution

- Involves measuring a fund's performance and determining the reasons for missing the target level of return (either higher or lower).
- It also helps differentiate between superior stock selection and pure luck.
- For implementations, attribution (or alternatively "post-trade analysis") refers to the measurement of *transaction costs* and assessment of *broker / trader performance*.
- The goal is to measure costs to *improve* future decisions and distinguish between exceptional broker / trader performance and luck.

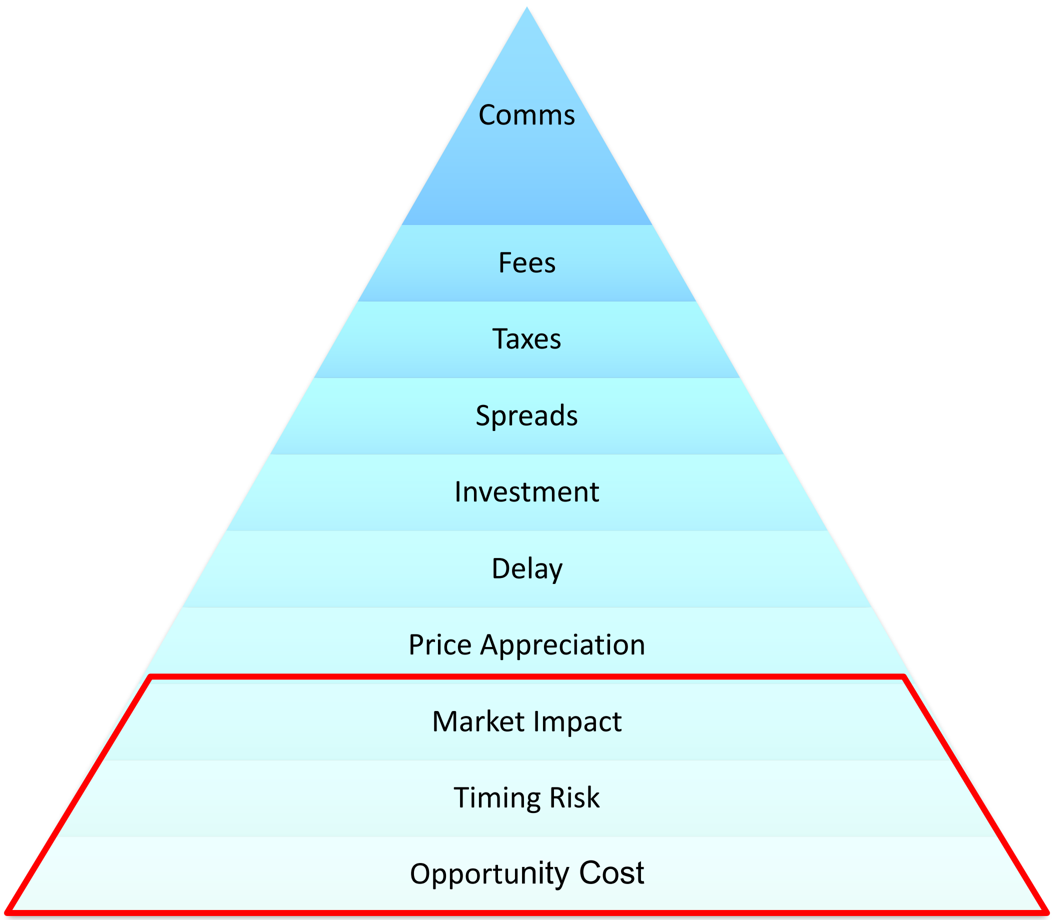
Transaction Cost Components

- Transaction cost is comprised of a number of components:
    - broker commissions
    - exchange fees
    - taxes
    - bid-ask spread
    - investment delay
    - price appreciation
    - market impact
    - timing risk
    - opportunity cost.
- Each adversely affects portfolio returns in different ways and in various degrees. Transaction cost are either *visible* or *hidden*. It is the *hidden costs* that make up the largest percentage of total transaction costs and visible costs that make up the smallest percentage of total transaction costs.
- We can describe these transaction costs as a pyramid, with the most visible costs on the top (e.g, they can only be seen from a distance) and the lease transparent costs shown on the bottom (they can be only seen up close)

The Transaction Cost Pyramid

Transaction Cost Components

- In this figure, the component costs *most visible* from a distance are those costs that contribute the *least* to the total transaction cost.
- Costs *least visible* (non-transparent) from a distance contribute *most* to the total transaction cost.
- Fortunately, these non-transparent cost components provide the greatest *opportunity* for *cost reduction* by skilled mangers / traders.
- Unfortunately, the cost reduction of one non-transparent cost is typically at the *expense* of another non-transparent cost.
- Therefore, traders need to understand *all* costs and how they interact with one another.
- For example, as we reduce market impact by trading more passively we expose the fund to greater risk.
- As we trade more aggressively we reduce risk but increase market impact.
- It is not possible to reduce all costs simultaneously. Proper transaction cost management requires careful balancing of al. costs.



The Trader's Dilemma

- During the implantation of a trade list, traders experience potentially *conflicting objectives:*
  - If the trader executes too *aggressively* they incur high *market impact* costs.
  - If execution is too *passive*, traders are exposed to significant *timing risk* that could result in even higher trading costs due to adverse price movement.
- Traders should *balance* the trade-off between cost and risk, and determine an optimal balancing point consistent with fund objectives.
- Balancing these conflicting costs can be very challenging but is a critical part of the investment and execution processes.

Best Execution

Best execution means different things to different people. But it can generally be categorized as *price*, *timing* and *size* factors.

For example:

- Value and passive investors are concerned with *price improvement* and *preservation* of asset value.
- Growth and momentum investors require *immediacy*.
- Still others, such as *block traders* and large *mutual funds*, require liquidity and *size improvements*.

More often than not, the prescribed best execution strategy ensures complete execution at fair market prices, and more importantly within cost and price guidelines specified by managers.

Only if a strategy provides the best opportunity to achieve one's implementation goals can it be considered a best execution strategy.

Goals of Implementation

- It is well accepted that the goal of *investment research* is the quest to uncover stocks (or other securities) most likely to achieve superior returns.
- When it comes to *implementation* of investment decisions, however, there are conflicting views regarding execution goals:
  - Some market participants believe the goal is to achieve the VWAP price because it represents a measure of fairness.
  - Some believe the goal is to achieve the closing price if that is the price that funds are valued or *marked* at.
  - Still others believe the goal is to achieve the opening price or some other price benchmark.
- These differing opinions can result in very conflicting implementation strategies.
- The one commonality, however, across all views is that <u>preservation of asset value</u> is vitally important. That is, we want to ensure that the new portfolio value is as close to the original ("decision time", or "arrival price") portfolio value as possible.

Goals of Implementation

- Thus, we can state the goal of implementation: to *minimize* the *difference* between the average execution price P_{avg} and the decision price at the time of investment decision P_d.
- This is formulated mathematically as:

$$Min\ \varphi = \left|P_{avg} - P_d\right|$$

Or in alternative quadratic form as:

$$Min\ \varphi = \left(P_{avg} - P_d\right)^2$$

- The value of φ is a function of random prices P_t at different times during the execution, and it is therefore itself a random variable.
- It can be best described as a distribution of expected costs and risk terms.
- The challenge is that the cost and risk terms compete with each other. Ideally both should be considered when developing an implementation strategy.
- For this reason, we will define three potential decision-making criteria incorporating cost and risk parameters that can be used by investors to develop an appropriate implementation strategy.

Goal 1: Minimize Cost

- The first criterion is to *minimize costs* within some *acceptable level of risk*.
- This quantity of risk may be specified by the firm, i.e. *maximum allowable risk exposure*, or may correspond to the level of risk from the investment model.
- This goal is formulated mathematically as:

$$Min\ \varphi = Cost$$

$$\text{Subject to: } Risk \leq R^*$$

where R* is the maximum allowable risk exposure specified by the firm.

Goal 2: Balance the Trade-off: Cost vs Risk

- The second goal is to balance the *tradeoff* between cost and risk.
- It is often preferred by investors who are unsure of a proper maximum level of risk exposure but have a preference to the amount of risk they will accept for a corresponding reduction in cost.
- This goal is formulated mathematically as:

$$Min\ \varphi = Cost + \lambda * Risk$$

where λ is the *risk aversion* factor that represent the investor's desired level of tradeoff between cost and risk.

Goal 3: Improving Return

- The third goal is to improve upon the price to *maximize* chances a trade is *better* than some specified cost.

- This goal is often selected by participants seeking to maximize *short-term returns* or investors seeking to maximize their chances of executing better than a principal bid. It is formulated mathematically as:

$$Min\ \varphi = Probability(Cost\ \leq C^{*})$$

where *C** is the *maximum acceptable transaction cost*, representing an upper bound on the average cost of the order.

Next Lecture

- Institutions and Mechanisms of Securities Trading
- Limit Order Book Mechanics