

The School of Mathematics



THE UNIVERSITY
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Incremental Learning of Recommender Systems

by

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Abstract

Here comes your abstract ...

Acknowledgments

Here come your acknowledgments ...

Own Work Declaration

Here comes your own work declaration

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1 Introduction

Here I will write a very good, precise and brief introduction. Particularly Section 2 is good!

2 Background

In the following, I explain some background stuff. I should really cut this short, but BlaBlaBlaBla
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Note that I start a new paragraph when I have an empty line like this. BlaBlaBlaBla BlaBlaBlaBlaBla
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But I can also end a line with a double backslash.

2.1 Models

Models are *very* helpful because.

- They're good.
- They're helpful.

2.2 Techniques

Techniques even better because.

1. They're magnificent.
2. If they work.

3 Technical Stuff

Now it's getting very technical ... I will cite [?, ?]. I will also show my incredible α , β and γ mathematics and do some other fancy stuff.

3.1 Formulae

For example look at this

$$\min \sum_{s \in \mathcal{S}} Pr_s \left[\sum_{t=1}^T \left(\sum_{g \in \mathcal{G}} \left(\alpha_{gts} C_g^0 + p_{gts} C_g^1 + (p_{gts})^2 C_g^2 \right) + \sum_{g \in \mathcal{C}} \gamma_{gts} C_g^s \right) \right], \quad (3.1)$$

and you will see that it has a little number on the side so that I can refer to it as equation (3.1). Now if I do this

$$\begin{aligned} \sum_{i=1}^n k_i &= 20 \\ \sum_{j=20}^m \delta_i &\geq \eta \end{aligned} \quad (3.2)$$

I can align two formulae and control which one has a number on the side. It is (3.2). I can also do something like this

$$Y_l = \begin{bmatrix} \left(y_s + i \frac{b_c}{2} \right) \frac{1}{\tau^2} & -y_s \frac{1}{\tau e^{-i\theta s}} \\ -y_s \frac{1}{\tau e^{i\theta s}} & y_s + i \frac{b_c}{2} \end{bmatrix},$$

and it won't have a number on the side. Now if I have to do some huge mathematics I'd better structure it a little and include linebreaks etc. so that it fits on one page.

$$\begin{aligned} p_l^f &= G_{l11} \left(2v_{F(l)} \bar{v}_{F(l)} - \bar{v}_{F(l)}^2 \right) \\ &+ \bar{v}_{F(l)} \bar{v}_{T(l)} \left[B_{l12} \sin(\bar{\delta}_{F(l)} - \bar{\delta}_{T(l)}) + G_{l12} \cos(\bar{\delta}_{F(l)} - \bar{\delta}_{T(l)}) \right] \\ &+ \begin{bmatrix} \bar{v}_{T(l)} \left[B_{l12} \sin(\bar{\delta}_{F(l)} - \bar{\delta}_{T(l)}) + G_{l12} \cos(\bar{\delta}_{F(l)} - \bar{\delta}_{T(l)}) \right] \\ \bar{v}_{F(l)} \left[B_{l12} \sin(\bar{\delta}_{F(l)} - \bar{\delta}_{T(l)}) + G_{l12} \cos(\bar{\delta}_{F(l)} - \bar{\delta}_{T(l)}) \right] \\ \bar{v}_{F(l)} \bar{v}_{T(l)} \left[B_{l12} \cos(\bar{\delta}_{F(l)} - \bar{\delta}_{T(l)}) - G_{l12} \sin(\bar{\delta}_{F(l)} - \bar{\delta}_{T(l)}) \right] \\ \bar{v}_{F(l)} \bar{v}_{T(l)} \left[-B_{l12} \cos(\bar{\delta}_{F(l)} - \bar{\delta}_{T(l)}) + G_{l12} \sin(\bar{\delta}_{F(l)} - \bar{\delta}_{T(l)}) \right] \end{bmatrix} \cdot \begin{bmatrix} v_{F(l)} - \bar{v}_{F(l)} \\ v_{T(l)} - \bar{v}_{T(l)} \\ \delta_{F(l)} - \bar{\delta}_{F(l)} \\ \delta_{T(l)} - \bar{\delta}_{T(l)} \end{bmatrix}, \end{aligned} \quad (3.3)$$

This is a lot of fun!

3.2 Important Things

Finally we should have a nice picture like this one. However, I won't forget that figures and table are environments which float around in my document. So LaTeX will place them wherever it thinks they fit well with the surrounding text. I can try to change that with a float specifier, e.g. [!ht]. Now I

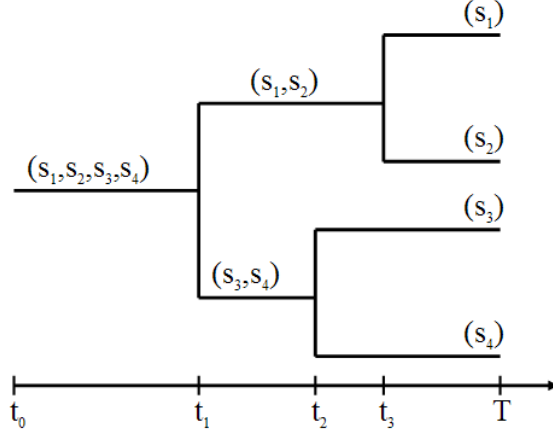


Figure 1: Look at this scenario tree with funny times t_1 and scenarios s_1 etc.

want to use one of my own environments. I want to define something.

Definition 3.1 *I define*

$$\Gamma_\eta := \sum_{i=1}^n \sum_{j=i}^n \xi(i, j)$$

I definitely need some good tables, so I do this. I should really refer to Table 1.

Case	Generators	Therm. Units	Lines	Peak load: [MW]	[MVar]
6 bus	3 at 3 buses	2	11	210	210
9 bus	3 at 3 buses	3	9	315	115
24 bus	33 at 11 buses	26	38	2850	580
30 bus	6 at 6 buses	5	41	189.2	107.2
39 bus	10 at 10 buses	7	46	6254.2	1387.1
57 bus	7 at 7 buses	7	80	1250.8	336.4

Table 1: Something that doesn't make sense.

3.3 And now something else

Let:

$$\begin{aligned}
\Omega_0 &= \{(x, y, z, f) : \text{satisfying (9) -- (19)}\}, \\
\Omega_1 &= \{(x, y, z, f) : \text{satisfying (9), (11) -- (20)}\}, \\
\bar{\Omega}_0 &= \{\mathbf{0} \leq (x, y, z, f) \leq \mathbf{1} : \text{satisfying (9) -- (18)}\}, \\
\bar{\Omega}_1 &= \{\mathbf{0} \leq (x, y, z, f) \leq \mathbf{1} : \text{satisfying (9), (11) -- (18), (20)}\}.
\end{aligned}$$

where $\mathbf{0}$ and $\mathbf{1}$ are vectors of appropriate dimensions with 0's and 1's, respectively. Next we see that both Ω_0 and Ω_1 give equivalent formulations for the A-MSSP. In particular, the following statements hold:

Proposition 1 $\Omega_0 \subseteq \Omega_1$.

Proof. Let us suppose there exists $(x, y, z, f) \in \Omega_1$ such that $(x, y, z, f) \notin \Omega_0$. Then, there exist indices $i \in I$ and $t \in \{0, \dots, |T| - s_i\}$ with $x_i^t > 0.5 \left(\sum_{h=1}^{s_i} x_i^{t+h} + 1 \right)$. By definition, $x_i^t = 1$ and $x_i^{t+h} = 0$ for all $h \in \{1, \dots, s_i\}$. By (11) and (12), $\sum_{h=1}^{s_i} f_i^{th} = 1$, so $f_i^{th'} = 1$ for some $h' \in \{1, \dots, s_i\}$. But then,

$$0 = x_i^{t+h'} = \sum_{h=\max\{1, t+h'-(|T|-s_i)\}}^{\min\{s_i, t+h'\}} f_i^{t+h'-h, h} \geq f_i^{th'} = 1,$$

as $h' \in [\max\{1, t+h'-(|T|-s_i)\}, \min\{s_i, t+h'\}]$. □

This immediately gives us

Corollary 1 *AS is a valid formulation for the A-MSSP.*

Next we compare the Linear Programming (LP) relaxations of the two formulations.

Proposition 2 $\overline{\Omega}_1 \subseteq \overline{\Omega}_0$.

Proof. Homework □

4 Conclusions

I have no idea how to conclude, so I don't write much. But the stuff that follows is important.

Appendices

A An Appendix

Some stuff.

B Another Appendix

Some other stuff.