

# One Max Problem

## Representation and evaluation

In the context of Keen, the representation of candidate solutions is fundamental. These solutions are modeled as collections of “genetic material,” mirroring the concept of genes and chromosomes in biological systems. This genetic material, depending on its organization and granularity, can represent different dimensions of solutions:

Genetic material	Mathematical construct
Gene	Scalar
Chromosome	Vector
Genotype	Matrix

Table 1: Genetic material and their mathematical equivalent

Given the binary nature of the problem domain in Keen, solutions can be effectively represented as arrays of boolean values, encapsulated by the `BooleanGene` and `BooleanChromosome` classes.

Consider the following Kotlin code snippet for defining a genotype:

```
// Define the size of each chromosome, here set to 50 genes.
private const val CHROMOSOME_SIZE = 50
// Initial probability for each gene to be `true`, set at 15%.
private const val TRUE_RATE = 0.15

// Constructing a genotype with specified characteristics.
val gt = genotypeOf {
    chromosomeOf {
        booleans {
            size = CHROMOSOME_SIZE // Number of genes in a chromosome.
            trueRate = TRUE_RATE    // Probability of a gene being `true`.
        }
    }
}
```

This code defines a genotype with chromosomes consisting of 50 genes each, where each gene has a 15% chance of being true. This setup is particularly useful in problems where the solution space can be binary encoded.

Evaluating the fitness of these genotypes is crucial for guiding the genetic algorithm towards optimal solutions. The fitness function, in this case, is designed to count the number of `TrueGenes` within a genotype:

```
// Function to evaluate the fitness of a genotype.
private fun count(genotype: Genotype<Boolean, BooleanGene>) =
    genotype.flatten() // Convert the genotype to a list of genes.
        .count { it } // Count the number of `TrueGenes`.
        .toDouble() // Convert the count to a Double for compatibility.
```

In this function, the genotype is first flattened to transform its structured genetic material into a linear list of genes, making it easier to apply operations like counting. The number of `TrueGenes` reflects the fitness of the genotype, with higher counts indicating potentially more optimal solutions.

By meticulously crafting the representation of solutions and defining a meaningful fitness function, Keen leverages the principles of genetic algorithms to efficiently navigate the solution space of complex optimization problems.

## Initialization

The initialization phase of a genetic algorithm marks the beginning of its process to find solutions. This phase involves creating an initial group of individuals, each representing a potential solution. The population's size and the method of initializing these individuals are critical. Generally, initialization uses randomness to ensure a wide exploration of solutions, but if specific knowledge about the problem is available, a more targeted approach can be taken.

Each individual is assessed for their fitness value after the population is created. This evaluation helps gauge the diversity and quality of solutions, setting the stage for the algorithm's evolution.

For problems where optimal solutions are unclear, like the OMP, initialization might involve generating random binary strings for each individual. For example, in a population of 100 individuals, each might start with a 50-bit string.

The Evolution Engine manages this process, overseeing initialization, evaluation, and evolution. It is configured through parameters like population size and the structure of individuals. Here's a Kotlin code example to illustrate the setup:

```
// Define the population size.
private const val POPULATION_SIZE = 100

// Configure and instantiate the Evolution Engine.
val engine = evolutionEngine(::count, genotypeOf {
    chromosomeOf {
        booleans {
            size = CHROMOSOME_SIZE // Length of each individual's chromosome.
            trueRate = TRUE_RATE // Initial probability of a gene being `true`.
        }
    }
}) {
    populationSize = POPULATION_SIZE // Set the population size.
    // Additional configurations can be added here.
}
```

This code sets up the Evolution Engine with a fitness evaluation function and the genetic structure of the population, readying the algorithm for its evolutionary journey.

## Genetic operators

Genetic operators are crucial components in genetic algorithms, directly manipulating genetic material to drive the evolutionary process. These operators, including mutation, crossover, and selection, modify genetic material to produce variability and innovation within a population.

In the Keen framework, all genetic operators implement the `GeneticOperator` interface, ensuring standardization and interoperability across different evolutionary strategies. Here's how this interface is defined:

```
// Interface for genetic operators where T represents the type of value in the gene,
// and G represents the gene itself.
interface GeneticOperator<T, G> where G : Gene<T, G> {

    // Function invoked to apply genetic operations. It takes the current
    evolutionary
    // state and the desired output size, returning the new evolutionary state.
    operator fun invoke(
        state: EvolutionState<T, G>, outputSize: Int
    )
```

```

    ): EvolutionState<T, G>
}

```

## Selection

Following the initialization phase, the GA enters its main evolutionary cycle, with selection being a pivotal process. This step mimics natural selection by preferentially choosing individuals with higher fitness for reproduction, thus steering the population towards more optimal solutions.

In this context, the concept of elitism is introduced through a survival rate  $\sigma$ , which determines the fraction of the population that advances to the next generation unchanged. Specifically, the top  $\lfloor \sigma N \rfloor$  individuals, based on their fitness, are preserved, while the rest are replaced by offspring generated through genetic operators. This blend of elitism and generation of new individuals helps balance exploration and exploitation in the search space.

**Definition 0.1** (Selection operator): The selection process is formalized through a selection operator, denoted as  $\Sigma$ , which is defined for a population  $P$  comprising  $N$  individuals, each with a fitness value  $\varphi_i$ . The operator is described as:

$$\Sigma(P : \mathbb{P}, n : \mathbb{N}, \dots) \rightarrow P'$$

where  $\mathbb{P}$  is the set of all possible populations,  $\mathbb{N}$  represents the set of natural numbers,  $P'$  is the selected subset of the population, and  $n$  is the number of selections to be made.

A commonly used method within this operator is the **roulette wheel selection**, where each individual's chance of being selected is proportional to its fitness. This can be mathematically expressed as:

$$\rho_{\Sigma(i)} = \varphi_i \sum_{j=1}^N \varphi_j$$

where  $\rho_{\Sigma(i)}$  represents the selection probability of the  $i$ -th individual.

In Keen, all selection methods conform to the Selector interface:

```

interface Selector<T, G> : GeneticOperator<T, G> where G : Gene<T, G> {

    override fun invoke(
        state: EvolutionState<T, G>, outputSize: Int
    ): EvolutionState<T, G> { ... }

    fun select(
        population: Population<T, G>, count: Int, ranker: IndividualRanker<T, G>
    ): Population<T, G>
}

```

Configuring the selection mechanism within a GA is typically straightforward, as demonstrated in the following Kotlin snippet for the Keen library:

```

val engine = evolutionEngine(::count, genotypeOf {
    chromosomeOf {
        booleans {
            size = CHROMOSOME_SIZE
            trueRate = TRUE_RATE
        }
    }
})

```

```

}) {
    // For selecting parents for crossover.
    parentSelector = RouletteWheelSelector()
    // For selecting individuals to survive to the next generation.
    survivorSelector = TournamentSelector()
    /* Additional configurations */
}

```

This configuration illustrates the use of `RouletteWheelSelector` for parent selection, where probabilities are aligned with individuals' fitness, and `TournamentSelector` for survivor selection, which involves selecting the best among a randomly chosen subset of individuals. The flexibility to use different selectors for these phases allows for a tailored approach, potentially enhancing the GA's ability to converge on optimal solutions.

## Variation

Variation is the cornerstone of GA, facilitating the creation of new individuals from existing ones to explore the solution space comprehensively. This process is essential to circumvent premature convergence to sub-optimal solutions, analogous to how genetic diversity in nature fosters adaptability and resilience in species.

The primary mechanisms of variation in GAs are crossover and mutation. **Crossover** resembles biological recombination, merging genetic information from two or more parents to produce offspring. **Mutation**, akin to spontaneous genetic mutations in nature, introduces random alterations to an individual's genetic makeup.

To formally define a variation operator, which is pivotal in generating new individuals within a population, consider the following:

**Definition 0.2** (Variation operator): A variation operator is a mechanism that derives new individuals from existing ones in a population. Formally, it can be represented as a function:

$$\phi : (P : \mathbb{P}, \rho_\phi : \mathbb{R}, \dots) \rightarrow \mathbb{P}$$

where:

- $\mathbb{P}$  represents the set of all possible populations.
- $\mathbb{R}$  denotes the set of real numbers, corresponding to the range of the probability parameter.
- $P$  specifies the particular population subject to variation.
- $\rho_\phi$  is the probability of applying the variation operator to an individual within  $P$ .

The ellipsis (...) signifies additional parameters that may be included based on the specific implementation and characteristics of the variation operator.

Variation operators in genetic algorithms are typically variadic, capable of accepting a variable number of parent individuals to produce offspring. This adaptability enables a diverse array of genetic combinations within the population, encouraging a thorough exploration of potential solutions.

Keen represents variation operators through the `Alterer` interface:

```

interface Alterer<T, G> : GeneticOperator<T, G> where G : Gene<T, G> {
    operator fun plus(alterer: Alterer<T, G>) = listOf(this, alterer)
}

```

## Crossover

A critical variation operator is the **crossover**, which mirrors genetic recombination observed in nature. This operator facilitates the exchange of genetic material between two parent individuals, leading to the generation of new offspring.

**Definition 0.3** (Crossover Operator): The crossover operator recombines genetic material from existing individuals to create new ones. It is formally represented as:

$$X(P : \mathbb{P}, \rho_X : \mathbb{R}, \dots) \rightarrow \mathbb{P}$$

where:

- $\mathbb{P}$  represents the set of all possible populations,
- $\mathbb{R}$  is the set of real numbers, indicating probabilities,
- $P$  denotes the current population under consideration,
- $\rho_X$  is the probability of applying the crossover to an individual.

In Keen, the crossover functionality is encapsulated within the following interface:

```
interface Crossover<T, G> : Alterer<T, G> where G : Gene<T, G> {
    val numOffspring: Int
    val numParents: Int
    val chromosomeRate: Double
    val exclusivity: Boolean

    override fun invoke(
        state: EvolutionState<T, G>, outputSize: Int
    ): EvolutionState<T, G> { ... }

    fun crossover(parentGenotypes: List<Genotype<T, G>>): List<Genotype<T, G>>
    { ... }

    fun crossoverChromosomes(
        chromosomes: List<Chromosome<T, G>>
    ): List<Chromosome<T, G>>
}
```

We'll use a **single-point crossover** operator in our example. This operator selects a random index within the parent chromosome to swap genes before and after this cut point. For instance, consider two parent individuals,  $I_1 = 1100$  and  $I_2 = 0001$ . Using the single-point crossover, if the cut is made after the second gene, the resulting offspring would be  $O_1 = 1101$  and  $O_2 = 0000$ .

Implementing this in Keen is straightforward:

```
val engine = evolutionEngine(::count, genotypeOf {
    chromosomeOf {
        booleans {
            size = CHROMOSOME_SIZE
            trueRate = TRUE_RATE
        }
    }
}) {
    alterers += SinglePointCrossover(chromosomeRate = 0.6)
    /* Other configurations */
}
```

Note the use of += for adding alterers, as they are initialized to an empty mutable list to provide flexibility in configuration.

## Mutation

While the crossover operator effectively recombines existing genetic material, it is limited by the genetic diversity already present in the population. This can sometimes lead to premature convergence, especially for complex problems characterized by numerous local optima.

To combat this and infuse fresh *diversity*, the **mutation** operator is employed. It introduces small, probabilistic changes to the genetic makeup of individuals.

**Definition 0.4** (Mutation operator): The mutation operator introduces variations in an individual's genetic material based on a predefined probability, resulting in a new population. Formally, a mutation operator can be represented as:

$$M(P : \mathbb{P}, \mu : [0, 1], \dots) \rightarrow \mathbb{P}$$

where:

- $\mathbb{P}$  – denotes the set of all possible populations
- $[0, 1]$  – is the range of valid probabilities
- $P$  – stands for the current population
- $\mu$  – indicates the mutation rate, i.e., the chance of an individual undergoing mutation

Additional parameters vary depending on the specific mutation operator in play.

## Termination

In genetic algorithms, termination criteria are pivotal as they dictate when the algorithm should stop. This is crucial for preventing unnecessary computations and focusing the search on promising areas of the solution space.

In Keen, we represent these criteria using Limits, which are defined with the following signature:

```
interface Limit<T, G> where G : Gene<T, G> {  
    var engine: Evolver<T, G>?  
    operator fun invoke(state: EvolutionState<T, G>): Boolean  
}
```

For this example, we utilize one of Keen's integrated limits to halt the algorithm when either the desired fitness is achieved or a maximum number of generations has been reached:

```
val engine = evolutionEngine(::count, genotypeOf {  
    chromosomeOf {  
        booleans {  
            size = CHROMOSOME_SIZE  
            trueRate = TRUE_RATE  
        }  
    }  
}) {  
    // Add termination conditions  
    limits += listOf(MaxGenerations(1000), TargetFitness(50.0))  
}
```

By setting a clear termination condition, such as achieving the highest possible fitness score, the algorithm efficiently navigates the search terrain. Although it might not explore every possible

solution, it strategically focuses on those that improve the fitness, effectively optimizing the search process.

## Full implementation

```
private const val POPULATION_SIZE = 100
private const val CHROMOSOME_SIZE = 50
private const val TRUE_RATE = 0.15
private const val TARGET_FITNESS = CHROMOSOME_SIZE.toDouble()
private const val MAX_GENERATIONS = 500

private fun count(genotype: Genotype<Boolean, BooleanGene>) =
    genotype.flatten().count { it }.toDouble()

fun main() {
    val engine = evolutionEngine(::count, genotypeOf {
        chromosomeOf {
            booleans {
                size = CHROMOSOME_SIZE
                trueRate = TRUE_RATE
            }
        }
    }) {
        populationSize = POPULATION_SIZE
        parentSelector = RouletteWheelSelector()
        survivorSelector = TournamentSelector()
        alterers += listOf(
            BitFlipMutator(individualRate = 0.5),
            SinglePointCrossover(chromosomeRate = 0.6)
        )
        limits += listOf(MaxGenerations(MAX_GENERATIONS),
            TargetFitness(TARGET_FITNESS))
        listeners += listOf(EvolutionSummary(), EvolutionPlotter())
    }
    engine.evolve()
    engine.listeners.forEach { it.display() }
}
```