

a

+	0.	1	2	3	4	5	6	7	8	9	0	1	2	3	4	0	0	10^{16}
---	----	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	-----------

b

-	0.	1	2	3	4	5	6	7	8	9	0	1	2	3	4	0	1	10^{16}
---	----	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	-----------

c

+	0.	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10^{-1}
---	----	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	-----------

I numeri a, b, c appartengono ad F. Calcoliamo $fl(fl(a + c) + b)$ e $fl(fl(a + b) + c)$

Il numero a appartiene al sottointervallo dell'asse reale $[2^{50}, 2^{51}]$. In questo sottointervallo lo spacing è dato da $s = 2^{50+1-53} = 2^{-2} = 0.25$

$fl(a + c) = a$ perché $c = 0.06$ è minore dello spacing

$$c = 0.6 \cdot 10^{-1} = 0.6 \cdot 10^{-1} \cdot (10^{-16}) \cdot 10^{16} = 0.000000000000000006 \cdot 10^{16}$$

a

+	0.	1	2	3	4	5	6	7	8	9	0	1	2	3	4	0	0	10^{16}
---	----	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	-----------

c

+	0.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	10^{16}
---	----	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	-----------

$$fl(a + c) = a$$

a

+	0.	1	2	3	4	5	6	7	8	9	0	1	2	3	4	0	0	10^{16}
---	----	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	-----------

b

-	0.	1	2	3	4	5	6	7	8	9	0	1	2	3	4	0	1	10^{16}
---	----	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	-----------

$$fl(fl(a + c) + b) = -0.000000000000000001 \cdot 10^{16} = -0.1 \cdot 10^{-15} \cdot 10^{16} = -0.1 \cdot 10^1 = -1$$

+	0.	1	2	3	4	5	6	7	8	9	0	1	2	3	4	0	0	10^{16}
---	----	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	-----------

-	0.	1	2	3	4	5	6	7	8	9	0	1	2	3	4	0	1	10^{16}
---	----	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	-----------

[illegible]

[illegible]
$$c = 0.6000000000000000 \cdot 10^{-1} \cdot (10^{-1}) \cdot 10^1$$
[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Valutare $y_1(x) = \sqrt{x^2 + 1} - x$ per $x = 7777$

$$x = 0.7777 \cdot 10^4 \quad x^2 = 0.60481729 \cdot 10^8$$

$$fl(x^2) =$$

0.	6	0	4	8	1	7	2	9	0	0	0	0	0	0	0	0	10^8
----	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	--------

1 in notazione scientifica normalizzata su un calcolatore a 64 bit si esprime come

0.	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10^1
----	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	--------

Ne modifico la mantissa in maniera tale che la parte esponente sia 10^8

$$1 = 1 * 10^0 = (0.100..00 \cdot 10^1) 10^{-8} \cdot 10^8 = 0.1 \cdot 10^{-7} \cdot 10^8$$

0.	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	10^8
----	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	--------

$$0.6048173 \cdot 10^8 \in [2^{25}, 2^{26}] \quad \text{Lo spacing in } [2^{25}, 2^{26}] \text{ è } s = 7.450580596923828e-09$$

$$fl(x^2)$$

0.	6	0	4	8	1	7	2	9	0	0	0	0	0	0	0	0	10 ⁸
----	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	-----------------

0.	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	10 ⁸
----	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	-----------------

$$t = fl(x^2 + 1) = \; 0.60481730 \; 10^8$$

$$fl(\sqrt{t}) = 0.7777000064292143 \cdot 10^4$$

$$x = \; \; \; 0.7777000000000000 \cdot 10^4$$

$$\sqrt{t} - x = \; 0.64292143 \; 10^{-8} \cdot 10^4 = 0.64292143 \; 10^{-4}$$

Valutare $y_1(x) = \sqrt{x^2 + 1} - x$ per $x = 77777777$

$$x = 0.77777777 \cdot 10^8 \quad x^2 = 0.6049382595061729 \cdot 10^{16}$$

$fl(x^2)$

0.	6	0	4	9	3	8	2	5	9	5	0	6	1	7	2	9	10^{16}
----	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	-----------

1 in notazione scientifica normalizzata su un calcolatore a 64 bit si esprime come

0.	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10^1
----	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	--------

Se lo voglio esprimere il numero in maniera tale che la parte esponente sia 10^{16}

$$1 = 1 * 10^0 = (0.100..00 \cdot 10^1) 10^{-16} \cdot 10^{16} = 0.1 \cdot 10^{-15} \cdot 10^{16}$$

0.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	10^{16}
----	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	-----------

$$0.6049382595061729 \cdot 10^{16} \in [2^{52}, 2^{53}]$$

Lo spacing in $[2^{52}, 2^{53}]$ è $s = 1$

$fl(x^2)$

0.	6	0	4	9	3	8	2	5	9	5	0	6	1	7	2	9	10^{16}
----	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	-----------

1

0.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	10^{16}
----	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	-----------

$$t = fl(x^2 + 1) = 0.6049382595061730 \cdot 10^{16}$$

$$\sqrt{t} = 0.77777777700000000 \cdot 10^{16}$$

$$x = 0.77777777700000000 \cdot 10^{16}$$

$$\sqrt{t} - x = 0$$

math.sqrt di Python implementa un algoritmo iterativo per calcolare la radice quadrata di un numero : fenomeno di **cancellazione di cifre significative**

$$y_2(x) = \frac{1}{\sqrt{x^2 + 1} + x}$$

Formula equivalente algebricamente alla precedente, ma non presenta fenomeni cancellazione di cifre significative, quando viene valutata in $x = 7777777$.