Q1 $A_1 = (2, 10)$, $A_2 = (2, 5)$, $A_3 = (8, 4)$

$B_1 = (5, 8)$

$C_1 = (1, 2)$, $C_2 (4, 9)$

A_1, B_1 & C_2 are the centers

First itieration

	A_1	A_2	A_3	B_1	C_1	C_2
Centroid A1	0	5	8.48	3.6	8.06	2.23
Centroid B1	3.6	4.24	5	0	7.21	1.41
Centroid C_2	2.23	4.47	6.4	1.41	7.61	0

1) Cluster after first iteration.

Cluster − 1 = { A1 (2, 10) }

Cluster - 2 = { A2 (2, 5), A3 (8, 4), B_1(5, 8), C_1(1, 2) }

Cluster −3 = { C_2(4, 9) }

2) Centroid after first iteration:

Center −1 = (2, 10)

Center −2 = $\left(\frac{2+8+5+1}{4}, \frac{5+4+8+2}{4}\right) = (4, 4.75)$

Center −3 = (4, 9)

Classification - Example.

Q3

i) $P(C_0) = P(C_1) = \frac{10}{20} = \frac{1}{2}$

Gender	C_0	C_1
M	6/10	4/10
F	4/10	6/10

Car type	C_0	C_1
Family	1/10	3/10
Sports	8/10	0
Luxury	1/10	7/10

Shirt Size	C_0	C_1
Small	3/10	2/10
Medium	3/10	4/10
Large	2/10	2/10
Extra Large	2/10	2/10

New sample → Gender = M, Cartype = Family, Shirt Size Large.

$P(C_0 | \text{new sample}) = P(C_0) \times P(\text{Gender} = M | C_0) \times P(\text{Cartype} = \text{Family} | C_0) \times P(\text{Shirt Size} = \text{Large} | C_0)$

$= \frac{1}{2} \times \frac{6}{10} \times \frac{1}{10} \times \frac{2}{10}$

$= 0.006$

$P(C_1 | \text{new sample}) = \frac{1}{2} \times \frac{4}{10} \times \frac{3}{10} \times \frac{2}{10}$

$= 0.012$

Since, $P(C_1) > P(C_0)$

New sample is classified as C_1

Q3

2) $P(C_0) = P(C_1) = \frac{1}{2}$

Car type → Shirt Size
↑
Gender → Shirt Size

Since Gender is independent

Gender	C_0	C_1
M	6/10	4/10
F	4/10	6/10

Since car type depends on gender

Cartype	C_0	C_1
family, M	1/10	3/10
family, F	0	0
Sports, M	5/10	0
Sports, F	3/10	0
Luxury, M	0	1/10
Luxury, F	1/10	6/10

Since, shirt Size depends on both car type = Gender.

Shirt Size	C_0	C_1
Small, M, Family	1/10	0
Small, M, sports	0	0
Small, M, Luxury	0	0
Small, F, Family	0	0
Small, F, sports	2/10	0
Small, F, Luxury	0	2/10

Shirt Size	C_0	C_1
Mid, M, Family	0	1/10
Mid M, sports	2/10	0
Mid M, Luxury	0	0
Mid F, Family	0	0
Mid F, Sports	1/10	0
Mid F, Luxury	0	3/10

shirt Size = Large

Shirt Size	C_0	C_1
M, Family	0	1/10
M, Sports	1/10	0
M Luxury	0	0
F, Family	0	0
F Sports	0	0
F Luxury	1/10	1/10

$\boxed{\text{Extra Large}}$

Shirt Size	C_0	C_1
M, Family	0	1/10
M, Sports	2/10	0
M, Luxury	0	1/10

Shirt Size	C_0	C_1
F, Family	0	0
F, Sports	0	0
F, Luxury	0	0

To Classify new sample.

$P(C_0 | \text{new sample}) = P(C_0) \times P(\text{Gender} = \text{Male} | C_0) \times P(\text{Cartype} = \text{family} | C_0) \times$
$P(\text{Shirt Size Large} | C_0)$

$= \frac{1}{2} \times \frac{6}{10} \times \frac{1}{10} \times 0$

$= 0$

$P(C_1 | \text{new sample}) = P(C_1) \times P(\text{Gender} = \text{Male} | C_1) \times P(\text{Cartype} = \text{Family Male} | C_1)$
$\times P(\text{Shirt Size} = \text{large, Male, family} | C_1)$

$= \frac{1}{2} \times \frac{4}{10} \times \frac{3}{10} \times \frac{1}{10}$

$= 0.006$

$P(C_1 | \text{new sample}) > P(C_0 | \text{new sample})$

$\Rightarrow$ New sample is classified as C1

Q4)

$$w^1, x \to \underbrace{w^1 x}_{z^1} \quad \underbrace{\sigma(z^1)}_{h^1} \to \underbrace{w^2 h^1}_{z^2}\underbrace{\sigma(z^2)}_{h^2} \to \underbrace{w^3 h^3}_{z^3}\ \underbrace{S(z^3)}_{h^3=\hat{y}} \to L(y,\hat{y})$$

$$L = (y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2$$

$$\frac{\partial L}{\partial w^3} = \frac{\partial L}{\partial z^3}\frac{\partial z^3}{\partial w^3} \to h^2$$

Here,

$$\frac{\partial L}{\partial z^3} = \frac{\partial L}{\partial \hat{y}}\frac{\partial \hat{y}}{\partial z^3}$$

And,

$$\frac{\partial L}{\partial \hat{y}} = \frac{\partial (y_1 - \hat{y}_1)^2}{\partial y'} + \frac{\partial (y_2 - \hat{y})^2}{\partial y^2} = -2(y_1 - \hat{y}) - 2(y_2 - \hat{y}_2)$$
$$= -2[(\hat{y}_1 - y_1) + \hat{y}_2 - y_2]$$

$$\frac{\partial \hat{y}}{\partial z^3} = \frac{\partial S(z^3)}{\partial z^3} = \frac{\partial S(z_1^3)}{\partial (z_1^3)} + \frac{\partial S(z_1^3)}{\partial z_2^3} + \frac{\partial S(z_2^3)}{\partial z_1^3} + \frac{\partial (z_2^3)}{\partial z_2^3}$$
$$= S(z_1^3)(1 - S(z_1^3)) - 2S(z_1^3)S(z_2^3) + S(z_2^3)(1 - S(z_2^3))$$

$$\Rightarrow \frac{\partial L}{\partial w^3} = 2[(\hat{y} - y_1) + (\hat{y}_2 - y_2)]h^2[S(z_1^3)(1 - S(z_1^3)) -$$
$$2S(z_1^3)S(z_3^3) + S(z_2^3)(1 - S(z_2^3))]$$

$$\frac{\partial L}{\partial w^2} = \frac{\partial L}{\partial z^2} \frac{\partial z^2}{\partial w^2} \text{ — } h^1$$

Here,

$$\frac{\partial L}{\partial z^2} = \frac{\partial L}{\partial z^3} \times \frac{\partial z^3}{\partial z^2}$$

Here,

w^3

$$\frac{\partial z^3}{\partial z^2} = \frac{\partial z^3}{\partial h^2} \frac{\partial h^2}{\partial z^2}$$

$$\frac{\partial h^2}{\partial z^2} = w^2 h^1 (1 - w^2 h^1)$$

$$\Rightarrow \frac{\partial L}{\partial w^2} = 2[(\hat{y} - y_1) + (\hat{y}_1 - y_2)] [S(z_1^3)(1 - S(z_1^3)) - 2S(z_1^3)S(z_2^3) + S(z_2^3)(1 - S(z_2^3))] h^1 \times w^3 w^2 h^1 (1 - w^2 h^1)$$

$$\frac{\partial L}{\partial w^1} = \frac{\partial L}{\partial z^2} \frac{\partial z^1}{\partial w^1} \longrightarrow x$$

Here,

$$\frac{\partial L}{\partial z^1} = \frac{\partial L}{\partial z^2} \frac{\partial z^2}{\partial z^1}$$

Here,

$$\frac{\partial z^2}{\partial z^1} = \frac{\partial z^2}{\partial h^1} \frac{\partial h^1}{\partial z^1}$$

$$\frac{\partial h^1}{\partial z^1} = w^1 x(1 - w^1 x)$$

$$\Rightarrow \frac{\partial L}{\partial w^1} = 2[(\hat{y} - y_1) + (\hat{y_2} - y_2)][S(z_1^3) - (1 - S(z_1^3)) - 2 S(z_1^3) S(z_2^3) + S(z_2^3)(1 - S(z_2^3))] x \times w^3 w^2 h^1 (1 - w^2 h^1) x \; w^2 w^1 x (1 - w^1 x)$$