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Q2)

1) Logistic regression:-

$$f(x) = P(C_1 | x) = \sigma(w^T x + w_0)$$

$$\frac{\partial}{\partial a} \sigma(a) = \sigma(a)(1-\sigma(a))$$

→ Taking -ve log likelihood function

$$= -\ln \prod_{n=1}^{N} P(C_1 | x_n)^{y_n} (1 - P(C_1 | x_n))^{1-y_n}$$

$$= -\ln \prod_{n=1}^{N} f(x_n)^{y_n} (1 - f(x_n))^{1-y_n}$$

$$= -\sum_{n=1}^{N} y_n \ln f(x_n) + (1-y_n) \ln (1 - f(x_n))$$

$$= -\sum_{n=1}^{N} y_n \ln f(x_n) + \ln(1 - f(x_n)) - y_n \ln(1 - f(x_n))$$

Now Taking derative w.r.t w

$$= -\frac{\partial}{\partial w} \sum_{n=1}^{N} y_n \ln f(x_n) + \ln(1 - f(x_n)) - y_n \ln(1 - f(x_n))$$

$$= -\left[\sum_{n=1}^{N} y_n \frac{1}{f(x_n)} f(x_n)(1-f(x_n)) x_n + \cancel{\sum_{n=1}^{N} \frac{1}{1-f(x_n)}}\right.$$

$$+ \sum_{n=1}^{N} \frac{1}{1-f(x_n)} [-f(x_n)(1-f(x_n)) x_n]$$

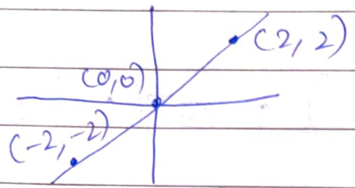
$$\left. - \sum_{n=1}^{N} y_n \frac{1}{1-f(x_n)} [-f(x_n)(1-f(x_n)) x_n] \right]$$

$$= -\sum_{n=1}^{N} x_n [y_n (1 - f(x_n)) - f(x_n) + y_n (f(x_n))]$$

$$= -\sum_{n=1}^{N} x_n (y_n - y_n f(x_n) - f(x_n) + y_n (f(x_n)))$$

$$= \sum_{n=1}^{N} (f(x_n) - y_n) x_n$$

Q.3)
2) Datapoints $(2,2)$, $(0,0)$, $(-2,-2)$



$\therefore$ Above line is $y = x$

$\therefore$ first principal component is $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\bar{x} = \frac{2+0-2}{3}$ $\quad \bar{y} = \frac{2+0-2}{3} = 0$

$cov(x,x) = \frac{1}{3-1}\left[(2-0)^2 + (0-0)^2 + (-2-0)^2\right] = 4$

$cov(y,y) = 4$

$cov(x,y) = cov(y,x) = 4$

$\therefore$ cov. matrix $\cdot \vec{v} = \lambda \vec{v}$

$\Rightarrow \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} \vec{v} = \lambda \vec{v}$

$\Rightarrow \begin{bmatrix} 4-\lambda & 4 \\ 4 & 4-\lambda \end{bmatrix} \vec{v} = \vec{0}$

$\Rightarrow (4-\lambda)^2 - 16 = 0$

$(4-\lambda)^2 = 16$
$4-\lambda = 4$
$\lambda = 0$ (eigen value)

$(4-\lambda)^2 = 16$
$4-\lambda = -4$
$\lambda = 8$ (eigen value)

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$\therefore$

$$\begin{bmatrix} 4-8 & 4 \\ 4 & 4-8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 4 \\ 4 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

or $\begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

which gives, $x - y = 0$

Eigen vector is unit vector of $(1, 1) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

b) Cordinates are given by $\vec{x}^T \vec{v}$

$\Rightarrow$ Projectn of $[-2, -2]^T = [-2, -2] \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = -2\sqrt{2}$

$\Rightarrow$ Projection of $[0, 0]^T = 0$

$\Rightarrow$ Projectn of $[2, 2] = [2, 2] \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = 2\sqrt{2}$

$\therefore$ $-2\sqrt{2}, 0, 2\sqrt{2}$ are the cordinates given by the $\vec{x}^T \vec{v}$

$$\text{Variance} = \frac{1}{3}\left[(-2\sqrt{2}-0)^2 + (0-0)^2 + (2\sqrt{2}-0)^2\right]$$

$$= \frac{16}{3}$$

c) So the points ~~line~~ lie on PC1, the ~~vac~~ varianc is therefore 100% of PC1.

There is no variance captured by PC1

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Q 4)

1) First we compute the weight vector for hyperplane.

$$W = \sum_{i, \alpha_i > 0} \alpha_i y_i X_i$$

$$= 0.414 \begin{vmatrix} 4 \\ 2.9 \end{vmatrix} + 0 + 0 - 0.018 \begin{vmatrix} 2.5 \\ 1 \end{vmatrix} + 0 + 0$$

$$+ 0.018 \begin{vmatrix} 3.5 \\ 4 \end{vmatrix} + 0 - 0.414 \begin{vmatrix} 2 \\ 2.1 \end{vmatrix} + 0$$

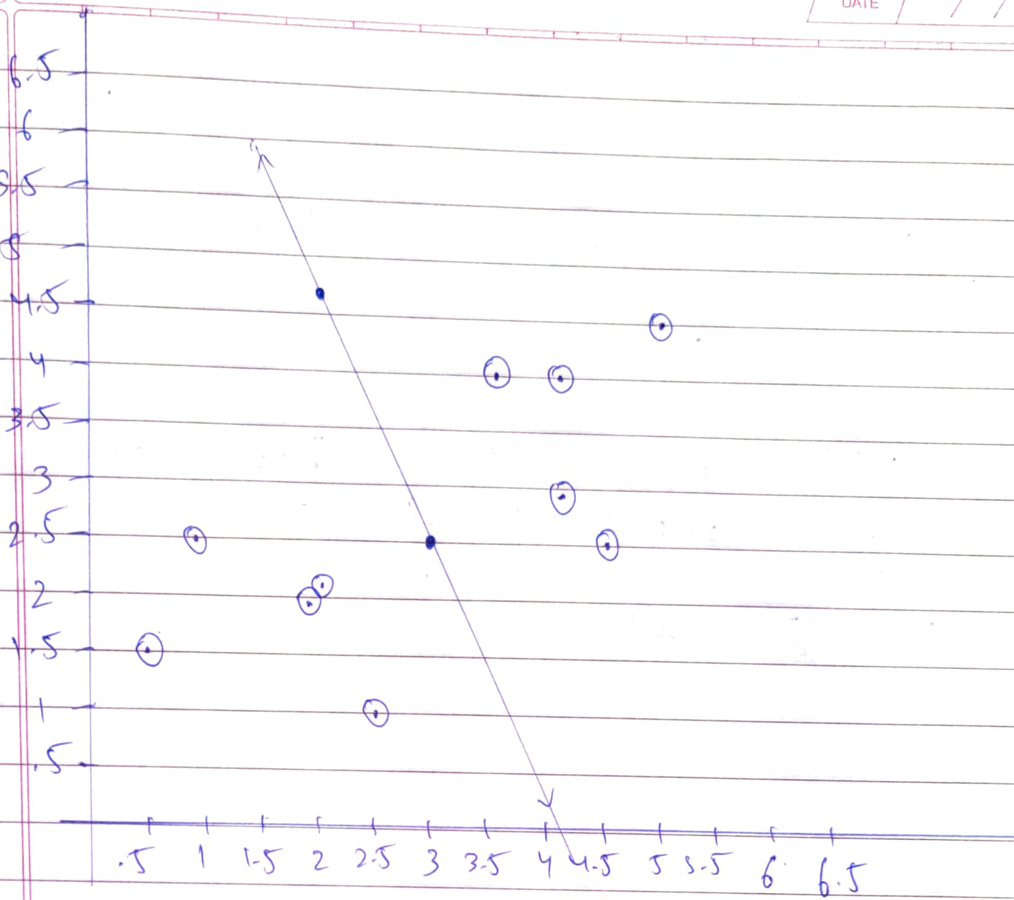
$$W = \begin{vmatrix} 0.846 \\ 0.385 \end{vmatrix}$$

→ To find the final bias we find the average bias obtained from each support vector.

x_i	$W^T x_i$	$b_i = y_i - W^T x_i$
X_1	4.5	$b_1 = -3.5$
X_4	2.5	$b_4 = -3.5$
X_7	4.5	$b_7 = -3.5$
X_9	2.5	$b_9 = -3.5$
		$\boxed{b_{Avg} = -3.5}$

Thus the optimal hyperplane is

$$h(x) = \begin{vmatrix} 0.846 \\ 0.385 \end{vmatrix}^T x - 3.5 = 0$$



2) $\therefore$ Our eqn of the line is

$$0.85x + 0.38y - 3.5 = 0$$

Distance formula $= \dfrac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$

$$= \frac{|0.85 \times 1.9 + 0.38 \times 1.9 - 3.5|}{\sqrt{0.85^2 + 0.38^2}}$$

$$= \frac{|-1.163|}{0.93}$$

$$= \frac{1.163}{0.93}$$

$$\approx 1.25$$

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3) Classify the point (3, 3)

$f(x) = 0.85(3) + 0.38(3) - 3.5$

$= 0.19$

$\therefore$ The point (3, 3) lies above the hyper plane ($\because f(x) > 0$)

$\therefore Z = +1$