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Q1)

1) Probability of standents in CS

$\Rightarrow$ P(S1) of CS students + P(S2) of CS students) + P(S3) of CS students.

$$\Rightarrow \left(\frac{0.2}{10} \times \frac{6}{20}\right) + \left(\frac{0.2}{10} \times \frac{10}{20}\right) + \left(\frac{0.6}{10} \times \frac{6}{20}\right)$$

$\Rightarrow$ 0.34

2) ~~Probabit~~

Probability of students in Session 3 from stats

$$\text{Probability of Stats} = \frac{8}{20} \times 0.2 + \frac{10}{20} \times 0.2 + \frac{6}{20} \times 0.6$$

$$= 0.36$$

Probability of studens of S3 from stats

$$\Rightarrow P(S3 \mid STAT) = \frac{\frac{6}{20} \times 0.6}{0.36}$$

$$= \frac{1}{2}$$

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Q2

1) $L(\mu,\sigma \mid x_1, x_2, \ldots x_n) = \prod_{n=1}^{10} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x_n-\mu)^2}$

2) log likelihood function :-

$\ln L(\mu,\sigma \mid x_1, x_2, \ldots x_n)$

$\Rightarrow \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x_1-\mu)^2} + \ldots + \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x_n-\mu)^2}\right)$

$\Rightarrow n\ln\left((2\pi\sigma^2)^{-\frac{1}{2}}\right) - \frac{1}{2\sigma^2}(x_1-\mu)^2 - \ldots - \frac{1}{2\sigma^2}(x_n-\mu)^2$

$\Rightarrow -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln\sigma^2 - \frac{1}{2\sigma^2}\left[(x_1-\mu)^2 + \ldots + (x_n-\mu)^2\right]$

$\Rightarrow -\frac{n}{2}\ln(2\pi) - n\ln\sigma - \frac{1}{2\sigma^2}\left[(x_1-\mu)^2 + \ldots + (x_n-\mu)^2\right]$

Derivative w.r.t μ :-

$\frac{\partial \ln L(\mu,\sigma \mid x_1, x_2, \ldots x_n)}{\partial \mu}$

$= \frac{-n}{2\sigma^2}\left[-2(x_1-\mu) - 2(x_2-\mu) \ldots -2(x_n-\mu)\right]$

$= \frac{n}{\sigma^2}\left[(x_1 + \ldots + x_n) - \mu n\right]$

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~~Q1) P(Students in CS)~~

equating to zero we get:-

$$\frac{n}{\sigma^2}\left[(x_1 + \dots + x_n) - \mu n\right] = 0$$

$$\Rightarrow \mu = \frac{x_1 + \dots + x_n}{n}$$

Derivative w.r.t σ:-

$$\frac{\partial}{\partial \sigma} \ln L(\mu, \sigma | x_1, \dots x_n) =$$

$$= \frac{-n}{\sigma} + \frac{2}{2} \times \frac{1}{\sigma^3}\left[(x_1-\mu)^2 + \dots + (x_n-\mu)^2\right]$$

$$= \frac{-n}{\sigma} + \frac{1}{\sigma^3}\left[(x_1-\mu)^2 + \dots + (x_n-\mu)^2\right]$$

equating to zero, we get

$$\frac{n}{\sigma} = \frac{1}{\sigma^3}\left[(x_1-\mu)^2 + \dots + (x_n-\mu)^2\right]$$

$$\Rightarrow \sigma = \sqrt{\frac{(x_1-\mu)^2 + \dots + (x_n-\mu)^2}{n}}$$

Hence,

$$\mu = \frac{1409}{10} = 140.9$$

$$\sigma = \sqrt{\frac{3466.9}{10}} = 18.619$$

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Q3 $P(X=1) = \frac{2q}{3}$, $P(X=2) = \frac{q}{3}$, $P(X=3) = \frac{2(1-q)}{3}$

$P(X=4) = \frac{1-q}{3}$

$$L(q|x_j) = \left(\frac{2q}{3}\right)^{n_1}\left(\frac{q}{3}\right)^{n_2}\left(\frac{2(1-q)}{3}\right)^{n_3}\left(\frac{1-q}{3}\right)^{n_4}$$

where, n_j = freq. of x_j in the distribution

$$\Rightarrow L(q|x_i) = \left(\frac{2q}{3}\right)^2\left(\frac{q}{3}\right)^3\left(\frac{2(1-q)}{3}\right)^3\left(\frac{1-q}{3}\right)^2$$

$$\Rightarrow \ln L(q|x_j) = 2\ln\left(\frac{2q}{3}\right) + 3\ln\left(\frac{q}{3}\right) + 3\ln\left(\frac{2}{3}(1-q)\right)$$

$$+ 2\ln\left(\frac{1-q}{3}\right)$$

$$\Rightarrow \ln L(q|X_j) = 2\ln\frac{2}{3} + 2\ln q + 3\ln\frac{1}{3} +$$

$$+ 3\ln\frac{1}{3} + 3\ln q + 3\ln\frac{2}{3} + 3\ln(1-q)$$

$$+ 2\ln\left(\frac{1}{3}\right) + 2\ln(1-q)$$

$$\Rightarrow \ln(q|x_i) = 5\ln q + 5\ln(1-q) + c$$

Differentiatin w.r.t (q) & equating to 0

$$\Rightarrow 0 = \frac{5}{q} - \frac{5}{1-q}$$

$$\Rightarrow 1-q = q$$

$$q = \frac{1}{2}$$

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Q4) $P(a/b) = \frac{P(b/a)\,P(a)}{P(b)}$

from Bayes theorm

$$P(w|x,y,\alpha,\beta) = \frac{P(y|w,\beta)\;P(w|\alpha)}{P(y)}$$

$$\therefore P(w|\alpha) = \left(\frac{\alpha}{2\pi}\right)^{(M+1)/2} e^{\left(-\frac{\alpha}{2} w^T w\right)}$$

$\therefore P(w|x,y,\alpha,\beta)$

$$\Rightarrow \left(\sqrt{\frac{\beta}{2\pi}}\right)^{n} e^{-\frac{(\Sigma(y_i - f(x,w))^2\beta}{2}} \times \left(\frac{\alpha}{2\pi}\right)^{\frac{M+1}{2}} e^{\left(\frac{-\alpha w^T w}{2}\right)}$$

$$\Rightarrow \frac{\partial}{\partial w} \log P(w/x,y,\alpha,\beta)$$

$$\Rightarrow -\frac{\beta}{2}\frac{\partial}{\partial w} \Sigma(y_i - f(x,w))^2 + \alpha w \quad \leftarrow (i)$$

Regularized sum of square error fun is,

$$\frac{\beta}{2}\Sigma(y_i - f(x,w))^2 + \frac{\alpha}{2} w^T w$$

$$\Rightarrow -\frac{\beta}{2}\frac{d}{dw} \Sigma(y_i - f(x,w))^2 + \alpha w$$

$\therefore$ We can say that derivative of regularized sum is equal eq^n (i)