rLoop 2D Rigid Body Dynamic Model

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This model will treat the rPod as a rigid 2 dimensional object, viewed from the top down, with a constant mass and moment of inertia. The spatial model states are position and rate of change along the x-y axis and yaw, totaling six states. Forces are applied to the body at given points, and these are translated into torque that affects the yaw. Both lateral drift and yaw motion will be modeled simultaneously, as these two spatial states of the pod likely cannot be treated separately. Lateral and yaw motion can be constrained by an infinitely rigid center beam. The model will be created in Simulink. A visualization tool is planned to play a movie of the lateral and yaw motion of the pod.

The primary objective of the model is to demonstrate yaw stability of the pod. Sources of drift and disturbances will have to be counteracted by the hover engines and by the lateral stability assembly. The model will then have to include realistic sensing from the accelerometers and gyroscopes, and realistic modeling of the shock absorbers/engines and relevant control loops if any.

Potential sources of disturbances

- Uneven drag force vector with off center positioning and lateral components
- Mass center of gravity off geometrical center of pod causing engines to one side to apply torque to pod
- Engine gimbaling error, uneven thrust from left to right engines
- Uneven force application of eddy brakes

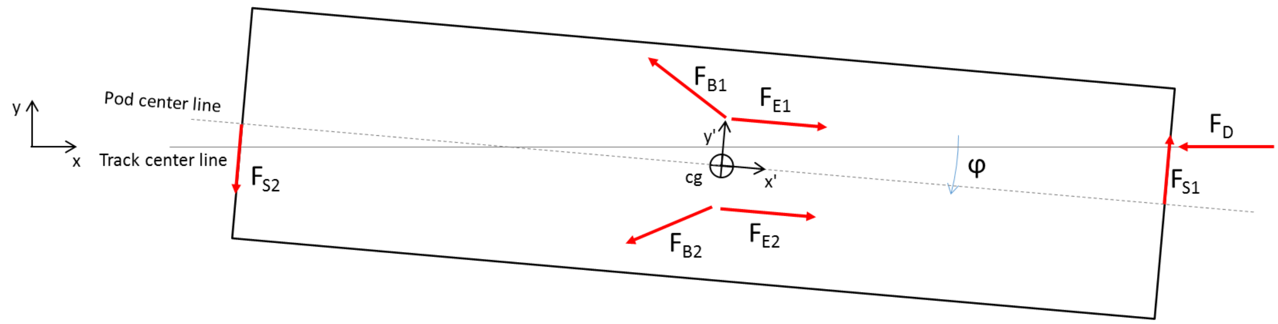
List of sensors

- Accelerometers
- MEMS gyroscopes
- Magnetometers

List of actuators

- Shock absorbers
- Hover Engines
- Eddy Brakes

Model 04/18/2016 file name "rigidBody2d04v.mdl": Simulink model that simulates lateral shock absorbers, two engine forces, two braking forces, and a drag force. CG assumed to be equal to coordinate system origin for simplicity. The file "runModel04v.m" initializes the model and runs the scenarios discussed below.



In the figure above, a free body diagram for the pod is shown. There are two coordinate systems, an inertial track centered coordinate (x and y) and a pod centered coordinate (x' and y'). The pod can move in all three of these degrees of freedom. In the shown picture, the pod is off center in the negative y-axis and has a negative yaw, φ. Seven force vectors are shown.

Shock absorbers will be modeled as linear spring/damper elements,

$$F_S = -kd - c\dot{d}$$

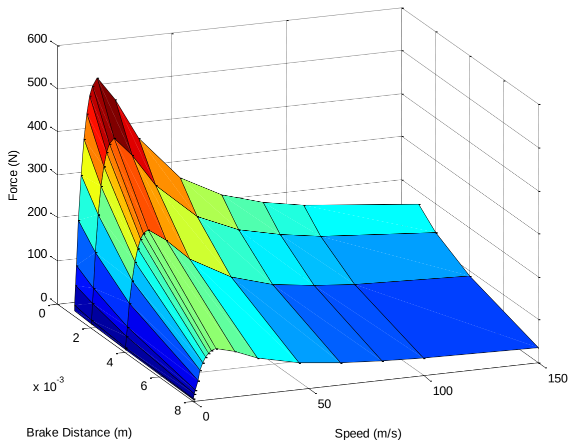
Here, d is the compression amount. For the forward suspension,

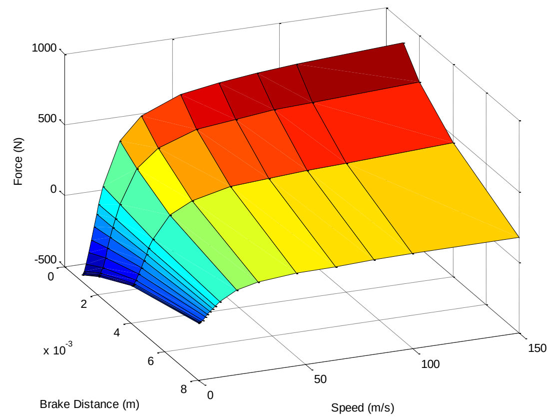
$$d = y + \frac{L}{2} sin\varphi \cong y + \frac{L}{2}\varphi$$

Where y is the lateral movement in the y-axis and L is the length of the pod. The equation is made simpler by the small angle approximation. Likewise, the rear suspension is the same, but 180° is added to the yaw angle.

$$d = y + \frac{L}{2}\sin(\varphi + \pi) = y - \frac{L}{2}\sin\varphi \cong y - \frac{L}{2}\varphi$$

The brakes are modeled as 48 total N42 magnets using a force lookup table. The brakes generate drag (in the negative x direction) and lift (in the positive y direction for the left brakes and in the negative y direction for the right brakes). The lift and drag amounts are a function of the distance to the rail and the speed of the pod. The model is in the "brakeModel.mat" file and is shown in the figure below.





The governing equations are the force sums impacting the pod's acceleration in the x, y and yaw direction. The vector sum of forces:

$$\sum \boldsymbol{F} = \boldsymbol{F_{E1}} + \boldsymbol{F_{E2}} + \boldsymbol{F_{B1}} + \boldsymbol{F_{B2}} + \boldsymbol{F_{S1}} + \boldsymbol{F_{S2}} + \boldsymbol{F_D} = m\boldsymbol{a}$$

These are calculated for each direction in the x-y coordinate system, such that the resultant acceleration, velocity and position are with respect to x-y. The forces application points and lines of force are attached to the pod, so these must be transformed to x-y, which is accounted for in the model.

The sum of moments:

$$\sum \boldsymbol{M} = \boldsymbol{r_{E1}} \times \boldsymbol{F_{E1}} + \boldsymbol{r_{E2}} \times \boldsymbol{F_{E2}} + \boldsymbol{r_{B1}} \times \boldsymbol{F_{B1}} + \boldsymbol{r_{B2}} \times \boldsymbol{F_{B2}} + \boldsymbol{r_{S1}} \times \boldsymbol{F_{S1}} + \boldsymbol{r_{S2}} \times \boldsymbol{F_{S2}} + \boldsymbol{r_D} \times \boldsymbol{F_D} = m\alpha$$

The moment and force sums are implemented in the "Sum of Forces/Torques" subsystem. The force locations are listed below

Symbol	Description	Location (x', y')
F_E1	Left engine	0 m, 0.5 m
F_E2	Right engine	0 m, -0.5 m
F_B1	Left brake	0 m, 0.5 m
F_B2	Right brake	0 m, -0.5 m
F_S1	Front suspension	1.9 m, 0 m
F_S2	Rear suspension	-1.9 m, 0 m
F_D	Drag force	1.9 m, 0 m (in x,y)

The remaining constants and data sources are listed below:

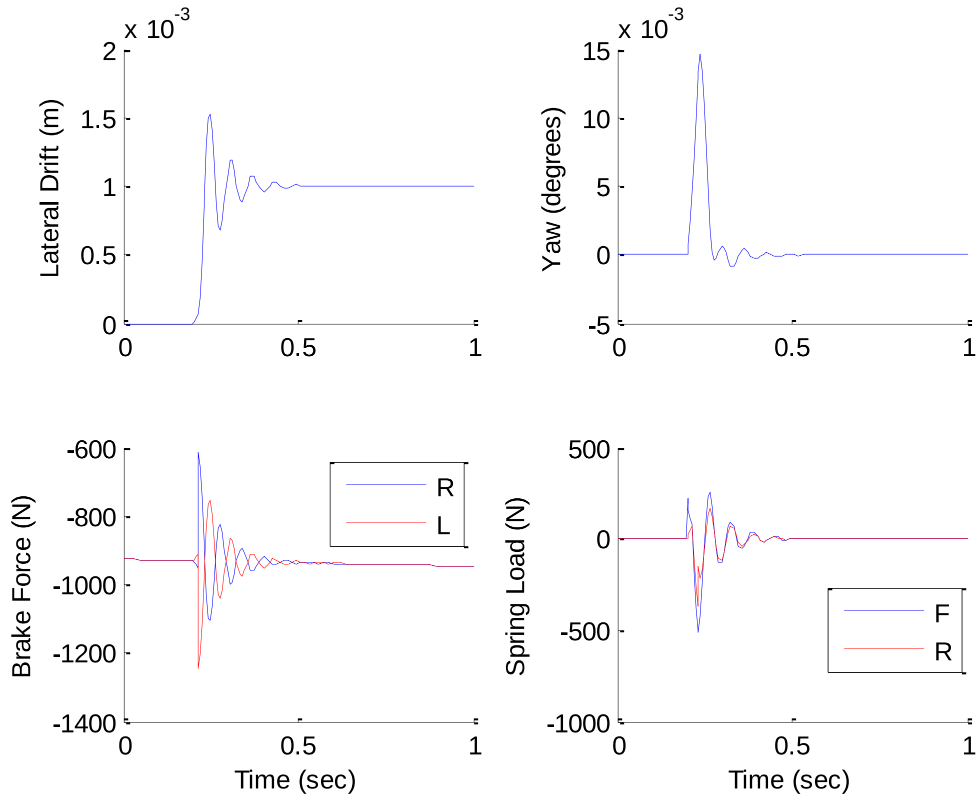
Description	Symbol	Value	Units	Source
Length of pod	L	3.8	m	SDD Sec 2.1
Diameter of pod	D	1	m	SDD Sec 2.1
Mass of pod	m	320	kg	SDD Sec 2.1
Drag Force	F_D	645	N	SDD Sec 2.1
Engine/Brake Lever Arm	rE1y, rE2y, rB1y, rB2y	0.5	m	SDD Fig 3.17 (approx.)
Cruise speed	v0	120	m/s	SDD Fig 3.18
Engine Thrust	FE1x, FE2x	60	N	SDD Sec 3.2.3.2
Drag Force	FDx	645	N	Amir 3/12
Spring Constant	k	111000	N/m	DV-22 stiffness (750 lbs by 30 mm)
Brake Force	FB1, FB2	Varies with distance & speed	N	Andi 4/10

We will consider four scenarios with an arbitrary amount of damping. Unfortunately no data exists on damping, however it is reasonable to expect some amount of damping.

- Track bump: the impact of hitting a bump assuming worst case construction of the track
- Engine failure: complete engine shutdown on one side
- Brake failure: brake application only on one side
- Brake delay: one brake is applied with a time delay

Track bump

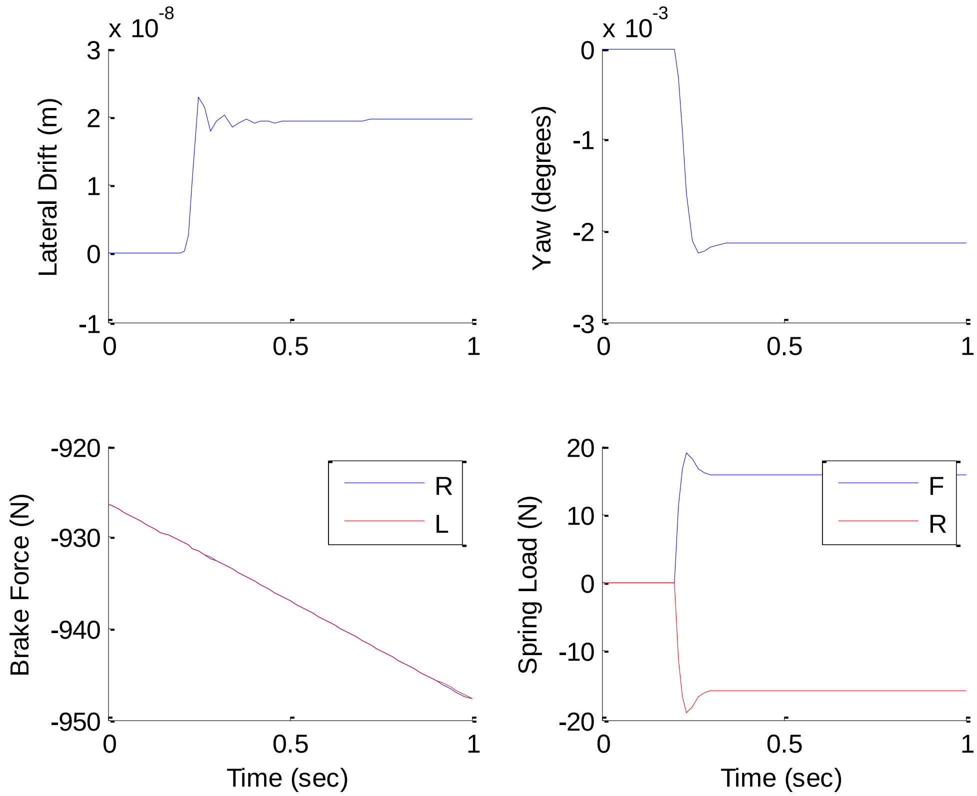
We will assume a worst case lateral misalignment occurs on the track. This will mean that the wheels will experience a forced step input, resulting in an instantaneous compression/expansion of the suspension. This translates to an instant force application on the pod. The front suspension is first impacted, then the rear is delayed by the time it takes to travel the length of the pod. The assumed misalignment is 1mm.



For this scenario, the pod lateral motion oscillates around 1mm in the positive direction (since the subsequent track is offset in that direction), and the yaw oscillates around zero (since at infinite time, no steady state change to the yaw should exist in this case).

Coast engine failure

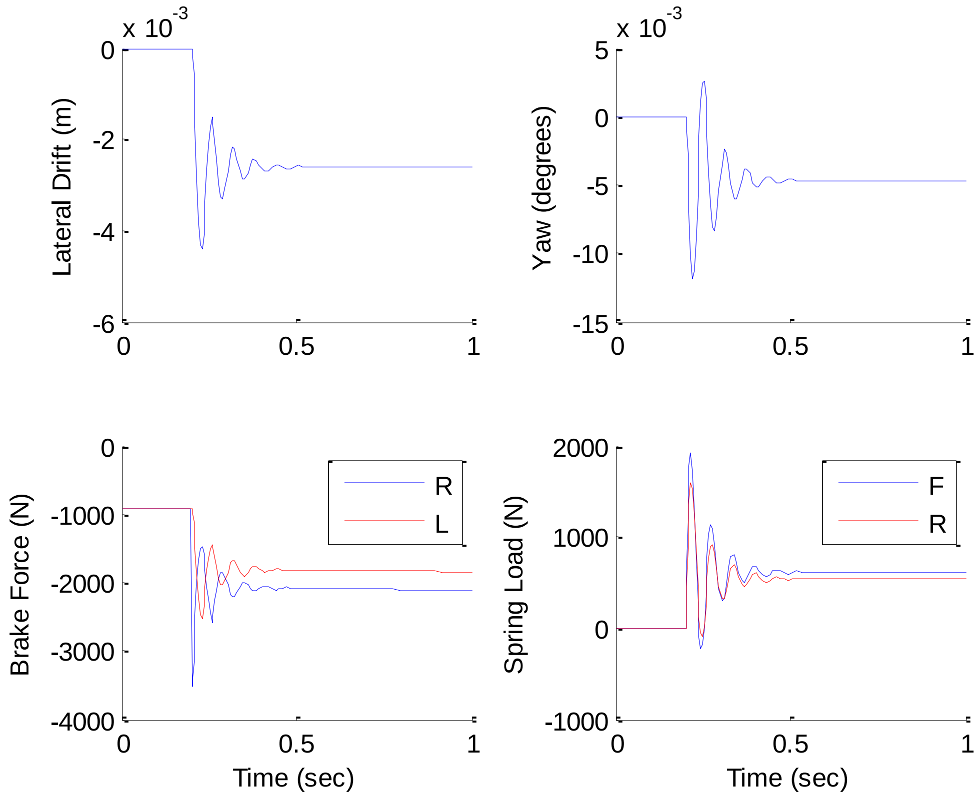
In the model, each pair of engines is modeled as a single force, one to the left and one to the right. We assume that the right engine fails to provide thrust suddenly, perhaps due to a gimbaling error.



Above is the result for this scenario. This failure causes the pod to yaw towards the right since the left engines are providing a net moment in that direction. This causes some of the remaining thrust vector on the left engines to provide some lateral motion, since they are no longer pointing perfectly forward. Both lateral motion and yaw are affected. The extremely small yaw and lateral motion simply highlight the high stiffness of the shock absorbers and the low amount of moment caused by the engines. Interestingly, the engine thrust vector points to the right due to yaw motion, and yet the lateral motion is to the left. This is due to the brake lift.

Brake failure

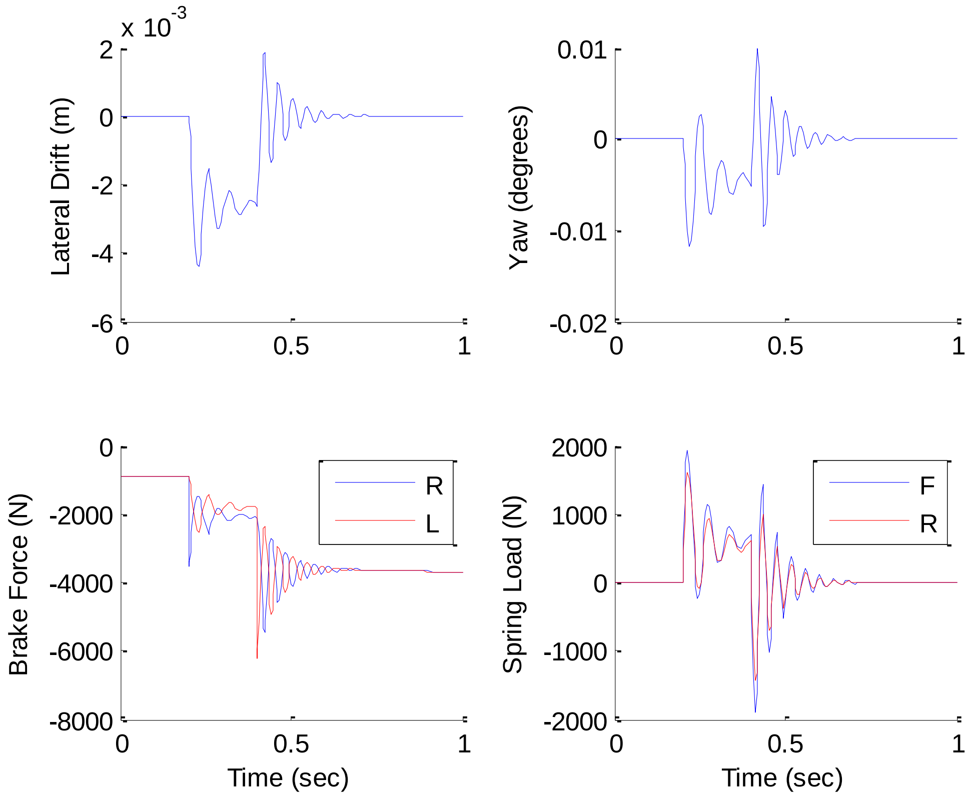
This case is similar to engine failure, but it is simulated in the braking portion of flight and simulates left side of the brakes failing to actuate. The brakes begin at 8mm distance and only the right brake drops to 2mm distance.



The results above show more of an impact, since the braking force is much higher than the engine thrust force. If the left side brakes fail, the pod will again yaw towards the right since that is where all the braking force is applied. The amount of yaw is comparable to the track bump scenario.

Brake delay

Just like the brake failure case, but the left brake engages at 0.4 seconds.



After the left brake is applied properly, the pod returns to centered position and zero yaw.