Happy Friday!!

Number Systems (in Binary)

Lab 1 part 2 due Sunday

CATME survey due Sunday

No lab or discussion section next week

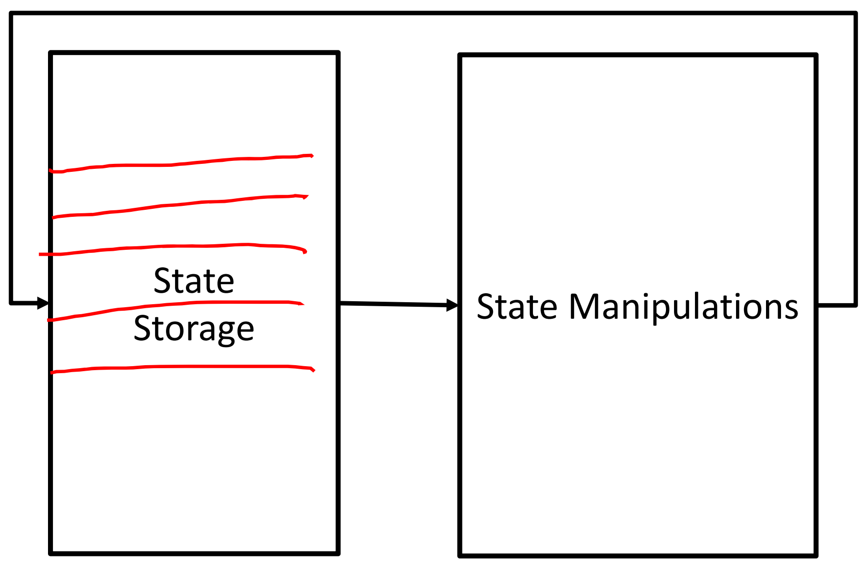
lab 2 part 1 is due Thursday

Office hours during scheduled section times

Stored state bits can be interpreted with many different encoding schemes

Computer can do 2 things

1) Store state (How do we interpret stored bits?)
2) Manipulate state



233 in one slide!

Today we introduce how
to interpret state as data

- The class consists roughly of 4 quarters: (Bolded words are the big ideas of the course, pay attention when you hear these words)
  1. You will build a simple computer processor
     Build and create **state** machines with **data**, **control**, and **indirection**
  2. You will learn how high-level language code executes on a processor
     Time limitations create **dependencies** in the **state** of the processor
  3. You will learn why computers perform the way they do
     Physical limitations require **locality** and **indirection** in how we access **state**
  4. You will learn about hardware mechanisms for parallelism
     Locality, dependencies, and **indirection** on performance enhancing drugs

- We will have a SPIMbot contest!

Today's lecture

- Representing things with bits
  - N bits gets you 2^N representations
- Unsigned binary number representation
  - Converting between binary and decimal
  - Hexadecimal notation
- Binary Addition & Bitwise Logical Operations
  - Every operation has a width
- Two's complement signed binary representation

A code maps each fixed-width string of bits to a meaning

(like a secret decoder ring…)

Bit pattern	Marine Mammal
0100101	Humpback Whale
0100110	Leopard Seal
0100111	Sea Otter
0101000	West Indian Manatee
0101001	Bottlenose Dolphin

- This mapping however is rarely stored explicitly
  - Rather it is used when we interpret the bits.

How many bits to encode N possible things?

- 1 bit can encode 2 possibilities (0, 1)

N

False 0 — 00 — 000, 001
01 — 010, 011 . . . . .

True 1 — 10, 11

Bits = 1   2   3   N

\# encodings = 2 2^2 $2^3 = 8$ 2^N

What is the minimum # of bits to encode?

- One of the U.S.'s 50 states?

a) 3 $= 2^3 = 8$

b) 4 $\Rightarrow 16$

c) 5 $\Rightarrow 32$

d) 6 $\Rightarrow 64$

e) 7

How many bits to encode?

- The list of Justin Bieber’s good songs?

a) 0

b) 0

c) 0

d) 0

e) 0

Unsigned numbers are the set of non-negative numbers

- 0, 1, 2, 3, 4, 5, …

- N bits ➔ store 2^N unsigned numbers ➔ what range should the bits encode?
  - 3 bits ➔ *8 representations* ➔ 0 to 7? 1 to 8? 32-40?
  - 8 bits ➔ 256 *representations* ➔ 0 to 255? 1 to 256? 1024 to 1280?

How does decimal representation work?

larger smaller

Consider 162.375

- Numbers consist of a bunch of digits, each with a weight:

1	6	2	.	3	7	5	Digits
100	10	1		1/10	1/100	1/1000	Weights

+1

- All weights are powers of the base, which is 10:

1	6	2	.	3	7	5	Digits
10^2 =100	10^1	10^0 = 1		10^{-1}	10^{-2}	10^{-3}	Weights

-1

- **To find the decimal value of a number**, multiply each digit by its weight and sum the products.

$$(1 \times 10^2) + (6 \times 10^1) + (2 \times 10^0) + (3 \times 10^{-1}) + (7 \times 10^{-2}) + (5 \times 10^{-3}) = 162.375$$

$$100 + 60 + 2 + \frac{3}{10} + \frac{7}{100} + \frac{5}{1000} =$$

Unsigned binary number representation uses a position-weighted encoding scheme

- The weights are powers of 2.
- For example, here is 1101 in binary:

1	1	0	1	Binary digits, or bits
2^3	2^2	2^1	2^0	Weights (in base ~~10~~) 2

- The decimal value is:

$$(1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) =$$
$$8 \quad + \quad 4 \quad + \quad 0 \quad + \quad 1 \qquad = 13$$

Powers of 2:

$2^0 = 1$	$2^4 = 16$	$2^8 = 256$
$2^1 = 2$	$2^5 = 32$	$2^9 = 512$
$2^2 = 4$	$2^6 = 64$	$2^{10} = 1024$
$2^3 = 8$	$2^7 = 128$	

Binary to Decimal

- What is the 5-bit unsigned number 01010 in decimal?

$8+2=10$

a) 2

b) 5

c) 10

d) 12

e) 18

Powers of 2:

$2^0 = 1$	$2^4 = 16$	$2^8 = 256$
$2^1 = 2$	$2^5 = 32$	$2^9 = 512$
$2^2 = 4$	$2^6 = 64$	$2^{10} = 1024$
$2^3 = 8$	$2^7 = 128$	

Fractional binary numbers use the same pattern as integer binary numbers

- For example, here is 1101.01 in binary:

1	1	0	1	.	0	1	Binary digits, or bits
2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	Weights (in base 10)

- The decimal value is:

-1

$$(1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) + (0 \times 2^{-1}) + (1 \times 2^{-2}) =$$
$$8 + 4 + 0 + 1 + 0 + 0.25 = 13.25$$

$\frac{1}{2}$ $\quad \frac{1}{2^2} = \frac{1}{4}$

An algorithm for converting decimal to binary

- Decimal integer ➔ binary: Keep dividing by 2 until the quotient is 0. Collect the remainders in *reverse* order.
- Example: 162:

162 / 2	= 81	rem	0
/ 2	= 40	rem	1
/ 2	= 20	rem	0
/ 2	= 10	rem	0
/ 2	= 5	rem	0
/ 2	= 2	rem	1
/ 2	= 1	rem	0
/ 2	= 0	rem	1

10100010

$\underbrace{128+32}_{160} + 2 = 162$

Converting decimal to binary

- Decimal integer → binary: Keep dividing by 2 until the quotient is 0. Collect the remainders in *reverse* order.
- Example: 162.375:

162 / 2 = 81	rem	0
81 / 2 = 40	rem	1
40 / 2 = 20	rem	0
20 / 2 = 10	rem	0
10 / 2 = 5	rem	0
5 / 2 = 2	rem	1
2 / 2 = 1	rem	0
1 / 2 = 0	rem	1

To convert a fraction, keep multiplying the fractional part by 2 until it becomes 0. Collect the integer parts in forward order.

0.375 x 2 = 0.750
0.750 x 2 = 1.500
0.500 x 2 = 1.000

- So, 162.375_{10} = 10100010.011_2

Converting Decimal to Binary

iclicker

- How do you represent 49 in 8-bit unsigned binary?

a) 110001

b) 100011

c) 00110001

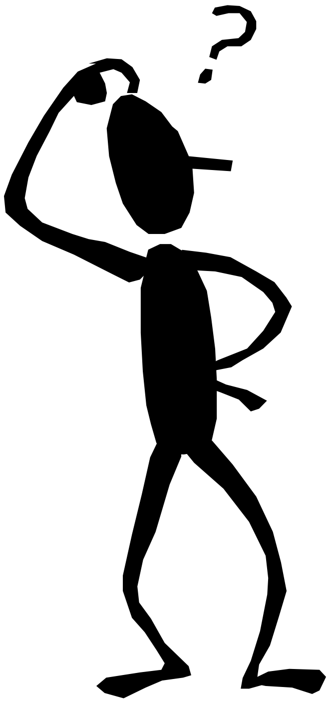
d) 10001100

0	0	1	1	0	0	0	1

Powers of 2:

$2^0 = 1$	$2^4 = 16$	$2^8 = 256$
$2^1 = 2$	$2^5 = 32$	$2^9 = 512$
$2^2 = 4$	$2^6 = 64$	$2^{10} = 1024$
$2^3 = 8$	$2^7 = 128$	

Why does this work?



- This works for converting from decimal to *any* base
- Why? Think about converting 162.375 from decimal to decimal.

162 / 10= 16 rem 2
16 / 10= 1 rem 6
1 / 10= 0 rem 1

- Each division strips off the rightmost digit (the remainder). The quotient represents the remaining digits in the number.
- Similarly, to convert fractions, each multiplication strips off the leftmost digit (the integer part). The fraction represents the remaining digits.

0.375 x 10 = 3.750
0.750 x 10 = 7.500
0.500 x 10 = 5.000

Writing binary numbers is tedious and error prone

char - 8 bits   int - 32-bit

Consider

$$\underbrace{10011010111001101011000111111101}_{32}$$

Use Hexadecimal (base-$\underbrace{16}_{2^4}$) as a shorthand for binary numbers

- The hexadecimal system uses 16 digits:

  0 1 2 3 4 5 6 7 8 9 A B C D E F

- We can write our 32-bit number:

  `1001 1010 1110 0110 1011 0001 1111 1101`

  9 A E 6

  as

  `0x9AE6B1FD` (C/Java style)

  `32'h9AE6B1FD` (Verilog style)

Fun fact: Hex is frequently used to specify things like 32-bit IP addresses and 24-bit colors.

Decimal	4 Binary	Hex
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A =10
11	1011	B =11
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Hexadecimal to Binary

- What is $B4_{16}$ in binary?
  - A: 10110100
  - B: 1010100
  - C: 1011100
  - D: 11000100

Binary and hexadecimal conversions

- Converting from hexadecimal to binary is easy: just replace each hex digit with its equivalent 4-bit binary sequence.

$$261.35_{16} = 2 \quad 6 \quad 1 \quad . \quad 3 \quad 5_{16}$$
$$= 0010 \quad 0110 \quad 0001 \quad . \quad 0011 \quad 0101_2$$

- To convert from binary to hex, make groups of 4 bits, starting from the binary point. Add 0s to the ends of the number if needed. Then, just convert each bit group to its corresponding hex digit.

$$10110100.001011_2 = 1011 \quad 0100 \quad . \quad 0010 \quad 1100_2$$
$$= B \quad 4 \quad . \quad 2 \quad C_{16}$$

Hex	Binary
0	0000
1	0001
2	0010
3	0011

Hex	Binary
4	0100
5	0101
6	0110
7	0111

Hex	Binary
8	1000
9	1001
A	1010
B	1011

Hex	Binary
C	1100
D	1101
E	1110
F	1111

Add binary numbers just like how you do with decimal numbers

- But remember that it's binary! For example, 1 + 1 = 10 and you have to carry!

2

1
5
+ 5
10

	1	1	1	0			
	0	1	0	1	1	Augend	(11)
+	0	1	1	1	0	Addend	(14)
	1	1	0	0	1	Sum	= 25

16 + 8 + 1 = 25 = 25 YAY

Add binary numbers just like how you do with decimal numbers

- But remember that it's binary! For example, 1 + 1 = 10 and you have to carry!

The initial carry in is implicitly 0

	1	1	1	0		Carry in
	0	1	0	1	1	Augend
+	0	1	1	1	0	Addend
	1	1	0	0	1	Sum

most significant bit, or MSb

least significant bit, or LSb

Computers restrict all binary numbers to use the same number of bits (i.e., fixed-width)

- What if we do that same addition, using only 4-bit numbers
  - (and where the result can only be 4 bits long…)

0 - 15

algorithm → last carry out

1	1	1	0			
	1	0	1	1	Augend	(11)
+	1	1	1	0	Addend	(14)
	1	0	0	1	Sum	(25)

8 + 1 = 9 ≠ 25

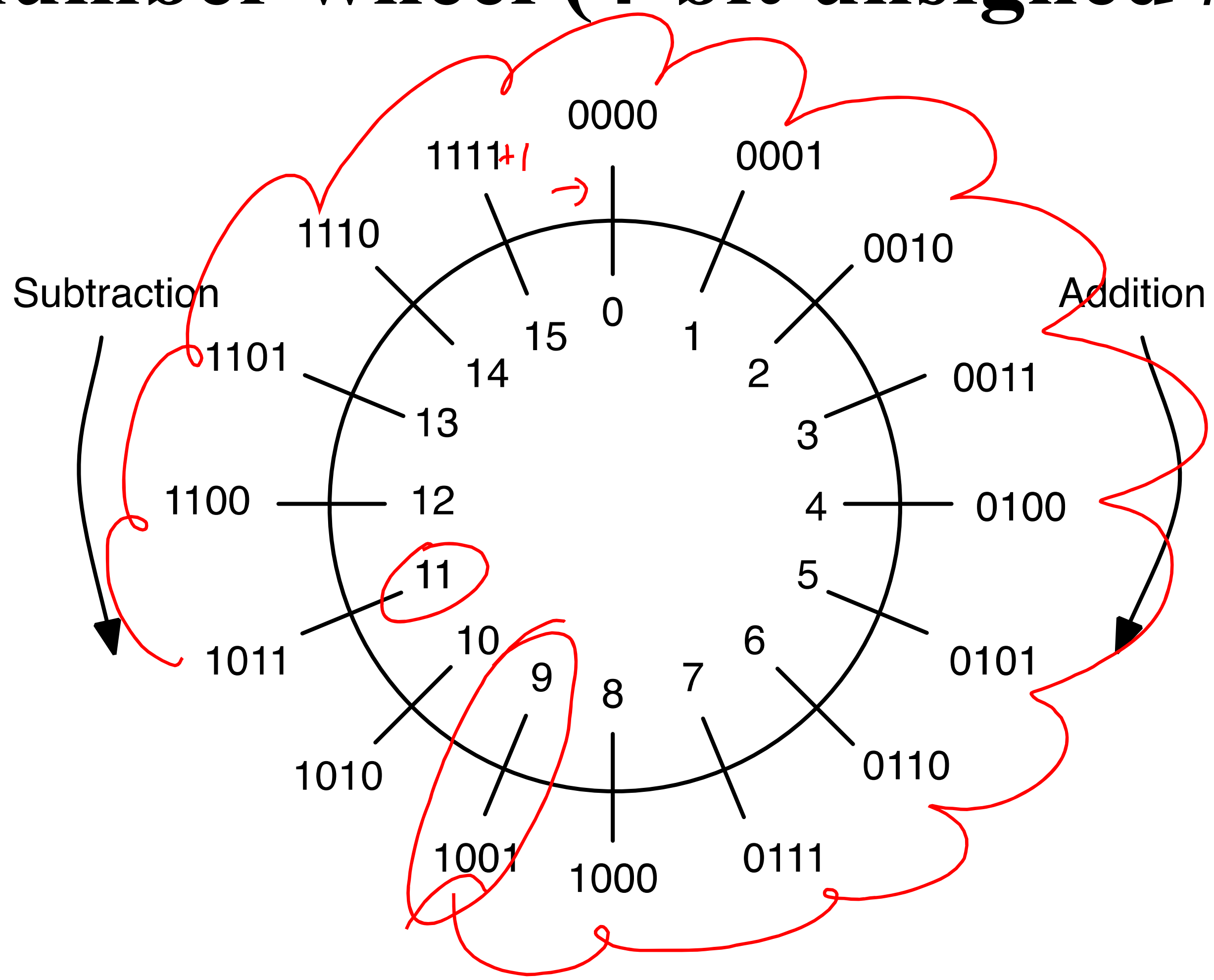
11

overflow

↑

interpretation

The number wheel (4-bit unsigned #'s)



“Carry-out” is a procedure, “Overflow” is an interpretation

Carry-out

- Occurs at every bit-position
- The process of moving larger numbers to higher bit positions
- Focuses on bit-wise operations

Overflow

- Can only be seen after completing an entire mathematical operation
- When the interpretation of a set of bits does not match the expected value after a mathematical operation
- Focuses on representational range (i.e., 4 bits represent 0-15)

Bitwise Logical operations support logical operations on multi-bit words

- To apply a logical operation to two words X and Y, apply the operation on each pair of bits X_i and Y_i:

```
      1 0 1 1           1 0 1 1           1 0 1 1
AND   1 1 1 0     OR    1 1 1 0     XOR   1 1 1 0
      -------           -------           -------
      1 0 1 0           1 1 1 1           0 1 0 1
```

Languages like C, C++ and Java provide bitwise logical operations:

X & Y

& (AND) | (OR)   ^ (XOR)   ~ (NOT)

- These operations treat each integer as a bunch of individual bits:

  13 & 25 = 9   because

```
  01101
& 11001
  01001
```

unsigned int X=13;
" " Y=25;

- Bitwise operators are often used in programs to set a bunch of Boolean options, or flags, with one argument.

- They are *not* the same as the operators &&, || and !, which treat each integer as a single logical value (0 is false, everything else is true):

  13 && 25 = 1   because   true && true = true

  true true true

Bit-wise XOR

001011 XOR 110011

- A: 111001
- B: 111011
- C: 111000
- D: 000110

Bitwise operations are used to find network information

- IP addresses are actually 32-bit (or 128-bit) binary numbers
- For example, you can bitwise-AND an address 192.168.10.43 with a "subnet mask" to find the "network address," or which network the machine is connected to.

```
  192.168. 10. 43 = 11000000.10101000.00001010.00101011
& 255.255.255.224 = 11111111.11111111.11111111.11100000
-------------------------------------------------------
  192.168. 10. 32 = 11000000.10101000.00001010.00100000
```

- You can use bitwise-OR to generate a "broadcast address," for sending data to all machines on the local network.

```
  192.168. 10. 43 = 11000000.10101000.00001010.00101011
|   0.  0.  0. 31 = 00000000.00000000.00000000.00011111
-------------------------------------------------------
  192.168. 10. 63 = 11000000.10101000.00001010.00111111
```

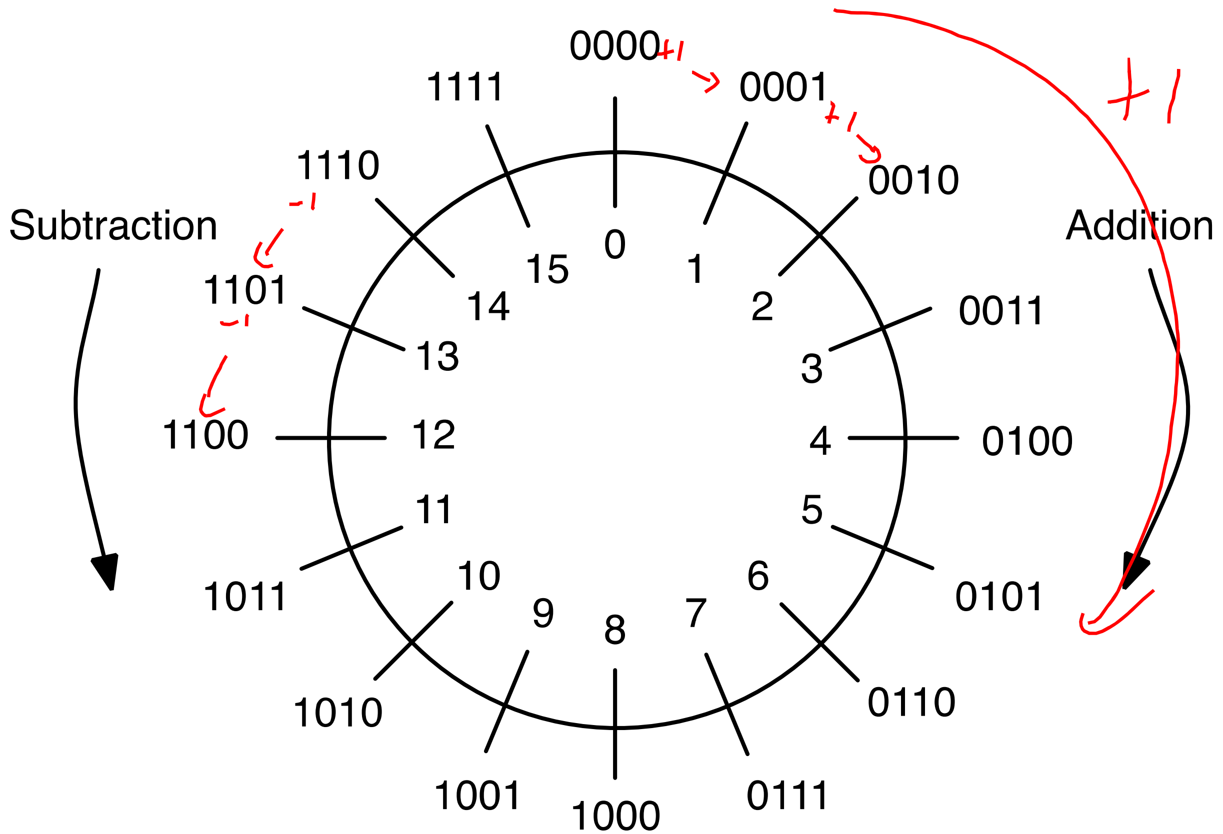
Preview for next time: How do we represent negative numbers

- It is useful to be able to represent negative numbers.

- What would be ideal is:
  - If we could use the **same algorithm to add signed numbers as we use for unsigned numbers**
  - Then our computers wouldn't need 2 kinds of adders, just 1.

- This is achieved using the 2's complement representation.

The number wheel (4-bit unsigned #'s)



The number wheel (4-bit 2's complement)