Writing Cache Friendly Code

Tonight: SPIMbot.review session @ 4-5 pm in 1304 SC

Will be recorded

Writing Cache Friendly Code

- Make the common case go fast
  - Focus on the inner loops of the core functions

- Minimize the misses in the inner loops
  - Repeated references to variables are good (temporal locality)
  - Stride-1 reference patterns are good (spatial locality)

Key idea: Our qualitative notion of locality is quantified through our understanding of cache memories.

Today

- Performance impact of caches
  - The memory mountain
  - Rearranging loops to improve spatial locality
  - Using tiling to improve temporal locality

The Memory Mountain

- Read throughput (read bandwidth)
  - Number of bytes read from memory per second (MB/s)

- Memory mountain: Measured read throughput as a function of spatial and temporal locality.
  - Compact way to characterize memory system performance.

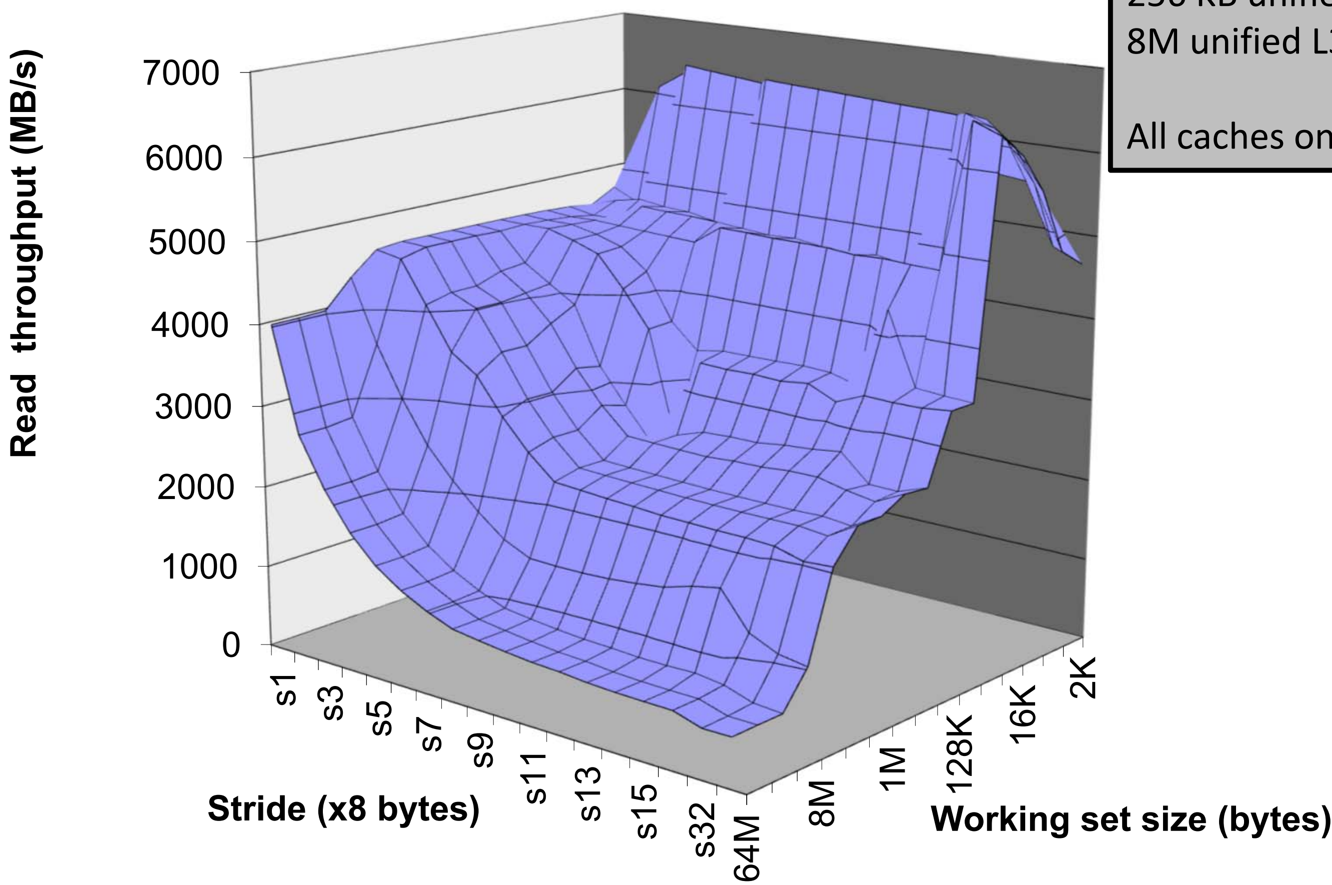
Memory Mountain Test Function

```
/* The test function */
void test(int elems, int stride) {
    int i, result = 0;
    volatile int sink;

    for (i = 0; i < elems; i += stride)
        result += data[i];
    sink = result; /* So compiler doesn't optimize away the loop */
}

/* Run test(elems, stride) and return read throughput (MB/s) */
double run(int size, int stride, double Mhz)
{
    double cycles;
    int elems = size / sizeof(int);

    test(elems, stride);                     /* warm up the cache */
    cycles = fcyc2(test, elems, stride, 0);  /* call test(elems,stride) */
    return (size / stride) / (cycles / Mhz); /* convert cycles to MB/s */
}
```

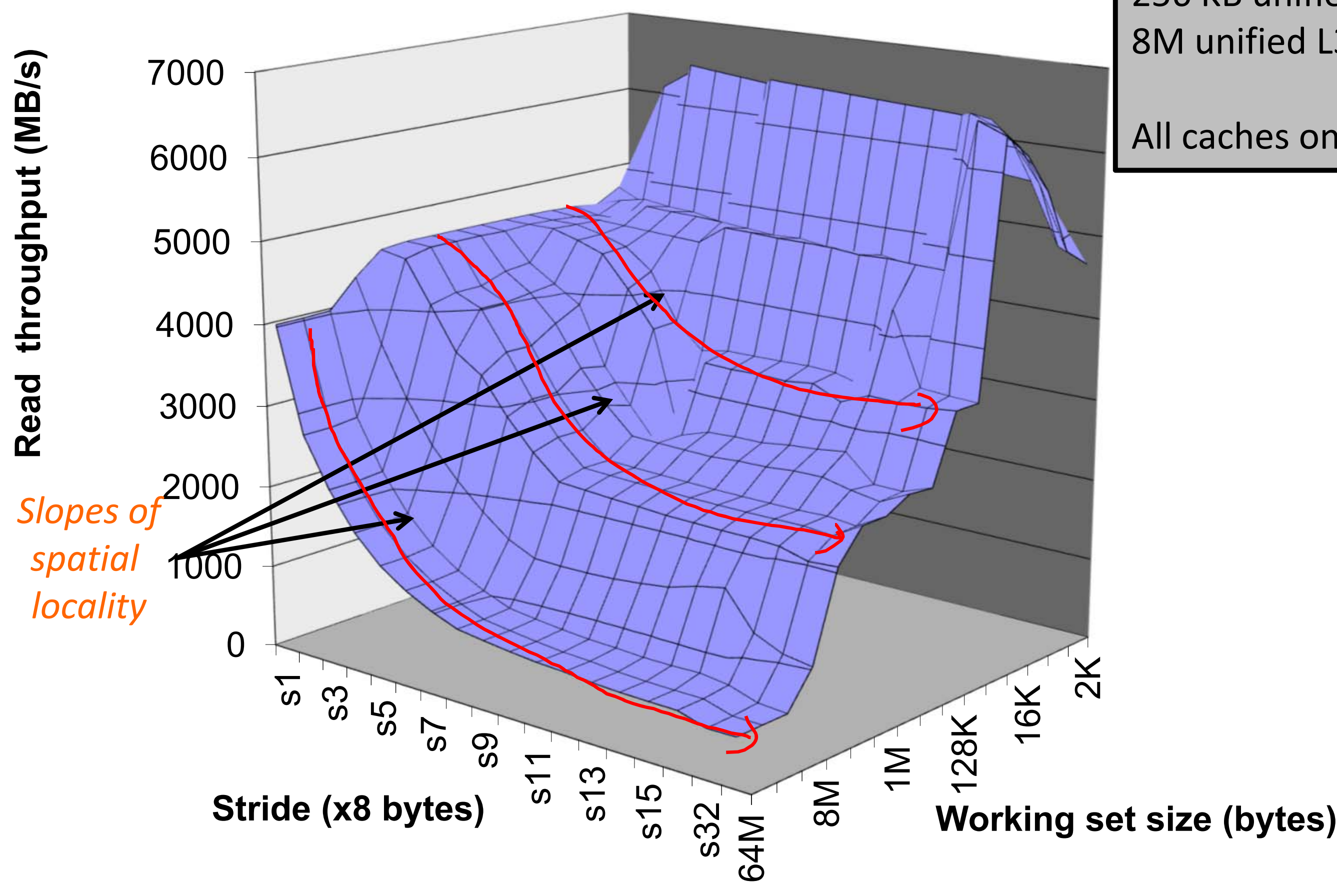
The Memory Mountain

Intel Core i7
32 KB L1 i-cache
32 KB L1 d-cache
256 KB unified L2 cache
8M unified L3 cache

All caches on-chip



The Memory Mountain



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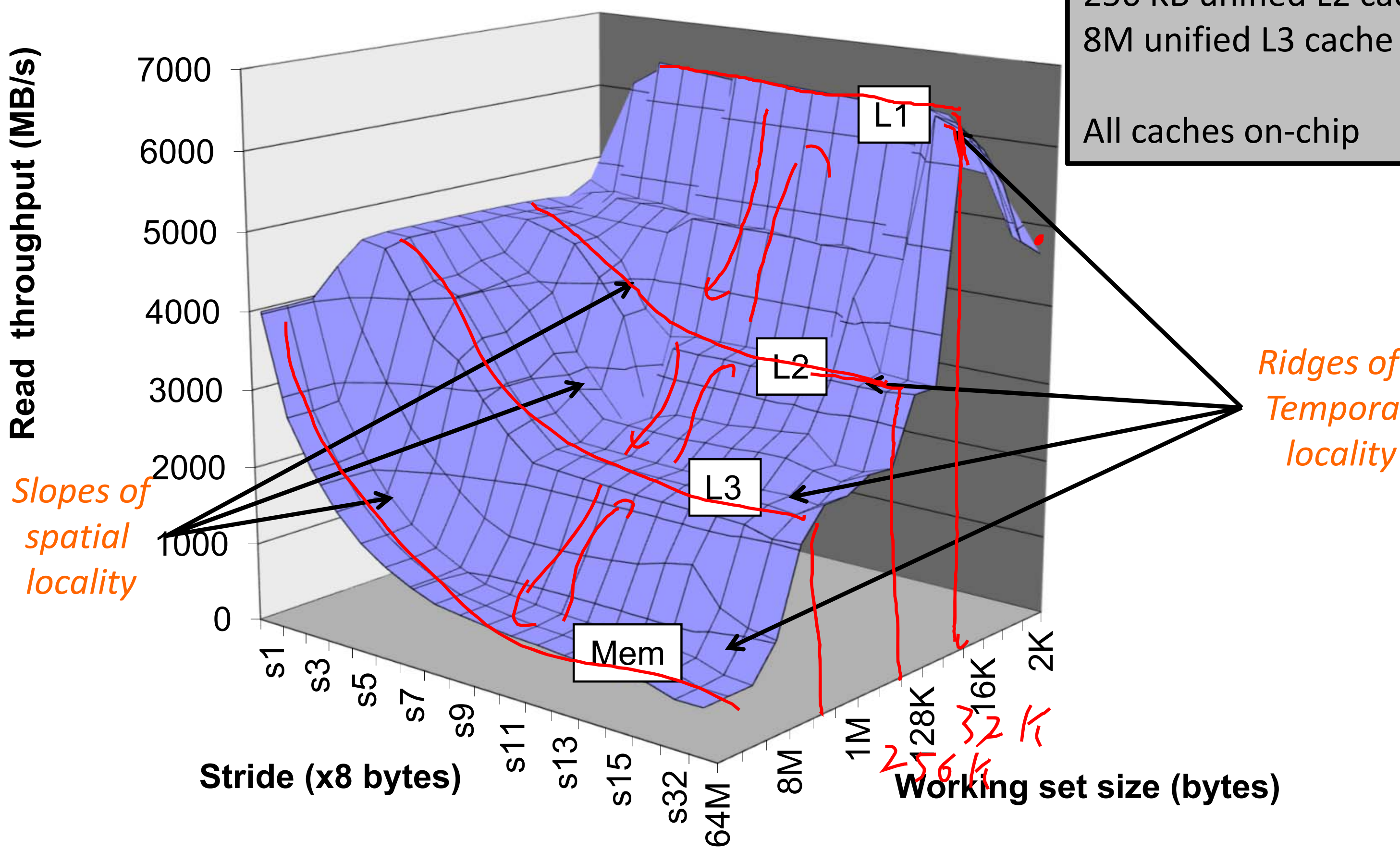
The Memory Mountain



Intel Core i7
32 KB L1 i-cache
32 KB L1 d-cache
256 KB unified L2 cache
8M unified L3 cache

All caches on-chip

Warm-up example

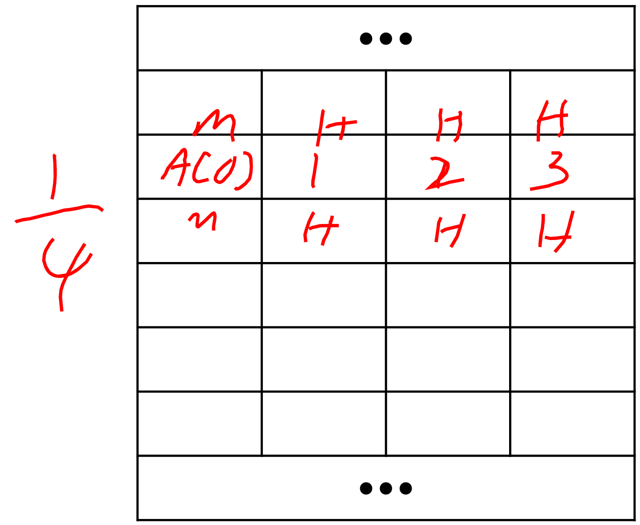
```
double sum = 0.0;
double a[n];

for (i = 0; i < m; i ++)  {
  for (j = 0; j < n; j ++) {
    sum += a[j];
  }
}
```

- Description:
  - 1 array of length n
  - Walk array m times

- Assumptions
  - 32B cache blocks (fit four, 64-bit (8B) doubles)
  - n is large (array much bigger than cache)
  - m is large (approximate 1/m as 0)

- Average miss rate?
  - A) 0   B) 1/8   C) 1/4   D) 1/2   E) 1



Loop inversion swaps the indexing variable of the inner loop (j indexes the inner loop)

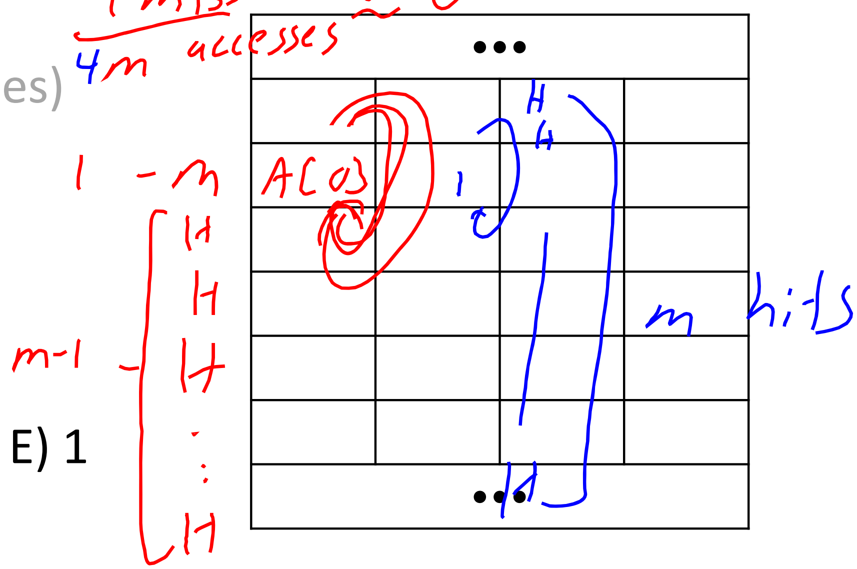
```
double sum = 0.0;
double a[n];

for (i = 0; i < m; i ++)  {
  for (j = 0; j < n; j ++) {
    sum += a[j];
  }
}
```

- Same Assumptions
  - 32B cache blocks (fit four, 64-bit (8B) doubles)
  - n is large (array much bigger than cache)
  - m is large (approximate 1/m as 0)

- Average miss rate?
  - A) 0   B) 1/8   C) 1/4   D) 1/2   E) 1

$\frac{1 \text{ miss}}{4m \text{ accesses}} \approx 0$



Loop Fusion joins two loops that traverse the same cache blocks, increasing temporal locality

```
for(int j = 0; j < LARGE; j++) {
      sum += A[j];
}
```

1/8

A

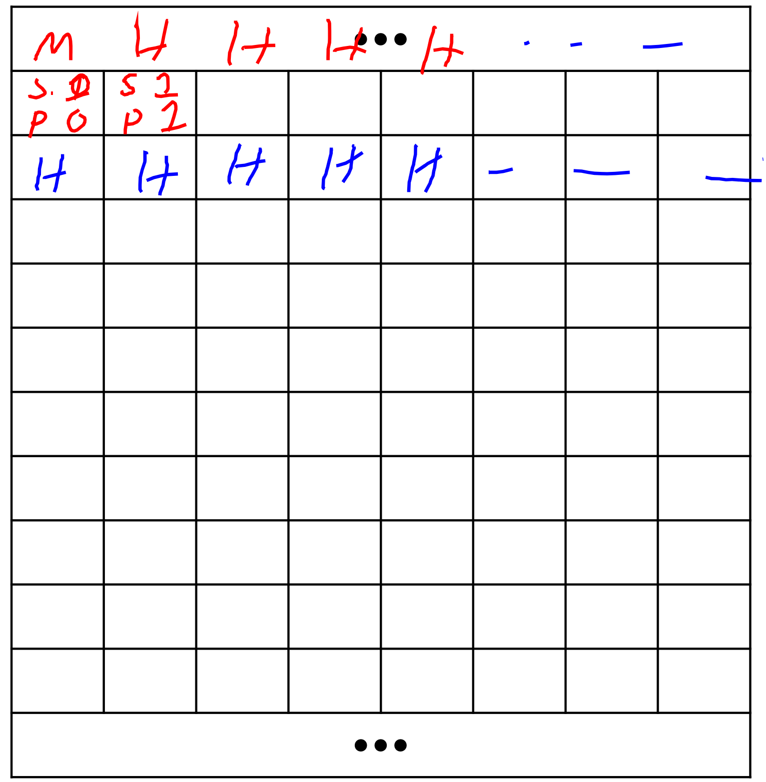
```
for(int j = 0; j < LARGE; j++){
      product *= A[j];
}
```

1/8

```
for(int j = 0; j < LARGE; j++){
      sum += A[j];
      product *= A[j];
}
```

1/8

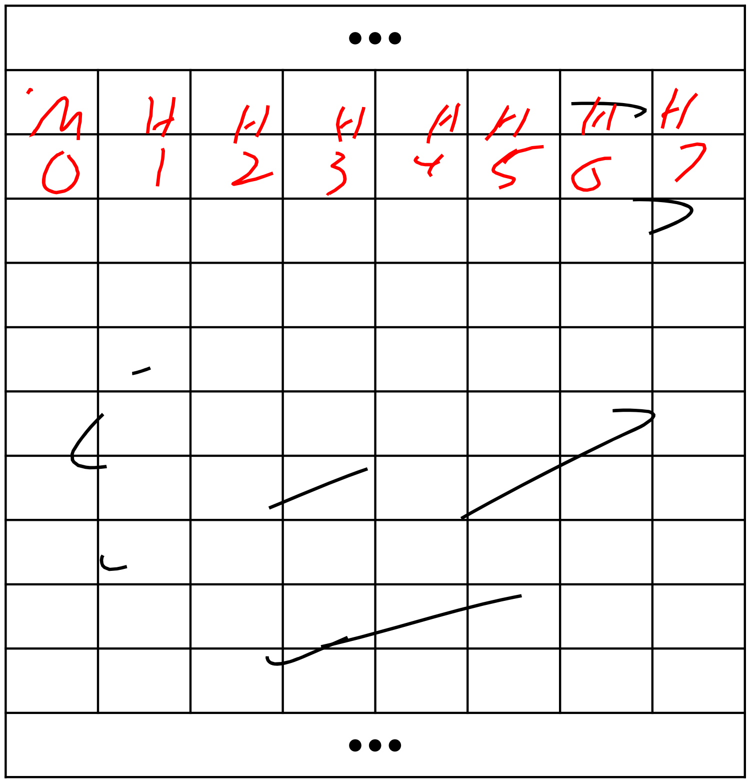
0 miss



Loop Fission separates loops that disrupt each other's temporal locality

```
for(int j = 0; j < LARGE; j++) {
      sum += A[j];      1
      for(int k = 0; k < LARGE; k++){
            other_sum += B[j][k];  1/8
      }
}
```

```
for(int j = 0; j < LARGE; j++)
      sum += A[j];      1/8
for(j = 0; j < LARGE; j++)
      for(int k = 0; k < LARGE; k++){
            other_sum += B[j][k]  1/8
      }
}
```



Accessing two arrays in the same inner loop

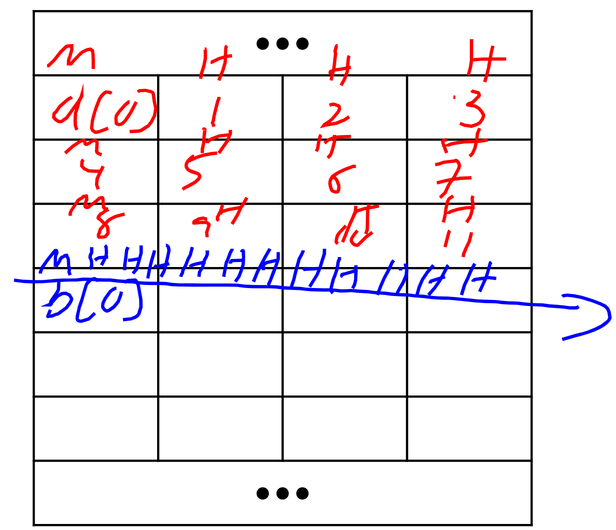
```
double sum = 0.0;
double a[n], b[m];

for (i = 0; i < m; i ++)  {
  for (j = 0; j < n; j ++) {
    sum += a[j] / b[i];
  }
}
```

L W   L W

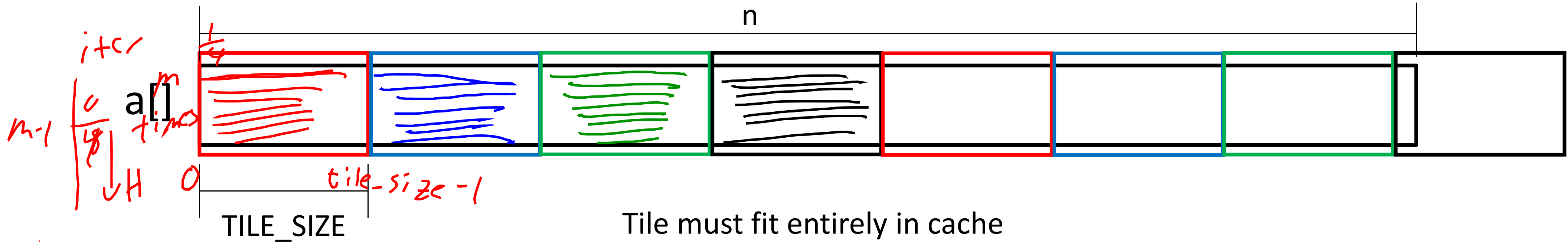
$\frac{1}{4} + \frac{0}{4} = \frac{1}{4}$

- Assumptions
  - 32B cache blocks (fit four, 64-bit (8B) doubles)
  - n & m are large (arrays much bigger than cache)



- Average misses per inner loop iteration?
  - A) 0   B) 1/8   C) 1/4   D) 1/2   E) >= 1

Tiling creates a "sliding window" of data, increasing temporal locality



$\frac{1}{4 \cdot m}$

```
double sum = 0.0;
double a[n], b[m];

for (j = 0; j < n; j += TILE_SIZE) {
  for (i = 0; i < m; i ++)  {
    for (jj = j; jj < j + TILE_SIZE; jj ++) {
      sum += a[jj] / b[i];
    }
  }
}
```

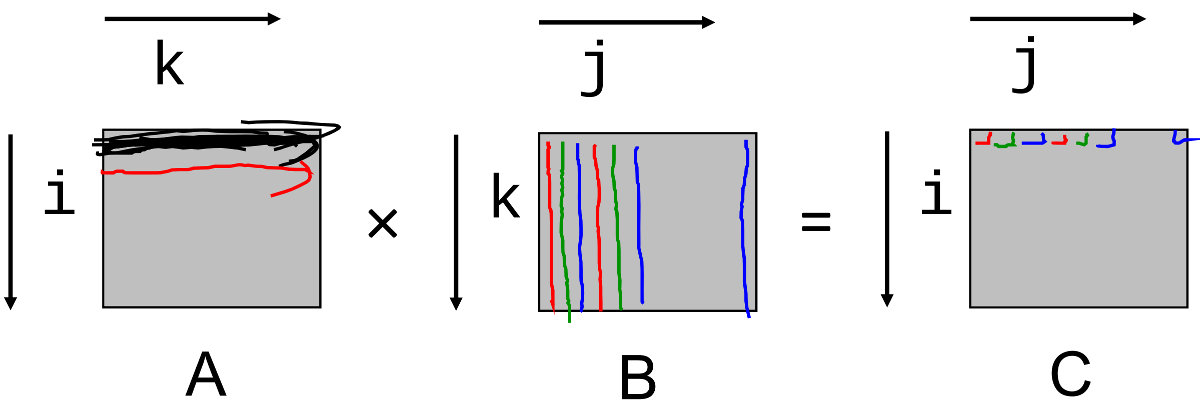
$\frac{0}{4}$ $\frac{0}{4}$

Average misses per inner loop iteration?

A) 0
B) 1/8
C) 1/4
D) 1/2
E) 1

Example: Multiply two NxN matricesMultiply

- Assume:
  - 32B cache blocks (fit four, 64-bit (8B) doubles)
  - Matrix dimension (N) is very large (Approximate 1/N as 0.0)
  - Cache is not even big enough to hold multiple rows




```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

Variable sum held in register

Matrices are allocated in row-major order in memory (rows are contiguous in memory)

```
for (i = 0; i < N; i++) //row traversal
    sum += a[0][i];
```

- accesses successive elements
- if block size (B) > 4 bytes, exploit spatial locality
  - compulsory miss rate = 8 bytes / B

$\frac{1}{4}$ $\frac{\text{miss}}{\text{access}}$

M a[0][0]	H a[0][1]	H a[0][2]
H a[1][0]	M a[1][1]	H a[1][2]
H a[2][0]	H a[2][1]	M a[2][2]

```
for (i = 0; i < n; i++) //col traversal
    sum += a[i][0];
```

- accesses distant elements
- no spatial locality! bad
  - compulsory miss rate = 1 (i.e. 100%)

N is large 1 $\frac{\text{miss}}{\text{access}}$

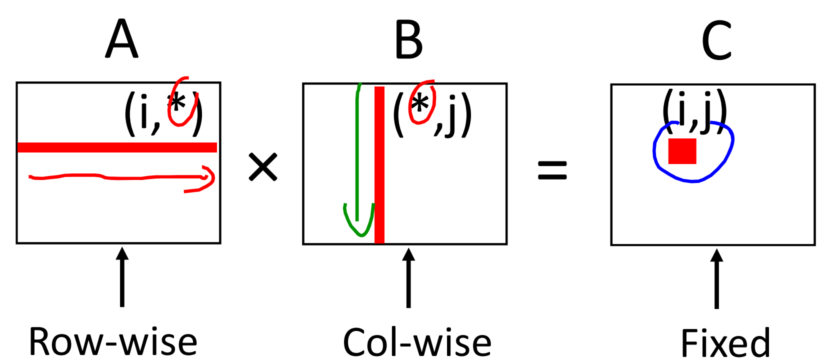
M	0xC1…00	a[0][0]
	0xC1…08	a[0][1]
	0xC1…10	a[0][2]
H	0xC1…18	a[1][0]
	0xC1…20	a[1][1]
M	•••	•••

Matrix Multiplication (ijk)

reg

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

Inner loop:



$\frac{1}{4}$ $\frac{\text{miss}}{\text{iLi}}$ 1 $\frac{\text{miss}}{\text{iLi}}$ 0 $\frac{\text{miss}}{\text{iLi}}$

Misses per inner loop iteration:

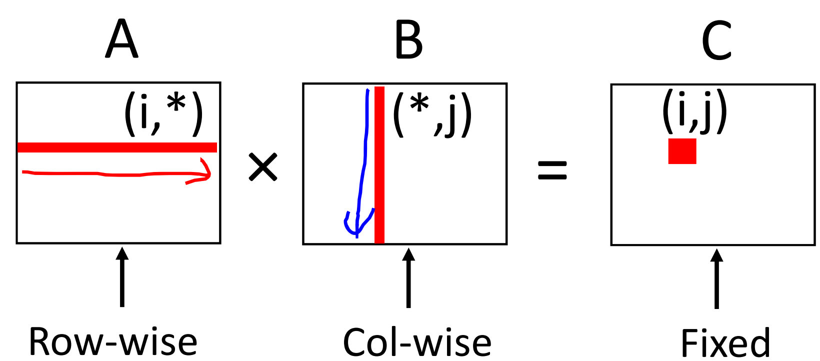
A		B		C		
0.25	+	1.0	+	0.0	=	1.25 $\frac{\text{misses}}{\text{iLi}}$

Matrix Multiplication (ijk)

ji

```
/* jik */
for (j=0; j<n; j++)  {
  for (i=0; i<n; i++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

Inner loop:



1/4 | 1 | 0

Misses per inner loop iteration:

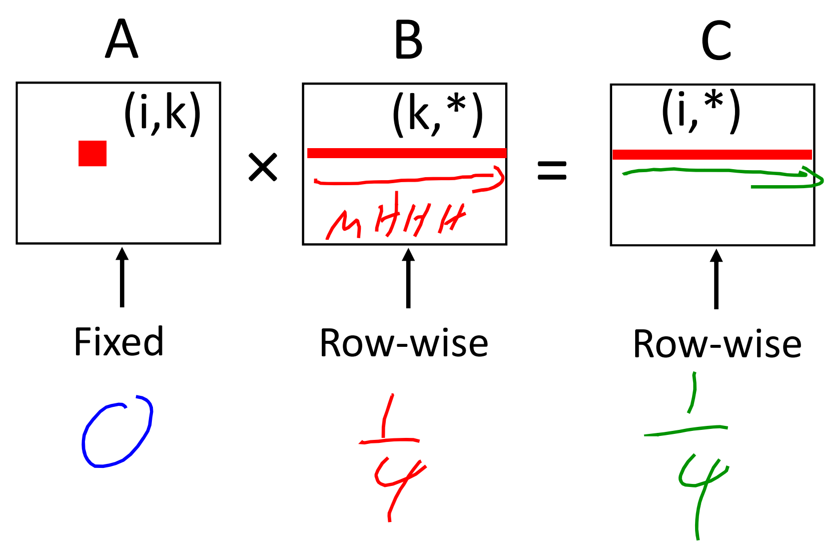
A	B	C
.25	+ 1	+ 0 = 1.25 misses/ic.

a) 0   b) .25   c) .5   d) 1   e) 1.25

Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++)  {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

Inner loop:



Misses per inner loop iteration:

A B C

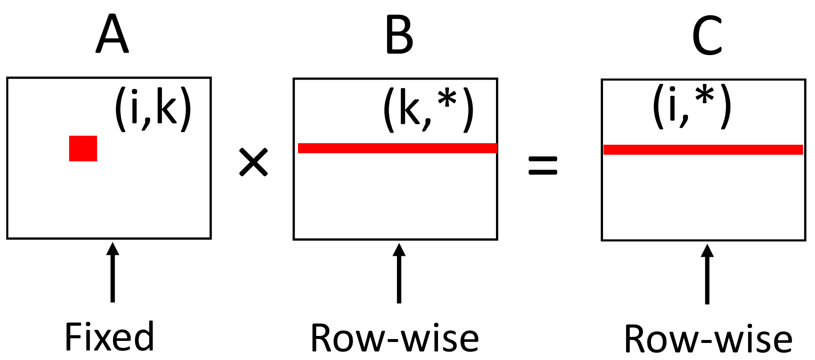
Total misses per inner loop iteration

a) .25 b) .5 c) .75 d) 1.25 e) 2

Matrix Multiplication (ikj)

```
/* ikj */
for (i=0; i<n; i++)  {
  for (k=0; k<n; k++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

Inner loop:



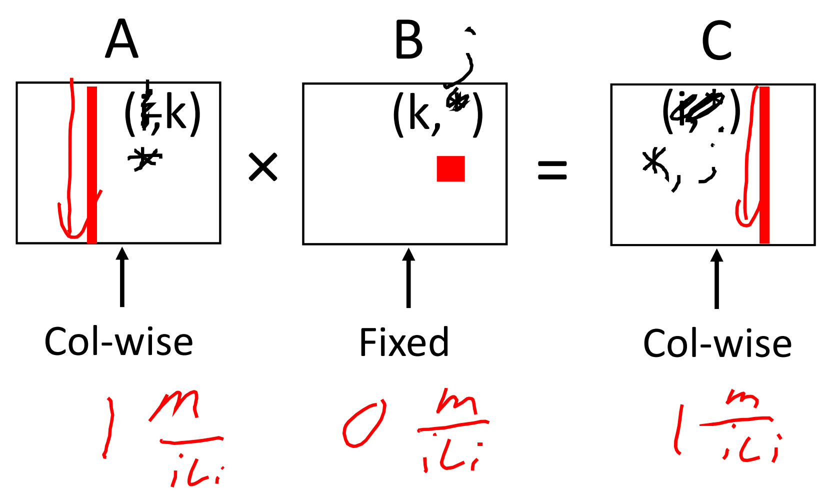
Misses per inner loop iteration:

A	B	C
0.0	0.25	0.25

Matrix Multiplication (jki)

```
/* jki */
for (j=0; j<n; j++)  {
  for (k=0; k<n; k++) {
    r = a[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

Inner loop:



Misses per inner loop iteration:

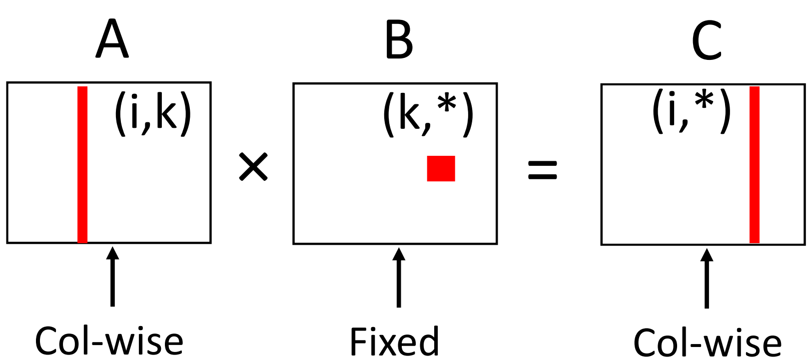
A B C

Total misses per inner loop iteration
a) .25 b) .5 c) .75 d) 1.25 e) 2

Matrix Multiplication (kji)

```
/* kji */
for (k=0; k<n; k++)  {
  for (j=0; j<n; j++) {
    r = a[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

Inner loop:



Misses per inner loop iteration:

A	B	C
1.0	0.0	1.0

```
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
   sum = 0.0;
   for (k=0; k<n; k++)
     sum += a[i][k] * b[k][j];
   c[i][j] = sum;
 }
}
```

ijk (& jik):

- 2 loads, 0 stores
- misses/iter = 1.25

```
for (k=0; k<n; k++) {
 for (i=0; i<n; i++) {
  r = a[i][k];
  for (j=0; j<n; j++)
   c[i][j] += r * b[k][j];
 }
}
```

kij (& ikj):

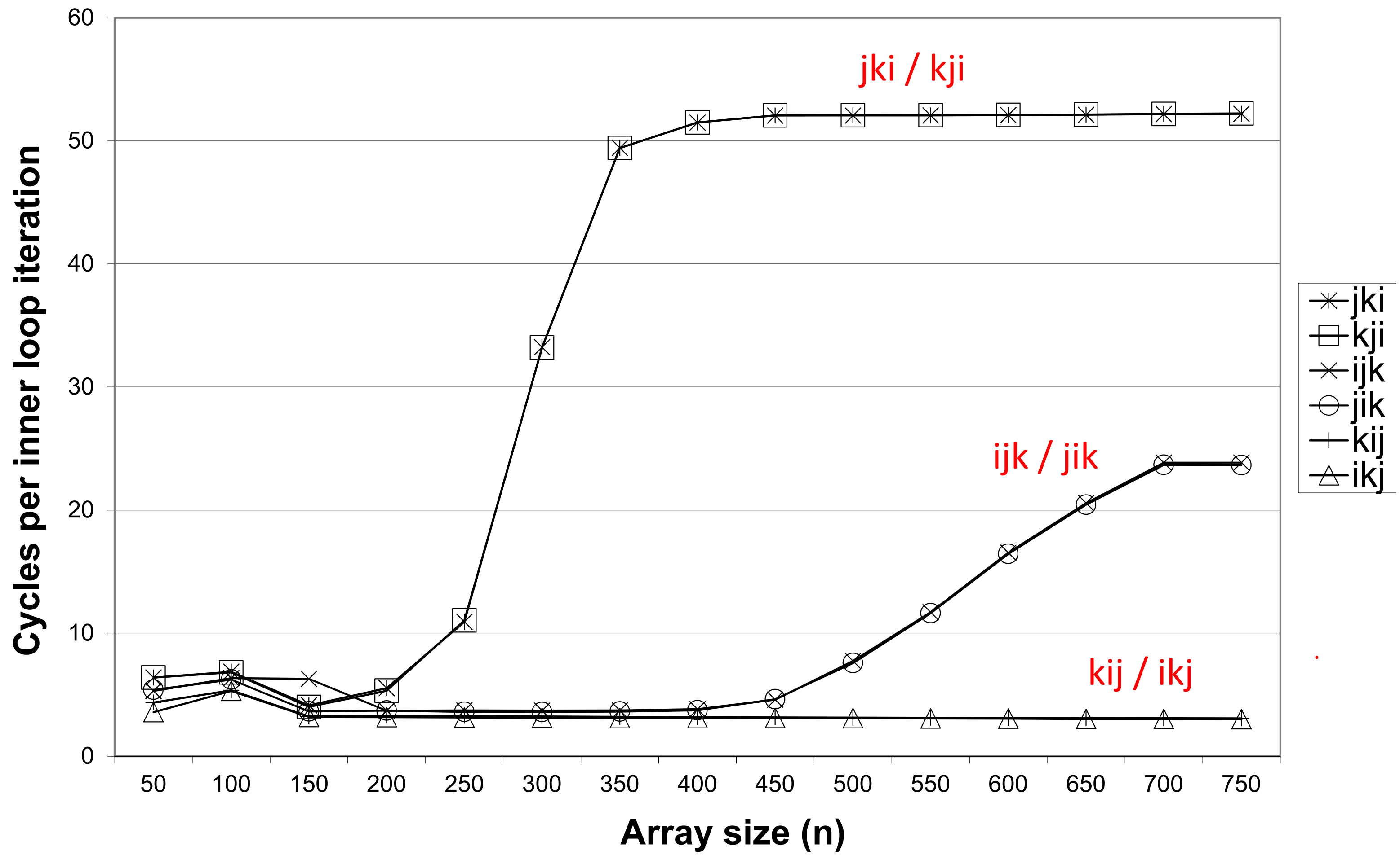
- 2 loads, 1 store
- misses/iter = 0.5

```
for (j=0; j<n; j++) {
 for (k=0; k<n; k++) {
   r = b[k][j];
   for (i=0; i<n; i++)
    c[i][j] += a[i][k] * r;
 }
}
```

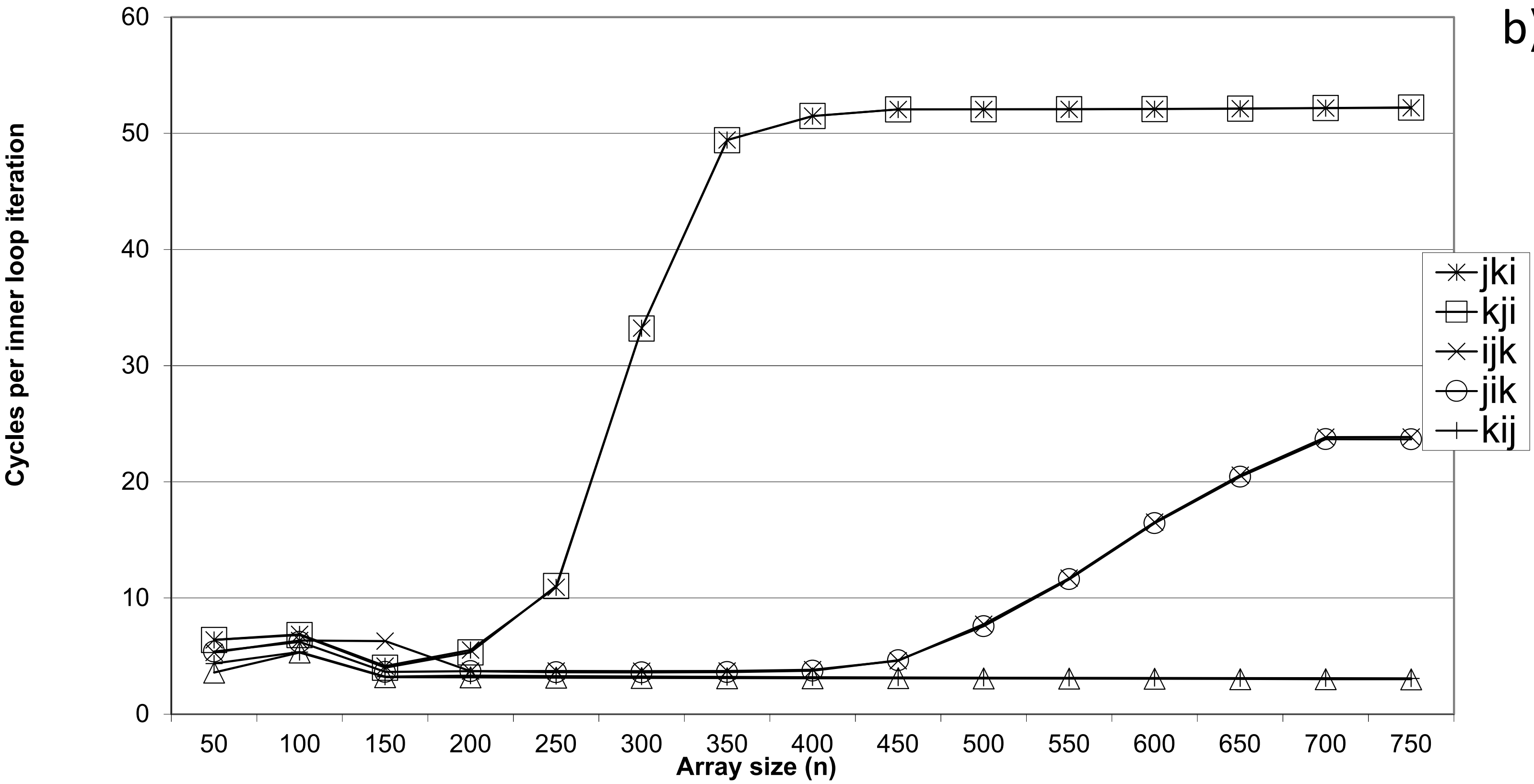
jki (& kji):

- 2 loads, 1 store
- misses/iter = 2.0

Core i7 Matrix Multiply Performance



From where comes the performance?

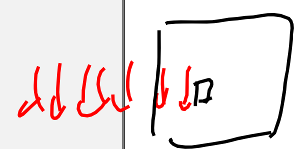


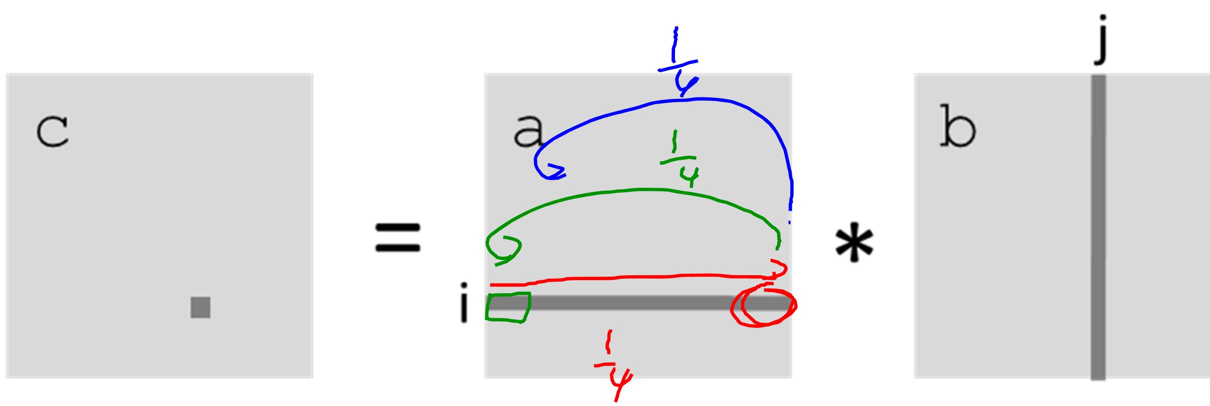
a) Spatial locality
b) Temporal locality

We can using tiling on matrices, just like we did with one-dimensional arrays

```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i++)
       for (j = 0; j < n; j++)
          for (k = 0; k < n; k++)
             c[i*n+j] += a[i*n + k]*b[k*n + j];
}
```



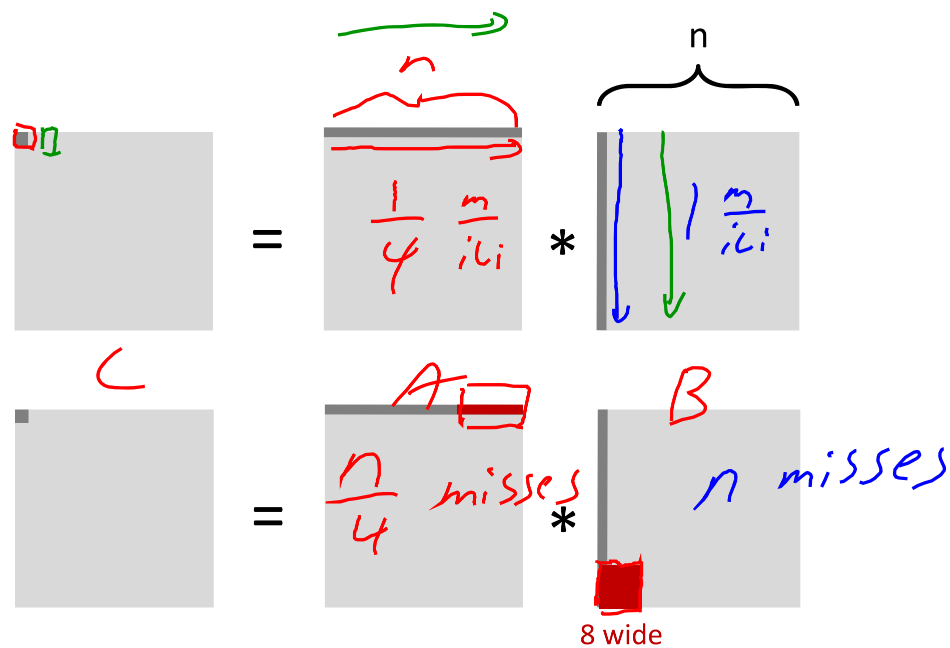


Cache Miss Analysis

- Assume:
  - Matrix elements are doubles
  - Cache block = 4 doubles
  - Cache size C << n (much smaller than n)

- First iteration:
  - n/4 + n = 5n/4 misses

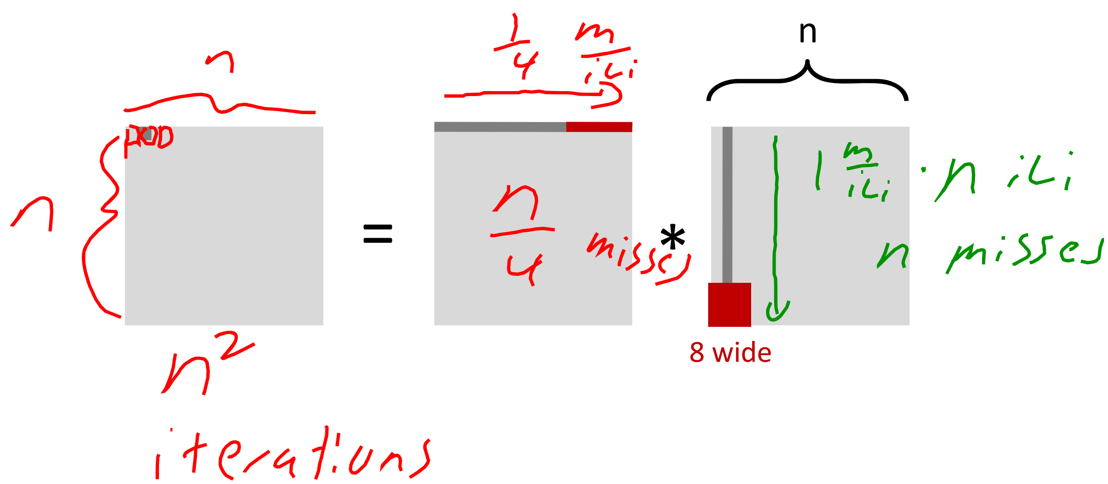
  - Afterwards in cache:
    (schematic)



Cache Miss Analysis

- Assume:
  - Matrix elements are doubles
  - Cache block = 4 doubles
  - Cache size C << n (much smaller than n)

- Second iteration:
  - Again:
    $n/4 + n = 5n/4$ misses

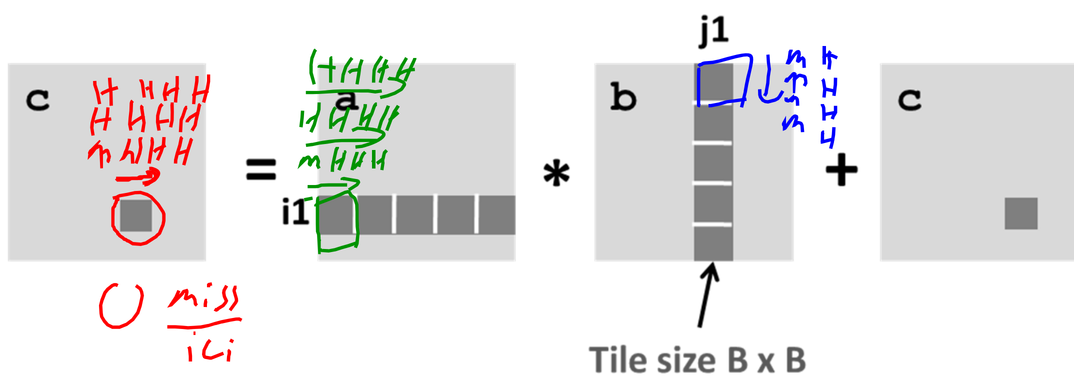


- Total misses:
  - $5n/4 * n^2 = (5/4) * n^3$

Create tiles in two dimensions (row & col)

```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i+=B)
        for (j = 0; j < n; j+=B)
            for (k = 0; k < n; k+=B)
                /* B x B mini matrix multiplications */
                for (i1 = i; i1 < i+B; i1++)
                    for (j1 = j; j1 < j+B; j1++)
                        for (k1 = k; k1 < k+B; k1++)
                            c[i1*n+j1] += a[i1*n + k1]*b[k1*n + j1];
}
```

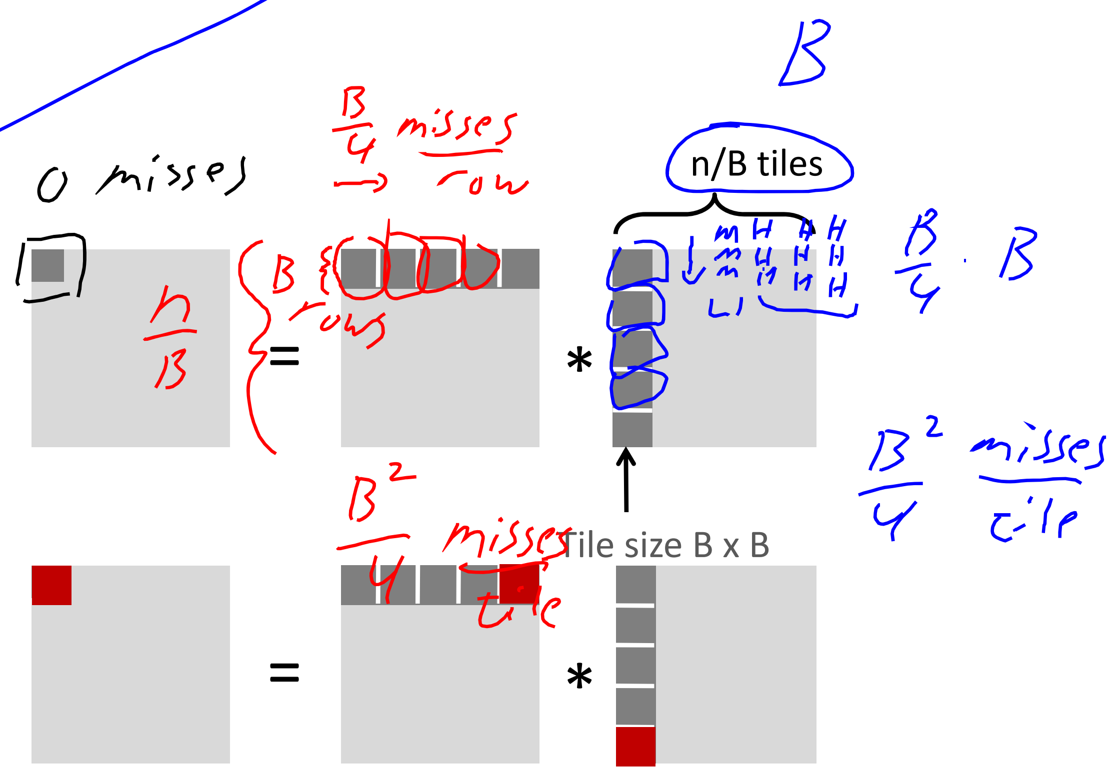


Cache Miss Analysis

$\left(\frac{n}{B}+\frac{n}{B}\right)$ tiles $\frac{B^2}{4}$ misses/~~tile~~

- Assume:
  - Cache block = 4 doubles
  - Cache size C << n (much smaller than n)
  - Three tiles ▪ fit into cache: $3B^2 < C$

- First (tile) iteration:
  - $B^2/4$ misses for each tile
  - $2n/B * B^2/4 = nB/2$ (omitting matrix c)

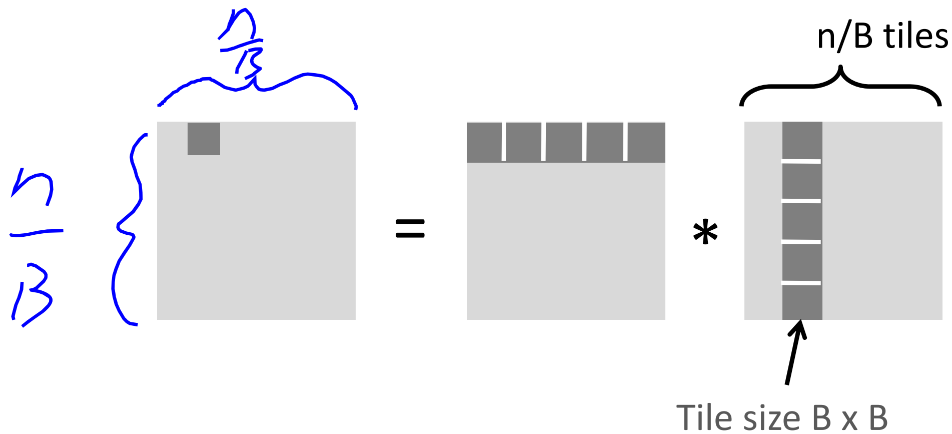


  - Afterwards in cache (schematic)

Cache Miss Analysis

- Assume:
  - Cache block = 4 doubles
  - Cache size C << n (much smaller than n)
  - Three tiles ■ fit into cache: $3B^2 < C$

- Second (tile) iteration:
  - Same as first iteration
  - $2n/B * B^2/4 = nB/2$



- Total misses:
  - $nB/2 * (n/B)^2 = n^3/(2B)$

Summary

- No tiling: (5/4) * n^3
- Tiling: 1/(2B) * n^3

$\frac{5}{4} > 1$

$\frac{1}{2B} << 1$

300 n
200

- Suggest largest possible tile size B, but limit $3B^2 < C$!

- Reason for dramatic difference:
  - Matrix multiplication has inherent temporal locality:
    - Input data: $3n^2$, computation $2n^3$
    - Every array elements used O(n) times!
  - But program has to be written properly

1) data struct >> cache

2) multiple iter on data struct

Concluding Observations

- Programmer can optimize for cache performance
  - How data structures are organized
  - How data are accessed
    - Nested loop structure
    - Tiling is a general technique
- All systems favor “cache friendly code”
  - Getting absolute optimum performance is very platform specific
    - Cache sizes, line sizes, associativities, etc.
  - Can get most of the advantage with generic code
    - Keep working set reasonably small (temporal locality)
    - Use small strides (spatial locality)