

Web Search

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Introduction



Web search



Game Theory



Auctions



Data flows



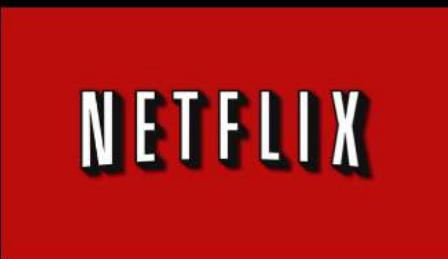
Privacy



Text Ads



Display Ads



Recommender systems



Behavioral targeting



Emerging areas



Final Presentations

Web Search

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some observations



apple







**with
the
beatles**

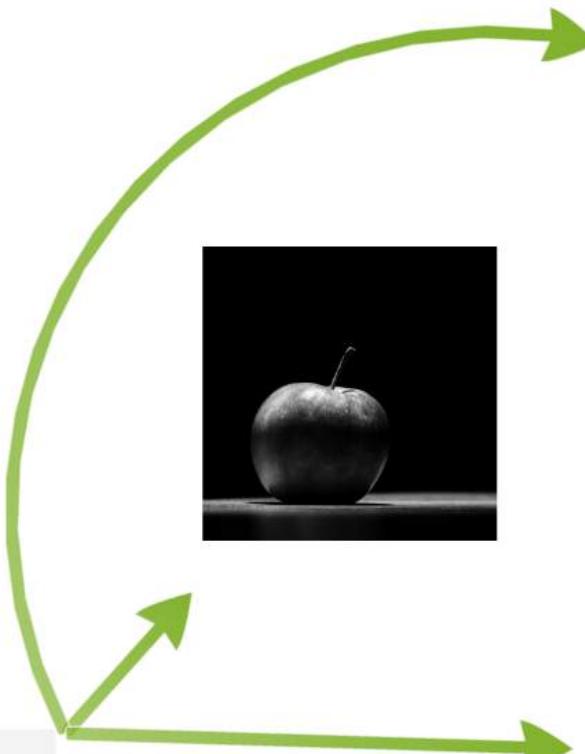
stereo



polysemy

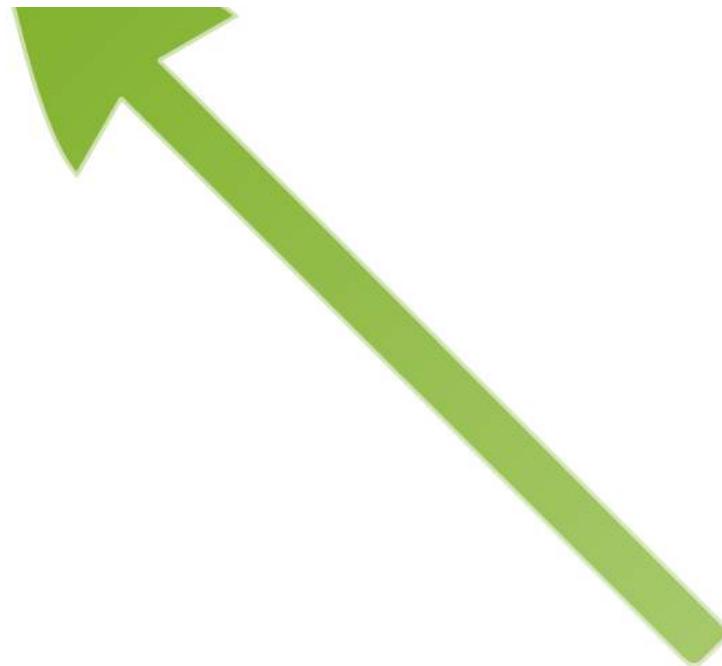
apple

multiple
meanings for the
same term





"okra"



"ladies' fingers"

synonymy



"okra"

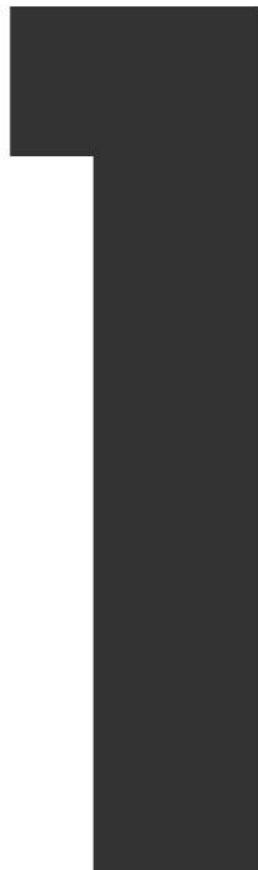


multiple terms
mean the same
thing

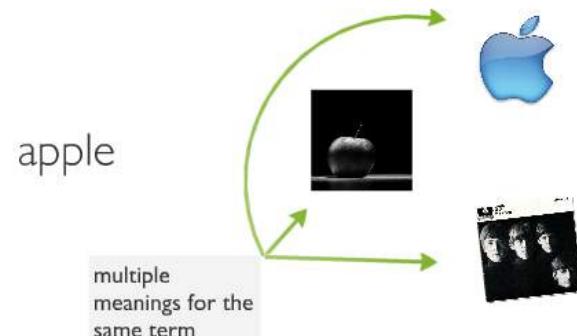
"ladies' fingers"



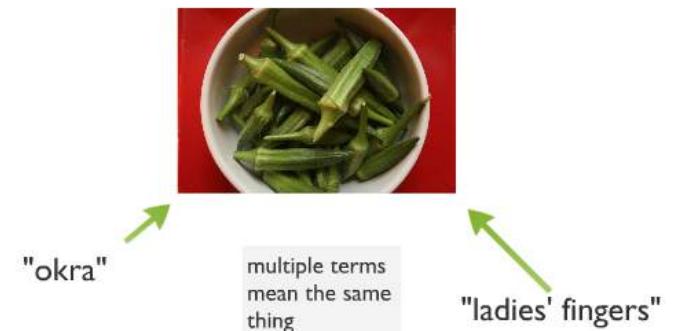
semantics



polysemy



synonymy





what happens
when everyone
is an author?

**who is
authoritative?**



production

2



what happens
when everyone
is an author?



who is
authoritative?

querying

3



people tend to use very
few words when they
issue a query

The New York Times

Friday, October 21, 2016 | [Today's Paper](#) | [Video](#) | 49°F | Nasdaq +0.16% ↑

World U.S. Politics N.Y. Business Opinion Tech Science Health Sports Arts Style Food Travel Magazine T Magazine Real Estate ALL

ELECTION 2016

Trump Says He Will Accept Election Result ... 'if'

By ALAN RABINOWITZ and ALEXANDRA GOLDBECK

• Donald Trump said that he would accept the results of the election if they came as he expected.

• But he also said he would "abide by the rules."

1214 Comments

Trump Jests, and a Tough Room Jeers

By MATT FLEGENHEIMER and ASHLEY PARKER

The Alfred E. Smith dinner is traditionally a chance for rivals to let off a little steam.



Luke Sharrett for The New York Times

Makyleigh Anderson survived a shooting that killed three of her family members.

What 130 of the Worst Shootings Say About Guns in America

A study of high-casualty attacks on gun control laws finds that firearms regulations are, but do not fully control them in a meaningful way.

By SHARON LAFRANCE and ERIC PALMER

Your Friday Briefing

By SEAN ALFANO 5:49 AM ET

Here's what you need to know to start your day.



The Opinion Pages

OP-ED CONTRIBUTOR

Late-Term Abortion Was the Right Choice for Me

By MEREDITH ISAKSEN

I made a decision many politicians don't think I should have the right to make.

OP-ED CONTRIBUTOR

'Too Bad You're Latin'

By JOHN LEGUIZAMO

A movie producer's words sum up America's view of its largest ethnic minority.

• **Blow: Trump vs. Democracy**

• **Brooks: How to Repair Moral Capital**

• **Cohen: The Anti-American**

• **Krugman: Why Hillary Wins**

• **Layard: Right Decisions, Wrong Democracy**

• **Egan: American Gut Check**

Sunday Review



NEWS ANALYSIS

Men Need Help. Is Hillary Clinton the Answer?

By SUSAN CHIRAS

We need "pink collar jobs" for everyone.

MORE IN OPINION

• **Editorial: Katie McGinty, the Best Choice for the Senate in Pennsylvania**

• **Room for Debate: Does ISIS Need Territory to Survive?**

• **David Brooks: The Best Way to Handle Nasty Women**

• **Room for Debate: Does ISIS Need Territory to Survive?**

THE CROSSWORD »
Play Today's Puzzle



dynamic content

MOVIES INSIDER »
Good, Bad and Mad: Andrew Rosenthal on the News

Election Forecast: Hillary Clinton has a 93% chance of winning.

Clinton

Trump

A search for
"University of
Illinois" yields the
UIUC page.



There is **nothing** internal
to the UIUC home page
that makes it special—it
uses UIUC as often as
other pages using the
term

T

links are essential
to ranking; they
form implicit
endorsements

Could we just
count the votes
from each page
linking to UIUC?

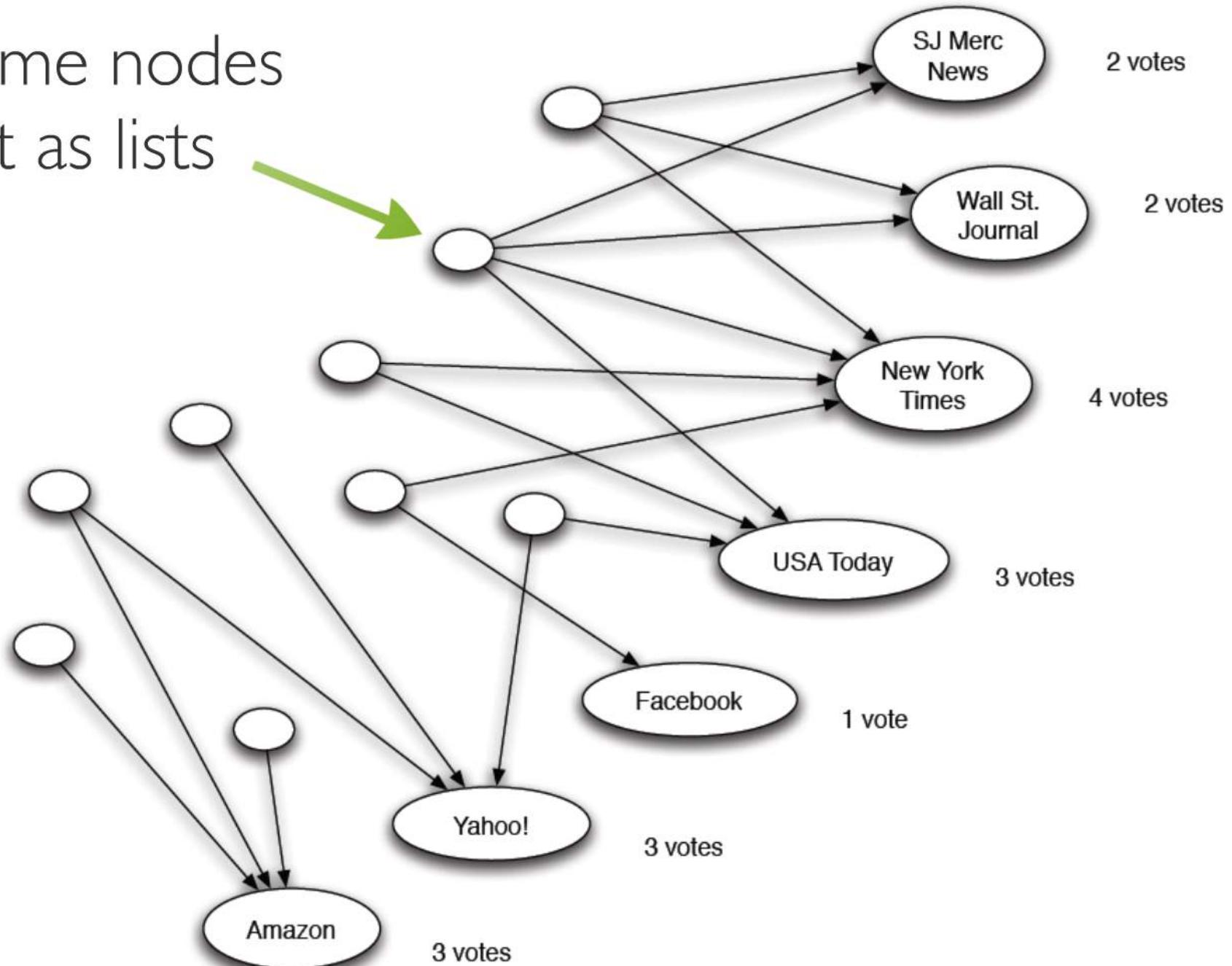


2

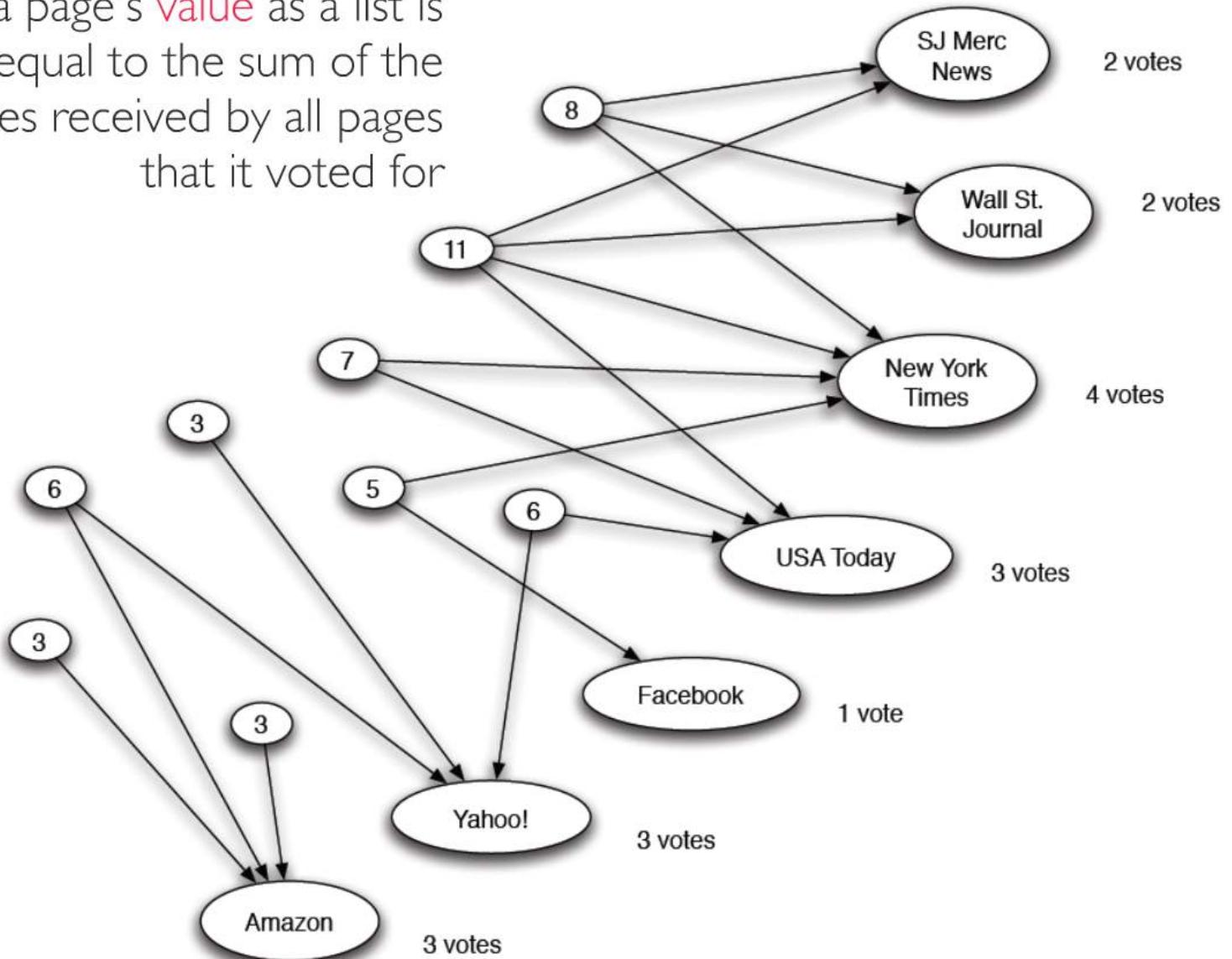
we can exploit
the network
structure to
improve results

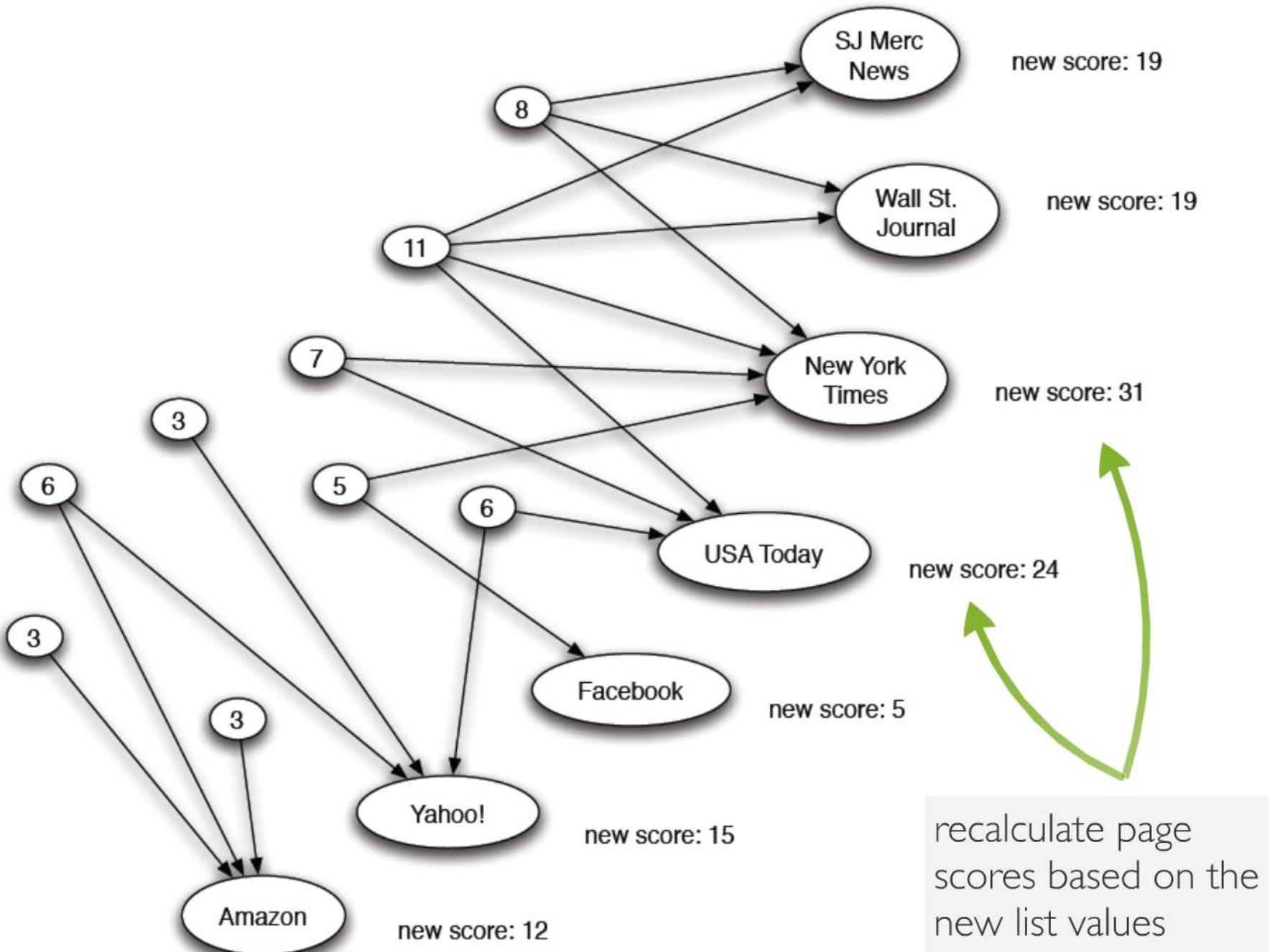
let's query for
"newspapers"

some nodes
act as lists



a page's **value** as a list is equal to the sum of the votes received by all pages that it voted for





Two kinds of attributes

H

hubs



high value
lists

&

every page has a
hub score and an
authority score

A

authorities



highly relevant
to the query

Two kinds of attributes

H & A

hubs
high value lists

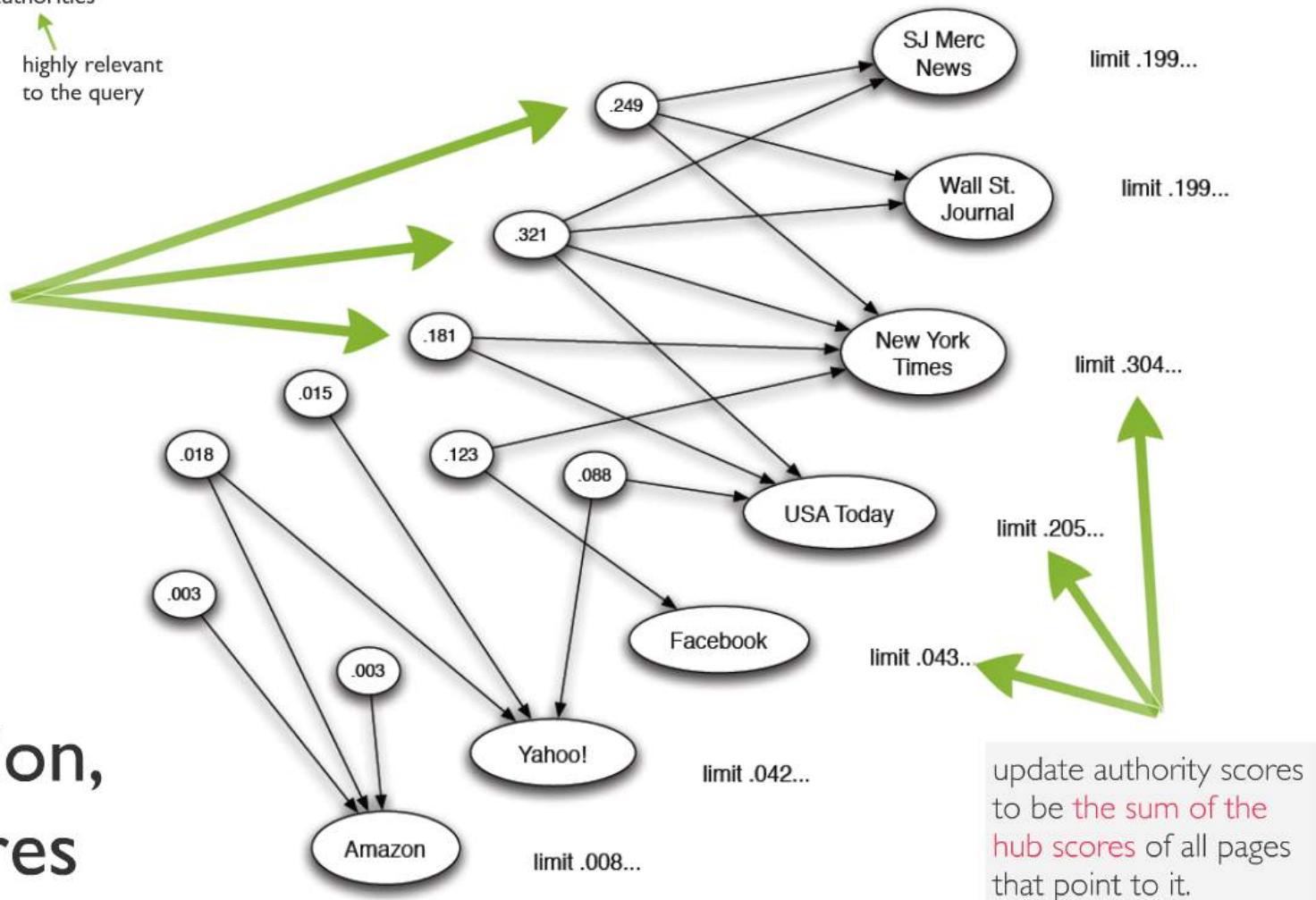
every page has a hub score and an authority score

authorities
highly relevant to the query

update hub score to be the sum of the authority scores of all pages to which it points.

at every iteration, normalize scores

for all nodes, set authority and hub scores to be 1



H & A

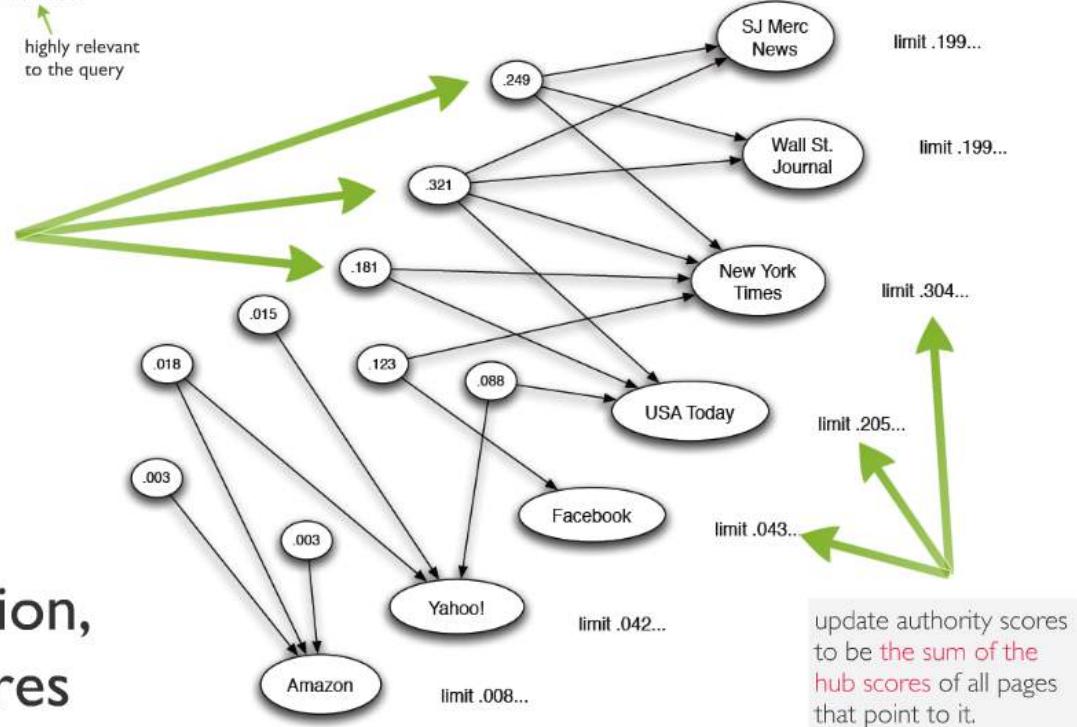
Two kinds of attributes



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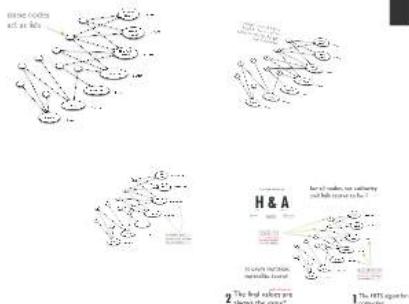
regardless of the initial values

2 The final values are
always the same*

1 The HITS algorithm
converges

let's query for
"newspapers"

Ranking Schemes



How can we
improve upon
HITS and
PageRank?



Let's make a **small** change



A page is important, if
it endorsed by other
important people

The PageRank algorithm

In a network with n
nodes, we assign all nodes
the same initial PageRank,
set to be $1/n$.

iterate
 k times.

Each page checks its
current PageRank value
and adds it to the
PageRank of each
page that links to it.
Each page's new
PageRank is the sum
of the pages it receives
times its pagerank.



1 links are essential
to ranking; they
form implicit
endorsements

2 we can exploit
the network
structure to
improve results



There is *nothing internal*
to the UIUC home page
that makes it special—it
uses UIUC as often as
other pages using the

A page is important, if
it endorsed by other
important people

The PageRank algorithm

In a network with n nodes, we assign all nodes the same initial PageRank, set to be $1/n$

If a page has **no outgoing links**, it passes all its current PageRank to itself.

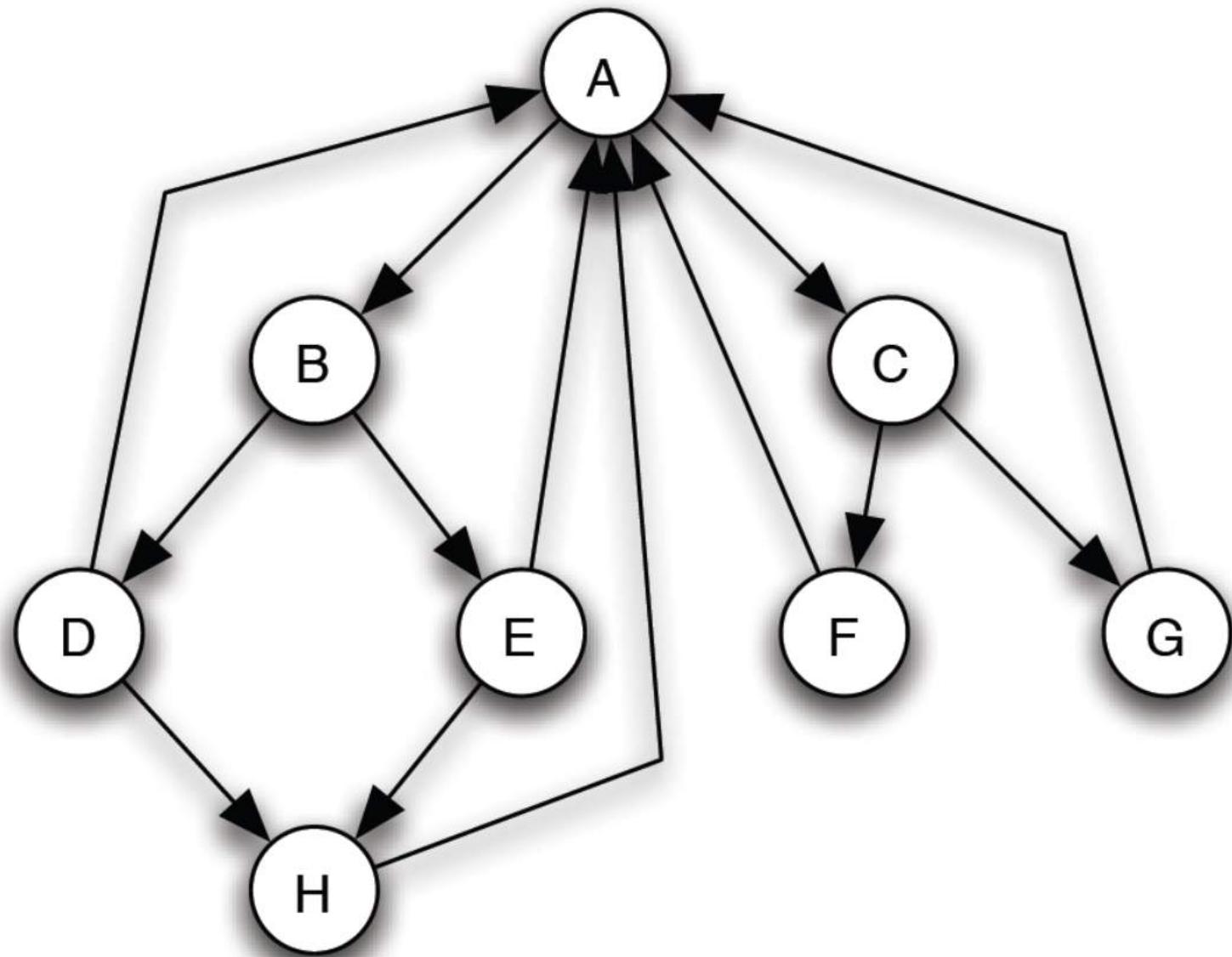
iterate
 k times

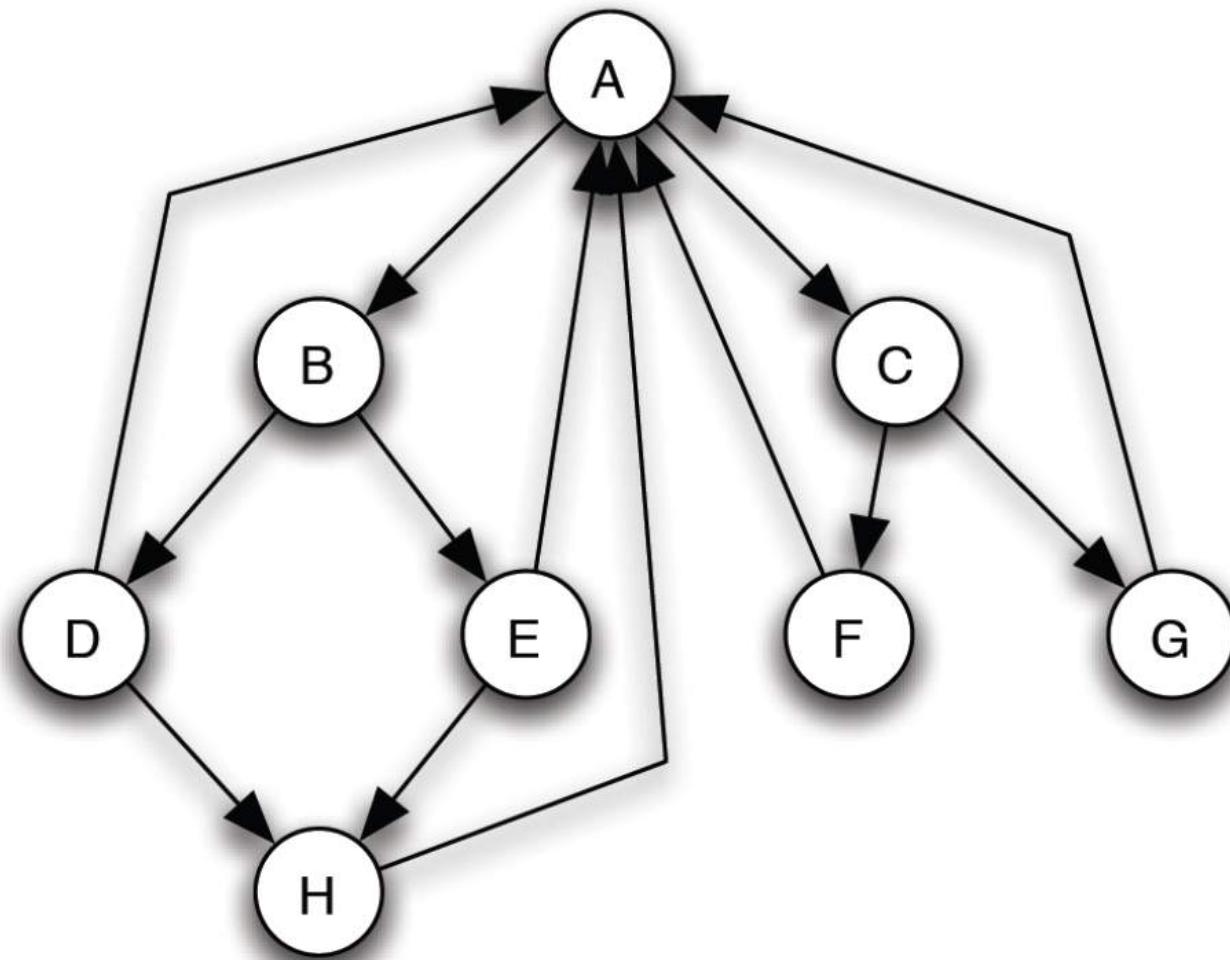
Each page **divides** its current PageRank **equally** across its out-going links

Each page updates its new PageRank to be the sum of the shares it receives.

basic update

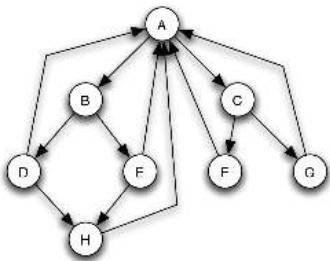
let's see an example



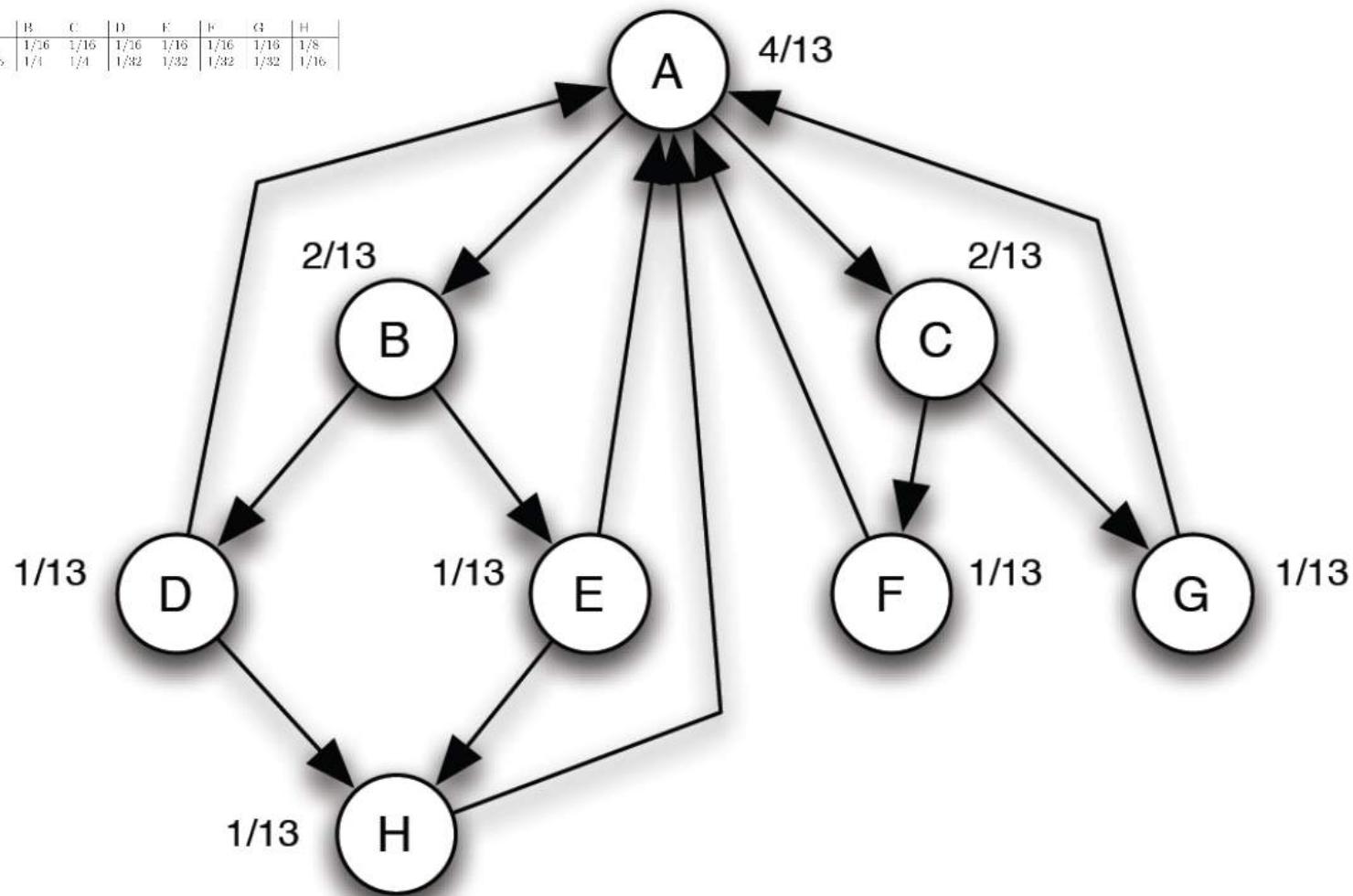


Step	A	B	C	D	E	F	G	H
1	$1/2$	$1/16$	$1/16$	$1/16$	$1/16$	$1/16$	$1/16$	$1/8$
2	$3/16$	$1/4$	$1/4$	$1/32$	$1/32$	$1/32$	$1/32$	$1/16$

let's see an example



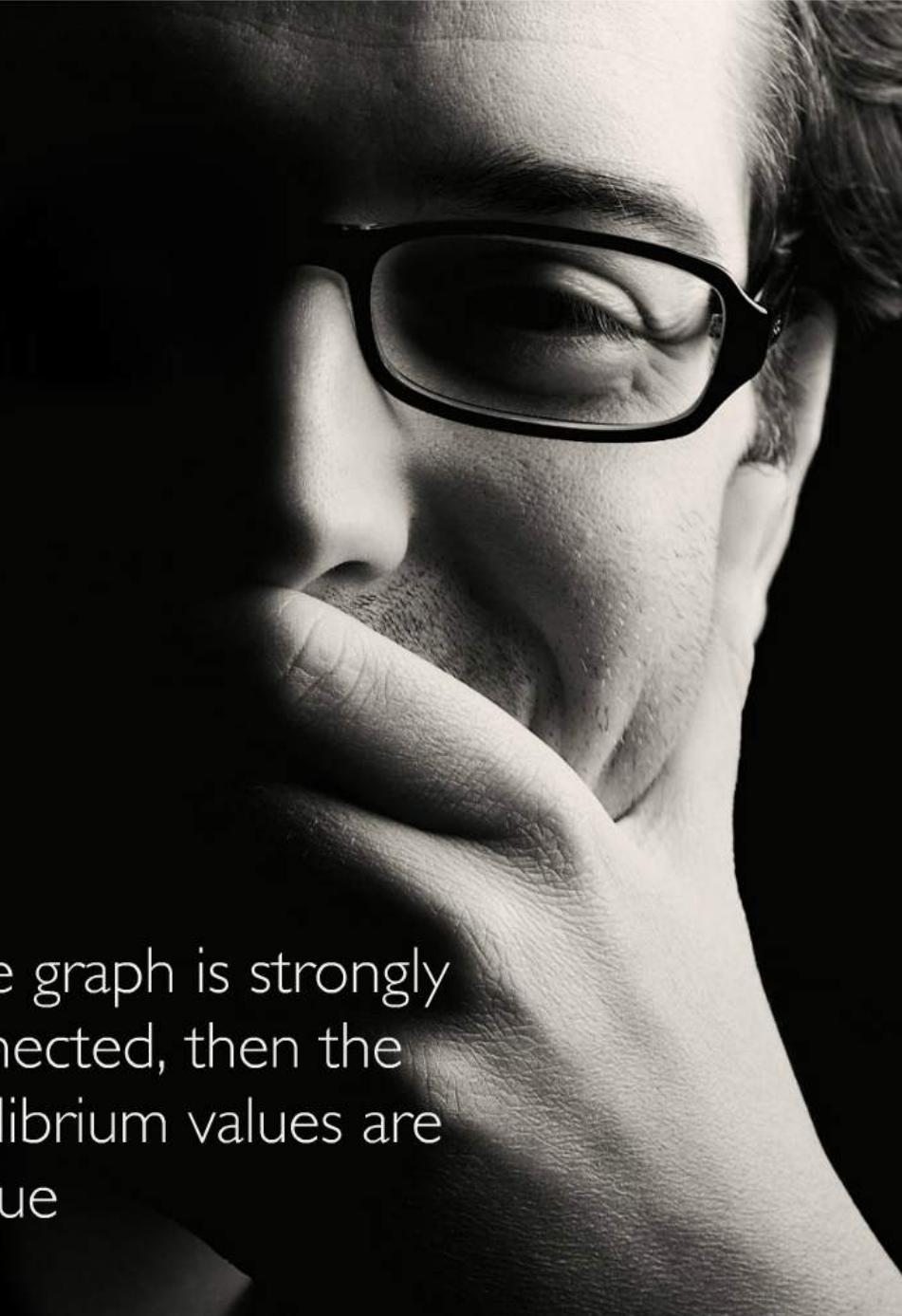
Step	A	B	C	D	E	F	G	H
1	1/2	1/16	1/16	1/16	1/16	1/16	1/16	1/8
2	3/16	1/1	1/4	1/32	1/32	1/32	1/32	1/16



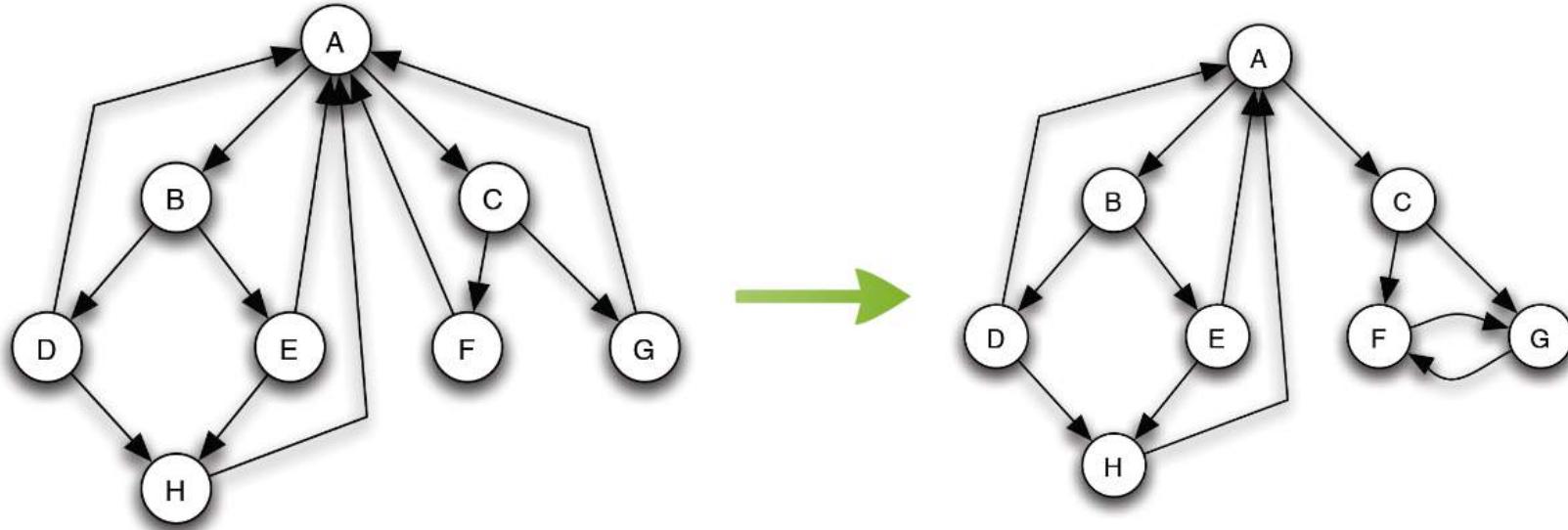
the algorithm is
guaranteed to
converge

Some things to keep in mind

if the graph is strongly
connected, then the
equilibrium values are
unique



Let's make a **small** change

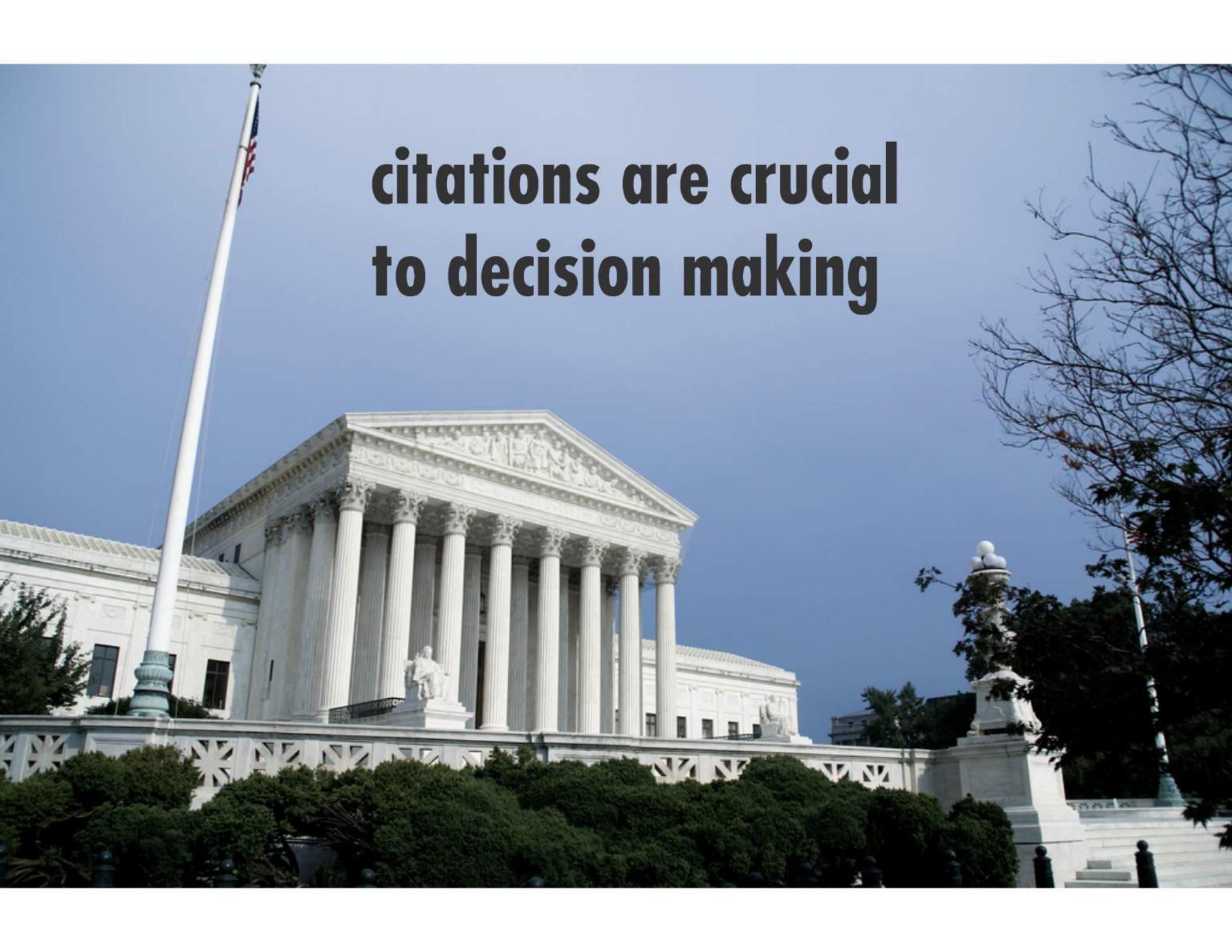


pick a scaling factor s

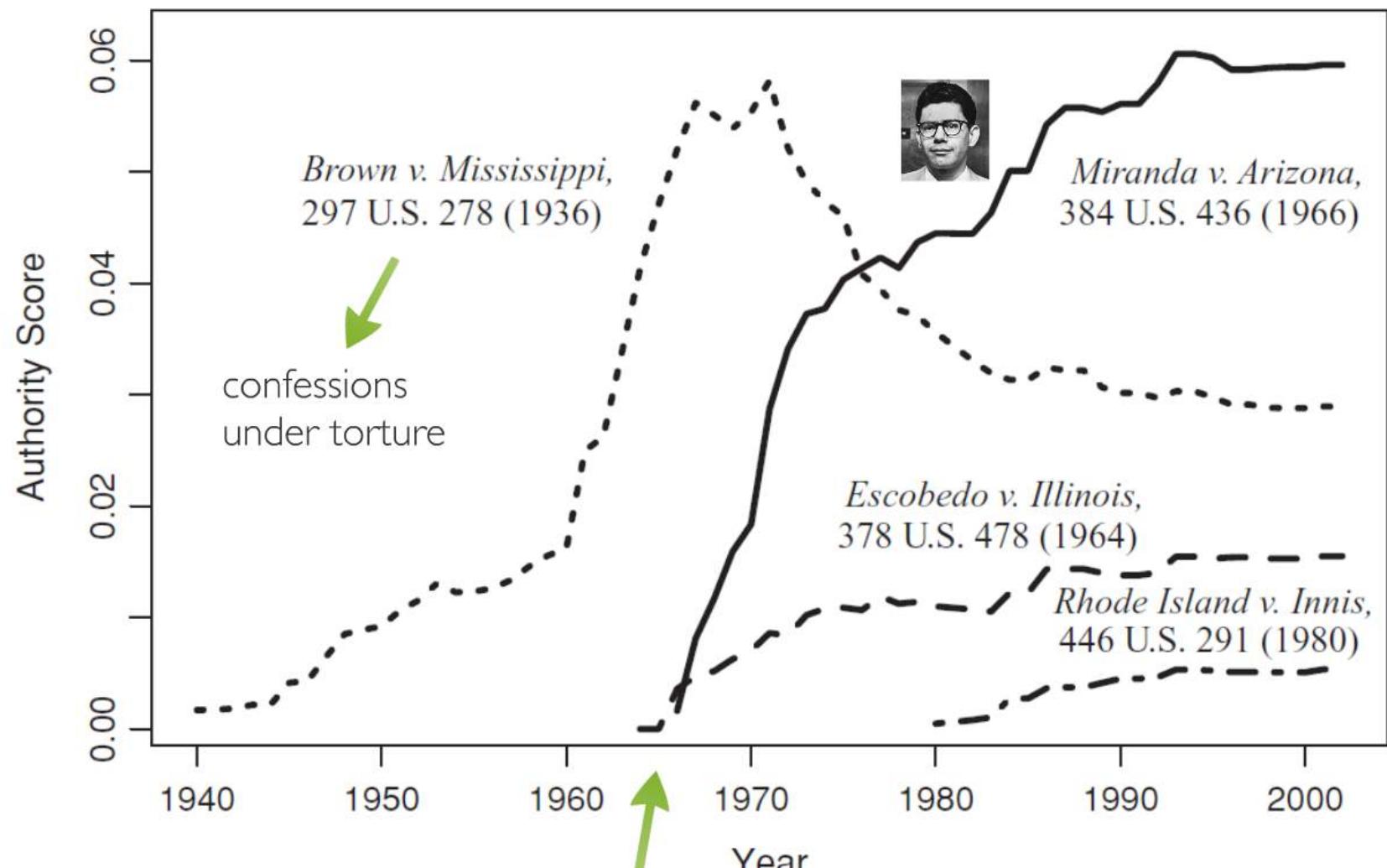
First apply the Basic PageRank Update Rule. Then scale down all PageRank values by a factor of s . We divide the residual $1-s$ units of PageRank equally over all nodes, giving $(1-s)/n$ to each.

**How can we
improve upon
HITS and
PageRank?**

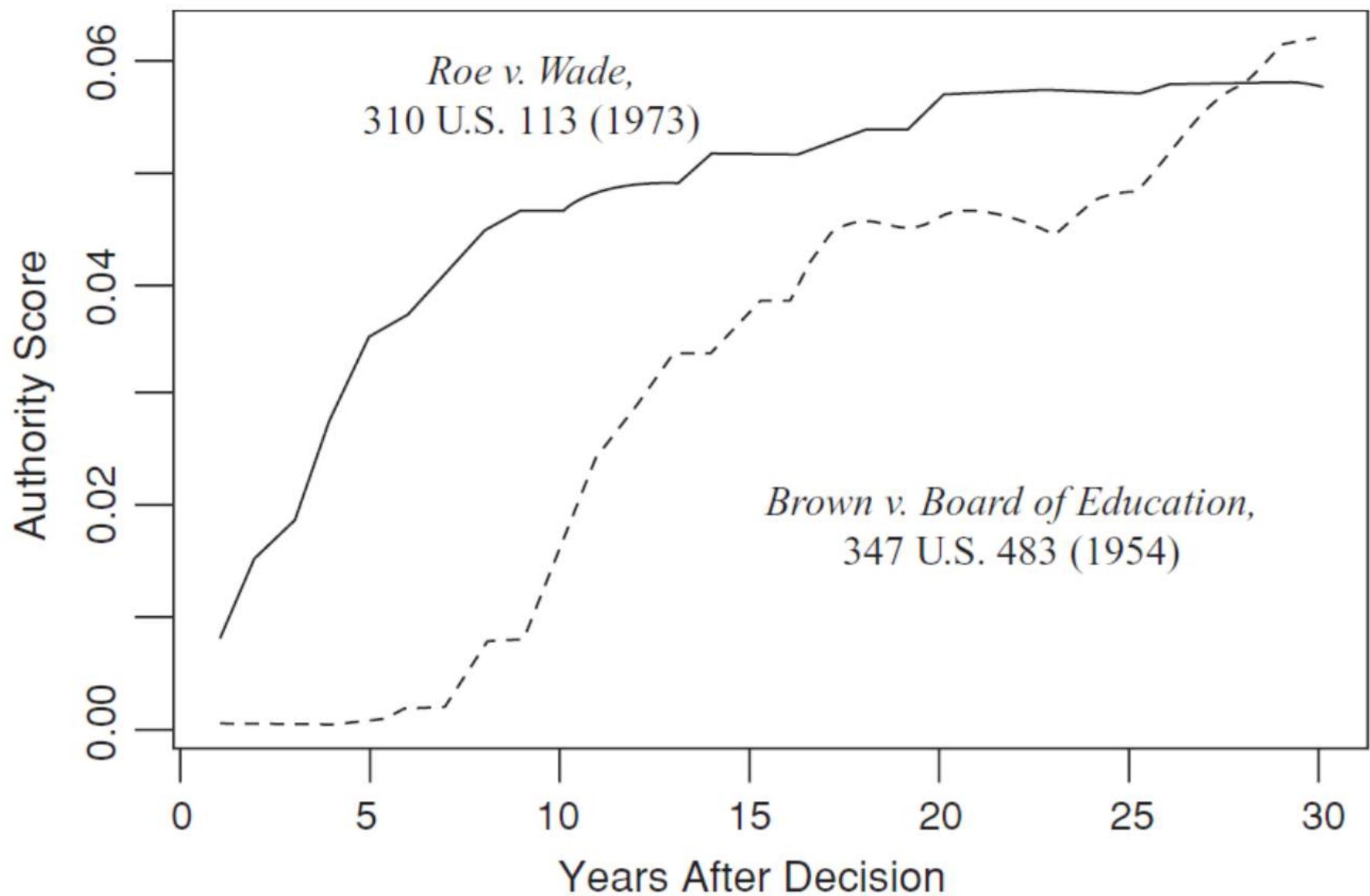




**citations are crucial
to decision making**



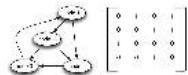
Warren court focuses on
fifth amendment issues



Details

Some basics

M_{ij}



We know that:
 $|c_1| > |c_2|$

since z form a basis:

$$x = p_1 z_1 + p_2 z_2 + \dots + p_n z_n$$

$$\begin{aligned} (MM^T)x &= (MM^T)(p_1 z_1 + p_2 z_2 + \dots + p_n z_n) \\ &= p_1 M M^T z_1 + p_2 M M^T z_2 + \dots + p_n M M^T z_n \\ &= p_1 c_1 z_1 + p_2 c_2 z_2 + \dots + p_n c_n z_n \end{aligned}$$

$$(MM^T)^k x = c_1^k p_1 z_1 + c_2^k p_2 z_2 + \dots + c_n^k p_n z_n$$

$$h^{(k)} = (MM^T)^k h^{(0)} = c_1^k q_1 z_1 + c_2^k q_2 z_2 + \dots + c_n^k q_n z_n$$

$$\frac{h^{(k)}}{q_1} = c_1^k + \left(\frac{c_2}{c_1}\right)^k q_2 z_2 + \dots + \left(\frac{c_n}{c_1}\right)^k q_n z_n$$

we still need to show two things:

$$\frac{h^{(k)}}{q_1} = q_1 z_1 + \left(\frac{c_2}{c_1}\right)^k q_2 z_2 + \dots + \left(\frac{c_n}{c_1}\right)^k q_n z_n$$

1
To show the second part:
constant is a positive vector

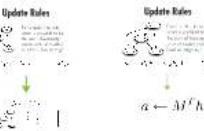
$$(MM^T)^k h^{(0)} = c_1^k q_1 z_1 + \dots + c_n^k q_n z_n$$

$p_1 z_1$ is a non-negative
vector, so is $c_1^k q_1 z_1$

We need to
show that
 c_1^k is
non-negative

$$(MM^T)^k z_1$$

$p_1 z_1$ is
non-negative
vector, so is
 $c_1^k q_1 z_1$



$$\begin{aligned} a^{(k)} &= M^T b \\ a^{(k+1)} &= a^{(k)} - \eta M^T g^{(k)} \\ &\vdots \\ a^{(n)} &= a^{(n-1)} - \eta M^T g^{(n-1)} \end{aligned}$$

$$a^{(k)} = M^T b$$

the general rule

$$a^{(k+1)} = a^{(k)} - \eta M^T g^{(k)}$$

$$\begin{aligned} h^{(k)} &= (MM^T)^k h^{(0)} \\ h^{(k)} &\text{ is a constant vector} \rightarrow (MM^T)^k h^{(0)} = \eta b^{(k)} \end{aligned}$$

Any nonnegative vector b with n rows and m columns has a set of n expressions that are all in vertices and all merely overlapping—that is, they form a basis for the space.

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Let there be n
pages





We can think of each page as a node, and each link as a directed edge

M_{ij}

**There is an adjacency matrix
associated with this graph**



the nodes are
associated with a hub
score (h) and an
authority score (a)

Let there be n
pages



We can think of each page as a node, and each link as a directed edge

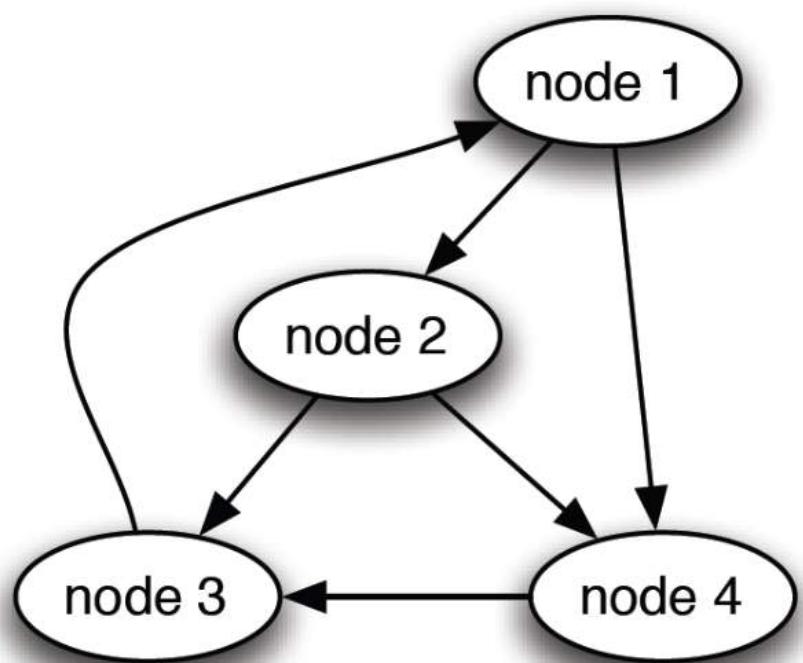
M_{ij}

There is an adjacency matrix associated with this graph



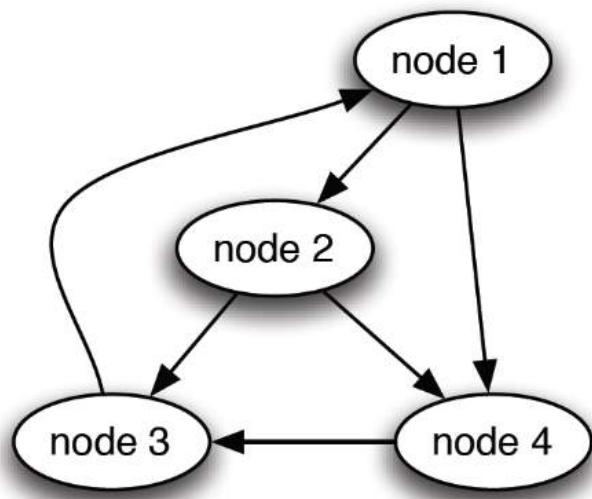
the nodes are associated with a hub score (h) and an authority score (a)

Some basics



0	1	0	1
0	0	1	1
1	0	0	0
0	0	1	0

Update Rules



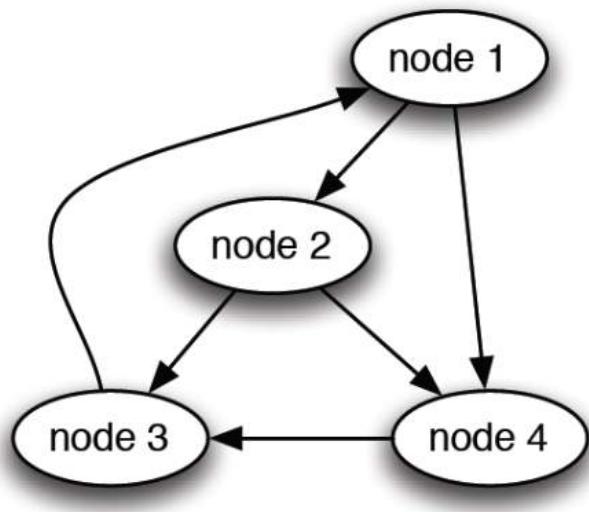
For a node i , its hub score is updated to be the sum of authority scores over all nodes j to which i has an edge.

$$h_i \leftarrow M_{i1}a_1 + M_{i2}a_2 + \cdots + M_{in}a_n$$



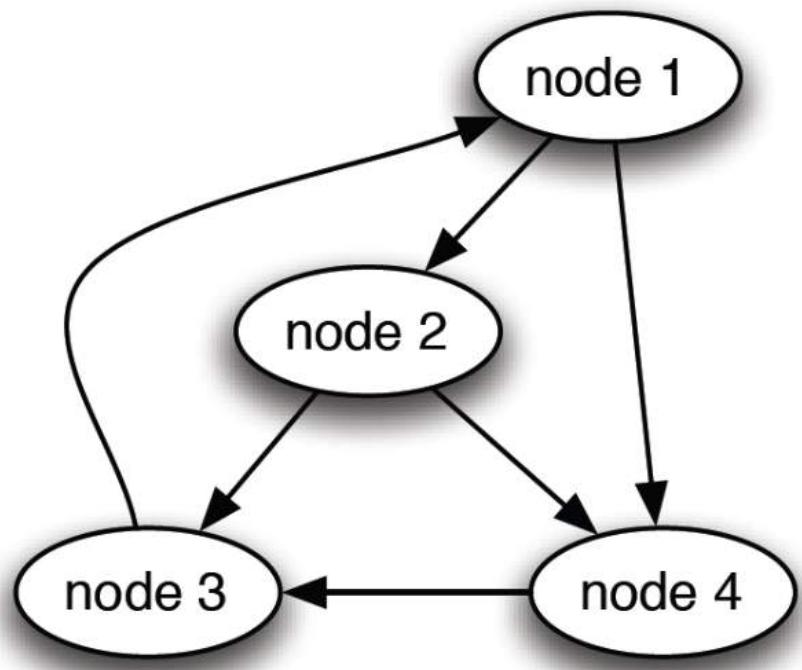


$$h \leftarrow Ma$$



$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 6 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 9 \\ 7 \\ 2 \\ 4 \end{bmatrix}$$

Update Rules



For a node i , its authority score is updated to be the sum of hub scores over all nodes j which have an edge to i

$$a_i \leftarrow M_{1i}h_1 + M_{2i}h_2 + \cdots + M_{ni}h_n$$



$$a \leftarrow M^T h$$



Let's examine
what happens at
each step.



step.



$$a^{(1)} = M^T h^{(0)}$$



$$Ma^{(1)} = MM^T h^{(0)}$$

$$a^{\langle 1 \rangle} = M^T$$



$$h^{\langle 1 \rangle} = Ma^{\langle 1 \rangle} = MM^T h^{\langle 0 \rangle}$$



$$a^{\langle 2 \rangle} = M^T h^{\langle 1 \rangle} = M^T M$$



$$Ma^{\langle 1 \rangle} = MM^T h^{\langle 0 \rangle}$$



$$a^{\langle 2 \rangle} = M^T h^{\langle 1 \rangle} = M^T MM^T h^{\langle 0 \rangle}$$



$$a^{\langle 2 \rangle} = MM^T MM^T h^{\langle 0 \rangle} = (MM^T)^2 h^{\langle 0 \rangle}$$

$$a^{\langle 2 \rangle} = M^T h^{\langle 1 \rangle} = M^T M M$$



$$h^{\langle 2 \rangle} = Ma^{\langle 2 \rangle} = MM^T MM^T h^{\langle 0 \rangle} = (MM^T)^2 h^{\langle 0 \rangle}$$





the general rule

$$a^{\langle k \rangle} = (M^T M)^{k-1} M^T h^{\langle 0 \rangle}$$

$$h^{\langle k \rangle} = (M M^T)^k h^{\langle 0 \rangle}$$



the general rule

$$a^{\langle k \rangle} = (M^T M)^{k-1} M^T h^{\langle 0 \rangle}$$

$$h^{\langle k \rangle} = (M M^T)^k h^{\langle 0 \rangle}$$

what can we
say about the
magnitudes of
 h and a ?

there exist
constants c
and d such
that

$$\frac{h^{(k)}}{c^k}$$

and

$$\frac{a^{(k)}}{d^k}$$

converge

$$\frac{h^{\langle k \rangle}}{c^k} = \frac{(MM^T)^k h^{\langle 0 \rangle}}{c^k}$$



$h^{(*)}$

what properties
should $h^{(*)}$ have?

$$(MM^T) h^{\langle * \rangle} = ch^{\langle * \rangle}$$

lets prove that $\frac{h^{\langle k \rangle}}{c^k}$
converges to
the eigenvector
of MM^T

Any symmetric matrix A with n rows and n columns has a set of n eigenvectors that are all unit vectors and all mutually orthogonal—that is, they form a basis for the space \mathbf{R}^n .

eigenvectors

z_1, z_2, \dots, z_n

$|c_1| \geq |c_2| \geq \dots \geq |c_n|$

eigenvalues

let's assume that

$$|c_1| > |c_2|$$

$$|c_1| > |c_2|$$

since $\textcolor{red}{z}$ form a basis:

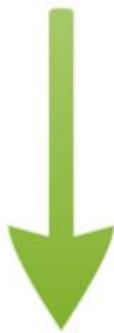
$$x = p_1 z_1 + p_2 z_2 + \cdots + p_n z_n$$

$$MM^T)x = (MM^T)(p_1 z_1 + p_2 z_2 + \cdots + p_n z_n)$$

SINCE \angle FORM A BASIS.

$$x = p_1 z_1 + p_2 z_2 + \cdots + p_n z_n$$

$$\begin{aligned}(MM^T)x &= (MM^T)(p_1 z_1 + p_2 z_2 + \cdots + p_n z_n) \\&= p_1 MM^T z_1 + p_2 MM^T z_2 + \cdots + p_n MM^T z_n \\&= p_1 c_1 z_1 + p_2 c_2 z_2 + \cdots + p_n c_n z_n,\end{aligned}$$



$$(MM^T)^k x = c_1^k p_1 z_1 + c_2^k p_2 z_2 + \cdots + c_n^k p_n z_n$$

$$\begin{aligned} &= p_1 M M^T z_1 + p_2 M M^T z_2 + \cdots + p_n M M^T z_n \\ &= p_1 c_1 z_1 + p_2 c_2 z_2 + \cdots + p_n c_n z_n, \end{aligned}$$

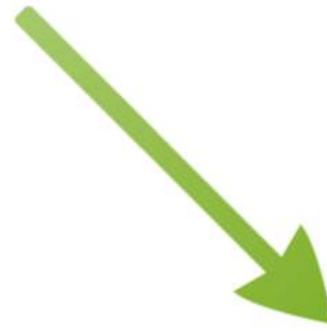


$$(M M^T)^k x = c_1^k p_1 z_1 + c_2^k p_2 z_2 + \cdots + c_n^k p_n z_n$$



$$h^{\langle k \rangle} = (M M^T)^k h^{\langle 0 \rangle} = c_1^k q_1 z_1 + c_2^k q_2 z_2$$

$$^k x = c_1^k p_1 z_1 + c_2^k p_2 z_2 + \cdots + c_n^k p_n z_n$$



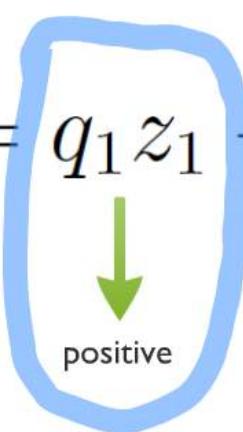
$$h^{(k)} = (MM^T)^k h^{(0)} = c_1^k q_1 z_1 + c_2^k q_2 z_2 + \cdots + c_n^k q_n z_n$$



$$\frac{h^{(k)}}{c_1^k} = q_1 z_1 + \left(\frac{c_2}{c_1}\right)^k q_2 z_2 + \cdots + \left(\frac{c_n}{c_1}\right)^k q_n z_n$$


$$\frac{h^{\langle k \rangle}}{c_1^k} = q_1 z_1 + \left(\frac{c_2}{c_1} \right)^k q_2 z_2 + \cdots + \left(\frac{c_n}{c_1} \right)^k q_n z_n$$

we still need to show two things:

$$\frac{h^{\langle k \rangle}}{c_1^k} = q_1 z_1 + \left(\frac{c_2}{c_1} \right)^k q_2 z_2 + \cdots + \left(\frac{c_n}{c_1} \right)^k q_n z_n$$


The term $q_1 z_1$ is highlighted with a blue rounded rectangle. A green arrow points downwards from this term to the word "positive".

2

and that we converge to
the same eigenvector,
independent of the
starting point.

to show the second point,
consider \mathbf{x} , a positive vector

$$(MM^T)^k \mathbf{x} = c_1^k p_1 z_1 + \cdots + c_n^k p_n z_n$$



$$p_1 z_1$$



so no matter how \mathbf{x}

$p_1 z_1$



so no matter how you start, you converge to the direction of z_1

we need to
show that
is nonzero!

p_1

or, we need to show
that no positive
vector is orthogonal
to z_1



$$(MM^T)^k x / c_1^k$$



$p_1 z_1$

hence **no** positive
vector can be
orthogonal to \tilde{z}_1

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