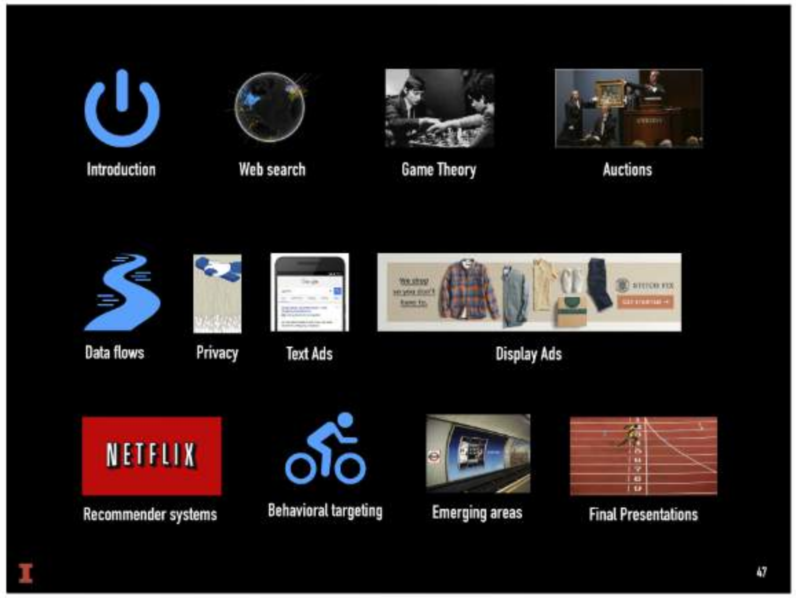
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Game Theory
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hs1@illinois.edu
introduction
nash equilibrium
mixed strategies

Introduction

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Game Theory

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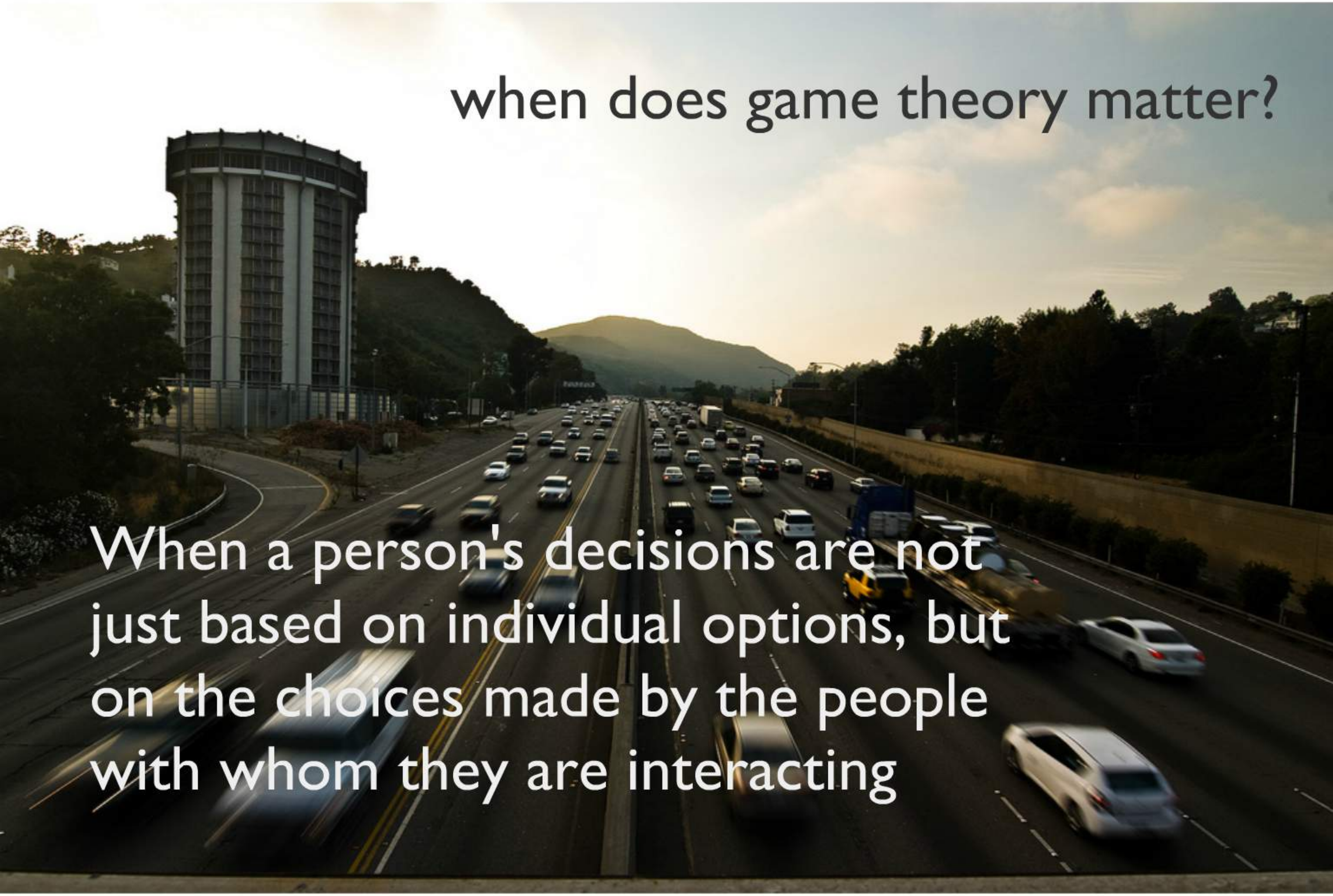
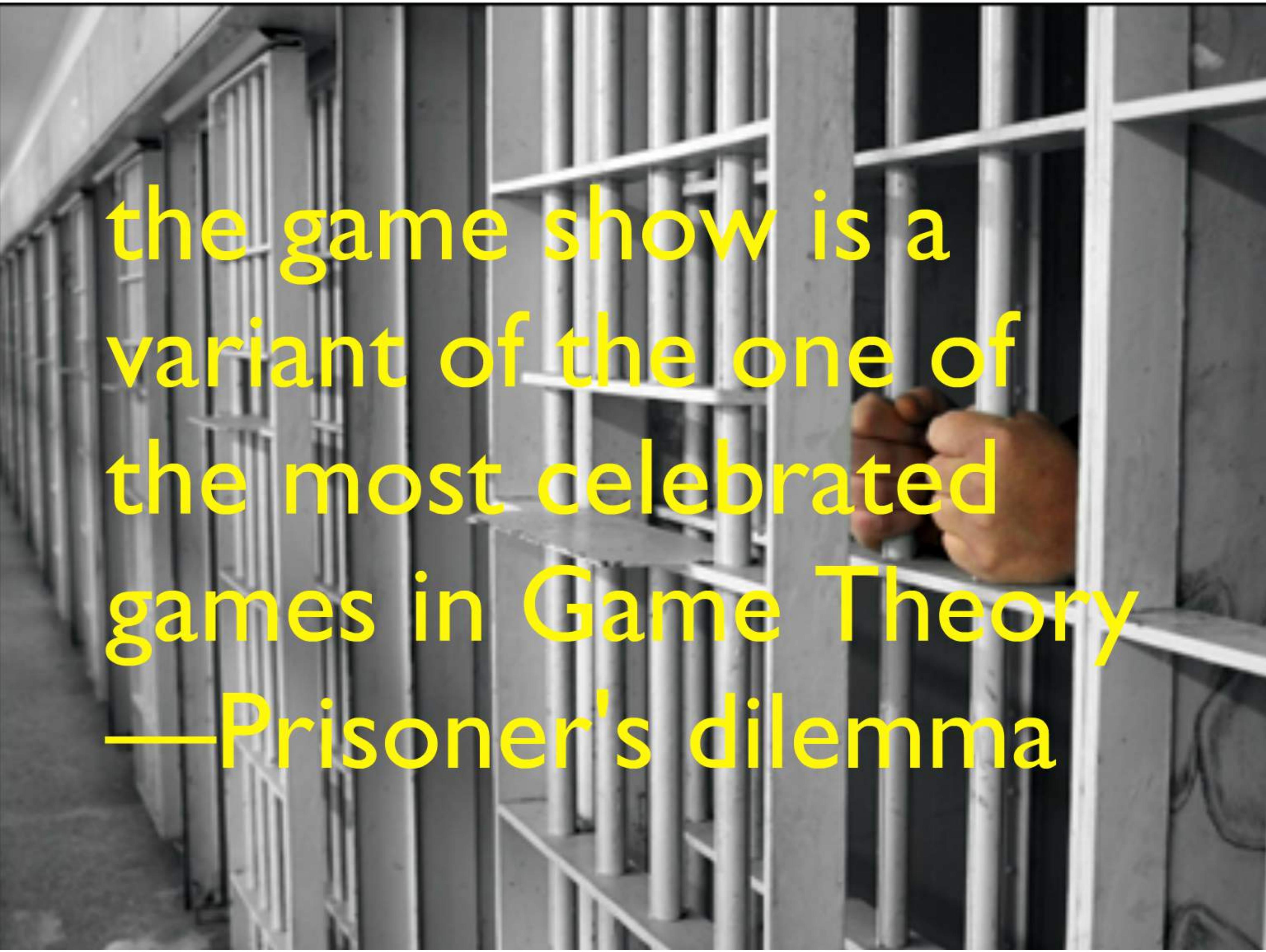
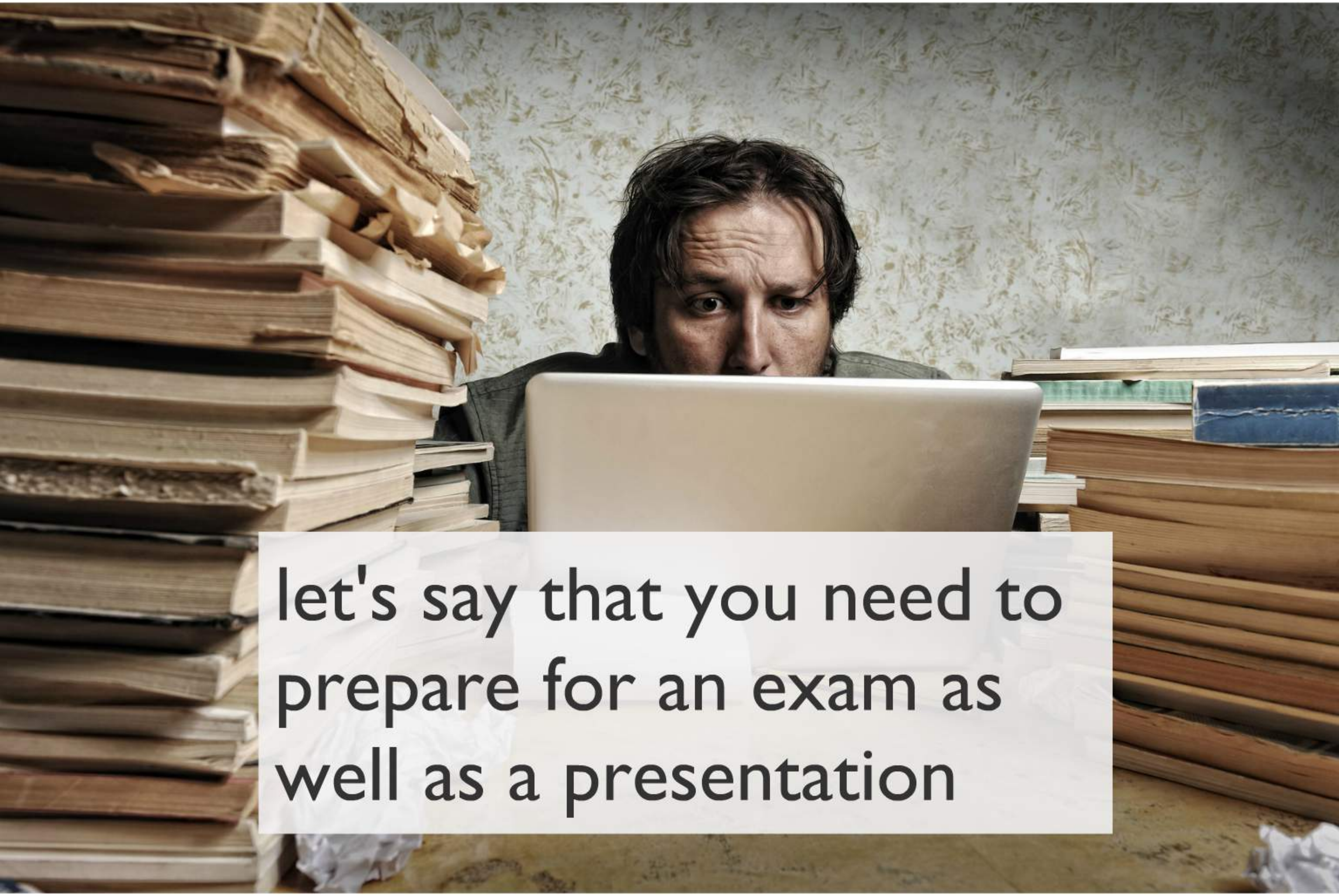

when does game theory matter?
When a person's decisions are not just based on individual options, but on the choices made by the people with whom they are interacting


YouTube


the game show is a
variant of the one of
the most celebrated
games in Game Theory
—Prisoner's dilemma

penalty kicks

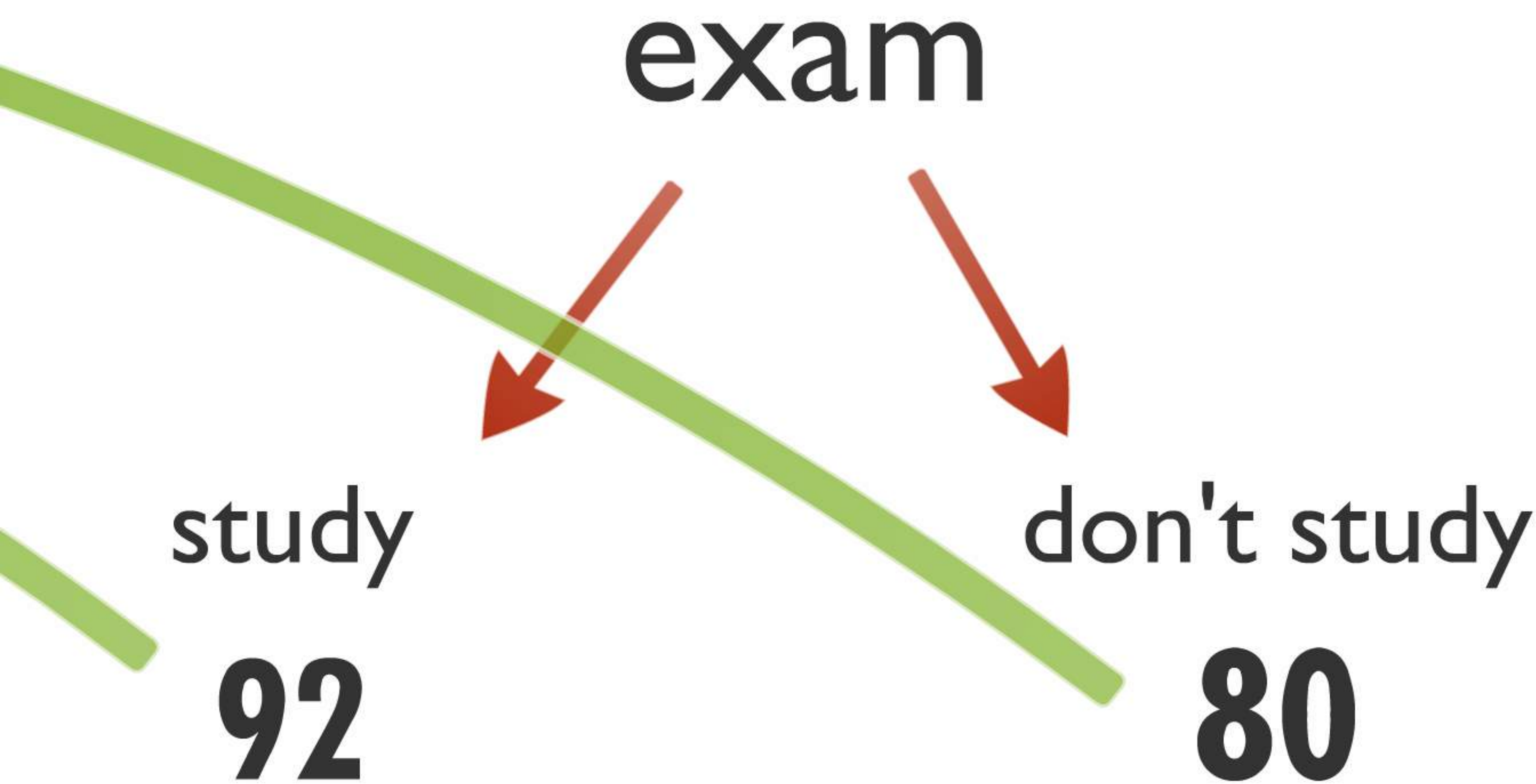
auctions

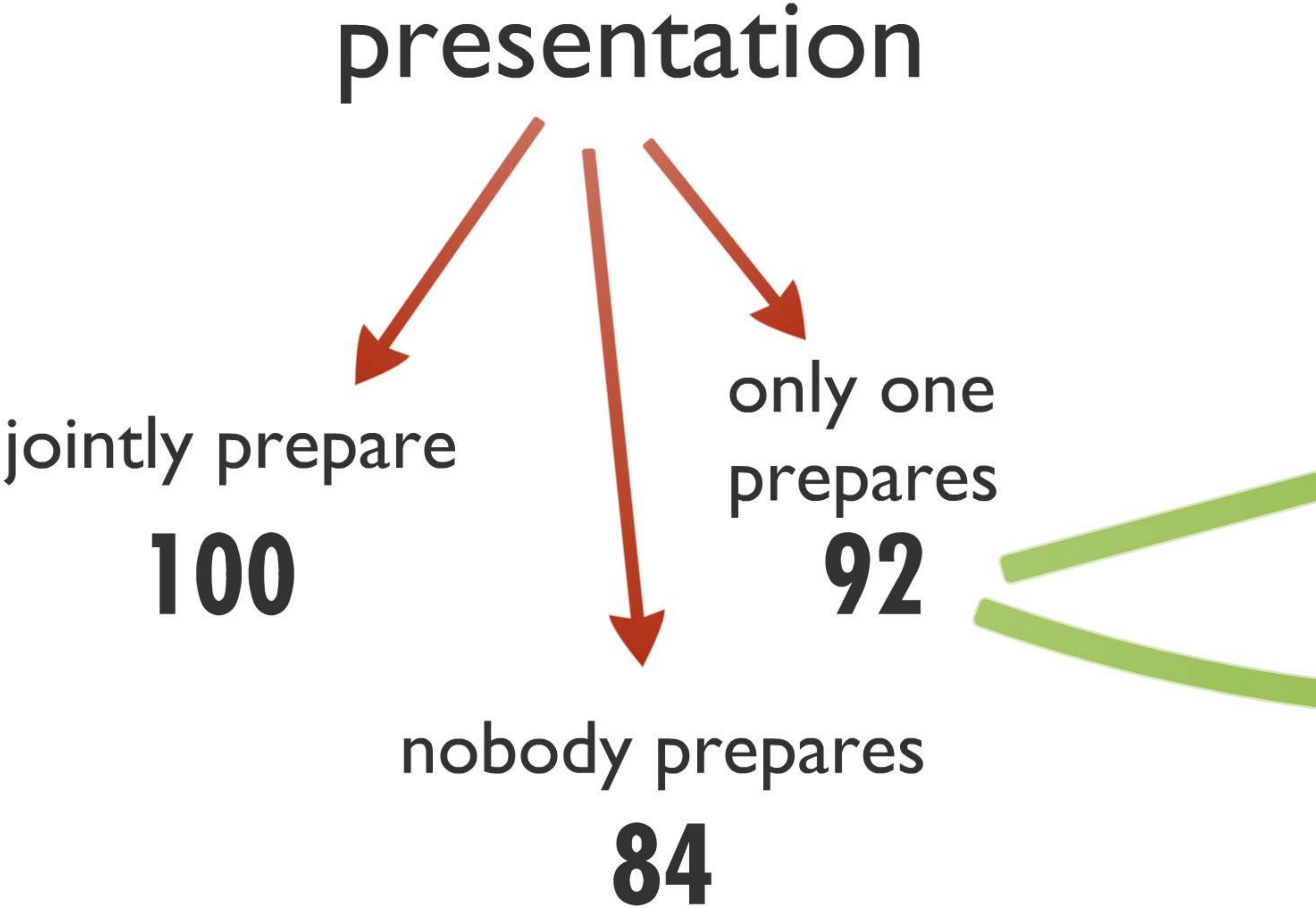

let's say that you need to prepare for an exam as well as a presentation

1 you don't have the time to prepare for both!

2

you have an accurate idea of the estimated grade under different situations


exam
study
don't study
92
80


presentation
jointly prepare
100
nobody prepares
84
only one
prepares
92

Payoff matrix

		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

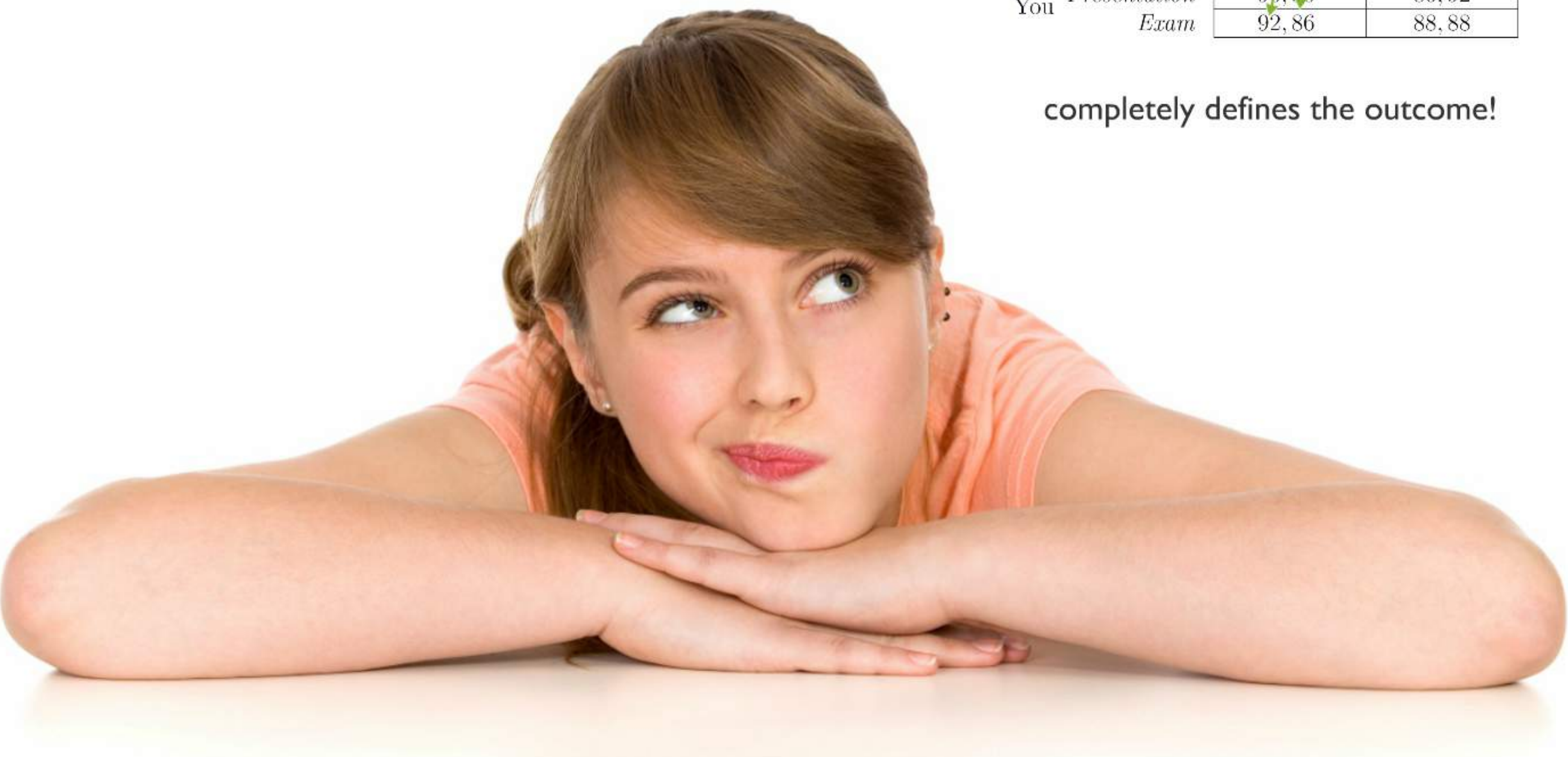
completely defines the outcome!

Payoff matrix

		Your Partner: *Presentation*	Your Partner: *Exam*
You	*Presentation*	$90, 90$	$86, 92$
	Exam	$92, 86$	$88, 88$

completely defines the outcome!

what should you do?



basic ingredients

a set of participants

each player has a set of options (strategies)

		Your Partner: *Presentation*	Your Partner: *Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

each strategy has a payoff

some key assumptions

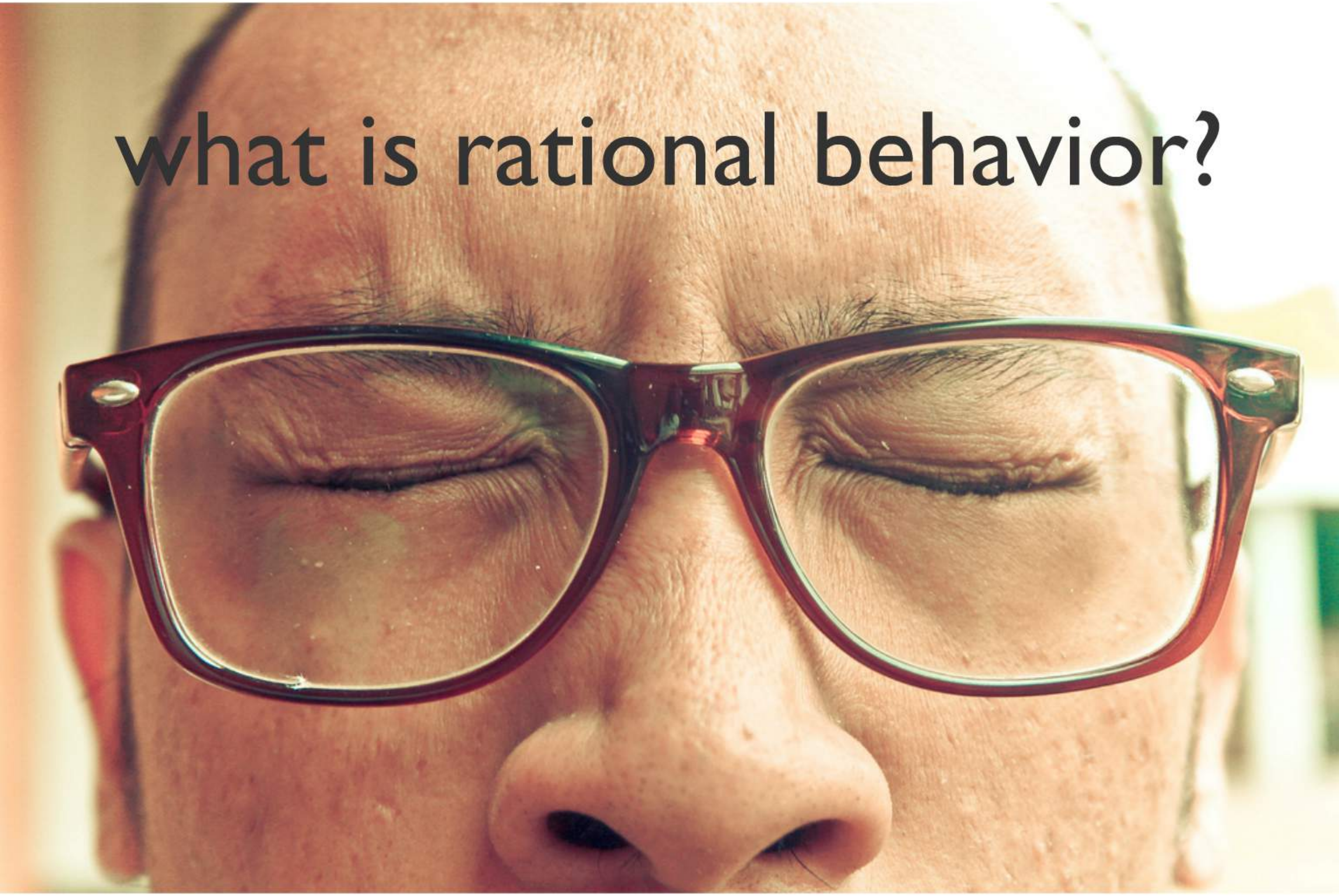
1 Everything that a player cares about is summarized in the player's payoffs.

		Your Partner: *Presentation*	Your Partner: *Exam*
You	*Presentation*	$90, 90$	$86, 92$
	Exam	$92, 86$	$88, 88$

2 Each player knows everything about the structure of the game

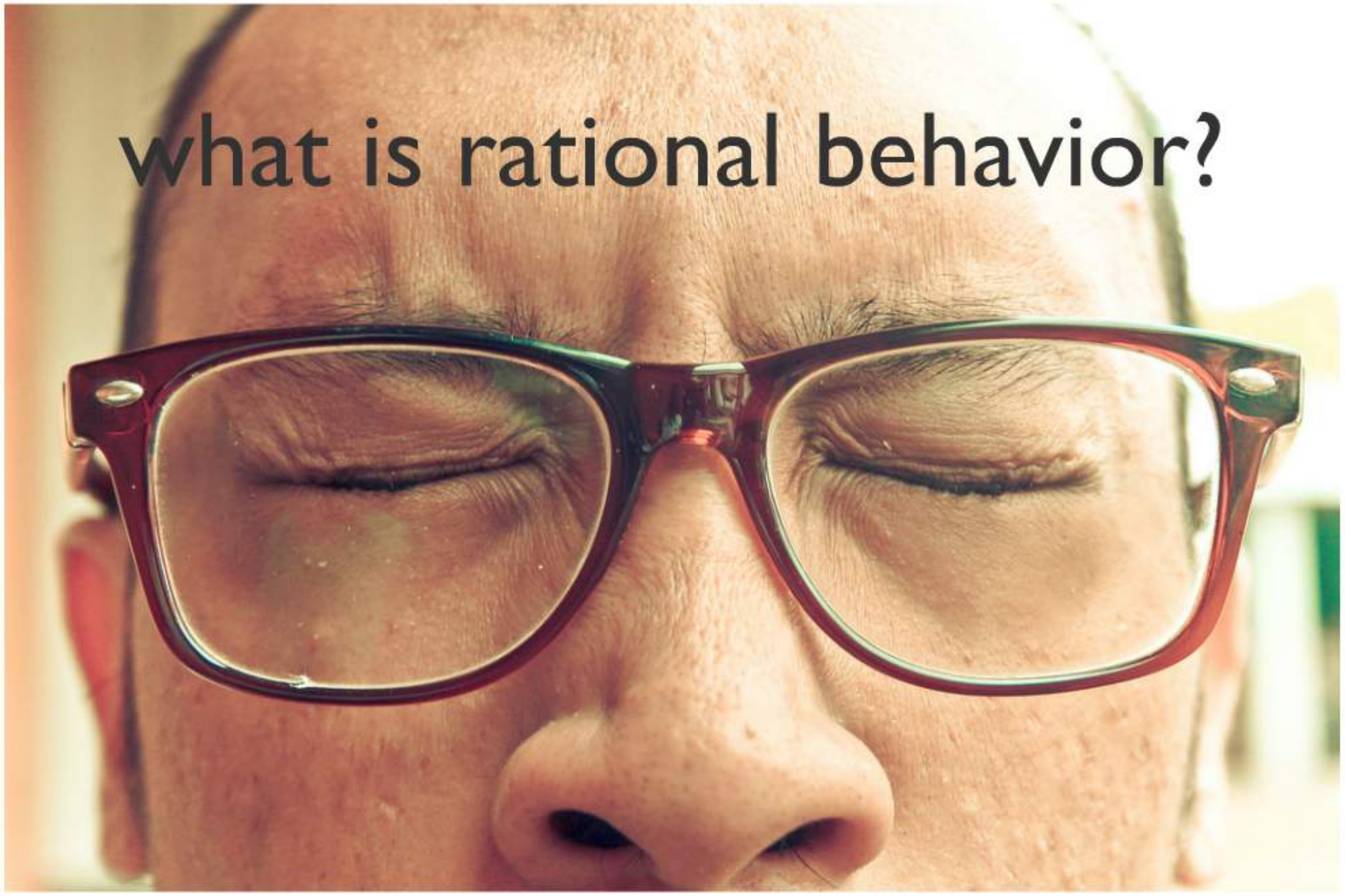
3 Each person is rational

these are the reasons why the conversations in the game of golden balls are irrelevant


what is rational behavior?

1 each player wants to maximize her own payoff

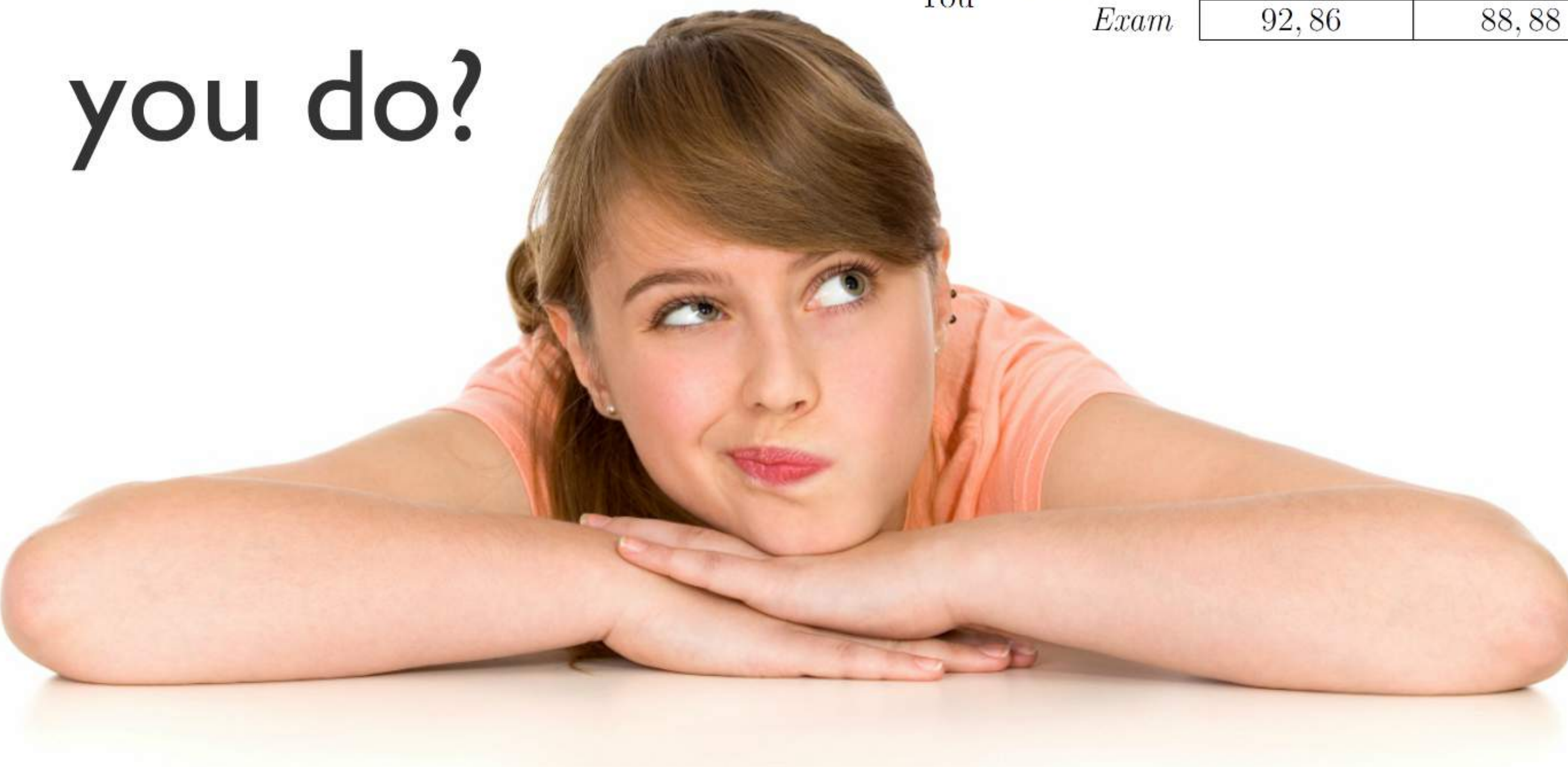
2 each player actually succeeds in selecting the optimal strategy



what should you do?

Payoff matrix

		Your Partner: *Presentation*	Your Partner: *Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88



dominant strategies

what should you do?

Payoff matrix

		Your Partner: *Presentation*	Your Partner: *Exam*
You	*Presentation*	$90, 90$	$86, 92$
	Exam	$92, 86$	$88, 88$

There is a optimal strategy, but neither player can select it!

limits of rational play

		Suspect 2	
		NC	C
Suspect 1	NC	$-1, -1$	$-10, 0$
	C	$0, -10$	$-4, -4$

A prisoner's dilemma

		Ibrahim	
		Split	**Steal**
Nick	**Split**	(5,5)	(0,10)
	Steal	(10,0)	(0,0)

both choices have the same value!

Nick \ Ibrahim	Split	Steal
Split	(5,5)	(0,10)
Steal	(10,0)	(0,0)

both choices have the same value!

Suspect 1 \ Suspect 2	NC	C
NC	$-1, -1$	$-10, 0$
C	$0, -10$	$-4, -4$

contrast with prisoner's dilemma

Assume that player 1 uses strategy S and player 2 uses strategy T

let's examine the case of one player with strictly dominant strategy

Firm 1 has a strictly dominant strategy, but Firm 2 doesn't

common knowledge assumptions

1 strict best response

2 dominant strategies

two concepts

do both players always have a dominant strategy?

anti-coordination

		Animal 2	
		D	H
Animal 1	D	$3,3$	$1,5$
	H	$5,1$	$0,0$

nash equilibrium

A reminder: all games analyzed thus far are simultaneous move, single shot games

We say that a pair of strategies (S,T) is a Nash equilibrium if S is a best response to T, and T is a best response to S.

let's look at some variants

what is the Nash equilibrium here?

coordination games

Assume that player 1 uses strategy S and player 2 uses strategy T

two concepts

1 strict best response

$$P_1(S,T) > P_1(S',T)$$

2 dominant strategies

2 dominant strategies

S is the best response for every T

S is the strict best response for every T

do both players always have a dominant strategy?

		Your Partner: *Presentation*	Your Partner: *Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

let's examine the case of one player with strictly dominant strategy


TM

Market Segments Size

Low priced: 60%
High priced: 40%

Same Segment Competition

Firm 1 dominates Firm 2, when they compete in the same segment

Firm 1: 80%
Firm 2: 20%

assume that profit per item is the same in both segments

Market Segments Size

Low priced: 60%
High priced: 40%

Same Segment Competition

Firm 1 dominates Firm 2, when they compete in the same segment

Firm 1: 80%
Firm 2: 20%

		Firm 2	
		Low-Priced	*Upscale*
Firm 1	*Low-Priced*	.48, .12	.60, .40
	Upscale	.40, .60	.32, .08

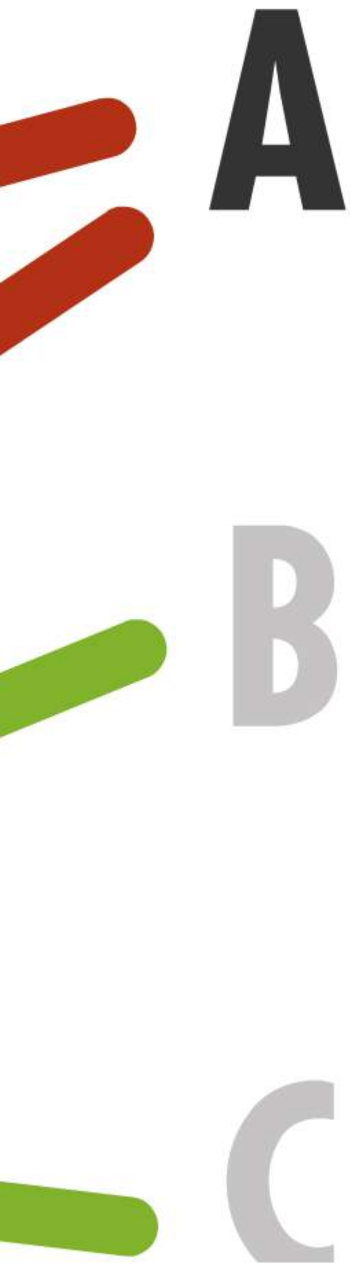
Firm 1 has a strictly dominant strategy, but Firm 2 doesn't

common knowledge assumptions

The Firms know the structure of the game, they know that each of them know the structure of the game, they know that each of them know that each of them know etc.

John C. Harsanyi. Game with incomplete information played by “Bayesian” players, I–III. Part I: The basic model. Management Science, 14(3):159–182, November 1967.

how realistic is this?



Let's now consider the case when two agencies are competing to do business with one of three clients (A,B and C)

Firm 2 is bigger than firm 1, and can attract business on its own. Firm 1 is too small to attract business on its own.

A will not give its business to a single client.

Let's now consider the case when two agencies are competing to do business with one of three clients (A,B and C)

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Firm 2 is bigger than firm 1, and can attract business on its own. Firm 1 is too small to attract business on its own.

A will not give its business to a single client.

		Firm 2		
		A	B	C
Firm 1	A	$4,4$	$0,2$	$0,2$
	B	$0,0$	$1,1$	$0,2$
	C	$0,0$	$0,2$	$1,1$

A reminder: all games analyzed thus far are simultaneous move, single shot games

We say that a pair of strategies (S,T) is a Nash equilibrium if S is a best response to T, and T is a best response to S.

Firm 1 \ Firm 2	A	B	C
A	$4, 4$	$0, 2$	$0, 2$
B	$0, 0$	$1, 1$	$0, 2$
C	$0, 0$	$0, 2$	$1, 1$

what is the Nash equilibrium here?

coordination games

		Your Partner	
		PowerPoint	*Keynote*
You	*PowerPoint*	1, 1	0, 0
	Keynote	0, 0	1, 1

Games can have multiple Nash equilibria; it becomes hard to predict how rational players would behave occurs in many scenarios: two manufacturers, attack maneuvers, waiting for someone at the mall etc..

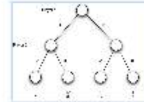
Games can have multiple Nash equilibria; it becomes hard to predict how rational players would behave

unbalanced coordination

		Your Partner	
		PowerPoint	*Keynote*
You	*PowerPoint*	$1, 1$	$0, 0$
	Keynote	$0, 0$	$2, 2$

	Your Partner	
	PowerPoint	*Keynote*
oint	$1, 1$	$0, 0$
ote	$0, 0$	$2, 2$

still Nash equilibria!



Dynamic games

Once you allow randomization, equilibria always exist.

this changes the game!

mixed strategies

pure strategies

mixed strategies

$q = 1/2$

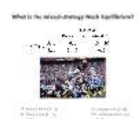
therefore, equality is the only possibility!

$q = 1/2$

Pareto Optimality

A choice of strategies—one by each player—is Pareto-inefficient if there is one other choice of strategies in which all players receive payoffs at least as high, and at least one player receives a strictly higher payoff.

A choice of strategies—one by each player—is Pareto-optimal if there is no other choice of strategies in which all players receive payoffs at least as high, and at least one player receives a strictly higher payoff.



what is the relationship between Pareto optimality and social optimality?

pareto optimal

nash

do Nash equilibria contradict social optimality?

- two player situations
- evolutionary biology
- equilibrium in beliefs

matching pennies

		Player 2	
		H	T
Player 1	H	$-1, +1$	$+1, -1$
	T	$+1, -1$	$-1, +1$

No pure Nash equilibrium

Once you allow randomization, equilibria always exist.

matching pennies

		Player 2	
		H	T
Player 1	H	$-1, +1$	$+1, -1$
	T	$+1, -1$	$-1, +1$

No pure Nash equilibrium

		Player 2	
		H	T
Player 1	H	$-1, +1$	$+1, -1$
	T	$+1, -1$	$-1, +1$

Say player 2 commits to H with probability q and to T with probability (1-q).

this changes the game!

		Player 2 H	Player 2 T
Player 1	H	$-1, +1$	$+1, -1$
	T	$+1, -1$	$-1, +1$

Say player 2 commits to H with probability q and to T with probability (1-q).

mixed strategies

$$q \in (0, 1)$$

pure strategies

Expected Payoffs for Player 1

H: $(-1)q + (1)(1-q) = 1 - 2q$

T: $(1)q + (-1)(1-q) = 2q - 1$

Player 2

Player 1	H	T
H	$-1, +1$	$+1, -1$
T	$+1, -1$	$-1, +1$

Pure strategies cannot be part of the matching pennies Nash equilibrium*

Expected Payoffs for Player 1

H: $(-1)q + (1)(1-q) = 1 - 2q$

T: $(1)q + (-1)(1-q) = 2q - 1$

		Player 2	
		H	T
Player 1	H	$-1, +1$	$+1, -1$
	T	$+1, -1$	$-1, +1$

but in general, you can have pure strategies as a solution when you randomize

- $(1)q + (-1)(1-q) = 2q - 1$

		Player 2	
		H	T
Player 1	H	$-1, +1$	$+1, -1$
	T	$+1, -1$	$-1, +1$

Expected Payoffs for Player 1

H: $(-1)q + (1)(1-q) = 1 - 2q$
T: $(1)q + (-1)(1-q) = 2q - 1$

		Player 2 H	Player 2 T
Player 1	H	$-1, +1$	$+1, -1$
	T	$+1, -1$	$-1, +1$

$$\text{if } 1 - 2q \neq 2q - 1$$

then a clear dominant strategy exists!

$$\therefore q = 1/2$$

so player 2 chooses q so that player 1 is indifferent to his choice

so player 2 chooses q so that player 1 is indifferent to his choice

		Player 2	
		H	T
Player 1	H	$-1, +1$	$+1, -1$
	T	$+1, -1$	$-1, +1$

What is the mixed-strategy Nash Equilibrium?

		Defense	
		Defend Pass	Defend Run
Offense	Pass	0, 0	10, −10
	Run	5, −5	0, 0

What is the mixed-strategy Nash Equilibrium?

		Defense	
		Defend Pass	Defend Run
Offense	Pass	0, 0	10, −10
	Run	5, −5	0, 0

$P : 0 \times q + 10 \times (1 - q)$

$R : 5 \times q + 0 \times (1 - q)$

q=2/3

$DP : 0 \times p + -5 \times (1 - p)$

$DR : -10 \times p + 0 \times (1 - p)$

p=1/3

is any of this selfishness good for society as a whole?

A choice of strategies—one by each player—is Pareto-inefficient if there is one other choice of strategies in which all players receive payoffs at least as high, and at least one player receives a strictly higher payoff.

A choice of strategies—one by each player—is Pareto-optimal if there is no other choice of strategies in which all players receive payoffs at least as high, and at least one player receives a strictly higher payoff.

Pareto Optimality

Vilfredo Pareto

A choice of strategies—one by each player—is Pareto-inefficient if there is one other choice of strategies in which all players receive payoffs at least as high, and at least one player receives a strictly higher payoff.

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pareto optimal

		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

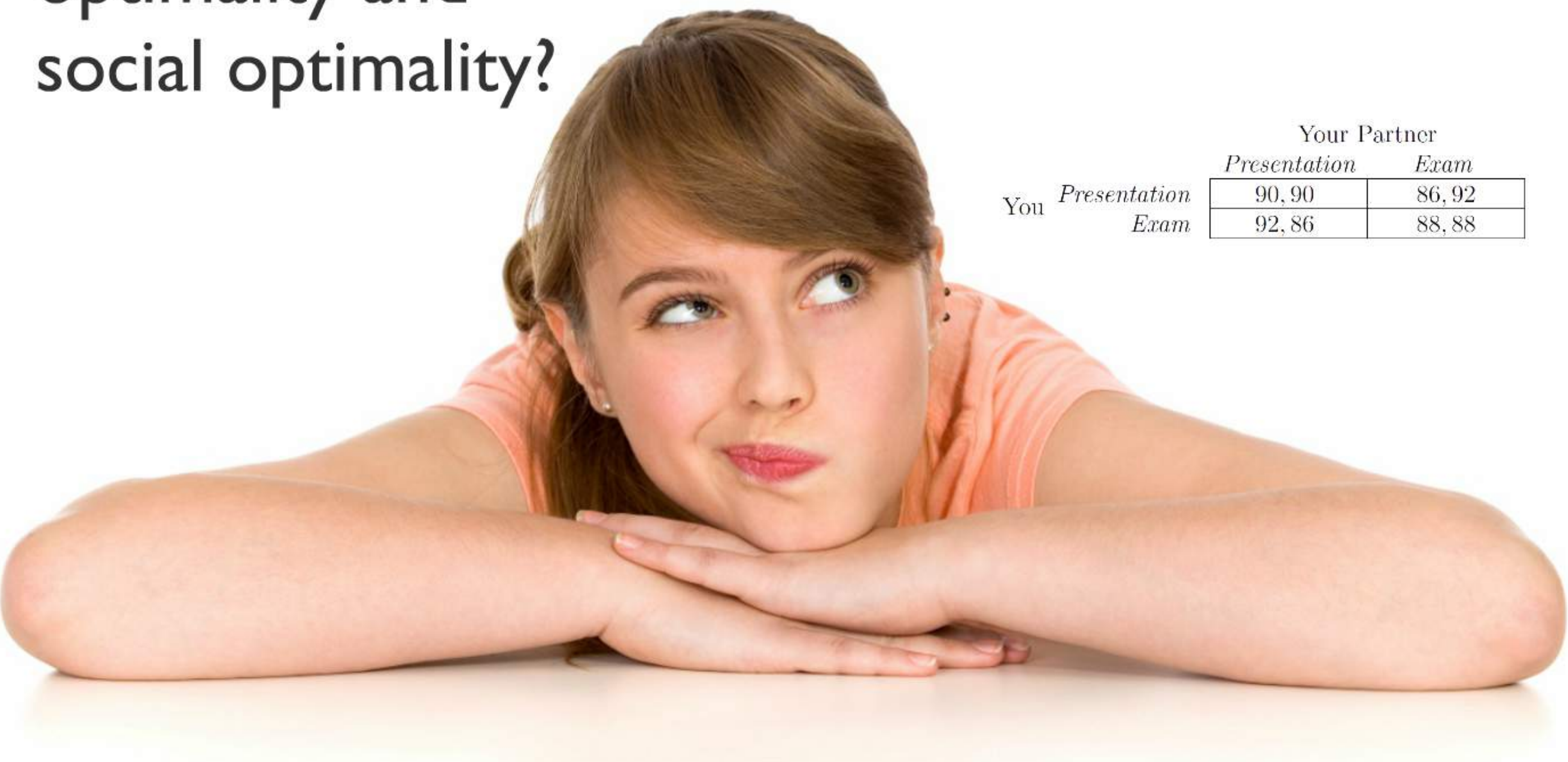
nash

A choice of strategies—one by each player—is a social welfare maximizer (or socially optimal) if it maximizes the sum of the players' payoffs.

what is the relationship between Pareto optimality and social optimality?

A choice of strategies—one by each player—is a social welfare maximizer (or socially optimal) if it maximizes the sum of the players' payoffs.

		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88



the set of strategies
adopted by n players

$$(S_1, S_2, \ldots, S_n)$$

$$P_i(S_1, S_2, \ldots, S_n)$$

payoff to i

best response

$$P_i(S_1, S_2, \ldots, S_{i-1}, S_i, S_{i+1}, \ldots, S_n) \geq P_i(S_1, S_2, \ldots, S_{i-1}, S_i', S_{i+1}, \ldots, S_n)$$

although dominant and strictly dominant strategies can exist in games with many players and many strategies, they are rare

A strategy is strictly dominated if there is some other strategy available to the same player that produces a strictly higher payoffs in response to every choice of strategies by the other players.

$$P_i(S_1, S_2, \ldots, S_{i-1}, S_i', S_{i+1}, \ldots, S_n) > P_i(S_1, S_2, \ldots, S_{i-1}, S_i, S_{i+1}, \ldots, S_n)$$

There are six towns and two Firms want to open stores. Firm 1 has the option of opening its store in any of towns A, C, or E, while Firm 2 has the option of opening its store in any of towns B, D, or F. These decisions will be executed simultaneously.

		Firm 2		
		B	D	F
Firm 1	A	$1, 5$	$2, 4$	$3, 3$
	C	$4, 2$	$3, 3$	$4, 2$
	E	$3, 3$	$2, 4$	$5, 1$

payoff matrix

		Firm 2	
		B	D
Firm 1	C	$4, 2$	$3, 3$
	E	$3, 3$	$2, 4$

Nash equilibria survive iterated deletion

1 We start with any n-player game, find all the strictly dominated strategies, and delete them.

2 We then consider the reduced game in which these strategies have been removed. In this reduced game there may be strategies that are now strictly dominated, despite not having been strictly dominated in the full game. We find these strategies and delete them.

3 We continue this process, repeatedly finding and removing strictly dominated strategies until none can be found.

1

We start with any n-player game, find all the strictly dominated strategies, and delete them.

2

We then consider the reduced game in which these strategies have been removed. In this reduced game there may be strategies that are now strictly dominated, despite not having been strictly dominated in the full game. We find these strategies and delete them.

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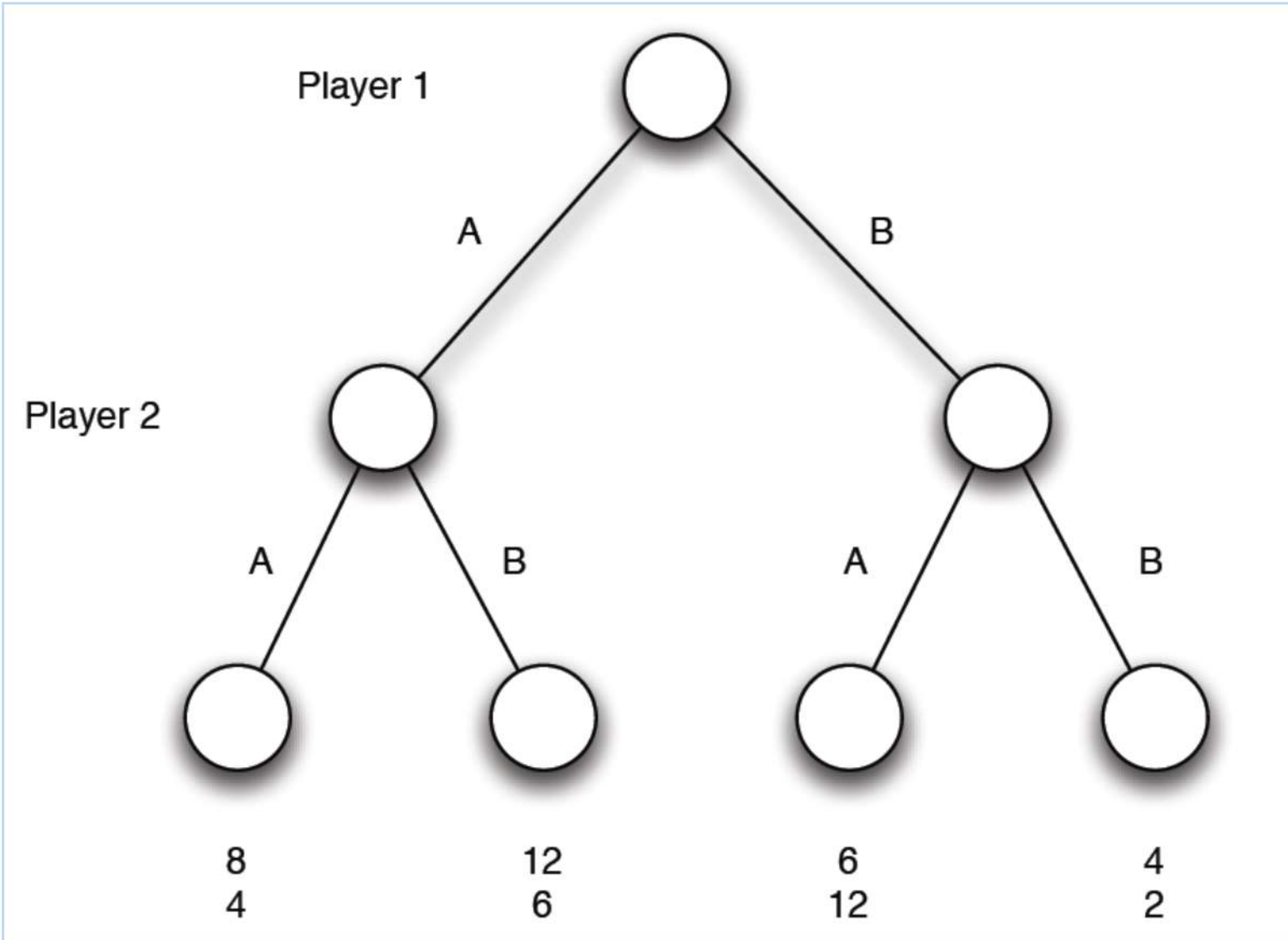

Dynamic games