



# Lecture 20: Message Integrity (contd), Pseudorandomness

Professor Adam Bates  
CS 461 / ECE 422  
Fall 2019

# Goals for Today



- Learning Objectives:
  - Briefly discuss midterm results
  - Conclude discussion of Message Integrity
- Announcements, etc:
  - Midterm grades released on Compass2G as of this morning.
  - MP3 Checkpoint #1: **Oct 14 at 6pm**



**Reminder:** Please put away devices at the start of class

# Midterm Results



- Exams were graded on Wednesday evening by Instructional Team.
- Entered and analyzed by me over the weekend.
- Initially...
  - Median Score: 58%
  - Max Score: 88%

	Total
Mean (Raw)	60.82
Median (raw)	60
Min (Raw)	17
Max (Raw)	92
Mean (%)	58%
Median (%)	58%
Min (%)	16%
Max (%)	88%
Max Possible	104

*Note: there was a typo on \_\_\_\_\_^  
Page 14; only 1 point (not 7) was possible on this page*

# Midterm Results



- Were the questions fair?

		Multiple Choice				Short Answer			MP1			MP2					
		Page 1	Page 2	Page 3	Page 4	Page 5	Page 6	Page 7	Page 8	Page 9	Page 10	Page 11	Page 12	Page 13	Page 14	Page 15	Page 16
Overall Performance	Q1	90%	69%	60%	73%	92%	98%	77%	77%	79%	84%	83%	68%	40%	60%	92%	70%
	Q2	86%	61%	50%	63%	85%	70%	68%	60%	71%	69%	69%	61%	29%	20%	93%	57%
	Q3	76%	56%	43%	61%	78%	60%	67%	48%	59%	54%	50%	63%	20%	27%	78%	30%
	Q4	76%	62%	30%	58%	76%	56%	63%	27%	43%	43%	39%	46%	13%	30%	62%	27%
	Q5	62%	48%	31%	54%	61%	42%	54%	14%	26%	5%	26%	45%	11%	11%	45%	7%

# Midterm Results



- Were the questions fair?
  - For the most part, it looks like it — best overall performers did well on a page-by-page basis with steady decrease in points awarded for lesser performers

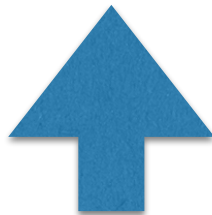
		Multiple Choice				Short Answer			MP1			MP2					
		Page 1	Page 2	Page 3	Page 4	Page 5	Page 6	Page 7	Page 8	Page 9	Page 10	Page 11	Page 12	Page 13	Page 14	Page 15	Page 16
Overall Performance	Q1	90%	69%	60%	73%	92%	98%	77%	77%	79%	84%	83%	68%	40%	60%	92%	70%
	Q2	86%	61%	50%	63%	85%	70%	68%	60%	71%	69%	69%	61%	29%	20%	93%	57%
	Q3	76%	56%	43%	61%	78%	60%	67%	48%	59%	54%	50%	63%	20%	27%	78%	30%
	Q4	76%	62%	30%	58%	76%	56%	63%	27%	43%	43%	39%	46%	13%	30%	62%	27%
	Q5	62%	48%	31%	54%	61%	42%	54%	14%	26%	5%	26%	45%	11%	11%	45%	7%

# Midterm Results

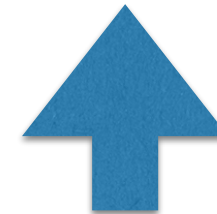


- Were the questions fair?
  - For the most part, it looks like it — best overall performers did well on a page-by-page basis with steady decrease in points awarded for lesser performers

		Multiple Choice				Short Answer			MP1			MP2					
		Page 1	Page 2	Page 3	Page 4	Page 5	Page 6	Page 7	Page 8	Page 9	Page 10	Page 11	Page 12	Page 13	Page 14	Page 15	Page 16
Overall Performance	Q1	90%	69%	60%	73%	92%	98%	77%	77%	79%	84%	83%	68%	40%	60%	92%	70%
	Q2	86%	61%	50%	63%	85%	70%	68%	60%	71%	69%	69%	61%	29%	20%	93%	57%
	Q3	76%	56%	43%	61%	78%	60%	67%	48%	59%	54%	50%	63%	20%	27%	78%	30%
	Q4	76%	62%	30%	58%	76%	56%	63%	27%	43%	43%	39%	46%	13%	30%	62%	27%
	Q5	62%	48%	31%	54%	61%	42%	54%	14%	26%	5%	26%	45%	11%	11%	45%	7%



*Top 30 performers averaged 60% credit on page 3....*



*Top 30 performers averaged 40% credit on page 14....*

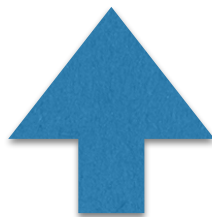
*This looks worst than it is;  
Page 14 is only 1 point.*

# Midterm Results

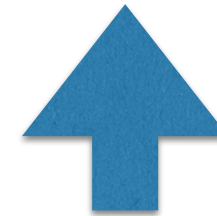


- Since Page 3 and Page 13 were poor predictors of overall performance, I decided to drop those questions from the point total of the exam

		Multiple Choice				Short Answer			MP1			MP2					
		Page 1	Page 2	Page 3	Page 4	Page 5	Page 6	Page 7	Page 8	Page 9	Page 10	Page 11	Page 12	Page 13	Page 14	Page 15	Page 16
Overall Performance	Q1	90%	69%	60%	73%	92%	98%	77%	77%	79%	84%	83%	68%	40%	60%	92%	70%
	Q2	86%	61%	50%	63%	85%	70%	68%	60%	71%	69%	69%	61%	29%	20%	93%	57%
	Q3	76%	56%	43%	61%	78%	60%	67%	48%	59%	54%	50%	63%	20%	27%	78%	30%
	Q4	76%	62%	30%	58%	76%	56%	63%	27%	43%	43%	39%	46%	13%	30%	62%	27%
	Q5	62%	48%	31%	54%	61%	42%	54%	14%	26%	5%	26%	45%	11%	11%	45%	7%



*Dropping this page,  
i.e., 8 point curve*



*Also dropping this page,  
i.e., another 6 point curve*

# Midterm Results



- After curve...
- Median Score: 67%
- Max Score: 102%
- **PICK-UP YOUR EXAMS IN DISCUSSION CLASS ON WEDNESDAY.**

	Total
Mean (Raw)	60.82
Median (raw)	60
Min (Raw)	17
Max (Raw)	92
Mean (%)	68%
Median (%)	67%
Min (%)	19%
Max (%)	102%
Max Possible	90



# Midterm Results



- **VERIFY OUR WORK**

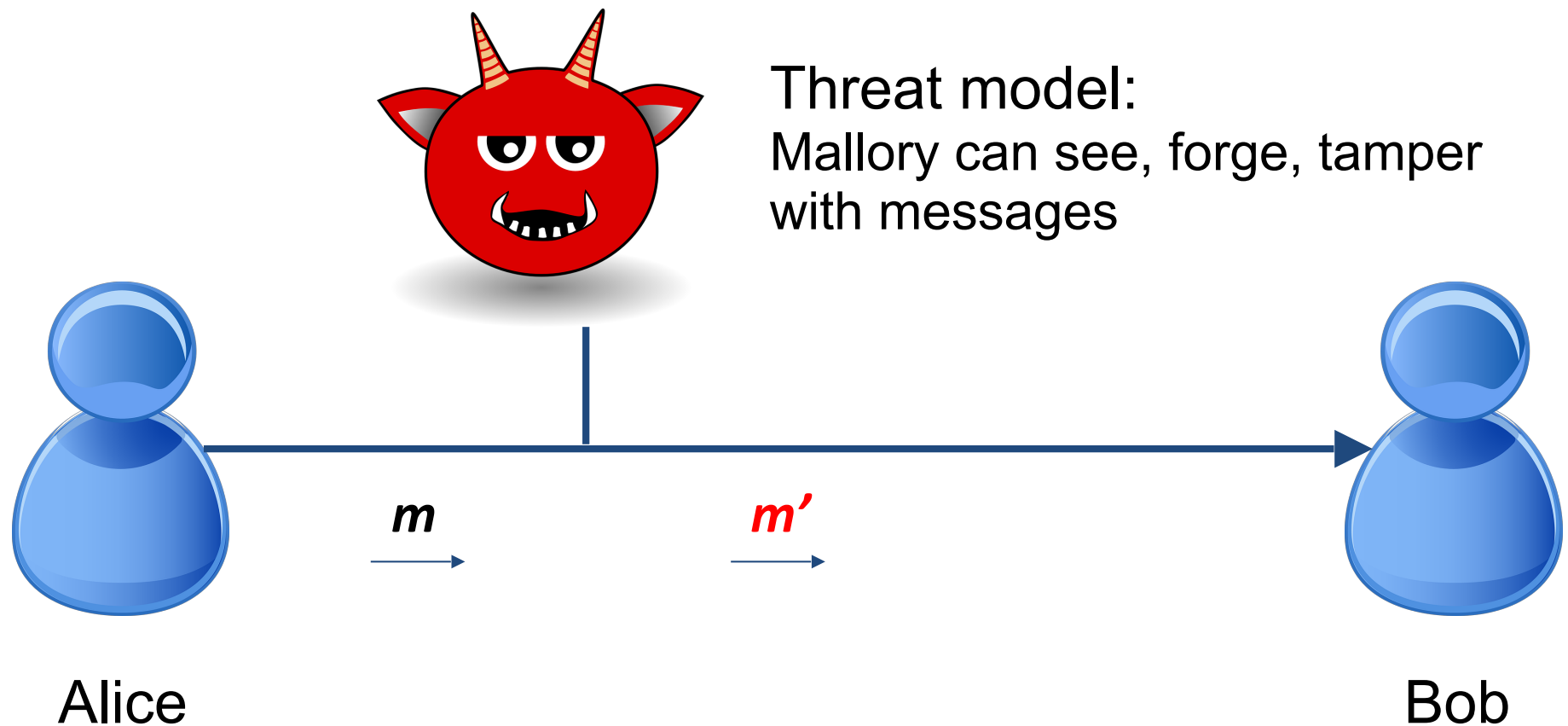
- Our team graded 2,400 pages of exams in 4 hours... mistakes were caught, perhaps some weren't.
- Check that point totals on cover page match bottom corner of each question page
- We will not be releasing answer key; go to TA office hours if you have a question.
- Email regrade requests to me ([batesa@illinois.edu](mailto:batesa@illinois.edu)) by **Wednesday, July 23rd.**

	Total
Mean (Raw)	60.82
Median (raw)	60
Min (Raw)	17
Max (Raw)	92
Mean (%)	68%
Median (%)	67%
Min (%)	19%
Max (%)	102%
Max Possible	90

# Goal: Secure File Transfer



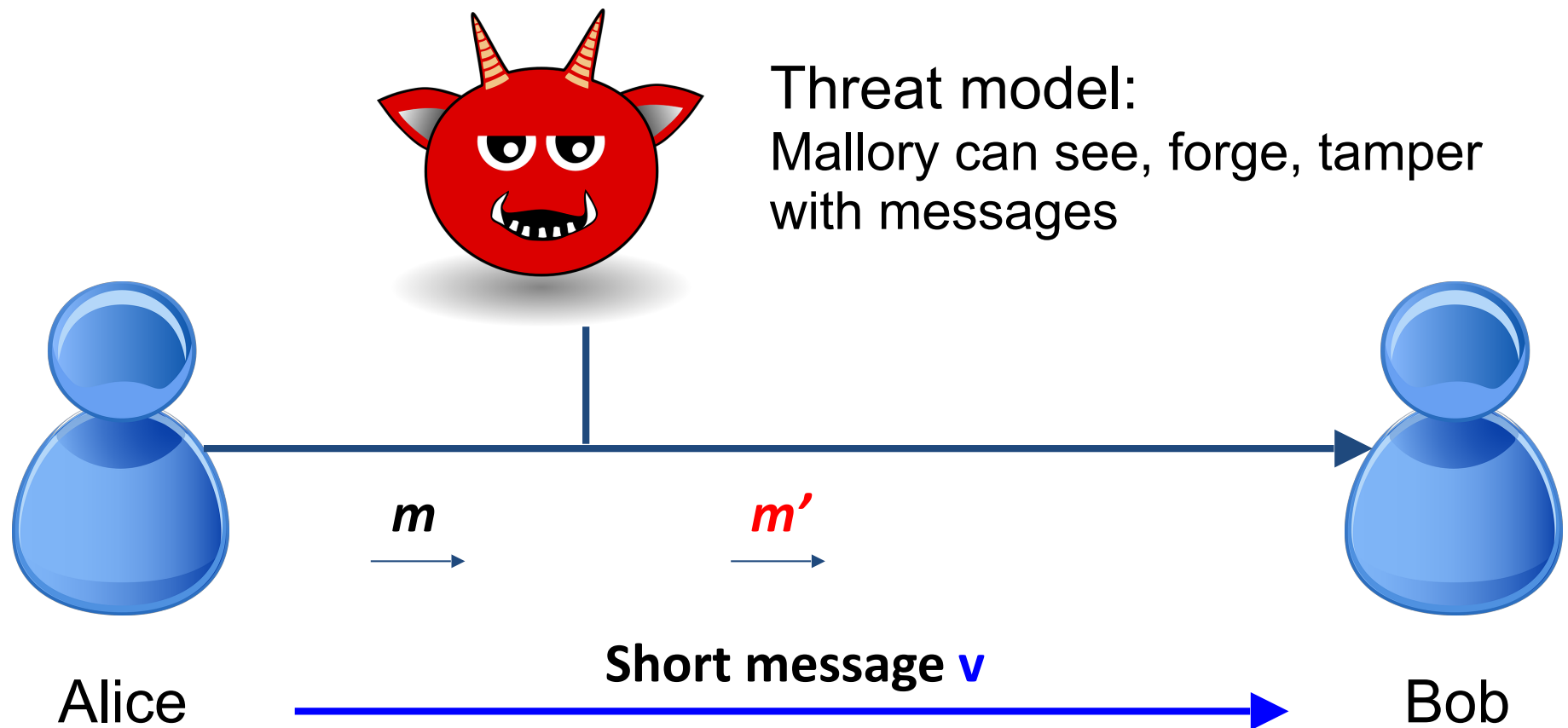
Alice wants to send file  $m$  to Bob (let's say, a 4 Gigabyte movie)  
Mallory wants to trick Bob into accepting a file Alice didn't send



# Goal: Secure File Transfer



Alice wants to send file  $m$  to Bob (let's say, a 4 Gigabyte movie)  
Mallory wants to trick Bob into accepting a file Alice didn't send



Setup assumption: *Securely transfer a short message!*

# Collision-Resistant Hash Function



Hash Function  $h: \{0,1\}^* \rightarrow \{0,1\}^{256}$  (or other fixed number)

1. Alice computes  $\mathbf{v} := h(\mathbf{m})$
2. Alice transfers  $\mathbf{v}$  over secure channel,  $\mathbf{m}$  over insecure channel



3. Bob verifies that  $\mathbf{v} = h(\mathbf{m}')$ , accepts file iff this is true

Function  $h$  ? We're sunk if Mallory can compute  $\mathbf{m}' \neq \mathbf{m}$   
where  $h(\mathbf{m}) = h(\mathbf{m}')$  *A collision!*

Contrast with: “checksums” e.g. CRC32.... defend against random errors, not a deliberate attacker!



Good hash functions should make it difficult to find ...

First pre-image:

given  $h(m)$ , find  $m$

Which of these properties  
implies which others?

Second pre-image:

given  $m_1$ , find  $m_2$  s.t.  $h(m_1) = h(m_2)$

Collision:

find *any*  $m_1 \neq m_2$  s.t.  $h(m_1) = h(m_2)$

# SHA256



`$ sha256sum file.dat` The **SHA256** compression function, **h**

Cryptographic hash

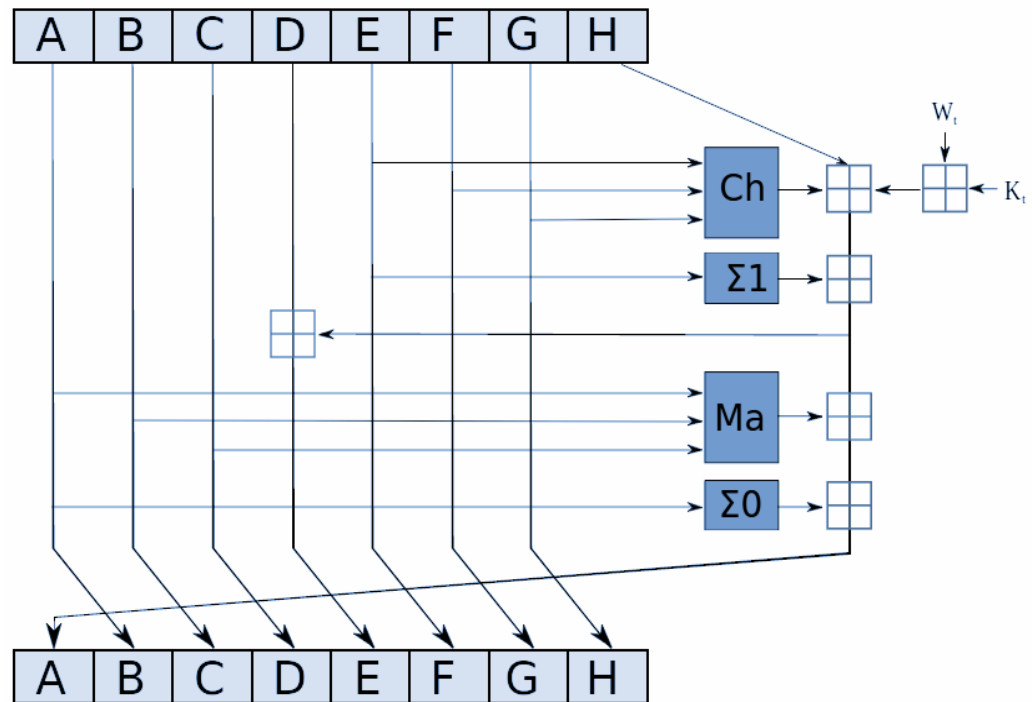
Input: arbitrary length data  
(No key)

Output: 256 bits

Built with compression  
function, ***h***

(256 bits, 512 bits) in →  
256 bits out

Designed to be really hairy  
(64 rounds of this)!



One iteration in a SHA-2 family compression function. The blue components perform the following operations:

$$\text{Ch}(E, F, G) = (E \wedge F) \oplus (\neg E \wedge G)$$

$$\text{Ma}(A, B, C) = (A \wedge B) \oplus (A \wedge C) \oplus (B \wedge C)$$

$$\Sigma_0(A) = (A \ggg 2) \oplus (A \ggg 13) \oplus (A \ggg 22)$$

$$\Sigma_1(E) = (E \ggg 6) \oplus (E \ggg 11) \oplus (E \ggg 25)$$

***Provides Confusion and  
Diffusion***

# How do you find a collision?



If Mallory finds an  $m'$  such that  $H(m') = v$ , they can swap  $m$  without  $m'$  without being detected... how to find?

- **Pigeonhole principle:** collisions must exist

Input space  $\{0,1\}^*$  larger than output  $\{0,1\}^{256}$

- **Birthday attack:** build a table with  $2^{128}$  entries

With  $\sim 50\%$  probability, have a collision

- **Cycle finding:** “Tortoise and hare” algorithm

$h(x), h(h(x)), h(h(h(x)), \dots, h^i(x)$

These are **generic**; actual attacks rely on **structure** of the particular function

# Birthday Problem



<i>n</i>	<i>Probability of two people sharing birthday</i>
1	0.0%
5	2.7%
10	11.7%
20	41.1%
23	50.7%
30	70.6%
40	89.1%
50	97.0%
60	99.4%
70	99.9%
75	99.97%
100	99.99997%



# Likelihood of Collision?



Most cryptographic primitives come with a **security parameter**... not the key, but an estimate of the difficulty of successful attack.

- Notation: Usually  $k$ , or  $\lambda$
- Often *corresponds* to a key size

Cryptography protocols run in **polynomial** time  
i.e., as a function of  $\lambda$ ,  $O(\text{poly}(\lambda))$

In a good cryptosystem, we should be able to show that the chance of failure is **negligible**, or **vanishingly** small, as a function of  $\lambda$ , i.e.,  $O(\text{negl}(\lambda))$



How large of a hash digest size should we choose?

## **1. Estimate an attacker's budget**

E.g., the entire NSA

## **2. Consider the best known attacks**

Reduction from protocol to well-studied problem

## **3. Add a safety margin**

If all goes well, adding 1 bit increases search space by 2x



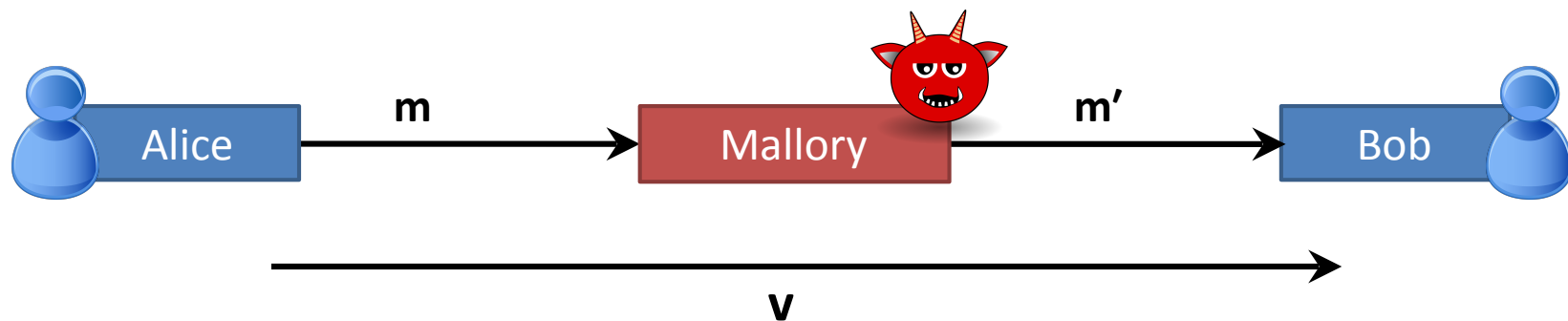
SHA512 provides a 512 bit output size. The security parameter  $k=512$  implies...

- $k/2$  bit collision resistance
- $k$  bit preimage resistance
- $k-N$  bit second preimage resistance
  - $N = \log(\text{target message length})$

# What's missing?



- In practice, we don't use a magically secure channel to send  $v$  over; it will have to traverse the same channel as  $m$



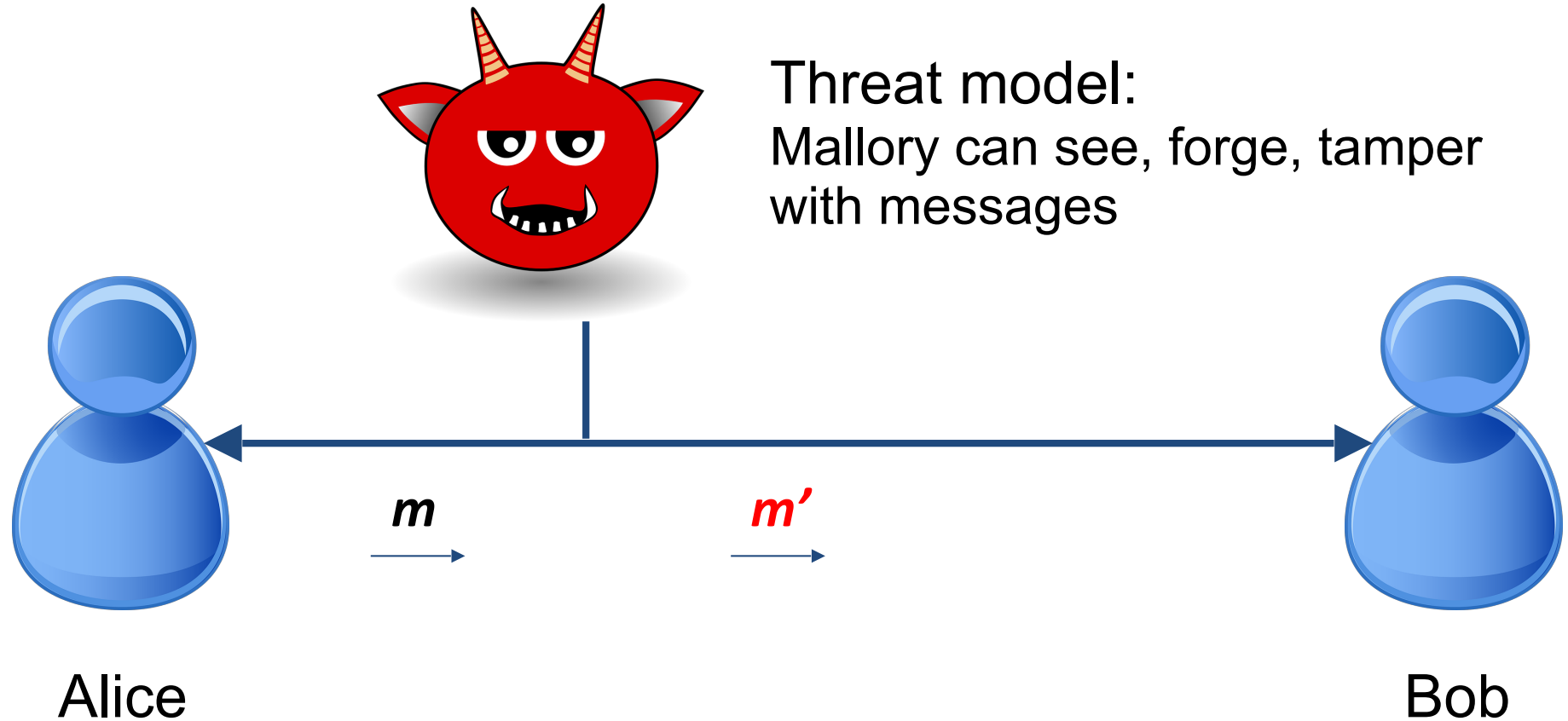
- If Mallory intercepts  $m$ ,  $v$ ; they can calculate a new  $v'$  to pass along with the tampered message  $m'$

# Message Integrity



Alice wants to send message  $m$  to Bob

Mallory wants to trick Bob into accepting a message Alice didn't send

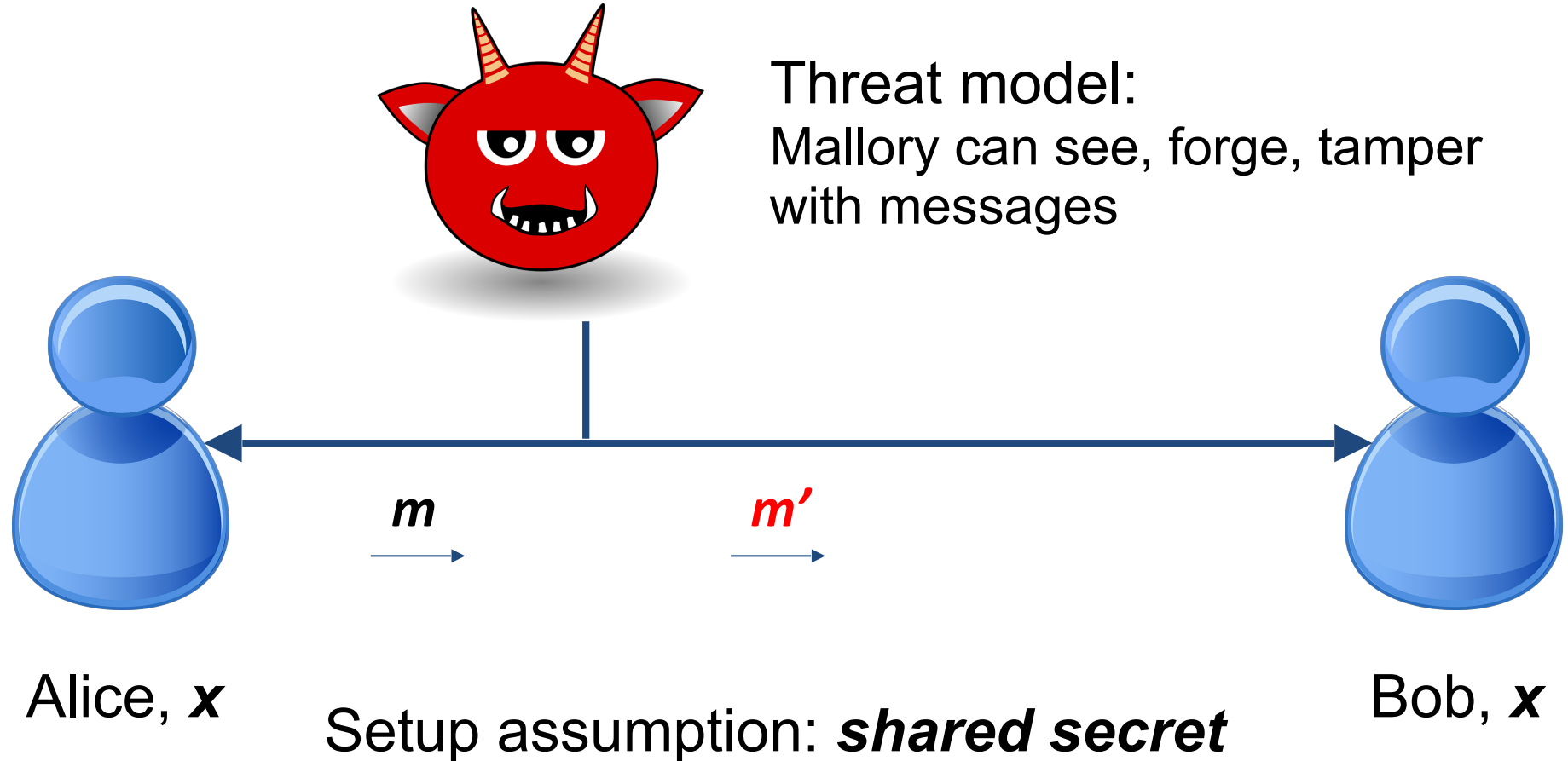


# Message Integrity



Alice wants to send message  $m$  to Bob

Mallory wants to trick Bob into accepting a message Alice didn't send



# Hash-Based MAC (HMAC)



## HMAC-SHA256

see RFC 2104

$\text{HMAC}_k(m) =$

$$\text{SHA256}\left(k \oplus c_1 \parallel \text{SHA256}(k \oplus c_2 \parallel m)\right)$$

Diagram illustrating the HMAC-SHA256 formula with annotations:

- An arrow points from the text "XOR" to the  $\oplus$  operator between  $k$  and  $c_1$ .
- An arrow points from the text "0x3636..." to the constant  $c_1$ .
- An arrow points from the text "Concatenation" to the  $\parallel$  operator between the inner SHA256 result and the outer SHA256 function.
- An arrow points from the text "0x5c5c..." to the constant  $c_2$ .

What are those two constant values for?

# Why not just use SHA256?



Is this a PRF?

$$f_k(m) = \text{SHA256}(k \parallel m)$$

- On its own, SHA256 is *not a strong PRF*
- **Length extension attacks:** If an attacker knows the value of  $m_1$  and the length of  $k$ , they can produce a valid hash for some message  $m_2$  that is an extension of  $m_1$ .
  - Problem for SHA2, addressed in SHA3 family.
- Can be used, e.g., to overwrite values for protocols that accept the last specified value for a field.

Original Data: `count=10&lat=37.351&user_id=1&long=-119.827&waffle=eggo`

Desired New Data:

`count=10&lat=37.351&user_id=1&long=-119.827&waffle=eggo&waffle=liege`





## Game against Mallory

1. Give Mallory  $\text{MAC}(k, m_i)$  for all  $m_i$  in  $M$

In other words, Mallory has an **oracle**

Mallory can choose next  $m_i$  after seeing answer

2. Mallory tries to discover  $\text{MAC}(k, m')$  for a new  $m'$  not in  $M$

We can show the **MAC game reduces** to the **PRF game**.  
Mallory wins MAC game  $\rightarrow$  she wins PRF game.

This is a **Security Proof**

# What is a security proof?



- A **reduction** from an **attack on your protocol** to an attack on a **widely studied, hard problem**
- Excludes large classes of attacks, guides **composition**
  - Proofs are in **models**. So, attack outside the model!
- It does **NOT prove** that your protocol is **secure**
- Also, we don't know if there are any hard problems!
- The field of **Modern Cryptography** is based on proofs
- Many widely used primitives (SHA-256, AES, DSA) have no security proof. We rely on them because they're widely studied

# ... so where do keys come from?



- To set up a MAC, we need a random key  $k$ ... how do we generate?
- **True Randomness:**
  - Output of a physical process that is inherently random
  - Scarce, and hard to get
- **Pseudorandom Function (PRF):**
  - Sampled from a family of functions using a key
- **Pseudorandom Generator (PRG):**
  - Takes small seed that is really random
  - Generates a stream (arbitrarily long sequence) of numbers that are “as good as random”

# Pseudorandom Generator



Definition: **PRG** is secure if it's indistinguishable from a random stream of bits

Similar game to PRF definition:

1. We flip a coin secretly to get a bit **b**
2. If **b**=0, let **s** be a truly random stream  
If **b**=1, let **s** be **g<sub>k</sub>** for random secret **k**
3. Mallory can see as much of the output of **s** as he/she wants
  1. Mallory guesses **b**,  
wins if guesses correctly

**g** is a secure PRG if no winning strategy for Mallory\*

# Pseudorandom Generator



Here's a *simple PRG that works*:

**For some random  $k$  and PRF  $f$ ,**

**output:  $f_k(0) \parallel f_k(1) \parallel f_k(2) \parallel \dots$**

**Theorem:** If  $f$  is a secure PRF, and  $g$  is built from  $f$  by this construction, then  $g$  is a secure PRG.

**Proof:** Assume  $f$  is a secure PRF, we need to show that  $g$  is a secure PRG.

Proof by contradiction:

1. Assume  $g$  is *not* secure; so Mallory can win the PRG game
2. This gives Mallory a winning strategy for the PRF game:
  - a. query the PRF with inputs 0, 1, 2, ...
  - b. apply the PRG-distinguishing algorithm
3. Therefore, Mallory can win PRF game; this is a contradiction
4. Therefore,  $g$  is secure

# How do we get true randomness?



Want “indistinguishable from random”  
which means: adversary can’t guess it

Gather lots of details about the computer that the adversary  
will have trouble guessing [\[Examples?\]](#)

Problem: Adversary can predict some of this

Problem: How do you know when you have enough randomness?

Modern OSes typically collect randomness, give you API calls  
to get it

e.g., Linux:

`/dev/random` a device that gives random bits, blocks until available

`/dev/urandom` gives output of a PRG, nonblocking, seeded from `/dev/random` *eventually*

# Review: Message Integrity



**Integrity** of message sent over an untrusted channel

Alice must append bits to message that only Alice (or Bob) can make

Idealized solution: Random function

Practical solution:



(Hash-based) MAC

$f_k$  is (we hope!) indistinguishable in practice from a random function, unless you know  $k$