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Outliers, Level Shifts, and Variance Changes in Time Series

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ABSTRACT

Outliers, level shifts, and variance changes are commonplace in applied time series analysis. However, their existence is often ignored and their impact is overlooked, for the lack of simple and useful methods to detect and handle those extraordinary events. The problem of detecting outliers, level shifts, and variance changes in a univariate time series is considered. The methods employed are extremely simple yet useful. Only the least squares techniques and residual variance ratios are used. The effectiveness of these simple methods is demonstrated by analysing three real data sets.

KEY WORDS Additive outlier ARIMA model Innovational outlier Intervention analysis Model change

Outliers and structure changes are commonly encountered in time series data analysis. The presence of those extraordinary events could easily mislead the conventional time series analysis procedure resulting in erroneous conclusions. The impact of those events is often overlooked, however, for the lack of simple yet useful methods available to deal with the dynamic behaviour of those events in the underlying series. The primary goal of this paper, therefore, is to consider unified methods for detecting and handling outliers and structure changes in a univariate time series. The outliers treated are the additive outlier (AO) and the innovational outlier (IO). The structure changes allowed for are level shift (LS) and variance change (VC). Level shift is further classified as permanent level change (LC) and transient level change (TC).

Several approaches have been considered in the literature for handling outliers in a time series. Abraham and Box (1979) used a Bayesian method, Martin and Yohai (1986) treated outliers as contamination generated from a given probability distribution, and Fox (1972) proposed two parametric models for studying outliers. Chang (1982) adopted Fox's models and proposed an iterative procedure to detect multiple outliers. In recent years, this iterative procedure has been widely used with encouraging results, see Chang and Tiao (1983), Hillmer, Bell and Tiao (1983), and Tsay (1986a). The methods mentioned above may be regarded as batch-type procedures for detecting outliers, because the full data set is used in detecting the existence of outliers. On the other hand, Harrison and Stevens (1976), Smith and West (1983), West, Harrison and Migon (1985) and West (1986) have considered sequential detecting methods for handling outliers. These sequential methods assume probabilistic models for outlier disturbances. The third method for handling outliers is the robust procedure advocated

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by Denby and Martin (1979). This approach is summarized in Martin and Yohai (1985). However, the study of Chang and Tiao (1983) shows that Denby and Martin's robust procedure is not powerful in handling innovational outliers. (Note that the effect of a single IO on estimation is usually negligible provided that the IO is not close to the end of the observational period. The effect of multiple IOs, however, could be serious. See Chang (1982).) There is no comparison available between the batch-type and the sequential procedures in handling outliers. The probabilistic treatment has its appeal but may not be easy to implement as it requires prior information of the underlying model to begin with. Since level shifts and variance changes are also considered, the approach of Chang and Tiao (1983) and Tsay (1986a) is adopted and generalized in this paper.

Level shift or drift in time series was investigated by Box and Tiao (1965). Their approach was generalized by Chen and Tiao (1986) by making use of the iterative method of Chang and Tiao (1983). Recently, Tsay (1986b) extended the method further to the case of a transient change in level. Variance change in first-order autoregressive models was considered by Wichern, Miller and Hsu (1976). These authors studied the effect of variance change on parameter estimation and suggested checking for variance change by examining the residual variances of approximately every 20 successive residuals. Hsu (1977) investigated tests for variance change when the time point of change is unknown.

In this paper, the batch-type techniques for detecting outliers and level shifts mentioned earlier are explored further and the techniques to cover the case of variance changes are extended. Thus, outliers, level shifts and variance changes are treated in a unified manner. This is achieved by adopting a parametric approach to describe the exogenous disturbances. Under the parametric model, impact of a disturbance is measured by a parameter which can be estimated and tested under certain mild conditions. Two iterative procedures to detect and handle outliers and model changes when the number and time points of disturbances are unknown are recommended. These iterative procedures are based on that of Chang (1982) and consist of specification-estimation-detecting-removing cycles to handle one-by-one the most significant disturbance. Real examples show that the proposed methods are effective and promising. They can identify extraordinary observations in a time series so that a thorough and careful analysis can be carried out.

The paper is organized as follows. Section 1 gives various parametric models for modeling outliers and structure changes. Section 2 discusses methods for estimating the impact parameters and for evaluating the significance of the parameter estimates. Section 3 considers two iterative procedures for detecting outliers and model changes in a time series, and Section 4 illustrates the proposed methods by analysing three real examples. Finally, some further use and remarks of the proposed methods are given in Section 5.

1. THE MODEL

The basic univariate time series model of this paper is the Gaussian autoregressive moving average (ARMA) model

$$\Phi(B)Z_t = \theta_0 + \theta(B)a_t \tag{1}$$

where $\Phi(B) = 1 - \Phi_1 B - \cdots - \Phi_p B^p$ and $\theta(B) = 1 - \theta_1 B - \cdots - \theta_q B^q$ are polynomials in B of degrees p and q, respectively, θ_0 is a constant, B is the backshift operator such that $BZ_t = Z_{t-1}$, and $\{a_t\}$ is a sequence of independent Gaussian variates with mean zero and variance σ_a^2. In model (1), it is assumed that all of the zeros of $\Phi(B)$ and $\theta(B)$ are on or outside the unit circle and that $\Phi(B)$ and $\theta(B)$ have no common factors. In addition, if some of the zeros of $\Phi(B)$ are on the unit circle, it is assumed that the series starts at a fixed time point with given starting values.

When outliers or structure changes are present, Z_t is disturbed and unobservable. In this case, it is assumed that the observed series $\{Y_t\}$ follows the model

$$Y_t = f(t) + Z_t \tag{2}$$

where $f(t)$ is a parametric function representing the exogenous disturbances of Z_t such as outliers or level changes. The function $f(t)$ may be deterministic or stochastic depending on the types of disturbances. In practice, $f(t)$ is specified by data analysts based on the substantive information of the disturbances and the process Y_t. For example, it may be a linear function of some exogenous variables such as the trading day and holiday effects in seasonal time series analysis, see Bell and Hillmer (1983). In what follows, the case of a single disturbance is considered in detail; the case of multiple disturbances can be treated exactly in the same manner through iteration, see the illustrative examples in Section 4.

For the deterministic model, it is assumed that $f(t)$ is of the form

$$f(t) = \omega_0 \frac{\omega(B)}{\delta(B)} \xi_t^{(d)} \tag{3}$$

where $\xi_t^{(d)} = 1$ if $t = d$ and $= 0$ if $t \neq d$ is an indicator variable signifying the occurrence of a disturbance at the time point d, $\omega(B) = 1 - \omega_1 B - \cdots - \omega_s B^s$ and $\delta(B) = 1 - \delta_1 B - \cdots - \delta_r B^r$ are polynomials in B of degrees s and r, respectively, and ω_0 is a constant denoting the initial impact of the disturbance. This $f(t)$ belongs to the intervention model of Box and Tiao (1975) and is a general form which can be tailored to describe many dynamic disturbances of a time series. Four special cases of (3) will be used later. Note that for deterministic $f(t)$, only the expectation of Z_t is disturbed. For the stochastic case, $f(t)$ assumed the form

$$f(t) = \omega_0 \frac{\omega(B)}{\delta(B)} e_t^{(d)} \tag{4}$$

where $e_t^{(d)} = 0$ if $t < d$ and $\{e_t^{(d)} \mid t \geqslant d\}$ is a sequence of independent and identically distributed random variables with mean zero and variance σ_e^2. This $f(t)$ may affect both the model and the variance of Z_t. $e_t^{(d)} = a_t$ for $t \geqslant d$ is used below to handle variance changes in Z_t.

Five special cases of $f(t)$ are considered in this paper. The first four are from equation (3) and the last one from (4).

(a) $\omega_0 = \omega_I$ and $\omega(B)/\delta(B) = \theta(B)/\Phi(B)$: This is the innovational outlier (IO) model considered by Fox (1972), Martin (1980), Chang and Tiao (1983), Hillmer *et al.* (1983), Hillmer (1984), and Tsay (1986a). It represents an outlier in the innovation series $\{a_t\}$ and has a dynamic effect on Z_t by propagating through the π-weight $\pi(B) = 1 - \pi_1 B - \pi_2 B^2 - \cdots = \Phi(B)/\theta(B)$ of Z_t. This outlier thus affects Z_t for every $t \geqslant d$. In practice, an IO often represents the onset of an external cause.

(b) $\omega_0 = \omega_A$ and $\omega(B)/\delta(B) = 1$: This is the additive outlier (AO) model discussed in the aforementioned papers. This outlier only affects Z_d. A typical example of AO is the recording error.

(c) $\omega_0 = \omega_L$ and $\omega(B)/\delta(B) = 1/(1 - B)$: This is a level change (LC) model because $Y_t = Z_t$ for $t < d$ but $Y_t = Z_t + \omega_L$ for $t \geqslant d$. The model says that a level shift of magnitude ω_L occurs at time $t = d$ and the change is permanent. See Box and Tiao (1965) and Chen and Tiao (1986).

(d) $\omega_0 = \omega_T$ and $\omega(B)/\delta(B) = 1/(1 - \delta B)$ where $0 < \delta < 1$: This model describes a disturbance that affects Z_t for all $t \geqslant d$. However, the effect decays exponentially with rate δ and an initial impact ω_T. Since $0 < \delta < 1$, the effect disappears eventually. For this reason, this model is referred to as a transient change (TC) model, see Tsay (1986b).

(e) $\omega_0 = \omega_V$, $\omega(B)/\delta(B) = \theta(B)/\Phi(B)$, and $e_t^{(d)} = a_t$ for $t \geqslant d$: This is a variance change (VC) model. Let $b_t = [\theta(B)]^{-1}\Phi(B)_t$. Then, $b_t = a_t$ for $t < d$, but $b_t = (1 + \omega_V)a_t$ for $t \geqslant d$. Consequently, the variance of b_t changes from σ_a^2 to $(1 + \omega_V)^2\sigma_a^2$ at the time point d. Without loss of generality, it is assumed that $-1 < \omega_V < \infty$.

In practice, a combination of the above special cases can be used to model the general response pattern of an exogenous disturbance. For instance, a combination of (c) and (d) allows for a disturbance to have both permanent and transient effects. Furthermore, there are many other choices of $\omega(B)$ and $\delta(B)$ available. For example, $\omega(B)/\delta(B) = (1 - \omega_1 B)/[(1 - B)(1 - \delta B)]$ with $0 < \delta < 1$ allows for a gradual response before reaching a new permanent level and $\omega(B)/\delta(B) = 1/(1 + B + \cdots + B^{11})$ denotes a permanent change in seasonal pattern of a monthly series. For simplicity, the above five special cases are considered and the use of these simple models to detect and remove disturbances of a time series are concentrated on.

2. PARAMETER ESTIMATION AND HYPOTHESIS TESTING

2.1. Time series parameters are known

To use model (2), it is necessary to estimate the impact parameters ω_0. This can be done by using the techniques of simple linear regression if the time series parameters Φ_i's, ϕ_j's, and d are known and $f(t)$ is deterministic. See Chang (1982) and Hillmer *et al.* (1983). Define

$$y_t = \frac{\Phi(B)}{\theta(B)} Y_t, \quad x_t = \frac{\Phi(B)}{\theta(B)} \frac{\omega(B)}{\delta(B)} \xi_t^{(d)}. \tag{5}$$

Then, models (2) and (3) imply that

$$y_t = \omega_0 x_t + a_t \tag{6}$$

which is precisely a simple linear regression equation. Therefore,

$$\hat{\omega}_0 = \frac{\sum_{t=1}^n y_t x_t}{\sum_{t=1}^n x_t^2} \quad \text{and } \operatorname{Var}(\hat{\omega}_0) = \frac{\sigma_a^2}{\sum_{t=1}^n x_t^2}, \tag{7}$$

where n is the sample size. Using this simple technique, one obtains the estimates of the impact parameter for the first four special cases of Section 1.

(a) IO case: $\hat{\omega}_{I,d} = y_d$ and $\operatorname{Var}(\hat{\omega}_{I,d}) = \sigma_a^2$.
(b) AO case: $\hat{\omega}_{A,d} = \rho_{A,d}^2(y_d - \sum_{i=1}^{n-d} \pi_i y_{d+i})$ and $\operatorname{Var}(\hat{\omega}_{A,d}) = \rho_{A,d}^2\sigma_a^2$ where π_i's are the π-weights of Z_t and $\rho_{A,d}^2 = (1 + \pi_1^2 + \cdots + \pi_{n-d}^2)^{-1}$.
(c) LC case: $\hat{\omega}_{L,d} = \rho_{L,d}^2(y_d - \sum_{i=1}^{n-d} \eta_i y_{d+i})$ and $\operatorname{Var}(\hat{\omega}_{L,d}) = \rho_{L,d}^2\sigma_a^2$ where η_i is the coefficient of B^i in the polynomial $\eta(B) = \eta_0 - \eta_1 B - \cdots = \pi(B)/(1 - B)$ and $\rho_{L,d}^2 = (1 + \eta_1^2 + \cdots + \eta_{n-d}^2)^{-1}$. This case was proposed by Chen and Tiao (1986).
(d) TC case: $\hat{\omega}_{T,d} = \rho_{T,d}^2(y_d - \sum_{i=1}^{n-d} \beta_i y_{d+i})$ and $\operatorname{Var}(\hat{\omega}_{T,d}) = \rho_{T,d}^2\sigma_a^2$ where β_i is the coefficient of B^i in the polynomial $\beta(B) = \beta_0 - \beta_1 B - \cdots = \pi(B)/(1 - \delta B)$, $\rho_{T,d}^2 = (1 + \beta_1^2 + \cdots + \beta_{n-d}^2)^{-1}$, and δ is a prespecified constant satisfying $0 < \delta < 1$.

Based on the above results, one may construct test statistics for testing the existence of an outlier or level change at the time point d. The hypotheses are (i) H_0: No disturbance at Z_d, (ii) H_I: An IO at Z_d, (iii) H_A: An AO at Z_d, (iv) H_L: A LC at Z_d, and (v) H_T: A TC at Z_d. And the likelihood ratio test statistics are

(a) Existence of an IO, H_I vs H_0: $\lambda_{I,d} = \dfrac{\hat{\omega}_{I,d}}{\sigma_a}$ (8)

(b) Existence of an AO, H_A vs H_0: $\lambda_{A,d} = \dfrac{\hat{\omega}_{A,d}}{\rho_{A,d}\sigma_a}$ (9)

(c) Existence of a LC, H_L vs H_0: $\lambda_{L,d} = \dfrac{\hat{\omega}_{L,d}}{\rho_{L,d}\sigma_a}$ (10)

(d) Existence of a TC, H_T vs H_0: $\lambda_{T,d} = \dfrac{\hat{\omega}_{T,d}}{\rho_{T,d}\sigma_a}$ (11)

Under the null hypothesis H_0 and the assumption of knowing the time series parameters and d, all of the test statistics are distributed as N(0, 1).

For the variance change model, define

$$b_t = \frac{\Phi(B)}{\theta(B)} Y_t. \tag{12}$$

Then, models (2) and (4) give

$$b_t = \begin{cases} a_t & \text{if } t < d \\ a_t[1 + \omega_V] & \text{if } t \geqslant d \end{cases}. \tag{13}$$

Consequently, the variance ratio of b_t before and after the time point d is

$$\hat{r}_d = \frac{(d-1)\sum_{t=d}^{n} b_t^2}{(n-d+1)\sum_{t=1}^{d-1} b_t^2} \tag{14}$$

where it is understood that $(d-1) > 0$ and $(n-d+1) > 0$. This ratio is an estimate of $(1+\omega_V)^2$ and is the likelihood ratio test statistic of a variance change at the given time point d under the assumption of normality. Under the null hypothesis of no variance change, $\hat{r}_d$ has a central F-distribution with degrees of freedom $(n-d+1)$ and $(d-1)$. This test is the most powerful test for step-change in variance if the time point d is *known*. When d is unknown, many other tests can be constructed, see Hsu (1977). None of the tests, however, has a closed form distribution when the sample size is greater than 2.

2.2. Time series parameters are unknown

In practice, the time series parameters are estimated from the data and the unknown parameters of the aforementioned test statistics, e.g. $\rho_{A,d}$ and σ_a, are replaced by some consistent estimates. Under the null hypothesis of no exogenous disturbances, the maximum likelihood estimates (MLE) are consistent and may be employed to obtain asymptotic justification for the use of those test statistics. More specifically, the test statistics of (8)–(11) are asymptotically N(0, 1) under the null hypothesis H_0 and the assumption that d is known. The consistency of MLE for stationary ARMA models is well known whereas that of the nonstationary case can be obtained by using the results of Tiao and Tsay (1983) and Tsay and Tiao (1984, 1986). Roughly speaking, the nonstationary factor of Z_t causes no difficulty in the consistency properties of the estimates because the convergence rate of the nonstationary parameters is faster than that of the stationary parameters.

3. DETECTING OUTLIERS, LEVEL SHIFTS AND VARIANCE CHANGES

In applications the ARMA model of Z_t, the number of disturbances, and the corresponding time points are unknown. Therefore, two iterative procedures for detecting and modeling the disturbances are considered. The first procedure is for variance change and the second for outlier and level shift. The second procedure is a combined version of those of Chang and Tiao (1983) and Tsay (1986a). It consists of *specification, estimation, detection and removal* cycles to build a time series model in the presence of exogenous disturbances. In each iteration, the maximum of a given test statistic is selected as the candidate for that type of disturbance, and the grand maximum across the tests is identified as the most likely exogenous disturbance. This grand maximum is then compared with a prespecified critical value so that the existence of an exogenous disturbance can be judged.

The first procedure is based on the same idea as the second one, but modifications are made to handle the variance change. For convenience, this procedure is referred to as *Procedure V*. It consists of the following steps.

Step 1. Assuming that there are no variance changes, specify an ARMA model for the observed series Y_t and obtain parameter estimates and residuals of the specified model.

Step 2. Compute the variance ratio using the residuals of Step 1 as estiamtes of the b_t's and obtain the minimum and maximum of the variance ratio, say $\lambda_{V,\min}$ and $\lambda_{V,\max}$ with

$$\lambda_{V,\min} = \mathrm{Min}\{\hat{r}_d \mid h \leqslant d \leqslant n-h\} \text{ and}$$
$$\lambda_{V,\max} = \mathrm{Max}\{\hat{r}_d \mid h \leqslant d \leqslant n-h\},$$

where h is a prespecified positive integer denoting the minimum number of residuals used to estimate the variance at the beginning and the end of the data set.

Step 3. Let $\lambda_V = \mathrm{Max}\{\lambda_{V,\max}, \lambda_{V,\min}^{-1}\}$ and compare λ_V with a prespecified critical value C. If $\lambda_V < C$, there is no significant variance change and the iteration is terminated. On the other hand, if $\lambda_V \geqslant C$, a variance change is detected. The time point of the change is the one at which λ_V occurs, say d_0.

Step 4. Adjust the variance change by constructing a modified process Y_t^* as

$$Y_t^* = \begin{cases} Y_t & \text{if } t < d_0 \\ \bar{Y} + \lambda_V^{-1/2}(Y_t - \bar{Y}) & \text{if } t \geqslant d_0 \end{cases}$$

where $\bar{Y}$ is the sample mean of Y_t. Then, go to Step 1 and treat Y_t^* as the observed process.

The critical value C of Step 3 may assume the value 3.5, 3.0, or 2.5. This is based on some simulation results shown in Table 1 which provides empirical percentiles of $\lambda_{V,\max}$ and $\lambda_{V,\min}^{-1}$ for Gaussian white noise and AR(1) processes. The results are based on 5000 replications and the sample sizes considered are 100, 200, and 300. In the AR(1) case, the ordinary least squares estimate was used in the simulation.

For the deterministic disturbances, there are four testing statistics involved and the iteration is divided into outer and inner loops, see Chang (1982) and Hillmer, *et al.* (1983). This modification simplifies the computation by reducing the number of times to estimate the time series parameters. This procedure is referred to as *Procedure $\dot{M}$* because its goal is to detect disturbances that affects the mean level of Z_t.

Table 1. Empirical percentiles of the λ statistic based on 5000 replications

	Sample Size					
	100		200		300	
Percentile	$\lambda_{\max}$	$\lambda_{\min}^{-1}$	$\lambda_{\max}$	$\lambda_{\min}^{-1}$	$\lambda_{\max}$	$\lambda_{\min}^{-1}$
(a) White Noise Series						
90%	2.33	2.53	2.19	2.42	2.22	2.38
95%	2.67	2.93	2.52	2.97	2.60	2.78
97.5%	3.14	3.36	2.91	3.26	3.01	3.24
99%	3.89	4.07	3.48	3.98	3.61	3.92
99.5%	4.30	4.63	3.96	4.44	4.19	4.44
(b) AR(1) Processes with $\Phi = 0.8$						
90%	2.34	2.51	2.25	2.41	2.24	2.36
95%	2.77	2.93	2.67	2.77	2.62	2.70
97.5%	3.21	3.46	3.14	3.23	3.13	3.10
99%	3.93	4.24	3.95	3.87	3.90	3.68
99.5%	4.49	4.81	4.69	4.44	4.38	4.08

Outer Iteration:

Step A. Assuming that there are no disturbances, specify an ARMA model for the observed series Y_t and obtain parameter estimates of the specified model. In addition, select a critical value C.

Inner Iteration:

Step A1. Using the parameter estimates of Step A and the entertained series Y_t, compute the residuals and the residual variance. Here the residual corresponds to y_t of (5) and the residual variance is used as an estimate of σ_a^2 of (7).

Step A2. Using the residuals and residual variance of Step A1 and the time series parameters of Step A, compute the test statistics of (8)–(11) for every time point t. Then, for each test statistic, locate its maximum in absolute value and identify the corresponding time point. For instance, consider the IO case. Define

$$\lambda_{I,\max} = \text{Max}\{|\lambda_{I,t}|\,|\,1 \leqslant t \leqslant n\}$$

and let d_I be the time point where $\lambda_{I,\max}$ occurs. Likewise, $\lambda_{A,\max}$ and d_A, $\lambda_{L,\max}$ and d_L, and $\lambda_{T,\max}$ and d_T are all defined in the same manner.

Step A3. Let λ be the maximum of $\{\lambda_{I,\max}, \lambda_{A,\max}, \lambda_{L,\max}, \lambda_{T,\max}\}$. Compare λ with the critical value C. If $\lambda < C$, no significant disturbance is found and go to Step B. On the other hand, if $\lambda \geqslant C$, a disturbance is detected. In this case, the type, the time point, and the impact parameter ω_0 of the identified disturbance are determined by those of the grand maximum λ.

Step A4. Remove the effect of the identified disturbance by subtracting from Y_t the dynamic impact as computed from the initial magnitude $\hat{\omega}_0$ and the dynamic weights
$[\omega(B)/\delta(B)]\xi_t^{(d)}$. Denote the adjusted series by Y_t^* and go to Step A1 by treating Y_t^* as the entertained series.

Step B. If there is no significant disturbance found in the inner iteration, stop the procedure. Otherwise, go to Step A with Y_t^* as the observed series.

Some remarks are in order here. First, the critical value C now may assume the value 4.0, 3.5, or 3.0. These three values were selected based on some simulation results of Chang (1982) and have provided satisfactory results, see Chang and Tiao (1983) and Tsay (1986a). The exact distributions of the statistics used, e.g. $\lambda_{A,\max}$ are intractable as they are the maximum of *dependent* variables. Some research, however, has been done to obtain good approximations of the percentiles of those statistics, e.g. Atwood (1986). Second, the computation of Step A2 can be carried out recursively for $t = 1, \ldots, n$ because all of the modifications needed from t to $t+1$ only involve the observation Y_{t+1}. Third, it is possible to construct likelihood ratio tests to distinguish the type of disturbance in Step A3, e.g. see Chang (1982) and Muirhead (1986). The method adopted here, however, has two advantages: it agrees with common sense by considering the most significant disturbance as the one that ought to be treated first, and it reduces substantially the number of hypothesis tests. Furthermore, experience shows that the method performs well in analysing real data sets, see Chang and Tiao (1983) and Tsay (1986a). Finally, it is possible to detect more than one disturbance within an inner iteration. However, experience suggests that the techniques of detecting one-at-a-time work nicely in applications. An alternative is to derive test statistics for multiple disturbances, but the possible combinations of multiple disturbances would then be substantial.

To provide further informaiton on the detection methods used in Procedure M, Table 2 shows some simulation results concerning the *correct* specification of the disturbance. The models used in the study are

$$Y_t = \omega_0 \xi_t^{(50)} + \frac{1}{1-\Phi B} a_t \quad (AO)$$

$$Y_t = \omega_0 \frac{1}{1-\Phi B} \xi_t^{(50)} + \frac{1}{1-\Phi B} a_t \quad (IO) \tag{15}$$

where $\sigma_a^2 = 1$. Four values of Φ and three values of ω_0 were employed. These results are the frequencies of detected disturbances based on 1000 replications each with 100 observations. For each series, 400 data points of the underlying AR(1) model were generated but only the last 100 observations were used in the study. The AR parameter is estimated by the ordinary least squares, and the critical value is $C = 3.5$. For TC, $\delta = 0.8$ is used in the detection. From parts (a) and (b) of the table, the detection methods work very well for most of the cases. There are some misclassifications between IO and TC when $\Phi = 0.95$ and 0.6. This type of misclassification, however, is not a serious problem because the TC model used is

$$Y_t = \omega_0 \frac{1}{1-0.8B} \xi_t^{(50)} + \frac{1}{1-\Phi B} a_t$$

which is very close to an IO model (15) when $\Phi = 0.95$ or 0.6. Similarly, the possibility of misclassifying an AO as an IO in the case of $\Phi = -0.5$ is understandable because for small $\hat{\Phi}$ the difference between an AO and an IO is rather minor when the sample size is not large. (Note that AO and IO are equivalent if the underlying time series is white noise.) Part (c) of Table 2 gives the frequencies of overspecification in detection. For the four AR(1) models considered the probabilities of overspecifying outliers or level shifts are quite small. The maximum probability of overspecification is only 3.6 per cent.

Table 2. Empirical frequencies of detected disturbances. The results are based on 1000 replications each with 100 observations. The underlying time series model is AR(1) with a disturbance at $t = 50$. The critical value used is $C = 3.5$, and for TC, $\delta = 0.8$

ω_0	3				4				5			
Φ	IO	AO	LC	TC	IO	AO	LC	TC	IO	AO	LC	TC
(a) The IO Case												
0.95	114	41	18	89	337	48	24	199	576	39	22	261
0.6	149	69	1	57	420	93	0	104	684	84	0	123
−0.5	220	47	0	11	515	73	0	8	802	85	0	2
−0.9	215	33	0	7	543	63	2	7	837	39	0	0
(b) The AO Case												
0.95	41	563	8	57	15	913	0	27	3	987	0	9
0.6	76	313	0	15	96	714	0	9	90	892	0	3
−0.5	107	265	0	14	158	580	0	14	130	812	0	6
−0.9	60	541	0	25	51	871	0	10	16	980	0	4

(c) No Exogenous Disturbances

Φ	0.95				0.6				−0.5				−0.9			
	IO	AO	LC	TC	IO	AO	LC	TC	IO	AO	LC	TC	IO	AO	LC	TC
Frequency	22	36	8	15	24	22	1	10	19	16	2	6	22	29	0	7

4. ILLUSTRATIVE EXAMPLES

To illustrate the proposed procedure, three real data sets are considered in this section. These three series all have been analysed in the literature. In what follows, the estimation is done by using the package of Scientific Computing Associates (SCA) and the detection is carried out by two Fortran routines developed by the author. The SCA has the capability of detecting IO, AO and LS but it cannot handle TS and VC at this moment.

Example 1. Consider the series of total monthly air-passager-miles within the United States from January 1960 through December 1977. The series consists of 216 observations and was listed and analysed in Cryer (1986). A multiplicative seasonal ARMA $(0, 1, 2) \times (0, 1, 1)_{12}$ with coefficients (standard deviations) $\hat{\theta}_1 = 0.284$ (0.069), $\hat{\theta}_2 = 0.234$ (0.069), and $\theta_{12} = 0.737$ (0.053) and residual variance $\hat{\sigma}_a^2 = 0.346 \times 10^{-2}$ was suggested there for the logged series. The log transformation was used to stabilize the variance. Cryer noticed that there are possible outliers in the series and the normal probability plot of residuals shows deviation from normality. He, however, did not refine the model.

Figure 1(a) plots the logged series Y_t and Figure 1(b) shows the differenced series $X_t = (1 - B)(1 - B^{12})Y_t$. Some unusual observations are clearly seen in Figure 1(b). Table 3 gives the detection results of Procedure M for the logged series Y_t. It took two outer iterations each with several inner iterations to identify two permanent level changes, four transient level changes, and four additive outliers. Figure 2(a) plots the residuals of the adjusted series of Table 3 and Figure 2(b) shows the associated normal probability plot. No model inadequacy emerges from the plots. The conventional diagnostics such as Box–Ljung Q-statistic also fail to reject the model. Procedure V is then applied but no significant variance changes are detected.

For this series, the proposed procedure reveals several interesting features of the data. First, there is a permanent level change at $t = 121$. This corresponds to January of 1970. A check on

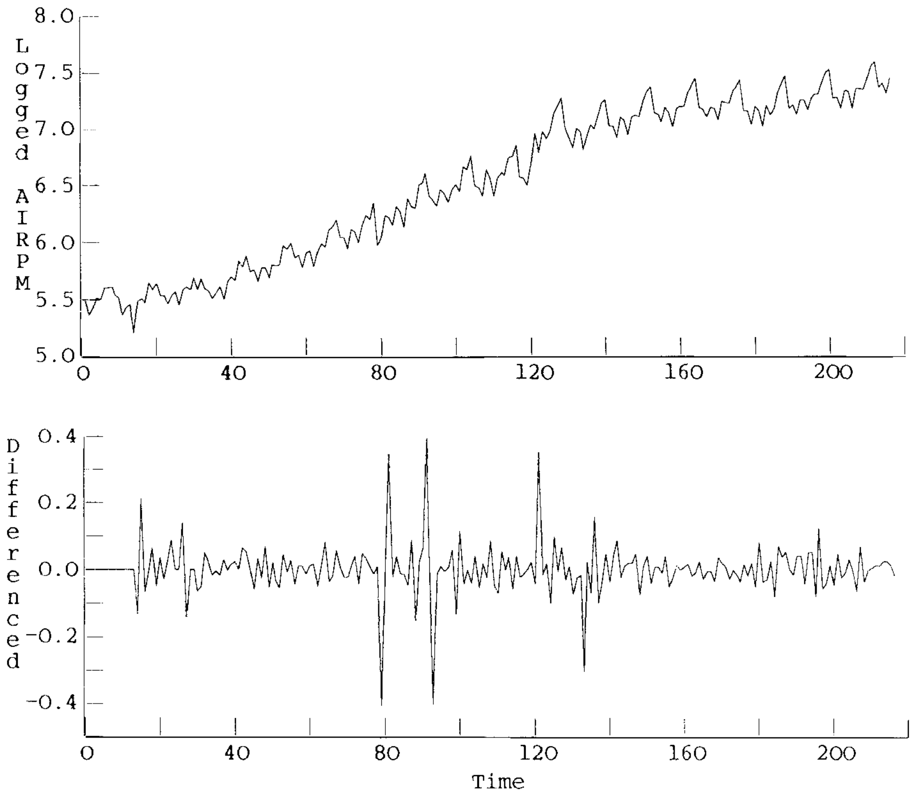

Figure 1. Plots of logged air-passenger-miles. (a) Original series and (b) first and seasonal differenced series

Table 3. Summary results of procedure M for air-passenger-miles. For TC, $\delta = 0.8$ was used. The critical values used are 4.0, 3.5 and 3.5 for the three outer iterations, respectively. The numbers in parentheses are standard errors and MSE is the residual mean squared error

Outer Iteration	Inner Iteration	MA Parameters θ_1	θ_2	θ_{12}	Type	Time	Magni.	MSE $\times 10^2$
I		0.285(0.068)	0.232(0.068)	0.744(0.047)				0.3498
	1				TC	79	−0.3491	0.2935
	2				LC	121	0.3048	0.2290
	3				TC	81	0.2781	0.1794
	4				AO	14	−0.1566	0.1579
II		0.481(0.061)		0.520(0.057)				0.1277
	1				AO	87	0.0965	0.1244
	2				AO	124	−0.0969	0.1161
	3				TC	31	−0.0959	0.1087
	4				LC	130	−0.0906	0.1017
	5				AO	184	−0.0867	0.0952
	6				TC	80	−0.0856	0.0893
III		0.498(0.061)		0.425(0.060)				0.0847
	1						Insignificant	

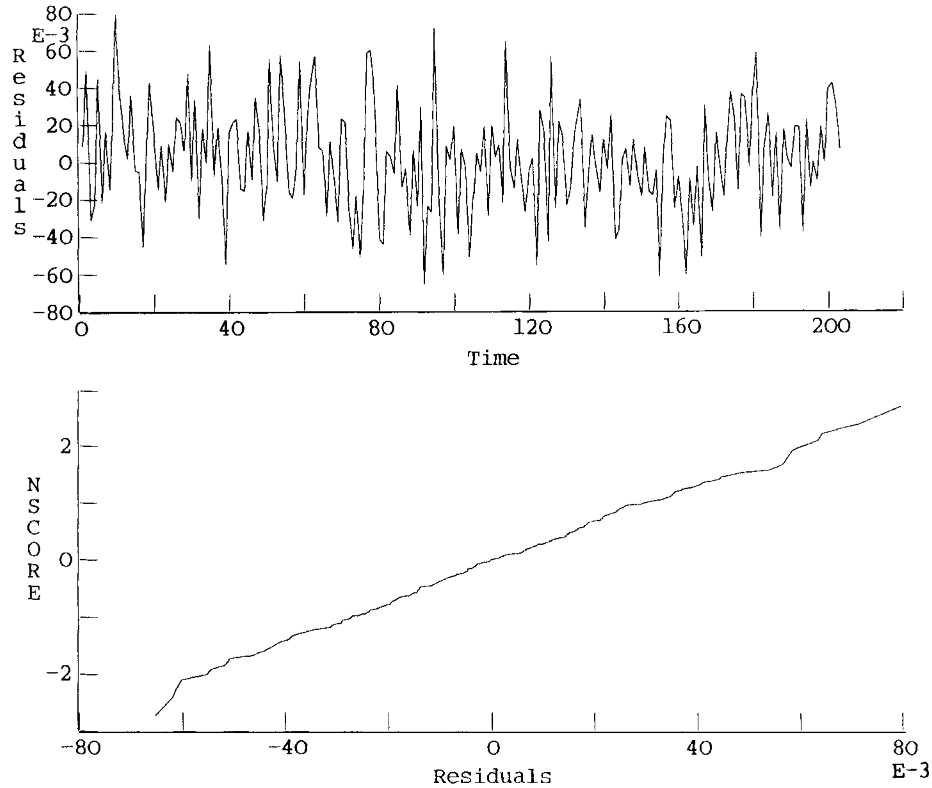

Figure 2. Residual analysis of logged air-passenger-miles. (a) Residual plot and (b) residual normal probability plot

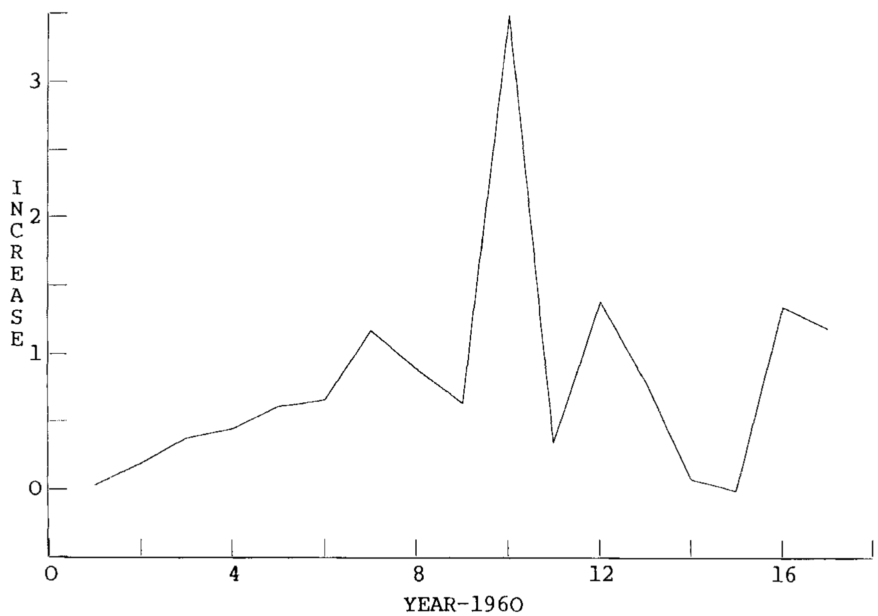

Figure 3. Increase of annual average of air-passenger-miles

the data or the data plot of Figure 1(a) shows that there was a big jump in the mean level from 1969 to 1970. Figure 3 shows the increase in annual average of the series as measured in the original scale. The increase from 1969 to 1970 is 3.495 while the second largest increase from 1971 to 1972 is only 1.38. Second, there were successive large disturbances at $t = 79$, 80, and 81 indicating that some external event occurred around these time points, see Figure 1(b) for further evidence. Third, the apparently significant MA(2) coefficient in Cryer's model disappears after the first outer iteration. This clearly demonstrates that outliers and level changes can easily distort model specification for the underlying model. In this particular instance, it would be unwise to believe that the MA(2) coefficient is significant because it depends solely on disturbances.

Discussions:

Overfitting: From Table 3, many disturbances are identified by Procedure M giving rise to the possibility of overfitting. An intervention model with the detected disturbances was estimated simultaneously to check for the overfitting. In the model, the three successive TCs at $t = 79$, 80 and 81 are treated as a single intervention and the δ of the remaining TC at $t = 31$ is regarded as a unknown parameter. The model is

$$\begin{aligned} Y_t = {} & \frac{0.313}{1-B}\xi_t^{(121)} - \frac{0.089}{1-B}\xi_t^{(130)} - \frac{0.091}{1-0.774B}\xi_t^{(31)} \\ & - 0.146\xi_t^{(14)} + 0.096\xi_t^{(87)} - 0.103\xi_t^{(124)} - 0.089\xi_t^{(184)} \\ & - \frac{0.397 - 0.378B^2}{1-B}\xi_t^{(79)} + \frac{(1-0.495B)(1-0.398B^{12})}{(1-B)(1-B^{12})}a_t \end{aligned} \tag{16}$$

with $\hat{\sigma}_a^2 = 0.852 \times 10^{-3}$, where all of the estimates are greater than their four standard errors in modulus. The residual analyses such as the Q-statistics and ACF of the residuals and the squared residuals all fail to suggest any model inadequacy. Thus, for this particular series there is no overfitting in the detection.

Forecasting: Next it is interesting to compare the forecasts of model (16) with those of Cryer's model. First of all, the width of the forecast intervals is very different. Consider, for instance, the one-step ahead forecast intervals. For Cryer's model the width of a 95 per cent forecast interval is approximately 0.115 whereas that of model (16) is only 0.057, a reduction of 50.4 per cent. Second, consider the point forecasts. Twenty one-step ahead forecasts for both models were calculated for the time origins t from 196 to 215. In the (out of sample) forecasting, the parameter estimates were updated every four observations. The resulting mean square error is 0.0229 for Cryer's model and 0.0232 for model (16). (Note that both mean square errors are much closer to the $\hat{\sigma}_a$ of model (16) than to that of Cryer's model.) Thus, the effect of the identified interventions on point forecasts is negligible. This is in agreement with the results of Ledolter (1987) who investigated the effect of an AO on forecasts. In general, the effect of an intervention on point forecasts is negligible provided that the forecast origin is not too close to the intervention. In this example, the closest intervention to the forecast origin $t = 196$ is an AO at $t = 184$.

Example 2. In this example, the series of 369 IBM closing stock prices given in Box and Jenkins (1976) is considered. The logarithms of the data were used by Wichern *et al.* (1976) (referred to as WMH in the sequel) to illustrate their theory of variance change. The logarithms Y_t are also used in the following analysis.

Figure 4(a) plots the residual of the ARIMA (0, 1, 1) model

$$(1-B)Y_t = (1 + 0.0245B)a_t, \quad \hat{\sigma}_a^2 = 0.3146 \times 10^{-3}. \tag{17}$$

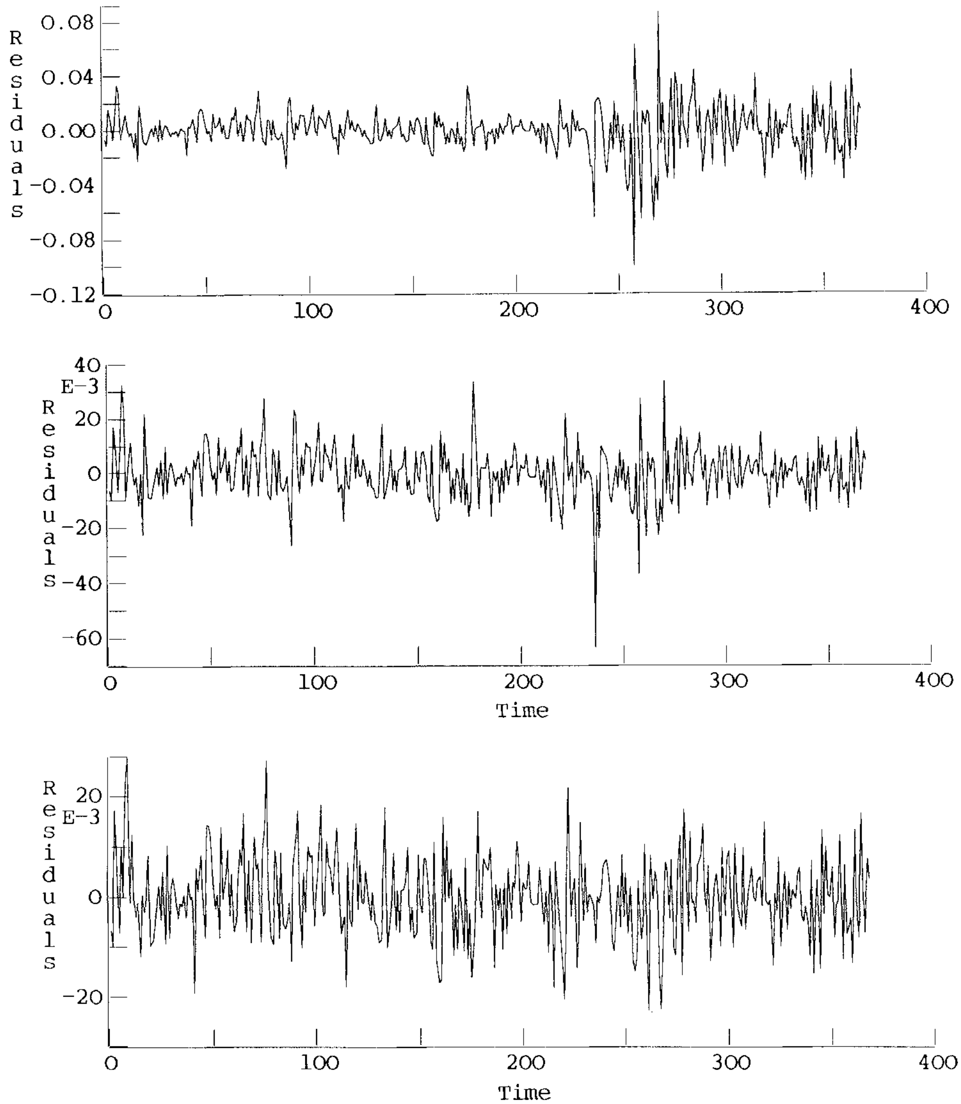

Figure 4. Plots of logged IBM closing stock price. (a) Residuals of logged data, (b) residuals after variance change adjusted, and (c) residuals after disturbances removed

From the plot, it is clear that the variability of the residual increases somewhere after $t = 231$. Hence, Procedure V of the preceding section was applied to the data. A value $h = 30$ is used in variance estimation, see Step 2 of the proposed procedure. Table 4 gives the summary of the detection result as well as the associated statistics. The procedure identified a variance change at $t = 237$ with a variance ratio 7.512 which is highly significant compared with the percentile of Table 1. Since the procedure fails to suggest any further variance change, the adjusted series Y_t^* is homogeneous where

$$Y_t^* = \begin{cases} Y_t & \text{if } t < 237 \\ \bar{Y} + (Y_t - \bar{Y})/\sqrt{7.512} & \text{if } t \geqslant 237 \end{cases}$$

Table 4. Summary of variance detection of IBM stock prices. The critical values 3.5 and 2.5 were used in iteration 1 and 2, respectively

Iteration	MA Parameter θ	Std	Time	Variance Ratio λ_{max} or λ_{min}^{-1}
1	−0.0245	0.0512	237	7.512
2	−0.1457	0.0516		Insignificant

with $\bar{Y}$ being the sample mean of Y_t. The model of Y_t^* is

$$(1 - B) Y_t^* = (1 + 0.1457B)e_t \quad \hat{\sigma}_e^2 = 0.1025 \times 10^{-3}. \tag{18}$$

Figure 4(b) plots the residual e_t of (18). As expected, the residuals appear to be homogeneous. The plot, however, shows that some possible outliers exist in the data.

Next Procedure M was applied to Y_t^*. Table 5 summarizes the results of the procedure. Six outliers and three level changes are identified as significant disturbances. A model for the series after removing the impacts of those disturbances is also given in Table 5. Figure 4(c) shows the residual plot of this model which fails to suggest any further discrepancy.

In summary, the proposed variance change procedure detected a variance change for the series at time $t = 237$. The outlier and level change procedure identified six possible outliers and three possible level changes at various time points. These findings suggest that further scrutiny into the series with emphasis on those extraordinary events is needed. The method used to stabilize the variance, i.e. Step 4 of the proposed procedure, may be changed. The current method treats the first part of the series as the standard series and adjusts the second part to achieve homogeneity. Alternatively, one can use the second part as the standard series and adjusts the first one.

Table 5. Summary results of Procedure M for IBM stock prices. For TC, $\delta = 0.8$ was used. The critical values used are 4.0, 3.5 and 3.5 for the three outer iterations, respectively

Outer Iteration	Inner Iteration	MA Parameter θ	Std	Type	Time	Magnitude	MSE $\times 10^3$
I		−0.1457	0.0516				0.1028
	1			IO	237	−0.0633	0.1164
	2			LC	239	−0.0971	0.0901
	3			AO	258	−0.0286	0.0849
	4			AO	270	−0.0250	0.0891
II		−0.1997	0.0512				0.0808
	1			TC	90	−0.0324	0.0776
	2			IO	178	−0.0341	0.0801
	3			TC	180	−0.0423	0.0743
	4			IO	8	0.0319	0.0730
	5			AO	18	−0.0191	0.0706
III		−0.1821	0.0512				0.0705
	1					Insignificant	

Discussions:

Model Change: In this example the variance change point $t = 237$ has also been identified as an IO point. There are two possible explanations. One is that the variance change may be a gradual one instead of an abrupt change suggested by model (13). The other is that there may be a model change at $t = 237$. The second explanation is supported by the following models

$$(1 - B)Y_t^* = (1 + 0.2580B)c_t, \quad \hat{\sigma}_c^2 = 0.8796 \times 10^{-4}, \quad \text{for } t = 1, \ldots, 236,$$
$$(1 - B)Y_t^* = (1 + 0.0313B)d_t, \quad \hat{\sigma}_d^2 = 0.9395 \times 10^{-4}, \quad \text{for } t = 237, \ldots, 369,$$

where Y_t^* is the adjusted series of Table 4. The standard deviations of the two MA coefficients are 0.0634 and 0.0871, respectively. These two models indicate that variance stabilization cannot account for all the changes at $t = 237$. For this reason, the nine disturbances detected in Table 5 may be interpreted as indications of a model change. This illustrates that the major contribution of the proposed procedures is that they can pinpoint those observations that deserve further investigation.

Comparison: For the logged process Y_t, WMH detected two major variance changes at time $t = 180$ and 235, respectively, and obtained the following models

$$(1 - 0.23B)(1 - B)Y_t = f_t, \quad \hat{\sigma}_f^2 = 0.99 \times 10^{-4} \quad (t = 1, \ldots, 179)$$
$$(1 - 0.13B)(1 - B)Y_t = g_t, \quad \hat{\sigma}_g^2 = 0.62 \times 10^{-4} \quad (t = 180, \ldots, 234)$$
$$(1 + 0.021B)(1 - B)Y_t = h_t, \quad \hat{\sigma}_h^2 = 0.71 \times 10^{-3} \quad (t = 235, \ldots, 369).$$

It is interesting to compare these results with that obtained earlier. First, both approaches identify a variance change around $t = 237$ and agree that the latter part of the series behaves as a random walk. Second, the variance change at $t = 180$ of WMH is specified as an IO (at $t = 178$) and a TC (at $t = 180$) by the proposed procedure. Thus, both methods were able to detect the discrepancy around $t = 180$ even though they suggested different possible causes. In applications, only further investigation and/or substantive information of the series can tell the most plausible cause.

Example 3. As a final example, the value of unfilled orders of radio and TV (UNFTV) from the U.S. Bureau of the Census is reanalysed. This is a monthly series for the period January 1958 to October 1980, a total of 274 data points. This data set was used by Martin, Samarov and Vandaele (1983) to demonstrate the effectiveness of their approximate conditional mean (ACM) type robust filter. Additional information of the series can be found there. Figure 5(a) plots the logged series Y_t and Figure 5(b) the first differenced series $X_t = (1 - B)Y_t$. It is clear from Figure 5(b) that the assumption of constant variance is in question. For this reason, the process X_t was analysed.

Following the proposed procedure of Section 4, a tentative model specification for X_t was first performed. In this instance, the multiplicative seasonal IMA $(0, 0, 1) \times (0, 1, 1)_{12}$ model appears to be reasonable. This is also the model used by Martin *et al.* Table 6 summarizes the detection results of Procedure V. Again, $h = 30$ was used to compute the residual variances. A variance change at $t = 224$ was detected with a variance ratio 5.608. This ratio is highly significant as compared with the empirical result of Table 1. Figure 6 plots the adjusted series X_t^* of Table 6. The variance now appears to be stable. Some outlying observations, however, are present. Procedure M was applied to X_t^*. Table 7 gives the results of outlier detection. No level change, permanent or transient, was detected. There are, however, two innovational outliers at times $t = 125$ and 256, respectively. An additive outlier appears at time $t = 77$ if the critical value is reduced to 3.0. Figure 7 shows the residual plot the outlier-adjusted series Z_t^* of Table 7. This plot fails to show any discrepancy from the

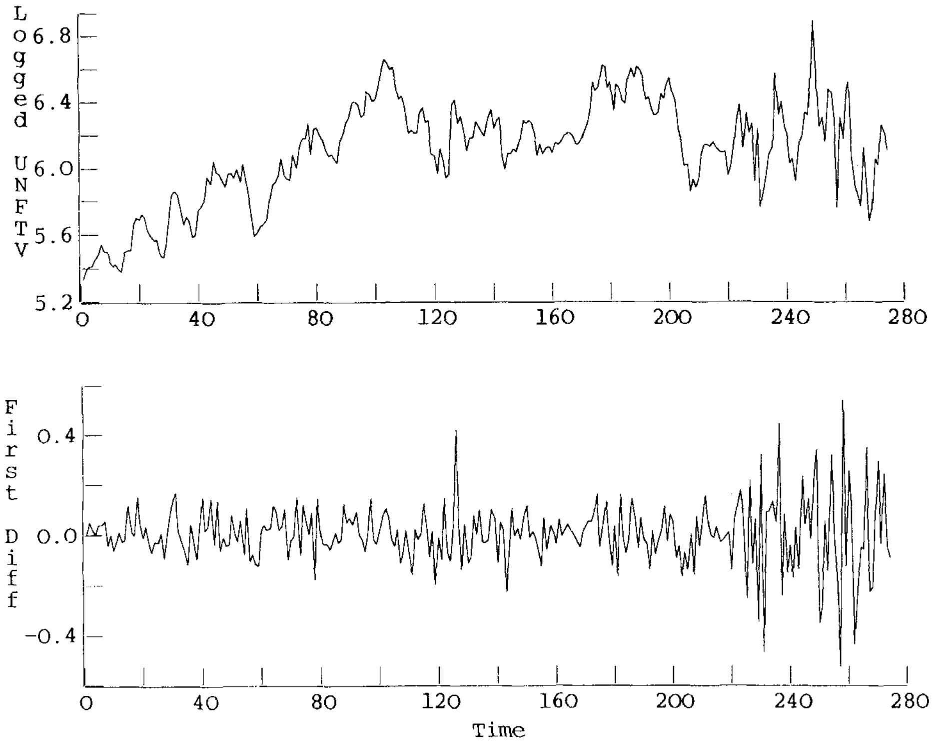

Figure 5. Plots of logged UNFTV. (a) Original series and (b) first differenced series

assumptions of an ARMA model and the residual autocorrlations are all small compared with their standard errors.

Again, this example highlights several aspects of time series analysis. First, if the variance change at $t = 224$ was ignored, then Procedure M would identify more than 15 outliers or level changes for the data. On the other hand, when one takes into account the variance change, there are only two outliers left. Thus, ignoring the variance change can be troublesome. Second, the final parameter estimates shown in Table 7 are different from those obtained by the ACM-type robust filter of Martin *et al.* (1983) who used the original data instead of the logged series. The seasonal MA(12) parameter estimates are close but the MA(1) coefficients are

Table 6. Summary of variance detection of UNFTV series. The critical values 3.5 and 2.5 were used in iteration 1 and 2, respectively

Iteration	Parameter Estimates θ_1(Std)	θ_{12}(Std)	Time	Variance Ratio λ_{max} or λ_{min}^{-1}
1	0.3735(0.0574)	0.7535(0.0495)	224	5.608
2	0.2313(0.0602)	0.8739(0.0355)		Insignificant

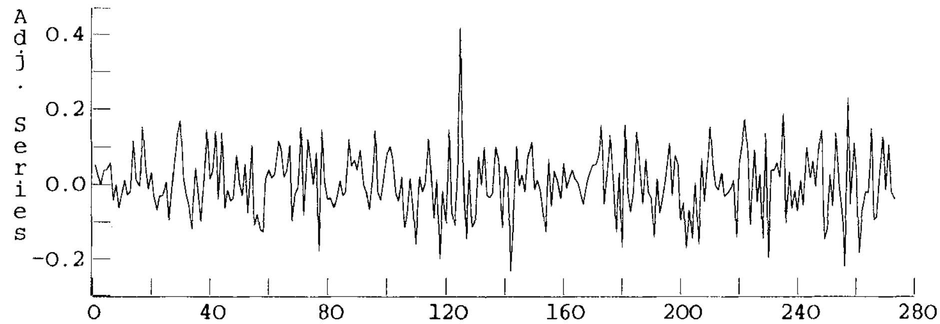

Figure 6. Plot of first differenced log UNFTV after variance change adjusted

Table 7. Summary of outlier detection of UNFTV series. For TC, $\delta = 0.8$ was used. The critical values used are 4.0, 3.5 and 3.5 for the three outer iterations, respectively

Outer Iteration	Inner Iteration	MA Parameters θ_1(Std)	θ_{12}(Std)	Type	Time	Magnit.	MSE $\times 10^2$
I		0.2313(0.0602)	0.8739(0.0355)				0.5885
	1			IO	125	0.3815	0.5532
II		0.2071(0.0606)	0.8556(0.0376)				0.5407
	1			IO	256	−0.2671	0.5311
III		0.1991(0.0607)	0.8600(0.0368)				0.5173
	1					Insignificant	

different. The MA(1) estimate in Table 7 is significant while that of Martin *et al.* is not. This suggests that the ACM-type robust filter might not be robust against variance change. As a final note, when the original data were used, Procedure *V* detectd two variance changes, one at $t = 44$ and the other at $t = 243$, and Procedure M identified three IO's at $t = 235$, 125, and 248. The adjusted series has MA coefficients $\theta_1 = 0.2343$ (0.0602) and $\theta_{12} = 0.8552$ (0.0358).

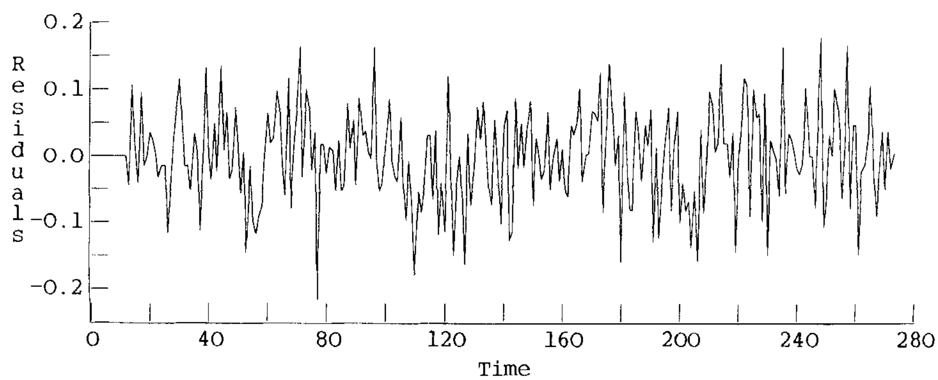

Figure 7. Residual plot of logged UNFTV after disturbances removed

5. CONCLUDING REMARKS

In this paper, procedures for detecting outliers, level shifts and variance changes in a univariate time series were proposed. The procedures were illustrated by analysing three real data sets. The major strength of these procedures is that they can identify those observations which need careful scrutiny. Consequently, the procedures can help data analysts focus on the special features hidden in a series.

Since the procedures are based on simple techniques, they are widely applicable. For instance, they can be used as data screening device in spectral density estimation and in robust time series analysis. They can also be used in biological study where exogenous disturbances are unavoidable. For example, Greenhouse, Kass and Tsay (1987) analysed body temperature of an individual involved in a psychiatric study where the observations clearly depended on the individual physical activities. A variance change from day to night seems highly plausible. A third application of the procedures is that they can be used to identify the time point of an intervention in the intervention analysis of Box and Tiao (1975). In the traditional intervention analysis, the time point of an intervention is assumed to be known.

Finally, two remarks are made on the procedures. First, in Section 4 the adjusted series was used in the detection process to demonstrate the usefulness of the suggested procedure. This, however, does not imply that one can rely on the adjusted series to make inferences. A more appropriate strategy would be (a) to search for the causes of the identified outliers, level changes and variance changes, (b) to specify a general model in the form of (2) based on causes of the exogenous disturbances, and (c) to estimate jointly the impact of disturbances and the time series parameters. This strategy allows for the use of prior information of the disturbances. It can also reduce the possibility of over parameterization that arises from the abuse of the detection procedure. Readers are referred to Tsay (1986a) for further discussion. Second, to detect the transient level change, $\delta = 0.8$ was used in Section 4. In fact, other values of δ can also be used. As an example, $\delta = 0.6$ was used to the air-passenger-miles data of Example 1. The procedure still identified exactly the same time points as significant disturbances even though some of the classifications between permanent and transient level changes are different. Similarly, to detect the variance changes, $h = 30$ was used to compute residual variances at both ends of a series. The choice of h is not critical as long as it is reasonable. For instance, the same detection results were obtained in Example 2 when $h = 20$ was used. In general, a h between 20 and 30 appears to be useful.

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