

# **COMP2300-COMP6300-ENGN2219**

## **Computer Organization &**

## **Program Execution**

Convenor: Shoib Akram  
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Australian  
National  
University

## **Shoaib Akram**

School of Computing, (Jan 2020 – )

Ph.D., 2019

**Teaching:** COMP2300, COMP2310, Computer Microarchitecture (3710)

**Interests:** Hardware/software interaction, storage systems

- Interesting time to learn and research about computer systems
- Technology (physical) limits vs. societal demand for more compute power; cost and energy efficiency; and sustainability
- Big Data, AI/ML, social media platforms, search engines, communications, mobile apps, drones, among others.

***My current research focus is on efficiently processing big datasets***

# Logistics

- Course webpage: <https://comp.anu.edu.au/courses/comp2300/>
- **Lectures (on the website)**
  - Lecture slides
  - Lecture videos are available via Echo360
- **Policies**
  - General conduct, assignment submissions, support, management, grading, late submissions
- **Resources**
  - Frequently asked questions
  - Writing design documents
  - Stuff needed to finish the assignments

# Communication

- We will use **Ed** for
  - Announcements
  - Addressing your concerns
  - Answering your questions
- Students are added automatically
  - If not send an email to [comp2300@anu.edu.au](mailto:comp2300@anu.edu.au)

# Communication

- The course email is an alternative form of communication
  - **comp2300@anu.edu.au**

# Tutorials

- Labs are a critical component of this course (one every week)
- Handouts should be posted on the website
- First six labs
  - Assignment 1
- Next six labs
  - Assignment 2
- Excellent tutors with deep technical knowledge of the subject
  - Almost all of them have taken multiple courses in the systems & architecture specialization

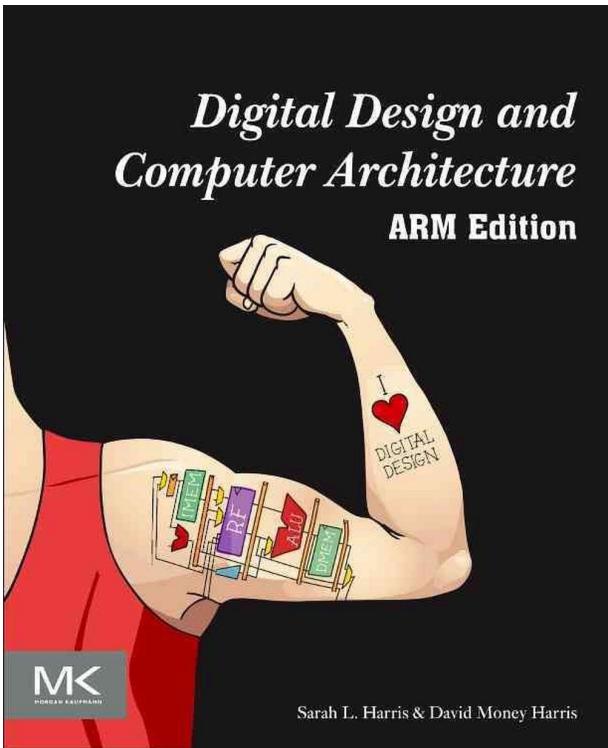
# Lectures

- Go well beyond what is covered in the tutorials
- Exam and quiz questions based on lectures
- Tutorials teach you how to build and program a simple computer to manage complexity
- Lectures will systematically build up from simple to a very complex, state of the art, computer

# Assignment Submission

- Assignment submissions are handled via Gitlab
  - You will learn about it in the labs
  - Make a habit of using Gitlab properly!
  - Push often, always pull the latest
- Extensions
  - <https://comp.anu.edu.au/courses/comp2300/policies/#extensions>

# Textbook



- Freely available online (check Ed or course webpage)
- I will post the chapters/sections on the Lectures page after the lecture

# COMP2300

- Asks a question
- Let's give it a shot!

# COMP2300

## How does a computer **actually** work?

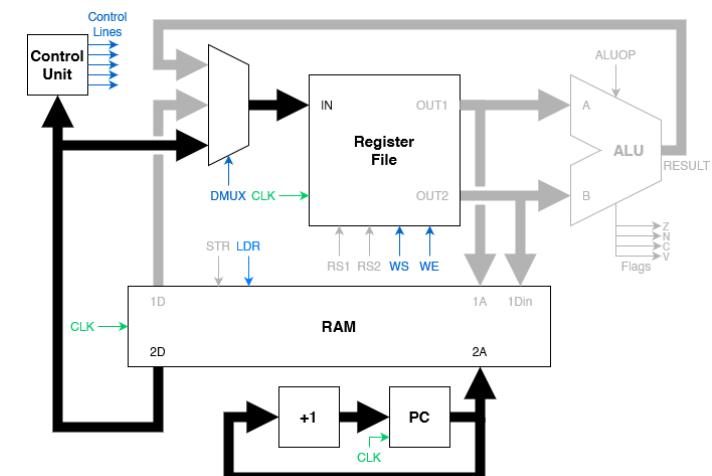
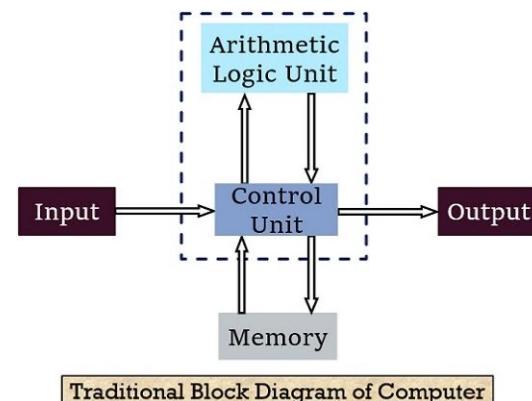
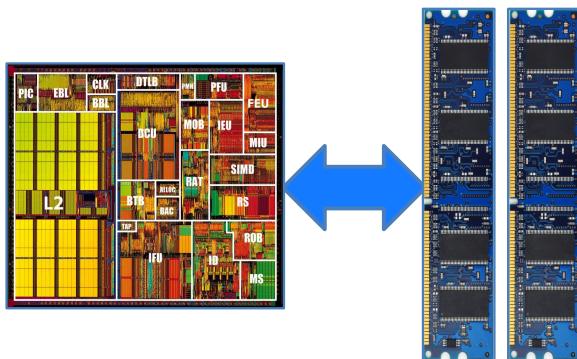


**COMP2300**

How does a computer  
work **under the hood?**

# COMP2300

## How does a computer work **under the hood**?

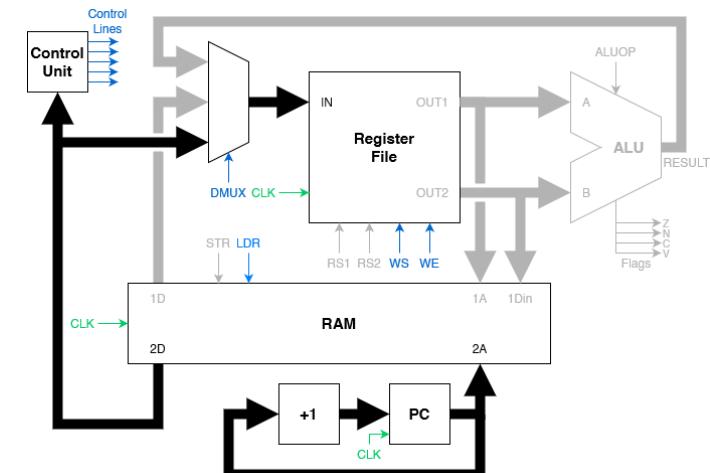
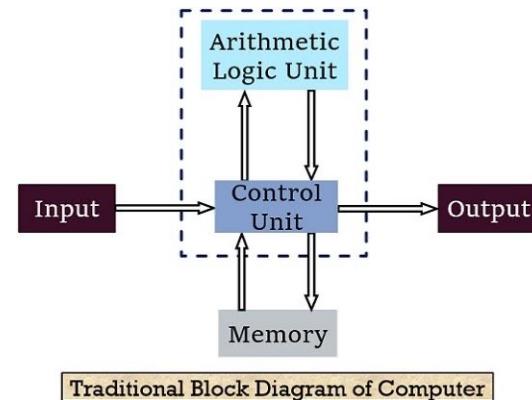
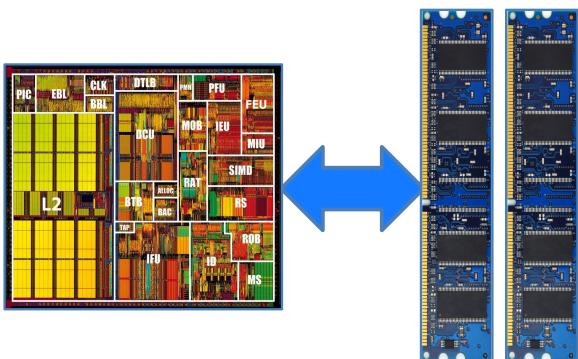


High abstraction level

Low abstraction level

# COMP2300

How does a computer **work under the hood?**



High abstraction level

Low abstraction level

**COMP2300**

Let's be more specific

• • •

**COMP2300**

How does a computer  
**perform** a useful task?



**COMP2300**



How does a computer  
perform a wide variety  
of useful tasks?

**COMP2300**

How do we make a  
computer perform a  
wide variety of useful  
tasks?

**COMP2300**

How do we make  
**electrons** perform a  
wide variety of useful  
tasks?

**COMP2300**

How do we make  
**electrons** ~~perform~~ a  
wide variety of useful  
~~tasks~~?

# COMP2300

How do we make electrons  
execute a wide variety of useful  
programs?

- Computer Organization & Program Execution

# How do we make electrons do the work?



Problem Statement: “Save the planet”

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---



# How do we make electrons do the work?



Problem Statement: “Save the planet”

---

The Algorithm

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# How do we make electrons do the work?



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Program in a High-Level Language

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Microarchitecture

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Circuits

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The Algorithm

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Program in a High-Level Language

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Instruction Set Architecture (ISA)

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Microarchitecture

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Circuits

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Devices



# How do we make electrons do the work?

- Using a *sequence of systematic transformations* developed over six decades
- Each step must be studied and improved for the whole “**compute stack**” to work/operate **efficiently**

# Transformation Hierarchy

- We call the steps of the process: **Levels of transformation OR transformation hierarchy**
- At each level of the stack, we have choices
  - Language → Java, Python, Ruby, Scala, C++, C#
  - ISA → ARM, x86, SPARC, PowerPC, RISC-V
  - Microarchitecture → Intel, AMD, IBM, Apple
- If we ignore any of the steps, then we cannot
  - Make the best use of computer systems
  - Build the best system for a set of programs



Problem

---

Algorithm

Program

---

Architecture

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micro-arch

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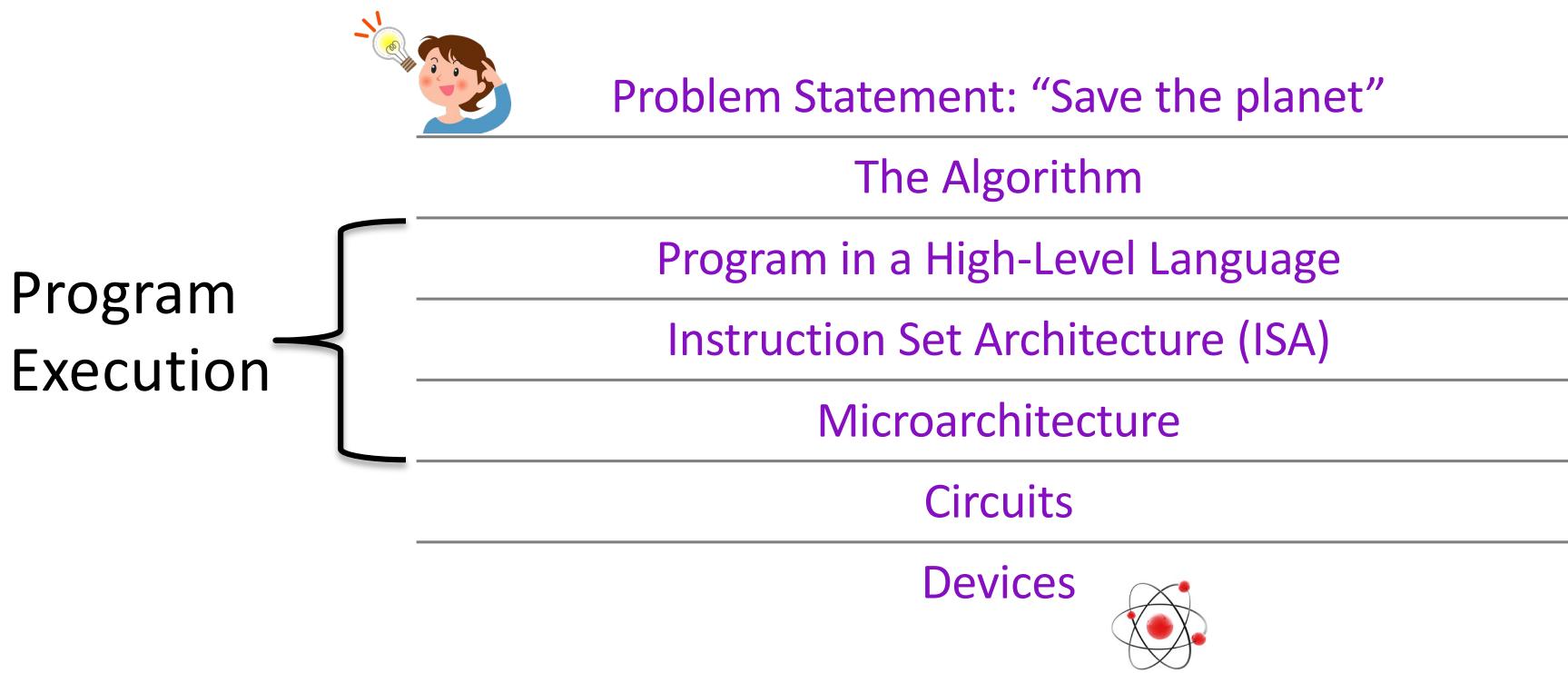
circuits

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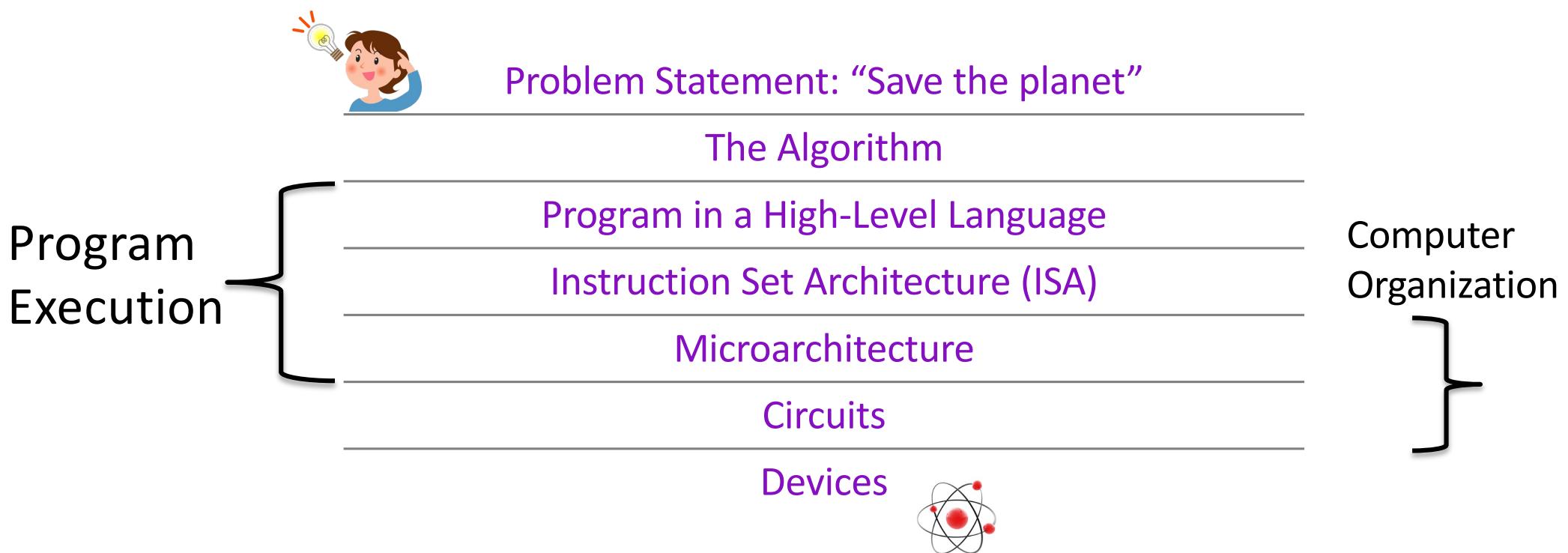
devices



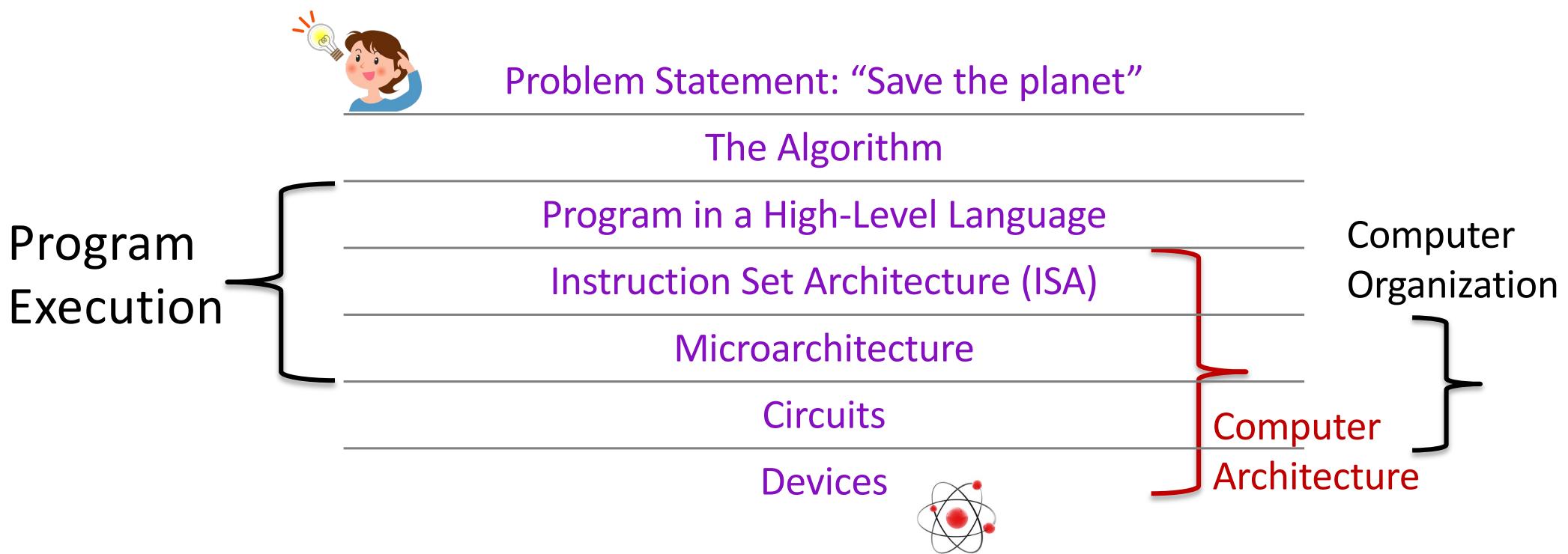
# Transformation Hierarchy



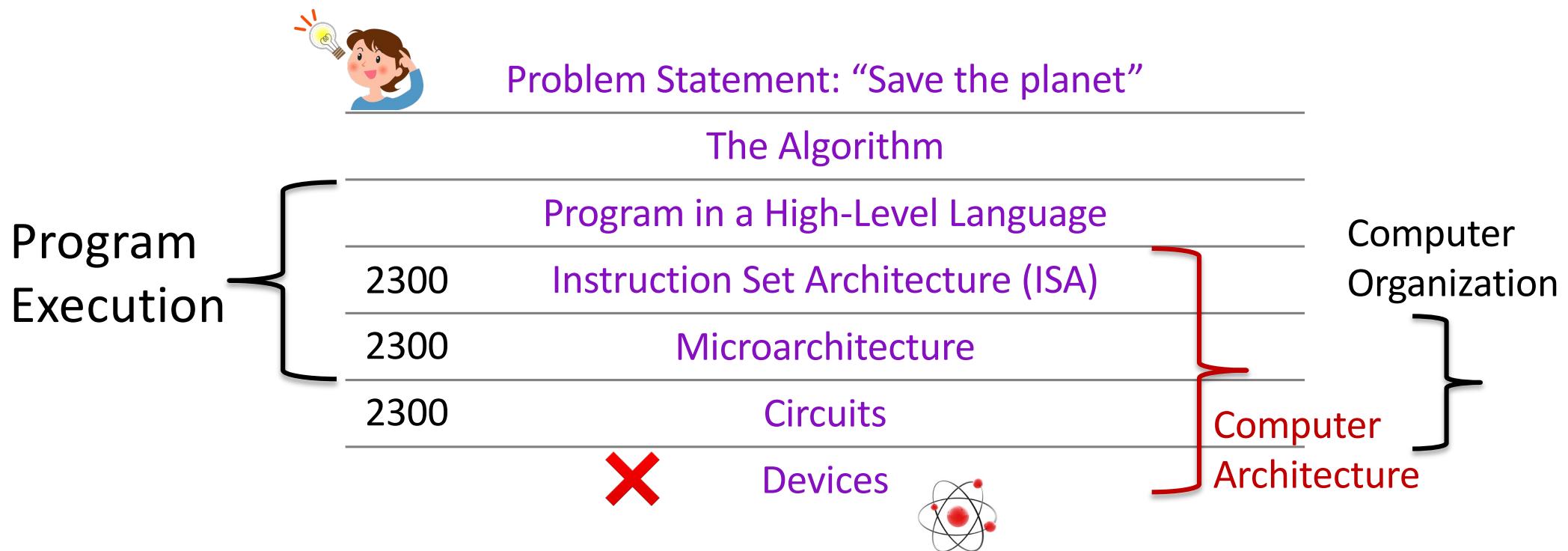
# Transformation Hierarchy



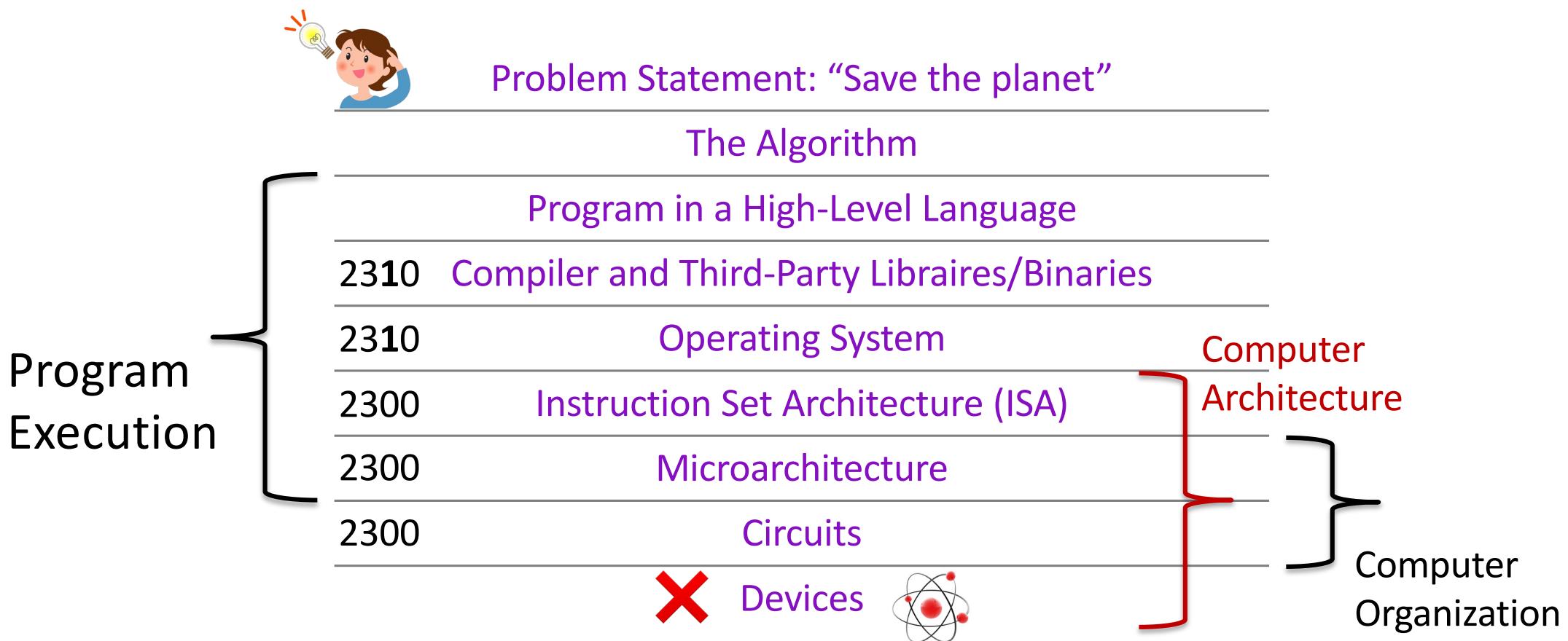
# Transformation Hierarchy



# Transformation Hierarchy & Us



# Transformation Hierarchy & Us



# Hardware and Software



Problem Statement: “Save the planet”

**ISA = Hw/Sw**

**boundary/interface**

**Software**

The Algorithm

Program in a High-Level Language

Compiler and Third-Party Libraries/Binaries

Operating System

Instruction Set Architecture (ISA)

**Hardware**

Microarchitecture

Circuits

Devices



# ISA vs. Microarchitecture

- **ISA:** **Specification** of a set of definite instructions that the computer can carry out
  - All computers (CPUs/microprocessors) perform the same set of basic instructions
  - ADD, MULTIPLY, DIVIDE, MOVE, BRANCH
  - Two manufacturers might differ in *which set* of basic instructions
- **Microarchitecture:** **Implementation** of the ISA using circuits

# ISA vs. Microarchitecture



- **ISA:** What the driver needs to know as she sits inside the automobile to make the automobile carry out the driver's wishes
- If the middle pedal (brake) is pressed, the car stops
- Steering wheel, ignition key, the gears, windshield wipers
- ISA specifies two things (?)
- All cars have the same ISA (hopefully). There could be differences!

- **Microarchitecture:** What goes underneath the hood
- Different cost/performance tradeoffs
- Some are turbocharged. Some have disc brakes. Some cost a million \$. Some are more fuel efficient than others
- But you don't need a separate license to drive a Honda and a BMW
- Must not take a Honda to a BMW factory for repair!

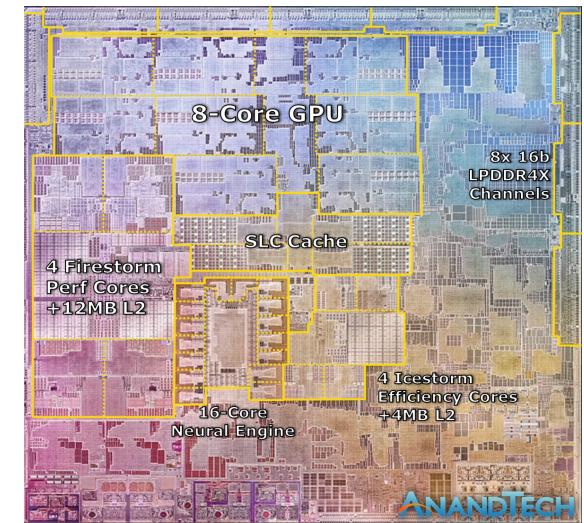
# Two Recurring Themes

- The notion of abstraction
- Hardware versus software

# The Notion of Abstraction

- **Abstraction:** Know components from a high level of detail

No human (programmer) can track  
10 billion elements. **Computer systems  
work because of abstraction!**



Apple M1 Chip  
Billions of transistors  
All working in parallel

# The Notion of Abstraction

- **Abstraction:** View the world from a higher level
- Focus on the important aspects
- It is a way to enhance productivity and efficiency

# The Notion of Abstraction



Put pressure on the pedal known as the accelerator

Drive 0.7 KM at an appropriate speed so we don't get fined

Turn the steering wheel to the left

Press the middle pedal so the automobile comes to a halt

....

# The Notion of Abstraction



First go straight

Then take a left

Then go straight again

...

# The Notion of Abstraction



Take me to 108 North Road, please!

# The Notion of Abstraction

- Important lesson in the previous example
- It is efficient to abstract and there is really nothing to be gained from the ridiculous elaboration to the driver
- **Except when**
  - Driver **does not know how to drive**
  - Driver **does not know where is 108 North Road**
  - This is where COMP2300 is **unique**. It teaches you to un-abstract when the world you are trying to abstract does not work as expected.

# The Notion of Abstraction

- Focus on the important aspects
  - What is input? What is output?
  - What is the function: Is X an ADD or MULTIPLY unit
- If the world below does not work as expected?
  - If the transistors X is built from do not behave as expected (unlikely in this course)
  - To deal with it, we need to go below the abstraction layer
- **Deconstruction:** To un-abstract when needed
  - Important skill
  - If the CPU does not work as expected, first test each of the big sub-components
  - Then, check to see if the next component in the hierarchy works as expected



# The Notion of Abstraction

- We will raise the level of abstraction in this course every couple weeks
- We start from the ground (up) because this is how computers have evolved
- Each layer in the *transformation hierarchy* is an abstraction layer



Problem

---

Algorithm

Program

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Architecture

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micro-arch

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circuits

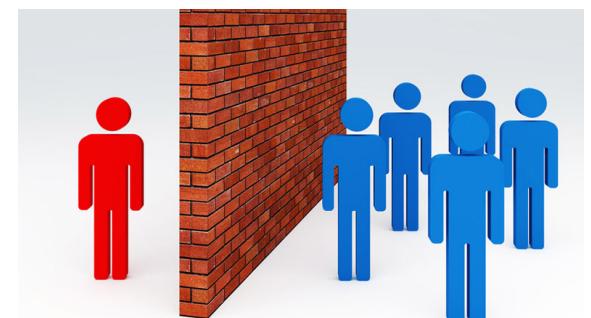
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devices



# Hardware versus Software

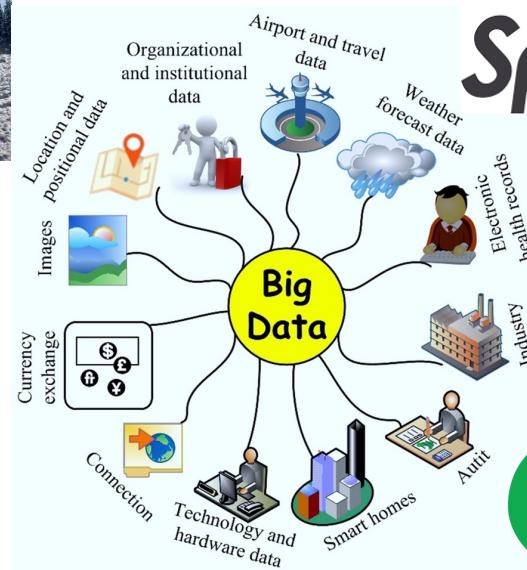
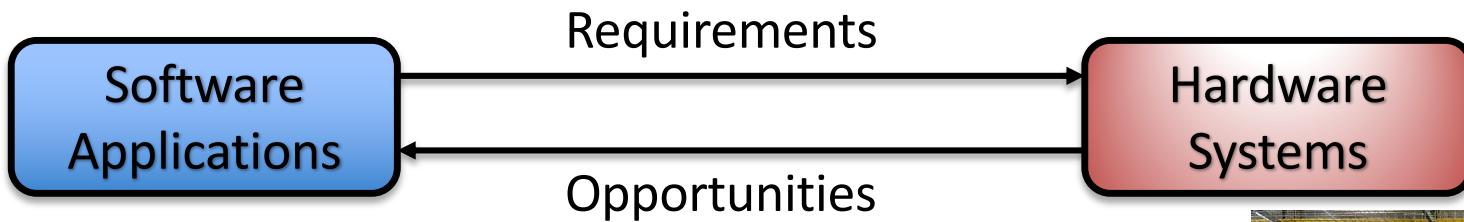
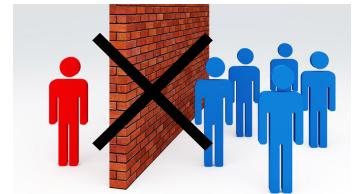
- **Hardware**
  - CPU, memory, storage device, disk, SSD, network card, USB device, AI accelerator, FPGA, PLA, motherboard, PCI express bus, SATA drive
- **Software**
  - Programs, operating systems, compilers, virtual machines, device drivers,
- **One view:** Ok to be an expert at one of these



# Hardware versus Software

- **Hardware** and **Software**
  - Two interacting parts of the computer system
- **COMP2300 view:** Knowing the capabilities and limitations of each leads to better overall systems

# Hardware-Software Interplay



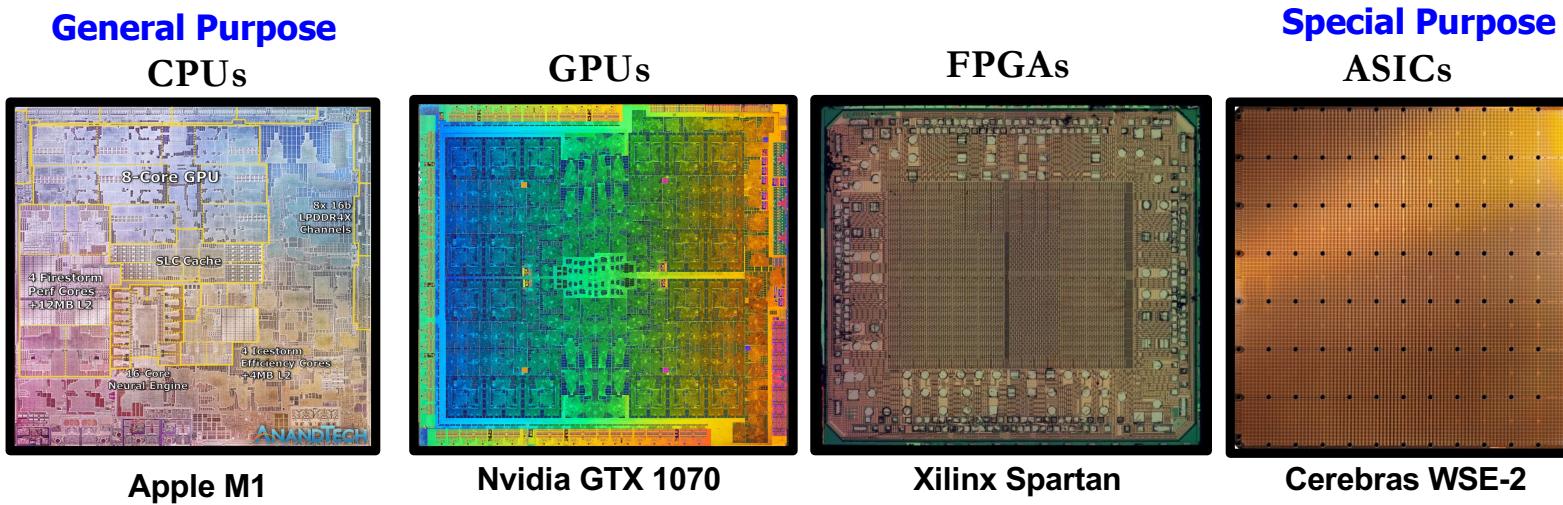
mongoDB®



# **Why so many hardware choices?**

# Hardware: General Purpose vs. Special Purpose

Common building block of computers



Flexible: Can execute any program  
Easy to program & use  
Not the best performance & efficiency  
C/C++/Java/...

Efficient & High performance  
(Usually) Difficult to program & use  
Inflexible: Limited set of programs  
Domain-specific, special purpose languages

# General Purpose vs. Special Purpose

## General Purpose



**Flexible:** Can work with any bolt

**Easy to use**

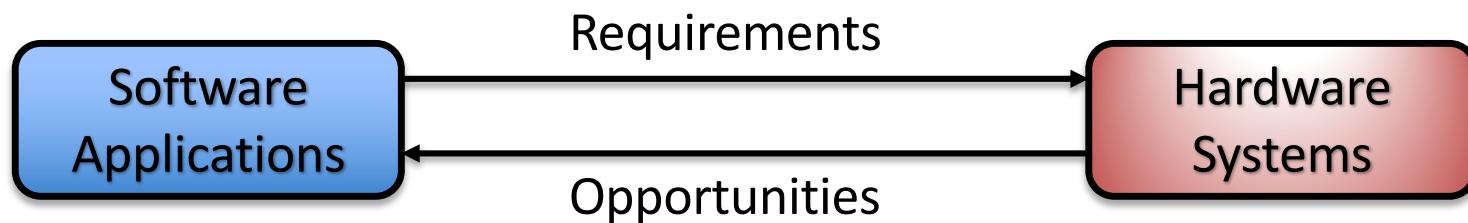
**Not the best fit, results or efficiency**

## Special Purpose



# Modern Software Trends

- Many important and emerging applications are data-intensive
- It is easier and convenient to produce data than to process, analyze, and store it
- This trend of producing data at high velocity is driving new applications and correspondingly novel hardware



# Two Key Ideas

- Key components of a computer are the same
- All computers can compute the same things

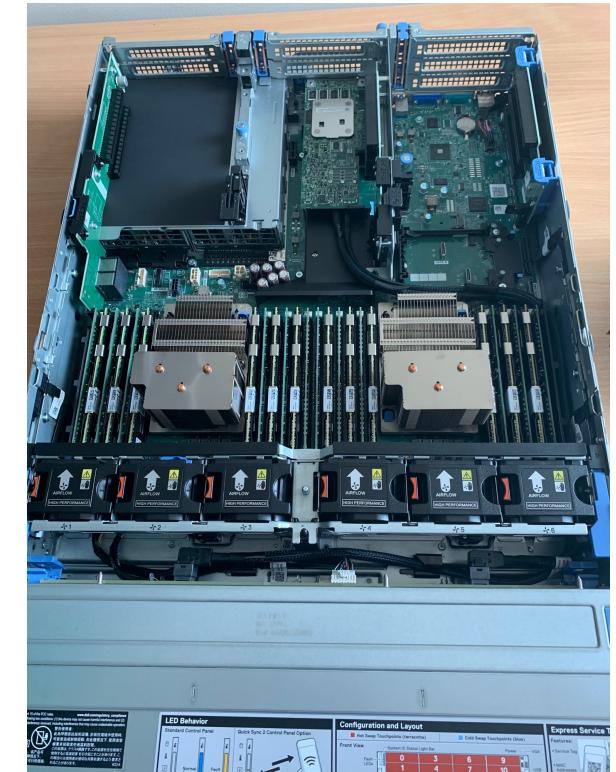
# Idea 1: Key components are same



Council Bluffs, Iowa  
data center, Google  
(115, 000 sq. feet)



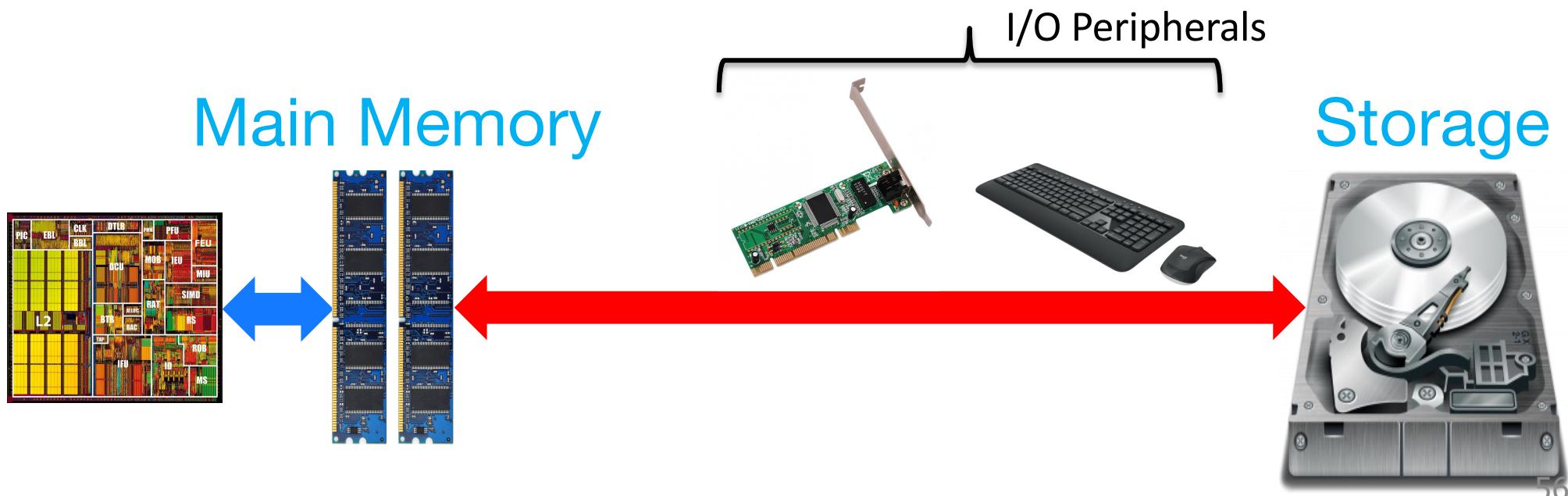
Self-flying nano drone  
94 milli-watts



Research server for my  
students with special memory  
& storage devices

# A Canonical Computer System

- Most computer systems can be viewed as below
  - Key resources: CPU, memory, I/O devices, storage
  - CPU (processor/microprocessor) does the actual computation
  - Processor can access memory much faster than storage



# Idea 1: Key components are same

- Key Idea 1: All computer systems, big or small, have a few fundamental components
  - **Microprocessor** (processor or central processing unit or CPU) for doing computation
  - **Main memory** for storing temporary information and program data close to the processor
  - **Storage devices** (disks or SSDs) for storing long-term or persistent information
  - **I/O devices** to communicate with the external environment
    - Sensors
    - Peripherals

# Idea 2: They all can solve the same problems

- Key Idea 2: All computers regardless of size, cost, and speed can compute the same things if they are given enough time and memory
  - Anything a fast computer can do, a slow computer can do
  - Let's explore this idea further ....

# Program Ex.

Application Software	<code>&gt;"hello world!"</code>
Operating Systems	
Architecture	
Micro-architecture	
Logic	
Digital Circuits	
Analog Circuits	
Devices	
Physics	

Programs

Device Drivers

Instructions Registers

Datapaths Controllers

Adders Memories

AND Gates NOT Gates

Amplifiers Filters

Transistors Diodes

Electrons

# Program Ex.

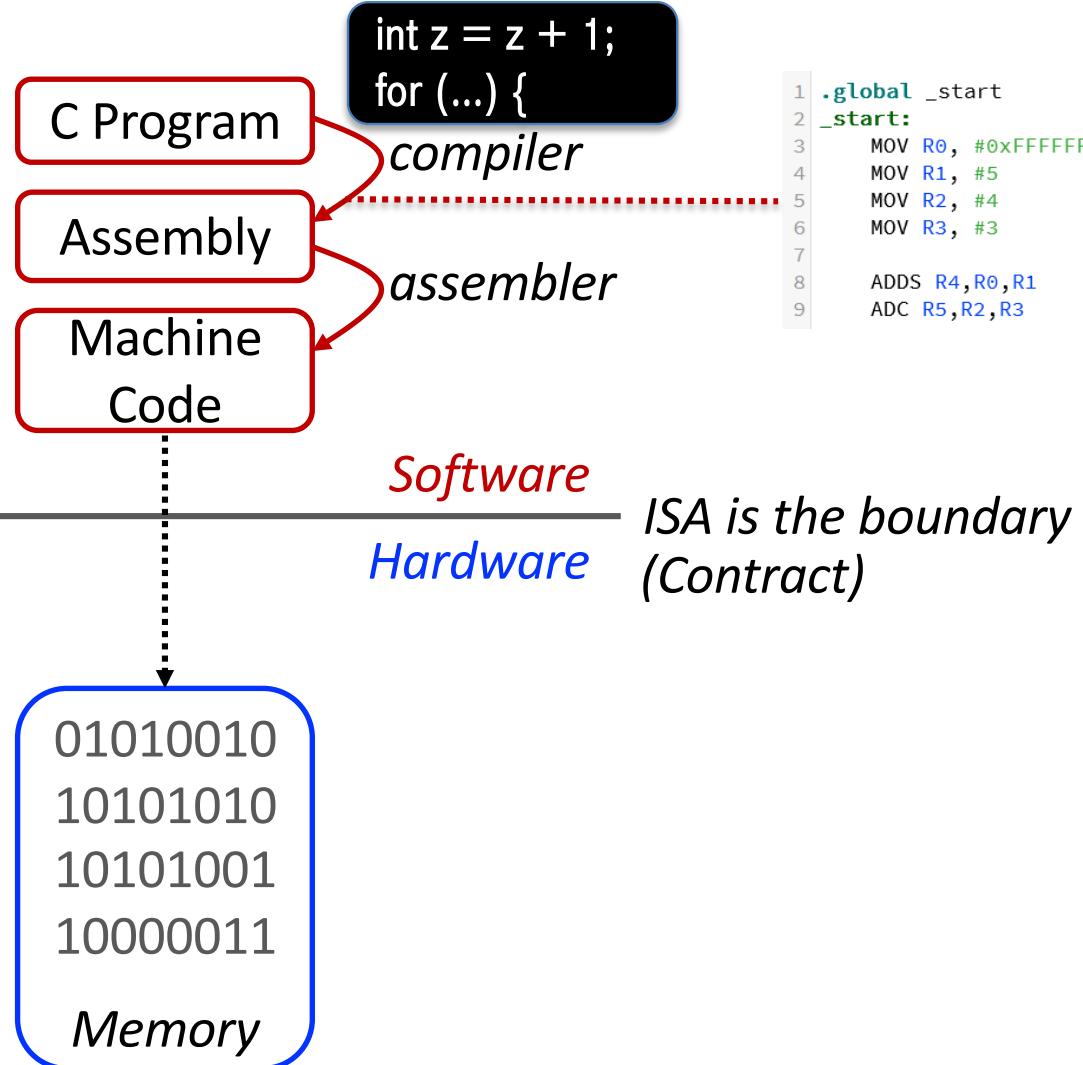
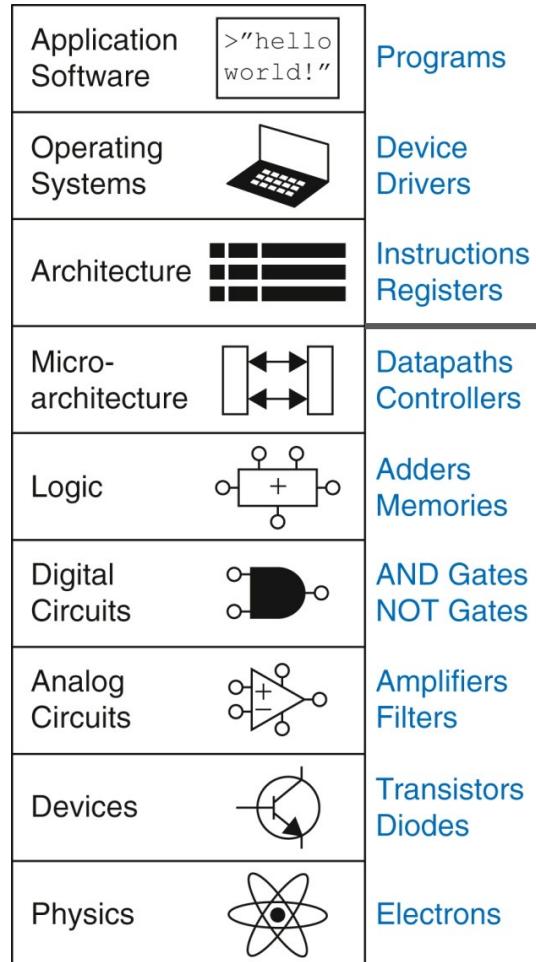
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Analog Circuits		Amplifiers Filters
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Physics		Electrons

*Software*  
*Hardware*

*ISA is the boundary  
(Contract)*



# Program Ex.



```

1 .global _start
2 _start:
3     MOV R0, #0xFFFFFFFF
4     MOV R1, #5
5     MOV R2, #4
6     MOV R3, #3
7
8     ADDS R4,R0,R1
9     ADC R5,R2,R3

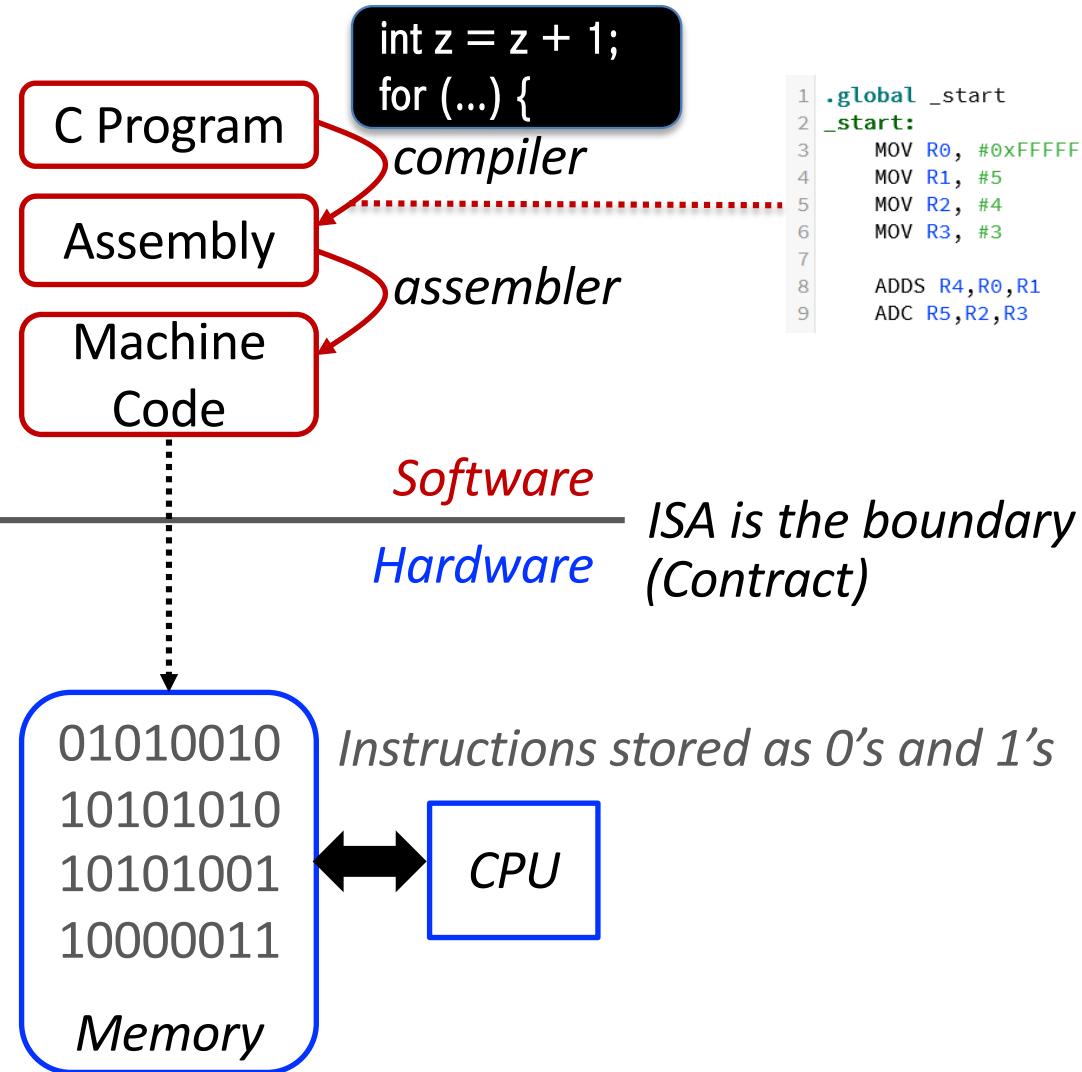
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```

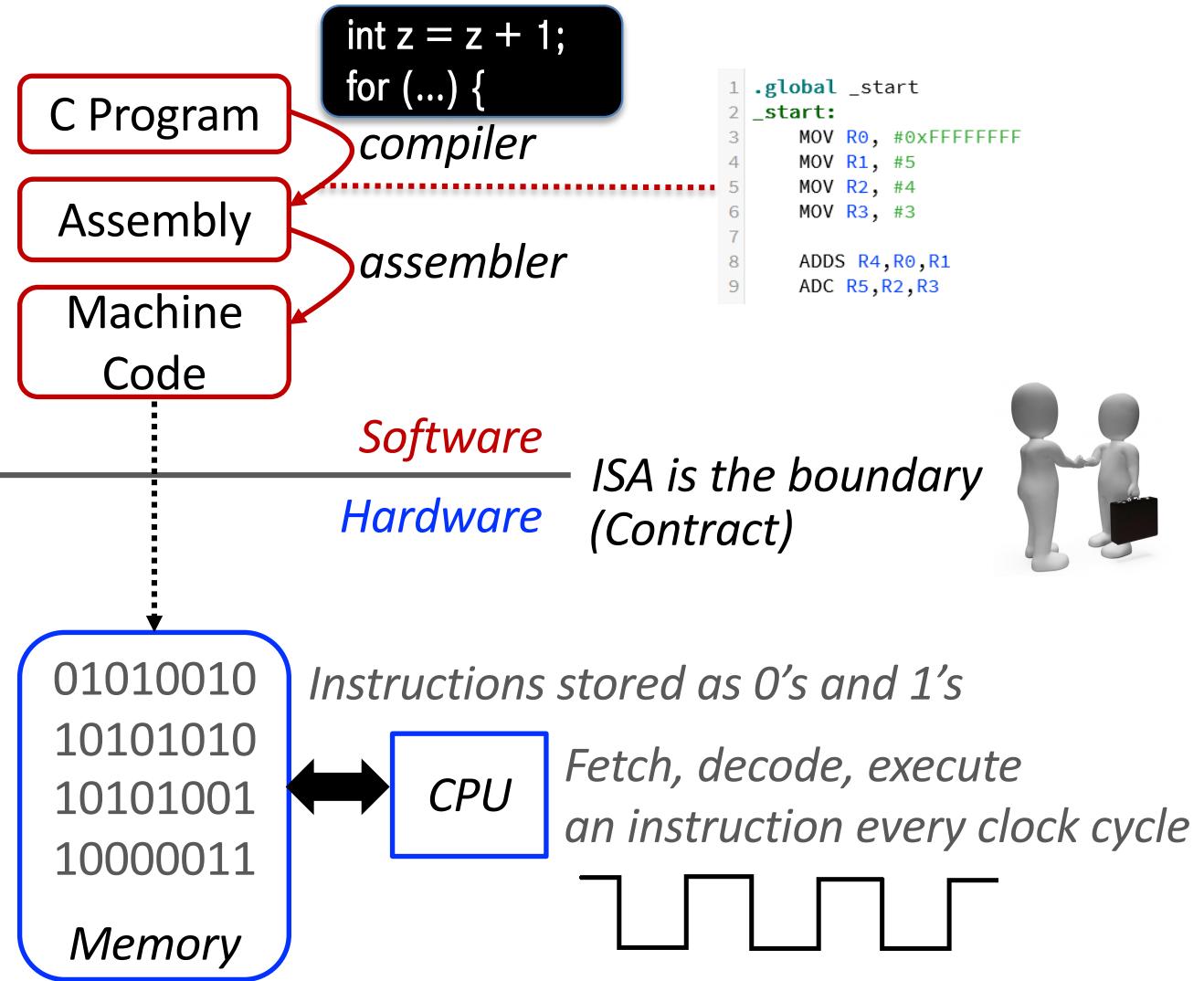
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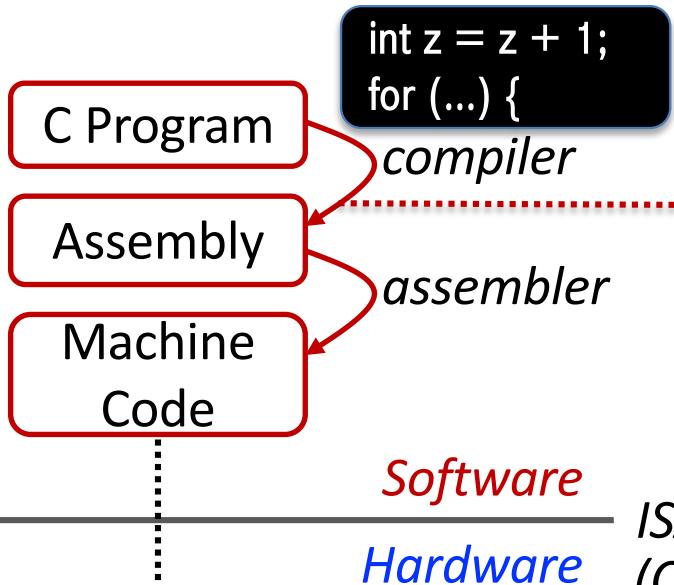
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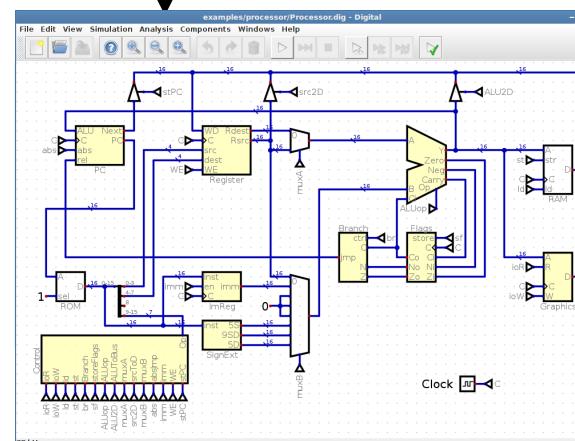
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```

*ISA is the boundary  
(Contract)*



*stored as 0's and 1's  
Fetch, decode, execute  
an instruction every clock cycle*



Assignment 1: Build CPU

# Program Ex.

Application Software	<code>&gt;"hello world!"</code>
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Architecture	
Micro-architecture	
Logic	
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Physics	

Programs

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Instructions  
Registers

Datapaths  
Controllers

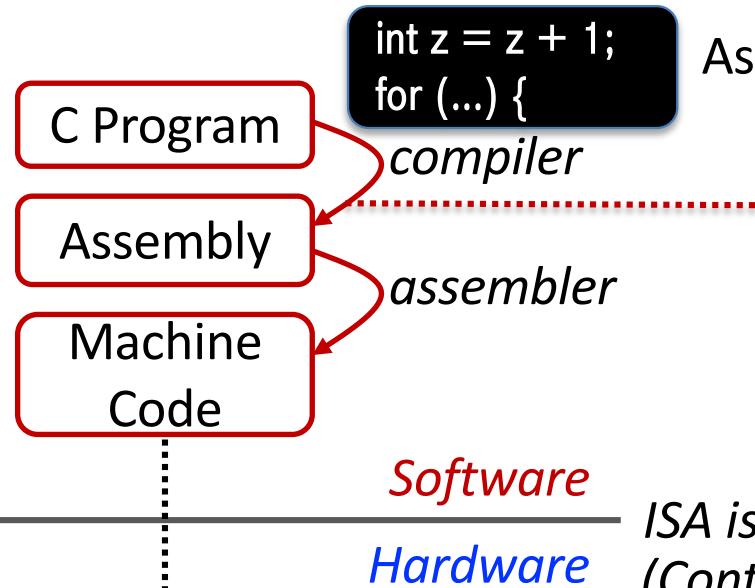
Adders  
Memories

AND Gates  
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Amplifiers  
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Transistors  
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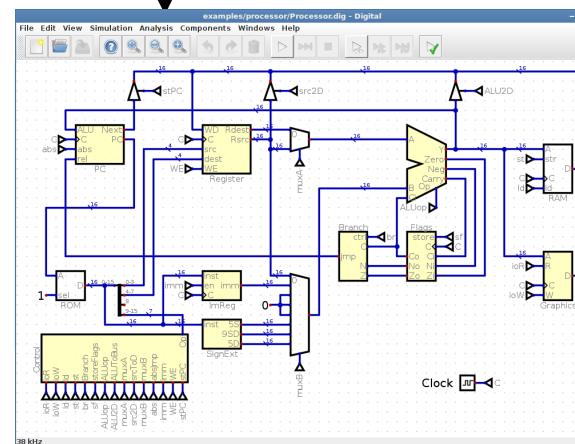


Assignment 2: Program CPU



*ISA is the boundary  
(Contract)*

*stored as 0's and 1's  
Fetch, decode, execute  
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Assignment 1: Build CPU

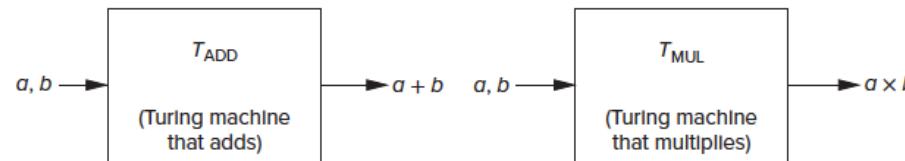
# Computer = Universal Computational Device

- Anything that can be computed, can be computed by a computer
- Studying computers is studying the fundamentals of all computing
- The idea of universal computational device is due to Alan Turing (1937)
- He gave the mathematical description of Turing machine
  - A particular kind of machine called the Turing machine can carry out all computations
  - Let's build this ultra powerful Turing machine



# Computer = Universal Computational Device

- First, we have two simple example Turing machines
  - One for addition, and one for multiplication



- What we now call a computer is a Turing machine that can simulate all Turing machines
- What does it need as inputs?
  - Inputs and the description of the Turing machine to simulate
  - Can you draw the black box model?
  - We say that Turing machines are programmable

# Universal Computational Device

- Anything that can be computed, can be computed by a computer provided it has enough time and enough memory
- We **instruct** the computer how to do **X**, and it obliges by **interpreting our instructions**
  - Instructions are stored in memory like regular data
- Computer is **programmable** because we can rewrite instructions to make it do something else

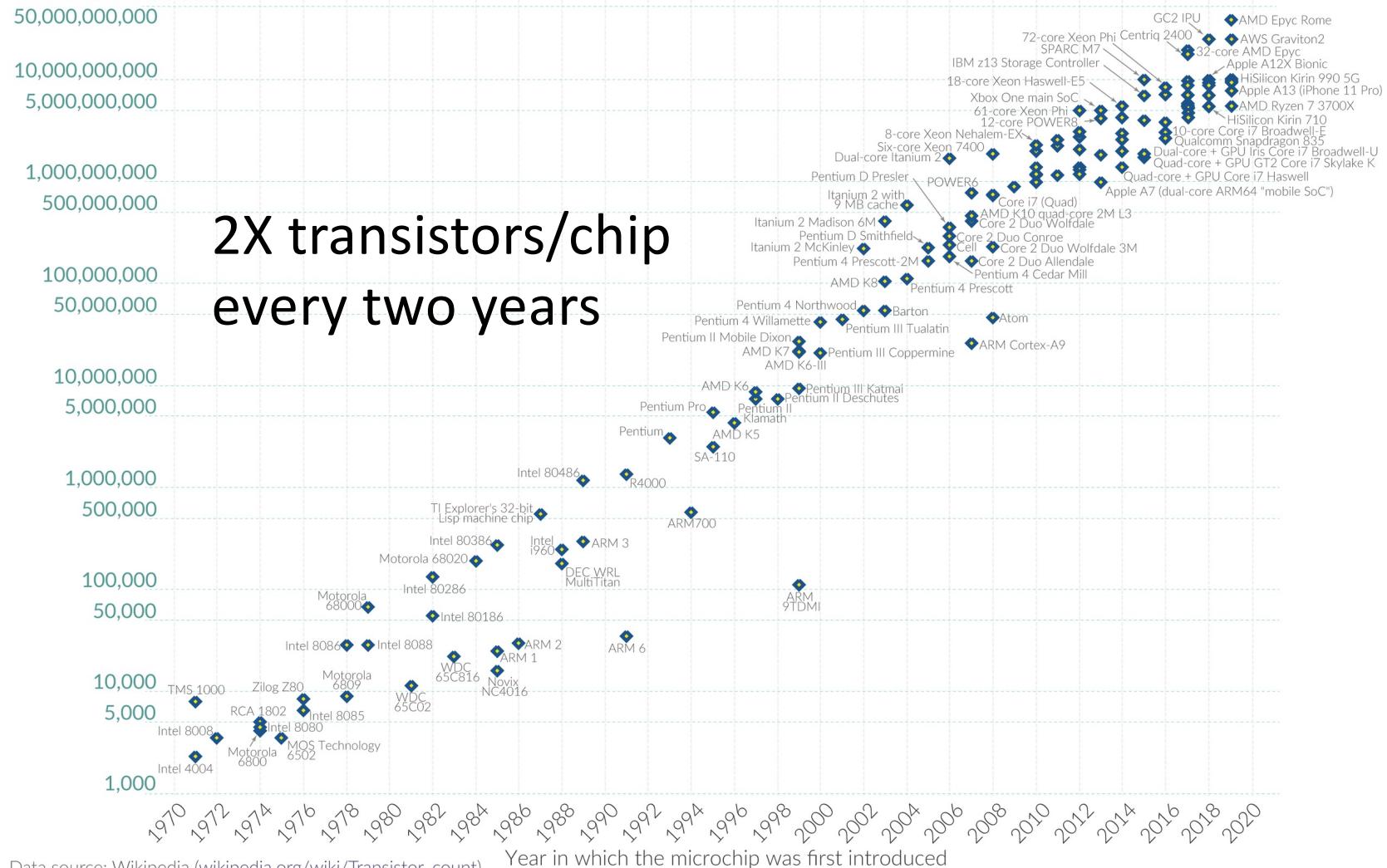
# Some Technology Trends

- Moore's Law
- Uniprocessor performance
- Memory wall
- Memory hierarchy

# Moore's Law: The number of transistors on microchips doubles every two years

Moore's law describes the empirical regularity that the number of transistors on integrated circuits doubles approximately every two years. This advancement is important for other aspects of technological progress in computing – such as processing speed or the price of computers.

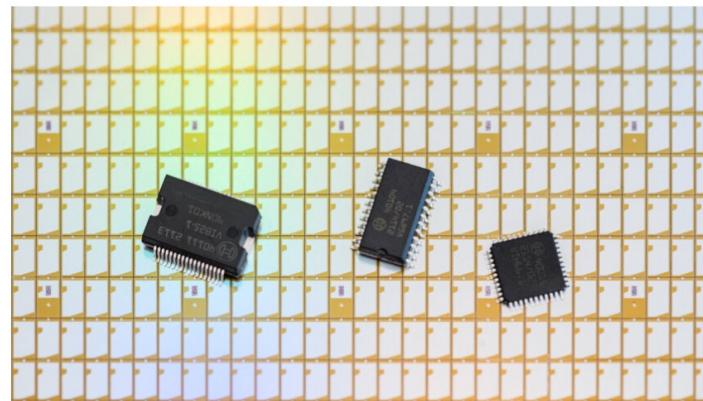
## Transistor count



## The semiconductor industry is ‘near its limit,’ says Nvidia CEO

CEO Jensen Huang believes that the long-held notion that the processing power of computers increases exponentially every couple of years has hit its natural limit.

17 October 2022



 Dashveenjit Kaur  
@DashveenjitK  
dashveen@hybrid.co

All stories

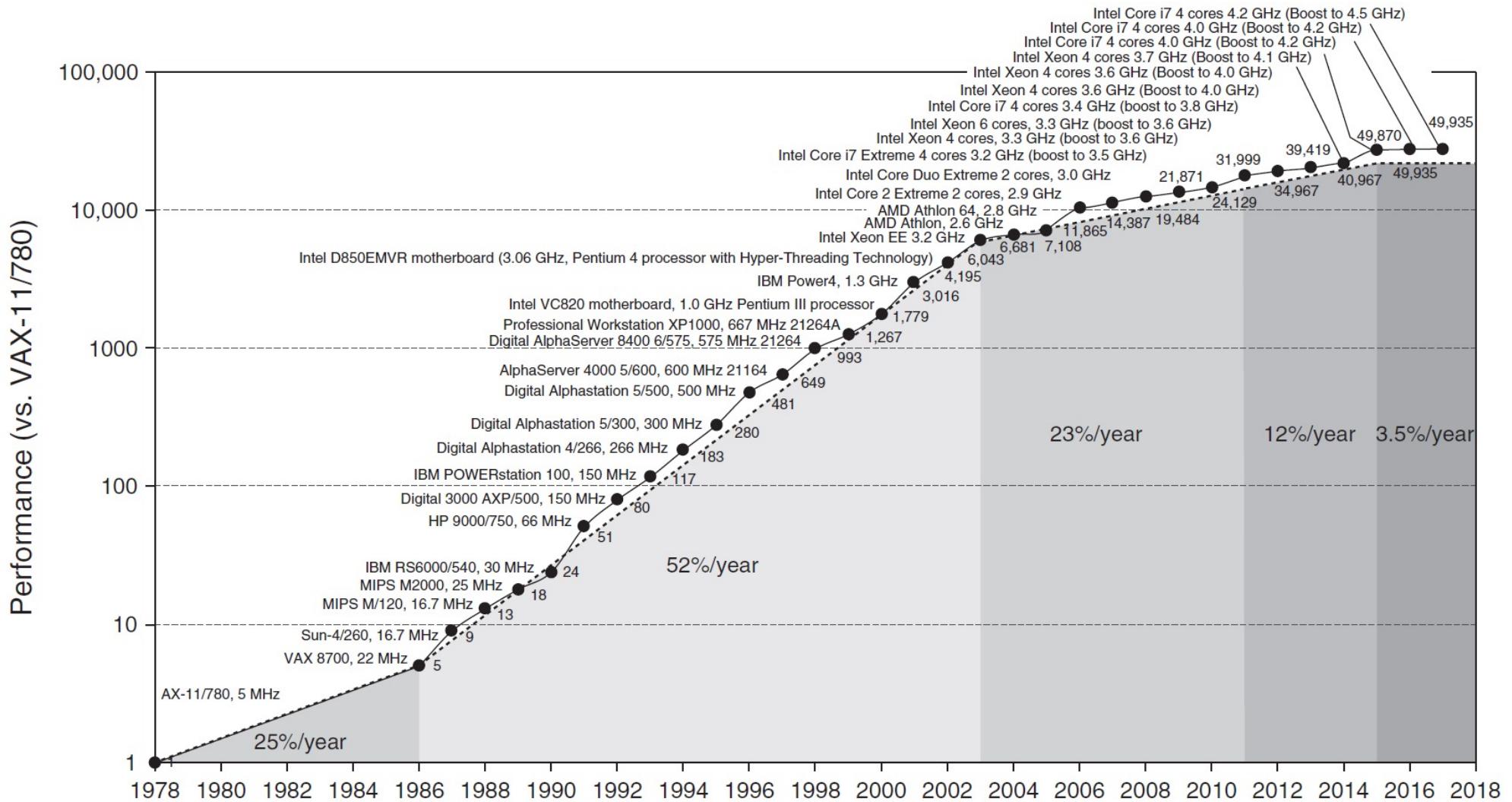
The semiconductor industry is near its limit, Nvidia CEO said. What does it mean? (Photo by JENS SCHLUETER / AFP)

- The semiconductor industry is at a point where the scale of chip components gets closer and closer to that of individual atoms, and that makes it harder to keep up the pace of Moore's Law, Huang said.
- Huang says “our transistors are going to find limits and we’re at atomic scales.”

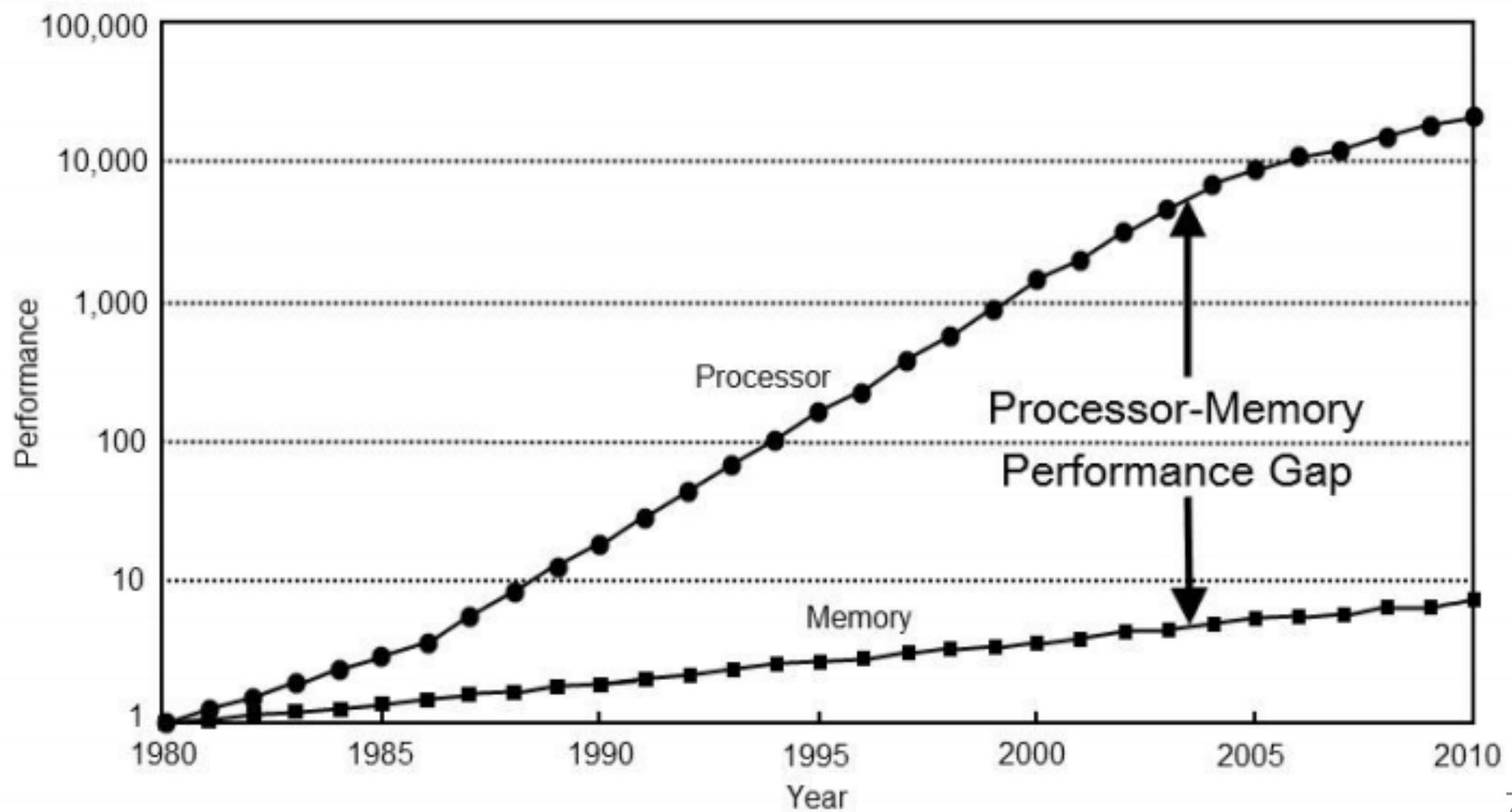
Within the technology industry, overall electronics innovation is highly dependable on semiconductor advancements. After all, it is the shrinking of processors that improves battery life, lowers costs and boosts performance of devices. As Moore’s Law suggests, the number of transistors packed onto the silicon chips that power the modern world has been steadily growing in

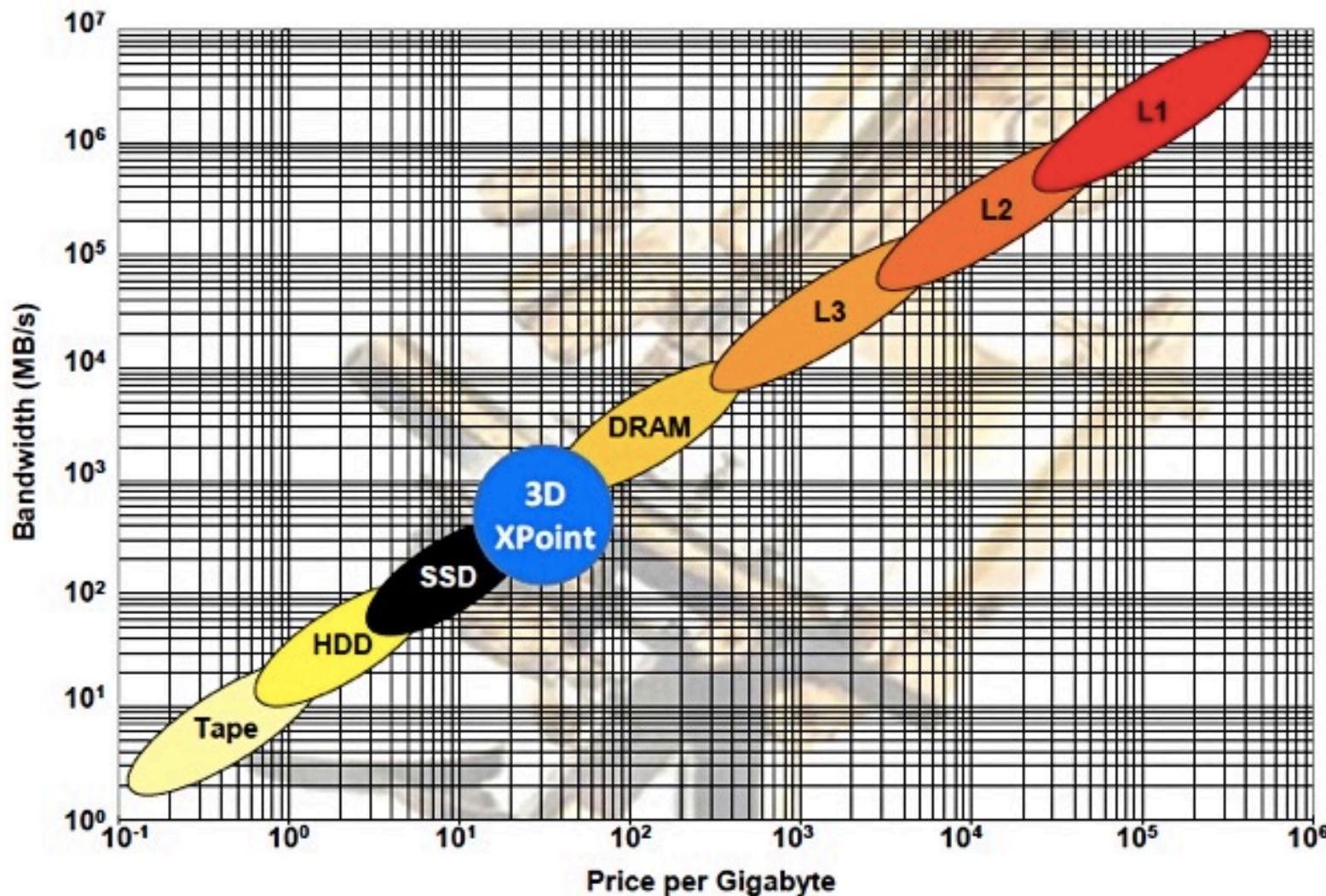
### READ NEXT





# CPU/Memory performance

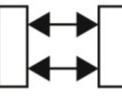
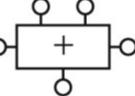
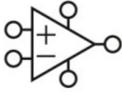




Source: Objective Analysis, 2019

# Course Plan

- Navigate the transformation hierarchy
- Bottom-up ✓
- Top-down

Application Software	>"hello world!"	Programs
Operating Systems		Device Drivers
Architecture		Instructions Registers
Micro-architecture		Datapaths Controllers
Logic		Adders Memories
Digital Circuits		AND Gates NOT Gates
Analog Circuits		Amplifiers Filters
Devices		Transistors Diodes
Physics		Electrons

Week 4

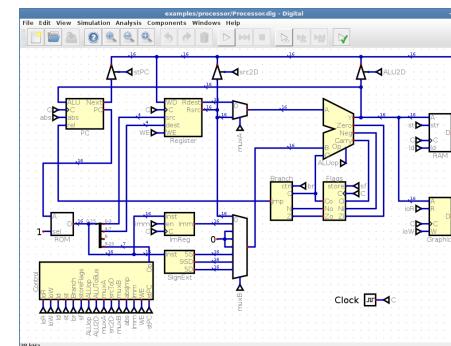
Week 5, 6

Week 2, 3

Week 1

Week 7, 8, 9

Week 10, 11, 12  
(I/O and Advanced  
microarchitecture optimizations)



Assignment 1: Logic simulator

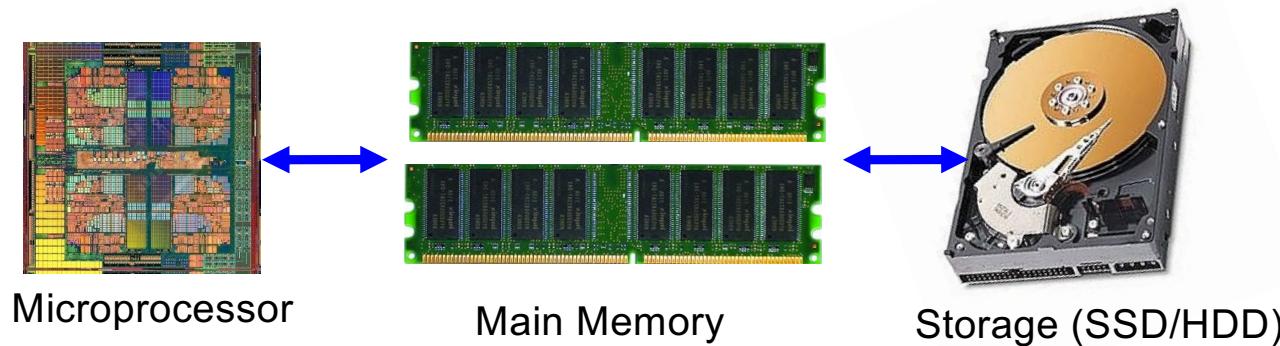
Assignment 2



# Hw/Sw Interaction an Important Skill

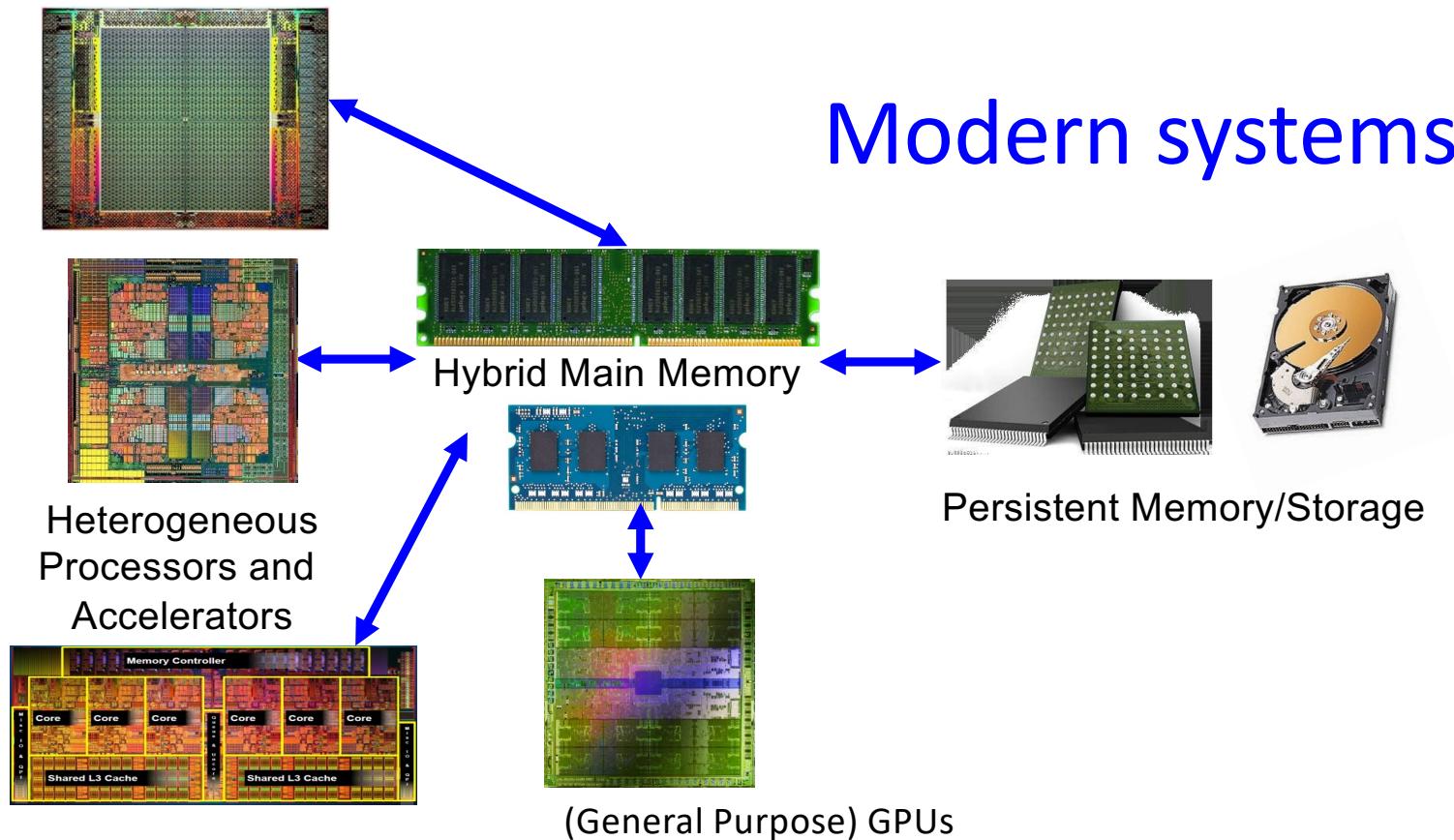
- Hardware is increasingly heterogeneous
- Programmers today need a good understanding of what the hardware offers

## Past Systems



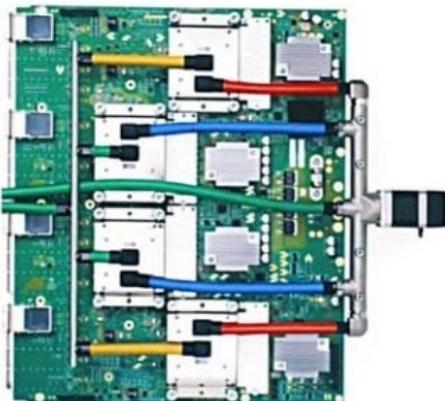
# Hw/Sw Interaction an Important Skill

- Programmers today need a good understanding of what the hardware offers
- FPGAs



# Hw/Sw Interaction an Important Skill

- Programming models are domain-specific
  - New ML/AI accelerators
  - Programming models intertwined with hardware details
  - No luxury of a commodity language



TPU<sup>v4</sup>



# New Important Metrics

- Traditional metrics to evaluate a computer
  - Performance
  - Energy efficiency
  - Battery life
  - Cost

# New Important Metrics

- Today, we care a lot about

- Reliability
- Sustainability
- Security

# Sustainability



- Sustainability is an important concern
  - ICT contribution to **global GHG emissions** around **4%**, and increasing
  - Need to understand emissions during manufacturing of computers and in deployment
  - Sustainability is the latest sub-area in the field of computer architecture

# Security Demands Robust Hardware

- Security trumps performance in many environments
  - Recent cyber attacks target vulnerabilities at the bottom
  - Early mitigations handled in software
  - Today's hardware built for performance
- Need to build fundamentally secure Hw/Sw systems



# Hw/Sw Interaction an Important Skill

- Writing compilers, operating systems, virtual machines



# Hw/Sw Interaction an Important Skill

- Writing compilers, operating systems, virtual machines



- Programming embedded computers
  - Nano drones, IoT devices, wearable computing



# Hw/Sw Interaction an Important Skill

- Writing compilers, operating systems, virtual machines



- Programming embedded computers
  - Nano drones, IoT devices, wearable computing



- Debugging performance issues better than the average programmer



# Programming Perspective

- Characteristics of great code
  - Is easy to read and maintain
  - Well-commented
  - Follows good style guidelines
  - Easy to modify
  - Well-documented
  - Well-tested and correct

# COMP2300 Emphasizes

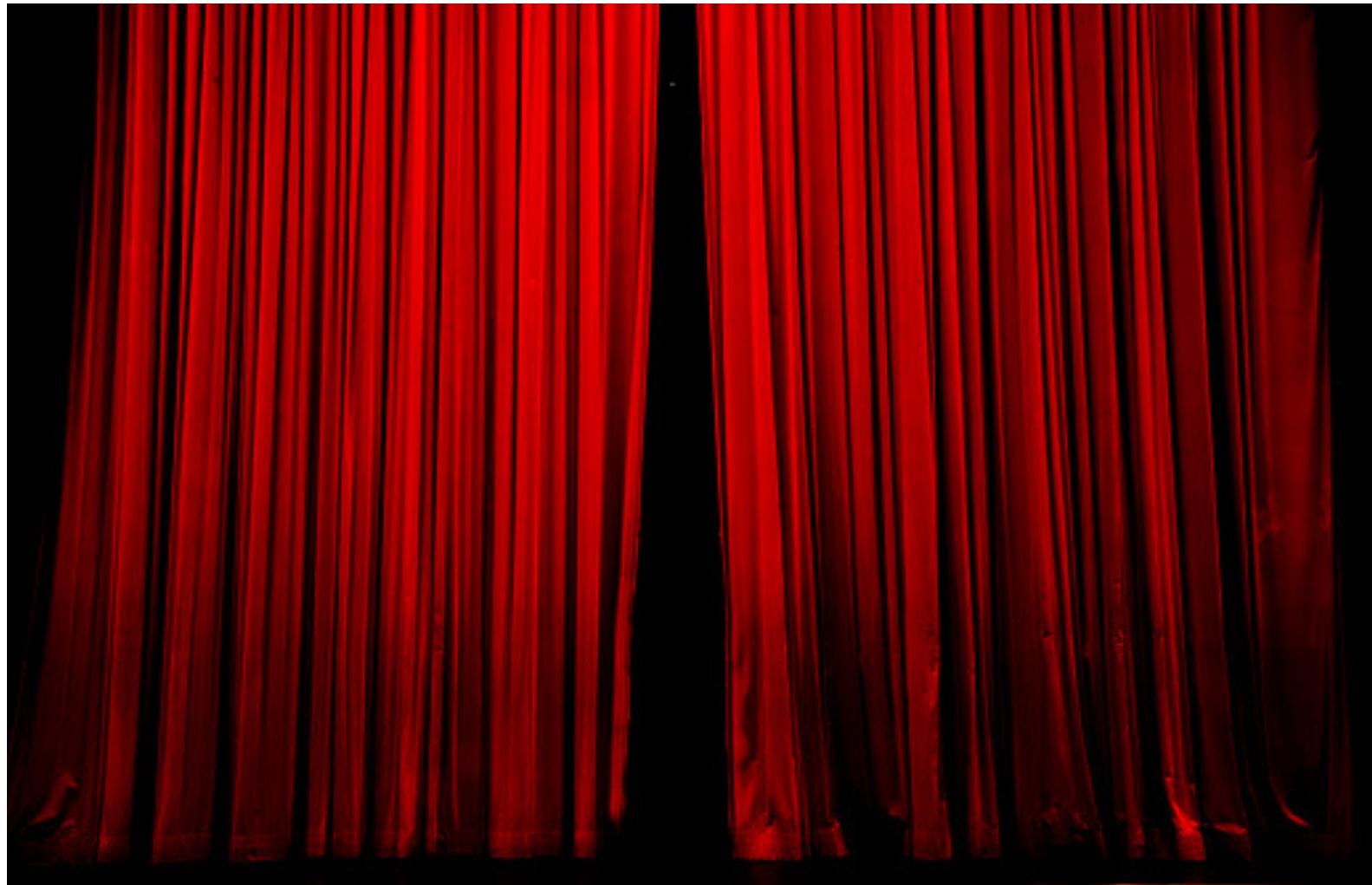
- Characteristics of great code
  - Is easy to read and maintain
  - Well-commented
  - Follows good style guidelines
  - Easy to modify
  - Well-documented
  - Well-tested and correct
  - Uses the CPU efficiently
  - Uses the memory efficiently
  - Uses system resources efficiently

# Our Ultimate Goals

- Make you think broadly *about computing*
- Make you think critically
- Give INSIGHT (into the NATURE of things)

# Some Comments from Last Year

- COMP2300 is “empowering”
- “**After building the CPU, I know exactly what my program is doing**”
- “When I engage with a computer expert, I know what they are talking about”
- “I use the knowledge of modern processors from this course every single day” *Now a Security professional*



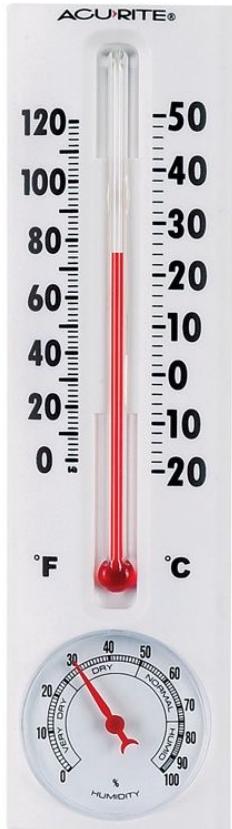
# Representing Information

- Continuous or analog variables can take an infinite number of values
  - Frequency of oscillation
  - Voltage
  - Position
  - Volume

# Representing Information

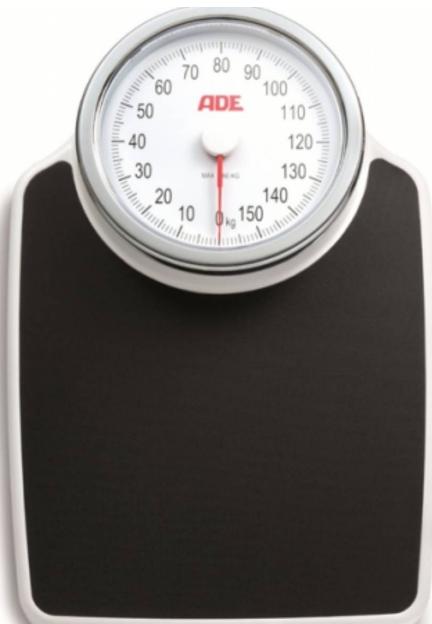
- **Using continuous variables to represent information is hard for the computer to deal with**
  - Sometimes difficult to measure precisely
  - Difficult to store
  - Difficult to copy

# Representing Information



- Mercury's volume represents temperature

# Representing Information



- Position of needle **represents** weight

# Representing Information



- Modulated grooves on vinyl record **represent** sound

# Representing Information



- Chemical properties of film **represent** captured image

# Representing Information

- Measuring things and storing data by **analogy** has been the **predominant** approach in history
- Engineers call signals that can take an **infinite number of values** analog even when they are not an analogy for something

# Representing Information

- If we don't want to represent data as something with **potentially infinitely varying analog values**, what can we do?
- Use the **Digital** approach

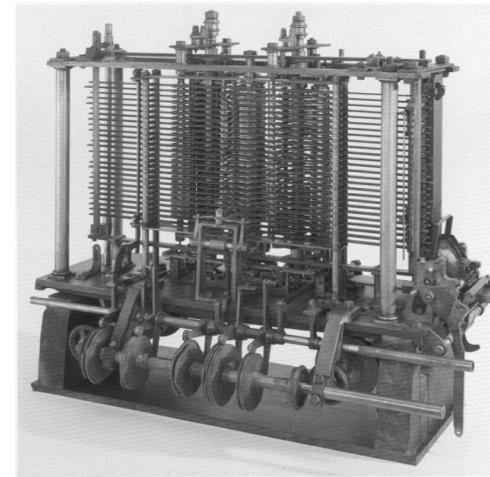
# Representing Information

**Digital Systems** represent information with discrete-valued variables

Variables with a finite number of distinct values

# Representing Information

- Charles Babbage's analytical engine used ten discrete values



- Used mechanical parts such as gears with ten positions 0 – 9

# Representing Information

Modern digital systems use a binary (*two-valued*) representation



0 1

0 1

0 1

# Representing Information

**High voltage:** Presence of something meaning 1

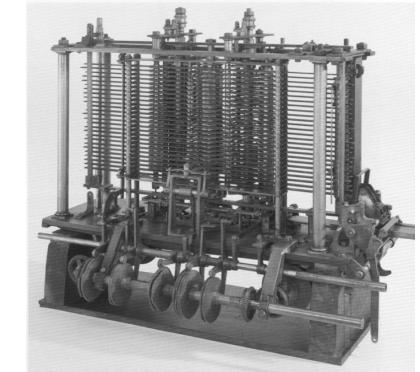
**Low voltage:** Absence of something meaning 0



0 1

# Why Voltage?

- Mechanical parts are not easy to scale to do large computations – circa 1850 Babbage engine
- Some 1964 computers (CDC 6600 & IBM 360)



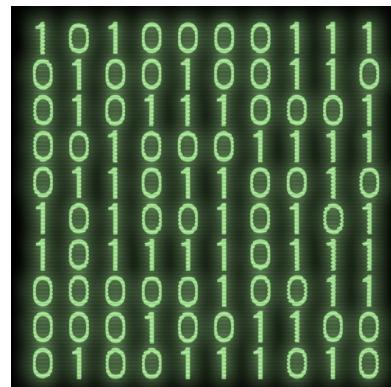
- 2020 Apple M1 400 mm<sup>2</sup> and 16 billion transistors



# Representing Information

We need more than **0** and **1** to represent large quantities and sets

**Data is represented using a sequence of symbols where each symbol is 0 or 1**



A 10x10 grid of binary digits (0s and 1s), representing data in binary format. The digits are arranged in 10 rows and 10 columns.

1	0	1	0	0	0	0	1	1	1
0	1	0	0	1	0	0	1	1	0
0	1	0	1	1	1	0	0	0	1
0	0	1	0	0	0	1	1	1	1
0	1	1	0	1	1	0	0	1	0
1	0	1	0	0	1	0	1	0	1
1	0	1	1	1	1	0	1	1	1
0	0	0	0	1	0	0	1	1	1
0	0	0	1	0	0	1	1	0	0
0	1	0	0	1	1	1	0	1	0

# Binary Representation

Digital systems internally use “voltages” for representing binary variables

- Low voltage means 0
- High voltage means 1

B I N A R Y   D I G I T

- A **bit** is a unit of information
- A binary variable represents one bit of information
- To represent discrete sets with more than two elements, we combine multiple bits into a binary code

# Binary Codes

Suppose we want to represent four colors: {red, blue, green, black}

- How many bits of information do I need?
- (00, 01, 10, 11)
- The assignment of the **2-bit binary code** to colors is *ad-hoc*
- Also legitimate is: (10, 11, 00, 01)

How many bits of information do we need to represent the alphabet set in English?

- **For 26 alphabets, we need 5 bits**

# Information Content in a Binary Code

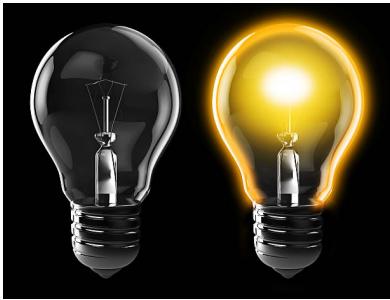
$$D = \log_2 N \text{ bits}$$

- The color set has four states:  $N = 4$ , # bits = 2
- The alphabet set has 26 states:  $N = 26$ , # bits = 5
- Conversely,
  - If  $D$  is 2,  $N = 4$
  - If  $D$  is 5,  $N = 32$

# Why do computers use binary?

- Two symbols enable **simplified hardware** and **improved reliability**
- Keep **complexity** and **cost** under control
- It is easy to use the amazing transistor as a **switch!**
  - We will see later

# TRUE and FALSE



0 1

F T

False Off  
True On

- **True and False** are called logical values
  - Logical variable is one that can be **1** or **0** (**True or False**)

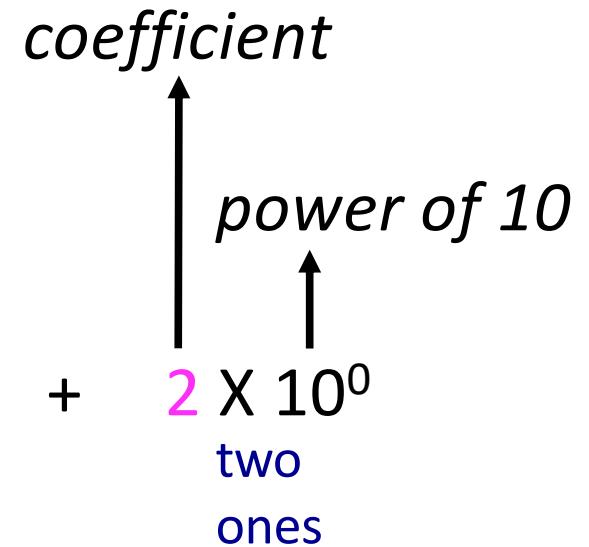
# Decimal Number System

- Base 10 means 10 digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)
- Multiple digits form longer decimal numbers
- Each column of a decimal number has 10 times the weight of the previous column

1's column  
10's column  
100's column  
1000's column

$$9742 = 9 \times 10^3 + 7 \times 10^2 + 4 \times 10^1 + 2 \times 10^0$$

nine thousands      seven hundreds      four tens      two ones



# Range of Decimal Numbers

- An N-digit decimal number represents one of  $10^N$  possibilities
  - $0, 1, 2, 3, \dots, 10^N - 1$
- **3 digits:** 1000 possibilities in the range 0 – 999

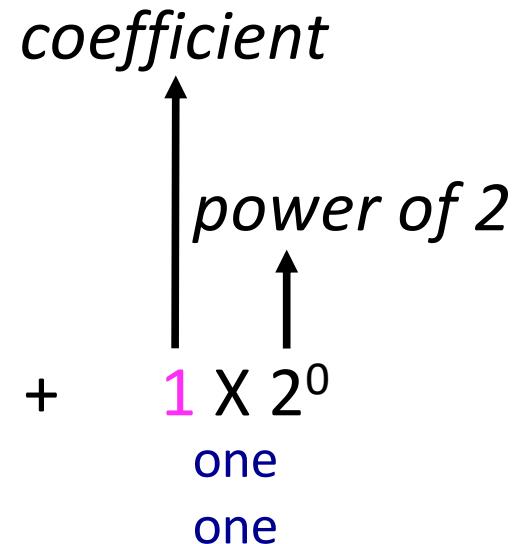
# Binary Numbers

- Base 2 means 2 digits (0, 1)
- Multiple bits form longer binary numbers
- Each column of a binary number has **2** times the weight of the previous column

8's  
4's  
2's  
1's  
column  
column  
column  
column

$$1001 = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

one                    one                    zero                    one  
      eight              four                    two                    one



# Range of Binary Numbers

An N-bit binary number represents one of  $2^N$  possibilities

- 0, 1, 2, 3, ...,  $2^N - 1$
- 3 bits: 8 ( $= 2 \times 2 \times 2$ ) possibilities in the range 0 – 7
- 4 bits: ?
- 5 bits: ?
- 10 bits: ?

# Powers of 2

Columns #	Power of 2	Weight
0	$2^0$	1
1	$2^1$	2
2	$2^2$	4
3	$2^3$	8
4	$2^4$	16
5	$2^5$	32
6	$2^6$	64
7	$2^7$	128
8	$2^8$	256
9	$2^9$	512

Columns #	Power of 2	Weight
10	$2^{10}$	1024
11	$2^{11}$	2048
12	$2^{12}$	4096
13	$2^{13}$	8192
14	$2^{14}$	16384
15	$2^{15}$	32768
16	$2^{16}$	65536

Kilo

# Powers of 2

Power of 2	Decimal Value	Abbreviation	
$2^{10}$	1024	Kilo (K)	~ 1000
$2^{20}$	1048576	Mega (M)	~ 1000, 000
$2^{30}$	1073741824	Giga (G)	~ 1000, 000, 000

What is  $2^{24}$  in decimal?

- $2^{20} \times 2^4 = 1 \text{ M} \times 16 = 16 \text{ M}$

What is  $2^{17}$  in decimal?

- $2^{10} \times 2^7 = 1 \text{ K} \times 128 = 128 \text{ K}$

# Terminology

Byte

- 8 bits

Nibble

- 4 bits

Word

- Machine-dependent
- 8 – 16 bits (gadgets)
- 32 – 64 bits (high-end)

Most Significant Bit

1 0 0 0 0 0 0 1

*The bit in the highest position*

Least Significant Bit

1 0 0 0 0 0 0 1

*The bit in the lowest position*

# Terminology

Most Significant Byte

0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1

*The byte in the **highest** position*

Least Significant Byte

0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1

*The byte in the **lowest** position*

# Rev: Binary Codes

Column weight  
Column #

$2^2$	$2^1$	$2^0$
2	1	0
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

3-bit  
8 elements

- Write combinations in a systematic way
- Note how often the bit flips in each column
- Can represent any arbitrary set with a code

# Rev: Binary Codes

0
1

1-bit

2 elements

0	0
0	1
1	0
1	1

2-bit

4 elements

0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

3-bit

8 elements

0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

4-bit

16 elements

- Write combinations in a systematic way
- Note how often the bit flips in each column
- Can represent any arbitrary set with a code

# Decimal to Binary Conversion

**Method # 1:** Find the largest power of 2, subtract, and repeat

**Example:** Convert  $53_{10}$  to binary

53	$32 \times 1$
$53 - 32 = 21$	$16 \times 1$
$21 - 16 = 5$	$4 \times 1$
$5 - 4 = 1$	$1 \times 1$

$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$
1	1	0	1	0	1

# Decimal to Binary Conversion

**Method # 2:** Repeatedly divide by 2, remainder goes in each column

**Example:** Convert  $53_{10}$  to binary

$53/2$	= 26	R: 1
$26/2$	= 13	R: 0
$13/2$	= 6	R: 1
$6/2$	= 3	R: 0
$3/2$	= 1	R: 1
$1/2$	= 0	R: 1

$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$
1	1	0	1	0	1

# Hexadecimal Numbers

**Motivation:** Tedious and error-prone to write long binary numbers

**Hexadecimal or base 16:** A group of four bits represent  $2^4$  or 16 possibilities

**16 digits:** 0 – 9, A, B, C, D, E, F

**Column weights:** 1, 16,  $16^2$  (or 256),  $16^3$  (or 4096)

# Hexadecimal Numbers

Hex Digit	Decimal Equivalent	Binary Equivalent
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

# Binary to Hexadecimal

Binary	1	1	1	1	0	1	1	1
--------	---	---	---	---	---	---	---	---

Hexa	F	7
------	---	---

Binary	1	1	1	1	1	1	1	0
--------	---	---	---	---	---	---	---	---

Hexa	F	E
------	---	---

# Hexadecimal to Binary

Hexa      D | 7 | 4 | 1

Binary    1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1

# Binary Addition

The diagram illustrates the concept of binary addition by comparing it with decimal addition. On the left, a decimal addition problem is shown:

$$\begin{array}{r} 4277 \\ + 5499 \\ \hline 9776 \end{array}$$

On the right, a corresponding binary addition problem is shown:

$$\begin{array}{r} 1011 \\ + 0011 \\ \hline 1110 \end{array}$$

A red arrow labeled "carries" points from the first column of the binary addition to the second column, indicating that a carry occurred from the first column to the second.

Decimal  
Addition

Binary  
Addition

$1 + 1 = 2$  (10 in binary), but a binary variable can either be 0 or 1

- We record the 1's digit (0), and carry the 2's digit (1) over to the next column

$1 + 1 + 1 = 3$  (11 in binary), but a binary variable can either be 0 or 1

- We record the 1's digit (1), and carry the 2's digit (1) over to the next column

# Overflow

- $A = 1111$  and  $B = 1111$ 
  - $A + B$  does not fit in the largest value four bits can represent
- **Overflow:** When the result is too big to fit inside the available bits
- **Detection:** If there is a carry bit out of the most significant column

$$\begin{array}{r} 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 15 \\ + & 1 & 1 & 1 & 1 & 15 \\ \hline 1 & 1 & 1 & 1 & 0 & 30 \end{array}$$

# Signed Binary Numbers

# Signed Binary Numbers

- We need both positive and negative numbers to solve real-world problems
- *How do we make a string of 1 and 0 represent both positive and negative numbers?*



- If we write all possible combinations of 0 and 1 in a disciplined fashion, maybe we can find a way

Most significant bit	least significant bit
0	0 0
0	0 1
0	1 0
0	1 1
1	0 0
1	0 1
1	1 0
1	1 1

# Signed Binary Numbers

- Use the *most significant bit* to represent the sign: 0 means positive and 1 means negative
- **N bit sign/magnitude system:** 1 bit for sign and N – 1 bits for magnitude (absolute)

			Decimal
0	0	0	+0
0	0	1	+1
0	1	0	+2
0	1	1	+3
1	0	0	-0
1	0	1	-1
1	1	0	-2
1	1	1	-3

# Drawbacks of Sign/Mag Rep.

- Ordinary binary addition does not work for sign/magnitude numbers
    - What is the sum of +3 and -3 and **does the result make sense?**
  - Zero has two representations (awkward)
- |   |     | Decimal |
|---|-----|---------|
| 0 | 0 0 | +0      |
| 0 | 0 1 | +1      |
| 0 | 1 0 | +2      |
| 0 | 1 1 | +3      |
| 1 | 0 0 | -0      |
| 1 | 0 1 | -1      |
| 1 | 1 0 | -2      |
| 1 | 1 1 | -3      |

# One's Complement

- 1's complement was tried in early computers, such as, Control Data Corporation (CDC) 6600
- **Negative number:** Flip all bits of the binary representation of a positive integer
- Suffers from the same problems as the sign/magnitude representation

			Decimal
0	0	0	+0
0	0	1	+1
0	1	0	+2
0	1	1	+3
1	0	0	-3
1	0	1	-2
1	1	0	-1
1	1	1	-0

# Two's Complement

- A third system of representation for signed integers
- Ordinary addition works
- There is a single representation for *zero*
- Used in almost all computers today

**Problem:** If  $\text{A} + \text{B} = \text{C}$ , and  $\text{A}$  is known, then find  $\text{B}$ , such that  $\text{C} = 0$

Binary						Decimal
A	=	0	1	0	1	?
+	B	=	?	?	?	?
C	=	0	0	0	0	0

**Problem:** If  $\text{A} + \text{B} = \text{C}$ , and  $\text{A}$  is known, then find  $\text{B}$ , such that  $\text{C} = 0$

Binary						Decimal
A	=	0	1	0	1	+5
+	B	=	?	?	?	?
C	=	0	0	0	0	0

**Problem:** If  $\text{A} + \text{B} = \text{C}$ , and  $\text{A}$  is known, then find  $\text{B}$ , such that  $\text{C} = 0$

Binary						Decimal
A	=	0	1	0	1	+5
+	B	=	?	?	?	-5
C	=	0	0	0	0	0

**Problem:** If  $\text{A} + \text{B} = \text{C}$ , and  $\text{A}$  is known, then find  $\text{B}$ , such that  $\text{C} = 0$

Binary						Decimal
A	=	0	1	0	1	+5
+	B	=	1	0	1	-5
C	=	0	0	0	0	0

**Problem:** If  $\text{A} + \text{B} = \text{C}$ , and  $\text{A}$  is known, then find  $\text{B}$ , such that  $\text{C} = 0$

*What is the relationship between  $\text{A}$  and  $\text{B}$ ?*

Binary						Decimal
A	=	0	1	0	1	+5
+	B	=	1	0	1	-5
C	=	0	0	0	0	0

# Some Observations

**Observation # 1:** If  $A + B = C$ , and  $A$  is +5, and  $C$  is 0, then  $B$  must be -5. (We have found a new representation for negative numbers.)

**Observation # 2:** To transform  $A$  to  $B$  (i.e., +5 to -5), we need to take 1's complement of  $A$  and then add +1. We say that  $B$  is **2's complement** of  $A$

**Observation # 3:** Like sign/magnitude numbers, positive numbers have the MSB set to 0, and negative numbers have the MSB set to 1

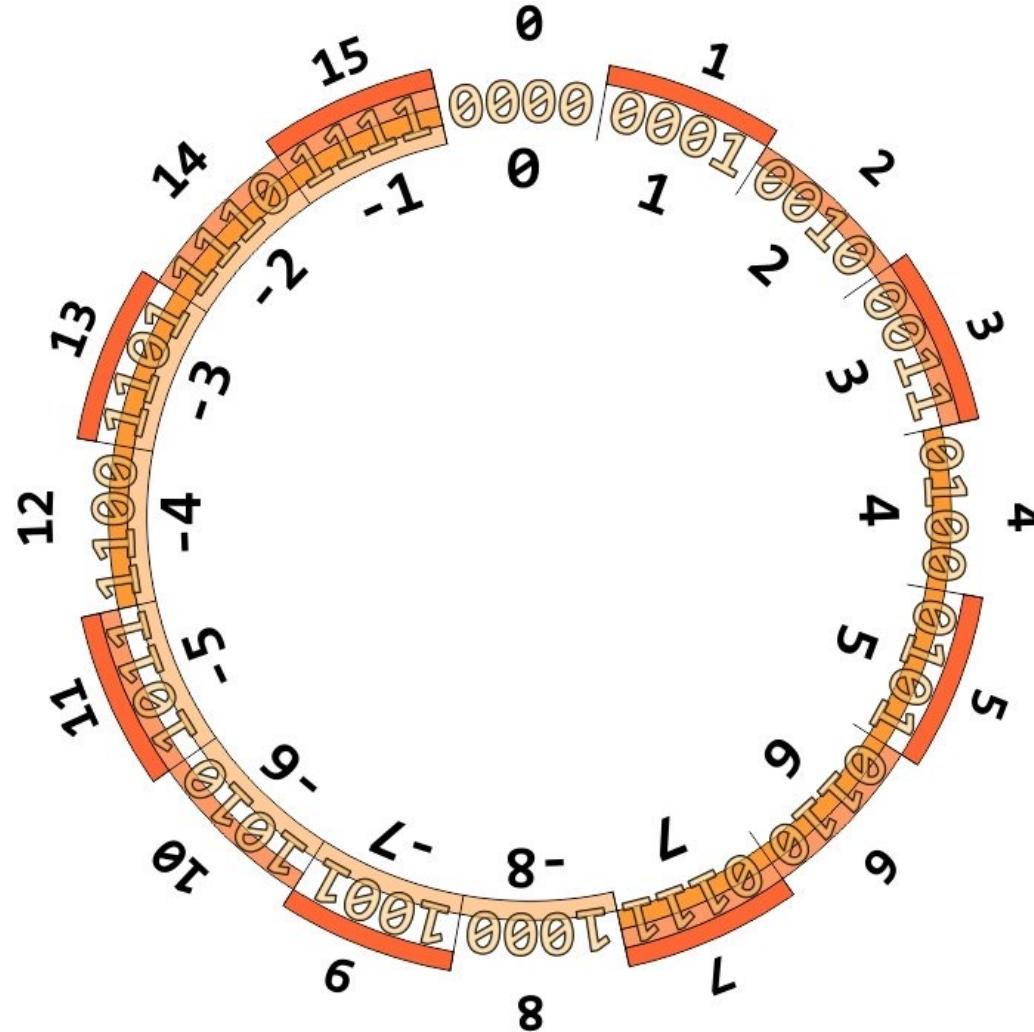
# Some Observations

**Observation # 4:** Ordinary addition works

- What is the sum of +3 and -3 in two's complement system, and *does the result make sense?*
- Since ordinary addition works, a circuit to add numbers can handle both addition and subtraction
  - Recall that,  $X - A$  is equivalent to  $X + (-A)$

$$\begin{array}{r} & & & 1 \\ & 0 & 1 & 1 & +3 \\ + & 1 & 0 & 1 & -3 \\ \hline 0 & 0 & 0 \end{array}$$

# 2's Complement Circle



# More Observations

**Observation # 5:** There is only one representation for *zero*

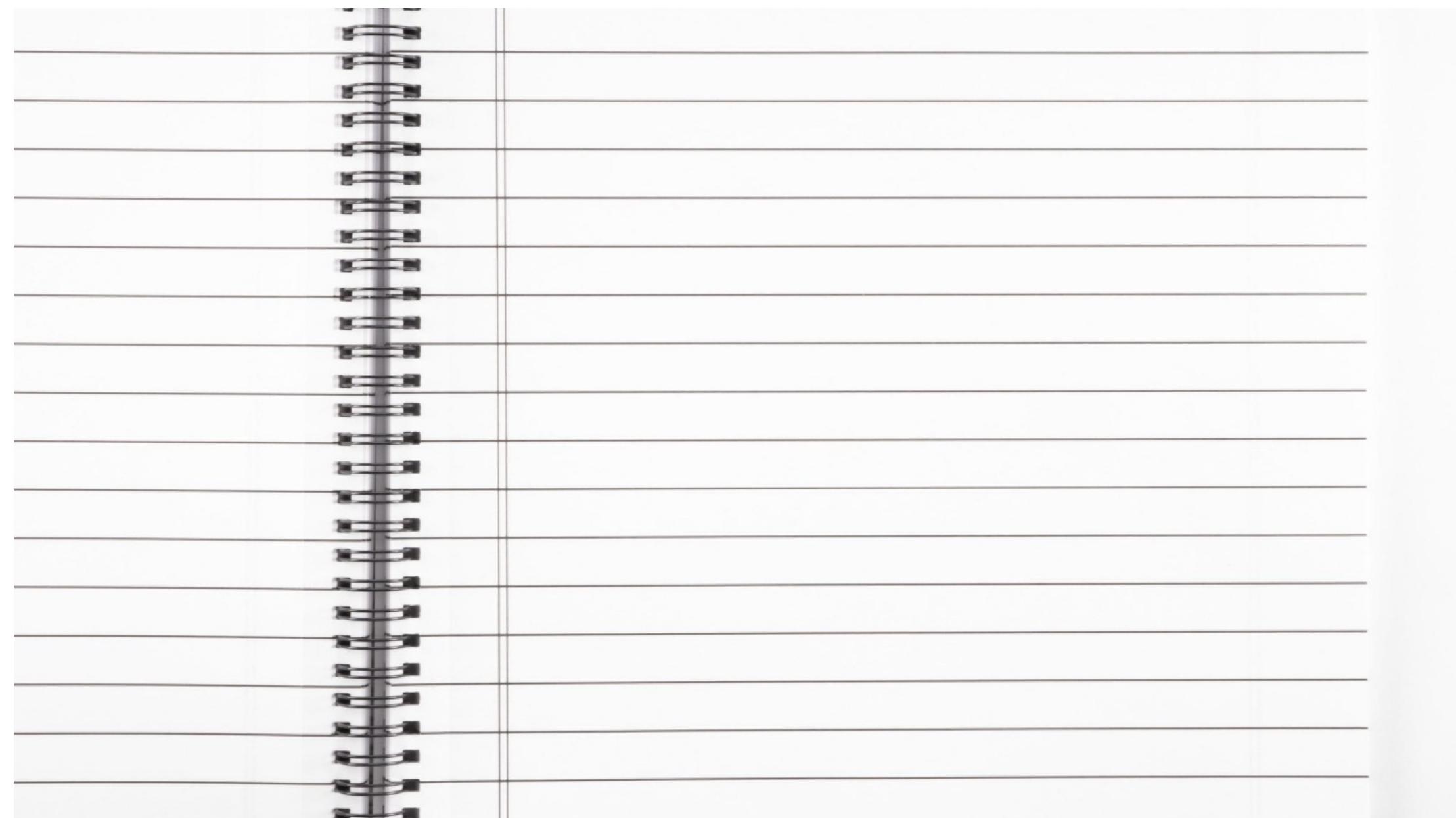
**Observation # 6:** There is one more negative number than positive number

- With 3 bits, this number is 100
- With 4 bits, this number is 1000
- This negative number has no positive counterpart
- It is called the *weird number*
- The 2's complement of the weird number is the weird number (verify!)

# 2's Complement to Decimal

- If MSB is **0**
  - It is a positive number. The magnitude is represented by the remaining  $N - 1$  bits
- If MSB is **1**
  - It is a negative number. Take the two's complement of the (binary) number. The magnitude of the negative number) is represented by the  $N - 1$  bits

**Practice and test your understanding using the two's complement circle**



# Overflow in 2's Complement

- Suppose we have two 5-bit numbers
  - $A = 01001$  and  $B = 01011$
  - What is  $A + B$ ?
  - What is the largest value 5 bits can represent in 2's complement?
- Overflow
  - The result is too big to fit inside the available bits
  - Sum of two positive integers cannot produce a negative integer!

$$\begin{array}{r} & 1 & 1 \\ & \hline & 0 & 1 & 0 & 0 & 1 & +9 \\ + & 0 & 1 & 0 & 1 & 1 & +11 \\ \hline & 1 & 0 & 1 & 0 & 0 & -12 \end{array}$$

The diagram shows a binary addition of two 5-bit numbers, 9 and 11, resulting in -12. The numbers are aligned by their most significant bits. The sum is calculated column by column from right to left. The first column (least significant) is 1 + 1 = 0. The second column is 0 + 0 = 0. The third column is 1 + 1 = 0. The fourth column is 0 + 1 = 1. The fifth column (most significant) is 0 + 0 = 0. A red box highlights the carry bit from the fourth column (1) and the result of the fifth column (0), which together form the 6th bit of the result, a 1. This indicates an overflow because the sum of two positive numbers (9 and 11) cannot be represented in 5 bits and resulted in a 6-bit value.

# Overflow in 2's Complement

- Suppose we have two 5-bit numbers
  - $A = 10100$  and  $B = 11010$
  - What is  $A + B$ ?
  - What is the *smallest* value 5 bits can represent in 2's complement?
- Overflow
  - The result is too big to fit inside the available bits
  - Sum of two negative integers cannot produce a positive integer!

$$\begin{array}{r} \boxed{1} 0 1 0 0 & -12 \\ + 1 \boxed{1} 0 1 0 & -6 \\ \hline 0 \boxed{1} 1 1 0 & 14 \end{array}$$

# Overflow in 2's Complement

**Observation # 1:** If two number being added have the same sign bit and the result has the opposite sign bit (easy!)

**Observation # 2:** Unlike unsigned numbers, a carry out of the most significant bit does not indicate overflow

**Observation # 3:** The sum of a negative number and a positive number never produces an overflow (**prove yourself!**)

# Range of Number Systems

Number System	Minimum	Maximum
Unsigned	0	$2^N - 1$
Sign/Magnitude	$-2^{N-1} + 1$	$2^{N-1} - 1$
Two's Complement	$-2^{N-1}$	$2^{N-1} - 1$

N = 3

Unsigned: 0 to 7

Sign/Magnitude: -3 to 3

2's Complement: -4 to 3

N = 4

Unsigned: 0 to ?

Sign/Magnitude: -? to ?

2's Complement: -? to ?

# Comparing Number Systems

Binary Representation	Decimal Value Represented			
	Unsigned	Signed Magnitude	1's Complement	2's Complement
000	0	0	0	0
001	1	1	1	1
010	2	2	2	2
011	3	3	3	3
100	4	-0	-3	-4
101	5	-1	-2	-3
110	6	-2	-1	-2
111	7	-3	-0	-1

## Quiz: See any errors?



**Unsigned**

**Signed**

**1's Comp.**

**2's Comp.**

0 0 0 0	0	0	0	0
0 0 0 1	1	1	1	1
0 0 1 0	2	2	2	2
0 0 1 1	3	3	3	3
0 1 0 0	4	4	4	4
0 1 0 1	5	5	5	5
0 1 1 0	6	6	6	6
0 1 1 1	7	7	7	7
1 0 0 0	8	-0	-7	-1
1 0 0 1	9	-1	-6	-2
1 0 1 0	10	-2	-5	-3
1 0 1 1	11	-3	-4	-4
1 1 0 0	12	-4	-3	-5
1 1 0 1	13	-5	-2	-6
1 1 1 0	14	-6	-1	-7
1 1 1 1	15	-7	-0	-8

# Sign Extension

**Question:** What is the difference between the 16-bit and 4-bit numbers below?

16-bit number	
4-bit number	

**Answer:** None. They both represent the positive number 5

*Leading zeros do not impact the magnitude of a binary number*

*There are times when it is useful to allocate a small number of bits to represent a value*

# Sign Extension

- What value does the two numbers below represent?

16-bit number (A)

0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1

4-bit number (B)

1 1 0 1

- What is the sum of **A** and **B**?

- **Scenario # 1:** Assume the absence of bits in **B** to be 0
- **Scenario # 2:** Assume the absence of bits in **B** to be 1

# Scenario # 1

$$\begin{array}{r} 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 \\ + 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 \\ \hline \end{array} \quad \begin{array}{l} +13 \\ -3 \end{array}$$

X

*The assumption that appending 0's will lead to correct addition was wrong*

# Scenario # 2

$$\begin{array}{r} 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 1 \\ + 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 1 \\ \hline \end{array} \quad \begin{array}{l} +13 \\ -3 \end{array}$$



*The assumption that appending 1's will lead to correct addition was right*

# Sign Extension

- Leading 0's do not change the magnitude of the positive number
- Leading 1's do not change the magnitude of the negative number

*When a 2's complement number is extended to more bits, the sign bit must be copied into the most significant bit positions.*

*We refer to the operation as Sign-EXTension or SEXT*

# Building Block of Computers

## Transistors

# We are Here.

---

# Transistors

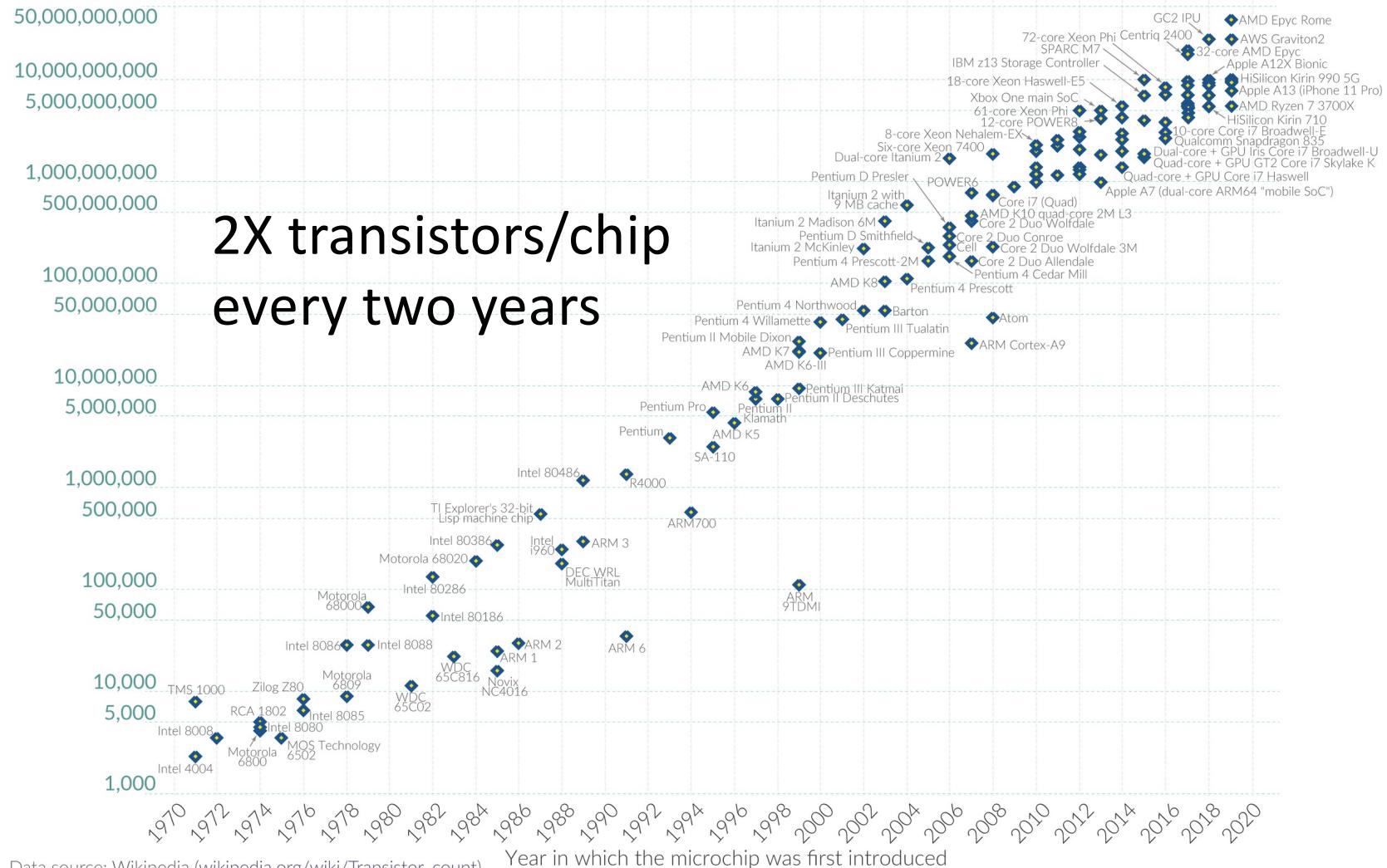
- Computers are built from billions of small and simple structures called transistors
  - 1970: Few 1000s of transistors
  - Apple's M2 Max: **50+ Billion** transistors
  - **Moore's Law:** Transistor count double in 18 months
  - Computers with improved capability over time due to a large # transistors at the device level
- We will cover
  - How MOS transistor works (as a logic element)?
  - How transistors are connected to form logic gates?
  - How logic gates are interconnected to form larger units that are needed to construct a computer

Problem
Algorithm
Program in C/Java
Runtime System (Operating system)
ISA (Architecture)
Microarchitecture
Logic
Devices
Electrons

# Moore's Law: The number of transistors on microchips doubles every two years

Moore's law describes the empirical regularity that the number of transistors on integrated circuits doubles approximately every two years. This advancement is important for other aspects of technological progress in computing – such as processing speed or the price of computers.

## Transistor count



Data source: Wikipedia ([wikipedia.org/w/index.php?title=Transistor\\_count](https://en.wikipedia.org/w/index.php?title=Transistor_count))

OurWorldInData.org – Research and data to make progress against the world's largest problems.

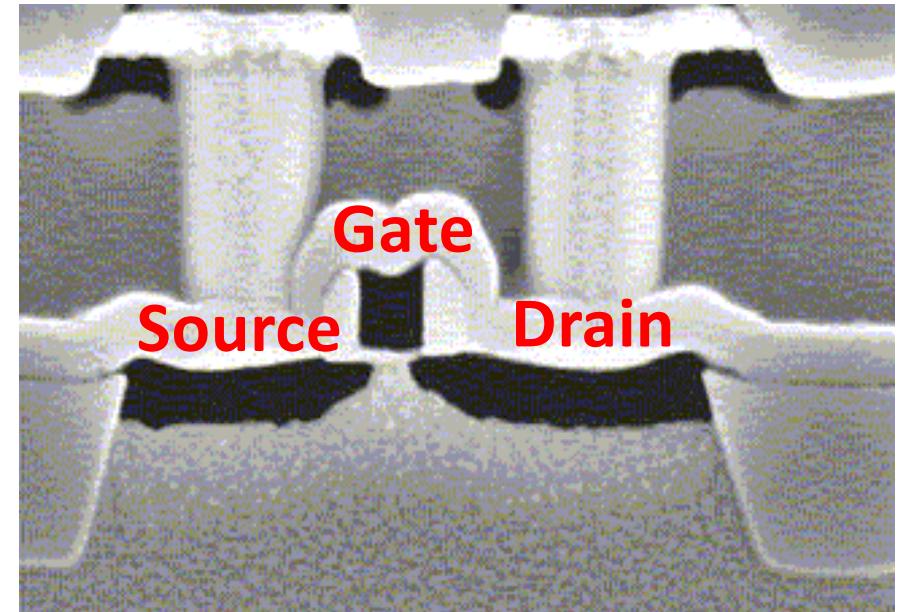
Licensed under CC-BY by the authors Hannah Ritchie and Max Roser.

# Transistors

- Sections 1.6 and 1.7 in Harris & Harris provide more technical explanations than what we will cover

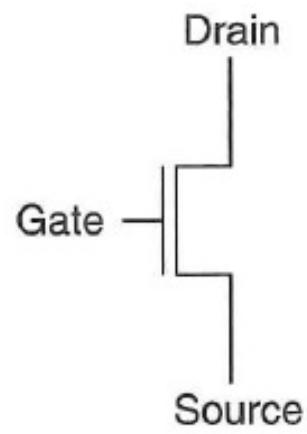
# MOS Transistor

- **MOS** stands for
  - Conductors (**M**etal)
  - Insulators (**O**xide)
  - **S**emiconductors
- MOS transistor has three terminals
- We can combine many of these to form logic gates
  - *The electrical properties of metal-oxide semiconductors are well beyond the scope of what we want to understand in this course*
  - They are below our **lowest level of abstraction**
  - If transistors **misbehave**, an architect is at their mercy (*unlikely to happen*)

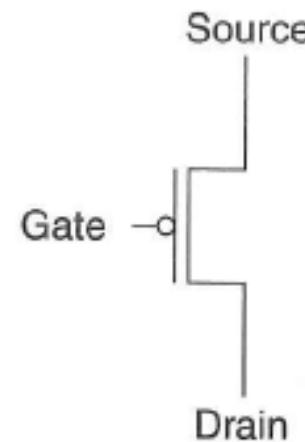


# Two Types of MOS Transistors

- Two types: n-type and p-type
- They both operate “logically,” very similar to the way wall switches work

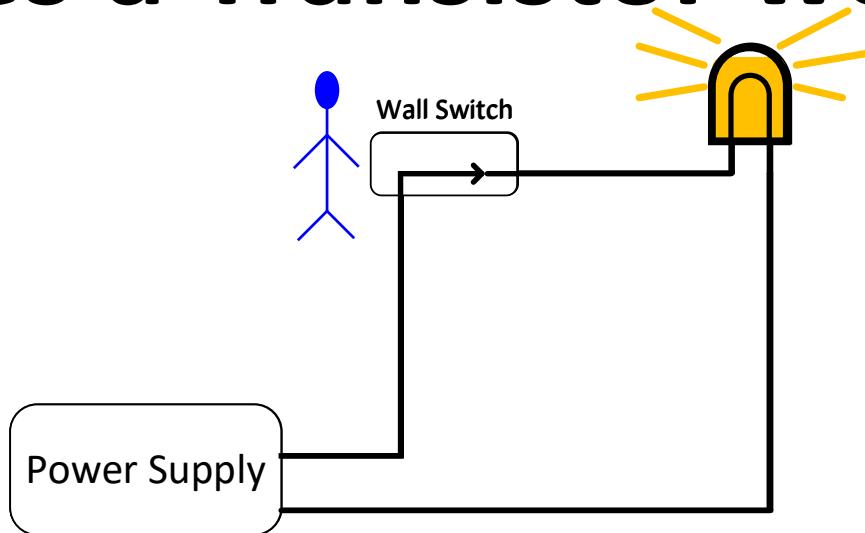


n-type



p-type

# How Does a Transistor work?



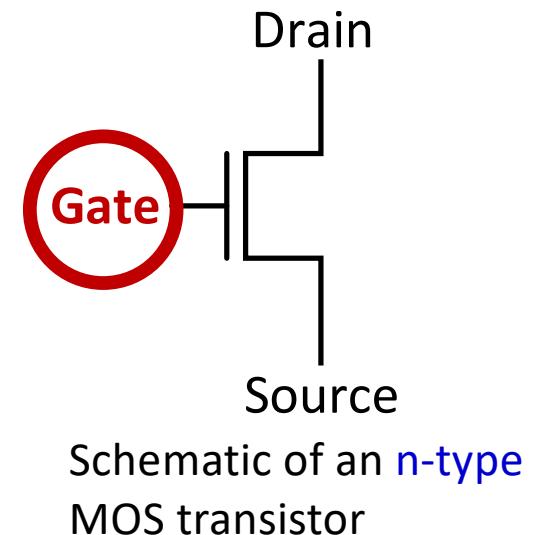
- For the lamp to glow, **electrons must flow**
- For electrons to flow, there must be a **closed circuit** from the power supply to the lamp and back to the power supply
- The lamp can be **turned on and off** by simply manipulating the wall switch to make or break the closed circuit

# How Does a Transistor work?

- Instead of the wall switch, we could use an **n-type** of a **p-type** MOS transistor to make or break the closed circuit

If the gate of the **n-type** transistor is supplied with a **high** voltage, the connection from source to drain acts like a **piece of wire (we have a closed circuit)**

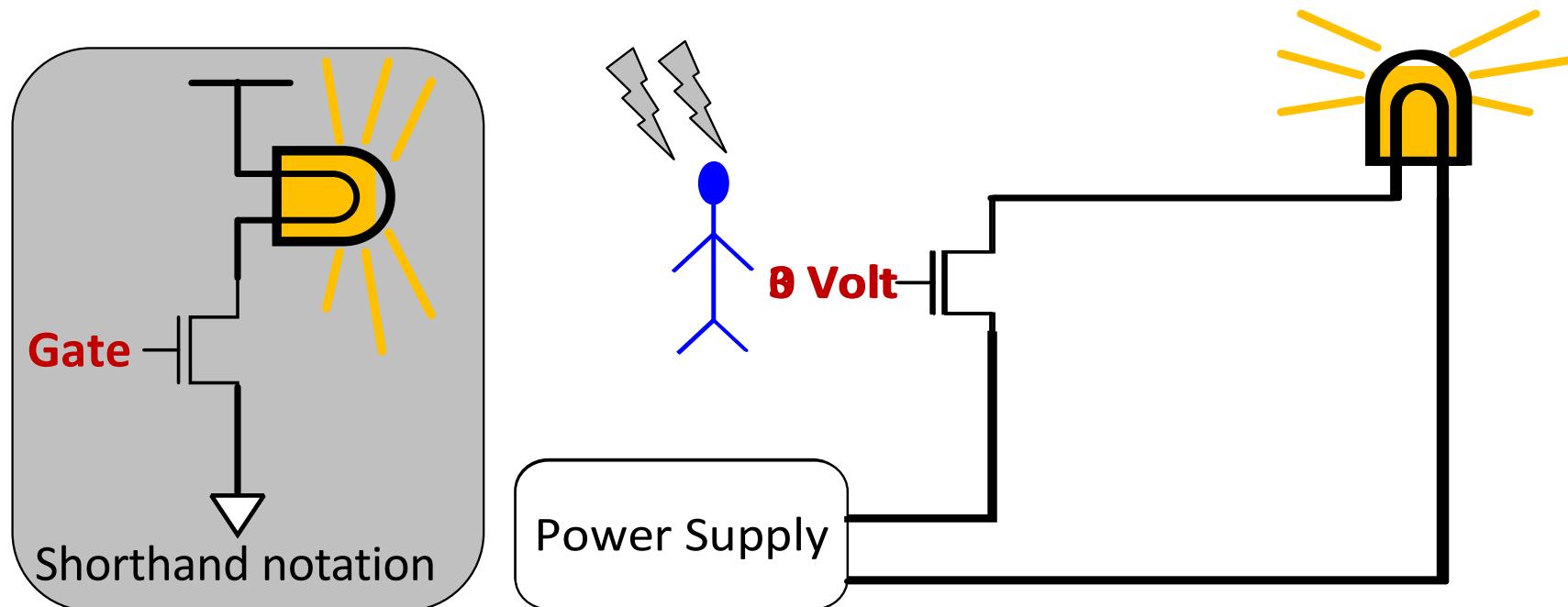
If the gate of the **n-type** transistor is supplied with **zero** voltage, the connection between source and drain is **broken (we have an open circuit)**



- Depending on the technology, high voltage can range from **0.3V** to **3V**

# How Does a Transistor work?

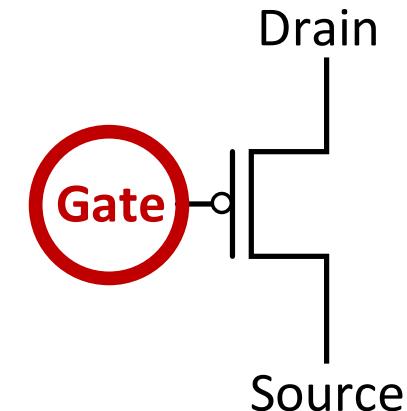
- The n-type transistor in a circuit with a battery and a bulb



# How Does a Transistor work?

- The **p-type** MOS transistor works in exactly the opposite fashion from the **n-type** transistor

The circuit is **open** when the gate is supplied with 3V



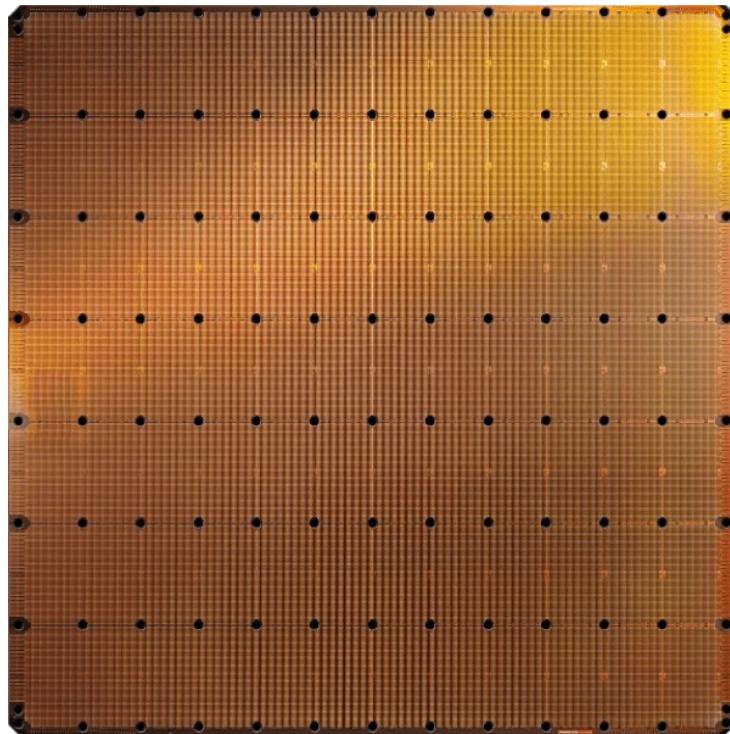
The circuit is **closed** when the gate is supplied with 0V

- Depending on the technology, high voltage can range from **0.3V** to **3V**

# Some Examples of Transistors as Building Blocks

---

# Modern Special-Purpose ASIC



**Cerebras WSE-2**  
2.6 Trillion transistors  
46,225 mm<sup>2</sup>

- The largest ML accelerator chip (2021)
- 850,000 cores

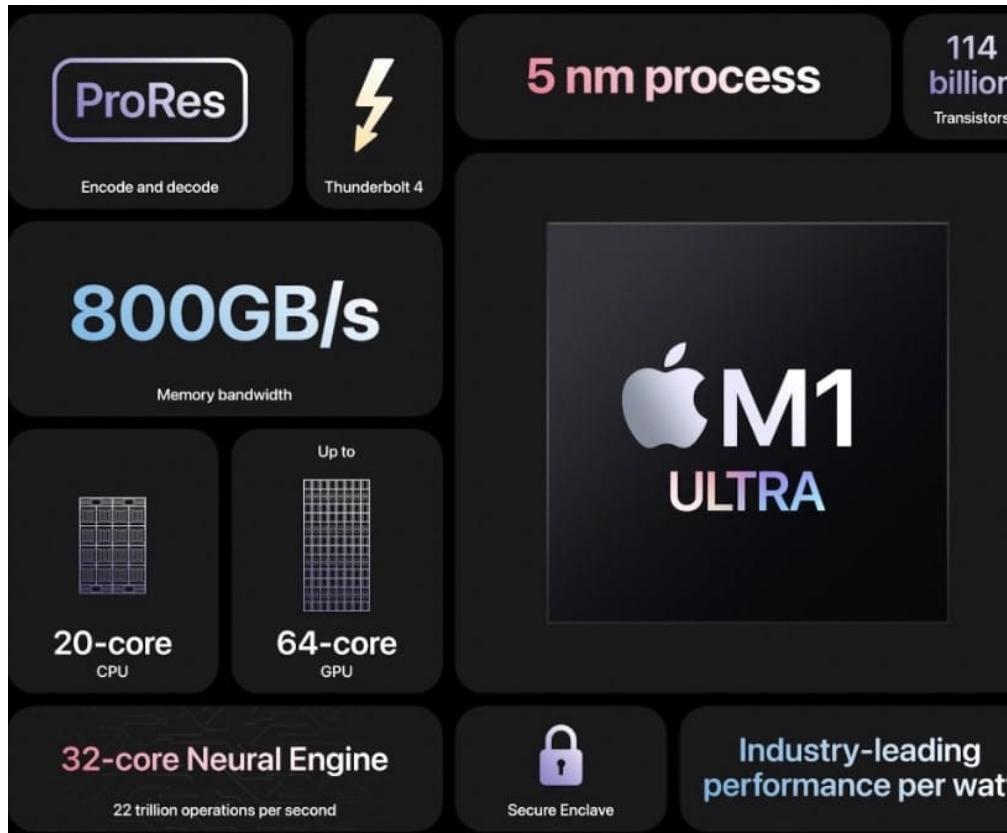


**Largest GPU**  
54.2 Billion transistors  
826 mm<sup>2</sup>  
NVIDIA Ampere GA100

<https://www.anandtech.com/show/14758/hot-chips-31-live-blogs-cerebras-wafer-scale-deep-learning/>

<https://www.cerebras.net/cerebras-wafer-scale-engine-why-we-need-big-chips-for-deep-learning/>

# Apple M1 Ultra (2022)

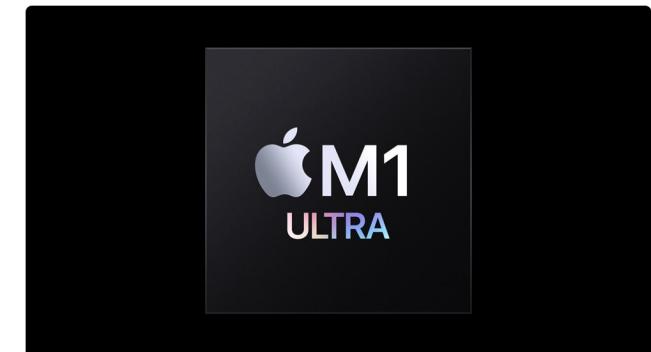


PRESS RELEASE  
09 March 2022

**Apple unveils M1 Ultra, the world's most powerful chip for a personal computer**

Available in the all-new Mac Studio, M1 Ultra brings unprecedented performance to the desktop

[Facebook](#) [Twitter](#) [Email](#) [RSS](#)



M1 Ultra is the world's most powerful and capable chip for a personal computer.

CUPERTINO, CALIFORNIA — Apple today announced M1 Ultra, the next giant leap for Apple silicon and the Mac. Featuring UltraFusion — Apple's innovative packaging architecture that interconnects the die of two M1 Max chips to create a system on a chip (SoC) with unprecedented levels of performance and capabilities — M1 Ultra delivers breathtaking computing power to the new [Mac Studio](#) while

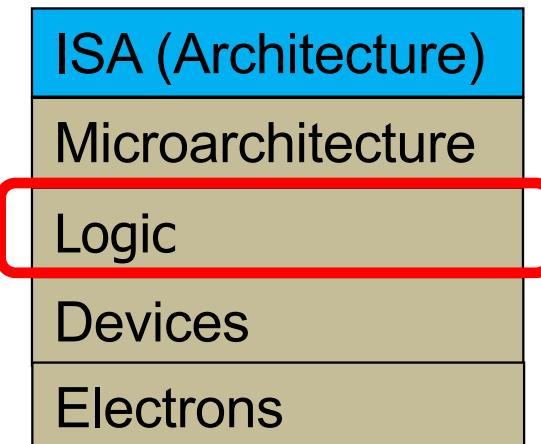
# Modern General-Purpose Microprocessor



# Logic Gates

# One Level Higher in Abstraction

- Now, we know how a **MOS transistor** works
- How do we build logic structures out of individual MOS transistors
- We called these logic structures **logic gates** and they implement simple **Boolean** functions



# Boolean Logic

- A system for describing logical statements/expressions where variables are **TRUE** or **FALSE**
- Defines *important but simple* logical operations on binary logical variables
- Boolean algebra defines manipulation rules similar to elementary algebra

# Logic

- Logic comes from reasoning or thinking
- When presented with some facts, how to derive a valid conclusion
- A statement is either TRUE or FALSE
- When many statements are combined, what is the conclusion?

# Origin of Logic Functions

- Canberra is the **CAPITAL** of Australia
  - **AND** today I am in Canberra
  - **Therefore, today, I am in the CAPITAL city of Australia**
- 
- When it rains, I am **NOT** in office
  - **AND** today it is raining
  - **Therefore, today, I am NOT in office**

# Boolean Logic Functions

- Logical operations are the **steppingstone** for composing **sophisticated digital circuits for performing arithmetic**
- Boolean logic is a system of logic for describing statements consisting of binary variables
  - Operations, rules, axioms, etc

# Logic Functions vs Gates

- Logic gates are digital circuits that take one or more **inputs** and produce a binary **output**
- Logic gate is the physical realization of a logical function built with transistors
- The **inputs** are to the left, and the **output** is to the right
- The relationship between **inputs** and the **output** is described by a **truth table** or a **Boolean equation**

# Truth Table

- A convenient way to describe the behavior of logical functions
- Suppose **A** and **B** are input operands and **Y** is the output
  - **A** can be 0 or 1
  - **B** can be 0 or 1
  - Four combinations (rows)
  - Three columns (2 inputs and an output)
- The Boolean equation for **Y**: **Y = 0**
  - The values of **A** and **B** does not matter

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	0

# Truth Table with More Inputs

A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

- Boolean Equation for output Y:  $Y = 1$

**Note:** Soon we will see more interesting logic functions than  $Y = 0$  and  $Y = 1$

# Logic Gates

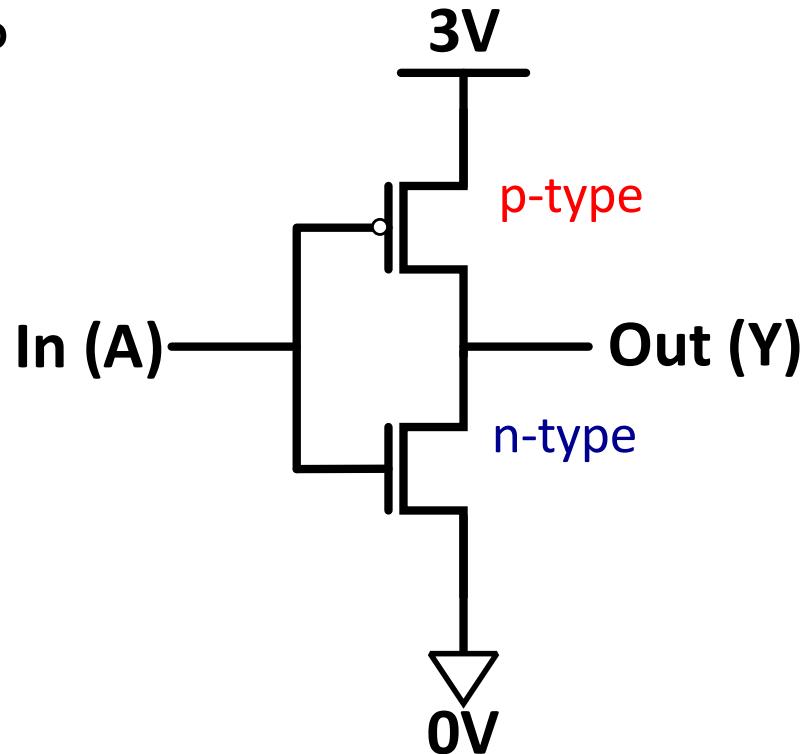
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# Making Logic Gates Using CMOS Technology

- Modern computers use both n-type and p-type transistors, called Complementary MOS (CMOS) technology
  - **nMOS + pMOS = CMOS**

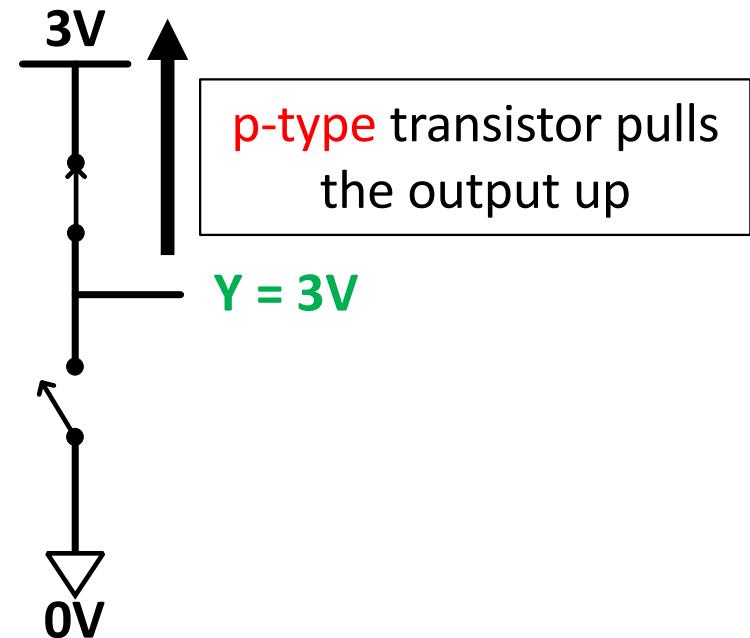
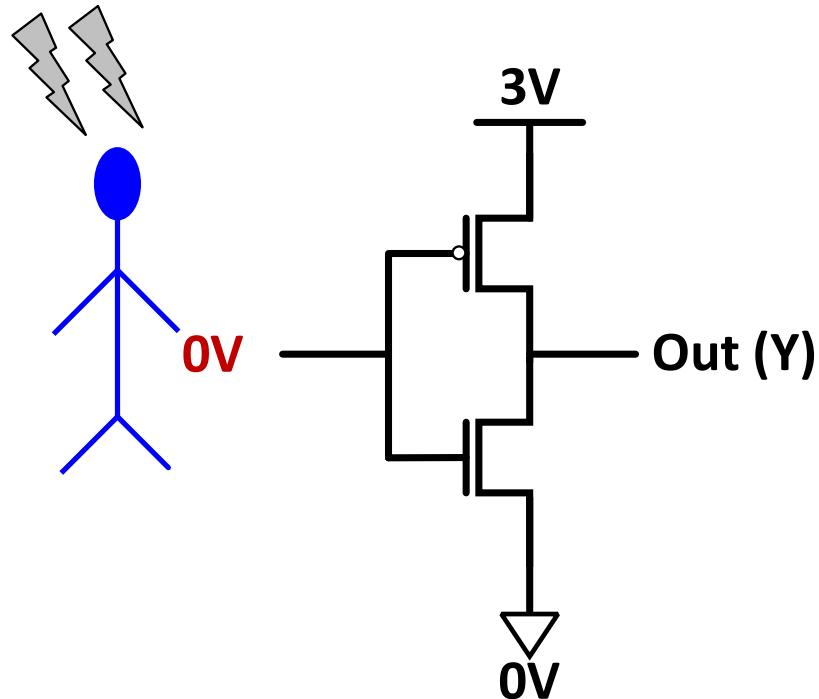
# CMOS Technology Fundamentals

- Let's look at the simplest logic structure that exists in a modern computer
  - What does this circuit do?



# CMOS Technology Fundamentals

- What happens when the input is connected to 0V?

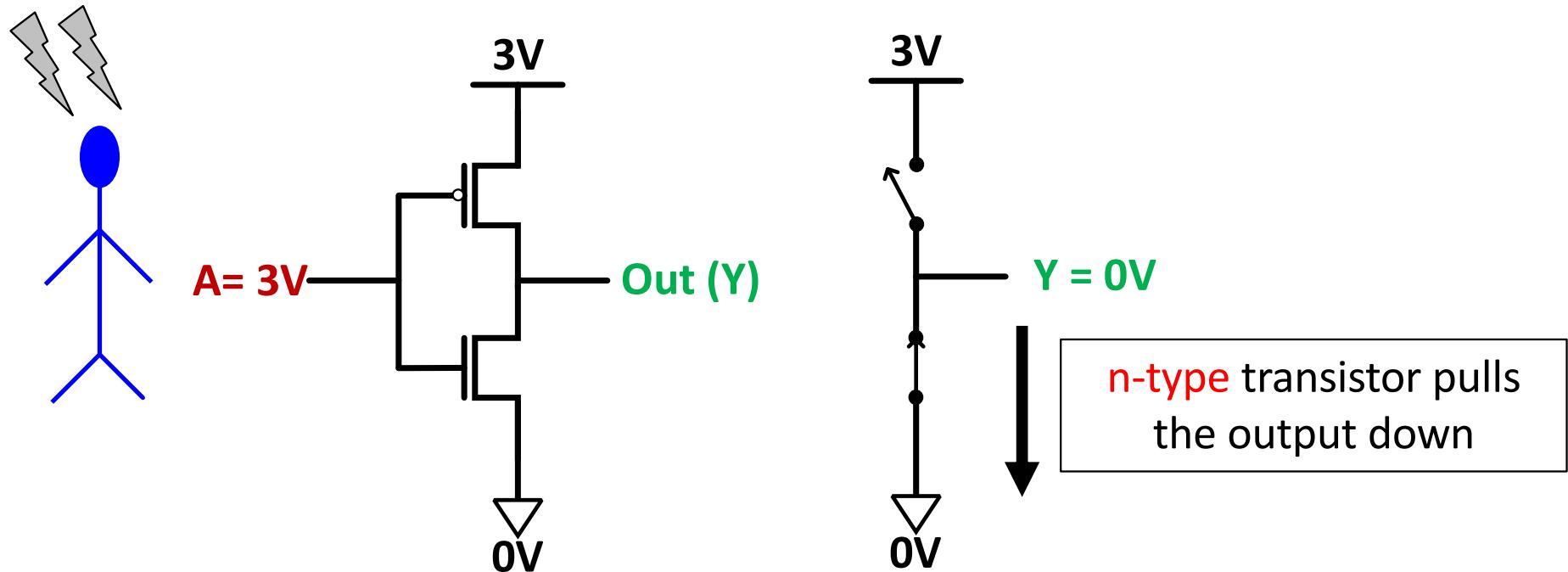


# CMOS Technology Fundamentals

- p-type transistors are good at pulling up the voltage

# CMOS Technology Fundamentals

- What happens when the input is connected to 3V?

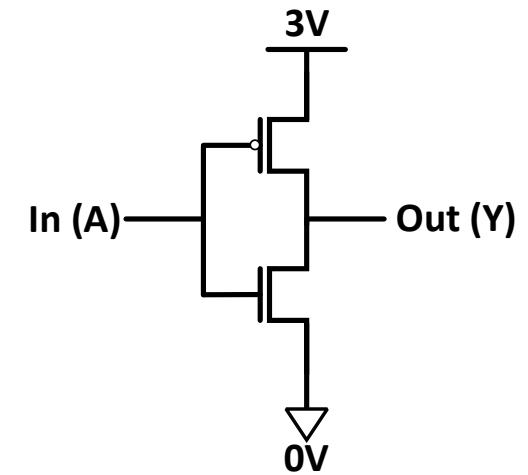


# CMOS Technology Fundamentals

- **n**-type transistors are good at pulling down**n** the voltage

# CMOS NOT Gate (Inverter)

- We have seen a NOT gate at the transistor level
  - If  $A = 0V$  then  $Y = 3V$
  - If  $A = 3V$  then  $Y = 0V$
- Interpretation of voltage levels
  - Interpret  $0V$  as logical (binary)  $0$  value
  - Interpret  $3V$  as logical (binary)  $1$  value



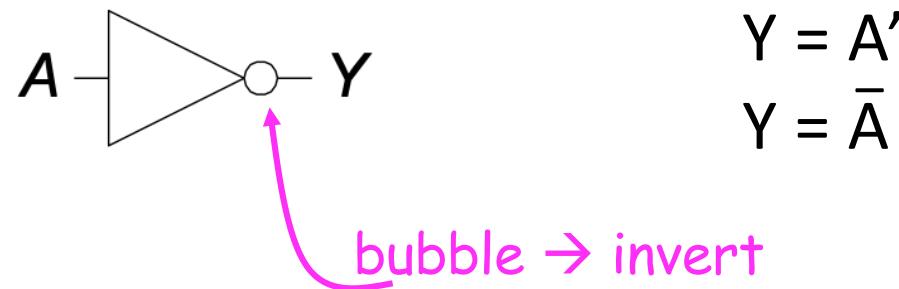
A	P	N	Y
0	ON	OFF	1
1	OFF	ON	0

$$Y = \bar{A}$$

# CMOS NOT Gate (Inverter)

A	Y
0	1
1	0

The NOT gate has only one input (unary)



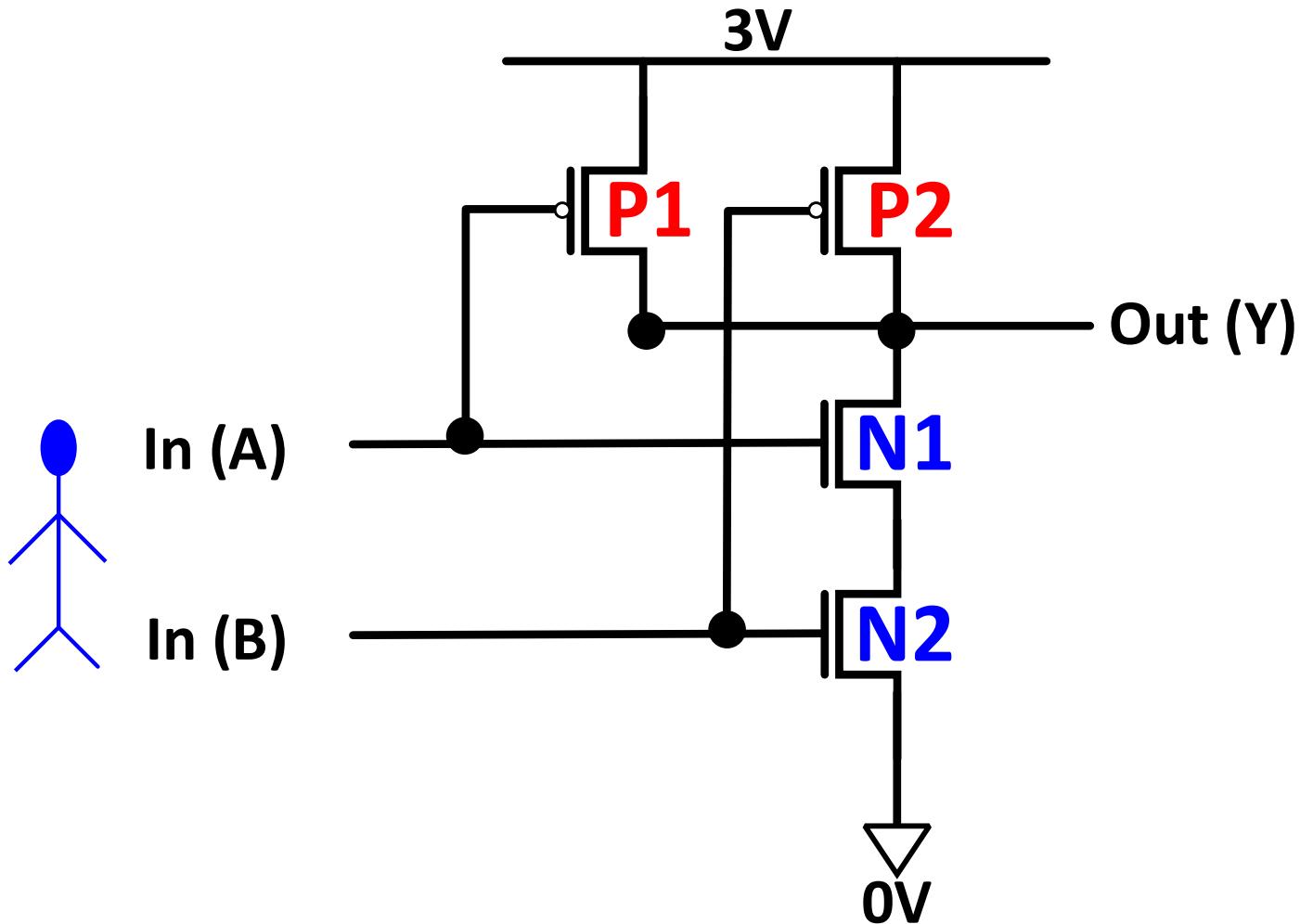
Truth Table

NOT Logic Gate

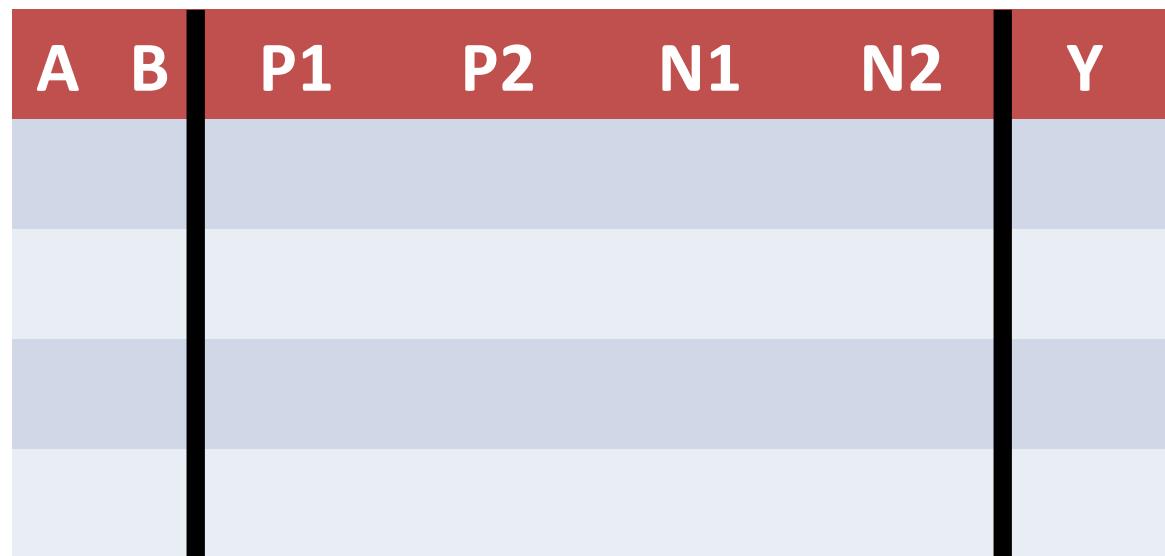
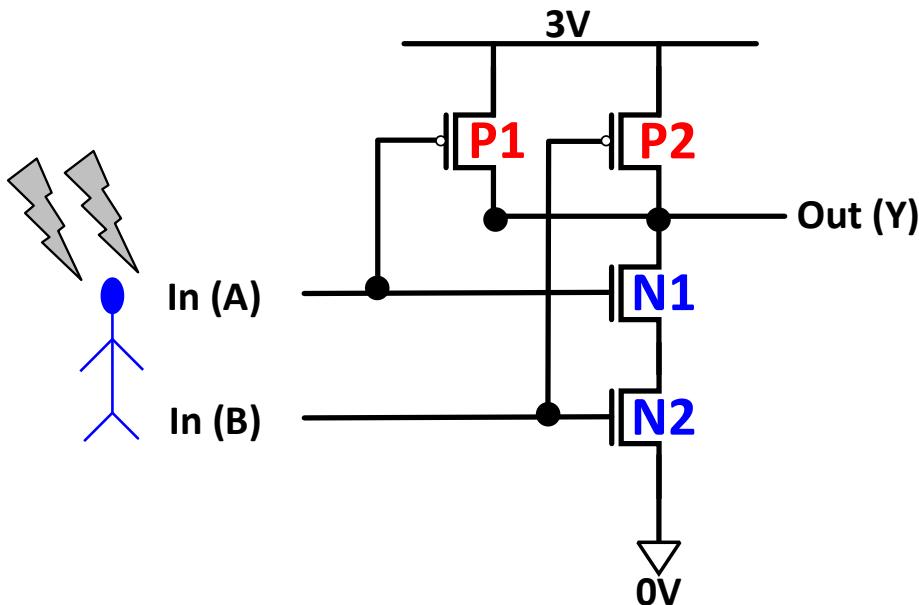
Boolean Equation

NOT Function: *The output Y is the inverse of the input A*

# Another CMOS Gate: What is this?

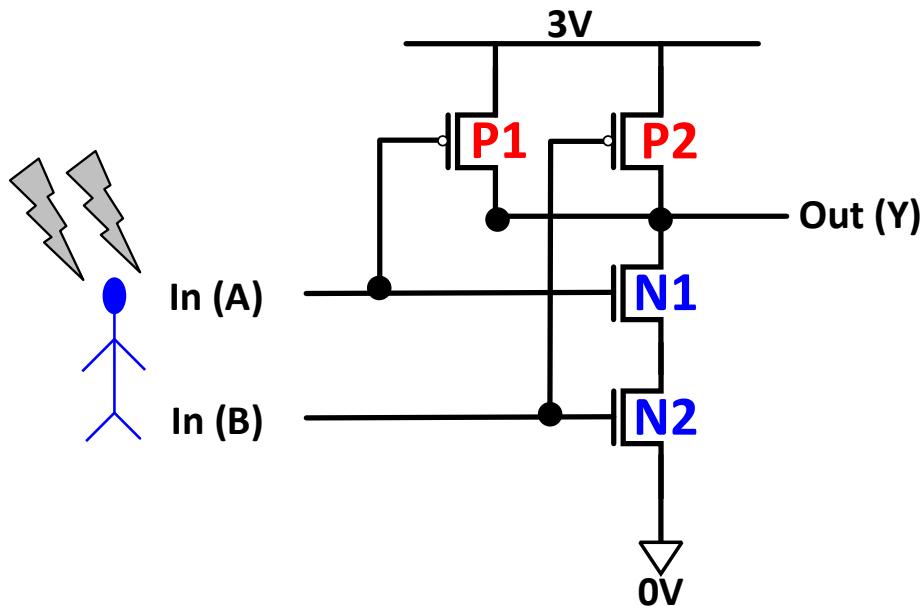


# CMOS NAND Gate



- P1 and P2 are in parallel; only one must be **ON** to pull up the voltage to **3V**
- N1 and N2 are connected in series; both must be **ON** to pull down the voltage to **0V**

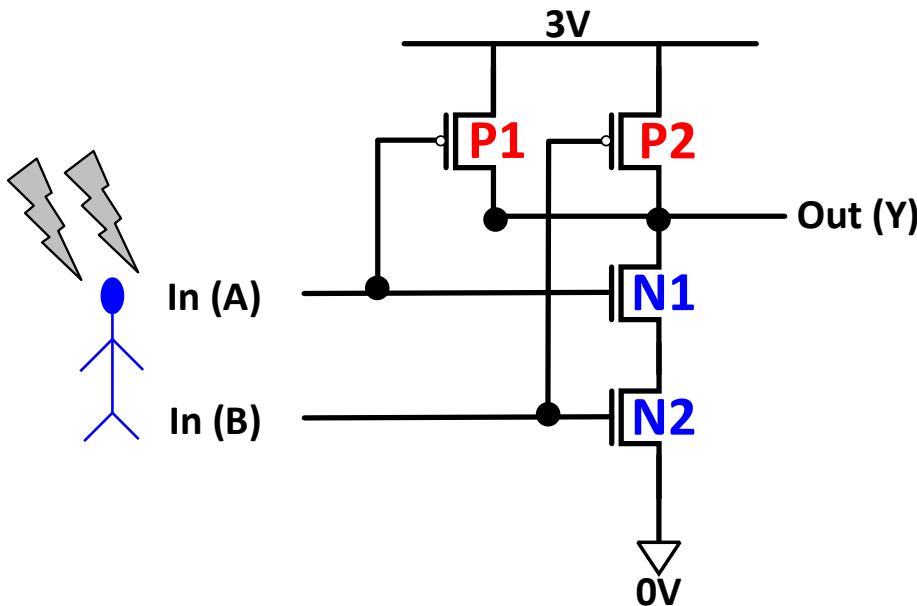
# CMOS NAND Gate



A	B	P1	P2	N1	N2	Y
0	0	ON	ON	OFF	OFF	1
0	1	ON	OFF	ON	OFF	0
1	0	OFF	ON	ON	OFF	0
1	1	OFF	OFF	OFF	ON	1

- P1 and P2 are in parallel; only one must be **ON** to pull up the voltage to **3V**
- N1 and N2 are connected in series; both must be **ON** to pull down the voltage to **0V**

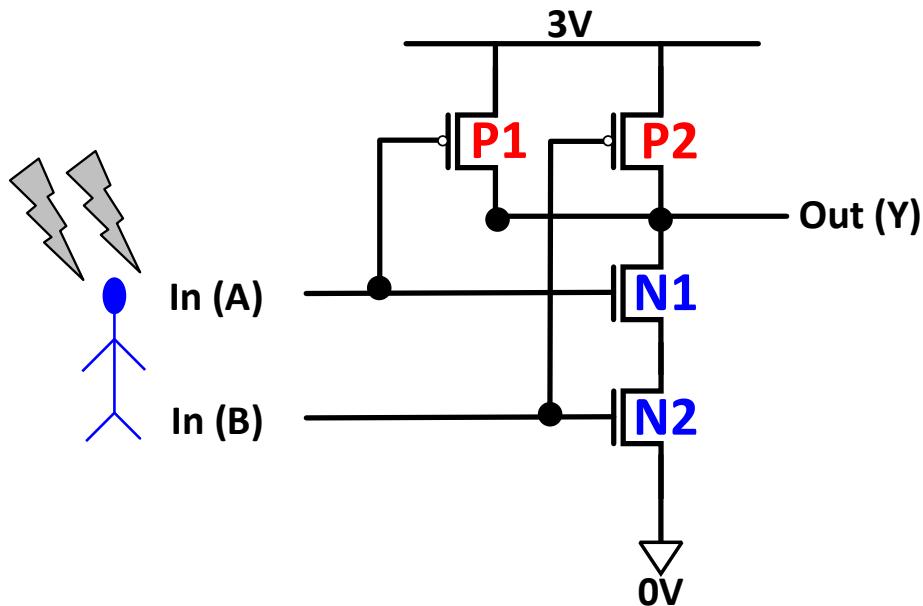
# CMOS NAND Gate



A	B	P1	P2	N1	N2	Y
0	0	ON	ON	OFF	OFF	1
0	1	ON	OFF	OFF	ON	1

- P1 and P2 are in parallel; only one must be **ON** to pull up the voltage to **3V**
- N1 and N2 are connected in series; both must be **ON** to pull down the voltage to **0V**

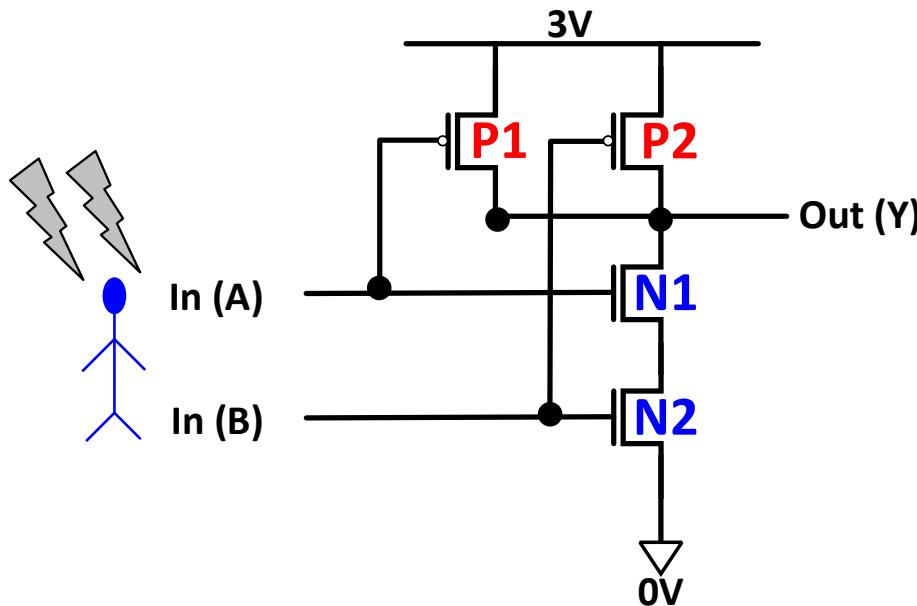
# CMOS NAND Gate



A	B	P1	P2	N1	N2	Y
0	0	ON	ON	OFF	OFF	1
0	1	ON	OFF	OFF	ON	1
1	0	OFF	ON	ON	OFF	1

- P1 and P2 are in parallel; only one must be **ON** to pull up the voltage to **3V**
- N1 and N2 are connected in series; both must be **ON** to pull down the voltage to **0V**

# CMOS NAND Gate



A	B	P1	P2	N1	N2	Y
0	0	ON	ON	OFF	OFF	1
0	1	ON	OFF	OFF	ON	1
1	0	OFF	ON	ON	OFF	1
1	1	OFF	OFF	ON	ON	0

- P1 and P2 are in parallel; only one must be **ON** to pull up the voltage to **3V**
- N1 and N2 are connected in series; both must be **ON** to pull down the voltage to **0V**

# CMOS NAND Gate

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

Truth Table



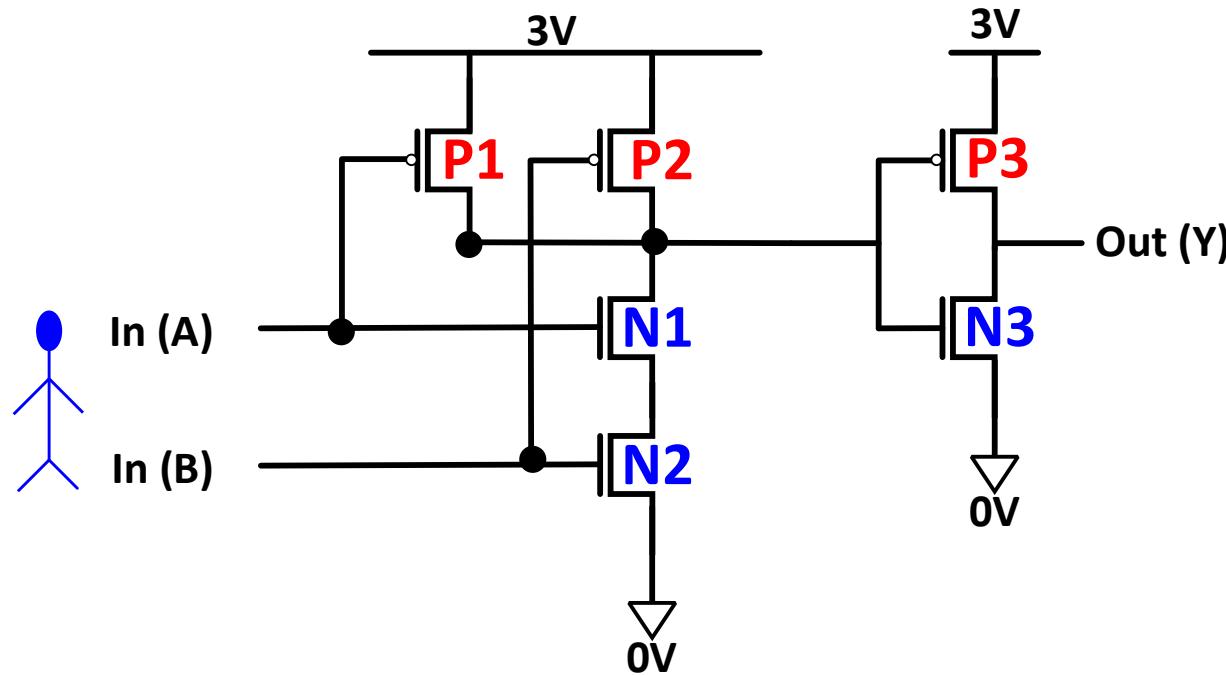
NAND Logic Gate

$$Y = (AB)'$$
$$Y = \overline{AB}$$

Boolean Equation

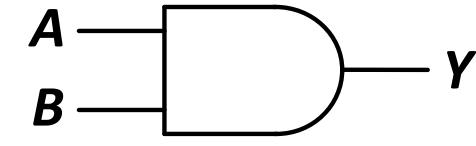
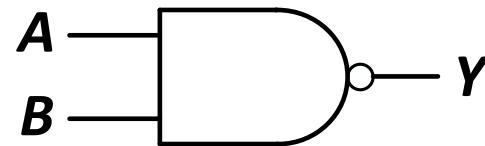
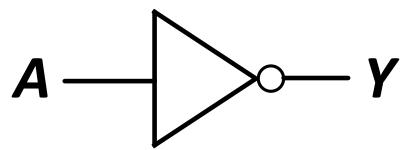
**NAND Function:** *The output Y is 1 unless both inputs are 1*

# CMOS AND Gate



- We make an AND gate using one NAND gate and one NOT gate
- **Homework:** Can we not use fewer transistors for the AND gate?

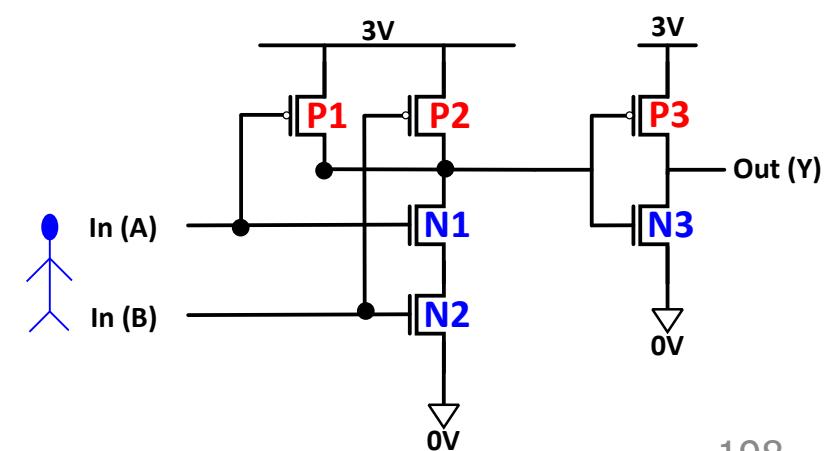
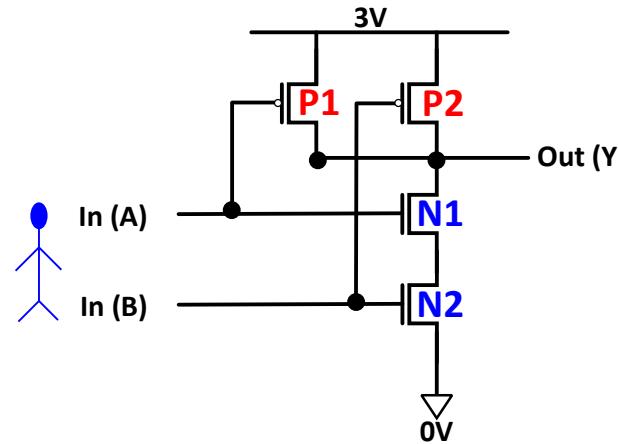
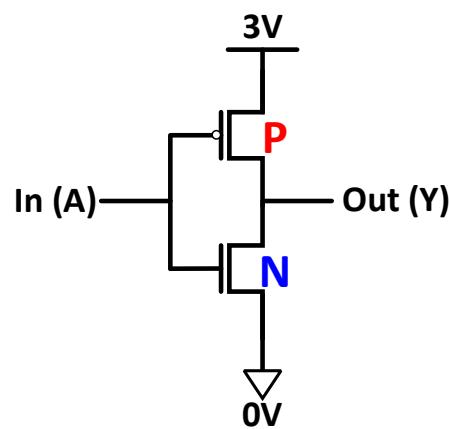
# CMOS NOT, NAND, and AND Gates



$A$	$Y$
0	1
1	0

$A$	$B$	$Y$
0	0	1
0	1	1
1	0	1
1	1	0

$A$	$B$	$Y$
0	0	0
0	1	0
1	0	0
1	1	1

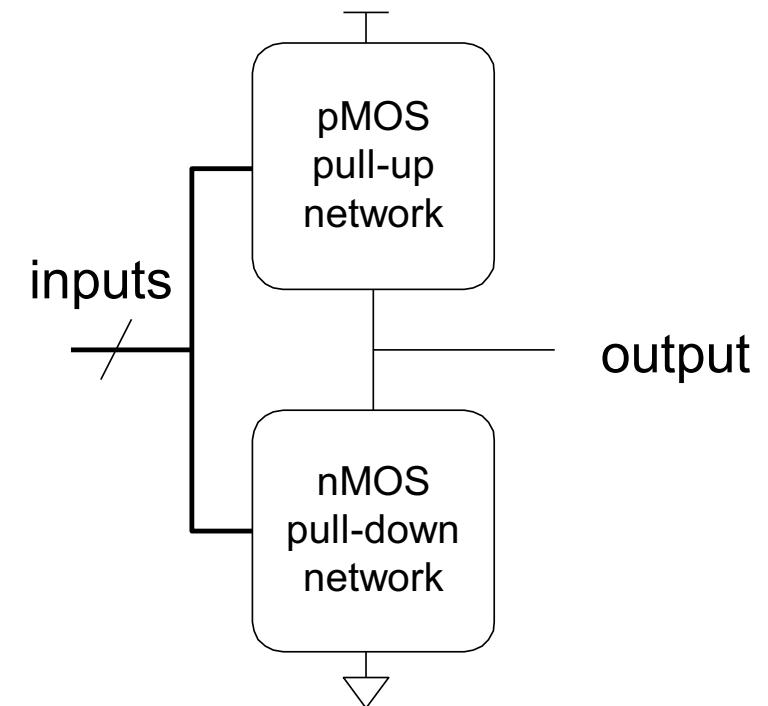


# General CMOS Gate Structure

- We have a general form to construct any inverting logic gate, such as, NOT, NAND, NOR
  - The networks may consist of transistors in **series** or in **parallel**
  - When transistors are in **parallel**, the network is **ON** if one of the transistors is **ON**
  - When transistors are in **series**, then network is **ON** only if all transistors are **ON**

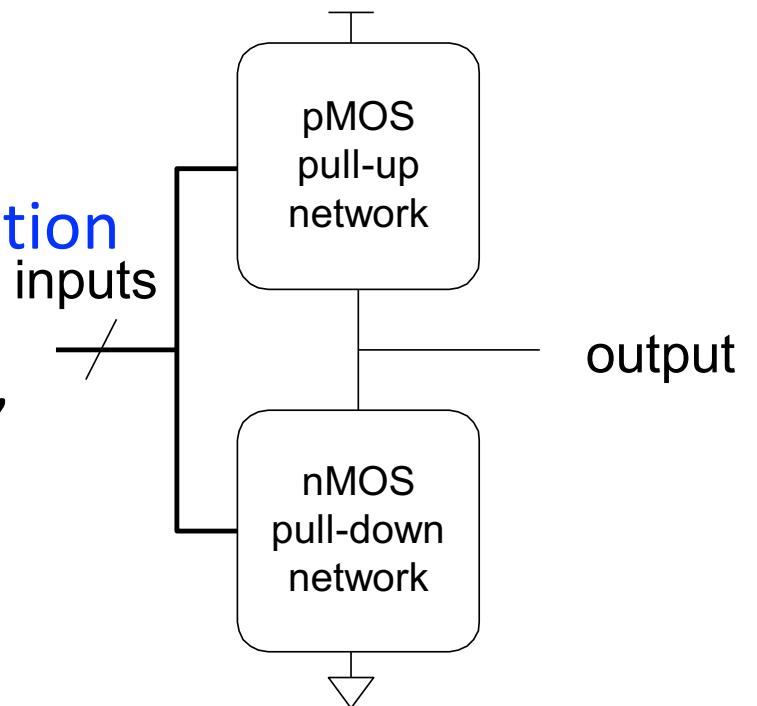
pMOS transistors are used for pull-up

nMOS transistors are used for pull-down



# General CMOS Gate Structure

- Exactly one network should be **ON**, and the other network should be **OFF** at any given time
- If both networks are **ON** simultaneously, there is a short circuit → **incorrect operation**
- If both networks are **OFF** simultaneously, the output is floating → **undefined**

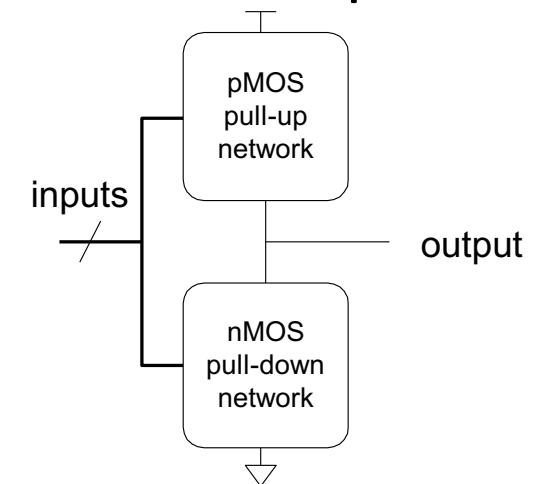
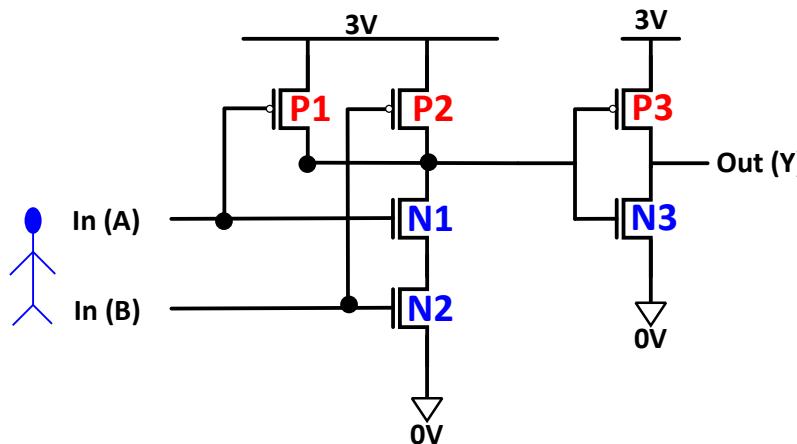


pMOS transistors are used for pull-up

nMOS transistors are used for pull-down

# Why This Structure?

- MOS transistors are imperfect switches
- pMOS transistors pass 1's well but 0's poorly
  - pMOS transistors are good at “pulling up” the output
- nMOS transistors pass 0's well but 1's poorly
  - nMOS transistors are good at “pulling down” the output

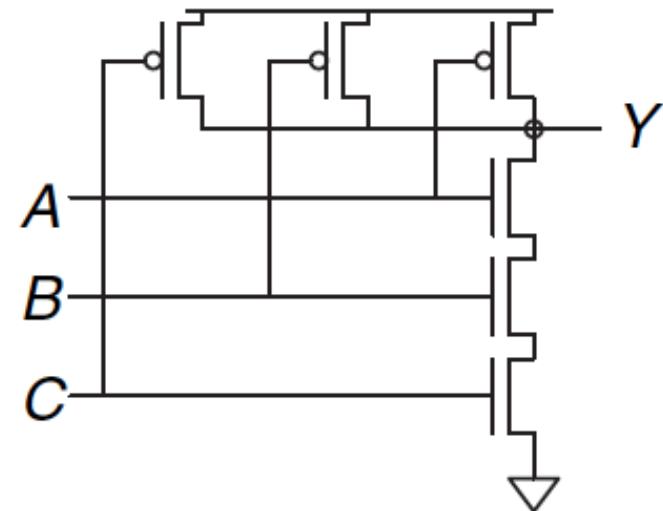


# Latency

- Which one is faster?
  - Transistor in **series**
  - Transistors in **parallel**
- Series connections are slower than parallel connections
  - More **resistance** on the wire
- Remember: **Latency of series vs. parallel circuits extend from transistors to gates and larger circuits**
- See Section 1.7.8 for more details

# Gates with More Than Two Inputs

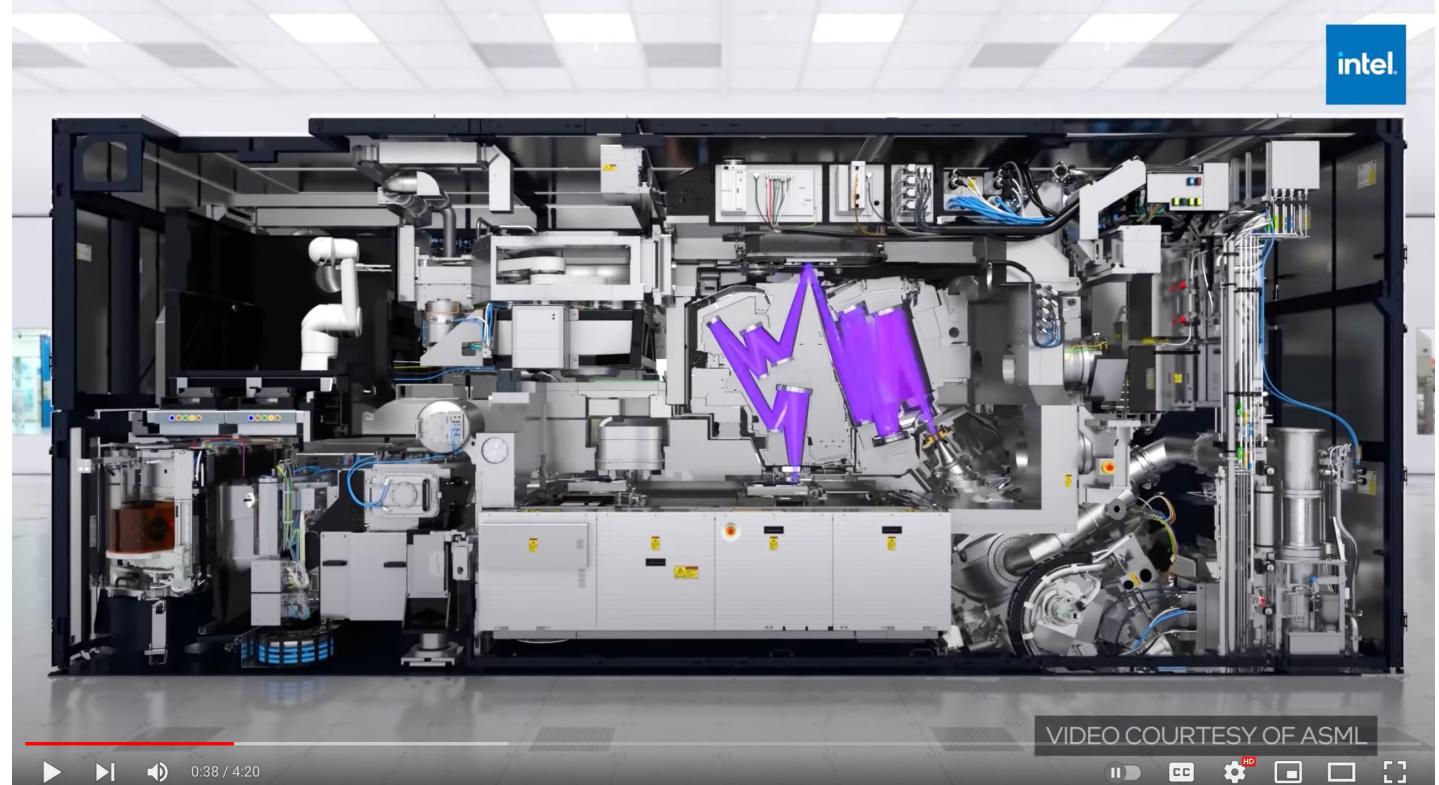
- We can create larger gates with more than 2 inputs
  - 3-input NOR gate or 11-input NAND gate



**Figure 1.35** Three-input NAND  
gate schematic

# Manufacturing Tech is the Enabler

- Precision Manufacturing
  - Extreme Ultraviolet (EUV) light to pattern <10nm structures



#EUV #chip #Intel

Behind this Door: Learn about EUV, Intel's Most Precise, Complex Machine

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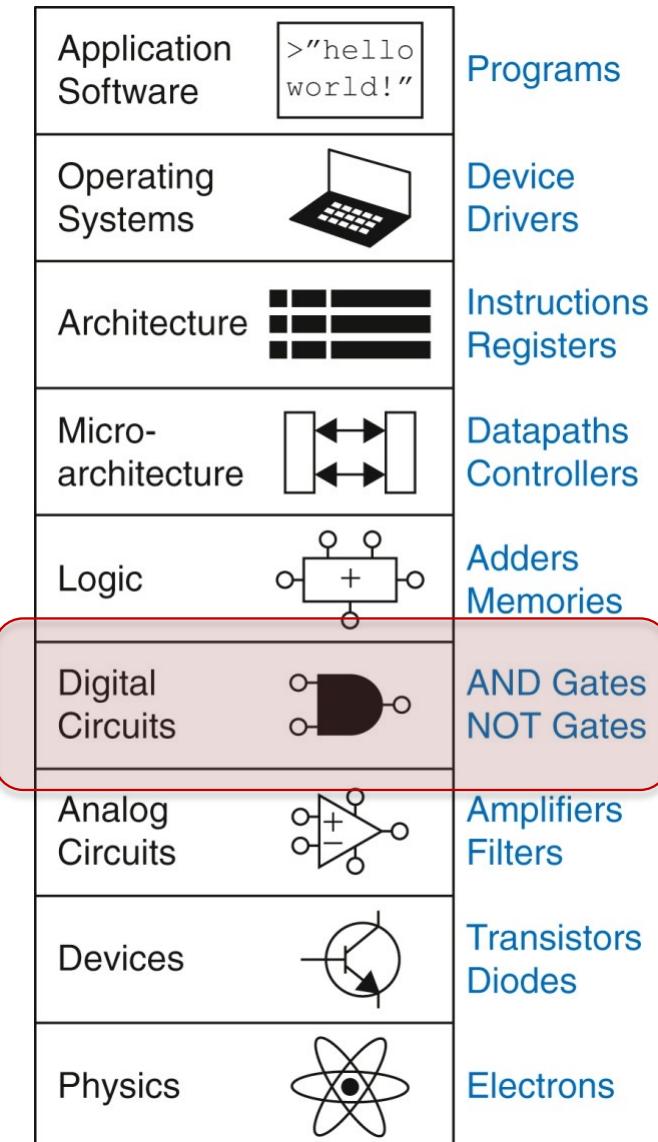


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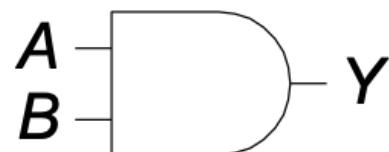


We are here

# The AND Function

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Truth Table



AND Logic Gate

$Y = AB$   
 $Y = A \cdot B$  (product)  
 $Y = A \cap B$  (intersection)

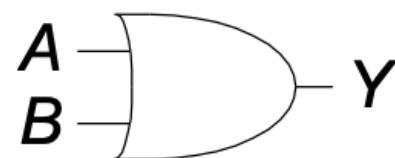
Boolean Equation

**AND Function:** *The output Y is 1 if and only if both A and B are 1*

# The OR Function

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

Truth Table



OR Logic Gate

$$Y = A + B \text{ (sum)}$$
$$Y = A \cup B \text{ (union)}$$

Boolean Equation

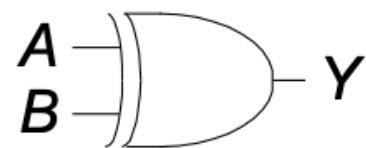
**OR Function:** *The output Y is 1 if either A or B are 1*

# The XOR Function

eXclusive-OR

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

Truth Table



XOR Logic Gate

$$Y = A \oplus B$$

Boolean Equation

**XOR Function:** *The output Y is 1 if A or B, but not both, are 1*

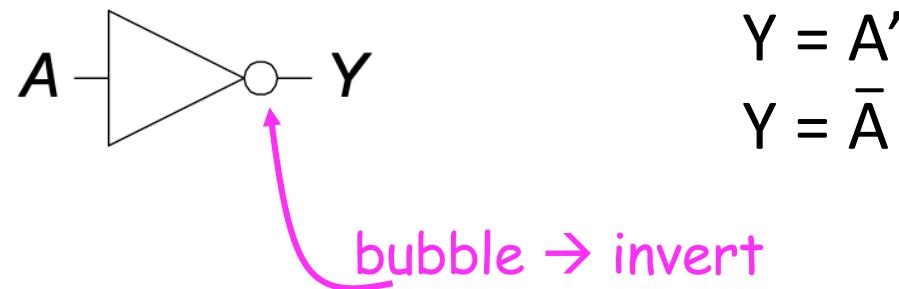
# OR and XOR

- The term **exclusive** is used because the output is **1** if only one of the inputs is **1** (mutually exclusive)
- OR produces an output **1**, if only one of the two sources is a **1**, or both sources are one (**inclusive OR**)

# The NOT Unary Function

A	Y
0	1
1	0

The NOT gate has only one input (unary)



Truth Table

NOT Logic Gate

Boolean Equation

**NOT Function:** *The output Y is the inverse of the input A*

# Inverting a Gate's Operation

Any gate can be followed by a bubble to invert its operation

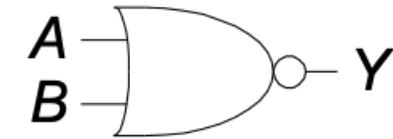
NOT AND →

NAND



NOT OR →

NOR



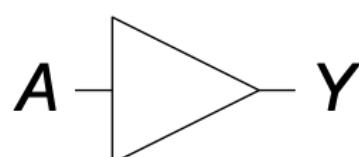
NOT XOR →

XNOR

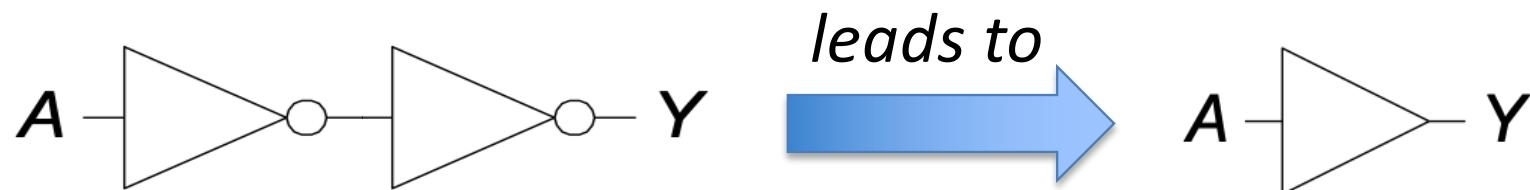


NOT NOT →

BUF



# In Boolean logic, two wrongs make a right!



We say that two bubbles cancel each other's effect

# The NAND Function

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

Truth Table



NAND Logic Gate

$$Y = (AB)'$$

Boolean Equation

NAND Function: *The output Y is 1 unless both inputs are 1*

# The NOR Function

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

Truth Table



NOR Logic Gate

$$Y = (A + B)'$$

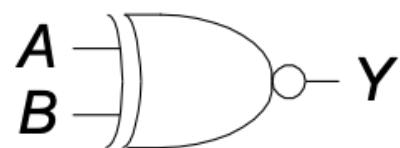
Boolean Equation

**NOR Function:** *The output Y is 1 if neither A nor B is 1*

# The XNOR Function

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Truth Table



XNOR Logic Gate

$$Y = (A \oplus B)'$$

Boolean Equation

**XNOR Function:** *The output Y is 1 if both A and B are 1*

# XOR and XNOR are special

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

XOR

**XOR:** Output is **1** when inputs are not equal (odd number of **1**'s)

**Parity Gate**

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

XNOR

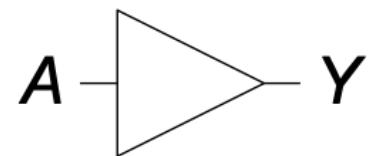
**XNOR:** Output is **1** when inputs are equal (even number of **1**'s)

**Equality Gate**

# Buffer (BUF)

A	Y
0	0
0	0
1	1
1	1

Truth Table



BUF Logic Gate

$$Y = A$$

Boolean Equation

**Buffer:** *The output Y is equal to the input A*

# Logic Gates

Name	NOT	AND	NAND	OR	NOR	XOR	XNOR																																																																																																
Alg. Expr.	$\bar{A}$	$AB$	$\overline{AB}$	$A+B$	$\overline{A+B}$	$A \oplus B$	$\overline{A \oplus B}$																																																																																																
Symbol																																																																																																							
Truth Table	<table border="1"> <tr> <th>A</th> <th>X</th> </tr> <tr> <td>0</td> <td>1</td> </tr> <tr> <td>1</td> <td>0</td> </tr> </table>	A	X	0	1	1	0	<table border="1"> <tr> <th>B</th> <th>A</th> <th>X</th> </tr> <tr> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <td>1</td> <td>1</td> <td>1</td> </tr> </table>	B	A	X	0	0	0	0	1	0	1	0	0	1	1	1	<table border="1"> <tr> <th>B</th> <th>A</th> <th>X</th> </tr> <tr> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <td>0</td> <td>1</td> <td>1</td> </tr> <tr> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <td>1</td> <td>1</td> <td>0</td> </tr> </table>	B	A	X	0	0	1	0	1	1	1	0	1	1	1	0	<table border="1"> <tr> <th>B</th> <th>A</th> <th>X</th> </tr> <tr> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>1</td> <td>1</td> </tr> <tr> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <td>1</td> <td>1</td> <td>0</td> </tr> </table>	B	A	X	0	0	0	0	1	1	1	0	1	1	1	0	<table border="1"> <tr> <th>B</th> <th>A</th> <th>X</th> </tr> <tr> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <td>1</td> <td>1</td> <td>0</td> </tr> </table>	B	A	X	0	0	1	0	1	0	1	0	0	1	1	0	<table border="1"> <tr> <th>B</th> <th>A</th> <th>X</th> </tr> <tr> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>1</td> <td>1</td> </tr> <tr> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <td>1</td> <td>1</td> <td>1</td> </tr> </table>	B	A	X	0	0	0	0	1	1	1	0	1	1	1	1	<table border="1"> <tr> <th>B</th> <th>A</th> <th>X</th> </tr> <tr> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <td>1</td> <td>1</td> <td>1</td> </tr> </table>	B	A	X	0	0	1	0	1	0	1	0	0	1	1	1
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# Multiple-Input Gates

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Gates with multiple inputs are possible

Looking at the truth table, can you guess the 3-input gate?



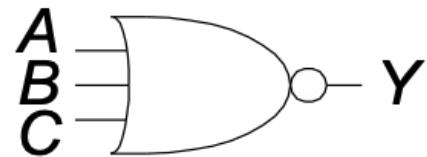
$$Y = ABC$$

# Multiple-Input Gates

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

Gates with multiple inputs are possible

Looking at the truth table, can you guess the 3-input gate?



$$Y = (A + B + C)'$$

# Bitwise Operations

- All logical operators are applicable to two bit-patterns (group of bits or **bit vectors**) of m bits each, m is any # bits (8, 16, ...)
  - We apply the operation individually to each pair of bits
  - If **A** and **B** are **8-bit input sources** (or source operands), then their **AND** or **product**, **C**, is also 8 bits

**C = AB** (bit-wise AND)

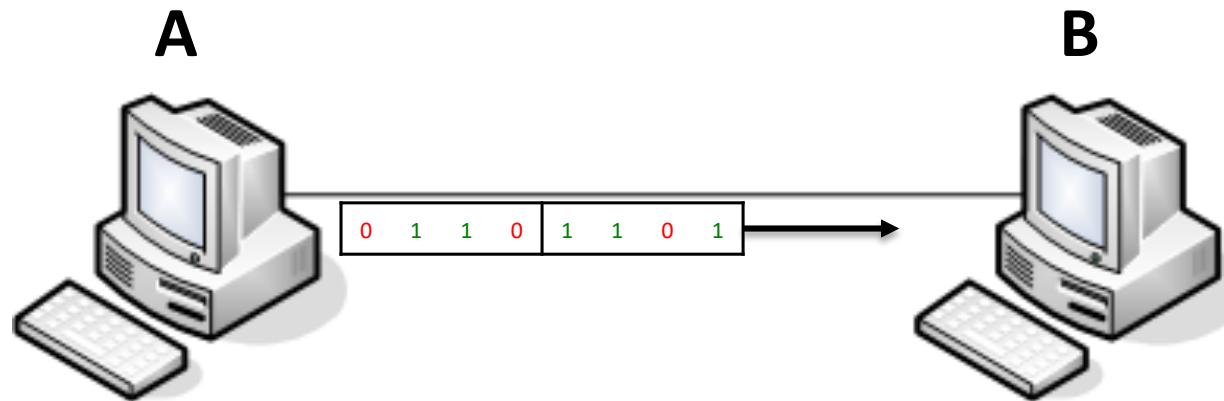
A	0	0	0	0	1	1	0	1
B	1	1	1	1	1	1	1	1
C	0	0	0	0	1	1	0	1

**C = A + B** (bit-wise OR)

A	0	0	0	0	1	1	0	1
B	0	0	0	0	0	0	0	0
C	0	0	0	0	1	1	0	1

# Bit Masks

	passwd				user id			
Packet	0	1	1	0	1	1	0	1



- **B** wants to create a new packet with user id set to **A**'s id and passwd bits set to 0 (e.g., to send a packet to another computer)

# Bit Masks

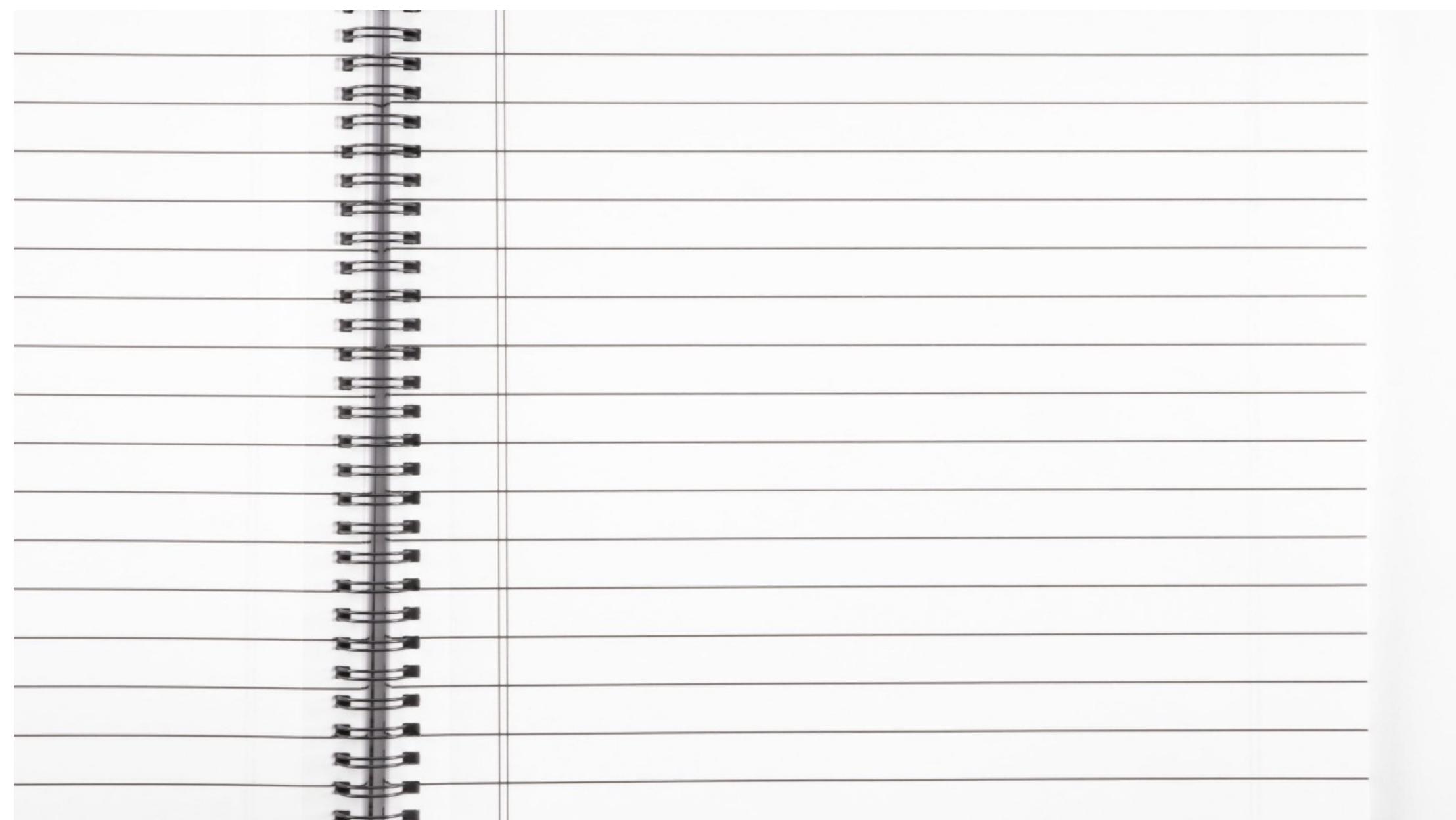
- **Bit mask:** A binary pattern (**B**) that separates the bits of **A** into two halves
- Suppose we are interested in extracting the least significant four bits from **A**, while ignoring the right-most four bits
  - If we **AND** **A** with **B**, and choose **B** as **00001111**, then we get the desired bit pattern in **C**

$$C = AB \text{ (bit-wise AND)}$$

A	0	1	1	0	1	1	0	1
B	0	0	0	0	1	1	1	1
C	0	0	0	0	1	1	0	1

# Exercises

- Suppose,  $A = \textcolor{green}{11000010}$ , and the rightmost two bits are of particular significance. Find a bitmask and a logical operation to mask out the values in the rightmost positions in a new bit pattern B. (**All other bits in B are set to 0.**)
- Suppose,  $A = \textcolor{green}{10110010}$ , and the leftmost two bits are of particular significance. Find a bitmask and a logical operation to mask out the values in the leftmost positions in a new bit pattern B. (**All other bits in B are set to 1.**)

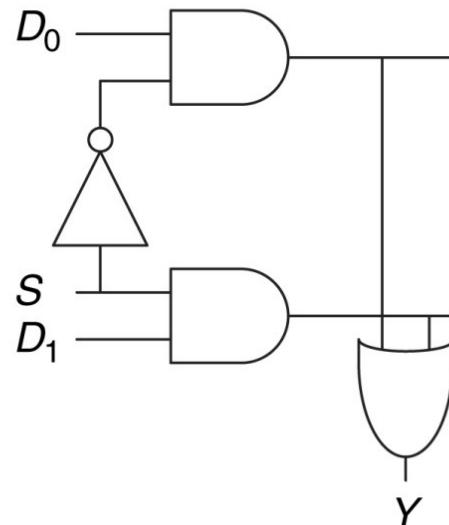


# Exercise

- How can we find out if two bit-patterns A and B are identical?
- Verify that,  $1 \text{ AND } X = X$ , where  $X$  is a binary variable. Also, verify that,  $0 \text{ OR } X = X$ .
- Verify that,  $B \text{ AND } B = B$ , where  $B$  is a binary variable. Also, verify that,  $B \text{ OR } B = B$ .
- Verify that,  $B \text{ AND } B' = 0$ , where  $B$  is a binary variable. Also, verify that,  $B \text{ OR } B' = 1$ .

# A Useful Circuit

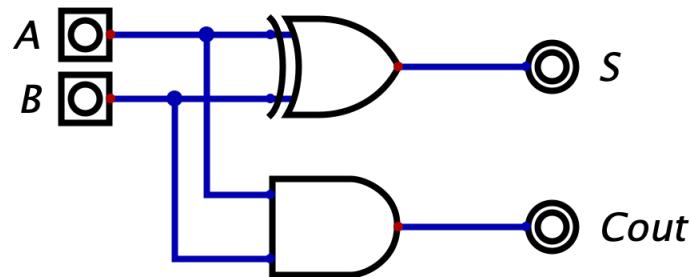
- What does this circuit do?



- Multiplexer
- Used for decision making and often found inside control logic

# Another Useful Circuit

- What does this circuit do?



- Half adder (no carry input)
- Used for making an ALU – Arithmetic & Logic Unit

# Coming Attractions

- We will learn to **systematically** build circuits from a specification
- We will look at many **useful** circuits
- We will study two types of logic circuits –  
Combinational and sequential