CSCI471/971
Modern Cryptography

PRG, Steam Cipher and Sematic Security

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This slide is made based the online course of Cryptography by Dan Boneh

Symmetric Ciphers: definition

<u>Def</u>: a **cipher** defined over (K, M, C)

is a pair of "efficient" algs (***E***, ***D***) where

$$E : \mathcal{K} \times \mathcal{M} \to \mathcal{C} \qquad D : \mathcal{K} \times \mathcal{C} \to \mathcal{M}$$

for all keys k and all messages m, we have

$$D(k,\ E(k,\ m)\ ) = m$$

- ***E*** is often randomized. ***D*** is always deterministic.

Pseudorandom Generators

Review

Cipher over (K,M,C): a pair of "efficient" algs (***E***, ***D***) s.t.

$$\forall\, m \in M,\ k \in K: \quad \boldsymbol{D}(k, \boldsymbol{E}(k, m)) = m$$

Weak ciphers: subs. cipher, Vigener, …

A good cipher: **OTP** $M=C=K=\{0,1\}^n$

$$E(k, m) = k \oplus m \quad , \quad D(k, c) = k \oplus c$$

<u>Lemma</u>: OTP has perfect secrecy (i.e. no CT only attacks)

Bad news: perfect-secrecy $\Rightarrow$ key-len $\geq$ msg-len

Stream Ciphers: making OTP practical

idea: replace "random" key by "pseudorandom" key

PRG is a function $G: \{0,1\}^s \to \{0,1\}^n$, $n \gg s$

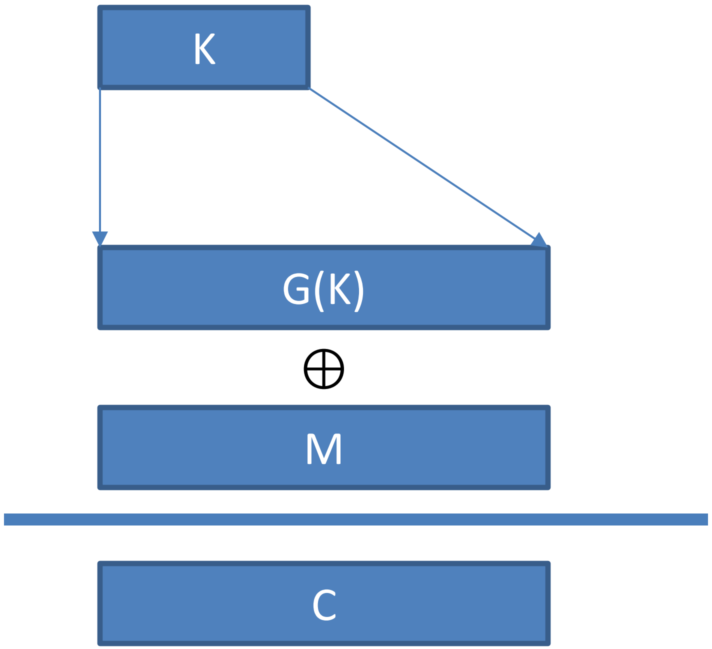
Seed space

Efficiently computable by a deterministic algorithm

Stream Ciphers: making OTP practical

$$C = E(K, m) = m \oplus G(k)$$

$$D(k, C) = C \oplus G(k)$$



Can a stream cipher have perfect secrecy?

- ○ Yes, if the PRG is really “secure”
- ○ No, there are no ciphers with perfect secrecy
- ○ Yes, every cipher has perfect secrecy
- ○ No, since the key is shorter than the message

Stream Ciphers: making OTP practical

Stream ciphers cannot have perfect secrecy !!

- Need a different definition of security

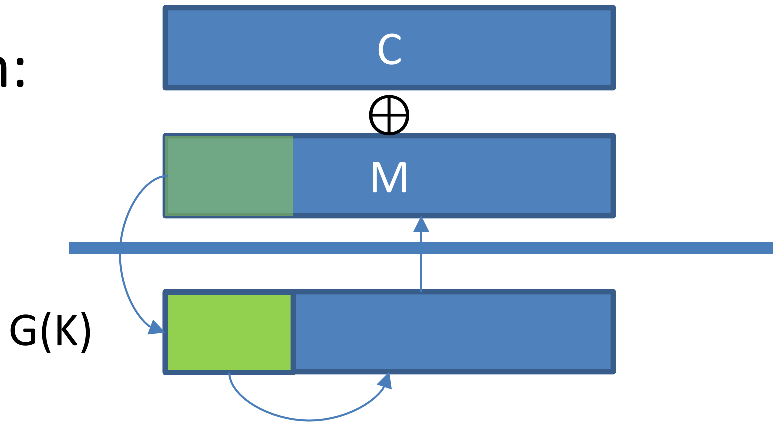
- Security will depend on specific PRG

PRG must be unpredictable

Suppose PRG is predictable.

$$\exists\, i: G(k)|_{1,\ldots,i} \longrightarrow G(k)|_{i+1,\ldots,n}$$

Then:



Even $G(k)|_{1,\ldots,i} \longrightarrow G(k)|_{i+1}$ is a problem!

PRG must be unpredictable

We say that G: K $\longrightarrow \{0,1\}^n$ is **predictable** if:

$\exists$ "eff" alg. A and $\exists\ 0 \leq i \leq n-1$

s.t. Pr[A($G(k)|_{1,\ldots,i}$)=$G(k)|_{i+1}$]> $\frac{1}{2} + \varepsilon$

for non-negligible ε (e.g. ε=1/2^{30})

Def: PRG is **unpredictable** if it is not predictable

$\Rightarrow$ $\forall$i: no "eff" adv. can predict bit (i+1) for "non-neg" ε

Suppose $G:K \longrightarrow \{0,1\}^n$ is such that for all k: **XOR(G(k)) = 1**

Is G predictable ?

- Yes, given the first bit I can predict the second
- No, G is unpredictable
- Yes, given the first (n-1) bits I can predict the n'th bit
- It depends

Weak PRGs (do not use for crypto)

Lin.Cong. Generator with parameters a,b,p:

r[i] $\longleftarrow$ $a \cdot r[i-1] + b \; mod \; p$ seed=r[0]

output bits of r[i]

i++

glibc random():

r[i] ← (r[i-3] + r[i-31]) % 2^{32}

output r[i] >> 1

never use random() for crypto!

(e.g. Kerberos V4)

Stream ciphers

Negligible vs. non-negligible

Negligible and non-negligible

- In practice: ε is a scalar and
  - ε non-neg: $\varepsilon \geq 1/2^{30}$ (likely to happen over 1GB of data)
  - ε negligible: $\varepsilon \leq 1/2^{80}$ (won't happen over life of key)

- In theory: ε is a function $\varepsilon: Z^{\geq 0} \longrightarrow R^{\geq 0}$ and
  - ε non-neg: $\exists d$: $\varepsilon(\lambda) \geq 1/\lambda^d$ inf. often ($\varepsilon \geq$ 1/poly, for many λ)
  - ε negligible: $\forall d, \lambda \geq \lambda_d$: $\varepsilon(\lambda) \leq 1/\lambda^d$ ($\varepsilon \leq$ 1/poly, for large λ)

Few Examples

$\varepsilon(\lambda) = 1/2^{\lambda}$: negligible

$\varepsilon(\lambda) = 1/\lambda^{1000}$: non-negligible

PRGs: the rigorous theory view

PRGs are "parameterized" by a security parameter λ

- **PRG** becomes "more secure" as λ increases

Seed lengths and output lengths grow with λ

For every λ=1,2,3,… there is a different PRG G_λ:

$$G_\lambda : K_\lambda \longrightarrow \{0,1\}^{n(\lambda)}$$

(in the lectures we will always ignore λ)

An example asymptotic definition

We say that $G_\lambda : K_\lambda \longrightarrow \{0,1\}^{n(\lambda)}$ is **predictable** at position i if:

there exists a polynomial time (in λ) algorithm A s.t.

$$\Pr_{k \leftarrow K_\lambda}\Big[\, A\big(\lambda,\ G_\lambda(k)\big|_{1,\dots,i}\big) = G_\lambda(k)\big|_{i+1} \,\Big] > 1/2 + \varepsilon(\lambda)$$

for some non-negligible function $\varepsilon(\lambda)$

End of Segment

Stream ciphers

Attacks on OTP and stream ciphers

Review

OTP: $E(k,m) = m \oplus k$, $D(k,c) = c \oplus k$

Making OTP practical using a PRG: $G: K \longrightarrow \{0,1\}^n$

Stream cipher: $E(k,m) = m \oplus G(k)$, $D(k,c) = c \oplus G(k)$

Security: PRG must be unpredictable (better def in two segments)

Attack 1: **two time** pad is insecure !!

Never use stream cipher key more than once !!

$$C_1 \leftarrow m_1 \oplus \text{PRG}(k)$$
$$C_2 \leftarrow m_2 \oplus \text{PRG}(k)$$

Eavesdropper does:

$$C_1 \oplus C_2 \quad \rightarrow \quad m_1 \oplus m_2$$

Enough redundancy in English and ASCII encoding that:

$$m_1 \oplus m_2 \quad \rightarrow \quad m_1 , \; m_2$$

Real world examples

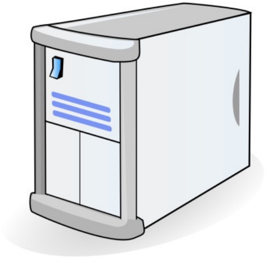
- Project Venona

- MS-PPTP (windows NT):
  - K=(Kc⟶s,Ks⟶c)

k 

m1 ⟶
m2 ⟶
m3 ⟶

⟵ s1
⟵ s2
⟵ s3

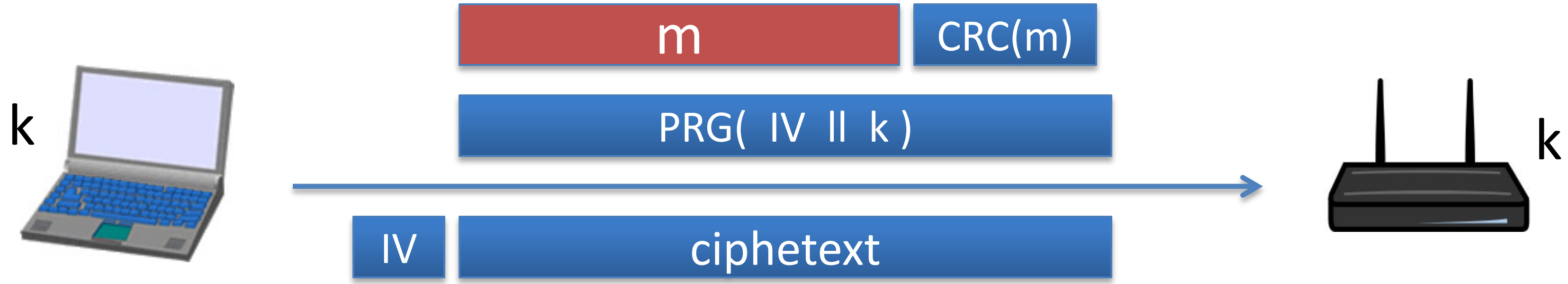
 k

- $[\mathrm{m1} \parallel m2 \parallel m3] \oplus G(k)$ $\quad$ $[s1 \parallel s2 \parallel s3] \oplus G(k)$

Need different keys for C⟶S and S⟶C

Real world examples

802.11b WEP:

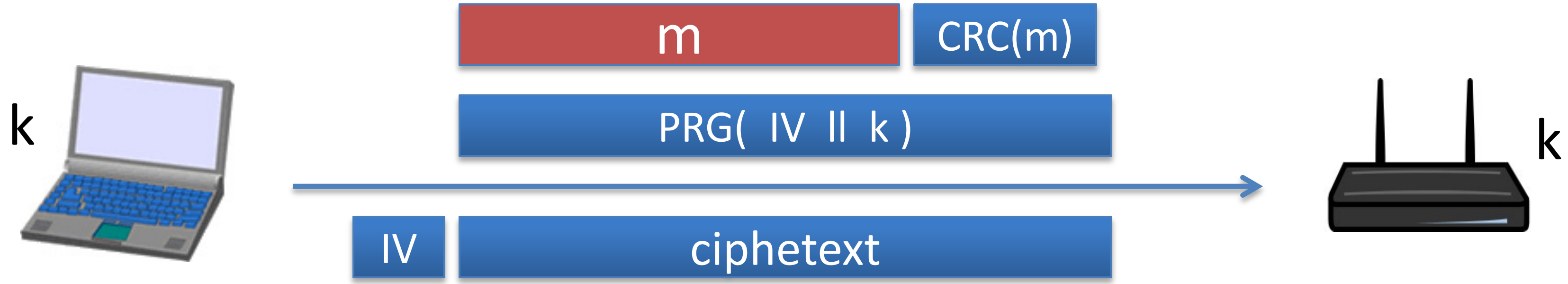


Length of IV: 24 bits

- Repeated IV after $2^{24} \approx$ 16M frames
- On some 802.11 cards: IV resets to 0 after power cycle

Avoid related keys

802.11b WEP:



key for frame #1: (1 ll k)

key for frame #2: (2 ll k)

⋮

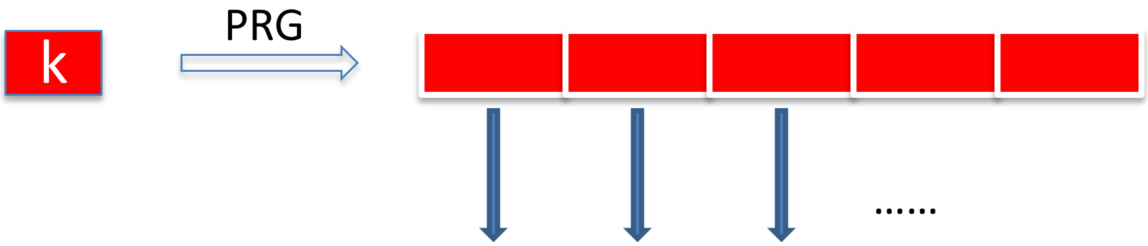
For the RC4 PRG:FMS2001 $\Longrightarrow$

can recover k after 10^6 frames

recent attacks $\approx$ 40000 frames

A better construction

k

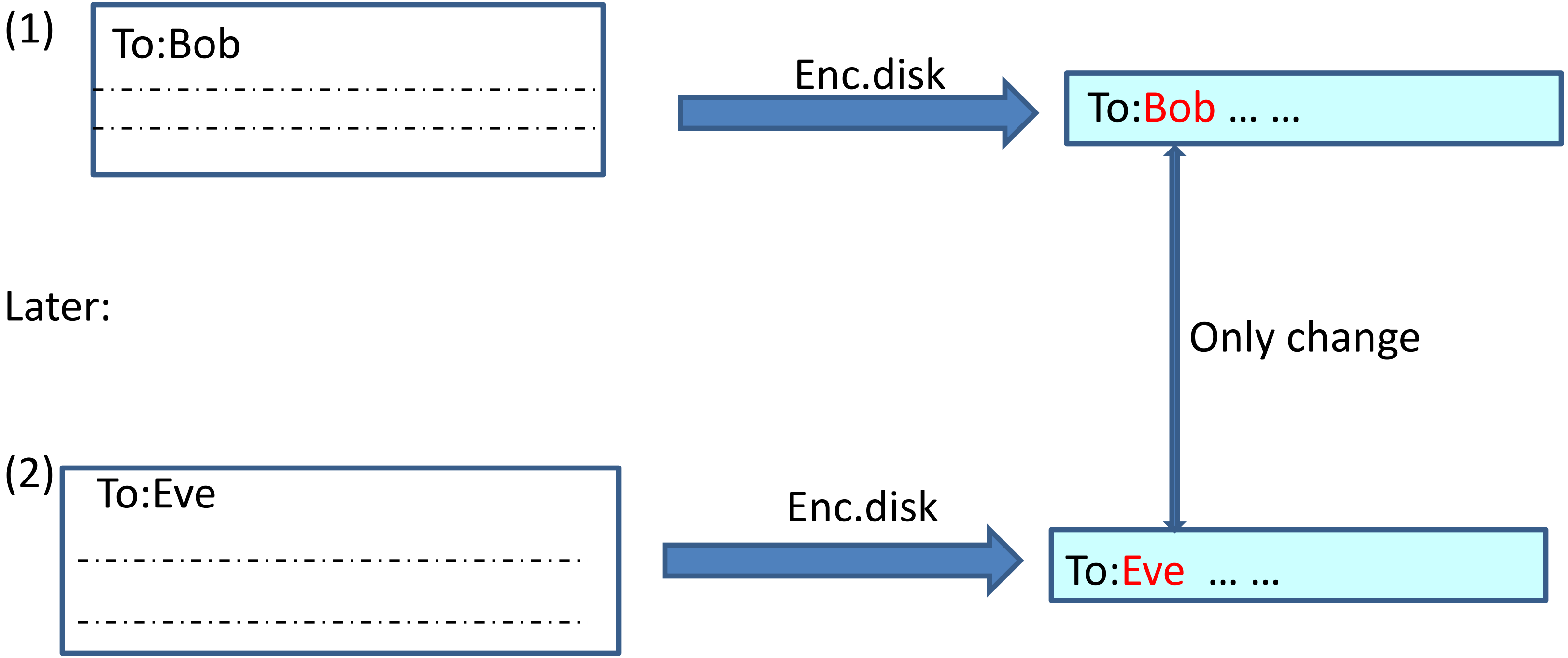


Key for frame #1   #2   #3

⇒ now each frame has a pseudorandom key

better solution: use stronger encryption method (as in WPA2)

Yet another example: disk encryption

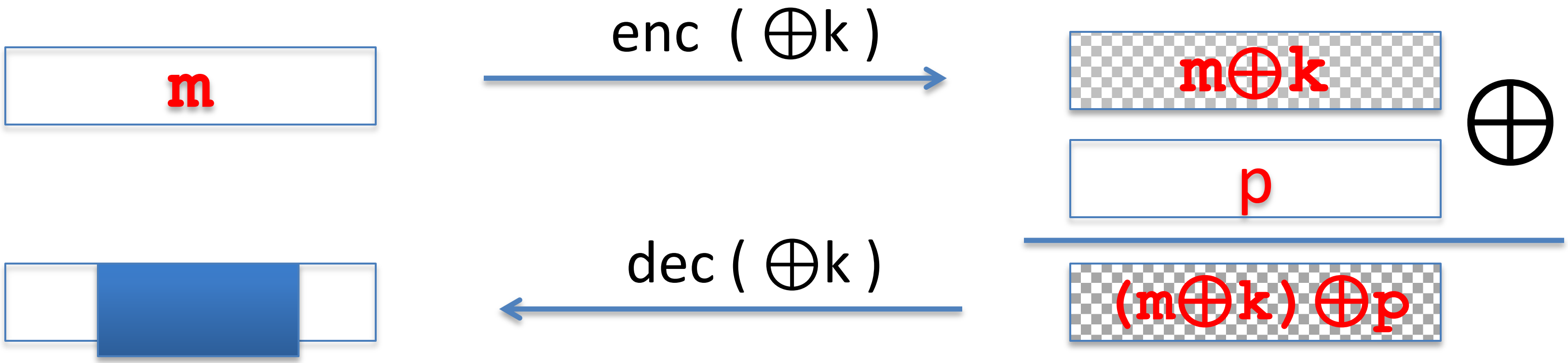


Two time pad: summary

Never use stream cipher key more than once !!

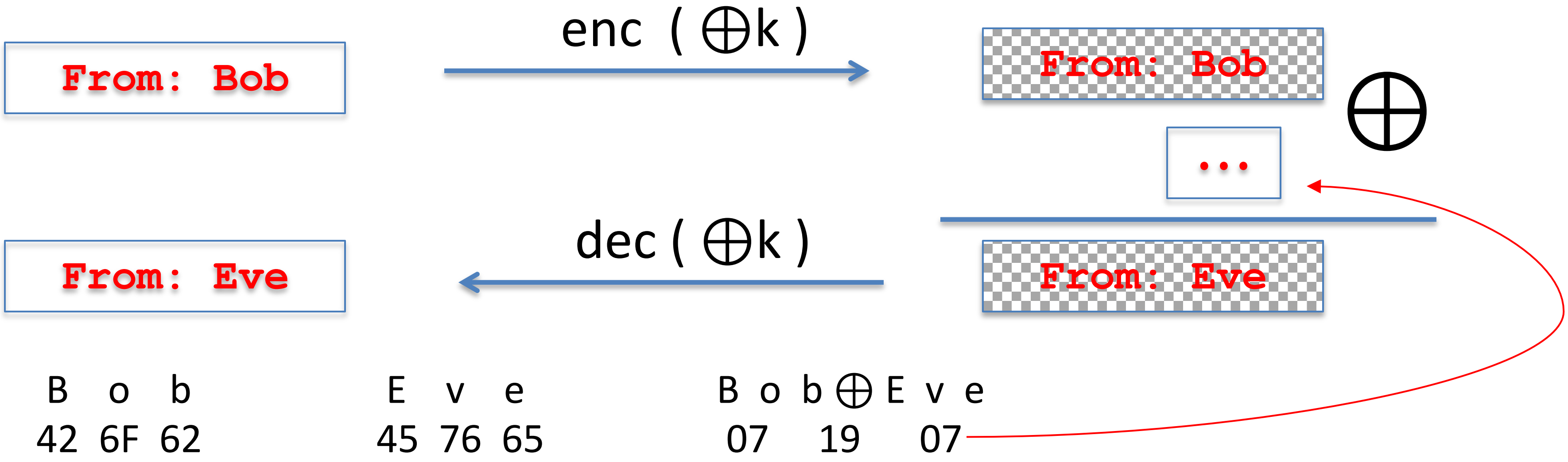
- Network traffic: negotiate new key for every session (e.g. TLS)
- Disk encryption: typically do not use a stream cipher

Attack 2: no integrity (OTP is malleable)



Modifications to ciphertext are undetected and have **predictable** impact on plaintext

Attack 2: no integrity (OTP is malleable)

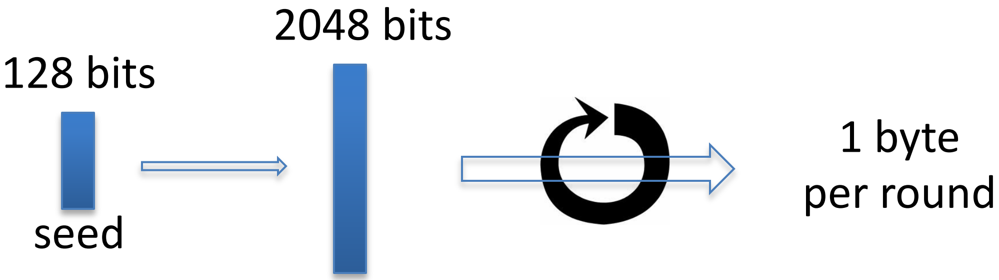


Modifications to ciphertext are undetected and have predictable impact on plaintext

End of Segment

Real-world Stream Ciphers

Old example (software): RC4 (1987)



- Used in HTTPS and WEP
- Weaknesses:
  1. Bias in initial output: Pr[2nd byte = 0] = 2/256
  2. Prob. of (0,0) is $1/256^2 + 1/256^3$
  3. Related key attacks

Modern stream ciphers: eStream

PRG: $\{0,1\}^s \times R \longrightarrow \{0,1\}^n$

Seed nonce

Nonce: a non-repeating value for a given key.

$$E(k, m\, ;\, r) \;=\; m \oplus PRG(k\, ;\, r)$$

The pair (k,r) is never used more than once.

Chacha20 (SW+HW)

Chacha20: $\{0,1\}^{256} \times \{0,1\}^{64} \longrightarrow \{0,1\}^n$

nonce

Padding function pad(s , j, n):

- 256-bit seed s0, s1,…,s7 in $\{0,1\}^{32}$
- 64-bit counter j0, j1 in $\{0,1\}^{32}$
- 64-bit nonce n0,n1 in $\{0,1\}^{32}$
- Output a 512-bit block x0,…,x15 in $\{0,1\}^{32}$

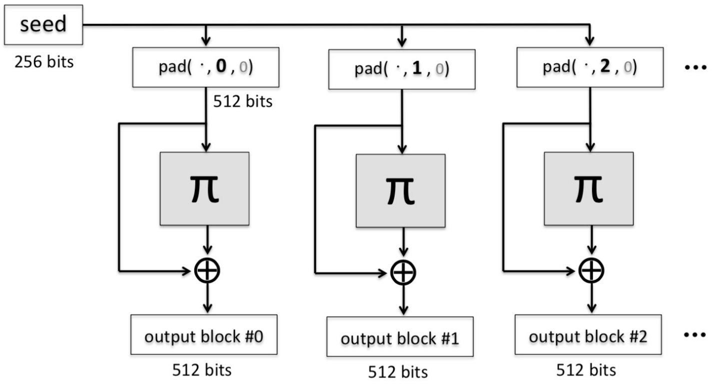
$$\begin{pmatrix} x_0 & x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 & x_7 \\ x_8 & x_9 & x_{10} & x_{11} \\ x_{12} & x_{13} & x_{14} & x_{15} \end{pmatrix} \longleftarrow \begin{pmatrix} c_0 & c_1 & c_2 & c_3 \\ s_0 & s_1 & s_2 & s_3 \\ s_4 & s_5 & s_6 & s_7 \\ j_0 & j_1 & n_0 & n_1 \end{pmatrix}$$

Permutation function π: $\{0,1\}^{512} \rightarrow \{0,1\}^{512}$

$$\begin{aligned}
&(1)\ \mathrm{QuarterRound}(x_0, x_4, x_8, x_{12}), && (2)\ \mathrm{QuarterRound}(x_1, x_5, x_9, x_{13}),\\
&(3)\ \mathrm{QuarterRound}(x_2, x_6, x_{10}, x_{14}), && (4)\ \mathrm{QuarterRound}(x_3, x_7, x_{11}, x_{15}),\\
&(5)\ \mathrm{QuarterRound}(x_0, x_5, x_{10}, x_{15}), && (6)\ \mathrm{QuarterRound}(x_1, x_6, x_{11}, x_{12}),\\
&(7)\ \mathrm{QuarterRound}(x_2, x_7, x_8, x_{13}), && (8)\ \mathrm{QuarterRound}(x_3, x_4, x_9, x_{14}).
\end{aligned}$$

QuarterRound(a, b, c, d):

```
a += b;  d ^= a;  d <<<= 16;
c += d;  b ^= c;  b <<<= 12;
a += b;  d ^= a;  d <<<= 8;
c += d;  b ^= c;  b <<<= 7;
```



Is Chacha20 secure (unpredictable) ?

- Unknown: no known **provably** secure PRGs

- In reality: no known attacks better than exhaustive search

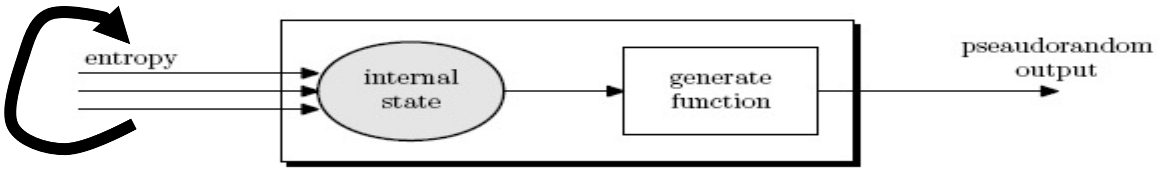
Performance:

Crypto++ 5.6.0 [Wei Dai]

AMD Opteron, 2.2 GHz (Linux)

	PRG	Speed (MB/sec)
	RC4	126
eStream	Salsa20/12	643
	Sosemanuk	727

Generating Randomness (e.g. keys, IV)



Pseudo random generators in practice: (e.g. /dev/random)

- Continuously add entropy to internal state
- Entropy sources:
  - Hardware RNG: Intel **RdRand** inst. (Ivy Bridge). 3Gb/sec.
  - Timing: hardware interrupts (keyboard, mouse)

NIST SP 800-90: NIST approved generators

PRG Security Defs

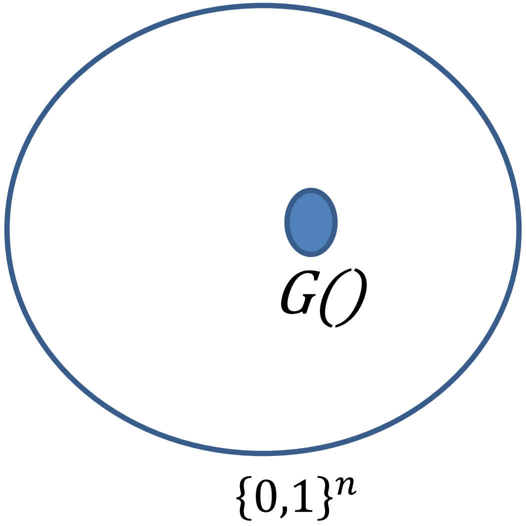
Let G:K ⟶ $\{0,1\}^n$ be a PRG

<u>Goal</u>: define what it means that

$$[k \xleftarrow{R} K, output\ \ G(k)]$$

is "indistinguishable" from

$$[r \xleftarrow{R} \{0,1\}^n, output\ r]$$



Statistical Tests

Statistical test on $\{0,1\}^n$:

an alg. A s.t. A(x) outputs "0"(not random) or "1"(random)

Examples:

(1) A(x)=1 iff $|\#0(x) - \#1(x)| \le 10 \cdot \sqrt{n}$

(2) A(x)=1 iff $|\#00(x) - n/4| \le 10 \cdot \sqrt{n}$

Statistical Tests

More examples:

(3) A(x)=1 iff max-run-of-0(x)<10$\cdot \log_2(n)$

…

…

Advantage

Let G:K $\longrightarrow \{0,1\}^n$ be a PRG and A a stat. test on $\{0,1\}^n$

Define:

$$Adv_{PRG}[A,G] = \left| Pr_{k \xleftarrow{R} K}\left[A\big(G(k)\big) = 1\right] - Pr_{r \xleftarrow{R} \{0,1\}^n}\left[A(r) = 1\right] \right| \in [0,1]$$

Adv close to 1 $\Longrightarrow$ A can dist. G from random

Adv close to 0 $\Longrightarrow$ A cannot dist. G from random

A silly example: A(x) = 0 $\Rightarrow$ Adv_{PRG} [A,G] =

Suppose $G: K \longrightarrow \{0,1\}^n$ satisfies **$msb(G(k)) = 1$** for 2/3 of keys in K

Define stat. test $A(x)$ as:

if [$msb(x)=1$] output "1" else output "0"

Then

$$Adv_{PRG}[A,G] = \big|\, Pr[\, A(G(k))=1] - Pr[\, A(r)=1 \,] \,\big| =$$

Secure PRGs: crypto definition

Def: We say that $G: K \longrightarrow \{0,1\}^n$ is a **secure PRG** if

$\forall$ "eff" stat. test A:

$\text{Adv}_{\text{PRG}}[A,G]$ is "negligible"

Are there provably secure PRGs?

but we have heuristic candidates.

More Generally

Let P_1 and P_2 be two distributions over $\{0,1\}^n$

Def: We say that P_1 and P_2 are

computationally indistinguishable (denoted $P_1 \approx_p P_2$)

if $\forall$ "eff" stat. test A:

$$\left| \Pr_{x \leftarrow P_1}[(A(x)=1] - \Pr_{x \leftarrow P_2}[(A(x)=1] \right| < \text{"negligible"}$$

Example: a PRG is secure if $\{ k \xleftarrow{R} K : G(k) \} \approx_p \text{uniform}(\{0,1\}^n)$

Semantic security

Goal: secure PRG $\Rightarrow$ "secure" stream cipher

What is a secure cipher?

Attacker's abilities: **obtains one ciphertext** (for now)

Possible security requirements:

attempt #1: **attacker cannot recover secret key** $E(k,m)=m$

attempt #2: **attacker cannot recover all of plaintext**

$$E(k, m_0 \,||\, m_1) = m_0 \,||\, m_1 \oplus k$$

Recall Shannon's idea:

CT should reveal no "info" about PT

Recall Shannon's perfect secrecy

Let (E,D) be a cipher over (K,M,C)

(E,D) has perfect secrecy if $\forall\, m_0, m_1 \in M \quad (\ |m_0| = |m_1|\ )$

$$\{\, E(k,m_0) \,\} \;=\; \{\, E(k,m_1) \,\} \qquad \text{where } \; k \leftarrow K$$

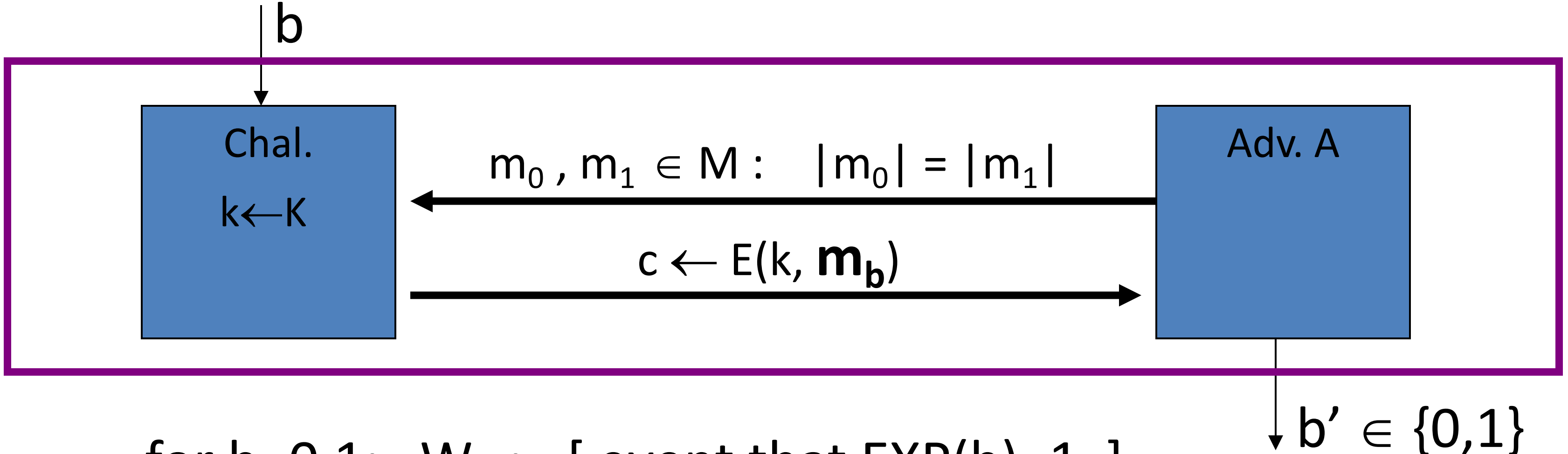
(E,D) has perfect secrecy if $\forall\, m_0, m_1 \in M \quad (\ |m_0| = |m_1|\ )$

$$\{\, E(k,m_0) \,\} \;\approx_p\; \{\, E(k,m_1) \,\} \qquad \text{where } \; k \leftarrow K$$

... but also need adversary to exhibit $m_0, m_1 \in M$ explicitly

Semantic Security (one-time key)

For $b=0,1$ define experiments EXP(0) and EXP(1) as:



for $b=0,1$: $\quad W_b :=$ [event that $\text{EXP}(b)=1$]

$$\text{Adv}_{SS}[A,E] := \Big| \Pr[W_0] - \Pr[W_1] \Big| \quad \in [0,1]$$

Semantic Security (one-time key)

Def: $\mathbb{E}$ is **semantically secure** if for all efficient A

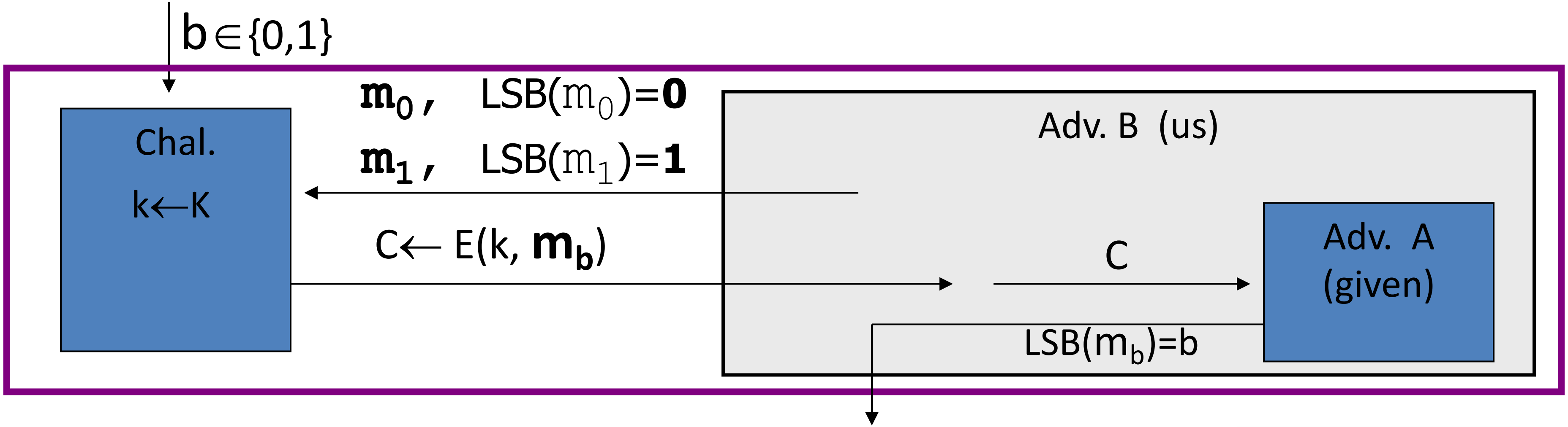
$\mathrm{Adv}_{SS}[A,\mathbb{E}]$ is negligible.

$\Rightarrow$ for all explicit $m_0 , m_1 \in M$: $\{ E(k,m_0) \} \approx_p \{ E(k,m_1) \}$

Examples

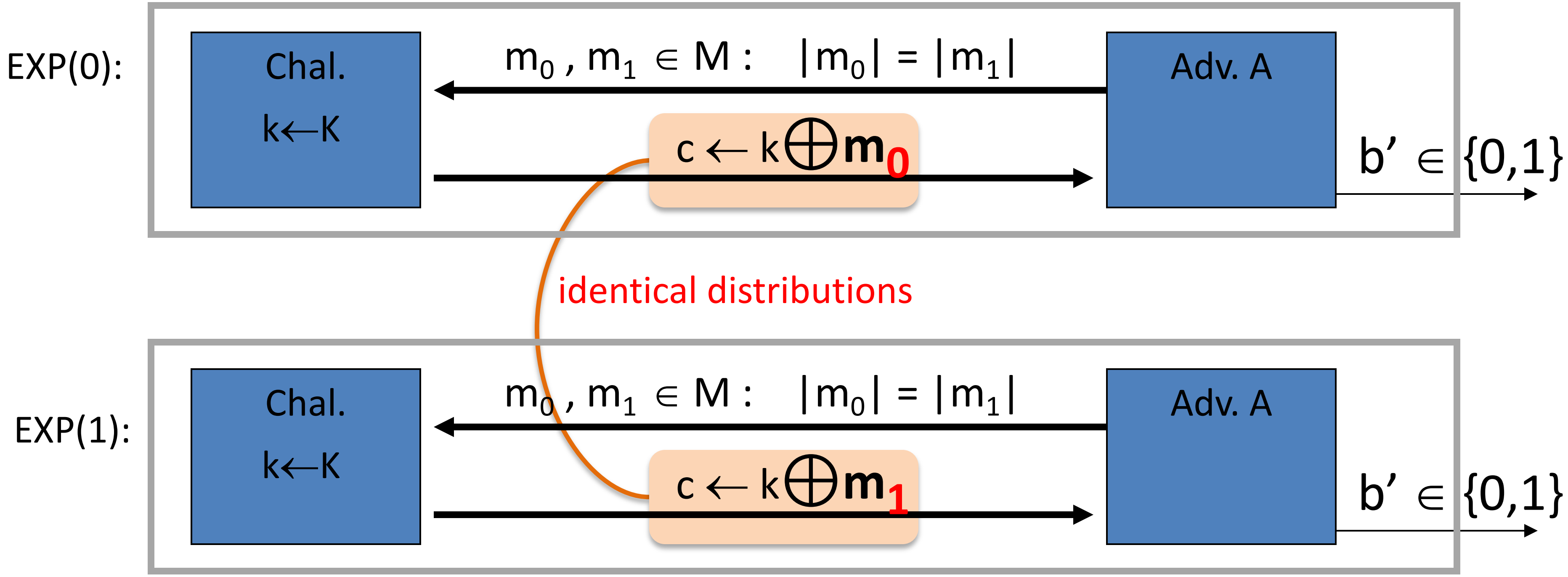
Suppose efficient A can always deduce LSB of PT from CT.

$\Rightarrow \quad \mathbb{E} = (E,D)$ is not semantically secure.



Then $\text{Adv}_{SS}[B, \mathbb{E}] = \Big| \Pr[\, \textbf{EXP(0)}=1 \,] - \Pr[\, \textbf{EXP(1)}=1 \,] \Big| =$

OTP is semantically secure



For **all** A: $\text{Adv}_{SS}[A,\text{OTP}] = \left|\, \Pr[\, \mathbf{A}(\mathbf{k} \oplus \mathbf{m_0}) = 1\,] - \Pr[\, \mathbf{A}(\mathbf{k} \oplus \mathbf{m_1}) = 1\,] \,\right|$

Stream ciphers are semantically secure

Goal: secure PRG $\Rightarrow$ semantically secure stream cipher

Stream ciphers are semantically secure

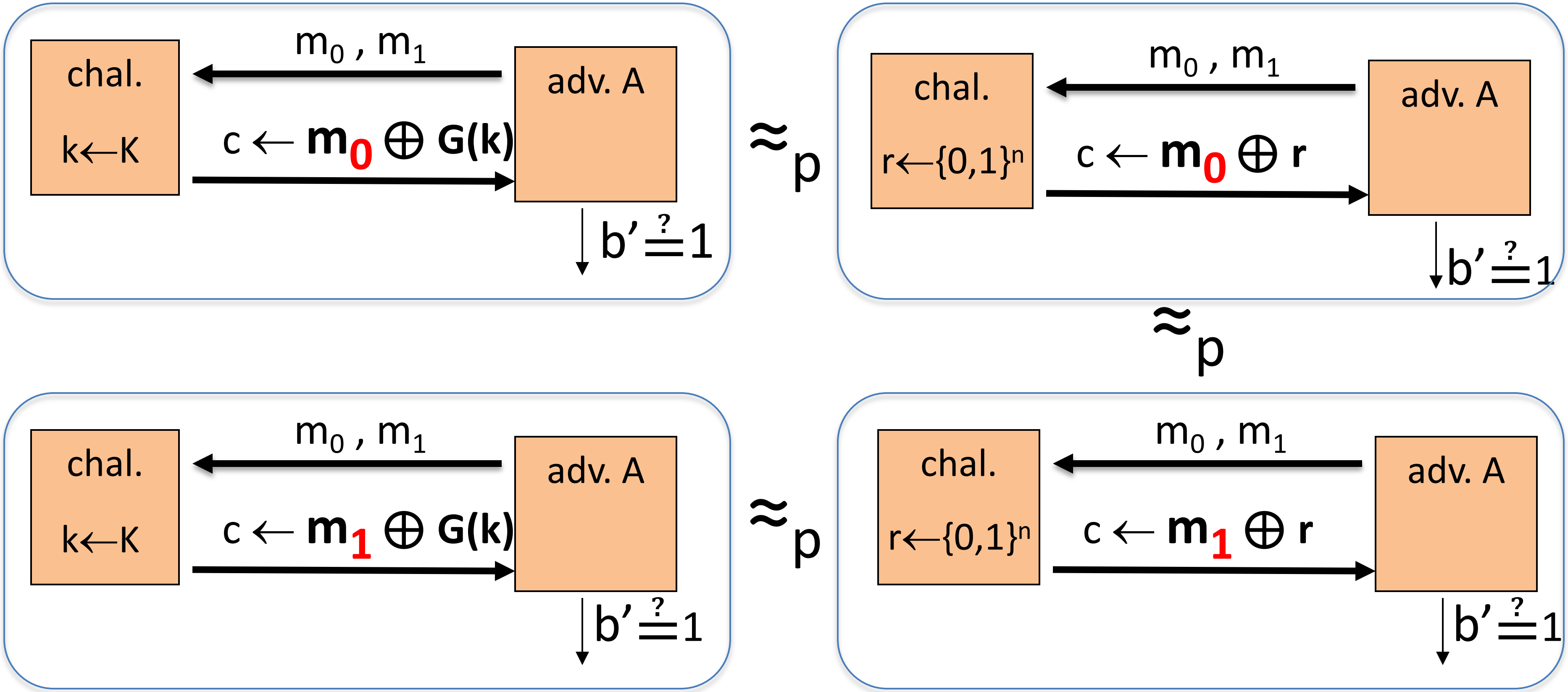
Thm: $G: K \longrightarrow \{0,1\}^n$ is a secure PRG $\Rightarrow$

stream cipher E derived from G is sem. sec.

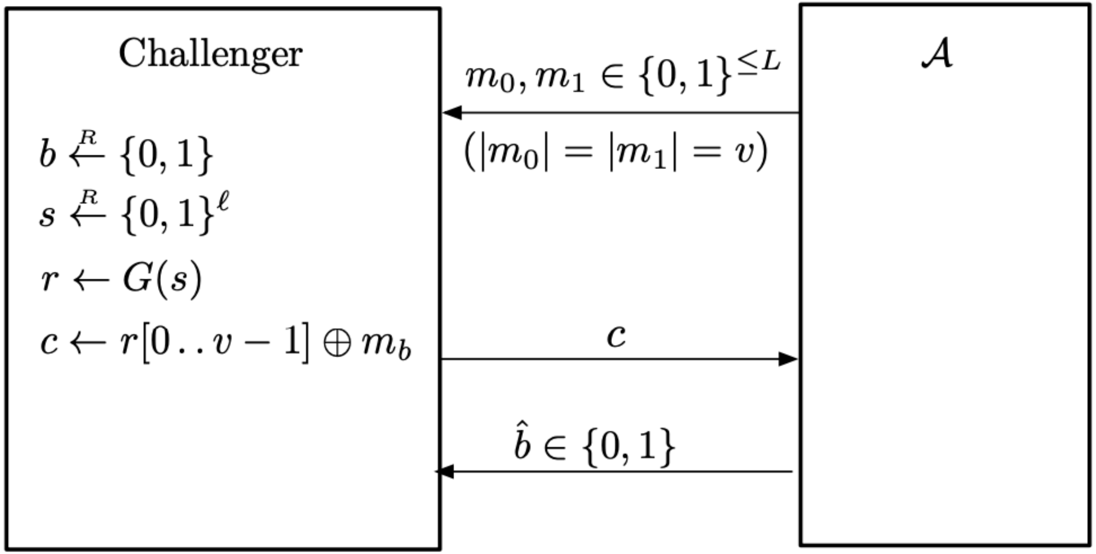
$\forall$ sem. sec. adversary A , $\exists$a PRG adversary B s.t.

$$\mathrm{Adv}_{SS}[A,E] \leq 2 \cdot \mathrm{Adv}_{PRG}[B,G]$$

Proof: intuition



Game 0



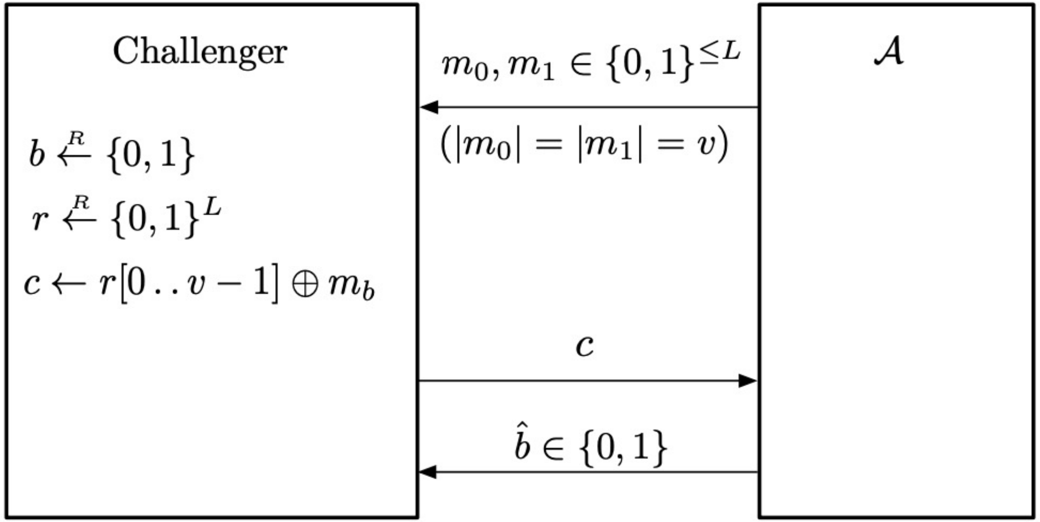
W_0: the event that $\hat{b} = b$ in Game 0

$$\text{SSadv}^*[\mathcal{A}, \mathcal{E}] = |\Pr[W_0] - 1/2|$$

Upon receiving $m_0, m_1 \in \{0,1\}^v$ from $\mathcal{A}$, for some $v \leq L$, do:

$b \xleftarrow{R} \{0,1\}$
$s \xleftarrow{R} \{0,1\}^\ell$, $r \leftarrow G(s)$
$c \leftarrow r[0..v-1] \oplus m_b$
send c to $\mathcal{A}$.

Game 1



Upon receiving $m_0, m_1 \in \{0,1\}^v$ from $\mathcal{A}$, for some $v \leq L$, do:

$b \xleftarrow{R} \{0,1\}$

$r \xleftarrow{R} \{0,1\}^L$

$c \leftarrow r[0..v-1] \oplus m_b$

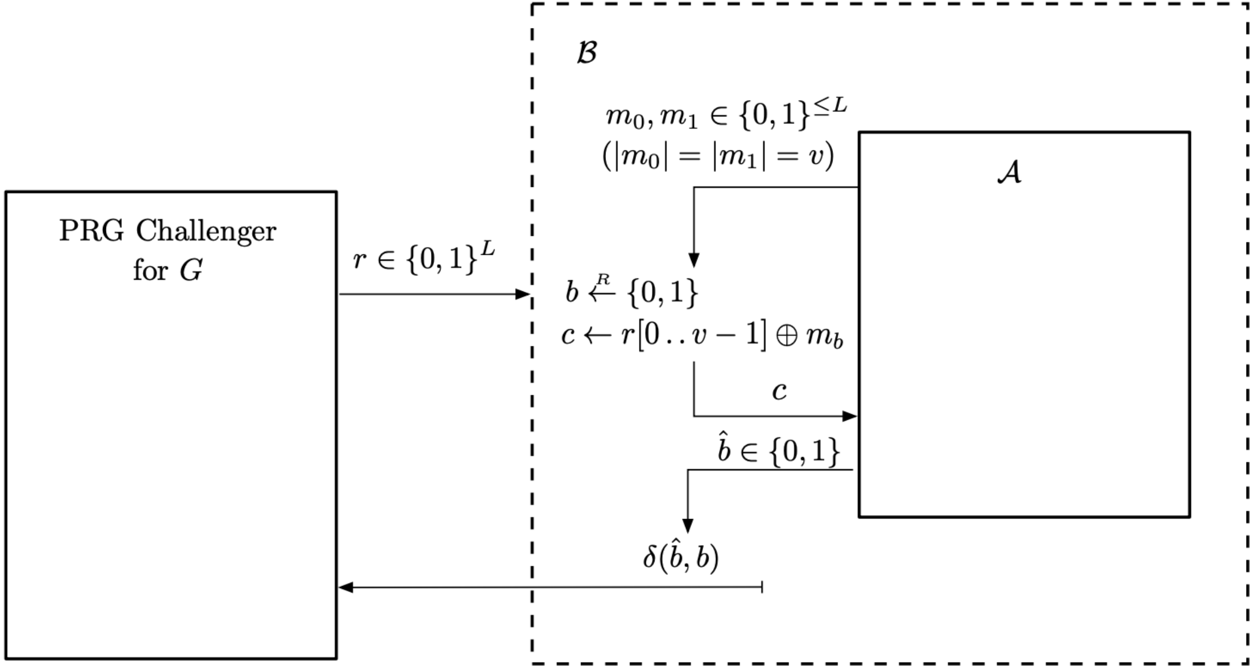
send c to $\mathcal{A}$.

W_1: the event that $\hat{b} = b$ in Game 1

$$\Pr[W_1] = 1/2.$$

Our goal: $|\Pr[W_0] - \Pr[W_1]| = \mathrm{PRGadv}[\mathcal{B}, G].$

Define $\delta(x,y) := \begin{cases} 1 & \text{if } x = y, \\ 0 & \text{if } x \neq y. \end{cases}$



P0: probability that B output 1 when PRG challenger is running exp 0.

P1: probability that B output 1 when PRG challenger is running exp 1.

$$\mathrm{PRGadv}[\mathcal{B}, G] = |p_1 - p_0|.$$

$$p_0 = \Pr[W_0], \quad p_1 = \Pr[W_1].$$

$\Downarrow$

$$|\Pr[W_0] - \Pr[W_1]| = \mathrm{PRGadv}[\mathcal{B}, G].$$

Upon receiving $m_0, m_1 \in \{0,1\}^v$ from $\mathcal{A}$, for some $v \leq L$, do:

$b \xleftarrow{R} \{0,1\}$
$c \leftarrow r[0 \mathinner{..} v-1] \oplus m_b$
send c to $\mathcal{A}$.

Finally, when $\mathcal{A}$ outputs a bit $\hat{b}$, $\mathcal{B}$ outputs the bit $\delta(\hat{b}, b)$.

Recall PRG game