

CSIT881

Programming and Data Structures

Sorting Algorithm



UNIVERSITY
OF WOLLONGONG
AUSTRALIA

Dr. Joseph Tonien

Objectives

- Bubble sort
- Selection sort
- Insertion sort

Python List

A list/array is used to hold a list of items:

```
animal_list = ["dog", "cat", "frog"]
```

```
fibonacci_numbers = [0, 1, 1, 2, 3, 5, 8, 13]
```

```
prime_numbers = [2, 3, 5, 7, 11, 13, 17]
```

```
subject_list = ["MATH101", "CS222", "PHY102", "ACCY203"]
```

```
selected_products = [] # this is an empty list
```

This is how we define a list:

```
list_variable = [item1, item2, ..., itemN]
```

Python List

List items can be accessed via **index**:

```
fibonacci_numbers = [0, 1, 1, 2, 3, 5, 8, 13]
```

```
print(fibonacci_numbers[0]) → 0
print(fibonacci_numbers[1]) → 1
print(fibonacci_numbers[2]) → 1
print(fibonacci_numbers[3]) → 2
print(fibonacci_numbers[4]) → 3
print(fibonacci_numbers[5]) → 5
print(fibonacci_numbers[6]) → 8
print(fibonacci_numbers[7]) → 13
```

items can be **appended** to the end of the list:

```
fibonacci_numbers.append(21)
fibonacci_numbers.append(34)
fibonacci_numbers.append(55)
fibonacci_numbers.append(89)
```

Python List

using `len` to find out how many items in the list:

```
animal_list = ["dog", "cat", "frog"]
```

```
animal_count = len(animal_list) → 3
```

Note that `len(animal_list)` is 3, but the **last index** is 2 because the index start at 0.

```
print(animal_list[0]) → "dog"  
print(animal_list[1]) → "cat"  
print(animal_list[2]) → "frog"
```

Bubble sort

Bubble (up) sort algorithm:

- Go through the list, compares adjacent items and swaps them if they are in the wrong order;
- Repeat this process until the list is sorted.

The name of the algorithm is derived from the fact that: after each round, the largest items are **bubbled up** towards the end of the list.

Bubble sort

Let's look at each round in details:

(compares adjacent items and swaps them if they are in the wrong order)

```
round 0 start [60, 10, 90, 50, 100, 80, 70, 30, 40, 20]
               [10, 60, 90, 50, 100, 80, 70, 30, 40, 20]
               [10, 60, 90, 50, 100, 80, 70, 30, 40, 20]
               [10, 60, 50, 90, 100, 80, 70, 30, 40, 20]
               [10, 60, 50, 90, 100, 80, 70, 30, 40, 20]
               [10, 60, 50, 90, 80, 100, 70, 30, 40, 20]
               [10, 60, 50, 90, 80, 70, 100, 30, 40, 20]
               [10, 60, 50, 90, 80, 70, 30, 100, 40, 20]
               [10, 60, 50, 90, 80, 70, 30, 40, 100, 20]
round 0 end   [10, 60, 50, 90, 80, 70, 30, 40, 20, 100]
```

Observe the movement of the largest item 100

We can see that after the 1st round, the largest item 100 is **bubbled up** to the end of the list.

Bubble sort

Let's look at each round in details:

(compares adjacent items and swaps them if they are in the wrong order)

round 1 start	[10, 60, 50, 90, 80, 70, 30, 40, 20, 100]
	[10, 60, 50, 90, 80, 70, 30, 40, 20, 100]
	[10, 50, 60, 90, 80, 70, 30, 40, 20, 100]
	[10, 50, 60, 90, 80, 70, 30, 40, 20, 100]
	[10, 50, 60, 80, 90, 70, 30, 40, 20, 100]
	[10, 50, 60, 80, 70, 90, 30, 40, 20, 100]
	[10, 50, 60, 80, 70, 30, 90, 40, 20, 100]
	[10, 50, 60, 80, 70, 30, 40, 90, 20, 100]
round 1 end	[10, 50, 60, 80, 70, 30, 40, 20, 90, 100]

We can see that after the 2nd round, the 2nd largest item 90 is **bubbled up** to the right place towards the end of the list.

Bubble sort

Let's look at each round in details:

(compares adjacent items and swaps them if they are in the wrong order)

round 2 start	[10, 50, 60, 80, 70, 30, 40, 20, 90, 100]
	[10, 50, 60, 80, 70, 30, 40, 20, 90, 100]
	[10, 50, 60, 80, 70, 30, 40, 20, 90, 100]
	[10, 50, 60, 80, 70, 30, 40, 20, 90, 100]
	[10, 50, 60, 70, 80, 30, 40, 20, 90, 100]
	[10, 50, 60, 70, 30, 80, 40, 20, 90, 100]
	[10, 50, 60, 70, 30, 40, 80, 20, 90, 100]
round 2 end	[10, 50, 60, 70, 30, 40, 20, 80, 90, 100]

We can see that after the 3rd round, the 3rd largest item **80** is **bubbled up** to the right place towards the end of the list.

Bubble sort

Let's look at each round in details:

(compares adjacent items and swaps them if they are in the wrong order)

round 3 start	[10, 50, 60, 70, 30, 40, 20, 80, 90, 100]
	[10, 50, 60, 70, 30, 40, 20, 80, 90, 100]
	[10, 50, 60, 70, 30, 40, 20, 80, 90, 100]
	[10, 50, 60, 70, 30, 40, 20, 80, 90, 100]
	[10, 50, 60, 30, 70, 40, 20, 80, 90, 100]
	[10, 50, 60, 30, 40, 70, 20, 80, 90, 100]
round 3 end	[10, 50, 60, 30, 40, 20, 70, 80, 90, 100]

We can see that after the 4th round, the 4th largest item **70** is **bubbled up** to the right place towards the end of the list.

Bubble sort

Let's look at each round in details:

```
round 4 start [10, 50, 60, 30, 40, 20, 70, 80, 90, 100]
               [10, 50, 60, 30, 40, 20, 70, 80, 90, 100]
               [10, 50, 60, 30, 40, 20, 70, 80, 90, 100]
               [10, 50, 30, 60, 40, 20, 70, 80, 90, 100]
               [10, 50, 30, 40, 60, 20, 70, 80, 90, 100]
round 4 end   [10, 50, 30, 40, 20, 60, 70, 80, 90, 100]
```

After the 5th round, the 5th largest item **60** is **bubbled up** to the right place towards the end of the list.

```
round 5 start [10, 50, 30, 40, 20, 60, 70, 80, 90, 100]
               [10, 50, 30, 40, 20, 60, 70, 80, 90, 100]
               [10, 30, 50, 40, 20, 60, 70, 80, 90, 100]
               [10, 30, 40, 50, 20, 60, 70, 80, 90, 100]
round 5 end   [10, 30, 40, 20, 50, 60, 70, 80, 90, 100]
```

After the 6th round, the 6th largest item **50** is **bubbled up** to the right place towards the end of the list.

Bubble sort

Let's look at each round in details:

```
round 6 start [10, 30, 40, 20, 50, 60, 70, 80, 90, 100]
               [10, 30, 40, 20, 50, 60, 70, 80, 90, 100]
               [10, 30, 40, 20, 50, 60, 70, 80, 90, 100]
round 6 end   [10, 30, 20, 40, 50, 60, 70, 80, 90, 100]
```

After the 7th round, the 7th largest item **40** is **bubbled up** to the right place towards the end of the list.

```
round 7 start [10, 30, 20, 40, 50, 60, 70, 80, 90, 100]
               [10, 30, 20, 40, 50, 60, 70, 80, 90, 100]
round 7 end   [10, 20, 30, 40, 50, 60, 70, 80, 90, 100]
```

After the 8th round, the 8th largest item **30** is **bubbled up** to the right place towards the end of the list.

```
round 8 start [10, 20, 30, 40, 50, 60, 70, 80, 90, 100]
round 8 end   [10, 20, 30, 40, 50, 60, 70, 80, 90, 100]
```

After the 9th round, the 9th largest item **20** is **bubbled up** to the right place towards the end of the list - and the sorting is **DONE!!!**

Bubble sort

pseudocode

```
n = length-of(intList)
FOR i from 0 to (n-2)
    FOR j from 1 to (n-i-1)
        // compare adj items, swap if in wrong order
        IF intList[j-1] > intList[j]:
            swap intList[j-1] and intList[j]
        END IF
    END FOR
END FOR
```

Bubble sort

Python implementation

```
def bubbleSort(intList):  
    n = len(intList)  
    for i in range(0, n-1):  
        #{  
            for j in range(1, n-i):  
                #{  
                    # compare adj items, swap if in wrong order  
                    if intList[j-1] > intList[j]:  
                        # swap intList[j-1] and intList[j]  
                        temp = intList[j-1]  
                        intList[j-1] = intList[j]  
                        intList[j] = temp  
                #}  
            #}  
        #}
```

Bubble sort

Let's look at another example:

```
round 0 start [ 90, 100, 10, 20, 30, 40, 50, 60, 70, 80]
               [90, 100, 10, 20, 30, 40, 50, 60, 70, 80]
               [90, 10, 100, 20, 30, 40, 50, 60, 70, 80]
               [90, 10, 20, 100, 30, 40, 50, 60, 70, 80]
               [90, 10, 20, 30, 100, 40, 50, 60, 70, 80]
               [90, 10, 20, 30, 40, 100, 50, 60, 70, 80]
               [90, 10, 20, 30, 40, 50, 100, 60, 70, 80]
               [90, 10, 20, 30, 40, 50, 60, 100, 70, 80]
               [90, 10, 20, 30, 40, 50, 60, 70, 100, 80]
round 0 end   [90, 10, 20, 30, 40, 50, 60, 70, 80, 100]
```

```
round 1 start [ 90, 10, 20, 30, 40, 50, 60, 70, 80, 100]
               [10, 90, 20, 30, 40, 50, 60, 70, 80, 100]
               [10, 20, 90, 30, 40, 50, 60, 70, 80, 100]
               [10, 20, 30, 90, 40, 50, 60, 70, 80, 100]
               [10, 20, 30, 40, 90, 50, 60, 70, 80, 100]
               [10, 20, 30, 40, 50, 90, 60, 70, 80, 100]
               [10, 20, 30, 40, 50, 60, 90, 70, 80, 100]
               [10, 20, 30, 40, 50, 60, 70, 90, 80, 100]
round 1 end   [10, 20, 30, 40, 50, 60, 70, 80, 90, 100]
```

Bubble sort

round 2 start	[10, 20, 30, 40, 50, 60, 70, 80, 90, 100]
	[10, 20, 30, 40, 50, 60, 70, 80, 90, 100]
	[10, 20, 30, 40, 50, 60, 70, 80, 90, 100]
	[10, 20, 30, 40, 50, 60, 70, 80, 90, 100]
	[10, 20, 30, 40, 50, 60, 70, 80, 90, 100]
	[10, 20, 30, 40, 50, 60, 70, 80, 90, 100]
	[10, 20, 30, 40, 50, 60, 70, 80, 90, 100]
round 2 end	[10, 20, 30, 40, 50, 60, 70, 80, 90, 100]

Notice that in round 2, NOT a single swap is needed. It means that the list has already been sorted. NO NEED TO GO ANY FURTHER TO round 3, round 4, round 5, ...

Bubble sort

Better algorithm:

pseudocode

```
n = length-of(intList)
FOR i from 0 to (n-2)
    swapped = false
    FOR j from 1 to (n-i-1)
        // compare adj items, swap if in wrong order
        IF intList[j-1] > intList[j]:
            swap intList[j-1] and intList[j]
            # remember that swap is needed
            swapped = true
        END IF
    END FOR
    BREAK IF swapped = false
END FOR
```

Bubble sort

Python implementation

better algorithm

```
def bubbleSort(intList):
    n = len(intList)
    for i in range(0, n-1):
        swapped = False
        for j in range(1, n-i):
            # compare adj items, swap if in wrong order
            if intList[j-1] > intList[j]:
                # swap intList[j-1] and intList[j]
                temp = intList[j-1]
                intList[j-1] = intList[j]
                intList[j] = temp
                # remember that swap is needed
                swapped = True
        if not swapped:
            # swap is NOT needed, so list is SORTED
            break
```

Bubble sort

Suggested activities:

- Make up a list of integers and write down in details each step in sorting this list of integers;
- Sort a list of integers in descending order;
- Write a Bubble (down) sort algorithm, so that after each round, the smallest items are **bubbled down** towards the start of the list;

Selection sort

Have a look at this example, can you figure out the process of selection sort?

round 0 start	[60, 10, 90, 50, 100, 80, 70, 30, 40, 20]
round 0 finish	[10, 60, 90, 50, 100, 80, 70, 30, 40, 20]
round 1 start	[10, 60, 90, 50, 100, 80, 70, 30, 40, 20]
round 1 finish	[10, 20, 90, 50, 100, 80, 70, 30, 40, 60]
round 2 start	[10, 20, 90, 50, 100, 80, 70, 30, 40, 60]
round 2 finish	[10, 20, 30, 50, 100, 80, 70, 90, 40, 60]
round 3 start	[10, 20, 30, 50, 100, 80, 70, 90, 40, 60]
round 3 finish	[10, 20, 30, 40, 100, 80, 70, 90, 50, 60]
round 4 start	[10, 20, 30, 40, 100, 80, 70, 90, 50, 60]
round 4 finish	[10, 20, 30, 40, 50, 80, 70, 90, 100, 60]
round 5 start	[10, 20, 30, 40, 50, 80, 70, 90, 100, 60]
round 5 finish	[10, 20, 30, 40, 50, 60, 70, 90, 100, 80]
round 6 start	[10, 20, 30, 40, 50, 60, 70, 90, 100, 80]
round 6 finish	[10, 20, 30, 40, 50, 60, 70, 90, 100, 80]
round 7 start	[10, 20, 30, 40, 50, 60, 70, 90, 100, 80]
round 7 finish	[10, 20, 30, 40, 50, 60, 70, 80, 100, 90]
round 8 start	[10, 20, 30, 40, 50, 60, 70, 80, 100, 90]
round 8 finish	[10, 20, 30, 40, 50, 60, 70, 80, 90, 100]

Selection sort

At each round i : find the minimum in $\{\text{item } i, \text{item } i+1, \dots\}$
and swap it to the position i

round 0 start	[60, 10, 90, 50, 100, 80, 70, 30, 40, 20]
round 0 finish	[10, 60, 90, 50, 100, 80, 70, 30, 40, 20]
round 1 start	[10, 60, 90, 50, 100, 80, 70, 30, 40, 20]
round 1 finish	[10, 20, 90, 50, 100, 80, 70, 30, 40, 60]
round 2 start	[10, 20, 90, 50, 100, 80, 70, 30, 40, 60]
round 2 finish	[10, 20, 30, 50, 100, 80, 70, 90, 40, 60]
round 3 start	[10, 20, 30, 50, 100, 80, 70, 90, 40, 60]
round 3 finish	[10, 20, 30, 40, 100, 80, 70, 90, 50, 60]
round 4 start	[10, 20, 30, 40, 100, 80, 70, 90, 50, 60]
round 4 finish	[10, 20, 30, 40, 50, 80, 70, 90, 100, 60]
round 5 start	[10, 20, 30, 40, 50, 80, 70, 90, 100, 60]
round 5 finish	[10, 20, 30, 40, 50, 60, 70, 90, 100, 80]
round 6 start	[10, 20, 30, 40, 50, 60, 70, 90, 100, 80]
round 6 finish	[10, 20, 30, 40, 50, 60, 70, 90, 100, 80]
round 7 start	[10, 20, 30, 40, 50, 60, 70, 90, 100, 80]
round 7 finish	[10, 20, 30, 40, 50, 60, 70, 80, 100, 90]
round 8 start	[10, 20, 30, 40, 50, 60, 70, 80, 100, 90]
round 8 finish	[10, 20, 30, 40, 50, 60, 70, 80, 90, 100]

Selection sort

Look at each round in details:

```
round 0 start    [ 60, 10, 90, 50, 100, 80, 70, 30, 40, 20 ]
```

```
find the minimum item in intList[0..9]
```

```
found minimum item: intList[1] = 10
```

```
swap intList[0] and intList[1]
```

```
round 0 finish   [ 10, 60, 90, 50, 100, 80, 70, 30, 40, 20 ]
```

```
round 1 start    [ 10, 60, 90, 50, 100, 80, 70, 30, 40, 20 ]
```

```
find the minimum item in intList[1..9]
```

```
found minimum item: intList[9] = 20
```

```
swap intList[1] and intList[9]
```

```
round 1 finish   [ 10, 20, 90, 50, 100, 80, 70, 30, 40, 60 ]
```

```
round 2 start    [ 10, 20, 90, 50, 100, 80, 70, 30, 40, 60 ]
```

```
find the minimum item in intList[2..9]
```

```
found minimum item: intList[7] = 30
```

```
swap intList[2] and intList[7]
```

```
round 2 finish   [ 10, 20, 30, 50, 100, 80, 70, 90, 40, 60 ]
```

Selection sort

Look at each round in details:

```
round 3 start    [10, 20, 30, 50, 100, 80, 70, 90, 40, 60]
```

```
find the minimum item in intList[3..9]
```

```
found minimum item: intList[8] = 40
```

```
swap intList[3] and intList[8]
```

```
round 3 finish   [10, 20, 30, 40, 100, 80, 70, 90, 50, 60]
```

```
round 4 start    [10, 20, 30, 40, 100, 80, 70, 90, 50, 60]
```

```
find the minimum item in intList[4..9]
```

```
found minimum item: intList[8] = 50
```

```
swap intList[4] and intList[8]
```

```
round 4 finish   [10, 20, 30, 40, 50, 80, 70, 90, 100, 60]
```

```
round 5 start    [10, 20, 30, 40, 50, 80, 70, 90, 100, 60]
```

```
find the minimum item in intList[5..9]
```

```
found minimum item: intList[9] = 60
```

```
swap intList[5] and intList[9]
```

```
round 5 finish   [10, 20, 30, 40, 50, 60, 70, 90, 100, 80]
```

Selection sort

Look at each round in details:

```
round 6 start    [10, 20, 30, 40, 50, 60, 70, 90, 100, 80]
```

```
find the minimum item in intList[6..9]
```

```
found minimum item: intList[6] = 70
```

```
swap intList[6] and intList[6] (so basically: do nothing)
```

```
round 6 finish   [10, 20, 30, 40, 50, 60, 70, 90, 100, 80]
```

```
round 7 start    [10, 20, 30, 40, 50, 60, 70, 90, 100, 80]
```

```
find the minimum item in intList[7..9]
```

```
found minimum item: intList[9] = 80
```

```
swap intList[7] and intList[9]
```

```
round 7 finish   [10, 20, 30, 40, 50, 60, 70, 80, 100, 90]
```

```
round 8 start    [10, 20, 30, 40, 50, 60, 70, 80, 100, 90]
```

```
find the minimum item in intList[8..9]
```

```
found minimum item: intList[9] = 90
```

```
swap intList[8] and intList[9]
```

```
round 8 finish   [10, 20, 30, 40, 50, 60, 70, 80, 90, 100]
```


Selection sort

Algorithm for Selection Sort:

pseudocode

```
n = length-of(intList)
FOR i from 0 to (n-2)
    among intList[i], intList[i+1], ..., intList[n-1]
    find the minimum item intList[kMin]

    swap intList[i] and intList[kMin]
END FOR
```

Selection sort

Python implementation

```
def selectionSort(intList):
    n = len(intList)
    for i in range(0, n-1):
        #{
            # find the minimum item in intList[i .. n-1]
            kMin = i
            for k in range(i+1, n):
                if (intList[k] < intList[kMin]):
                    kMin = k

            # swap intList[i] and intList[kMin]
            if (kMin != i):
                temp = intList[i]
                intList[i] = intList[kMin]
                intList[kMin] = temp

        # }
```

Insertion sort

Have a look at this example, can you figure out the process of insertion sort?

round 1 start	[60, 10, 90, 50, 100, 80, 70, 30, 40, 20]
round 1 finish	[10, 60, 90, 50, 100, 80, 70, 30, 40, 20]
round 2 start	[10, 60, 90, 50, 100, 80, 70, 30, 40, 20]
round 2 finish	[10, 60, 90, 50, 100, 80, 70, 30, 40, 20]
round 3 start	[10, 60, 90, 50, 100, 80, 70, 30, 40, 20]
round 3 finish	[10, 50, 60, 90, 100, 80, 70, 30, 40, 20]
round 4 start	[10, 50, 60, 90, 100, 80, 70, 30, 40, 20]
round 4 finish	[10, 50, 60, 90, 100, 80, 70, 30, 40, 20]
round 5 start	[10, 50, 60, 90, 100, 80, 70, 30, 40, 20]
round 5 finish	[10, 50, 60, 80, 90, 100, 70, 30, 40, 20]
round 6 start	[10, 50, 60, 80, 90, 100, 70, 30, 40, 20]
round 6 finish	[10, 50, 60, 70, 80, 90, 100, 30, 40, 20]
round 7 start	[10, 50, 60, 70, 80, 90, 100, 30, 40, 20]
round 7 finish	[10, 30, 50, 60, 70, 80, 90, 100, 40, 20]
round 8 start	[10, 30, 50, 60, 70, 80, 90, 100, 40, 20]
round 8 finish	[10, 30, 40, 50, 60, 70, 80, 90, 100, 20]
round 9 start	[10, 30, 40, 50, 60, 70, 80, 90, 100, 20]
round 9 finish	[10, 20, 30, 40, 50, 60, 70, 80, 90, 100]

Insertion sort

At each round i : {item 0, item 1, ..., item i } are sorted

round 1 start	[60, 10, 90, 50, 100, 80, 70, 30, 40, 20]
round 1 finish	[10, 60, 90, 50, 100, 80, 70, 30, 40, 20]
round 2 start	[10, 60, 90, 50, 100, 80, 70, 30, 40, 20]
round 2 finish	[10, 60, 90, 50, 100, 80, 70, 30, 40, 20]
round 3 start	[10, 60, 90, 50, 100, 80, 70, 30, 40, 20]
round 3 finish	[10, 50, 60, 90, 100, 80, 70, 30, 40, 20]
round 4 start	[10, 50, 60, 90, 100, 80, 70, 30, 40, 20]
round 4 finish	[10, 50, 60, 90, 100, 80, 70, 30, 40, 20]
round 5 start	[10, 50, 60, 90, 100, 80, 70, 30, 40, 20]
round 5 finish	[10, 50, 60, 80, 90, 100, 70, 30, 40, 20]
round 6 start	[10, 50, 60, 80, 90, 100, 70, 30, 40, 20]
round 6 finish	[10, 50, 60, 70, 80, 90, 100, 30, 40, 20]
round 7 start	[10, 50, 60, 70, 80, 90, 100, 30, 40, 20]
round 7 finish	[10, 30, 50, 60, 70, 80, 90, 100, 40, 20]
round 8 start	[10, 30, 50, 60, 70, 80, 90, 100, 40, 20]
round 8 finish	[10, 30, 40, 50, 60, 70, 80, 90, 100, 20]
round 9 start	[10, 30, 40, 50, 60, 70, 80, 90, 100, 20]
round 9 finish	[10, 20, 30, 40, 50, 60, 70, 80, 90, 100]

Insertion sort

At each round i : {item 0, item 1, ..., item i } are sorted

round 1 start	[60, 10, 90, 50, 100, 80, 70, 30, 40, 20]
round 1 finish	[10, 60, 90, 50, 100, 80, 70, 30, 40, 20]
round 2 start	[10, 60, 90, 50, 100, 80, 70, 30, 40, 20]
round 2 finish	[10, 60, 90, 50, 100, 80, 70, 30, 40, 20]
round 3 start	[10, 60, 90, 50, 100, 80, 70, 30, 40, 20]
round 3 finish	[10, 50, 60, 90, 100, 80, 70, 30, 40, 20]
round 4 start	[10, 50, 60, 90, 100, 80, 70, 30, 40, 20]
round 4 finish	[10, 50, 60, 90, 100, 80, 70, 30, 40, 20]
round 5 start	[10, 50, 60, 90, 100, 80, 70, 30, 40, 20]
round 5 finish	[10, 50, 60, 80, 90, 100, 70, 30, 40, 20]

helicopter view pseudocode of Insertion Sort Algorithm of a list of integers

```
n = length-of(intList)
FOR i from 1 to (n-1)
    // sort intList[0], intList[1], ..., intList[i]
END FOR
```

Insertion sort

Look at each round in details:

```
round 1 start    [ 60, 10, 90, 50, 100, 80, 70, 30, 40, 20]
```

```
are 10 and 60 in order? No
```

```
swap 10 and 60: [10, 60, 90, 50, 100, 80, 70, 30, 40, 20]
```

```
round 1 finish   [ 10, 60, 90, 50, 100, 80, 70, 30, 40, 20]
```

```
round 2 start    [ 10, 60, 90, 50, 100, 80, 70, 30, 40, 20]
```

```
are 90 and 60 in order? Yes
```

```
round 2 finish   [ 10, 60, 90, 50, 100, 80, 70, 30, 40, 20]
```

```
round 3 start    [ 10, 60, 90, 50, 100, 80, 70, 30, 40, 20]
```

```
are 50 and 90 in order? No
```

```
swap 50 and 90: [10, 60, 50, 90, 100, 80, 70, 30, 40, 20]
```

```
are 50 and 60 in order? No
```

```
swap 50 and 60: [10, 50, 60, 90, 100, 80, 70, 30, 40, 20]
```

```
are 50 and 10 in order? Yes
```

```
round 3 finish   [ 10, 50, 60, 90, 100, 80, 70, 30, 40, 20]
```

Insertion sort

Look at each round in details:

```
round 4 start      [10, 50, 60, 90, 100, 80, 70, 30, 40, 20]
```

```
are 100 and 90 in order? Yes
```

```
round 4 finish     [10, 50, 60, 90, 100, 80, 70, 30, 40, 20]
```

```
round 5 start      [10, 50, 60, 90, 100, 80, 70, 30, 40, 20]
```

```
are 80 and 100 in order? No
```

```
swap 80 and 100: [10, 50, 60, 90, 80, 100, 70, 30, 40, 20]
```

```
are 80 and 90 in order? No
```

```
swap 80 and 90:  [10, 50, 60, 80, 90, 100, 70, 30, 40, 20]
```

```
are 80 and 60 in order? Yes
```

```
round 5 finish     [10, 50, 60, 80, 90, 100, 70, 30, 40, 20]
```

Insertion sort

Look at each round in details:

```
round 6 start      [10, 50, 60, 80, 90, 100, 70, 30, 40, 20]

are 70 and 100 in order? No
swap 70 and 100: [10, 50, 60, 80, 90, 70, 100, 30, 40, 20]
are 70 and 90 in order? No
swap 70 and 90:  [10, 50, 60, 80, 70, 90, 100, 30, 40, 20]
are 70 and 80 in order? No
swap 70 and 80:  [10, 50, 60, 70, 80, 90, 100, 30, 40, 20]

are 70 and 60 in order? Yes
round 6 finish    [10, 50, 60, 70, 80, 90, 100, 30, 40, 20]
```


Insertion sort

Look at each round in details:

```
round 7 start      [10, 50, 60, 70, 80, 90, 100, 30, 40, 20]

are 30 and 100 in order? No
swap 30 and 100: [10, 50, 60, 70, 80, 90, 30, 100, 40, 20]
are 30 and 90 in order? No
swap 30 and 90:  [10, 50, 60, 70, 80, 30, 90, 100, 40, 20]
are 30 and 80 in order? No
swap 30 and 80:  [10, 50, 60, 70, 30, 80, 90, 100, 40, 20]
are 30 and 70 in order? No
swap 30 and 70:  [10, 50, 60, 30, 70, 80, 90, 100, 40, 20]
are 30 and 60 in order? No
swap 30 and 60:  [10, 50, 30, 60, 70, 80, 90, 100, 40, 20]
are 30 and 50 in order? No
swap 30 and 50:  [10, 30, 50, 60, 70, 80, 90, 100, 40, 20]

are 30 and 10 in order? Yes
round 7 finish    [10, 30, 50, 60, 70, 80, 90, 100, 40, 20]
```

Insertion sort

Look at each round in details:

```
round 8 start      [10, 30, 50, 60, 70, 80, 90, 100, 40, 20]
```

```
are 40 and 100 in order? No
```

```
swap 40 and 100: [10, 30, 50, 60, 70, 80, 90, 40, 100, 20]
```

```
are 40 and 90 in order? No
```

```
swap 40 and 90:  [10, 30, 50, 60, 70, 80, 40, 90, 100, 20]
```

```
are 40 and 80 in order? No
```

```
swap 40 and 80:  [10, 30, 50, 60, 70, 40, 80, 90, 100, 20]
```

```
are 40 and 70 in order? No
```

```
swap 40 and 70:  [10, 30, 50, 60, 40, 70, 80, 90, 100, 20]
```

```
are 40 and 60 in order? No
```

```
swap 40 and 60:  [10, 30, 50, 40, 60, 70, 80, 90, 100, 20]
```

```
are 40 and 50 in order? No
```

```
swap 40 and 50:  [10, 30, 40, 50, 60, 70, 80, 90, 100, 20]
```

```
are 40 and 30 in order? Yes
```

```
round 8 finish    [10, 30, 40, 50, 60, 70, 80, 90, 100, 20]
```

Insertion sort

Look at each round in details:

```
round 9 start      [ 10, 30, 40, 50, 60, 70, 80, 90, 100, 20 ]

are 20 and 100 in order? No
swap 20 and 100: [10, 30, 40, 50, 60, 70, 80, 90, 20, 100]
are 20 and 90 in order? No
swap 20 and 90:  [10, 30, 40, 50, 60, 70, 80, 20, 90, 100]
are 20 and 80 in order? No
swap 20 and 80:  [10, 30, 40, 50, 60, 70, 20, 80, 90, 100]
are 20 and 70 in order? No
swap 20 and 70:  [10, 30, 40, 50, 60, 20, 70, 80, 90, 100]
are 20 and 60 in order? No
swap 20 and 60:  [10, 30, 40, 50, 20, 60, 70, 80, 90, 100]
are 20 and 50 in order? No
swap 20 and 50:  [10, 30, 40, 20, 50, 60, 70, 80, 90, 100]
are 20 and 40 in order? No
swap 20 and 40:  [10, 30, 20, 40, 50, 60, 70, 80, 90, 100]
are 20 and 30 in order? No
swap 20 and 30:  [10, 20, 30, 40, 50, 60, 70, 80, 90, 100]

are 20 and 10 in order? Yes
round 9 finish    [ 10, 20, 30, 40, 50, 60, 70, 80, 90, 100 ]
```

Insertion sort

helicopter view pseudocode of Insertion Sort Algorithm of a list of integers

```
n = length-of(intList)
FOR i from 1 to (n-1)
    // sort intList[0], intList[1], ..., intList[i]
END FOR
```

Insertion sort

At each round i : {item 0, item 1, ..., item i } are sorted

pseudocode of round i

```
// sort intList[0], intList[1], ..., intList[i]
k = i
WHILE k > 0 and intList[k-1] > intList[k]
    swap intList[k] and intList[k-1]
    k = k - 1
END WHILE
```

Insertion sort

Put it together, we have the algorithm for Insertion Sort:

pseudocode

```
n = length-of(intList)
FOR i from 1 to (n-1)
    // sort intList[0], intList[1], ..., intList[i]
    k = i
    WHILE k > 0 and intList[k-1] > intList[k]
        swap intList[k] and intList[k-1]
        k = k - 1
    END WHILE
END FOR
```

Insertion sort

Python implementation

```
def insertionSort(intList):  
    n = len(intList)  
    for i in range(1, n):  
        #{  
            # sort intList[0], intList[1], ..., intList[i]  
            k = i  
            while (k > 0) and (intList[k-1] > intList[k]):  
                #{  
                    # swap intList[k] and intList[k-1]  
                    temp = intList[k]  
                    intList[k] = intList[k-1]  
                    intList[k-1] = temp  
  
                    k = k - 1  
                #}  
            #}  
        #}
```

Insertion sort

Suggested activities:

- Make up a list of integers and write down in details each step in sorting this list of integers;
- Sort a list of integers in descending order;
- Sort a list of decimal numbers;
- Sort a list of strings.

Suggested activities:

- Write a program to generate a random list of integers of length N ;
- Write a program to count how many comparison operations, and how many swap operations are needed to sort this random list using each sorting algorithms;
- Repeat this program many times with a large sample of random lists of integers and display the statistics.

References

- Python 3 documentation
<https://docs.python.org/3/>
- NumPy Reference
<https://numpy.org/doc/stable/reference/>