CSCI933: Machine Learning - Algorithms and Applications
Assignment #1 - Review of Fundamentals

Due date: Beijing Time 23:59, March 24, 2024

1. (5 Marks) If

$$S = \begin{bmatrix} \dfrac{1}{\sqrt{2}} & -\dfrac{1}{\sqrt{2}} \\ \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \end{bmatrix}$$

   show that S is an orthogonal matrix. Hence show that if $P = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$, then SPS^t is a diagonal matrix.

2. (10 Marks) Let

$$S = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$$

   (a) (3 Marks) Show that S is orthogonal.

   (b) (4 Marks) Now consider a 2×2 symmetric matrix A with elements a_{ij}; show that $B = SAS^t$ is diagonal if

$$tan2\alpha = \frac{2a_{12}}{a_{11} - a_{22}}.$$

   (c) (3 Marks) Verify that $Tr[B] = Tr[A]$

3. (4 Marks) We have two coins; the first is fair and the second is two-headed. We pick one of the coins at random, we toss it twice and heads shows both times. Find the probability that the coin picked is fair.

4. (6 Marks) There are two boxes, one named A and the other B. Box A contains 1000 bulbs of which 10 percent are defective. Box B contains 2000 bulbs of which 5 percent are defective. Two bulbs are picked from a randomly selected box.

   (a) (3 Marks) Find the probability that both bulbs are defective.

   (b) (3 Marks) Assuming that both are defective, find the probability that they come from box A.

Note: Please submit your answers in PDF format.