Stream ciphers

The One Time Pad

This slide is made based the online course of Cryptography by Dan Boneh

Symmetric Ciphers: definition

Def: a **cipher** defined over (K, M, C)

is a pair of "efficient" algs (***E***, ***D***) where

$$E : \mathcal{K} \times \mathcal{M} \to \mathcal{C} \qquad D : \mathcal{K} \times \mathcal{C} \to \mathcal{M}$$

for all keys k and all messages m, we have

$$D(k,\ E(k,\ m)\ ) = m$$

- ***E*** is often randomized. ***D*** is always deterministic.

The One Time Pad (Vernam 1917)

First example of a "secure" cipher

$$M = C = \{0,1\}^n, \; K = \{0,1\}^n$$

key = (random bit string as long the message)

The One Time Pad (Vernam 1917)

$$E(k, m) := k \oplus m$$

$$D(k, c) := k \oplus c$$

msg:	0	1	1	0	1	1	1	$\oplus$
key:	1	0	1	1	0	1	0	
CT:								

Indeed, we have

$$D(k,\ E(k,\ m)\ ) = D(k,\ k \oplus m) = k \oplus (k \oplus m) = (k \oplus k) \oplus m = 0^L \oplus m = m$$

You are given a message (m) and its OTP encryption (c).

Can you compute the OTP key from m and c ?

No, I cannot compute the key.

Yes, the key is $k = m \oplus c$.

I can only compute half the bits of the key.

Yes, the key is $k = m \oplus m$.

The One Time Pad (Vernam 1917)

Very fast enc/dec !!

… but long keys (as long as plaintext)

Is the OTP secure? What is a secure cipher?

What is a secure cipher?

Attacker's abilities: **CT only attack** (for now)

Possible security requirements:

attempt #1: **attacker cannot recover secret key**

$E(k, m)=m$ would be secure

attempt #2: **attacker cannot recover all of plaintext**

$E(k, m0||m1) = m0||k \oplus m1$ would be secure

Shannon's idea:

CT should reveal no "info" about PT

Information Theoretic Security

(Shannon 1949)

Definition (perfect security). *Let $\mathcal{E} = (E, D)$ be a Shannon cipher defined over $(\mathcal{K}, \mathcal{M}, \mathcal{C})$. Consider a probabilistic experiment in which the random variable* k *is uniformly distributed over $\mathcal{K}$. If for all $m_0, m_1 \in \mathcal{M}$, and all $c \in \mathcal{C}$, we have*

$$\Pr[E(\mathsf{k}, m_0) = c] = \Pr[E(\mathsf{k}, m_1) = c],$$

then we say that $\mathcal{E}$ is a ***perfectly secure*** *Shannon cipher.*

Information Theoretic Security

<u>**Def**</u>: A cipher ***(E,D)*** over (K,M,C) has **perfect secrecy** if

$$\forall m_0, m_1 \in M \quad (|m_0| = |m_1|) \quad \text{and} \quad \forall c \in C$$

$$\boldsymbol{Pr[\ E(k,m_0)=c\ ] \ = \ Pr[\ E(k,m_1)=c\ ]} \qquad \text{where } k \xleftarrow{R} K$$

1. Given CT cannot tell if message is m0 or m1 (For all m0 and m1)
2. Most powerful adversary learns nothing about Plaintext from ciphertext
3. No Ciphertext only attack

Lemma: OTP has perfect secrecy.

Proof: $\forall\, m, c: \Pr[E(k,m) = c] = \frac{\#\, keys\ \ k \in K\ \ s.t.\, E(k,m)=c}{|K|}$

So: if $\forall\, m, c: \exists \Pr[\, k \in K: E(k,m) = c]$ =const.

$\Longrightarrow$ Cipher has perfect secrecy.

Let $m \in \mathcal{M}$ and $c \in \mathcal{C}$.

How many OTP keys map $\boldsymbol{m}$ to $\boldsymbol{c}$?

None

1

2

Depends on $\boldsymbol{m}$

Lemma: OTP has perfect secrecy.

Proof:

For OTP: $\forall\, m, c: if\ E(k,m) = c$

$k \oplus m = c \Longrightarrow k = c \oplus m$

$\exists\ \Pr[\, k \in K: E(k,m) = c] = 1/|K|$

OTP has perfect secrecy.

The bad news …

Thm: perfect secrecy $\Rightarrow$ $|\mathcal{K}| \geq |\mathcal{M}|$

Perfect secrecy ⇒ Key length $\geq$ message length

Hard to apply in practice!

End of Segment

Stream ciphers

Pseudorandom Generators

Review

Cipher over (K,M,C): a pair of "efficient" algs (***E***, ***D***) s.t.

$\forall\, m \in M,\ k \in K: \quad \boldsymbol{D}(k, \boldsymbol{E}(k, m)) = m$

Weak ciphers: subs. cipher, Vigener, …

A good cipher: **OTP** $M=C=K=\{0,1\}^n$

$E(k, m) = k \oplus m \quad , \quad D(k, c) = k \oplus c$

<u>Lemma</u>: OTP has perfect secrecy (i.e. no CT only attacks)

Bad news: perfect-secrecy $\Rightarrow$ key-len $\geq$ msg-len

Stream Ciphers: making OTP practical

idea: replace "random" key by "pseudorandom" key

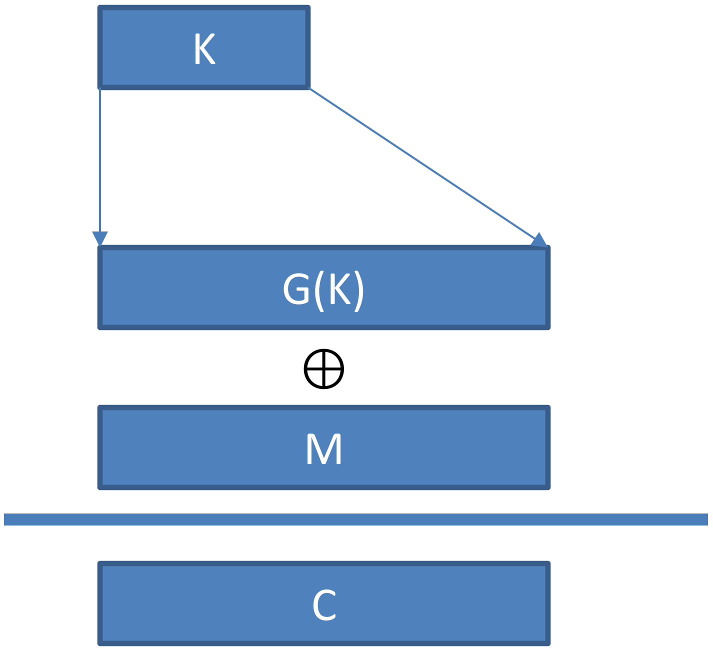
PRG is a function $G: \underbrace{\{0,1\}^s}_{\text{Seed space}} \rightarrow \{0,1\}^n$, $n \gg s$

Efficiently computable by a deterministic algorithm

Stream Ciphers: making OTP practical

$$C = E(K,m) = m \oplus G(k)$$

$$D(k,C) = C \oplus G(k)$$



Can a stream cipher have perfect secrecy?

- Yes, if the PRG is really “secure”
- No, there are no ciphers with perfect secrecy
- Yes, every cipher has perfect secrecy
- No, since the key is shorter than the message

Stream Ciphers: making OTP practical

Stream ciphers cannot have perfect secrecy !!

- Need a different definition of security

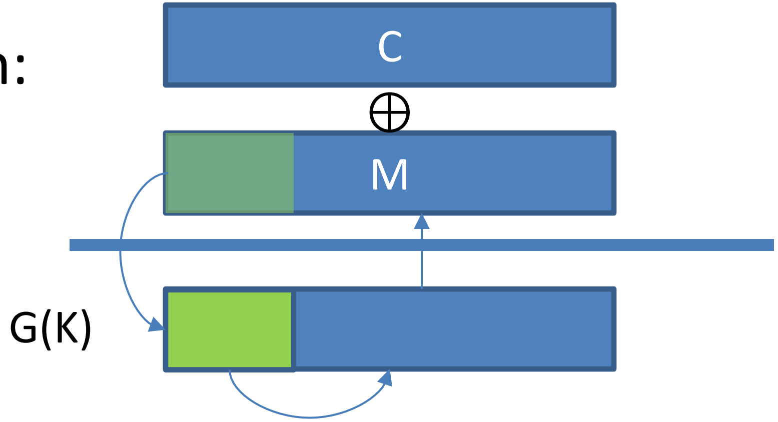
- Security will depend on specific PRG

PRG must be unpredictable

Suppose PRG is predictable.

$$\exists\ i: G(k)|_{1,\dots,i} \longrightarrow G(k)|_{i+1,\dots,n}$$

Then:



Even $G(k)|_{1,\dots,i} \longrightarrow G(k)|_{i+1}$ is a problem!

PRG must be unpredictable

We say that G: K $\longrightarrow \{0,1\}^n$ is **predictable** if:

$\exists$ "eff" $alg.$ A and $\exists\ 0 \le i \le n-1$

s.t. $\Pr[A(G(k)|_{1,\ldots,i}) = G(k)|_{i+1}] > \frac{1}{2} + \varepsilon$

for non-negligible ε (e.g. $\varepsilon = 1/2^{30}$)

Def: PRG is **unpredictable** if it is not predictable

$\Rightarrow$ $\forall$i: no "eff" adv. can predict bit (i+1) for "non-neg" ε

Suppose $G:K \longrightarrow \{0,1\}^n$ is such that for all k: **$XOR(G(k)) = 1$**

Is G predictable ??

Yes, given the first bit I can predict the second

No, G is unpredictable

Yes, given the first (n-1) bits I can predict the n'th bit

It depends

Weak PRGs (do not use for crypto)

Lin.Cong. Generator with parameters a,b,p:

r[i] $\longleftarrow$ $a \cdot r[i-1] + b \bmod p$ seed=r[0]

output bits of r[i]

i++

glibc random():

r[i] $\leftarrow$ (r[i-3] + r[i-31]) % 2^{32}

output r[i] >> 1

never use random() for crypto!

(e.g. Kerberos V4)

Stream ciphers

Negligible vs. non-negligible

Negligible and non-negligible

- In practice: ε is a scalar and
  - ε non-neg: $\varepsilon \geq 1/2^{30}$ (likely to happen over 1GB of data)
  - ε negligible: $\varepsilon \leq 1/2^{80}$ (won't happen over life of key)

- In theory: ε is a function $\varepsilon: Z^{\geq 0} \longrightarrow R^{\geq 0}$ and
  - ε non-neg: $\exists d: \varepsilon(\lambda) \geq 1/\lambda^d$ inf. often ($\varepsilon \geq$ 1/poly, for many λ)
  - ε negligible: $\forall d, \lambda \geq \lambda_d: \varepsilon(\lambda) \leq 1/\lambda^d$ ($\varepsilon \leq$ 1/poly, for large λ)

Few Examples

$\varepsilon(\lambda) = 1/2^{\lambda}$: negligible

$\varepsilon(\lambda) = 1/\lambda^{1000}$: non-negligible

PRGs: the rigorous theory view

PRGs are "parameterized" by a security parameter λ

- **PRG** becomes "more secure" as λ increases

Seed lengths and output lengths grow with λ

For every $\lambda=1,2,3,\ldots$ there is a different PRG G_λ:

$$G_\lambda : K_\lambda \longrightarrow \{0,1\}^{n(\lambda)}$$

(in the lectures we will always ignore λ)

An example asymptotic definition

We say that $G_\lambda : K_\lambda \longrightarrow \{0,1\}^{n(\lambda)}$ is **predictable** at position i if:

there exists a polynomial time (in λ) algorithm A s.t.

$$\Pr_{k \leftarrow K_\lambda}\left[A\left(\lambda,\ G_\lambda(k)\big|_{1,\ldots,i}\right) = G_\lambda(k)\big|_{i+1} \right] > 1/2 + \varepsilon(\lambda)$$

for some non-negligible function $\varepsilon(\lambda)$

End of Segment

Stream ciphers

Attacks on OTP and stream ciphers

Review

OTP: $E(k,m) = m \oplus k$, $D(k,c) = c \oplus k$

Making OTP practical using a PRG: $G: K \longrightarrow \{0,1\}^n$

Stream cipher: $E(k,m) = m \oplus G(k)$, $D(k,c) = c \oplus G(k)$

Security: PRG must be unpredictable (better def in two segments)

Attack 1: **two time** pad is insecure !!

Never use stream cipher key more than once !!

$$C_1 \leftarrow m_1 \oplus PRG(k)$$

$$C_2 \leftarrow m_2 \oplus PRG(k)$$

Eavesdropper does:

$$C_1 \oplus C_2 \quad \rightarrow \quad m_1 \oplus m_2$$

Enough redundancy in English and ASCII encoding that:

$$m_1 \oplus m_2 \quad \rightarrow \quad m_1, m_2$$

Real world examples

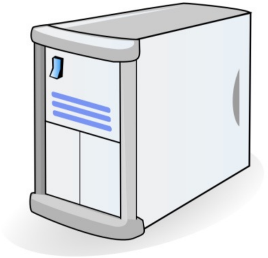
- Project Venona

- MS-PPTP (windows NT):
  - K=(Kc⟶s,Ks⟶c)

k 

m1 ⟶
m2 ⟶
m3 ⟶

⟵ s1
⟵ s2
⟵ s3

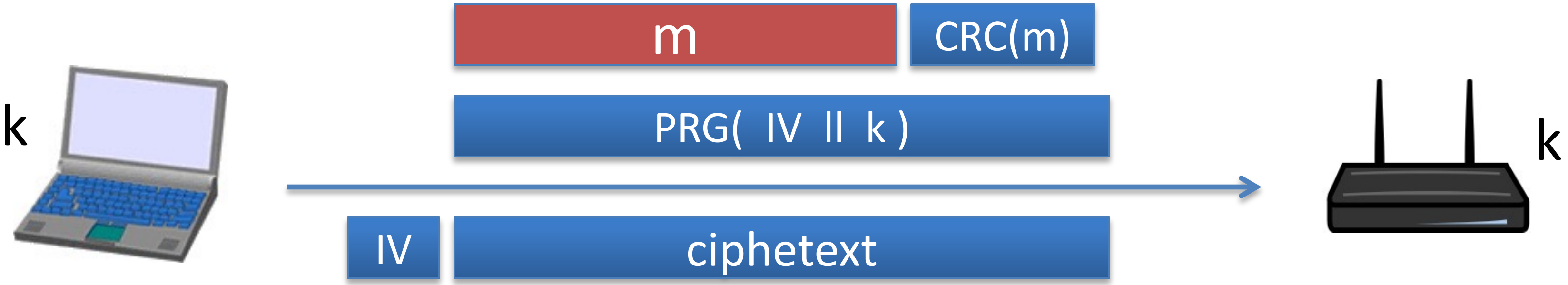
 k

- $[\mathrm{m1} \parallel m2 \parallel m3] \oplus G(k)$ $\qquad$ $[\mathrm{s1} \parallel s2 \parallel s3] \oplus G(k)$

Need different keys for C⟶S and S⟶C

Real world examples

802.11b WEP:

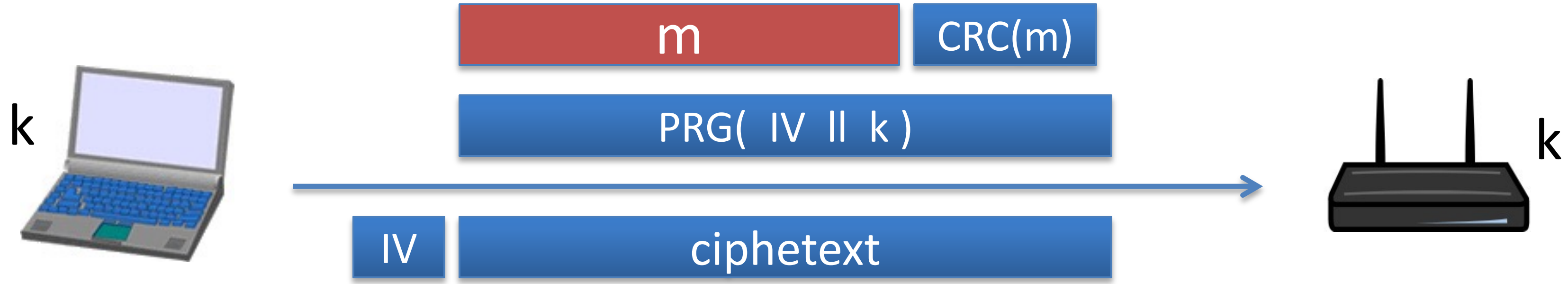


Length of IV: 24 bits

- Repeated IV after $2^{24} \approx$ 16M frames
- On some 802.11 cards: IV resets to 0 after power cycle

Avoid related keys

802.11b WEP:



key for frame #1: (1 ll k)

key for frame #2: (2 ll k)

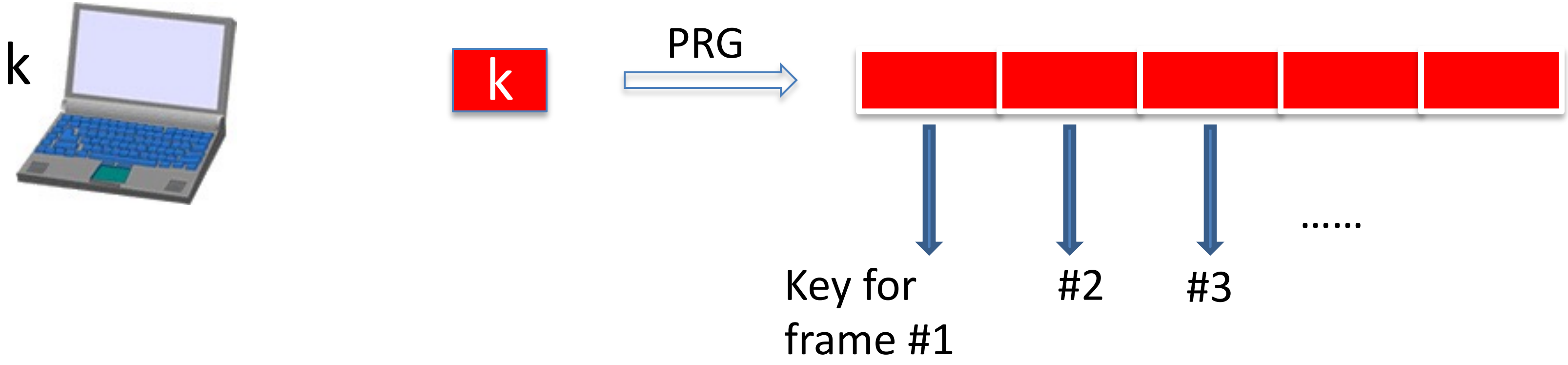
⋮

For the RC4 PRG:FMS2001 $\Longrightarrow$

can recover k after 10^6 frames

recent attacks $\approx$ 40000 frames

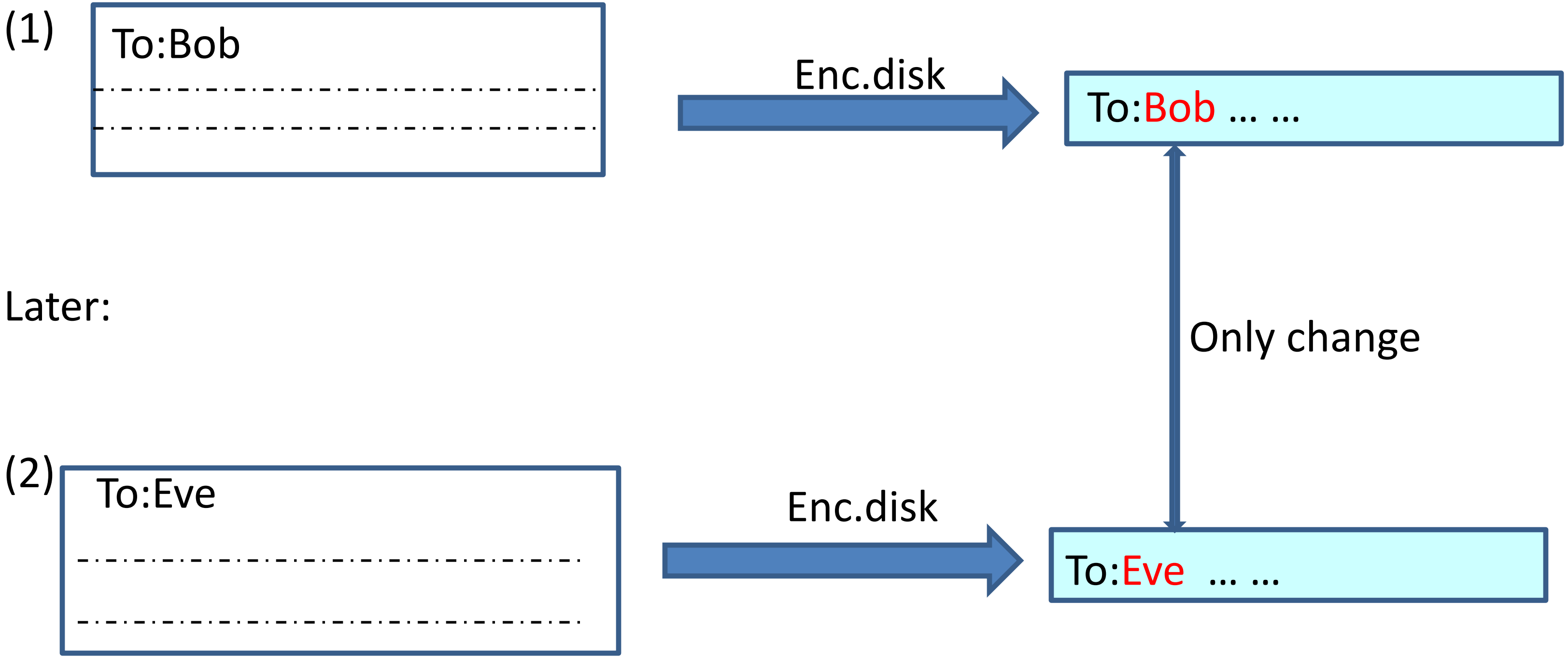
A better construction



⇒ now each frame has a pseudorandom key

better solution: use stronger encryption method (as in WPA2)

Yet another example: disk encryption



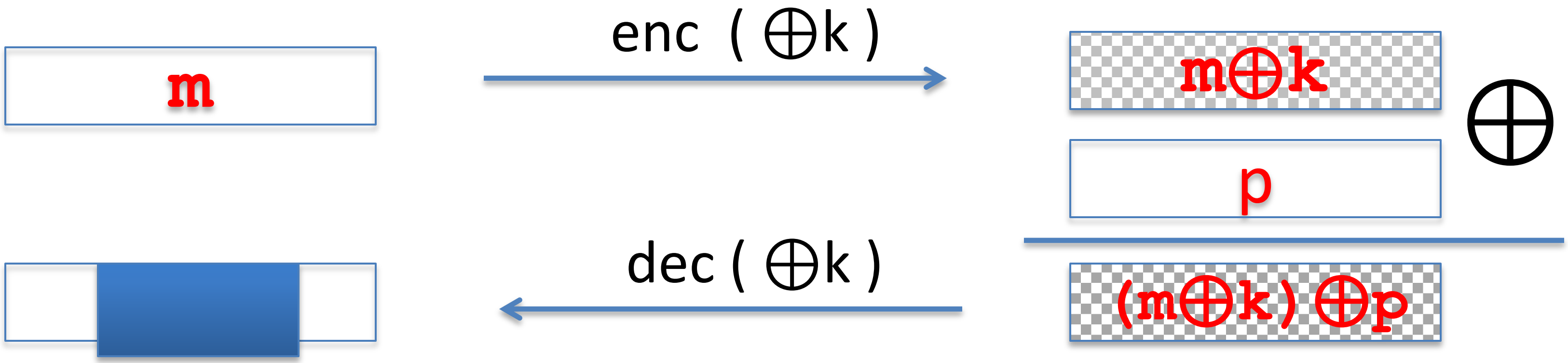
Two time pad: summary

Never use stream cipher key more than once !!

- Network traffic: negotiate new key for every session (e.g. TLS)

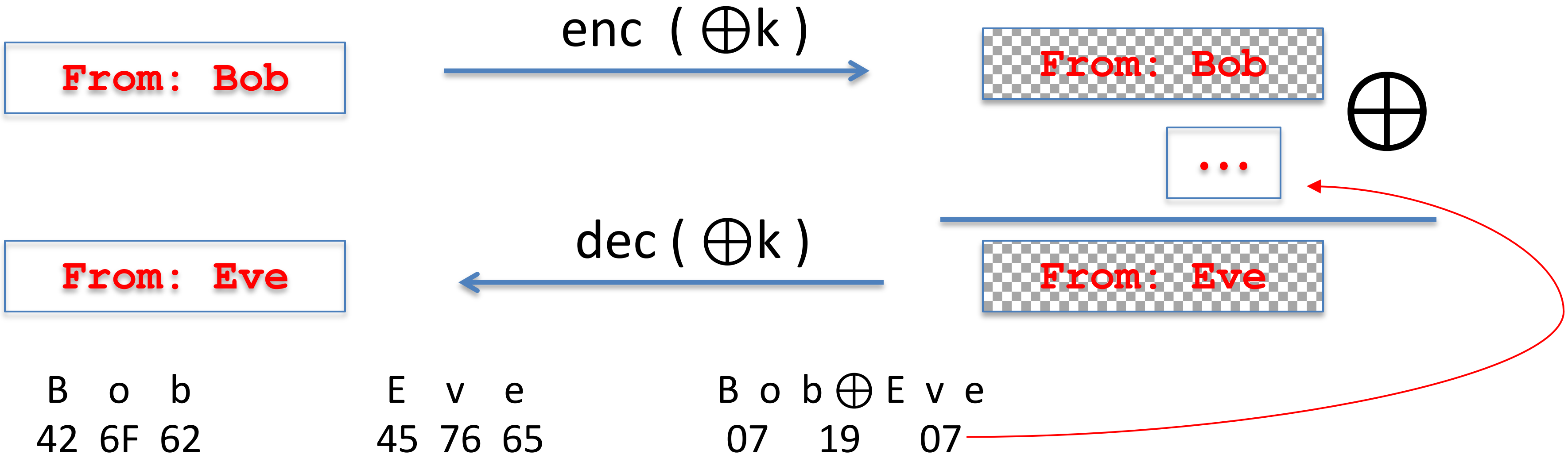
- Disk encryption: typically do not use a stream cipher

Attack 2: no integrity (OTP is malleable)



Modifications to ciphertext are undetected and have **predictable** impact on plaintext

Attack 2: no integrity (OTP is malleable)



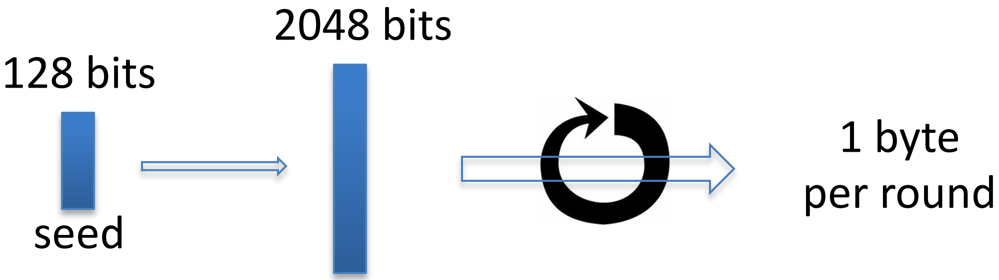
Modifications to ciphertext are undetected and have predictable impact on plaintext

End of Segment

Stream ciphers

Real-world Stream Ciphers

Old example (software): RC4 (1987)



- Used in HTTPS and WEP

- Weaknesses:
  1. Bias in initial output: $\Pr[\, 2^{nd} \text{ byte} = 0 \,] = 2/256$
  2. Prob. of (0,0) is $1/256^2 + 1/256^3$
  3. Related key attacks

Modern stream ciphers: eStream

PRG: $\{0,1\}^s \times R \longrightarrow \{0,1\}^n$

Seed nonce

Nonce: a non-repeating value for a given key.

$$E(k, m\, ; r) = m \oplus PRG(k\, ; r)$$

The pair (k,r) is never used more than once.

Chacha20 (SW+HW)

Chacha20: $\{0,1\}^{256} \times \{0,1\}^{64} \longrightarrow \{0,1\}^n$

nonce

Padding function pad(s , j, n):

- 256-bit seed s0, s1,…,s7 in $\{0,1\}^{32}$
- 64-bit counter j0, j1 in $\{0,1\}^{32}$
- 64-bit nonce n0,n1 in $\{0,1\}^{32}$
- Output a 512-bit block x0,…,x15 in $\{0,1\}^{32}$

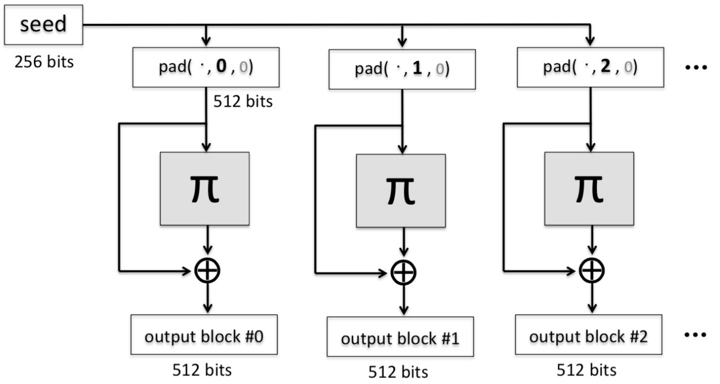
$$\begin{pmatrix} x_0 & x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 & x_7 \\ x_8 & x_9 & x_{10} & x_{11} \\ x_{12} & x_{13} & x_{14} & x_{15} \end{pmatrix} \longleftarrow \begin{pmatrix} c_0 & c_1 & c_2 & c_3 \\ s_0 & s_1 & s_2 & s_3 \\ s_4 & s_5 & s_6 & s_7 \\ j_0 & j_1 & n_0 & n_1 \end{pmatrix}$$

Permutation function π: $\{0,1\}^{512} \rightarrow \{0,1\}^{512}$

$$\begin{array}{ll}
(1)\ \mathrm{QuarterRound}(x_0, x_4, x_8, x_{12}), & (2)\ \mathrm{QuarterRound}(x_1, x_5, x_9, x_{13}), \\
(3)\ \mathrm{QuarterRound}(x_2, x_6, x_{10}, x_{14}), & (4)\ \mathrm{QuarterRound}(x_3, x_7, x_{11}, x_{15}), \\
(5)\ \mathrm{QuarterRound}(x_0, x_5, x_{10}, x_{15}), & (6)\ \mathrm{QuarterRound}(x_1, x_6, x_{11}, x_{12}), \\
(7)\ \mathrm{QuarterRound}(x_2, x_7, x_8, x_{13}), & (8)\ \mathrm{QuarterRound}(x_3, x_4, x_9, x_{14}).
\end{array}$$

QuarterRound(a, b, c, d):

```
a += b;  d ^= a;  d <<<= 16;
c += d;  b ^= c;  b <<<= 12;
a += b;  d ^= a;  d <<<= 8;
c += d;  b ^= c;  b <<<= 7;
```



Is Chacha20 secure (unpredictable) ?

- Unknown: no known **provably** secure PRGs

- In reality: no known attacks better than exhaustive search

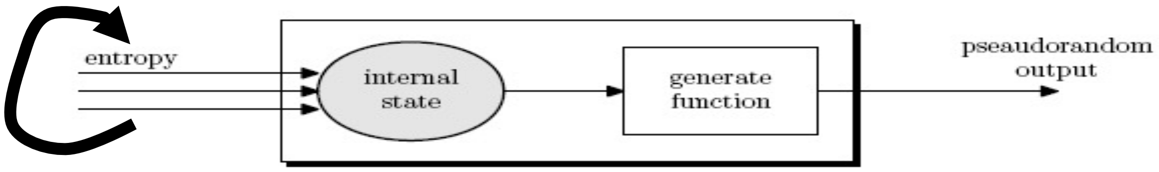
Performance:

Crypto++ 5.6.0 [Wei Dai]

AMD Opteron, 2.2 GHz (Linux)

	PRG	Speed (MB/sec)
	RC4	126
eStream	Salsa20/12	643
eStream	Sosemanuk	727

Generating Randomness (e.g. keys, IV)



Pseudo random generators in practice: (e.g. /dev/random)

- Continuously add entropy to internal state
- Entropy sources:
  - Hardware RNG: Intel **RdRand** inst. (Ivy Bridge). 3Gb/sec.
  - Timing: hardware interrupts (keyboard, mouse)

NIST SP 800-90: NIST approved generators

End of Segment

Stream ciphers

PRG Security Defs

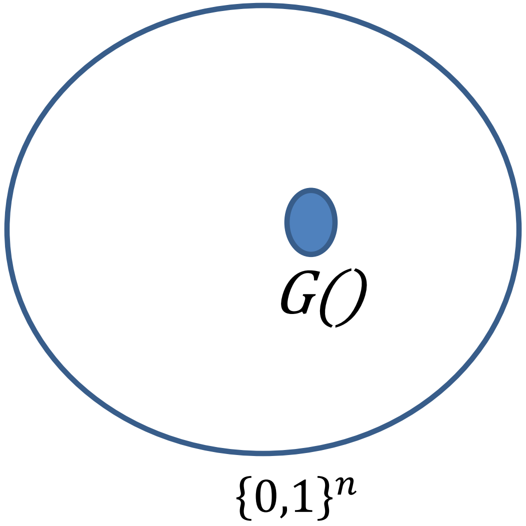
Let G:K $\longrightarrow$ $\{0,1\}^n$ be a PRG

<u>Goal</u>: define what it means that

$$[k \overset{R}{\leftarrow} K, output\ \ G(k)]$$

is "indistinguishable" from

$$[r \overset{R}{\leftarrow} \{0,1\}^n, output\ r]$$



Statistical Tests

<u>Statistical test</u> on $\{0,1\}^n$:

an alg. A s.t. A(x) outputs "0"(not random) or "1"(random)

Examples:

(1) A(x)=1 iff $|\#0(x)-\#1(x)| \le 10\cdot\sqrt{n}$

(2) A(x)=1 iff $|\#00(x)-n/4| \le 10\cdot\sqrt{n}$

Statistical Tests

More examples:

(3) A(x)=1 iff max-run-of-0(x)<10$\cdot \log_2(n)$

…

…

Advantage

Let $G: K \longrightarrow \{0,1\}^n$ be a PRG and A a stat. test on $\{0,1\}^n$

Define:

$$Adv_{PRG}[A,G] = \left| Pr_{k \xleftarrow{R} K}\left[A\left(G(k)\right) = 1\right] - Pr_{r \xleftarrow{R} \{0,1\}^n}\left[A(r) = 1\right] \right| \in [0,1]$$

Adv close to 1 $\Longrightarrow$ A can dist. G from random

Adv close to 0 $\Longrightarrow$ A cannot dist. G from random

A silly example: $A(x) = 0 \Rightarrow Adv_{PRG}[A,G] =$

Suppose $G:K \longrightarrow \{0,1\}^n$ satisfies **msb(G(k)) = 1** for 2/3 of keys in K

Define stat. test $A(x)$ as:

if [msb(x)=1] output "1" else output "0"

Then

$$\text{Adv}_{\text{PRG}}[A,G] = \left| \Pr[A(G(k))=1] - \Pr[A(r)=1] \right| =$$

Secure PRGs: crypto definition

Def: We say that $G: K \longrightarrow \{0,1\}^n$ is a **secure PRG** if

$\forall$ "eff" stat. test A:

$Adv_{PRG}[A,G]$ is "negligible"

Are there provably secure PRGs?

but we have heuristic candidates.

More Generally

Let P_1 and P_2 be two distributions over $\{0,1\}^n$

Def: We say that P_1 and P_2 are

computationally indistinguishable (denoted $P_1 \approx_p P_2$)

if $\forall$ "eff" stat. test A:

$$\left| \Pr_{x \leftarrow P_1}[(A(x)=1] - \Pr_{x \leftarrow P_2}[(A(x)=1] \right| < \text{“negligible”}$$

Example: a PRG is secure if $\{ k \xleftarrow{R} K : G(k) \} \approx_p \text{uniform}(\{0,1\}^n)$

End of Segment

Stream ciphers

Semantic security

Goal: secure PRG $\Rightarrow$ "secure" stream cipher

What is a secure cipher?

Attacker's abilities: **obtains one ciphertext** (for now)

Possible security requirements:

attempt #1: **attacker cannot recover secret key** $E(k,m)=m$

attempt #2: **attacker cannot recover all of plaintext**

$$E(k, m_0 \,||\, m_1) = m_0 \,||\, m_1 \oplus k$$

Recall Shannon's idea:

CT should reveal no "info" about PT

Recall Shannon's perfect secrecy

Let (E,D) be a cipher over (K,M,C)

(E,D) has perfect secrecy if $\forall\, m_0, m_1 \in M$ ($|m_0| = |m_1|$)

$$\{ E(k,m_0) \} \;=\; \{ E(k,m_1) \} \quad \text{where } k \leftarrow K$$

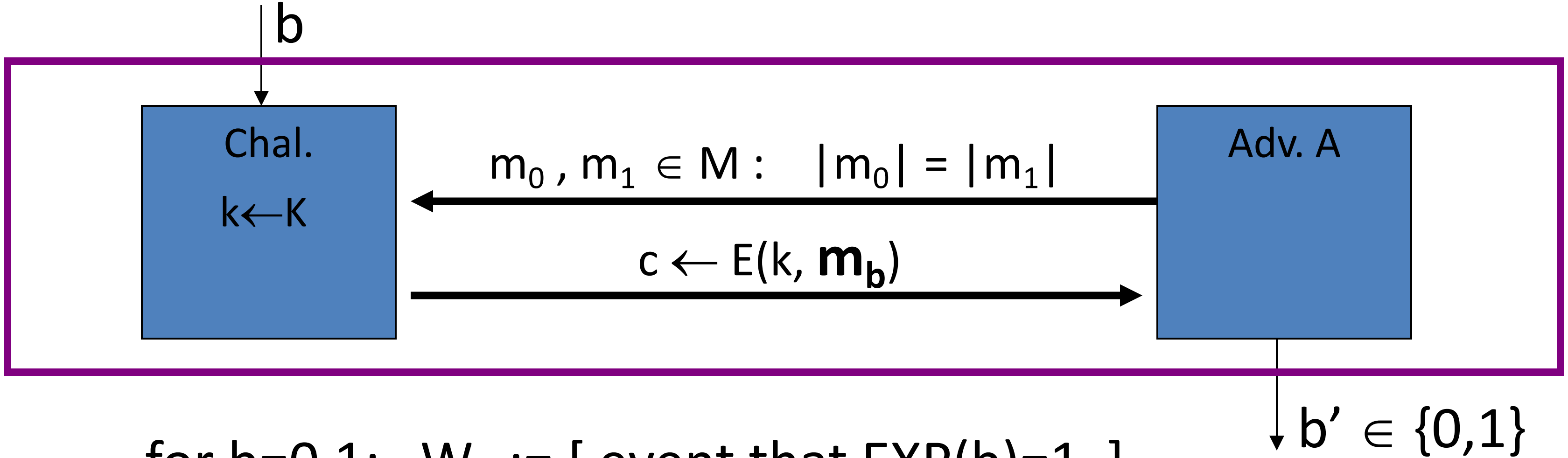
(E,D) has perfect secrecy if $\forall\, m_0, m_1 \in M$ ($|m_0| = |m_1|$)

$$\{ E(k,m_0) \} \;\approx_p\; \{ E(k,m_1) \} \quad \text{where } k \leftarrow K$$

... but also need adversary to exhibit $m_0, m_1 \in M$ explicitly

Semantic Security (one-time key)

For $b=0,1$ define experiments EXP(0) and EXP(1) as:



for $b=0,1$: W_b := [event that EXP(b)=1]

$$\mathrm{Adv}_{SS}[A,E] := \Big|\ \Pr[W_0] - \Pr[W_1]\ \Big| \quad \in [0,1]$$

Semantic Security (one-time key)

Def: $\mathbb{E}$ is **semantically secure** if for all efficient A

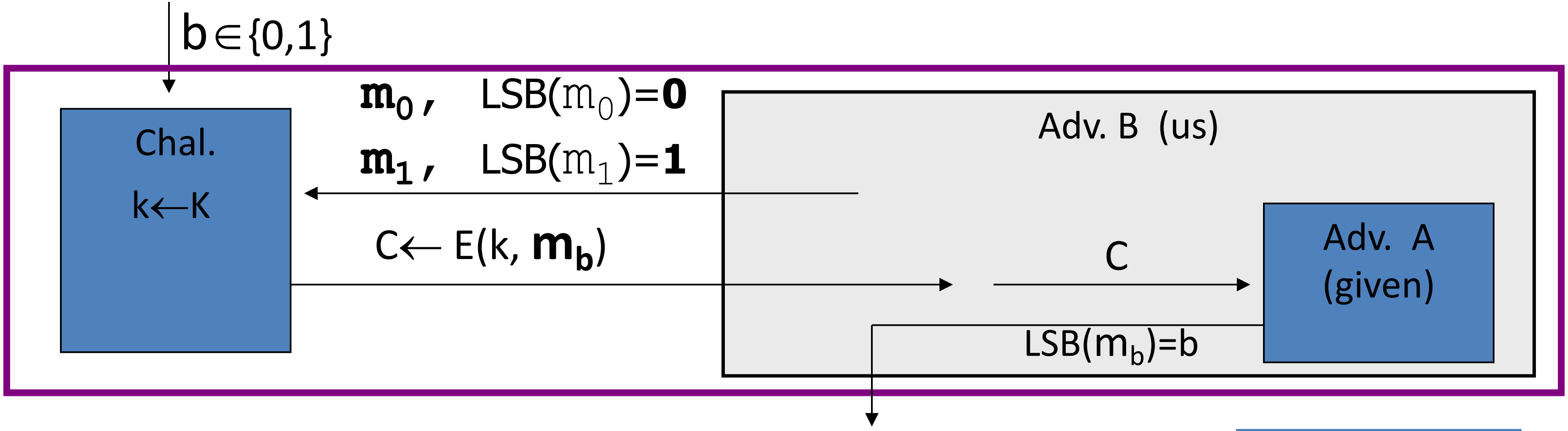
$\mathrm{Adv}_{SS}[A,\mathbb{E}]$ is negligible.

$\Rightarrow$ for all explicit m_0, $m_1 \in M$: $\{ E(k,m_0) \} \approx_p \{ E(k,m_1) \}$

Examples

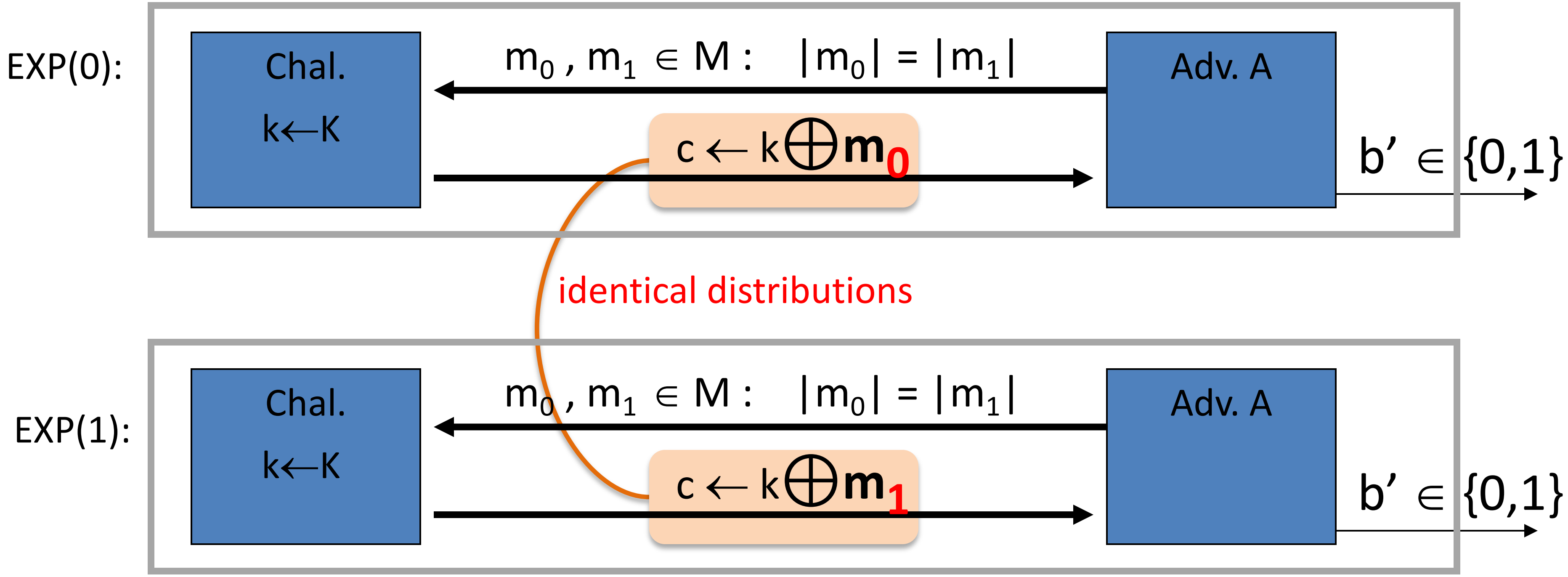
Suppose efficient A can always deduce LSB of PT from CT.

$\Rightarrow \quad \mathbb{E} = (E,D)$ is not semantically secure.



Then $\text{Adv}_{SS}[B, \mathbb{E}] = \big|\, \Pr[\, \textbf{EXP(0)}=1 \,] - \Pr[\, \textbf{EXP(1)}=1 \,] \,\big| =$

OTP is semantically secure



For **<u>all</u>** A: $\mathrm{Adv}_{SS}[A,OTP] = \big|\ \Pr[\ \mathbf{A(k\oplus m_0)}=1\ ] - \Pr[\ \mathbf{A(k\oplus m_1)}=1\ ]\ \big|$

End of Segment

Stream ciphers

Stream ciphers are semantically secure

Goal: secure PRG $\Rightarrow$ semantically secure stream cipher

Stream ciphers are semantically secure

(Theorem 3.1 from GCAC, refer to pp.49)

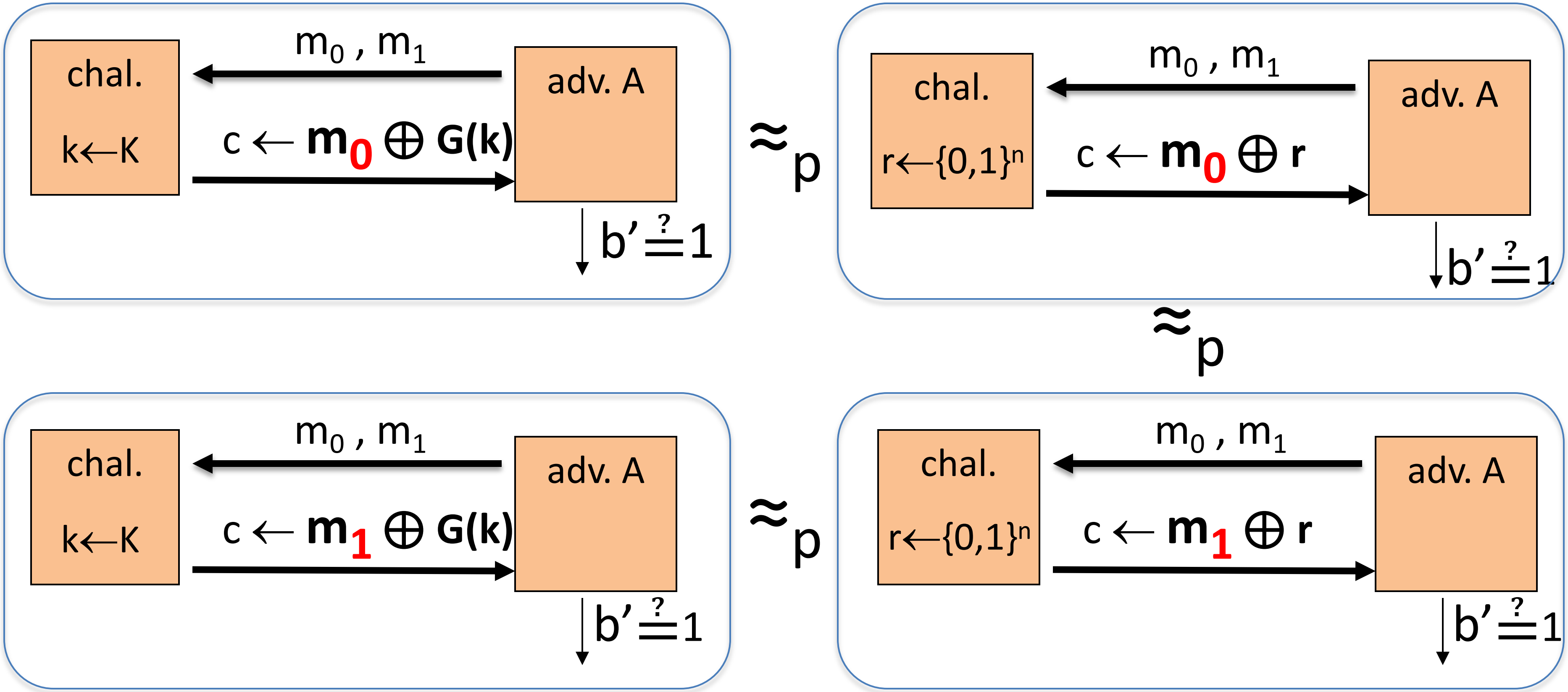
Thm: G:K $\longrightarrow \{0,1\}^n$ is a secure PRG $\Rightarrow$

stream cipher E derived from G is sem. sec.

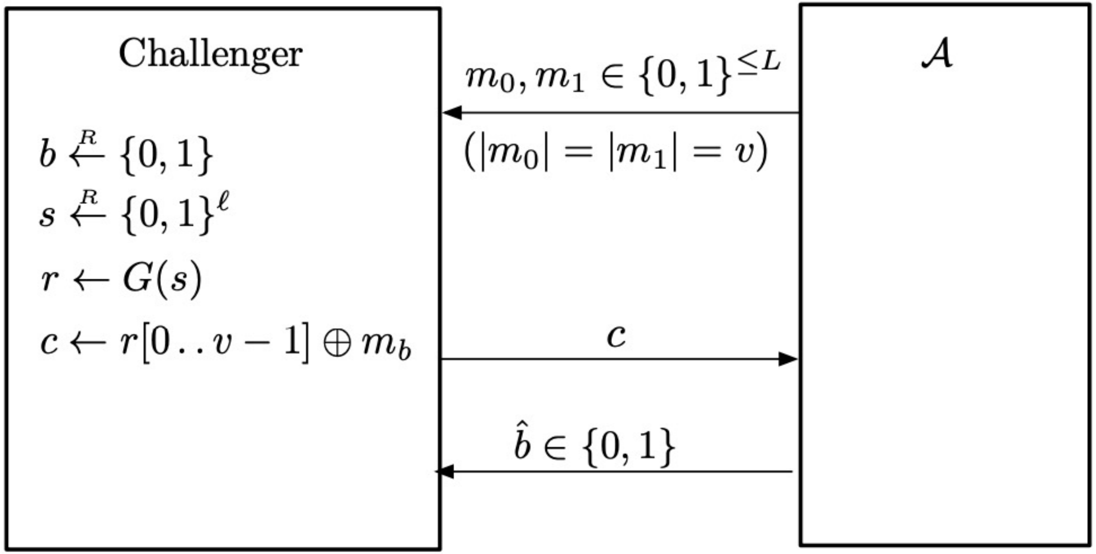
$\forall$ sem. sec. adversary A , $\exists$a PRG adversary B s.t.

$$\mathrm{Adv}_{SS}[A,E] \leq 2 \cdot \mathrm{Adv}_{PRG}[B,G]$$

Proof: intuition



Game 0



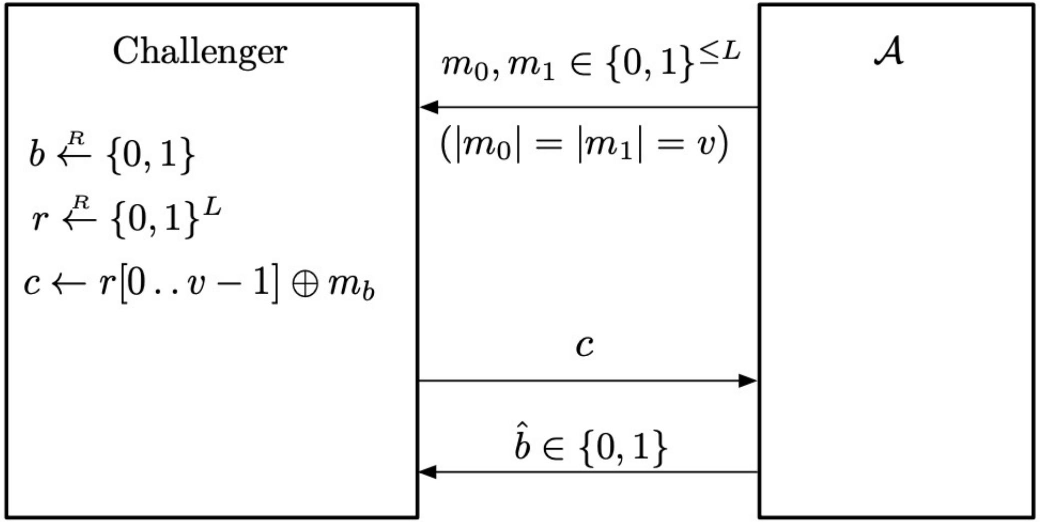
W_0: the event that $\hat{b} = b$ in Game 0

$$\mathsf{SSadv}^*[\mathcal{A}, \mathcal{E}] = |\Pr[W_0] - 1/2|$$

Upon receiving $m_0, m_1 \in \{0,1\}^v$ from $\mathcal{A}$, for some $v \leq L$, do:

$b \xleftarrow{R} \{0,1\}$
$s \xleftarrow{R} \{0,1\}^\ell$, $r \leftarrow G(s)$
$c \leftarrow r[0..v-1] \oplus m_b$
send c to $\mathcal{A}$.

Game 1



Upon receiving $m_0, m_1 \in \{0,1\}^v$ from $\mathcal{A}$, for some $v \leq L$, do:

$b \xleftarrow{R} \{0,1\}$

$r \xleftarrow{R} \{0,1\}^L$

$c \leftarrow r[0 \mathinner{..} v-1] \oplus m_b$

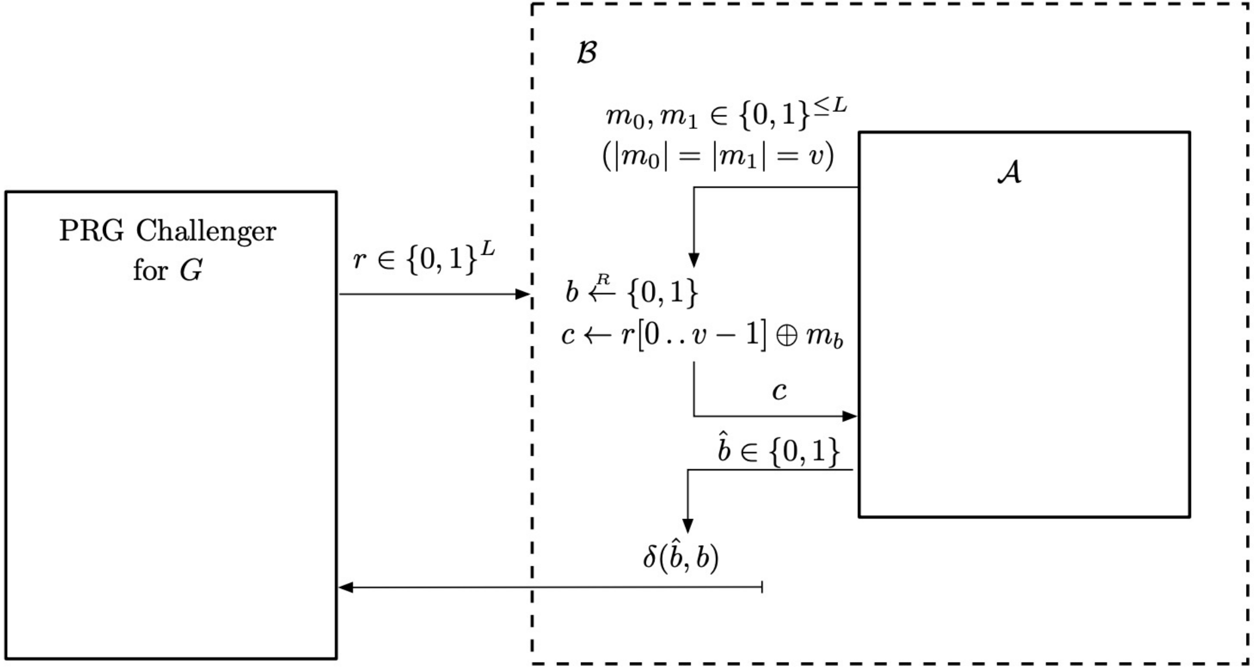
send c to $\mathcal{A}$.

W_1: the event that $\hat{b} = b$ in Game 1

$$\Pr[W_1] = 1/2.$$

Our goal: $|\Pr[W_0] - \Pr[W_1]| = \mathsf{PRGadv}[\mathcal{B}, G].$

Define $\delta(x,y) := \begin{cases} 1 & \text{if } x = y, \\ 0 & \text{if } x \neq y. \end{cases}$



Upon receiving $m_0, m_1 \in \{0,1\}^v$ from $\mathcal{A}$, for some $v \leq L$, do:

$b \xleftarrow{R} \{0,1\}$

$c \leftarrow r[0\,..\,v-1] \oplus m_b$

send c to $\mathcal{A}$.

Finally, when $\mathcal{A}$ outputs a bit $\hat{b}$, $\mathcal{B}$ outputs the bit $\delta(\hat{b}, b)$.

P0: probability that B output 1 when PRG challenger is running exp 0.

P1: probability that B output 1 when PRG challenger is running exp 1.

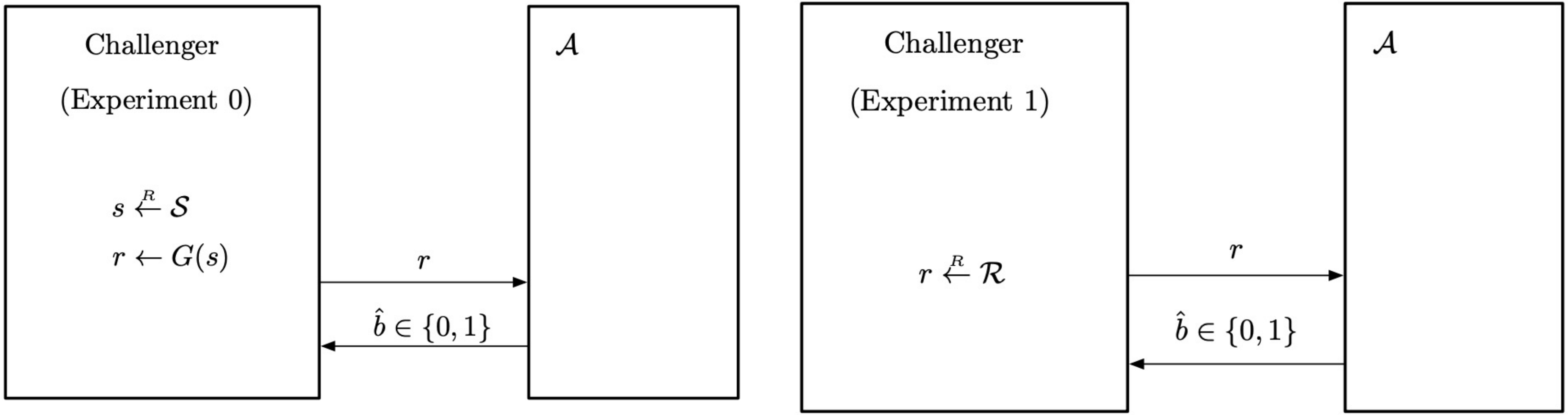
$$\mathsf{PRGadv}[\mathcal{B}, G] = |p_1 - p_0|.$$

$$p_0 = \Pr[W_0], \quad p_1 = \Pr[W_1].$$

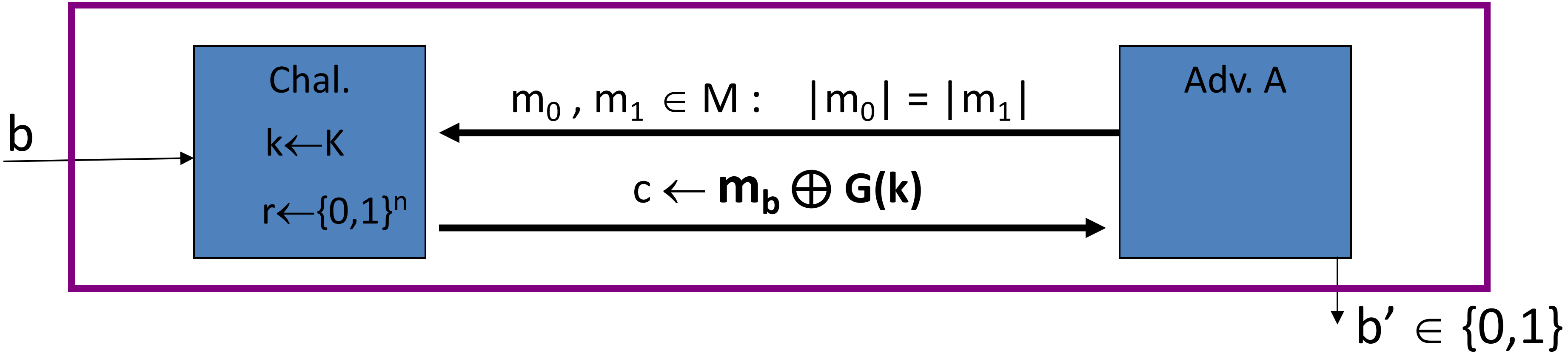
$\Downarrow$

$$|\Pr[W_0] - \Pr[W_1]| = \mathsf{PRGadv}[\mathcal{B}, G].$$

Recall PRG game



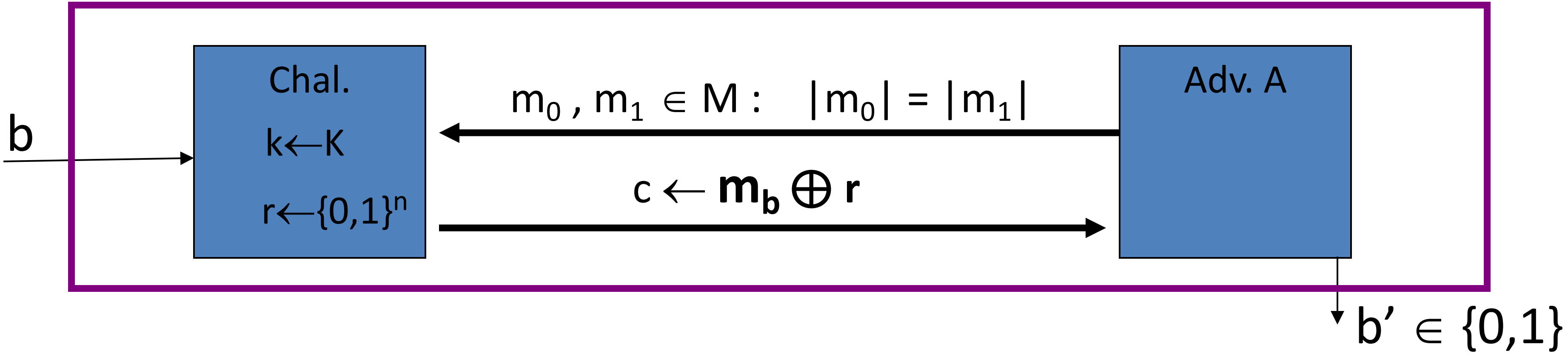
Proof: Let A be a sem. sec. adversary.



For b=0,1: $W_b :=$ [event that b'=1].

$$\mathrm{Adv_{SS}}[A,E] = \Big|\ \Pr[W_0] - \Pr[W_1]\ \Big|$$

Proof: Let A be a sem. sec. adversary.



For b=0,1: $W_b :=$ [event that b'=1].

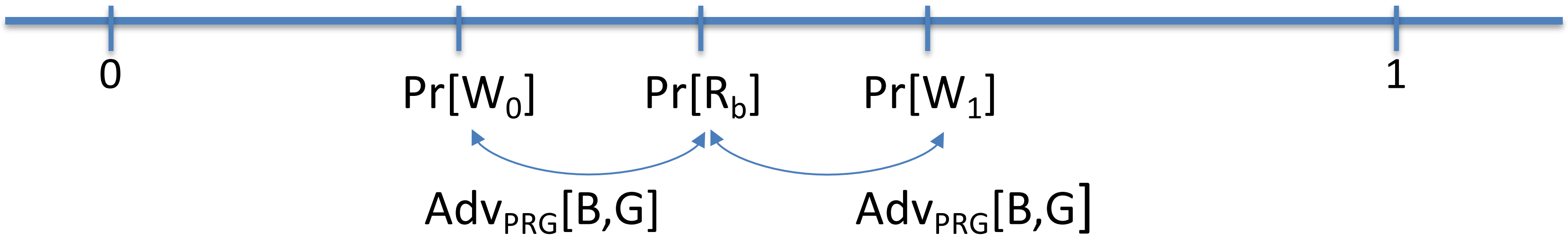
$$Adv_{SS}[A,E] = \Big| \Pr[W_0] - \Pr[W_1] \Big|$$

For b=0,1: $R_b :=$ [event that b'=1]

Proof: Let A be a sem. sec. adversary.

Claim 1: $\left| \Pr[R_0] - \Pr[R_1] \right| = \mathrm{Adv}_{SS}[A,\mathrm{OTP}] = 0$

Claim 2: $\exists B$: $\left| \Pr[W_b] - \Pr[R_b] \right| = \mathrm{Adv}_{PRG}[B,G]$ for b=0,1



$\Rightarrow \mathrm{Adv}_{SS}[A,E] = \left| \Pr[W_0] - \Pr[W_1] \right| \le 2 \cdot \mathrm{Adv}_{PRG}[B,G]$

Proof of claim 2: $\exists B: \left| \Pr[W_0] - \Pr[R_0] \right| = \mathrm{Adv}_{PRG}[B,G]$

Algorithm B:

$$\mathrm{Adv}_{PRG}[B,G] = \left| \Pr_{k \xleftarrow{R} K}\left[B\big(G(k)\big) = 1\right] - \Pr_{r \xleftarrow{R} \{0,1\}^n}\left[B(r) = 1\right] \right| = \left| \Pr[W_0] - \Pr[R_0] \right|$$

End of Segment