Analysis and design of relay feedback systems

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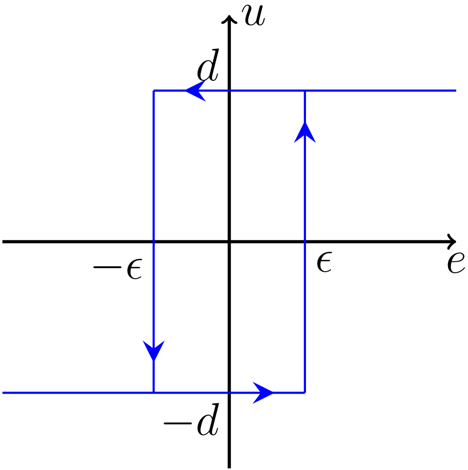
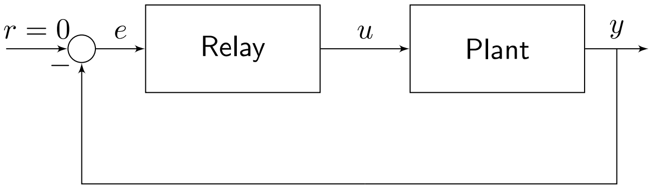
Outline

- Relay feedback
- Biological oscillations
- Conclusions

Relay



Relay feedback



Motivation

- Historically classical field
- Auto-tuning of process controllers
- Simplify biological oscillations
- Unsolved research problems exist
    - Why do the oscillations converge towards the limit cycle so quickly?
    - Is it possible to have several limit cycles depending on the initial conditions?

Theory

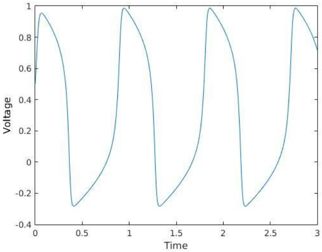
K.J.Åström, *Oscillations in systems with relay feedback*, (1995)

- Conditions for limit cycles
- Conditions for local stability
- Initial conditions for oscillations

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

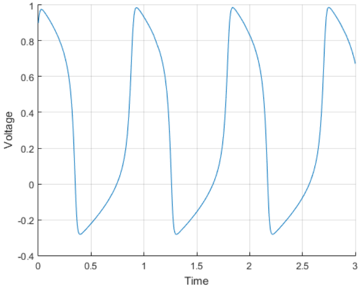
FitzHugh-Nagumo model for action potentials

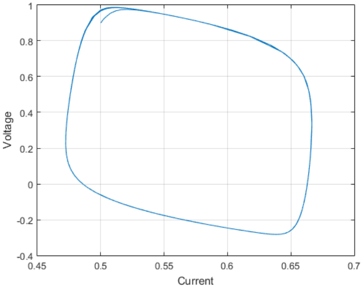
- Action potential
- Hodgkin-Huxley model
    - Four variables
    - Fast and slow variables
- FitzHugh-Nagumo model extracts the essential behaviour
- Only two variables



FitzHugh-Nagumo

$$\epsilon \frac{dv}{dt} = f(v) - i + I_{\text{app}}$$
$$\frac{di}{dt} = v - \gamma i$$





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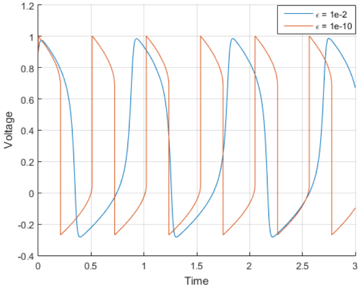
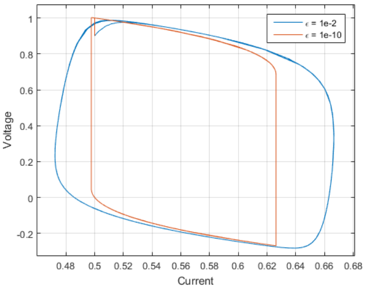
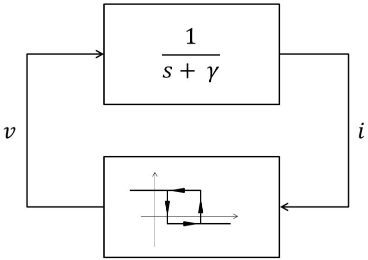
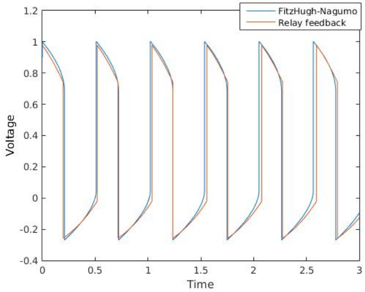




FitzHugh-Nagumo and Relay feedback



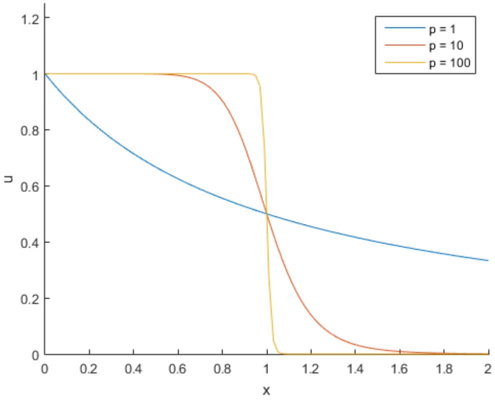


Goodwin Oscillator

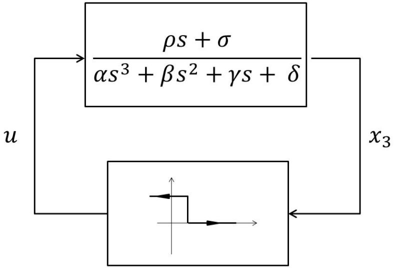
- Biochemical oscillator based on negative feedback
- Concentration of mRNA, protein and end product

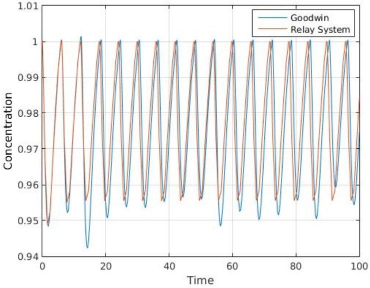
$$\begin{aligned} &\text{mRNA} & \frac{dx_1}{dt'} &= \frac{1}{1+x_3^p} - b_1 x_1 \\ &\text{Protein} & \frac{dx_2}{dt'} &= b_2(x_1 - x_2) \\ &\text{Product} & \frac{dx_3}{dt'} &= b_3(x_2 - x_3) \end{aligned}$$

Goodwin Oscillator



Goodwin Oscillator and Relay Feedback





Next Term

- Study differential positivity and its application to relay feedback
- Apply this analysis tool to biological oscillations approximated by relay feedback

Conclusions

- Studied oscillations of relay feedback systems
- Bridged classical control theory with non-linear oscillations currently being studied in biology
  - FitzHugh-Nagumo model
  - Goodwin Oscillator model
- Study differential positivity
- Apply this analysis to relay feedback systems