Design and Analysis of Algorithms Assignment - 6

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Abstract: Transportation problem is a linear problem of cost minimization in the transportation from a set of suppliers to a set of destinations. This paper discusses north-west corner solution for the given problem as well as propose an alternative approach. The time complexity of this approach is measured to be O(N).

Index Terms: Arrays, Minimum cost,

INTRODUCTION

Transportation problem is a linear problem in which goods are transported from a set of sources to a set of destinations subject to the supply and demand of the sources and destination respectively such that the total cost of transportation is minimized. There are several methods to solve this problem and we will be discussing the north-west corner approach.

This report further contains:

- Algorithm Designs
- Algorithm Analysis
- Experimental Study and Profiling
- Conclusion
- References
- Appendix

ALGORITHM DESIGN

The North-West Corner Rule is a method adopted to compute the initial feasible solution of the transportation problem. The name North-west corner is given to this method because the basic variables are selected from the extreme left corner. The prerequisite condition for solving the transportation problem is that demand should be equal to the supply. In case the demand is more than supply, then dummy origin is added to the table.

Algorithmic Steps: We basically select the north-western most corner available, compute the supply and demand operations and move to the next box accordingly.

1. We take input for the size of the cost matrix.
2. We store n X n randomly generated values in an 2D array.
3. We then elect the north-west or extreme left corner of the matrix, assign as many units as possible to the cell, within the supply and demand constraints.
4. Either the demand is satisfied or the supply is finished or both. Based on either of these 3 possibilities, we either move 1 column across (if the demand is satisfied) or 1 row downwards(if the supply is finished) or both 1 row down and 1 column across (if supply and demand are equal).
5. This process is repeated until the the demand and supply are saturated and compute the total cost associated with the transport.

Presenting below is the pseudo code for the above given algorithm.

```
Int:
Function main()
    int n
    print Enter n:
    Input n

    arr :array
    supply :array
    demand :array

    print Generated post matrix:
    loop i=0 to n with i++
        arr[i][j] = rand() % 9 + 1
        print arr[i][j]

    loop i=0 to n with i++
        demand[i] = rand() % 250 + 1
        supply[i] = demand[i]

    shuffle(demand)

    print Demands:
    loop i=0 to n with i++
        print demand[i]

    print Supply:
    loop i=0 to n with i++
```

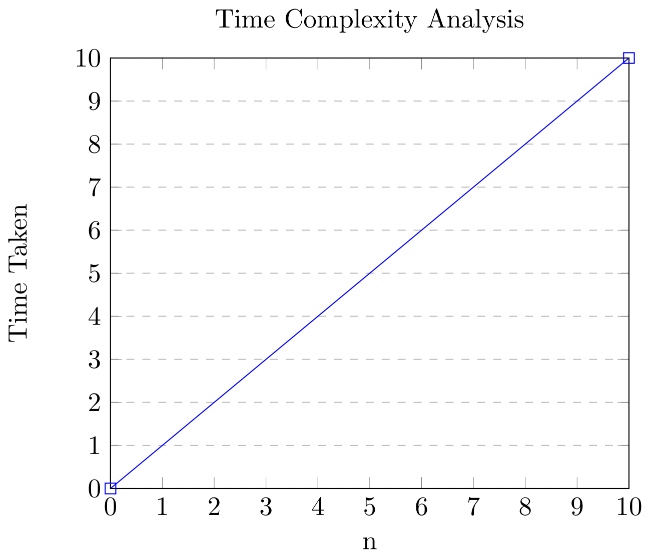
```
    print supply[i]

i :int
j :int
cost :int

while(i != n && j!= n)
    if(demand[i]==supply[j])

            cost+= demand[i]*arr[i][j]
            i++
            j++
            demand[i] = 0
            supply[j] = 0

        else if(demand[i]<supply[j])

            supply[j]-=demand[i]
            cost+= demand[i]*arr[i][j]
            demand[i] = 0
            i++

        else if(demand[i]>supply[j])

            demand[i]-=supply[j];
            cost+= supply[i]*arr[i][j];
            supply[j]=0;
            j++;

cost += demand[n-1] * arr[i-1][j-1]
demand[n-1] -= supply[n-1]
supply[n-1] = 0

    print cost
return 0
```

ALGORITHM ANALYSIS

APRIORI ANALYSIS : This is the analysis performed prior to running in a stage where the function is defined using a theoretical model. Therefore, complexity is determined by just examining the algorithm rather than running it on a particular system with a different memory,processor, and compiler. So,as we discussed under the heading complexity analysis we arrived at the conclusion that the best time complexity is O(n) and the space complexity O(n).

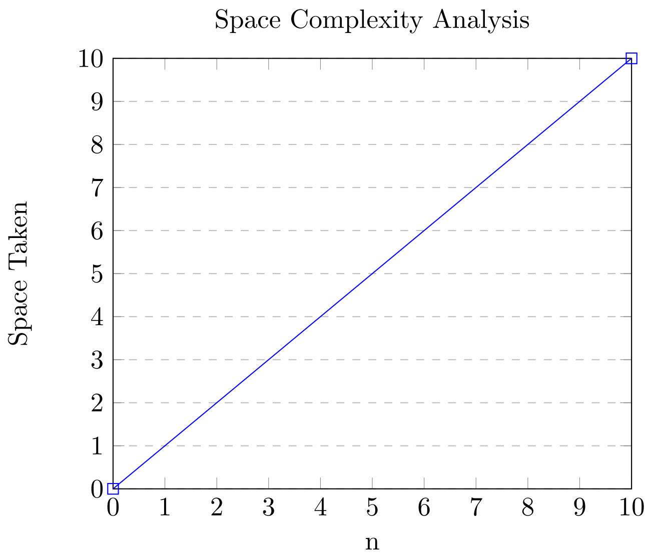
Time complexity Derivation: Since we run a single loop to compute the cost, the time complexity is O(n).

Time Analysis: Following is the graph representing the time complexity of the algorithm.



By the experimental analysis, we found that in case of optimized approach, on increasing the number of numbers the graph is strictly increasing. Thus the overall time increases with an increase in size.

Space Analysis: Following is the graph representing the space complexity of the algorithm.



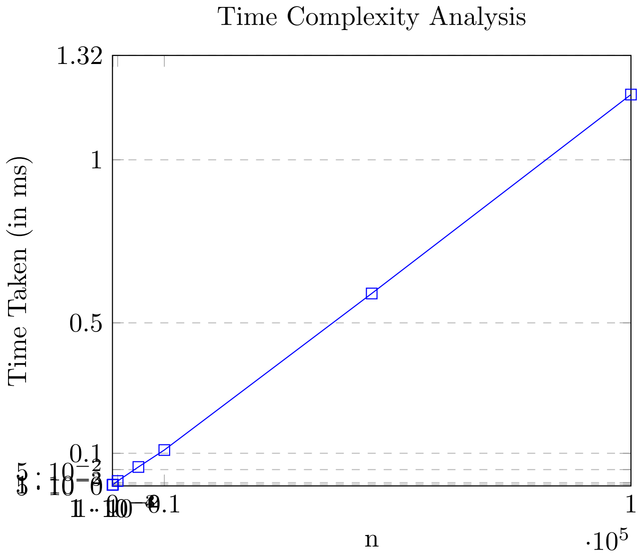
By the experimental analysis, we found that in case of optimized approach, on increasing the number of numbers the graph is strictly increasing. Thus the overall space increases with an increase in size.

APOSTERIORI ANALISIS: Aposteriori analysis of an algorithm means we per- form analysis of an algorithm only after running it on a system. It directly depends on the system and changes from system to system. So for the a aposteriori analysis of the algorithm,we have run our code on the compiler and get values of the time.

Experimental Analysis

In the following table some cases are plotted for the algorithm on our local machine,

n	Time Taken (in ms)
10	0.004
100	0.0049
1000	0.025
5000	0.068
10000	0.12
50000	0.71
100000	1.32
1000000	11.2



Alternative Algorithm: An alternative algorithm can be proposed which is based on the minimum cell algorithm. In this algorithm, we choose the least cost in the given row. If the supply is greater than 0, we satisfy the demand associated with that column. If the supply is zeroed, we move to the minimum in the next row, else we find the next smallest element in the same row. We continue this procedure until all the demands are satisfied. This has the same time complexity of O(n) and space complexity of O(n).

CONCLUSION

So, with the north-west corner algorithm and its profiling, we come to the conclusion that this classical problem of minimum cost in the optimal transport problem has best time complexity of O(n) and space complexity of O(n).

REFERENCES

1. Transportation Problem: https://www.geeksforgeeks.org/transportation-problem-set-1-introduction/
2. Introduction to Algorithms by Cormen,Charles, Rivest and Stein. https://web.ist.utl.pt/ fabio.ferreira/material/asa

APPENDIX

To run the code, follow the following procedure:

1. Download the code(or project zip file) from the github repository.
2. Extract the zip file downloaded above.
3. Open the code with any IDE like Sublime Text, VS Code, Atom or some online compilers like GDB.
4. Run the code following the proper running commands(vary from IDE to IDE)
   (a) **For VS Code:** Press Function+F6 key and provide the input on the terminal.
   (b) **For Sublime Text:** Click on the Run button and provide the input.

Code for Implementation is:

```
#include <bits/stdc++.h>
using namespace std;

int main()
{
        int arr[100][100];
        int n;
        cout<<"Enter n: ";
        cin>>n;
        int demand[n];
        int supply[n];
        cout<<"\nGenerated cost matrix: \n";
        for(int i=0;i<n;i++)
        {
                for(int j=0;j<n;j++)
                {
                 arr[i][j] = rand() % 9 + 1;
                 cout<<arr[i][j]<<"  ";
                }
                cout<<endl;
        }
        cout<<endl;
        for(int i=0;i<n;i++)
        {
                demand[i] = rand() % 250 + 1;
                supply[i] = demand[i];
        }
        shuffle(demand,demand+n,
            default_random_engine(0));
        cout<<"\nDemands: ";
        for(int i=0;i<n;i++)
        {
                cout<<demand[i]<<" ";
        }
        cout<<endl;
        cout<<"\nSupply: ";
        for(int i=0;i<n;i++)
        {
                cout<<supply[i]<<" ";
        }
        cout<<endl;

        int i=0,j=0,cost=0;
        cout<<endl;
        while(i!=n && j!=n)
        {
                if(demand[i]==supply[j])
                {
                 cost+= demand[i]*arr[i][j];
                 i++;
                  j++;
                 demand[i] = 0;
                 supply[j] = 0;
                }
                else if(demand[i]<supply[j])
                {
                 supply[j]-=demand[i];
                 cost+= demand[i]*arr[i][j];
                 demand[i] = 0;
                 i++;
                }
                else if(demand[i]>supply[j])
                {
                 demand[i]-=supply[j];
                 cost+= supply[i]*arr[i][j];
                 supply[j]=0;
                 j++;
                }
        }
        cost+=demand[n-1]*arr[i-1][j-1];
        demand[n-1] -=supply[n-1];
        supply[n-1] = 0;
        cout<<"Final cost: "<<cost<<endl;
        return 0;
}
```