GRE Math Basics

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Manhattan 1 - Algebra

Important notes and equations:

1. The order of operations : PEMDAS

   p= parenthesis [{ ( ) }].

   Exponents = 2^2 = 4

   Multiplications / divisions

   Addition / Subtraction

2. Quadratic equation : the power of a variable increases to 2.

   When we raise variables to exponents the number of possible solutions increases.

   - Quadratic equations might have 2 solutions but not always

     E. g : x^2 = 25

     X = +5 / -5

3. One solution quadratics is known as perfect square quadratics.
4. Zero in denominator is undefined.
5. x^2 - y^2 = (x+y)(x-y)
6. x^2 +2xy + y^2 = (x+y)^2 / (x+y)(x+y)
7. x^2 -2xy + y^2 = (x-y)^2 / (x-y)(x-y)
8. ⚫ solid circle means included.
9. O unsolid circle means not included.

10. Equations have only 1 or just a few solutions

11. Inequalities give a whole range of values as solutions .

    E.g $x+3 < 8$

    $X < 8-3$

    $X < 5$

    So x could be anything less than 5 up to $-\infty$

12. Multiplying and dividing by a negative number in inequality changes the inequality sign.

    E. g $-2x > 10$

    $X > 10/2$

    $X > 5$

    $-2x > 10$

    $X < 10/-2$

    $X < -5$

13. Absolute value $|-3| = 3$

14. $|y| = 3$

    $Y = 3$ or $-Y = 3$

    $Y = -3$

15. $|2x+5| < 15$

    $2x+5 < 15$ or $-(2x+5) < 15$

$2x < 15-5$ $-2x - 5 < 15$

$X < 10/2$ $-2x < 15+5$

$X < 5$ $x > 20/-2$

So $x < 5$ and $x > -10$

16. The only numbers that make the original inequality true are those that are true for both inequality.
17. If a range is given in inequality equations then find the lowest possible values as well as highest possible values then perform multiplication for both scenarios to find which would be the highest and lowest value in terms of optimization problem.
18. Percent increase = (new - old)/old.
19. Percent decrease = (old - new)/old.
20. Distance = rate ✖ time.
21. Work = rate ✖ time.
22. Finding the unit digit problems :
    - First try to find a pattern
    - What will be the unit digit of 3^23
    - 3^1 = (3)
    - 3^2 = (9)
    - 3^3 = 2(7)
    - 3^4 = 8(1)
    - 3^5 = 24(3)

- 3^6 = 72(9)
- 3^7 = 218(7)
- 3^8 = 656(1)

As we can see a pattern (3,9,7,1) is repeating. After 4 times each unit digits are repeating so divide 23/4 and find the remainder 23/4 = result = 5, remainder = 3

So, in the repeating digits (3,9,7,1) the 3rd one would be the result.

So 3^23's unit digit would be 7.

23. Sequence problem :

If the number in a sequence is 3 more than the previous number and 6th number is 32. What is the 100th number?

Eqn :

Difference = (the position's value needed to find that position - the position's value given that position)

=(100-6) = 94

Value = (difference ✖ increment)

Value = 94 ✖ 3 = 282

The intended position's value = value + given position's value.

100th positions value = 282 + 32 = 314

24. f(x) = 2x + 3

Domain = possible input values of x

Range = possible outputs of $f(x)$.

25. Absolute value functions are typically V shaped.

Manhattan 2 - Fraction, Decimal, Percents

1. Fractions consists of 2 parts : numerator, denominator (numerator/denominator) e.g - $\frac{2}{3}$
2. As the denominator of a number gets bigger the value of the fraction gets smaller.

   E.g - $\frac{3}{4} > \frac{3}{5} > 3/6$

   Denominator increases (4,5,6) and the fraction value decreases if the numerator is fixed.
3. When the denominator gets smaller and the numerator is fixed then fraction value in creases. E.g - $3/6 < \frac{3}{5} < \frac{3}{4}$
4. If the numerator and denominator are the same then fraction yields as one.

   E.g - $3/3 = 1$
5. Mixed number consists of an integer and a fraction . e.g - $3\frac{3}{4} = 3+\frac{3}{4} = (12+3)/4 = 15/4$.
6. If numerator is larger than denominator then this kind of fractions are known as improper fractions. E.g = $3/2$.

7. If numerator is smaller than denominator then this kind of fraction is known as proper fractions , e.g - $\frac{2}{3}$
8. Improper fractions and mixed numbers express the same thing.
9. If the numerator gets bigger and the denominator is constant then the fraction value gets bigger. E.g - 5/4< 6/4 < 7/4 —---------<n/4.
10. If the numerator is constant and the denominator is getting bigger then the fraction value gets smaller. E.g - ¾> ⅗ > 3/6 —---- > 3/n.
11. In terms of proper fractions , in other words when numerator is smaller than denominator the fraction value will always be less than 1. ½ = .5 , 3/7 = 0.428...
12. When the numerator is larger than the denominator then the faction value yields larger than 1. E.g- 5/3 = 1.66......., 2/1 = 2.
13. Fraction addition and subtraction affects the numerator.

    E.g - ⅓ + ⅗

    =(5+3)/15

    = 8/ 15 (when denominator is different we need to perform lcm(least common multiple) of the both fraction's denominator)
14. Subtraction : 5/7 - ⅓

    => 15-7/21

    => 8/21

15. While multiplying a fraction and an integer the denominator tells us how many parts to divide the number into and the numerator tells us how many parts to keep. $\frac{2}{3}$ X 12 = 8.
16. Both multiplication and division change the numerator and the denominator.
17. Multiplication : 33/7 X 14/3 = 11 X 2 = 22.
18. We can transform a division into a multiplication problem by only just performing reciprocals.
19. Reciprocal of 3 is $\frac{1}{3}$, 2/4 is 4/2.
20. Every integer can be thought of as a fraction e.g - 2 = 2/1.
21. Dividing a number by another number is the same as multiplying by its reciprocal. E.g - 6 /2 = 6 X $\frac{1}{2}$ = 3
22. Always replace the divisor (number under the '/' or right side of the division sign (➗) by its reciprocal.
23. A number multiplied by its reciprocal is equal 1. 3 X $\frac{1}{3}$ = 1.
24. Fractions and Equations :

    $(4/3)\,x = 15/8$

    4x/3 = 15/8

    4x = 45/8

    X = 45/32.
25. Switching between improper fraction and mixed number:

    We can always split the numerator of a fraction into different parts.

5/4 => (4+1)/4 => 4/4 + ¼ = 1+¼ =>1 ¼

15/4 => (12+3)/4 => 12/4 + ¾ => 3 + ¾ => 3 ¾ .

26. Changing mixed number to improper fraction:

5 ⅔ => (15+2)/3 => 17/3

27. Division in disguise :

(½)/(¾)

=> ½ x 4/3

=> ⅔

28. Fraction operations :

- Adding 2 positive fractions we get a larger number.
  ½ + ⅔ => (3+4)/6 => 7/6.
- Subtracting a positive fraction from something else we get a smaller fraction.
  ⅔ - ½ => 4-3/6 = ⅙
- Multiplying a number by a fraction between 0 and 1 creates a product smaller than the original number. E.g- 7 x ½ = 3.5.
- If a fraction is multiplied by a fraction between 0 and 1 creates a product smaller than the original fraction value. E.g - ½ x ½ = ¼. ½ = .5 ¼ = .25
- Conversely dividing a number or a fraction by a fraction between 0 and 1 yields a higher result. E.g - 7 ÷ ½ = 7 x 2/1 = 14 > 7. ½ ÷ ½ => ½ x 2/1 => 1 > .5.

29. Comparing fractions :

Which fraction is greater 7/9 or ⅘

=> 7/9 + ⅘

=> 35+36/ 45

=> 35/45 + 36/45

As the denominator is equal and the numerator increases so the fraction value increases . For this reason 36/45 is greater.

Or we can find it very easily -

7/9 , ⅘ by cross multiplication.

=> 7 x 5 , 4 x 9

=> 35 , 36

As 36 is greater than 35 so 4/5 is greater than 7/9

30. Never split the denominator - 15+10/ 5 => 15/5 + 10/5

31. To perform approximation :

127/255 or 162/320

0.49 < 0.50

127*2 = 254 (1< 255)

162*2 = 324 (4>320)

We know that the denominator increases fraction value decreases.

As 127*2 less than 1 of 255 so denominator increases.

As 162*2 greater than 4 of 320 so denominator decreases .

So 127/255 will be less than 162/320.

10/22 of 5/18 of 2000 find approximate value-

10*2 = 20 so 10/22 ≅1/(2)+2

5*4 = 20 so 5/18 ≅ 1/(4)-2

Excluding both +2 and -2 we get ½ x ¼ x 2000 = 250(approximate).

32. Dividing by a larger denominator decreases the value and dividing by smaller denominator increases the value.

33. While performing a fractional problem if there is not any total amount then find LCM using all the fractions we can make and pick this lcm as a smart number.

E.g -

For example, consider this problem: The Crandalls's hot tub is halfway filled. Their swimming pool, which has a capacity four times that of the hot tub, is filled to four fifths of its capacity. If the hot tub is drained into the swimming pool, to what fraction of its capacity will the swimming pool be filled? The denominators in this problem are 2 (from 1/2 of the hot tub) and 5 (from 4/5 of the swimming pool). The Smart Number in this case is the least common denominator, which is 10. Therefore, assign the hot tub, the smaller quantity, a capacity of 10 units. Since the swimming pool has a capacity 4 times that of the hot tub, the swimming pool has a capacity of 40 units. The hot tub is only halfway filled; therefore, it has 5 units of water in it. The swimming pool is four-fifths of the way filled, so it has 32 units of water in it. Add the 5 units of water from the hot tub to the 32 units of water that are already in the swimming pool: 32 + 5 = 37. With 37 units of water and a total capacity of 40, the pool will be filled to of its total capacity

34. If there is a total number in the question then do not pick a smart number. E.g-

    Mark's comic book collection contains 1/3 Killer Fish comics and 3/8 Shazam Woman comics. The remainder of his collection consists of Boom comics. If Mark has 70 Boom comic books, how many comic books does he have in his entire collection? Even though you do not know the number of comics in Mark's collection, you can see that the total is not completely unspecified. You know a piece of the total: 70 Boom comics. You can use this information to find the total. Do not use Smart Numbers here. Instead, solve problems like this one by figuring out how big the known piece is; then, use that knowledge to find the size of the whole. You will need to set up an equation and solve: Killer Fish + Shazam Woman = comics that are not Boom Therefore, of the books are in fact Boom comic books. Thus, Mark has 240 comics.

35. In summary, do pick Smart Numbers when no amounts are given in the problem, but do not pick Smart Numbers when any amount or total is given!

36. Digits of decimals-

37. There are only 10 digits in our number system, 0,1,2,3,4,5,6,7,8,9.

38. We can add as many as zeros at the end of a fraction number. e.g- 9.1 = 9.100000000

39. Non-integers can be classified by how many non zero digits to the right of a decimal point. e.g-

    0.23 —-----------> 2 digits.

    8.014—-----------> 2 digits.

    0.00000079—---> 2 digits.

40. Decimals fall between the integers. For example between 0 and 1 0.5 is situated.

41. Decimal can be any decimal point number -∞ to +∞.

42. An integer can be represented as decimal e.g- 8 = 8.0, -123 = -123.0

43. Place value :

    452—---- ones.

    | |--------- tens.   4x100 + 5x10 + 2x1= 452.

    |---------- hundreds.

    .453 —----------- thousandth   3/1000 + 5/100+4/10= .453

    | |----------------- hundredth

    |------------------- tenths

44. 0.8 might mean 8/10 out of 1 dollar = 8dimes or 80 cents .

45. Adding or taking away zeros from the end of a decimal does not change the value of the decimal. E.g - 3.6 = 3.60= 3.60000.

46. When we multiply any number by the positive power of 10 then move the decimal point to the certain number of places. Which also makes the positive number larger. E.g - 3.9742 x 10^3 = 3974.2
89.507 x 10 = 895.07.

47. When we divide any number by a positive power of 10 we need to move the decimal point to the left the specified number of places. Which also makes the positive number smaller. E.g-
4169.2 / 10^2 = 41.692, 83.708/10 = 8.3708.

48. If we need to add zeros in order to shift a decimal we should do so. E.g- 2.57 x 10^6 = 2570000
14.29/ 10^5 = 0.0001429

49. When we divide any number by a negative power of 10 we need to move the decimal point to the right the specified number of places. Which also makes the positive number bigger. E.g-
53.0447/10^-2 = 5304.47

50. When we multiply any number by a negative power of 10 we need to move the decimal point to the left the specified number of places. Which also makes the positive number smaller. E.g-
6782.01 x 10^-3 = 6.78201

51. In summary, dividing by the positive power of 10 shifts the decimal to the left, multiplying moves the decimal point to right.
    Dividing by Negative power of 10 shifts the decimal point to right, multiplying shifts the decimal point to left.
52. Division shortcut-
    If we need to perform a long division
    - Transform the problem with necessary adjustment.
      1530794/31.49 x 10^4
        - 1530794/314900 (shift decimal point 4 place to the right)
        - Now see 7 digits at the dividend and 6 digits at the divisor.
        - Our goal is to get a single digit at the denominator
        - If we keep a single digit at the denominator then we left out 5 digits so for this reason we need to leave the same number of digits at the numerator then we will get only (7-5) = 2 digits at numerator.
        - 15/3 = 5 which will be the approximate result not exact quotient of (1530794/314900).
        - For more precision we can take 153/31 = 4.9
53. Decimal operations -

- To add or subtract decimals make sure to line up the decimal points then add zeros to make the right side of the decimals the same length.

  4.319 + 221.8

  4.319

  221.800

  226.119

  10 - 0.063

  10.000

  0.063

  9.937

54. Multiplication of decimals is the same as normal multiplication. Count the total number of digits after the same number of digits we should put the decimal point. E.g - 0.02 x 1.4

 14

 002

 28

 000

 0000

 0.028

 .003 x 40000 = 120.000

55. Division :

- When divisor is a whole number and dividend is a decimal Number

  12.42 / 3

  3|12.42|4.14

  12

  .4

  3

  12

  12

  0

- When divisor and dividend both decimal number

  1. Move divisor's decimal point necessary spaces needed to make it a whole number
  2. Move the decimal point to the same number of spaces for the dividend as well.

     12.42 / .3

     3 | 124.2 | 41.4

     12

     4

     3

     12

$$\begin{array}{r} \underline{\quad 12} \\ 0 \end{array}$$

- When divisor is decimal and dividend is whole number
    1. Move divisor's decimal point necessary spaces needed to make it a whole number
    2. Add zeros for the same number of spaces for the dividend as well.

    21/.3

    210/3 = 70

    9/.15

    900/15 = 60

56. Terminating vs non terminating decimals:

- Repeating decimals

  2/9 = 9 | 20 | .22

$$\begin{array}{r} \underline{18} \\ 20 \\ \underline{\qquad 18} \\ 2 \end{array}$$

It will go on and on for infinite time; these are known non terminating decimals.

$.222222222 = .\overline{2}$

57. If the denominator is 9,99,999,9999 or another number equal to a power of 10 -1 then the numerator gives us the repeating digits; these aren't the only denominators that result in repeating digits though.
58. Some numbers like √2 or π have decimals that never end and never repeat themselves.
59. Terminating decimals 0.2 = 2/10

    0.47 = 47/100

    0.375 = 375/1000
60. Positive powers of 10 composed of only 5's and 2's prime factors.
61. If a denominator with a prime factor that only includes 2's and 5's the decimal form the fraction will terminate.
62. If denominator is prime factored but not a 2 and 5 thus the decimal will repeat infinitely (Numerator and denominator both have to be integers)

63. Percents : percent literally means per one hundred.

    75% = 75/100 =3/4 = .75
64. 100% means 1
65. Necessary equation to find is,of,percent

    p/100 = is/off

    Here p = percent amount

    Is = is value if given

of= of value if given

E.g- 75% of what number is 21?

p/100 = is/of

75/100 = 21/of

75 of = 21 x 100

Of = 2100/75

So of = 28

66. To find the 10% of any number just move the decimal point to the left 1 place. 10% of 500 = 50.0

    10% of 34.99 = 3.499

    10% of 0.978 = 0.0978

    - 5% is the half of 10% so 10% of 300 = 30.0 thus 5% of 300 = 15.0
    - 35 % of 640

      10 % of 640 = 64.0

      5 % of 640 = 32.0

      So, 35% = 3 x 10% + 5%

      = 3 x 64 + 32 = 224

67. Percent change = p/100 = change/original

    E.g - original price = 80, price increased = 84

    So, change = 84-80 = 4

    p/100 = 4/80

So p = (100 x 4)/80 =5%

Now we can say price increases 5%.

68. p/100 = new/old

Or if increases then (old + old x p%)

Else decreases then (old - old x p%)

Note : if in the question it says that increase then we need to add, if it says decrease then we need to subtract.

E.g-

- Original cost = 35, percent increased 20% new cost?

  We know that,

  p/100 = new/old

  20/100 = new/35

  20 x 35/100 = new

  7000/100 = new

  So new = 7

  Then new price = 35 + 7 = $42 (addition cause increased)

69. 100 is the easiest number to choose for percent problems.

70. Increases 20% multiply it by 1.20, increase 5% multiply it by 1.05

71. Successive percent = original price x (1+% increase) x (1 + % increase)..................

If some point decreases just use - instead of +

E.g price 100 then it increased by 25% then decreased by 20% what is its current value?

SI = original price x (1+25%) x (1-20%)

= 100 X (1+.25) x (1-.2)

= 100

72. New = original x (1+p/100) or original x (1+p%)

73. Interest formulas :

Interest 2 types :

- Simple
- Compound

Compound interest express the idea of "successive percentages"

74. Simple interest = P x r x t

Here p = principal

R = rate

T = time

E.g- given principal=5000, rate = 7% or .07, time = 6 months = 6/12 = .5 year

So simple interest = 5000 x 7/100 x .5

= 350 x .5 = 175

75. Compound interest = p(1 + r%/n)^n*t or p(1+(r/100)/n)^n*t

Here p = principal

R = rate

N = number of times per year

T = number of years

E.g- given p=5000, t = 1 year, r = 8% = .08 n=quarter=4 times

So compound interest = p(1+r%/n)^n*t

=> 5000(1+.08/4)^4*1

=>412

76. Fraction decimal percentage connection :
    - ½ full = .5 filled of its total capacity = 50% full
    - ¼ = 25/100 = 25%

77. Percent to decimal :
    - Mode decimal point to the left 3 places.

      E.g - 53% = .53

      40.57% = .4057

      3% =.03

78. From percent to fraction :
    - Always divide by 100 and remove the % sign.

      E.g - 45% = 45/100 = 9/20

      8% = 8/100 = 2/25

79. Decimal to percent :
    - Move decimal point 2 spots right and add %

      E.g - .53 = 53%

      .4057 = 40.57%

80. Decimal to fraction :

- Add 1 and number of zeros same as the number of digits after decimal point at denominator

  E.g - .3 = 3/10

  .23 = 23/100

  .007 = 7/1000

81. From fraction to decimal :

- Divide the numerator by denominator

  E.g - 3/8 =.375

82. Fraction to percent :

- 1st fraction to decimal
- Decimal to percent

  E.g : 1/2 = .5 = 50%

  3/5 = .6 = 60%

83. FDP and algebraic translations:

- x% = x/100
- Of means multiply
- Of z = z is the whole percent
- Y is x% of z = y is the x percent of the whole part of z.
- Y = z x (x)% or z x x/100
- A is 1/6 of b => a = 1/6 x b
- C is 20% of d => c = d x 20% or d x 20/100

- E is 10% greater than f => e = f x (1+10%)
- G is 30% less than H => G = H x (1-30%)
- Profit = (original price + original price x percent increase) - original price.
- Ratio of x to y = x/y or x:y

84. FDP word problems :
    - Part = Fraction x Whole
    - Part = decimal x whole
    - Part = percent/100 x whole

    E.g total student 200 attended ¼

    So part = fraction x whole

    = ¼ x 200

    = 50

    Commission 30% total commission 4500

    So part = percent x whole

    Part = 30/100 x 4500

    Part = 1350

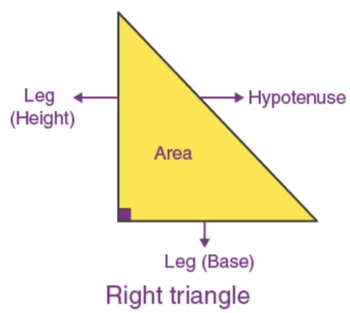
85. X is 40% of what number?

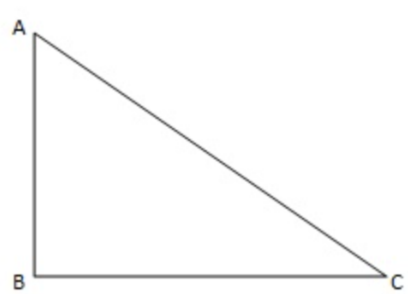
    Let number is Y so x = 40% x y

Manhattan 3 - Geometry

1. Area = length x height.
2. If any triangle has a right angle then it is a right triangle.
3. Hypotenuse of a right triangle is, hypo = $\sqrt{(edge^2 + another\ edge^2)}$

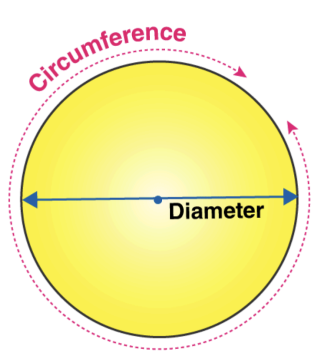
   E.g if a triangle has





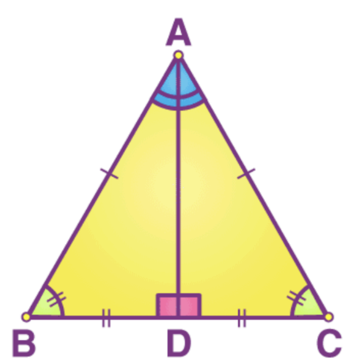
$ac = \sqrt{(ab^2+bc^2)}$

4. Circumference of a circle is $2\pi r$.

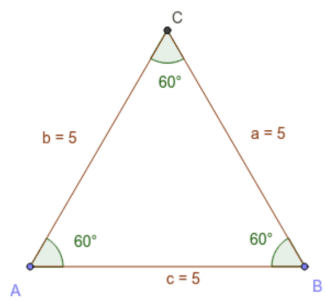


Here r = radius.

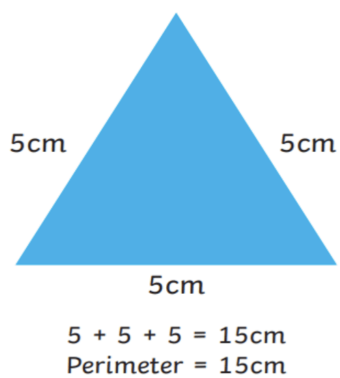
5. Diameter of a circle is d = 2r.
6. Radius of a circle r = 2r/2.
7. Diagonal = √(length^2+width^2)
8. Area of square = edge^2. (because all of the edges are equal)
9. Triangles & diagonals:
10. Sum of any 2 side lengths will always be greater than the third side length.
11. The 3rd side length will always be greater than the difference of the other two side lengths.
12. The internal angles of a triangle must be added up to 180°.
13. Angle upon a straight line is always 180°.
14. Sides corresponds to their opposite angles of a triangle
    - The largest side is opposite the largest angle of that triangle.
    - The smallest side is opposite the smallest angle.
15. A triangle that has two equal angles and two equal sides (opposite of the equal angles) is an isosceles triangle.



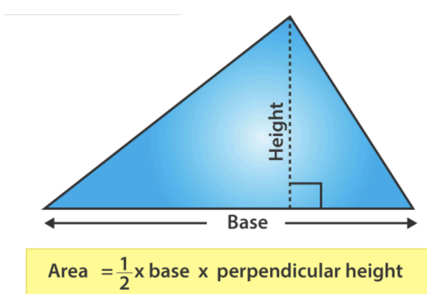
16. A triangle that has 3 equal angles (all 60°) and 3 equal sides is an equilateral triangle.



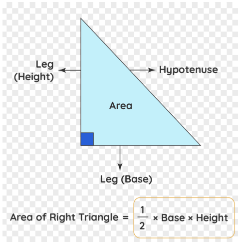
17. Perimeter of a triangle is the sum of all lengths of all 3 sides.



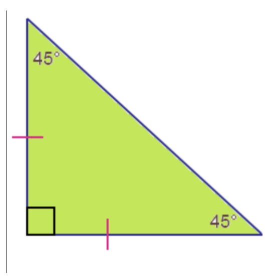
18. Area of a triangle = ½ * base * height.



19. Right triangles:
    - One of the angles is 90° or right angle.
20. Area of right triangle :



21. Pythagorean Theorem : a^2 + b^2 = c^2
22. Pythagorean triplets
    - 3,4,5
    - 5,12,13 (first 2 are the sides which make the right angle, 3rd one is the length of hypotenuse)
    - 8,15,17
23. Isosceles triangle and 45-45-90 triangle.
24. Isosceles' right angle has one 90° angle and two 45° angles.



25. 45° —----> 45°—------->90°

    Leg —----> leg —--------> hypotenuse

    1 : 1 : √2

    x : x : x√2
26. Given a 90° right isosceles triangle's one age ab =5

    So we know that x : x : x√2

    So 5 : 5 : 5√2

27. Equilateral triangle or 30-60-90 triangle.

28. Equilateral triangle's all 3 sides are equal.

29. All 3 internal angles are 60°

30. 30°-60°-90°

    Short leg - long leg - hypotenuse

    1 - $\sqrt{3}$ - 2

    x - x $\sqrt{3}$ - 2x

    E.g - given short leg = 6

    We know tha

    x - x $\sqrt{3}$ - 2x

    6 - 6$\sqrt{3}$ - 2*6

    6 - 6$\sqrt{3}$ - 12

31. Area of a equilateral triangle is = $(\sqrt{3}/4)* a^2$.

32. Side of an equilateral triangle is the same as the hypotenuse of the 30-60-90 triangle.

33. Height of an equilateral triangle is the same as the long leg of a 30-60-90 triangle.

34. E.g - given hypotenuse = 10.

    We know that,

    For 30-60-90 triangle = x : x$\sqrt{3}$: 2x

    As 2x = 10

Thus $x = 5$

So $5 : 5\sqrt{3} : 10$

So height = $5\sqrt{3}$ cause short - long/height - hypotenuse.

35. Important equations :
    - Area of triangle = ½ * base * height.
    - Right angle pythagorean theorem :

      hypotenuse^2 = long edge^2 + short edge^2
    - Isosceles triangle 45-45-90, $1:1:\sqrt{2}$, $x:x:\sqrt{2}$
    - Equilateral triangle 30-60-90, $1:\sqrt{3}:2$, $x:x\sqrt{3}:2x$
    - Area of square = a^2 or edge^2
    - Perimeter of a square = 4a or summation of all the 4 edges.
36. Diagonals of a polygon :
    - Diagonal of square = $s\sqrt{2}$ here s = 1 edge of a square.
    - Any square can be divided into 2 isosceles 45-45-90, $1\text{-}1\text{-}\sqrt{2}$, $x:x:x\sqrt{2}$.
    - Main diagonal of a cube = $S\sqrt{3}$, here, S = an edge of a cube.
    - To find the diagonal of a rectangle we need to know the length and width of a rectangle then we need to use pythagorean theorem to find the hypotenuse or diagonal.
    - Perimeter of a rectangle = sum of all 4 sides or 2(length + width).
    - Bisection of a square creates 2 equilateral triangles.

30-60-90

1: 1√3 : 2

x : x√3 : 2x

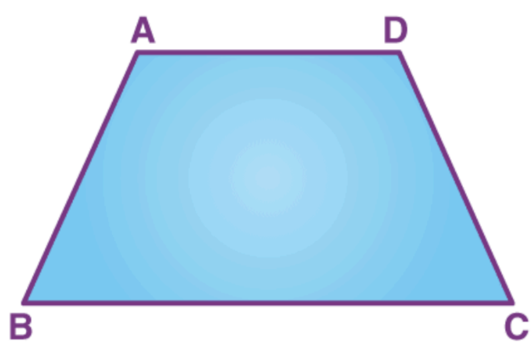
37. Chapter 4 Polygons:

38. Polygons 3 types :
    - Triangles .
    - Quadrilaterals.
    - N sides 5 or more.

39. 4 or more side polygons :
    - Squares.
    - Rectangles.
    - Trapezoids.
    - parallelogram

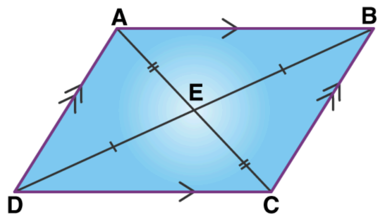
40. Polygons are 2D.

41. Trapezoid :



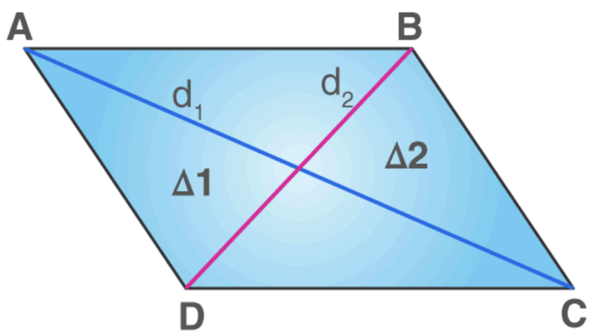
AD || BC parallel. AB and DC are not parallel.

42. Parallelogram :



- Opposite sides are equal
- Opposite angles are equal
- AB == DC, AD == BC, AB || DC, AD || BC, ∠A == ∠C , ∠B == ∠D.

43. Rhombus :

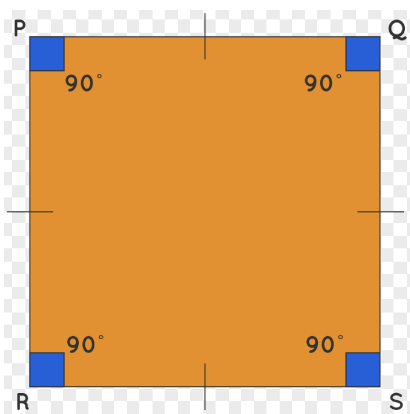


- All sides are equal.
- Opposite angles are equal.
- AB == BC == CD == DA.
- ∠A == ∠C , ∠B == ∠D .

44. Rectangle :

- Opposite sides are equal & parallel.
- All angles are equal and 90°.
- AB || DC & AD || BC.
- AB == DC, AD == BC.
- $\angle A = \angle C = \angle B = \angle D = 90°$.

45. Square :



- All sides are equal
- PQ = QS = SR = RP AND PQ || RS , PR || QS.
- All angles are equal and 90°.

46. Sum of interior angles :

(n - 2) x 180. Here n = number of edges.

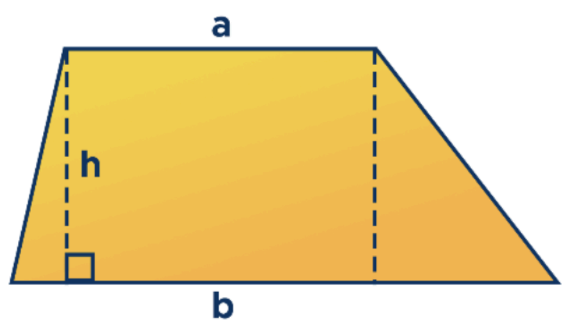
- Triangle : n = 3, (3-2) x 180 = 180°.

- Quadrilateral : n = 4, (4-2) x 180 = 360°.
- Pentagon : n = 5, (5-2) x 180 = 540°.
- Hexagon : n = 6, (6-2) x 180 = 720°.
- Octagon : n = 8, (8-2) x 180 = 1080°.

47. Perimeter = sum of all sides.
48. Area of polygon : amount of space is occupied.
49. Area measured in squared unit.
50. Area of triangle = ½ * base * height.

    Height always refers to a line that is perpendicular to its base.

51. Area of rectangle = length * width.
52. Area of trapezoid = ((base1 + base2)/2) * height.

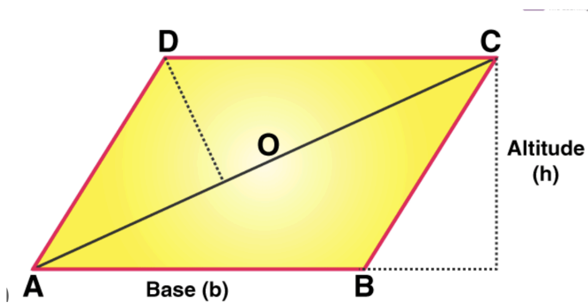


    Here a = base 1

    B = base 2

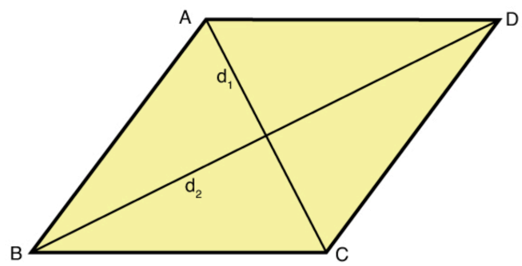
    H = height.

53. Area of parallelogram = base * height.



Here ab = base and h = height

54. Area of rhombus = ½ * diagonal1 * diagonal2



Here diagina1 = d1 and diagonal2 = d2.

55. Area of square = a^2 / side^2 / edge^2.

56. 3D surface area is measured in square units.

57. Surface area = sum of the areas of all of the faces.

58. Both the rectangle solid and cube have 6 faces.

59. Surface area of a rectangle solid = (top face ares + bottom face area) + (back face area + front face area) + (left side area + right side area)

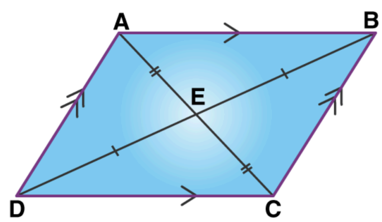
Or surface area of a rectangular solid = 2*top face/ bottom face area + 2*back face/front face area + 2* right side/left side area.

60. Area of cube = 6 * length * width

61. 3D volume :

62. Volume means amount of stuff it can hold (milk inside carton)
63. Volume measured in cubic units.
64. Volume = length * height * width.
65. Volume of a cube = 3 * side.
66. Quadrilaterals : has 4 sides.
67. Parallelogram : opposite sides are equal and parallel as well as opposite angles are equal.

Diagonal will divide the parallelogram into 2 equal triangles.

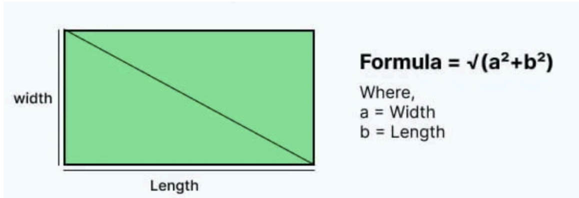


Diagonals also cut each other into half. AE = EC and BE = ED.

But diagonal ac is not equal to bd.

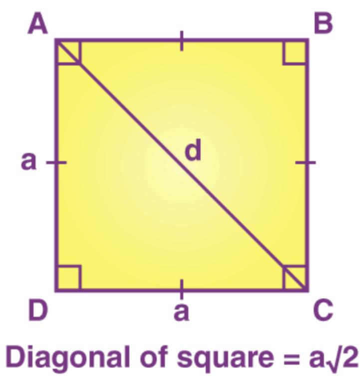
- perimeter of parallelogram = sum of all 4 sides.
- Area of parallelogram = base* height.

68. Rectangles :
    - All opposite sides are equal and parallel.
    - All internal angles are equal and 90°.
    - Perimeter of a rectangle = sum of all 4 sides.
    - Area of a rectangle = length * width.
    - Diagonals are equal in length



69. Squares :

- Length of all 4 sides are equal.
- All diagonals are of the same length.
- Area of a square = a^2 or side^2.
- Perimeter of a square = sum of all 4 sides.



70. Maximum area of a polygon :

71. Maximum area of a quadrilateral with a given perimeter square has the highest area.

72. Of all quadrilaterals with a given area the square has the lowest perimeter.

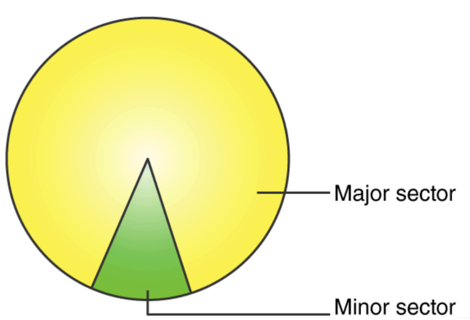
73. Maximum area of a parallelogram or triangle :

- If 2 sides of a triangle or parallelogram are given we can maximize the area by placing the 2 sides perpendicular to each other. In other words when 2 given sides are perpendicular to each other then they will give maximum area.

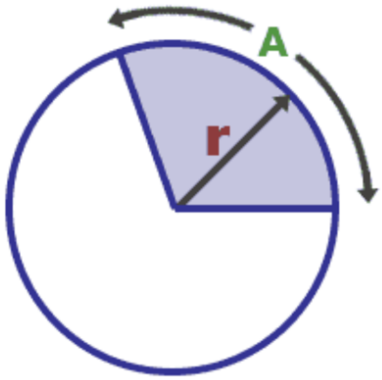
74. Chapter 5 : Circles & Cylinders.
75. Every circle has a center.
76. Radius of a circle is the distance between a point on the circle and the center of the circle.
77. All radius in a circle has the same length.
78. Diameter of circle = 2 radius. D = 2r here r = radius
79. Circumference = perimeter of a circle.
80. If we divide the circumference by diameter we will get π (pi).
    circumference/diameter = π, π = 3.1416..........................
81. Circumference = π * diameter

    => π * 2r here r = radius.

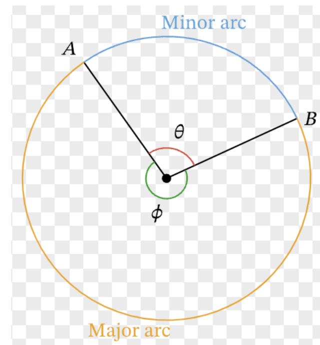
    => $2\pi r$
82. Area of a circle is the space inside the circle.
    - Area = πr^2
83. Sectors : fractional position inside a circle is known as sector.



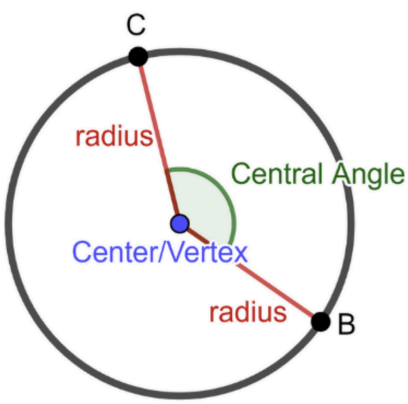
84. Arc length : in a semicircle the portion of the circumference that remains is called arc length.



Here, A is the arc length.

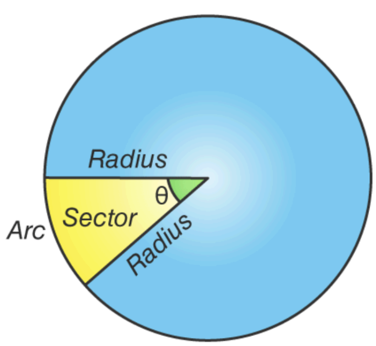


85. Central angle of a sector is the degree measured between 2 radii.



- Full circle has a central angle of 360 degrees.

86. Sector area :



87.Central angle/360° = sector area /circle area(πr^2) = arc length/ circumference ($2\pi r$)

E.g - given arc length = 4π

Radius = 3

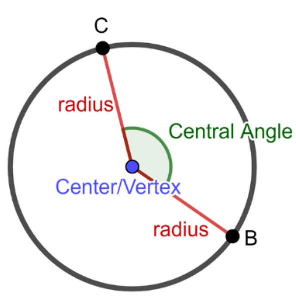
We know that,

Central angle/360 = arc length/circumference

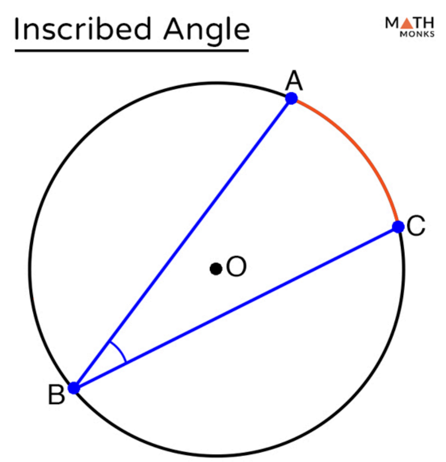
ca/360 = $4\pi/2\pi r$

Thus ca = $(4\pi/2\pi r)*360$ = 240°

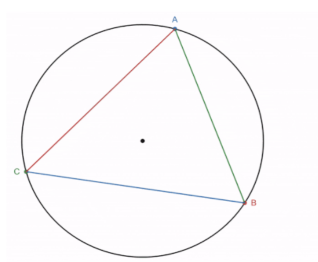
88. Inscribed vs central angle :
89. Central angle's vertex (highest point) lies on the center point of the circle.



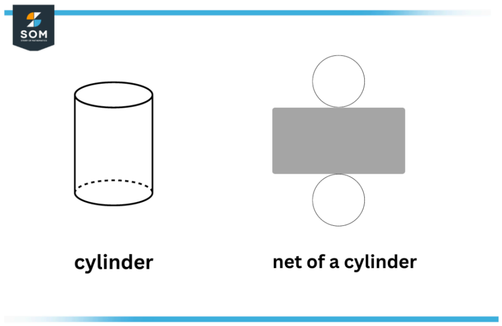
90. Inscribed angle's vertex lies on the circle itself.



91. Inscribed angle is half of the central angle.
92. Central angle = 2* inscribed angle.
93. Inscribed triangle :
    - A triangle is said to be inscribed in a circle if all of the vertex of the triangle is on the circle.



94. Inscribed angle = ½ of the arc it intercepts in degrees.
95. If one of the sides of the triangle is the diameter of the circle then the triangle must be a right triangle.
96. Cylinder & surface :
    - 2 circles and a rectangle combined to form a 3 dimensional shape called a right circular cylinder or cylinder.



97. Cylinder = area of 2 circle + area of rectangle.

    => 2*πr^2 + length * width.

    => 2*πr^2 + 2πr * h

    - Length of the rectangle = circumference of the circle = 2πr
    - Width of the rectangle = height of the cylinder.
    - Surface area of cylinder = 2πr^2 + 2πrh

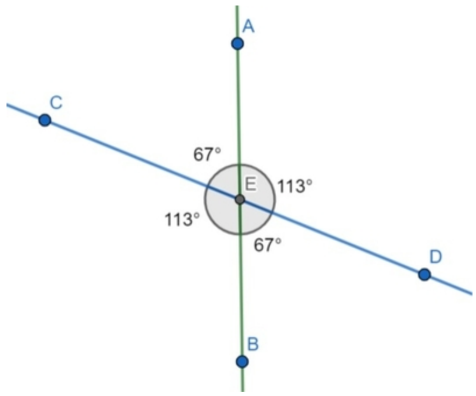
      => 2πr(r+h)

98. Cylinders and volume :
    - Volume of a cylinder measures how much stuff it can store.
    - Volume of a cylinder = πr^h

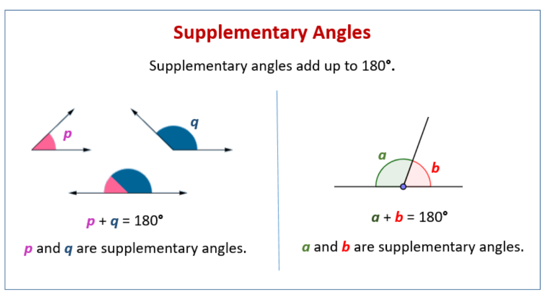
      Here r = radius, h = height

99. Chapter 6 : lines and angles -
100. A straight line is 180° , half of a circle.
101. Parallel lines never intersect.
102. Perpendicular lines intersect at 90°
103. Intersecting lines has 3 important properties :

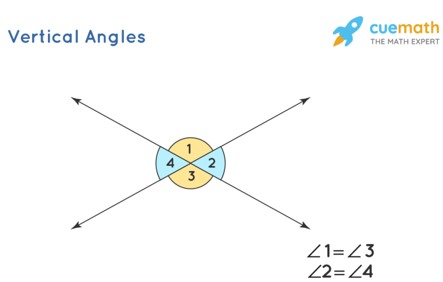
- The interior angles formed by intersecting 2 lines form a circle, so the sum of these angles is 360°.

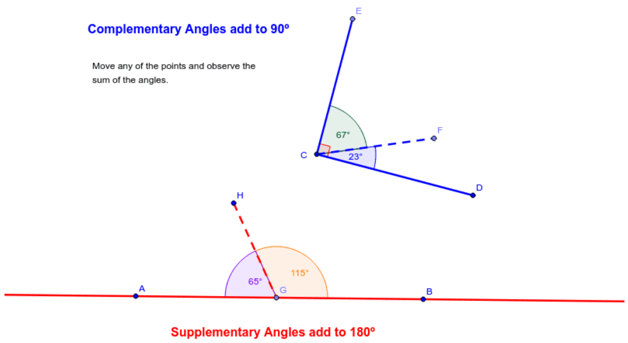


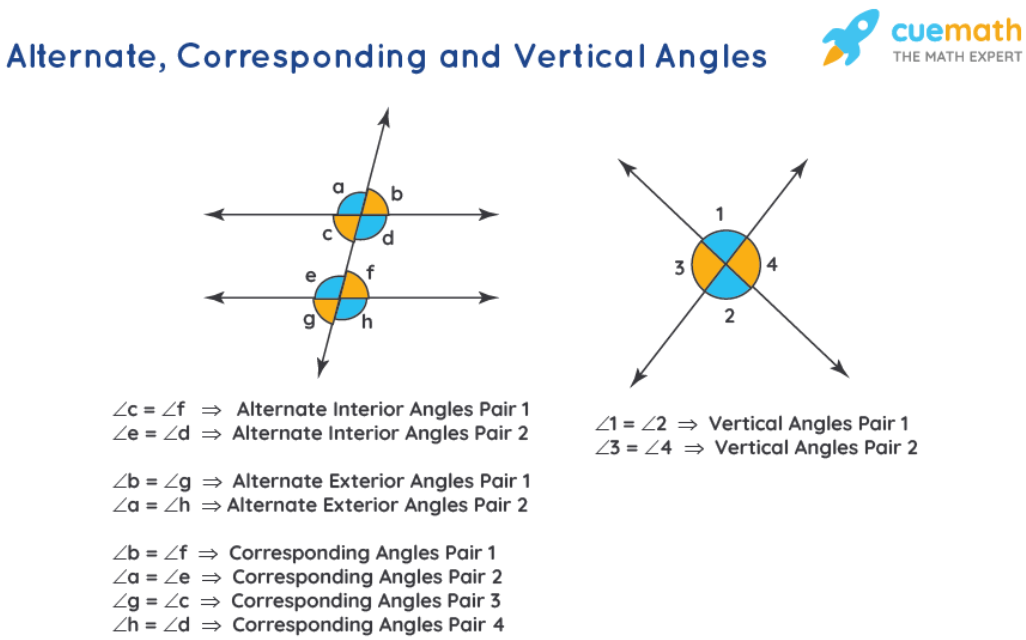
- Interior angles that combine to form a line sum to 180° are called supplementary angles.



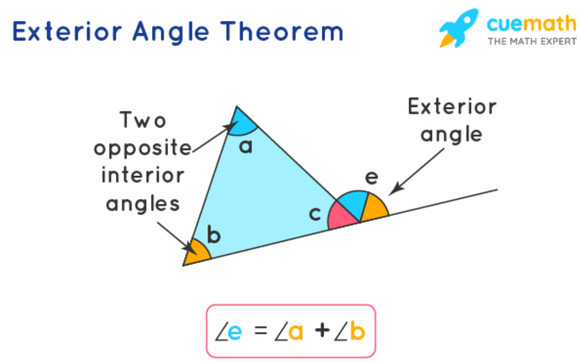
- Angles found opposite of each other where these two lines intersect are equal. They are called vertical angles.



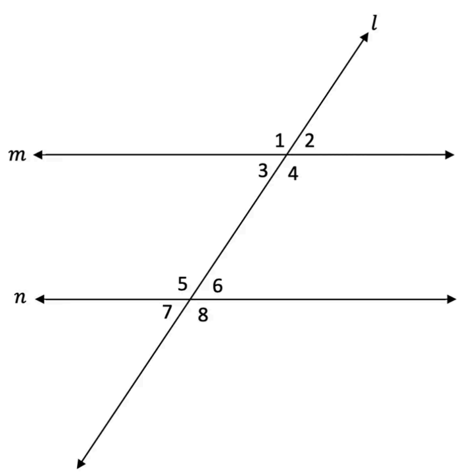




104. Exterior angles of a triangle -
    - An exterior angle of a triangle is equal to the sum of the two non adjacent (opposite) interior angles of the triangle.



105. Parallel lines cut by a transversal :



Notice!

Notice lines *m* and *n* are parallel to each other and cut but the transversal line *l*, forming angles 1 through 8.

- All of the acute angles (less than 90°) are equal.
- All of the obtuse angles (more than 90° but less than 180°) are equal.
- Acute angles are supplementary to the obtuse angles .

  ⦣1 (obtuse) + ⦣2 (acute) = 180°.

106. Chapter 7 : the coordinate plane -
107. Open circle means number is not included.
108. By combining two number lines horizontal and vertical we can determine the location of a point.

109. The x coordinates gives the left or right numbers

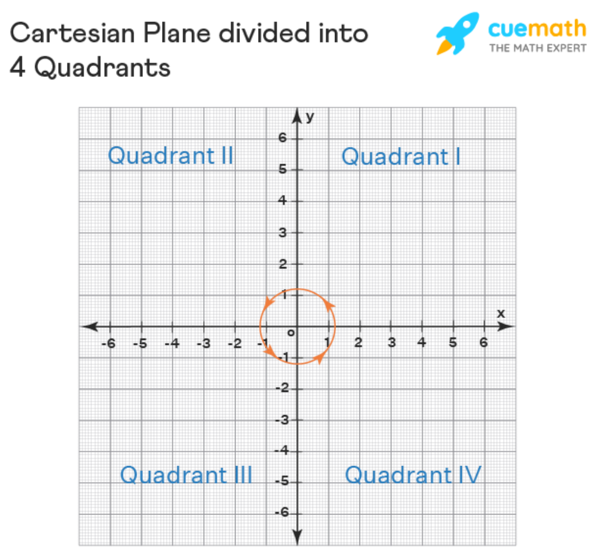
Numbers left of the zero are negative and right of the zero is positive.

110. The y coordinates gives the up or down value-

Number above 0 is positive and below zero is negative.

111. Line 1D, plane 2D.

112. 4 quarters of the coordinate plane are called quadrants.



113. Equation of a line y =mx +b

114. If y =2x+1 then m = 2, b =1

115. Y = 3x-2 (linear equation), y = x^2 (non linear equation)

116. y = mx + b

Here m = slope, it tells how steep the line is and whether the line is rising or falling.

117. M > 0 = positive slope. (1,3,q)

- M<0 = negative slope (2,4,q)

- M>1 = steep slope
- 0 < M <1 = gentle slope.

118. B = y intercept , this tells where the line crosses the y-axis.
119. Y = 3x + 4 , here m = 3, b = 4.
120. Plotting a line using line equation m and b

Y = ½ x -2

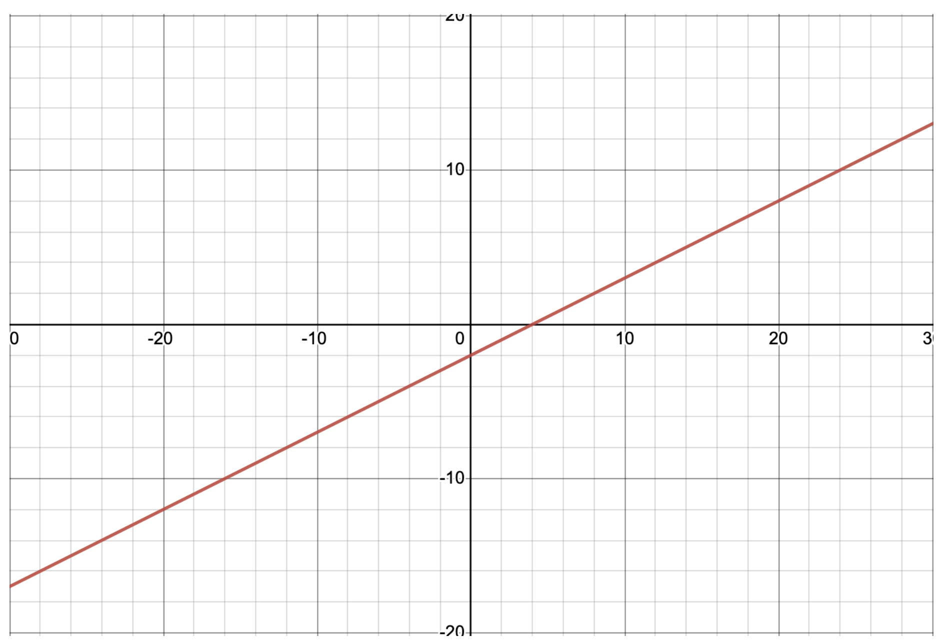
Here b = 2

First b = y intercept value if positive plot at the positive part of the y, else plot at the negative part of the y.

M = ½ or rise/ run

Here rise value =1 which means from the y point it will rise towards the y axis 1 point.

Run = 2 which means from the risen point it will go towards the x axis 2 points depending on the positive or negative sign.



121. A point where a line intercept a coordinate axis is called an intercept.

    - X intercept —--> intersects the x axis (x,0) here y=0.
    - Y intercept —--> intersects the y axis (0,y) here x=0.

122. To find x intercepts plug in 0 for y to find y intercept plug in 0 for x. E.g -

    X - 2y = 8

    So x - 2 x 0 = 8

    X = 8.

    X - 2y = 8

    -2y = 8

    Thus y = -4

    So x,y = (8,-4)

123. 2 equations might represent the same line, in this case infinitely many points (x,y) along the line satisfy the two equations.

124. Distance between 2 points = √(x1-x2)^2 + (y1-y2)^2

    E.g - (1,3) (7,-5)

    => √(x1-x2)^2 + (y1-y2)^2

    => √(1-7)^2 + (3-(-5))^2

    => √(1-7)^2 + (3+5))^2

    => 10

Manhattan 4 - Number Properties

1. Divisibility & Prime chapter 2:
2. Integers are the whole number.
    - Do not have decimals.
    - Do not have fractions
    - Also known as counting numbers. E.g - 1,2,3 etc.
    - Integers can be positive or negative. E.g - 1,23,-23,-100 etc.
    - Zero is also integer.
    - Integer example : 7,15,003,-346,0 etc.
    - Not integer : 1.3, ¾, π etc.
    - Integer x integer = integer, 13 x 3 = 39.
    - Integer + integer = integer, 13 + 3 = 16.
    - Integer - integer = integer, -5 - 32 = -37.
    - Dividing an integer by another integer won't always give an integer. E.g 18/3 = 6 (integer), 5/2 = 2.5 (not integer) .
3. Divisibility rules :
    - An integer is divisible by 2 if the integer is even.
        - Always check the last unit digit of a number.
        - 172—---(unit digit 2), 388—---------(unit digit 8) as 2 and 8 are even so 172 and 388 would be divisible by 2.

- An integer is divisible by 3 if the sum of that integer's digits is divisible by 3.
    - 72, 7 + 2 = 9—---- and 9 is divisible by 3 thus 72 divisible by 3.
- An integer is divisible by 4 if the integer is divisible by 2 twice or if the last 2 digit number is divisible by 4.
    - 28/2 = 14, 14/2 = 7 so 28 is divisible by 4
    - 23456 —---- last 2 digits = 56 and 56/4 = 14, so 23456 is divisible by 4.
- An integer can be divisible by 5 if that integer's last digit is either 0 or 5.
    - 75/5 = 15, 80/5= 16.
- An integer is divisible by 6 if the integer is divisible by 2 (if the last digit of that number is even) and divisible by 3 (if the sum of all digits is divisible by 3).
    - 48, 48/2 = 24, 48—4+8=12, 12/3 = 4. So 48 is divisible by 6, 48/6 = 8.
- An integer is divisible by 8 if that integer is divisible by two 3 times in succession or if the last 3 digits of that number is divisible by 8.
    - 32/2 = 16, 16/2 = 8, 8/2 = 4 —--- so 32/8 = 4
    - 23456 —-- last 3 digits 456/8 = 57.

- If the sum of digits is divisible by 9 then the number is also divisible by 9.
    - 4185 —- 4+1+8+5 = 18, 18/9=2 so 4185 is divisible by 9.
- If an integer ends in 0 then it is divisible by 10.
    - 670/10 =67.

4. Factors-
5. Any number divided by 1 equals itself.
6. Any number divided by itself = 1.
7. Factors are those numbers by which a certain number is divisible.

   E.g - 6—---- 6 / 1 = 6

   6 / 2 = 3

   6 / 3 = 2

   6 / 6 = 1

   So 6 is divisible by 1,2,3,6 these are the factors of 6.
8. Numbers that only have 2 factors 1 and itself are known as prime numbers.

   2/1 = 2 , 2/2 =1

   7/1 = 7, 7/7 =1
9. 1 is not a prime number.
10. 2 is the only even prime number.
11. Integers can be prime and not prime.
12. Primes - 2,3,5,7,11,13,17,19,23,29,31,37,41,43,47.

13. Prime factorization -

14. Prime factors are the DNA of numbers.

```
15—-----60—----4
 |             |
3,5           2,2
```

Factor until all the factors are prime numbers.

So 3,5,2,2 or 2^2, 3,5 are the prime factors of 60.

15. Factor foundation rule :

    - if a number x is divisible by y and y is divisible by z then x is also divisible by z.

      E.g - 12 divisible by 6 and 6 divisible by 2 so 12 divisible by 2.

    - All of the factors of a number can be built with different combinations of its prime factors except 1.
    - Every even number is multiple of 2.

16. Factors are always fewer, divisible numbers .

17. Multiples are infinite.

18. Factors make the number small.

    - Factors of 8 - 1,2,4,8.

19. Multiples make the number big.

    - Multiples of 8 - 8,16,24,32,40,48,-------------- ∞.

20. Following statements say exactly the same things -

    - 12 is divisible by 3

- 12 is multiple of 3
- 12/3 = 4 is an integer.
- 12 = 3n →n = 4 is an integer.
- 12 items can be shared among 3 people so that each person has the same number of items.
- 3 is divisor of 12 or factor of 12.
- 3 divides 12.
- 12/3 yields a remainder of 0.
- 3 goes into 12 evenly.
- If we add or subtract multiples of N the result is a multiple of N.

21. Remainder :

    5 | 17 | 3

    15

    2

    Here 5 is the divisor 17 is the dividend 3 is quotient and 2 is remainder.

22. Range of possible remainder if we divide an integer by x then possible remainder could be 0 - x-1.
23. Negative and greater than x remainder is not possible.
24. Odd and evens :

    Odd numbers are not divisible by 2, e.g - 1,3,5,7,9
25. Even numbers are divisible by 2, e.g - 2,4,6,8,10.

26. Adding or subtracting two even numbers results in even. 8+2=10, 8-2=6.
27. Adding and subtracting an even and an odd number results in odd. 1+2=3, 5-2=3
28. While multiplying multiple integers if any of the integers is even then the multiplication result will be an even number. 2x3x5=30
29. If all the integers are odd then after multiplication the result would be odd. 1x3x5=15
30. If we multiply several numbers (odd as well as even) then the multiplication result will be divisible by the total even integers to the power of 2. E.g - 2x5x6 = 60

    Here 2 even integers 2,6 so 2^2 = 4 . so 60 will be divisible by 4.
31. Even odd equations :
    - Odd ± odd = even. 1+1 = 2, 1-1 =0
    - Even ± even = even. 2+4=6, 2-4=-2
    - Odd ± even / even ± odd = odd. 1+2=3, 1-2= -1
    - Odd x odd = odd. 3*5 = 15, 1*1=1.
    - Even x even = even. 2*2 = 4, 2*4=8.
    - Odd x even = even. 2*3 = 6, 7*2=14.
32. Positive and negative : zero (0) is not positive nor negative.
33. |-5| = 5, means distance from 0 to 5.
34. |5| = 5, means distance from 0 to 5.

35. Double negative yields positive -

- Adding 2 negative numbers results in negative sum

  E.g (-2) + (-3) = -5

- Multiplying 2 negative numbers yields a positive product.

  E.g (-2) x (-3) = 10.

36. Exponents : $4^3 = 64$ or $4^3 = 4x4x4 = 64$.

37. The greater the exponent the faster the rate of increase (positive base).

38. Exponents has higher precedence than subtraction so $-4^2 = -(4)^2$ not $(-4)^2$

39. $0^3 = 0$

40. $1^3 = 1$

41. $(-1)^n$ if n = even number then 1, if n = odd number then -1. This rule is applicable for all numbers.

42. If we are told $x^6 = x^7 = x^{16}$ here x either 0 or 1.

43. If we are told $x^6 = x^8 = x^{10}$ here x either 0 or 1 or -1.

44. For positive proper fractions as the exponent increases the value of that expression decreases.

    E.g - $(3/4)^1 = 3/4$, $(3/4)^2 = 9/16$, $(3/4)^3 = 27/64$

    Here $3/4 > 9/16 > 27/64$.

45. Just like proper fractions, decimals between 0 and 1 decrease as their exponents increase.
(0.6)^2 = 0.36, (0.6)^3= 0.216, (0.6)^4 = 0.1296.
Here 0.36 > 0.216 > 0.1296.

46. For improper fractions as the power increases the value get bigger and bigger. 10/7 < (10/7)^2.

47. (n x m)^x = n^x * m^x or (n.m)^3
(2*5)^3 = (2^3 x 5^3) = 10^3 = 1000.

48. (2+5)^3 = 7^3.
Not (2^3+5^3)

49. Negative exponent is the reciprocal of what that expression would be with a positive expression.

50. (¾)^-3 = 1/(¾)^3 or 1 x (4/3)^3 = 64/27

51. When we see a base without an exponent always consider there is an exponent of 1. 3 x 3^4 = 3^1 x 3^4

52. If the bases are the same but exponents are different and both the bases are in multiplication format then add the exponents.
3^1 x 3^4 => 3^(1+4) => 3^5

53. 5^(y+2)/5^3
=> 5^(y+2-3)

54. When multiplying an exponential term that shares a common base add the exponent. 3^2 * 3^3 = 3^(2+3) = 3^5.

55. When dividing an exponential term that shares a common base subtract the exponent. $3^2 / 3^3 = 3^{(2-3)} = 3^{-1}$.

56. When something with an exponent is raised to another power multiply the two exponent.

    $(a^2)^3 = a^6$

57. $4^2 = 16$
    - $5^2 = 25$
    - $6^2= 36$
    - $7^2 = 49$
    - $8^2 = 64$
    - $9^2 = 81$
    - $10^2 = 100$
    - $11^2=121$
    - $12^2=144$
    - $13^2=169$
    - $14^2=196$
    - $15^2= 225$
    - $2^3 = 8$
    - $3^3= 27$
    - $4^3= 64$
    - $5^3=125$

58. When can we simplify exponent expression ?

- If they are linked by multiplication or division , but can not be simplified when they are in addition or subtraction form.
- We need to have a common base or exponent.

59. Roots : √2 x √2 = 2
60. Root and power 2 cancel each other out.
61. Use prime factorization while simplifying roots.

    E.g - x = √360 —---- 360 = 36 * 10 = 6 * 6 * 5*2 = 3*2,* 3*2, * 5*2 . so prime factors = 2*2*2*3*3*5

    Thus x = √2*2*2*3*3*5

    X = √2*2 * √3*√3 * √2*5

    X = 2 * 3 * √10

    X = 6√10
62. Consecutive integers : 4,5,6
63. Consecutive even integers : 0,2,4,6,8
64. Consecutive odd integers : 1,3,5,7,9
65. Evenly separated : these are sequences of numbers whose values go up and down by the same amount.

    E.g - 4,7,10,13,16 { each value increases by 3}
66. Consecutive multiples : 12,16,20,24 each number is a multiple of 4
67. All sequences of multiple integers are sequences of consecutive multiples.

68. All sequences of consecutive multiples are evenly spaced sequences.

69. All evenly spaced sequences are fully defined if the following three parameters are known.

    - The smallest (first) or the largest (last) number in the sequence.
    - The increment or decrement (always 1 for consecutive integers)
    - The numbers of items in the sequence.

70. Properties of evenly spaced sequences :

    - Arithmetic mean and median are equal to each other.

      4,8,12,16,20→ here 12 is the middle number so 12 is median. Thus the arithmetic mean is also 12.

      Average = (4+8+12+16+20)/5 =60/5 = 12.

      Or average = (4+20)/2 = 12.

71. Mean & median of a sequence are equal to the average of the first and last term.

    4,8,12,16,20→ average = (4+20)/2 = 12.

72. Sum of the elements in the sequence equals the arithmetic mean times the number of items in the sequence.

    4,8,12,16,24 → average = (4+8+12+16+20)/5 =60/5 = 12

    Here n = 5

    Thus sum = n*average = 5*12 = 60.

73. Counting integers : how many integers from 6-10

Ans : 6,7,8,9,10 = 5.

Add 1 with the difference between the last and first number.

Number of integers = (last - first)+1

E.g - how many integers between 765 and 14

Number of integers = (last - first)+1

Number of integers = (765 - 14)+1 = 751+1 = 752

74. If the numbers are not consecutive but it follows a certain increment or decrement pattern then,

Number of integers = ((last - first)/increment)+1

12 to 24

Number of integers = ((24 - 12)/2)+1 = 7

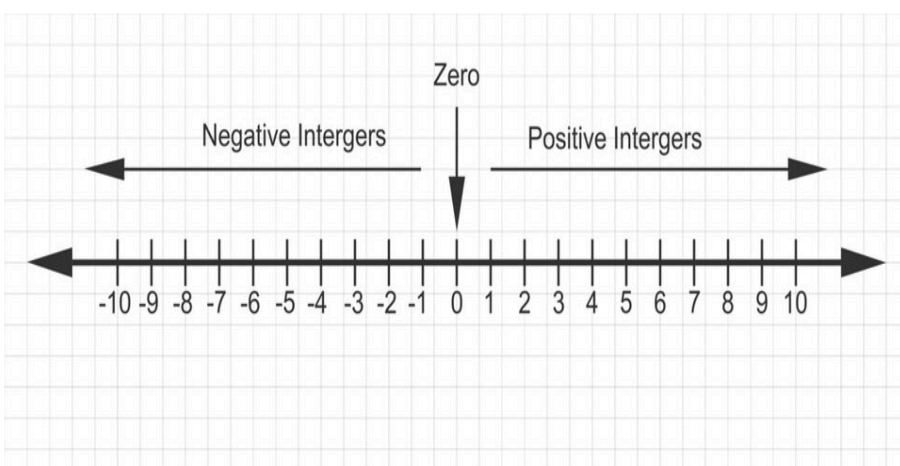
- How many multiples of 7 are there between 100 and 150?

  First 105

  Last 147

  Thus = ((147-105)/7)+1 = 7.

75. Number line : distance is always the absolute value.



76. Number of ticks in a number line is 1 more than the number of intervals. If 4 ticks then 3 intervals.

Manhattan 5 - Word Problem

1. Algebraic translation strategy step:
    - Identify unknown
    - Identify relationship and create equation
    - Identify what the equation is asking for
    - Solve for the unwanted element.
2. Rate of work-
    - Rate x time = work → RT = W
    - Rate x time = distance → RT = D
3. Given, r = 30 miles per hour

   D = 75 miles.

   T = ?

   We know thad,

   RT = D

   So, t = d/r

   T = 75/30

   Thus t = 15/6 = 2.50 hours.
4. Always express rates as distance over time.

5. Second to minute = multiply time in second with 60.
6. Minute to hour = multiply time in minute with 60.
7. If car A & B driving towards each other
   - Shrinking distance in a rate of (a+b) rate.

     E. g car A going 30 miles per hour, car B going 40 miles per hour towards each other. A→ ←B

     Thus (a+b) = (30+40) = 70 miles per hour.
8. If car a and b both are driving away from each other then distance grows at a rate of (a+b) miles/hour as well. A ← → B
9. If car a is chasing car b and catching up A → B →

   Then distance grows at a rate (a-b) miles/hour.

   If car A is failing to chase or falling behind then distance grows at a rate (b-a) miles/hour.
10. If one goes at a faster rate and comes back at a slower rate then the average rate will be closer to the slower rate.
11. In average speed math, pick a smart number as distance. W or d = RT

    Here r = rate = work per time
12. When 2 or more workers are performing the same task for the same duration their work rates can be added together.

    A 5 boxes in 1 hour

    B 12 boxes in 1 hour

    Thus together a and b make 5+12 = 17 boxes per hour.

13. When a worker is undoing the work of the other at the same amount of time then work rate will be subtracted.

    A fills 3 gallons in 1 min

    B drains 1 gallon in 1 min

    Then work rate = 3-1 = 2 gallons/ min

14. Ratios : 3 to 4 = 3 : 4 or ¾ (this only works for ratios with 2 quantities)

15. If 2 ratios have a constant ratio they are directly proportional to each other.

    g/b = 4/7

    g/35 = 4/7

    G = (4 x 35)/7

    Thus g = 20

16. The unknown multiplier :

    If the number of ratios is not given, instead the whole population quantity is given then we will use the unknown multiplier method.

    E.g - given,

    Lemon : wine : water = 2 : 5 : 7

    Total combined = 35 ml

    Let,

    2x + 5x + 7x = 14x

So 14x = 35

X = 35/14

Thus x = 2.5

Then wine = 5x = 5*2.5 = 12.5 ml.

17. In the problems of multiple ratios make a common term.

C:A = 3:2 C:L = 5:4

A : C : L

2 : ⅗ : 4

2*5 : 3*5 : 3*4

10 : 15 : 12

Thus A : C : L = 10 : 15 : 12

After combining we can extract the numbers corresponding to the quantities.

18. Statistics :

19. Average = sum/number of terms

Avg = s/n

S = sum of all terms in the set

N = number of total terms in the set.

20. Sum = average * number of terms.

21. Distance = rate * time.

22. Evenly spaced set : if total number of terms is evenly spaced and odd then average value would be (n+1)/2 th term.

E.g - {3,5,7,9,11}

Here n = 5 (odd)

Then avg would be = (5+1)/2 = 3rd term of the set which is 7.

- If the total number of terms is evenly spaced and even then the average value would be (n/2 th term value + ((n/2)+1) term value)/2 .

  E.g - {5,10,15,20,25,30}

  Avg = ((6/2)th term + (6/2)+1 th term)/2

  => (3rd term + 3+1 th term) / 2

  => (15 + 20)/2 = 17.5

23. The most easy approach for the evenly spaced problems :

    Add first and the last term and divide by 2 odd even doesn't matter here.

    {3,5,7,9,11}

    Avg = (3+11)/2 = 7

24. Weighted average : a weighted average of only two values will fall closer to whichever value is weighted more heavily.

25. Weighted average will always be the closer which number of terms are greater as well as holds the highest weights.

    E.g - 2 shots with 15% alcohol, 3 shots with 20% alcohol. Their average weighted value would be closer to 20%.

26. Median : the middle number.

27. If data set contains odd number of values

- Median = (no of terms + 1)/2 term value
- E.g {5,17,24,25,28} = (5+1)/2 = 3 rd term = 24.

28. Median is another type of average.
29. If data set contains even number of values

- Medina = ((no of terms)/2 + (no of terms +1)/2)/2
- E.g {3,4,9,9}
- Median = ((4/2) + (4+1)/2)/2 = 6.5

30. If n/2 th value and (n+1)/2 th values are same then the median would be the same as well.
31. Median of sets containing unknown values.

E.g - {x,2,5,11,11,12,33} first organize the set in ordered form.

X would be less than or equal or greater than the medina value, which is =(n+1)/2 = (7+1)/2 = 4th term = 11.

32. Standard deviation : to describe the spread or variation of the data in a list use a different measure called standard deviation.
33. Standard deviation indicates how far from the average (mean) the data points typically fall.
34. A small sd indicates that a list is clustered closely around the average value or mean value.
35. A large sd indicates that the list is spread out widely with some point appearing far from the mean.

36. {5,5,5,5}
   - It's mean value = (5+5+5+5)/5 = 5
   - Sd = 0 (no spread at all)
   - {2,4,6,8}
   - Mean = (2+8)/2 = 5
   - Sd = moderate sd (the larger the sd the data set is more spread out)
   - {0,0,10,10}
   - Mean = (10+10+0+0+)/4 = 5
   - Sd = has large sd
37. Standard deviation formula,
   - First organize the order of the data
   - Find mean of the data set
   - The find variance valu (each data point - mean value)^2
   - Then find the sum of all the variance values and divide this with the number of total data which will give the standard deviation value.
38. More spread out the data set the greater the standard deviation would be.
39. Difference between the data points -
   - If the difference is higher then standard deviation would be higher.

40. The standard deviation will not change if all the data points are added with the same number.
41. The standard deviation will not change if all the data points are multiplied with the same number.
42. Range = (largest number in the list - smallest number in the list)
43. The range of a list of numbers is another measure of the dispersion of the list of numbers.
44. Range -

E.g {3,6,-1,4,12,8}

{-1,3,4,6,8,12}

Thus range = (12 - (-1))

=> (12+1)=> 13.

E.g {2,-1,x,5,3}

{-1,2,3,5,x}

Range = x -(-1) = x+1

Largest value = x+1

13 = x+1

X = 13-1 = 12.

If 5 is the largest value in the list then range = (5-x)

=> 13 = 5-x

=> 13-5 = -x

=> x = -8

Thus the answer would be -8 or 12.

45. Quartiles and percentiles :

46. {0,1,2,2,3,4,5,5,5,6,7,8,10,11,13,14}

Quartile1|Quartile2|Quartile3|Quartile4

{0,1,2,2 | 3,4,5,5 | 5,6,7,8 | 10,11,13,14}

Q1 Q2 Q3

47. Q1 = (highest item of the quartile 1 + lowest item of the quartile 2)/2

Q1 = (2+3)/2 = 5/2 = 2.5

48. Q2 = (highest item of the quartile 2 + lowest item of the quartile 3)/2

Q2 = (5+5)/2 = 5

49. Q3 = (highest item of the quartile 3 + lowest item of the quartile 4)/2

Q3 = (8+10)/2 = 18/2 = 9.

50. Q2 is the same as the median of the list.

51. For large dataset percentiles can be used.

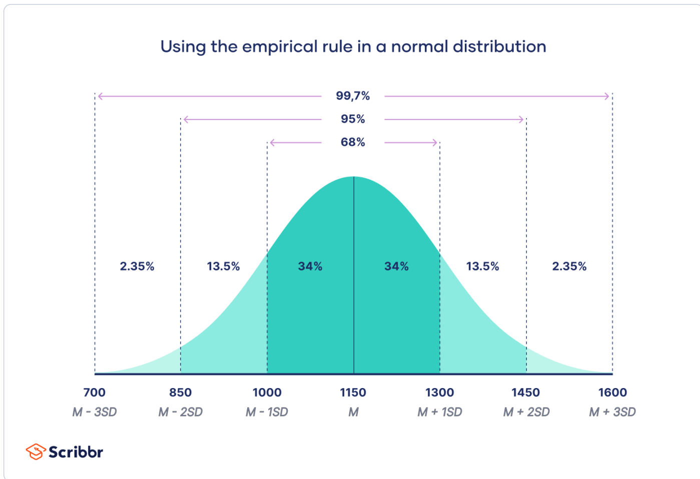
52. the 10 smallest items will be in percentile 1.

P1 = (10th and 11th smallest item)/2

53. P25 = Q1, P50 = Q2, P75 = Q3

54. P50 = Q2 = median.

55. Normal distribution : normal or gaussian distribution or bell curve is a continuous probability distribution that is symmetrical around its mean which is also its median and mode.



56. The mean is at the center and the curve is symmetric, meaning that the probabilities of events occurring to the left and right of the mean are equal.

57. Equal or approximately equal normal distribution has the following characteristics :

- The mean and median are equal or almost exactly equal.
- Data is exactly or almost exactly symmetric around the mean/median.

58. Z score = $(x - \mu)/\sigma$

59. Combinatorics :

60. Fundamental counting principle : if we need to make multiple separate decisions then multiply the number of ways to make each

individual decision to find the number of ways to make all the decisions.

E.g - make a sandwich with 1 type of bread out of 2 types, 1 type of filling out of 3 types.

Here total choices we have 2 type bread and 3 type fillings

So 2 * 3 = 6 combinations possible.

61. The number of plotting n distinct objects in order there are no restrictions is n!

N! = (n-1)x(n-2)----------3*2*1.

Here n counts the possible rearrangements of n.

62. Anagrams : it is a rearrangement of the letter in a word or phrase.

E.g - deductions is an anagram of discontinued.

Deductions is a meaningful word but it also could make the gibberish word “cddeinostu”.

63. If there are n distinct letters then n!

E.g - GRE 3 distinct letters so 3! = 3 x 2 x 1 = 6

64. Combinatorics with repetition :

E.g - 7 people 3 seats

1st seat → 1 among 7 people can seat.

2nd seat → 1 among 6 people can seat. 7*6*5 = 210

3rd seat → 1 among 5 people can seat.

7!/(7-3)! = 7*6*5*4*3*2*1 / 4*3*2*1

=> 210.

65. Permutation use case :
    - When arrangement matters.
    - Distinct items.
    - nPr = (n!)/(n-r)!

66. Combination use case :
    - Selection order is irrelevant.
    - Similar items.
    - nCr = n!/r! x (n-r)!

67. Multiple arrangements : if it is required to choose 2 or more sets of items from separated pools, count the arrangements separately. Then multiply the numbers of possibilities for each step.

    E.g - 3 senior need to be chosen from 12 seniors

    2 juniors need to be chosen from 11 juniors.

    As there is no condition during choosing and no selection order then we will use a combination formula.

    12!/3! * (12-3)! = 220

    11!/2! * (11-2)! = 55

    So ans = 220*55 = 12100

68. Probability : for events with countable outcome probability

    =>(no of desired or successful outcome)/(total number of possible outcomes) this assumes all outcomes are equally likely.

69. Probability of getting 5 after rolling a dice = $\frac{1}{6}$

Probability of prime number in a dice = 3/6 = $\frac{1}{2}$

Coin tossed 3 times probability of getting head twice exactly

=> $\frac{3}{8}$

70. The greatest probability of an event that will certainly occur is 1.

E.g - the probability of rolling a dice once and it lands on a number less than 7 = 6/6 = 1.

71. The lowest probability is 0. Which means an event won't occur.

E.g - a fair dice is rolled and outcome of number 9 = 0/6 = 0.

72. Probabilities can be expressed inclusive of 0 to 1, or fractions or 0% to 100%.

73. More than 1 event : and, or

74. Probability problems that deal with multiple events usually involve 2 operations: multiplication and addition.

75. For independent events we multiply

Probability of a coin flipped twice will land on heads both times.

1st time p(head) = $\frac{1}{2}$

2nd time p(head) = $\frac{1}{2}$

Thus $\frac{1}{2}$ x $\frac{1}{2}$ = $\frac{1}{4}$

Multiplication means lower probability.

76. When events are dependent we add their probability.

E.g - probability of a fair dice rolled once will land on either 4 or 5.

p(4) = ⅙

p(5) = ⅙

Thus p = ⅙ + ⅙ = 2/6 = ⅓

77. If events are not mutually exclusive then

p(X or Y) = p(X) + p(Y) - p(X & Y)

78. Probability of success and probability of failure = 1

79. If a problem contains phrases “at least” and “at most” then consider finding the probability that success doesn’t happen by finding failure probability (x). Then perform (1-x) would be our desired probability.

E.g - what is the probability that on 3 rolls of a single dice at least 1 of the rolls will be 6?

1st roll p(not 6) = ⅚

2nd roll p(not 6) = ⅚

3rd roll p(not 6) = ⅚

Thus p (not 6) = ⅚ x ⅚ x ⅚ = 125/216

Thus p(at least 1 six) = 1 - 125/216

=> 216-125/216 => 91/216.

80. Domino effect : in a box 10 blocks, 3 of them are red (no replacement scenario)

p(1st red) = 3/10

p(2nd red) = 2/9

Thus p(red) = 3/10 x 2/9 = 1/15

Manhattan 6 - Quantitative Comparison and Data Interpretation

1. While proving the 'D' option just plug in numbers like (-1, -0.5, 0, 0.5, 1).
2. Number less than the numerator in the denominator makes the number bigger. 3/2
3. Number greater than the numerator in the denominator makes the number smaller ½, ⅔ etc.
4. Any 2 sides of a triangle must be added up to greater than the 3rd side otherwise we can't connect all 3 sides.
5. When to plug in numbers :
    - Have 1 unique value (find x,y or something else)
    - Have a range of possible values (i.e - 3<z <2)
    - Have no constraints.
    - Be defined in terms of other variables.

      E.g - x-3=12   y+2x=40

      A = y   b=9

      X = 12+3 = 15

      Y+ 2*15 = 40

Y = 40-30

Thus y = 10.

So a = y=10, b = 9

So a>b.

6. If a variable has a defined range of values, test the upper and lower boundaries .

   2 <= z <=4

   Here upper 4 and lower 2

7. If a variable is defined in terms of another variable then simplify and find a direct comparison.

   E.g - a = 6x , b=5y

   x+5/5 = y+6/6

   => 6x+30 = 5y+30

   => 6x = 5y

   Thus a = b ©

8. If a variable has no constraints try to prove (D)

   - Another way to prove (D) is to check negative possibilities.

     E.g - a = x/2, b=2x

     Let x = 1

     So a= ½ = .5 , b = 2*1 = 2 (a<b)

     Let x = -1

     A = -½ = -.5, b = 2*-1 = -2 (a>b)

So undetermined then answer is (D)

9. If a variable has a certain property then try to prove (D).

   E.g - x is positive

   $x(x+1)$ b) $x(x^2+1)$

   $\Rightarrow x^2 + x$ $\Rightarrow x^3+x$

   Let $x = 2$

   $x^2 = 2^2 = 4$ $x^3=2^3=8$ $(a<b)$

   Let $x = 1$

   $x^2 = 1^2 = 1$ $x^3=1^3=1$ $(a=b)$

   Let $x = .5$

   $x^2 = .5^2$ $x^3=.5^3=8$ $(a<b)$

   $\Rightarrow 5^2 \times 10^{-2}$ $\Rightarrow 5^3 \times 10^{-3}$

   $\Rightarrow 25 \times 10^{-2}$ $\Rightarrow 125 \times 10^{-3}$

   $\Rightarrow .25$ $\Rightarrow .125$ $(a>b)$

   So undetermined (D).

10. When squaring a fraction or while powering up the fraction the value decreases.

11. Strategies to follow :

    - Has unique value (e.g - $x+3 = -5$)

      Solve fo the value of the variable

      $X+3 = -5$

      Thus $X = -8$

- Has a defined range (e.g - $-4 \le w < 3$)

  Test the boundaries - test upper value in this scenario (3)

  Test the lower value, here (4)

- Has a relationship with another variable e.g - $2p=r$

  Simplify the equation and make a direct comparison of the variables

  $2p=r$

  $p=r/2$

  R is equal to 2 p's

  P is equal to half of r.

- Has no constraints

  Try to prove (D)

- Has specific properties (e.g x is negative)

  Try to prove (D)

12. Quadratic equations :
13. When quadratic equations appear in 1 or both quantities ()() cross multiply or "FOIL" it down and try to eliminate common terms and compare the quantities with a question mark inequality sign (?).

    $Pq \neq 0$

    a) $(2p+q)(p+2q)$ b) $p^2+5pq+q^2$

    $\Rightarrow 2p^2 + 4pq+pq+2q^2$

    $\Rightarrow 2p^2 + 4pq+pq+2q^2$? $p^2+5pq+q^2$

$=> 2p^2 + 5pq + 2q^2 \quad ? \quad p^2 + 5pq + q^2$

$=> 2p^2 + 2q^2 \quad ? \quad p^2 + q^2$

$=> 2(p^2 + q^2) \quad ? \quad p^2 + q^2$

Thus $2(p^2 + q^2) > p^2 + q^2$

14. Has specific properties or conditions try to prove (D) (-1,-.5,0,.5,1) or negative positive value.
15. When the common information contains a quadratic equation, solve for both possible values and put them into the quantities.
16. If a quadratic appears in one or both quantities :
    - Foil or cross multiply.
    - Eliminate common terms.
    - Compare quantities
    - E.g a) $(x+y)(x-y)$ b) $(y+x)(y-x)$
17. If a quadratic appears in the common information
    - Factor the equation and find both solution
    - Plug both solutions into quantities.
    - E.g → $x^2 + 5x + 6=0$
18. If a quantitative comparison question with a strange symbol contains numbers, plug in the numbers and evaluate the formula. If the question doesn't contain numbers plug the given variable directly into the formula.

19. When absolute values contain a variable, minimize the absolute value by making the expression inside as far away from ‘0’ as possible. Add positive to positives or negative to negatives.

20. Sometimes inequalities are used to order variables from least to greatest.

   - Use the invisible inequality (?).
   - Eliminate common terms.
   - Try to discern a pattern if one is present.

   E.g - 0<p<q<r

   a) P+q   b) q+r

   P+q ? q+r

   => p ? r

   => p<r   so a<b as p<r

21. If the denominator is bigger than the numerator then fraction value will be less than 1. (proper fraction) ½

22. When multiplying complex fractions -

   - Split the numerator when the denominator is one term
   - Turn division into multiplication by reciprocal
   - As the positive denominator gets larger the fraction value gets smaller.

   E.g - (2+2/3x)/2

   => 2/2 + (2/3x)/2

=> $1 + 2/3x * ½$

=> $1+ 1/3x$

23. When a fraction contains exponents and you have to plug in numbers for the exponents, always plug in 0 and 1 first to save some time.

24. When dealing with percents, always pay attention to the size of the original value. Thus 20% of a small number is less than 20% of a larger number.

    E.g - let, original price 100

    Discount 20% so = 100 - (100 x 20%)

    => 100 - (100 x 20/100)

    => 100 - 20 = 80

    Now adding charge of 20%

    Thus new price = 80 + (80 x 20%)

    => 80 + (80 x 20/100)

    => 80 + 16 = 96

25. If a triangle has 2 same size angles then it is an isosceles triangle and the side opposite of these 2 angles are equal.

26. Area of a triangle = ½ *base*height.

27. When an angle gets bigger and bigger its opposite edge also gets bigger and bigger and vice versa.

28. Tips :

- Do not trust the diagram
- Establish what you know from the given question
- Establish what you do not know from the given question
- Establish what you need to know from the given question

29. Exterior angle is equal to the sum of 2 non adjacent interior angles.
30. When in geometry word problems reference specific dimensions (length, width, radius) but do not provide actual numbers, using numbers is a viable strategy.
31. If $pq > 0$
    - Either pq both positive
    - Or both negative.
32. When exponent value is even then $(-x)^{even}$ power becomes positive.
33. When the product of more than 1 variable is either greater than or less than 0, consider all possible signs and test all possible scenarios.
    - If $xy>0 \rightarrow x,y$ both positive or both negative.
    - If $xy<0 \rightarrow$ either x positive , y negative or x negative, y positive.
34. Negative number raised to odd power gives a negative result.
35. Negative number raised to even power gives a positive result.
36. Even x even = even, even * odd = even, odd * odd = odd.
37. Number greater than 1 gets bigger when we raise them to a higher power. $2^2 = 4 < 2^3 = 8$

38. The Numbers between 0 and 1 get smaller when we raise them to higher power. $(\frac{1}{2})^2 = \frac{1}{4} > (\frac{1}{2})^3 = 1/8$

39. When questions involve variables and exponents try to prove D. try using numbers greater than 1 and numbers between 0 and 1.

40. To compare the sum or products of sets of consecutive integers eliminate overlap in order to make a direct comparison.

41. Whenever you see word problems make sure you have enough information you need before doing any computation. If you don't have enough information then answer is (D)

42. For the ratios they do not provide information about actual values. To try to prove (D) on a ratio problem -

    - Choose one scenario in which the actual values are the same values as the ratios.
    - Choose another scenario in which the numbers are much larger (pick numbers that are easy to work with)

43. In any question that involves 2 groups that have some kind of average value, use the principle of weighted average.

44. If 2 groups have an equal number of members, the total average will be the average of the 2 groups.

45. If any group has more members the total average will be closer to the average of that group. Group with the highest member gets priority

46. Distance = rate * time

47. Data interpretation solving procedure :

- Scan the graph
- Type of graph
- Percentage or absolute value
- Have any overall total value.

48. Figure out what is asking in the question

49. Find the graphs with needed information.

50. Column chart :

- Shows amounts as heights
- Typically the x axis represents time.
- Shows trends over time.
- Use a piece of paper or even a finger to make a straight edge.
- Approximate percentage of sales increase from april to may
  => (may sales - april sales)/april sales.

51. Stacked column chart -

- Where 2 components are indicated in 1 bar.

52. Clustered column chart - any given amount is a greater percentage of a small number then it is of a larger number.

53. Formula of fruit sales in december (any certain period of time)
  => % fruit sale * total sales in december.

54. Line charts -

- Know the percent increase or decrease formula
- % increase = (new -old)/old
- Try visual eliminations before performing calculations.

55. Gross profit = sales revenue - cost.
56. Sometimes it is easier to calculate the percentage that does not satisfy a condition rather than calculate a percentage directly.