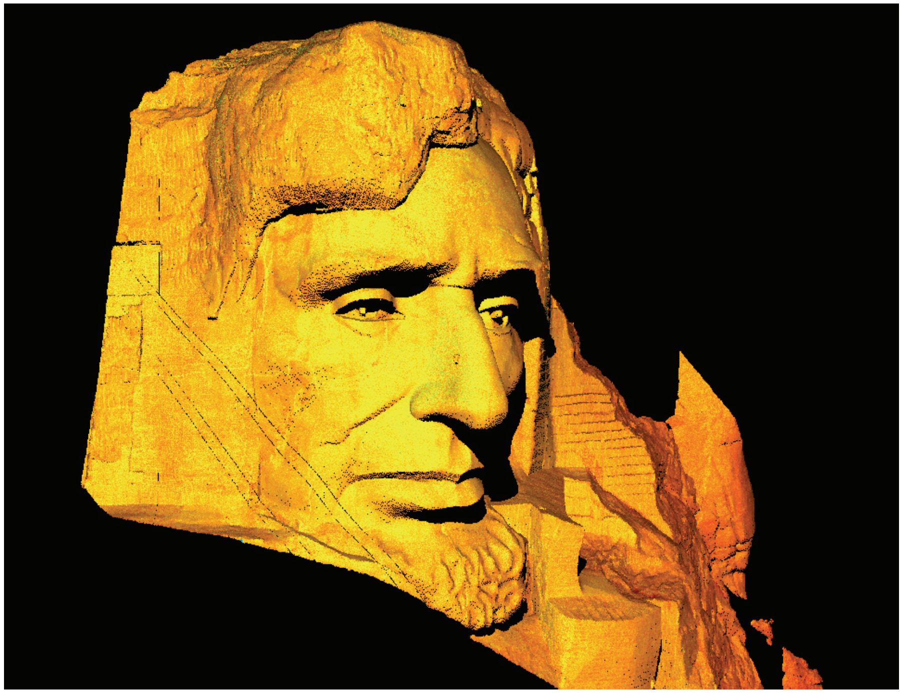
Surface Reconstruction: Part I

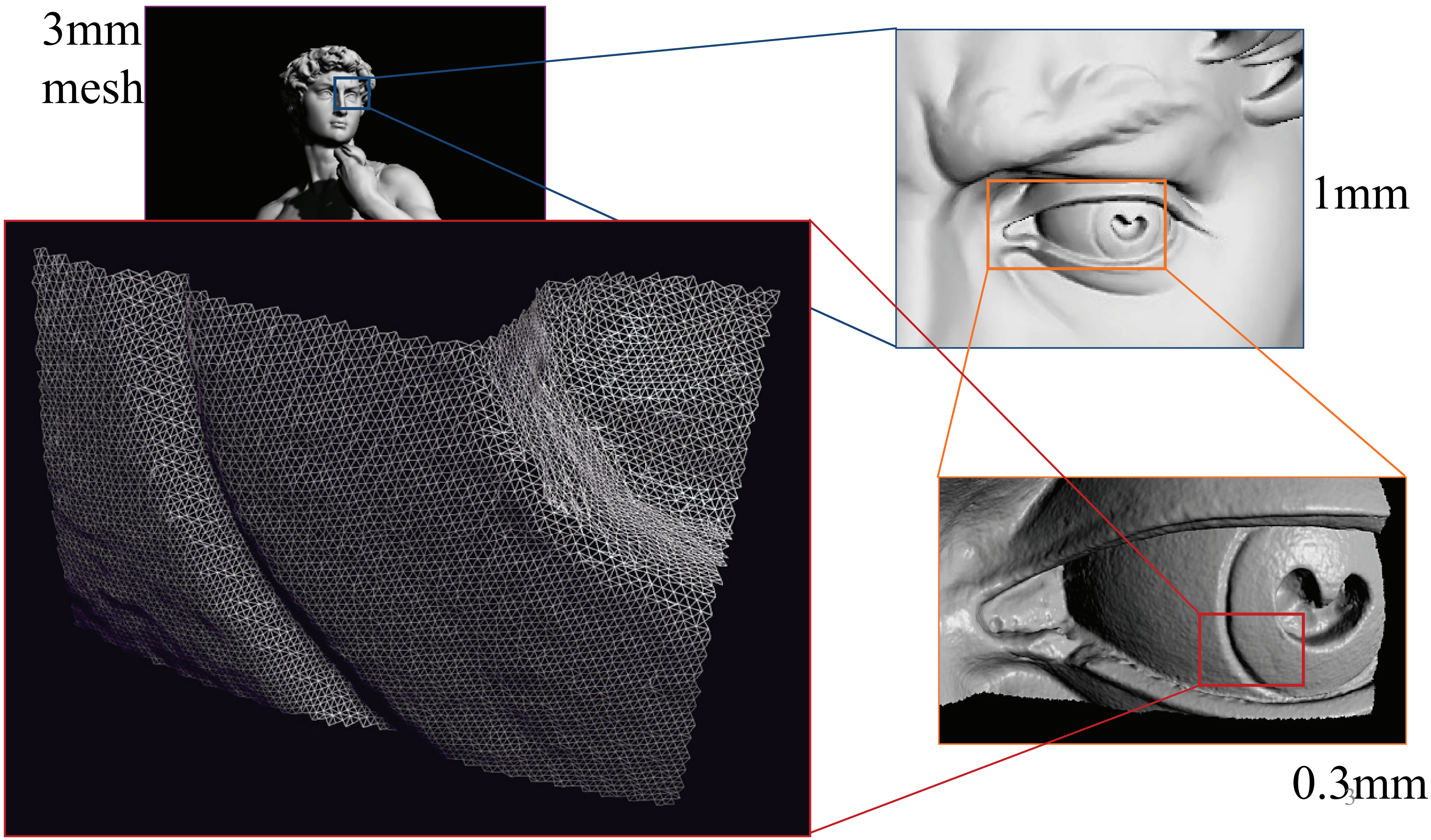


(Stanford's) Digital Michelangelo Project

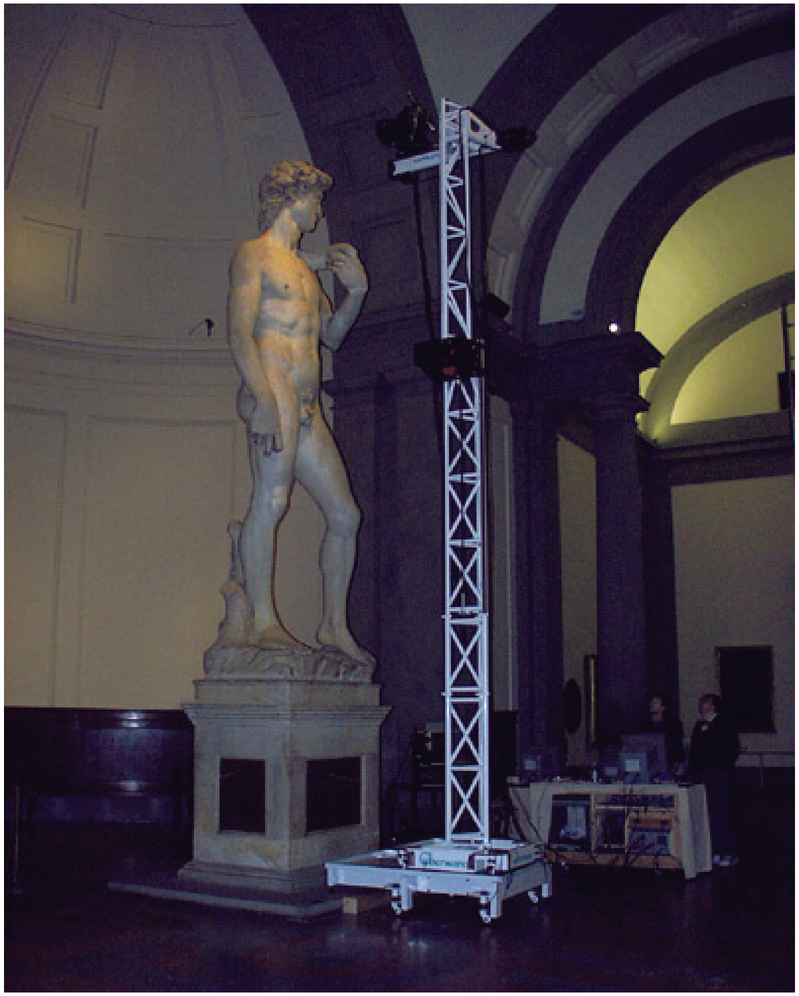
1G sample points → 8M triangles

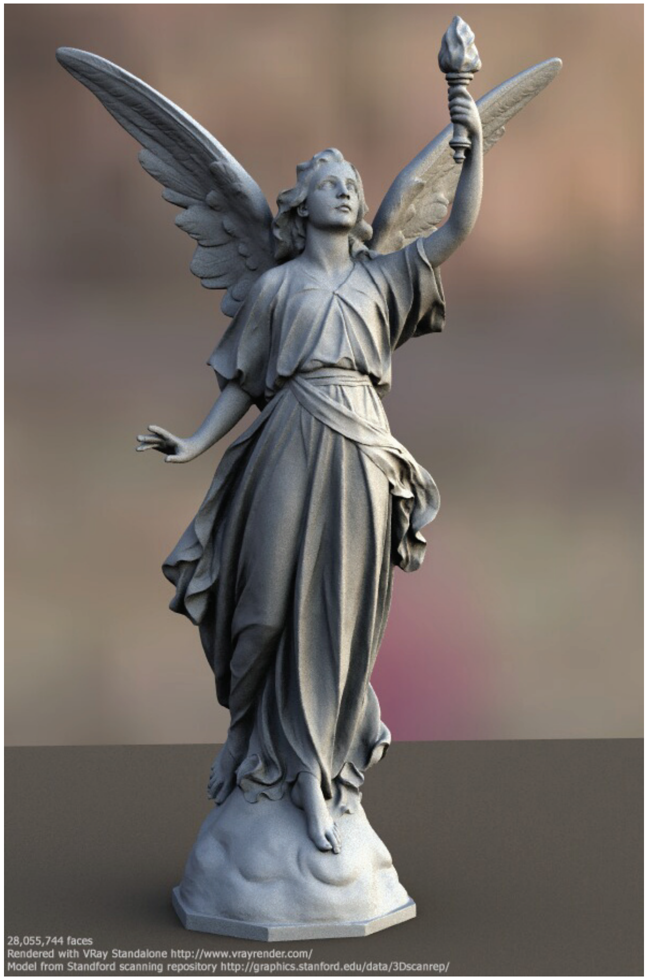
4G sample points → 8M triangles

(Stanford's) Digital Michelangelo Project

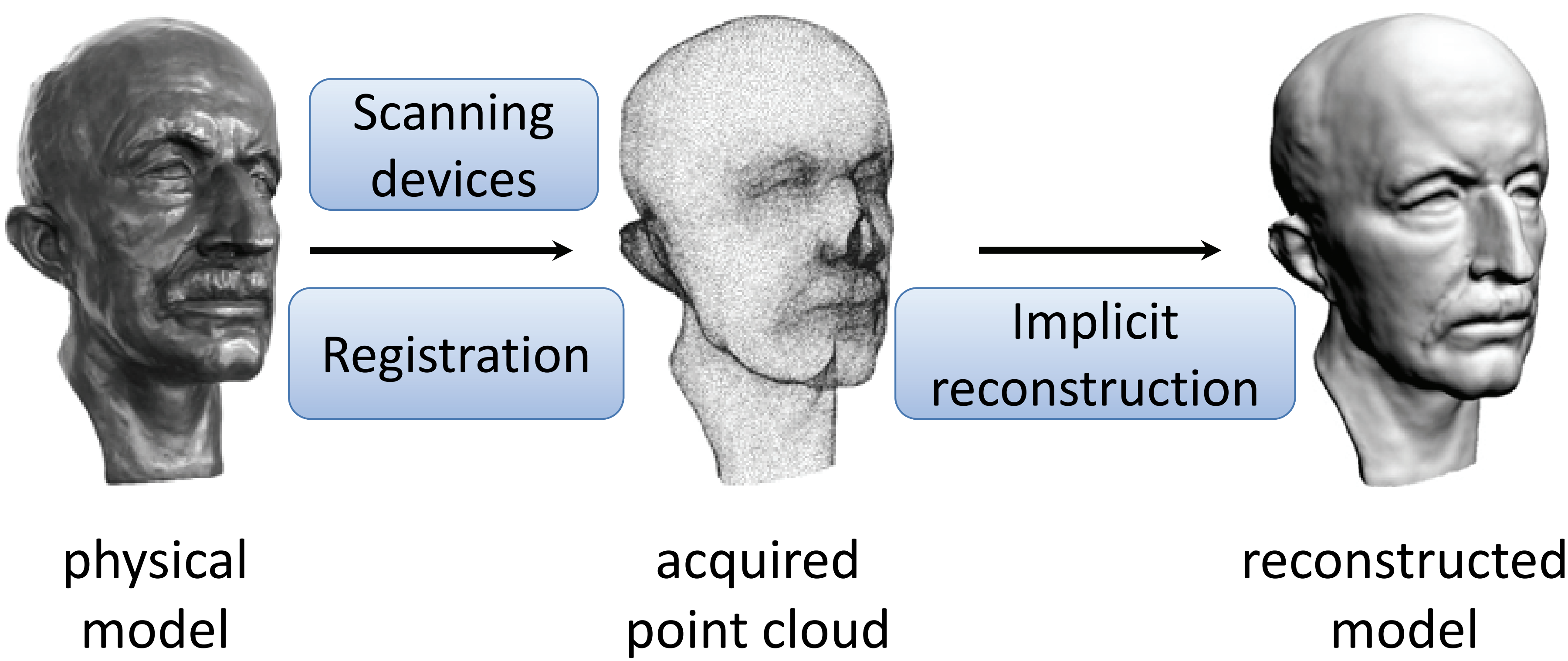


Local Sightseeing





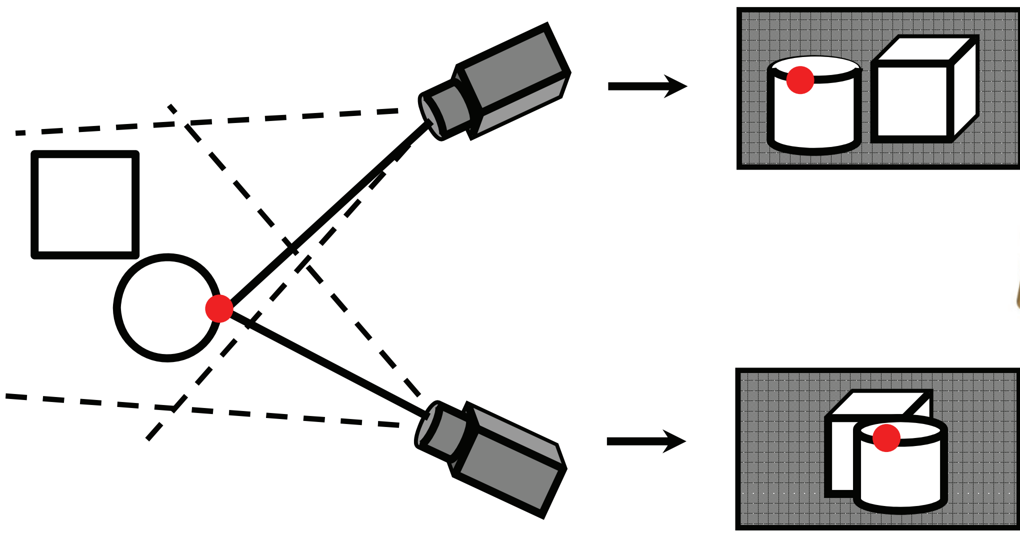
Surface Reconstruction



Range Scanning Systems

Passive: Stereo matching

Find and match features in both images

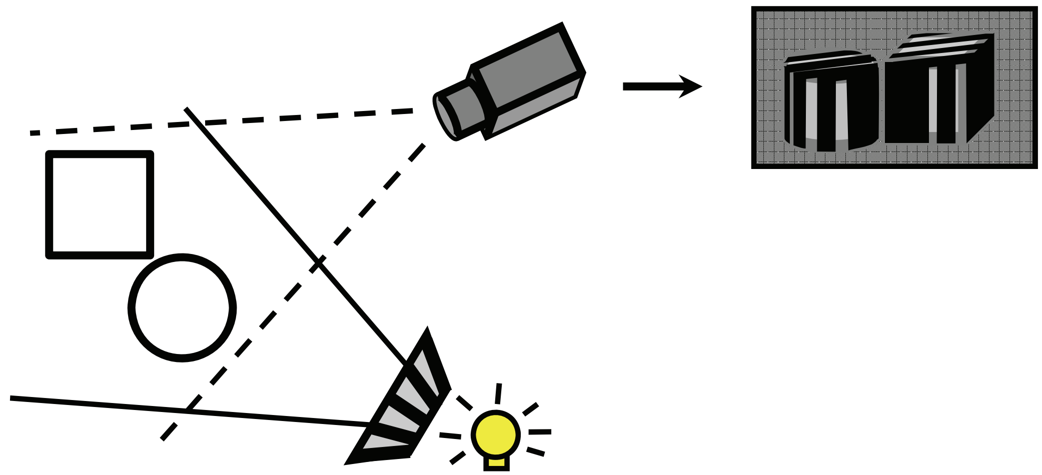


Problem: Needs features to match

Range Scanning Systems

Active: Structured light

Project special b/w patterns to identify pixels



Problematic for materials / textures having strong color differences.

Range Scanning Systems

Active: Laser scanning, time-of-flight

Send laser pulse, time how long it takes to return

$$r = \frac{1}{2} c \Delta t$$

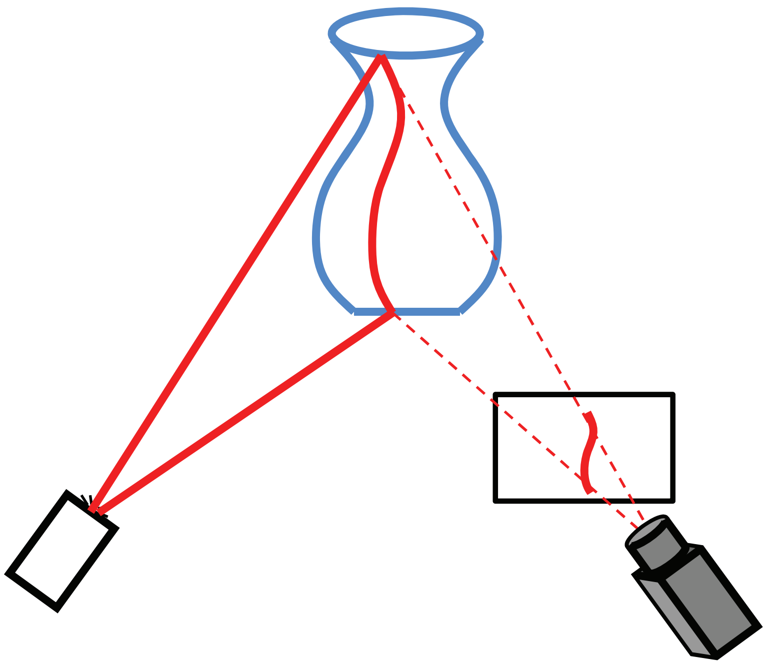
Accuracy depends on how precisely can measure time

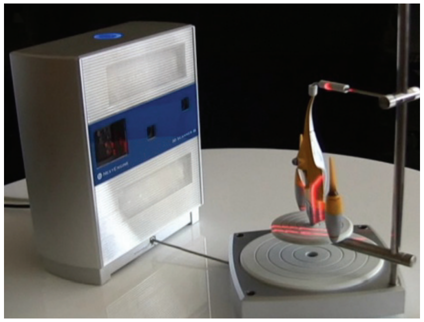
Operates on large distances – good for buildings, large scale objects

Range Scanning Systems

Active: Laser scanning triangulation

Sweep laser, record where pixel intensity is max.





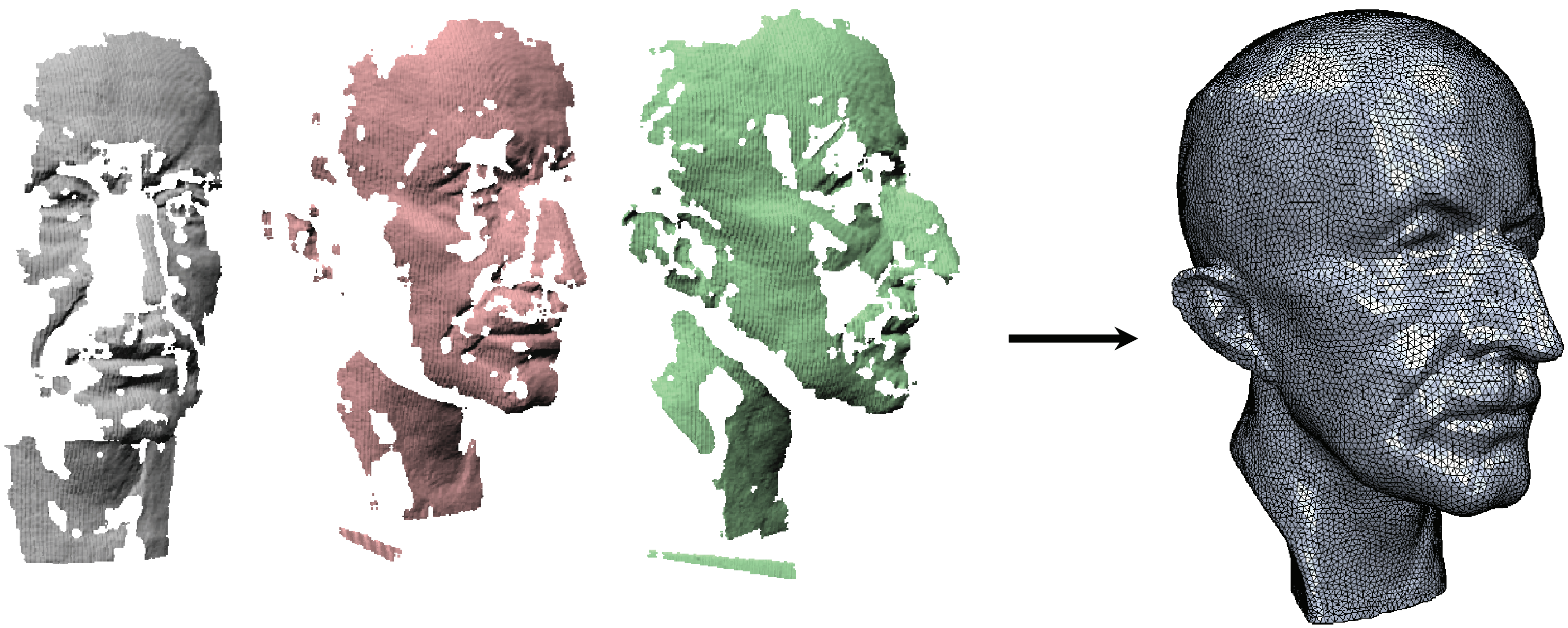
Problematic for difficult reflectance properties (highly specular, hair)
Line of sight required to camera and laser

High accuracy

Range Scans

- Scanning generates multiple range images
- Each contain 3D points for different parts of the model in local coordinates of the scanner
- Need registration to one point cloud

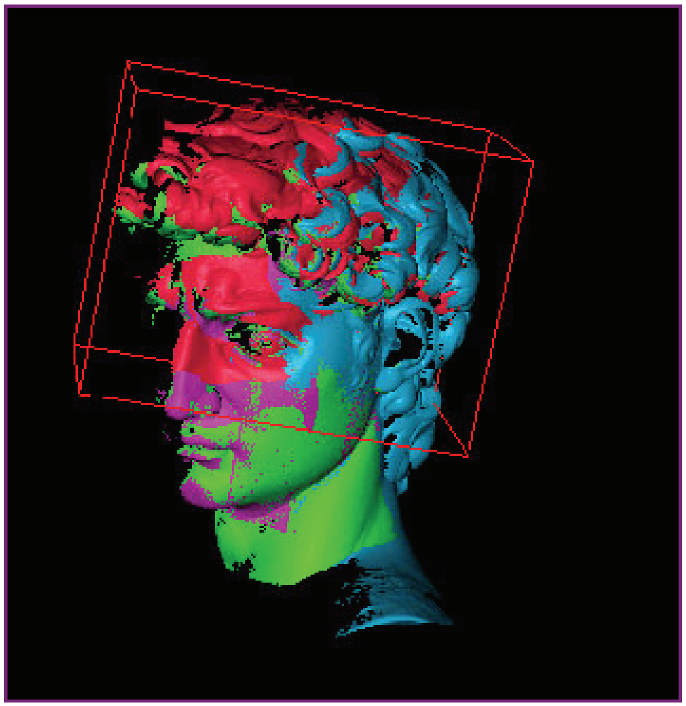
Goal



set of raw scans

reconstructed model

Range Processing Pipeline

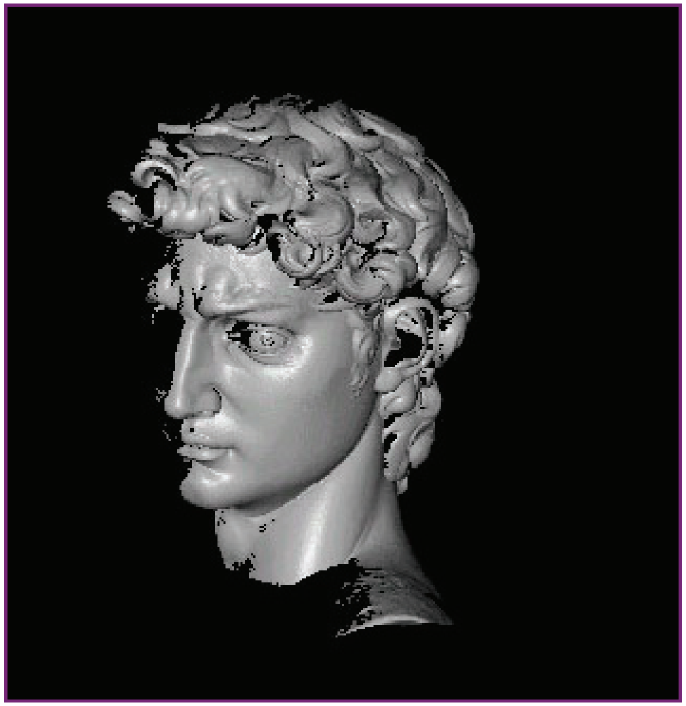


Steps

1. Initial registration
2. Pairwise registration
3. Global relaxation to spread out error
4. Generate surface

Slides from: http://www.cs.princeton.edu/~bjbrown/iccv05_course/

Range Processing Pipeline

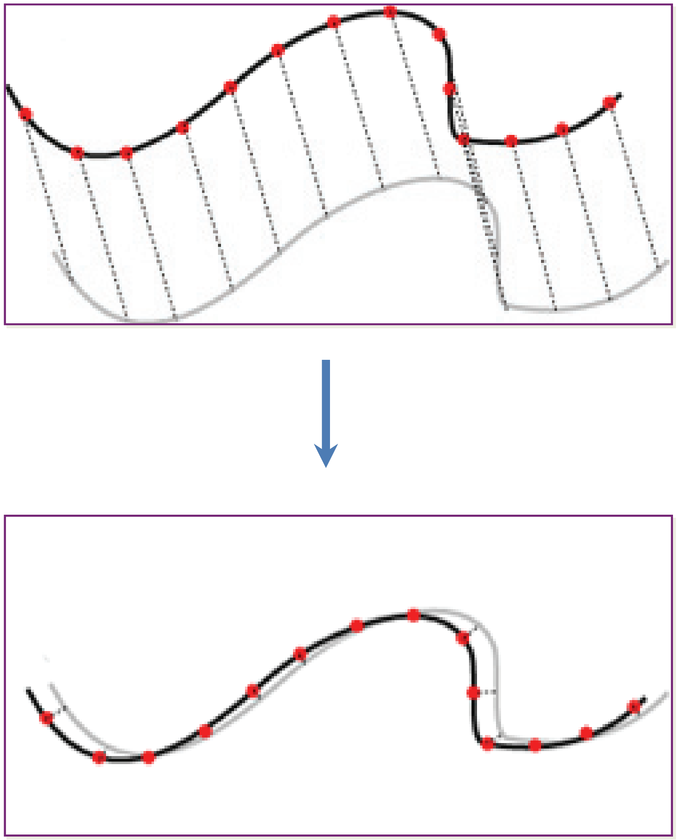


Steps

1. Initial registration
2. Pairwise registration
3. Global relaxation to spread out error
4. Generate surface

Slides from: http://www.cs.princeton.edu/~bjbrown/iccv05_course/

Range Processing Pipeline

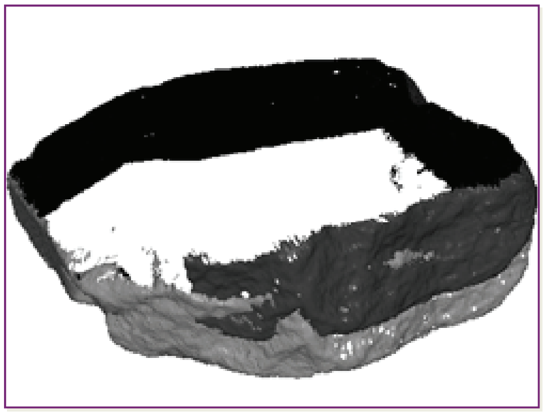


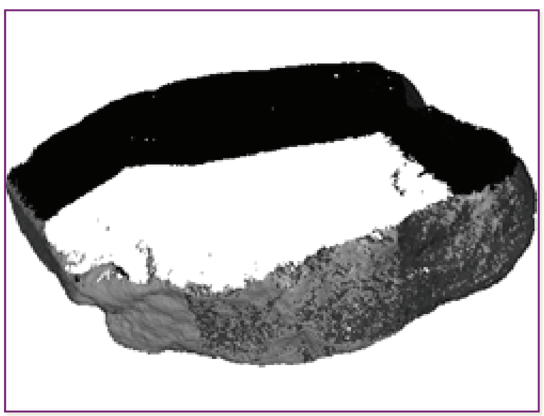
Steps

1. Initial registration
2. Pairwise registration
3. Global relaxation to spread out error
4. Generate surface

Slides from: http://www.cs.princeton.eu/~bjbrown/iccv05_course/

Range Processing Pipeline



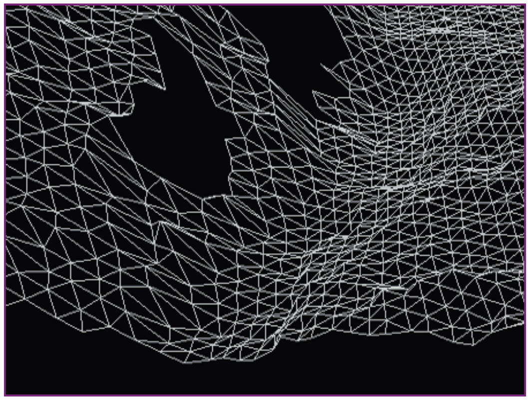


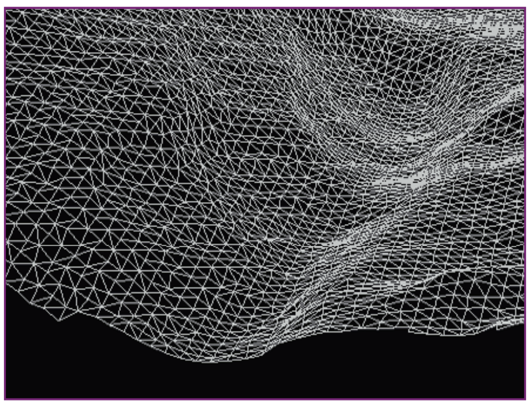
Steps

1. Initial registration
2. Pairwise registration
3. Global relaxation to spread out error
4. Generate surface

Slides from: http://www.cs.princeton.edu/~bjbrown/iccv05_course/

Range Processing Pipeline





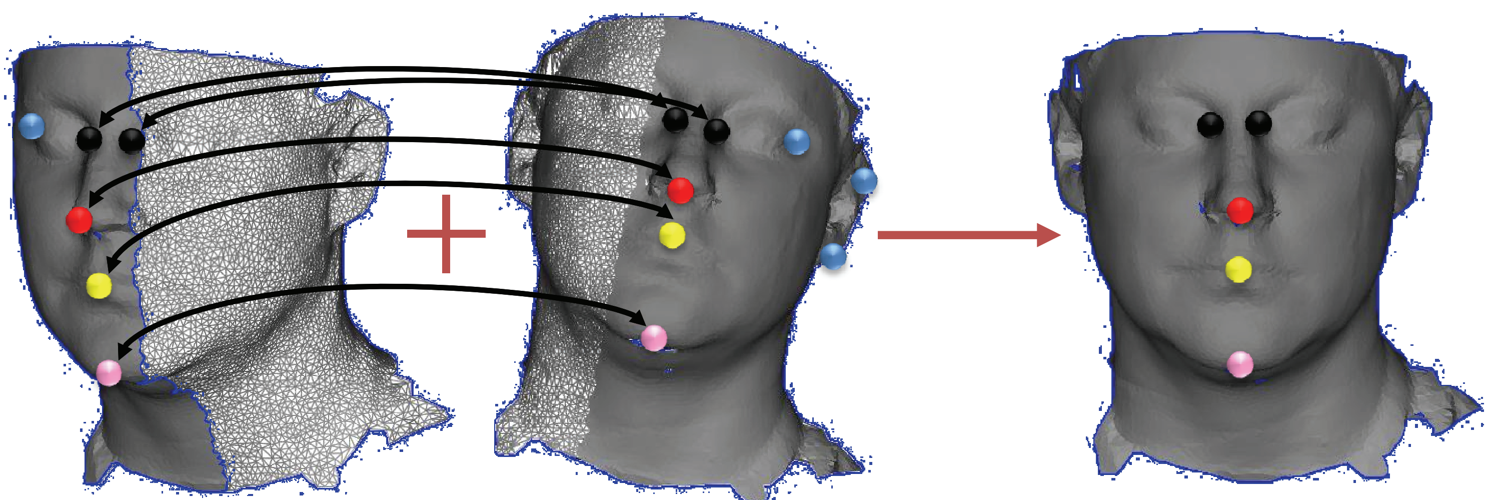
Steps

1. Initial registration
2. Pairwise registration
3. Global relaxation to spread out error
4. Generate surface

Slides from: http://www.cs.princeton.edu/~bjbrown/iccv05_course/

Initial Registration

- Options:
  - Scans are calibrated – scan on a turntable
  - Manual feature selection, global coarse alignment
  - Automatic feature selection and matching

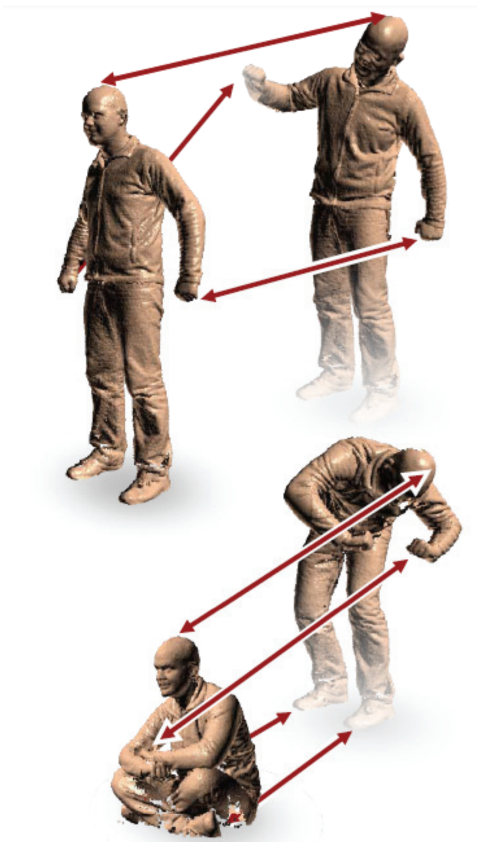


Partially Overlapping Scans

Aligned Scans

Slides from: http://www.cs.princeton.edu/~bjbrown/iccv05_course/

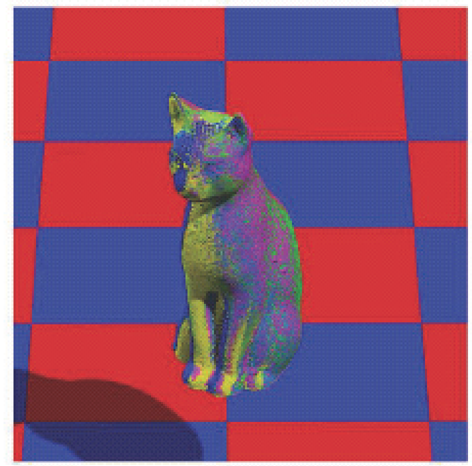
Non-Rigid Registration

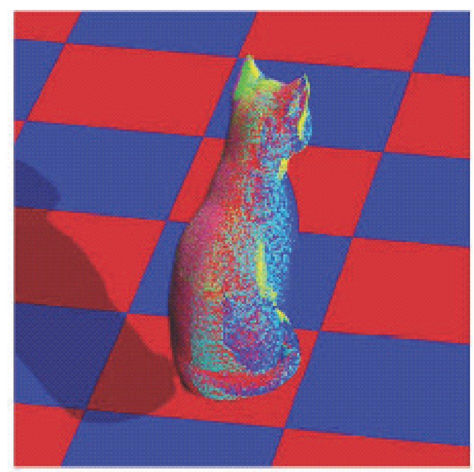


Data courtesy of C. Stoll, MPI Informatik

Video

Pairwise Rigid Registration

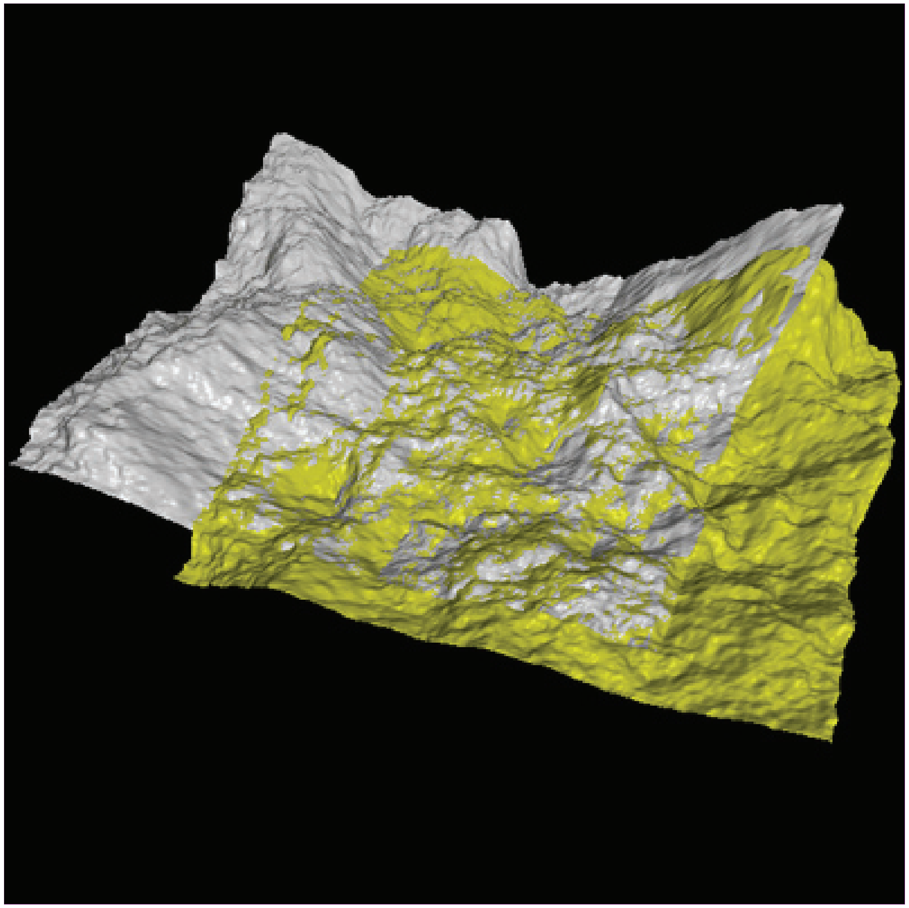




Images from: "Geometry and convergence analysis of algorithms for registration of 3D shapes" by Pottman

Pairwise Rigid Registration Goal

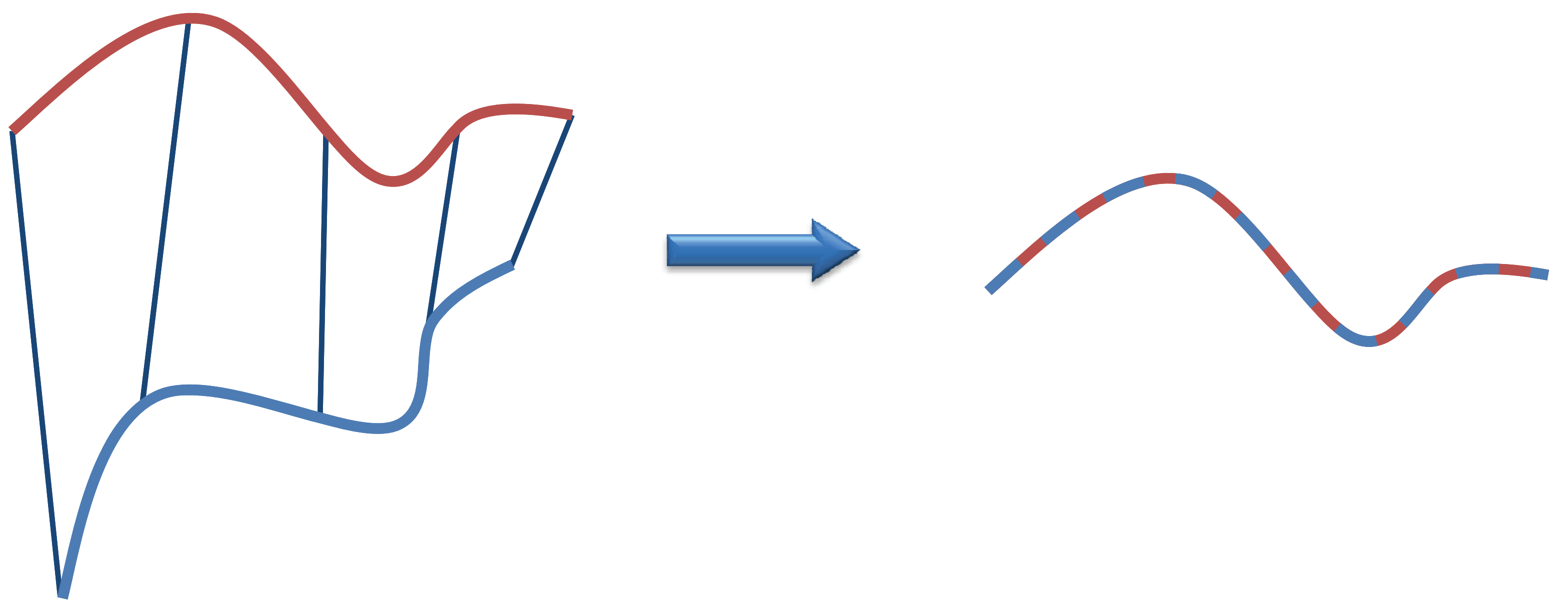
Align two partially-
overlapping point clouds
given initial guess
for relative transform



Slides from: http://www.cs.princeton.edu/~bjbrown/iccv05_course/

Aligning 3D Data

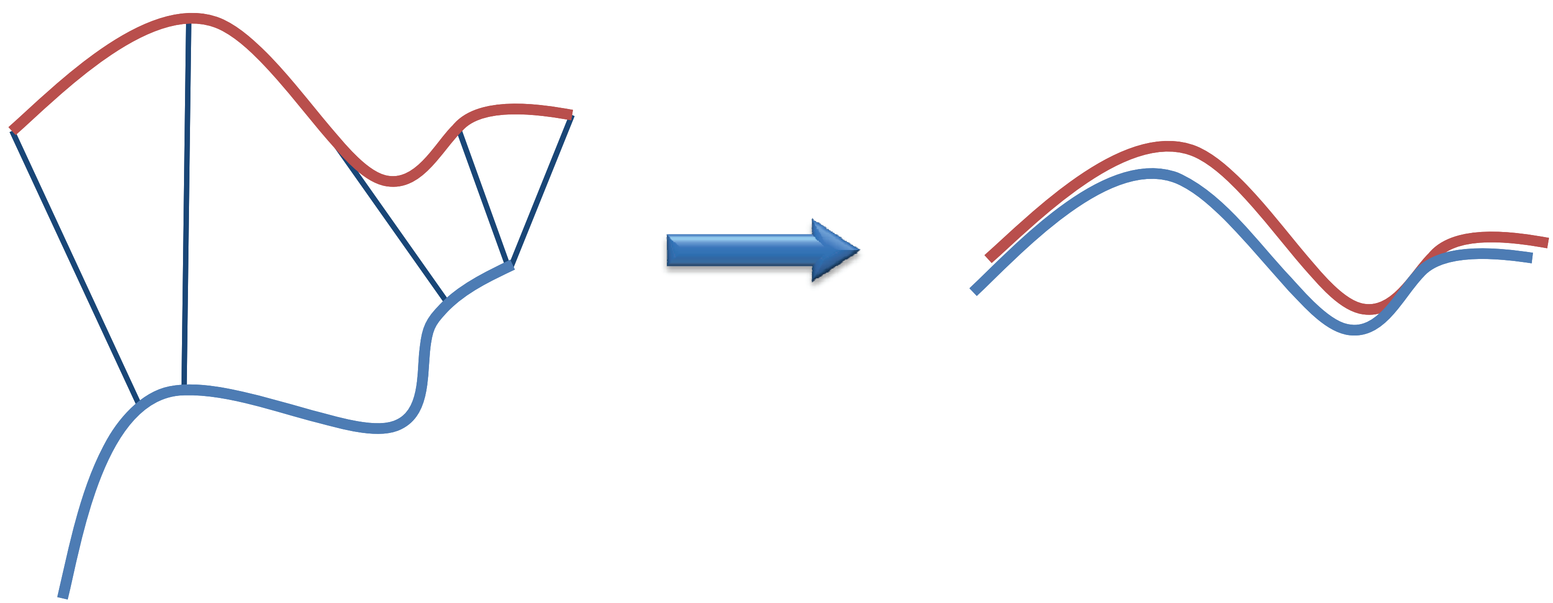
If correct correspondences are known, can find correct relative rotation/translation



Slides from: http://www.cs.princeton.edu/~bjbrown/iccv05_course/

Aligning 3D Data

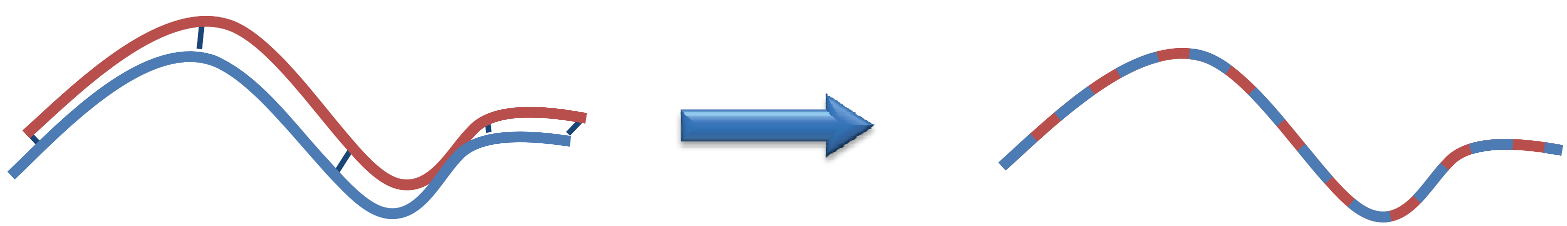
- How to find correspondences: User input? Feature detection? Signatures?
- Alternative: assume closest points correspond



Slides from: http://www.cs.princeton.edu/~bjbrown/iccv05_course/

Aligning 3D Data

- … and iterate to find alignment
  - Iterative Closest Points (ICP) [Besl & McKay 92]
- Converges if starting position "close enough"



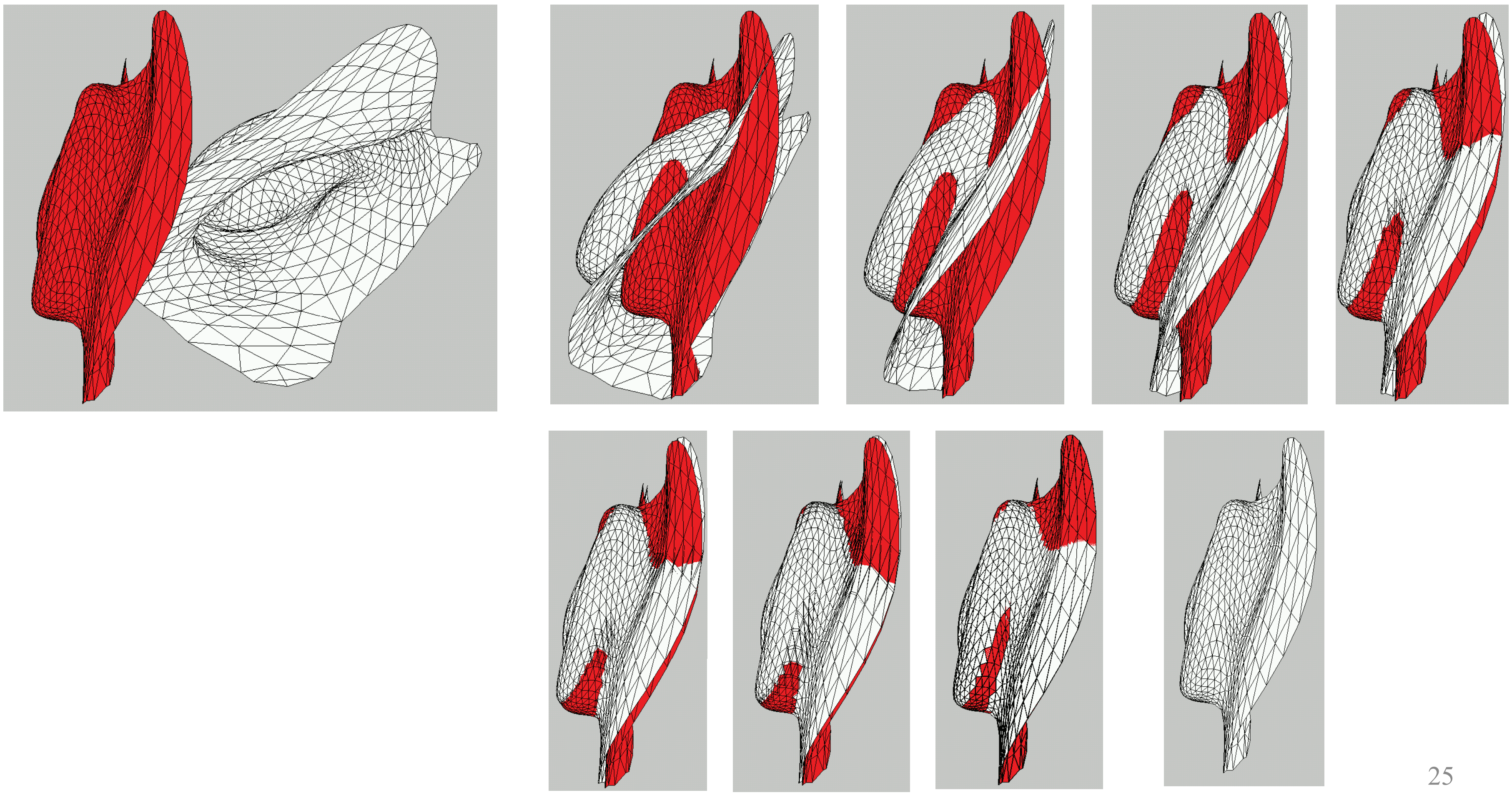
Slides from: http://www.cs.princeton.edu/~bjbrown/iccv05_course/

Iterative Closest Point (ICP)

- Given two scans *P* and *Q*.
- Iterate:
  1. Find some pairs of closest points $(\mathbf{p}_i, \mathbf{q}_i)$
  2. Find rotation $\mathbf{R}$ and translation $\mathbf{t}$ to minimize

$$\min_{\mathbf{R},\mathbf{t}} \sum_i \|\mathbf{p}_i - \mathbf{R}\mathbf{q}_i - \mathbf{t}\|^2$$

Example



Finding t

- Define barycentered point sets
- Optimal translation vector **t** maps barycenters onto each other

$$\begin{aligned} \bar{\mathbf{p}} &:= \frac{1}{m}\sum_{i=1}^{m}\mathbf{p}_i & \bar{\mathbf{q}} &:= \frac{1}{m}\sum_{i=1}^{m}\mathbf{q}_i \\ \hat{\mathbf{p}}_i &:= \mathbf{p}_i - \bar{\mathbf{p}} & \hat{\mathbf{q}}_i &:= \mathbf{q}_i - \bar{\mathbf{q}} \end{aligned}$$

$$\mathbf{t} = \bar{\mathbf{p}} - \mathbf{R}\bar{\mathbf{q}}$$

- Proof [Horn 88]

Finding $\mathbf{R}$

- In matrix notation

Frobenius norm

$$\hat{\mathbf{P}} = \begin{pmatrix} \hat{\mathbf{p}}_1^T \\ \dots \\ \hat{\mathbf{p}}_m^T \end{pmatrix}_{m\times 3}, \hat{\mathbf{Q}} = \begin{pmatrix} \hat{\mathbf{q}}_1^T \\ \dots \\ \hat{\mathbf{q}}_m^T \end{pmatrix}_{m\times 3} \qquad \left\|A_{n\times m}\right\|_F^2 = \sum_{i=1}^{m}\sum_{j=1}^{n} A_{ij}^2$$

- The problem:

$$\min_{\mathbf{R}} \left\|\hat{\mathbf{P}}\mathbf{R} - \hat{\mathbf{Q}}\right\|_F^2 \quad s.t. \quad \mathbf{R}^T\mathbf{R} = \mathbf{I}$$

- Equivalent to finding rotation matrix $\mathbf{R}$ closest to: $\hat{\mathbf{P}}^T\hat{\mathbf{Q}}$

Orthogonal Procrustes Problem

Finding $\mathbf{R}$

- Compute matrix

$$\mathbf{S} = \hat{\mathbf{P}}^T \hat{\mathbf{Q}}$$

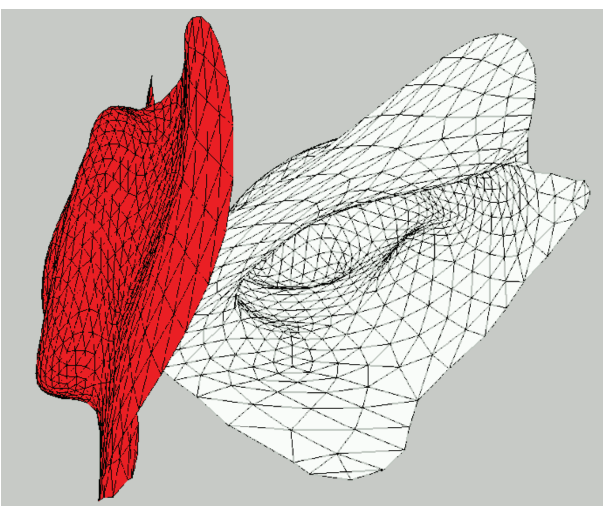
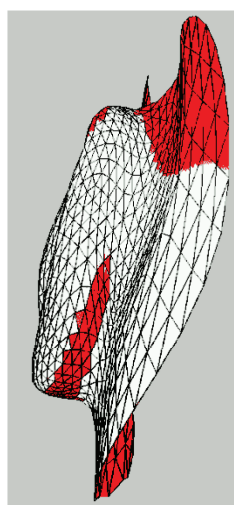
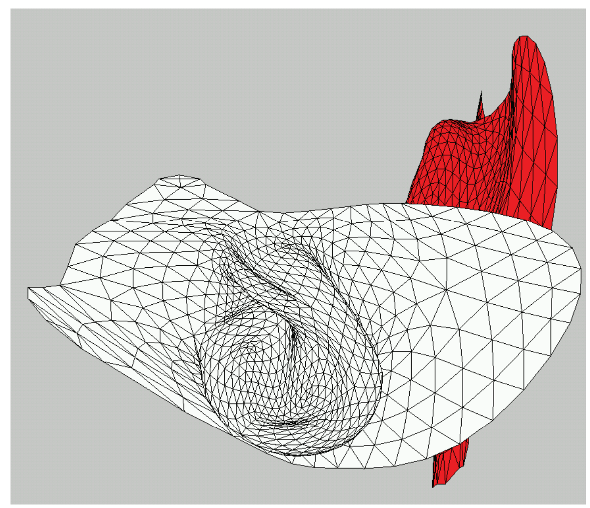
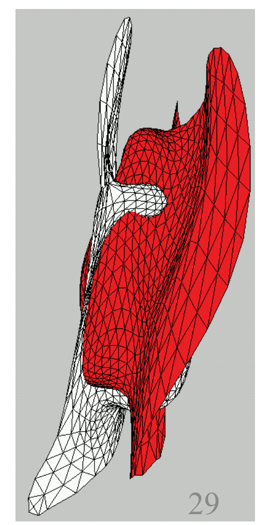
- Singular value decomposition (SVD) extracts rotation from $\mathbf{S}$

$$\mathbf{S} = \mathbf{U\Sigma V}^T \longrightarrow \mathbf{R} = \mathbf{UV}^T$$

[Schonemann ’66 - “A generalized solution of the orthogonal Procrustes problem”]

Converges?

- Errors decrease monotonically [Besl & McKay 92]
- Converges to local minimum
- Good initial guess → Converges to global minimum

ICP Variants

Variants on the following stages of ICP have been proposed:

1. Selecting source points (from one or both meshes)
2. Matching to points in the other mesh
3. Weighting the correspondences
4. Rejecting certain (outlier) point pairs
5. Assigning an error metric to the current transform
6. Minimizing the error metric w.r.t. transformation

Slides from: http://www.cs.princeton.edu/~bjbrown/iccv05_course/

ICP Variants

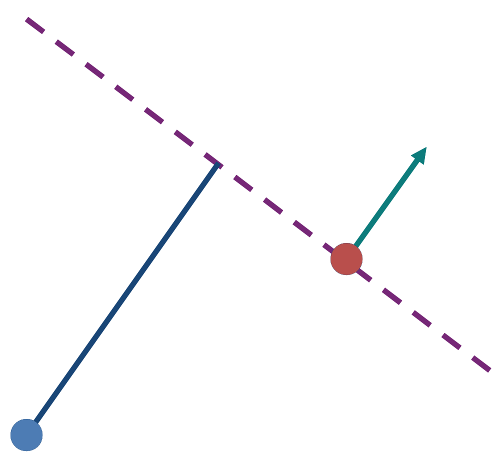
Variants on the following stages of ICP have been proposed:

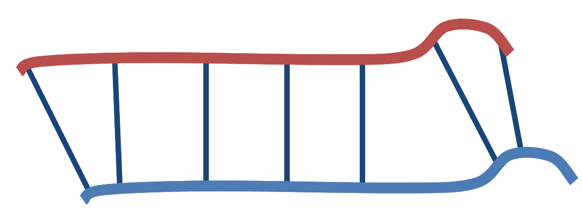
1. Selecting source points (from one or both meshes)
2. Matching to points in the other mesh
3. Weighting the correspondences
4. Rejecting certain (outlier) point pairs
5. Assigning an error metric to the current transform
6. Minimizing the error metric w.r.t. transformation

Slides from: http://www.cs.princeton.edu/~bjbrown/iccv05_course/

Point-to-Plane Error Metric

Using point-to-plane distance instead of point-to-point lets flat regions slide along each other
[Chen & Medioni 91]





Slides from: http://www.cs.princeton.edu/~bjbrown/iccv05_course/

Point-to-Plane Error Metric

- Error function:

$$E = \sum \left((\mathbf{R}\mathbf{p}_i + \mathbf{t} - \mathbf{q}_i) \cdot \boxed{\mathbf{n}_i} \right)^2$$

- Linearize (assume rotation is small)

$$\mathbf{R} = e^{\mathbf{H}} = 1 + \mathbf{H} + \cancel{\frac{1}{2}\mathbf{H}^2 + \ldots}$$

Skew-Symmetric Matrix

$$\mathbf{H} = \begin{pmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{pmatrix}, \quad \mathbf{r} = \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix}$$

$$\mathbf{H}\mathbf{p}_i = \mathbf{r} \times \mathbf{p}_i$$

Point-to-Plane Error Metric

$$E = \sum \left((\mathbf{R}\mathbf{p}_i + \mathbf{t} - \mathbf{q}_i) \cdot \mathbf{n}_i \right)^2$$

$$\mathbf{R}\mathbf{p}_i \approx \left(\mathbf{I} + \mathbf{H} \right) \mathbf{p}_i = \mathbf{p}_i + \mathbf{r} \times \mathbf{p}_i$$

Result: over-constrained linear system

$$E \approx \sum \left((\mathbf{p}_i - \mathbf{q}_i) \cdot \mathbf{n}_i + \mathbf{r} \cdot (\mathbf{p}_i \times \mathbf{n}_i) + \mathbf{t} \cdot \mathbf{n}_i \right)^2 \quad , \quad \mathbf{r} = \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix}$$

Point-to-Plane Error Metric

- Over-constrained linear system

$$\mathbf{A}\mathbf{x} = \mathbf{b},$$

$$\mathbf{A} = \begin{pmatrix} \leftarrow & \mathbf{p}_1 \times \mathbf{n}_1 & \rightarrow & \leftarrow & \mathbf{n}_1 & \rightarrow \\ \leftarrow & \mathbf{p}_2 \times \mathbf{n}_2 & \rightarrow & \leftarrow & \mathbf{n}_2 & \rightarrow \\ & \vdots & & & \vdots & \end{pmatrix}, \qquad \mathbf{x} = \begin{pmatrix} r_x \\ r_y \\ r_z \\ t_x \\ t_y \\ t_z \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} -(\mathbf{p}_1 - \mathbf{q}_1) \cdot \mathbf{n}_1 \\ -(\mathbf{p}_2 - \mathbf{q}_2) \cdot \mathbf{n}_2 \\ \vdots \end{pmatrix}$$

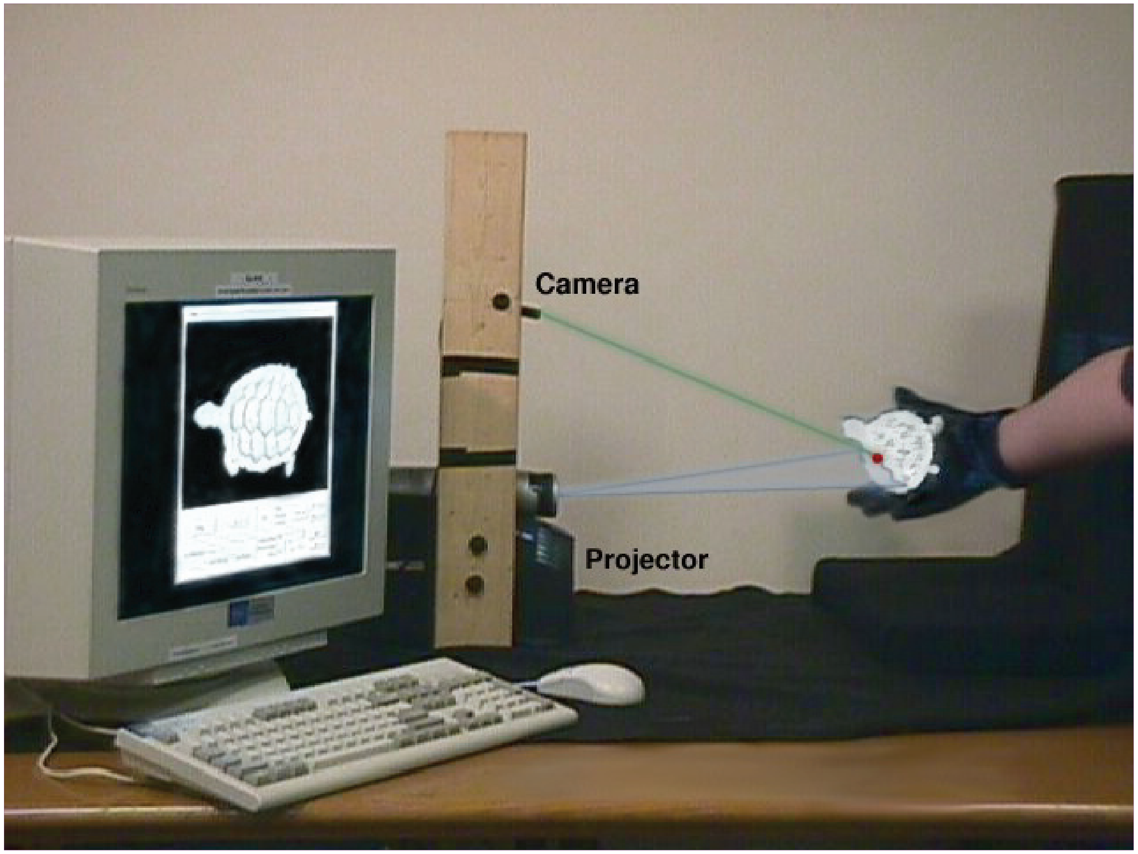
- Solve using least squares

$$\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x} = \mathbf{A}^{\mathrm{T}}\mathbf{b}$$

$$\mathbf{x} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b}$$

Slides from: http://www.cs.princeton.edu/~bjbrown/iccv05_course/

Real-time Example



Slides from: http://www.cs.princeton.edu/~bjbrown/iccv05_course/

Photograph

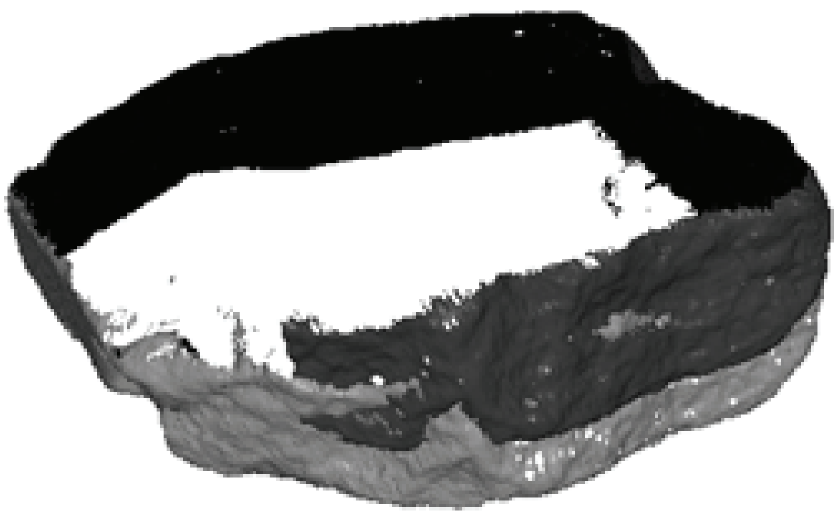
Slides from: http://www.cs.princeton.edu/~bjbrown/iccv05_course/

Real-Time Result

Slides from: http://www.cs.princeton.edu/~bjbrown/iccv05_course/

Global Registration Goal

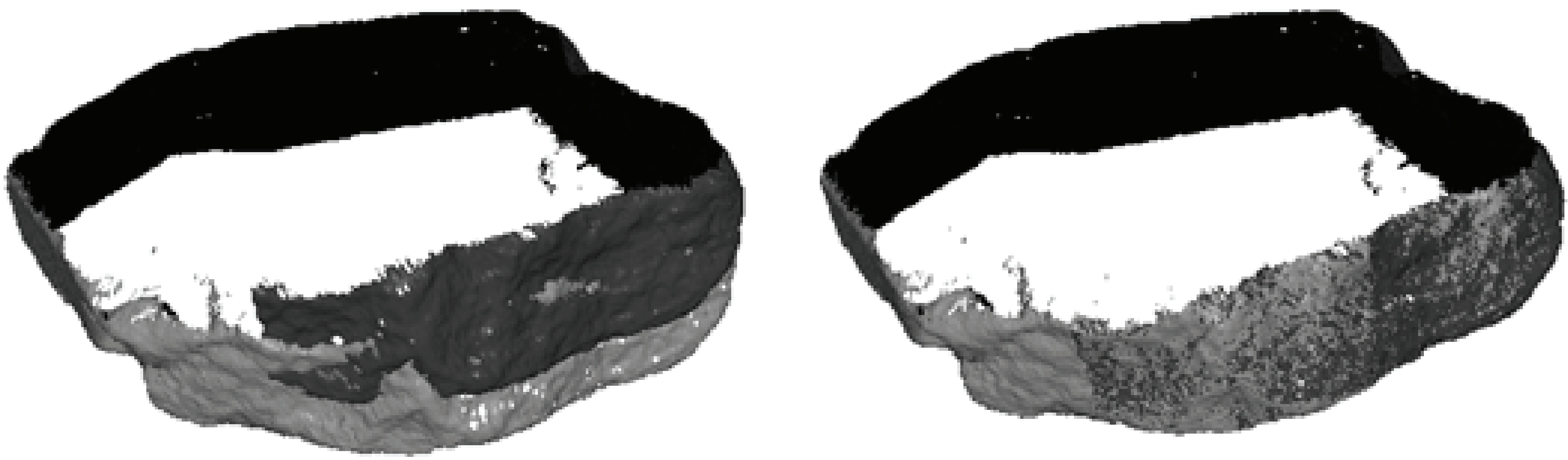
- Given: *n* scans around an object
- Goal: align them all
- First attempt: ICP each scan to one other



Slides from: http://www.cs.princeton.edu/~bjbrown/iccv05_course/

Global Registration Goal

- Want method for distributing accumulated error among all scans



Approach #1: Avoid the Problem

- In some cases, have 1 (possibly low-resolution) scan that covers most of surface
- Align all other scans to this “anchor” [Turk 94]
- Disadvantage: not always practical to obtain anchor scan

Approach #2: The Greedy Solution

- Align each new scan to all previous scans [Masuda 96]
- Disadvantages:
  - Order dependent
  - Doesn’t spread out error

Slides from: http://www.cs.princeton.edu/~bjbrown/iccv05_course/

Approach #3: The Brute-Force Solution

- While not converged:
  - For each scan:
    - For each point:
      - For every other scan
        - » Find closest point
  - Minimize error w.r.t. transforms of all scans
- Disadvantage:
  - Solve $(6n)\times(6n)$ matrix equation, where n is number of scans

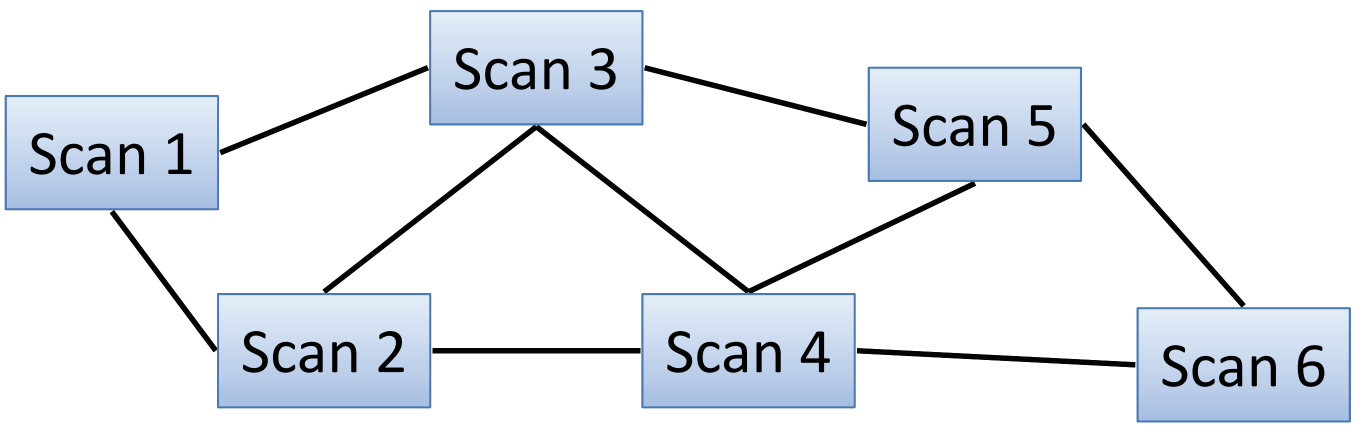
Slides from: http://www.cs.princeton.edu/~bjbrown/iccv05_course/

Approach #3a: Slightly Less Brute-Force

- While not converged:
  - For each scan:
    - For each point:
      - For every other scan
        - » Find closest point
    - Minimize error w.r.t. transform of this scan
- Faster than previous method (matrices are 6×6) [Bergevin 96, Benjemaa 97]

Graph Methods

- Speedup previous by creating a graph of pairwise alignments between overlapping scans



- Find transformations consistent as possible with all pairwise ICP

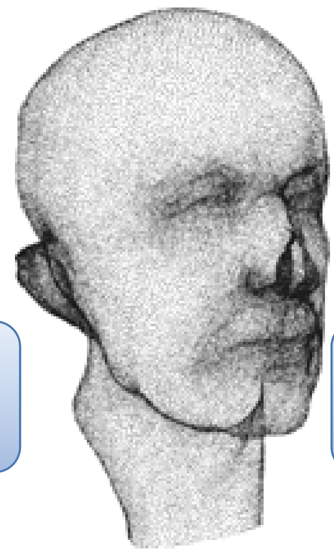
Slides from: http://www.cs.princeton.edu/~bjbrown/iccv05_course/

Surface Reconstruction

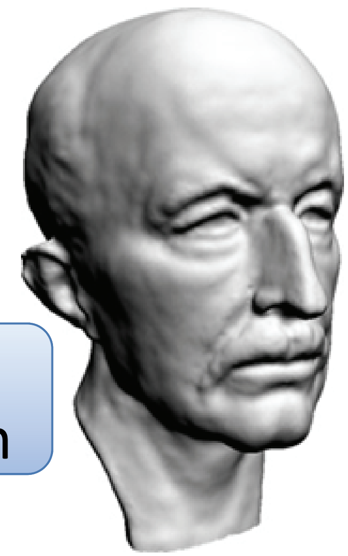
Next time:

Scanning devices

Registration



Implicit reconstruction



physical model

acquired point cloud

reconstructed model

References

- “The 3D Model Acquisition Pipeline”, Bernardini et al, ‘02
- “A method for registration of 3-D shapes”, Besl et al., ’92 ← ICP paper
- “Surface reconstruction from unorganized points”, Hoppe et al., ’92
- “Closed-form solution of absolute orientation using orthonormal matrices”, Horn et al., ‘88
- “A general solution of the orthogonal Procrustes problem”, Schönemann ’66