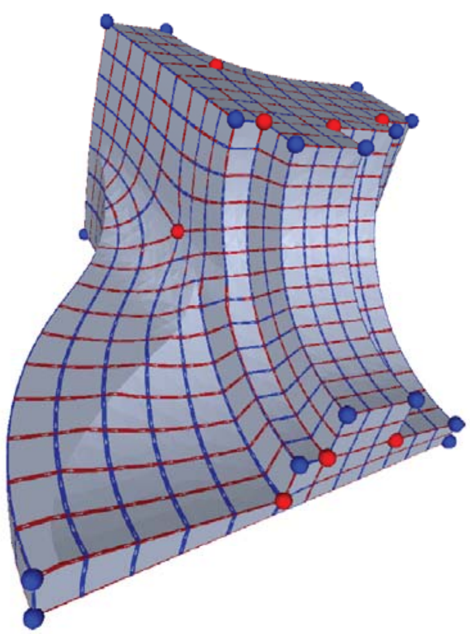
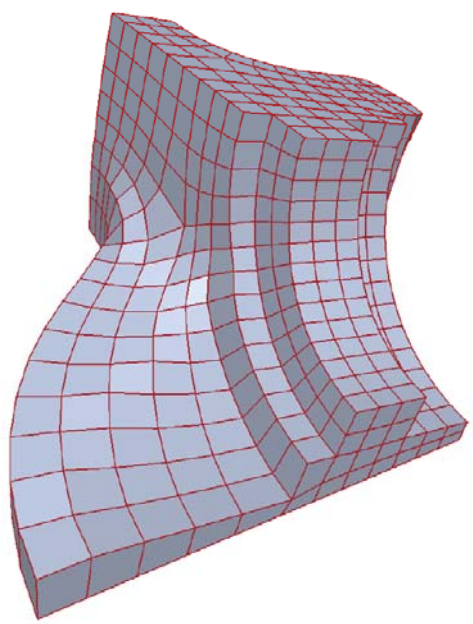
Remeshing II

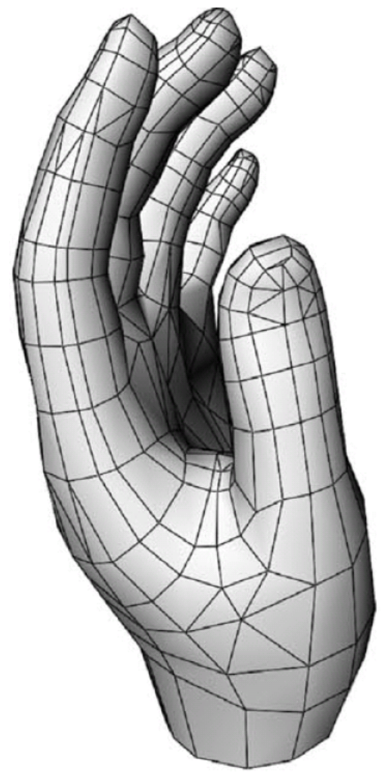




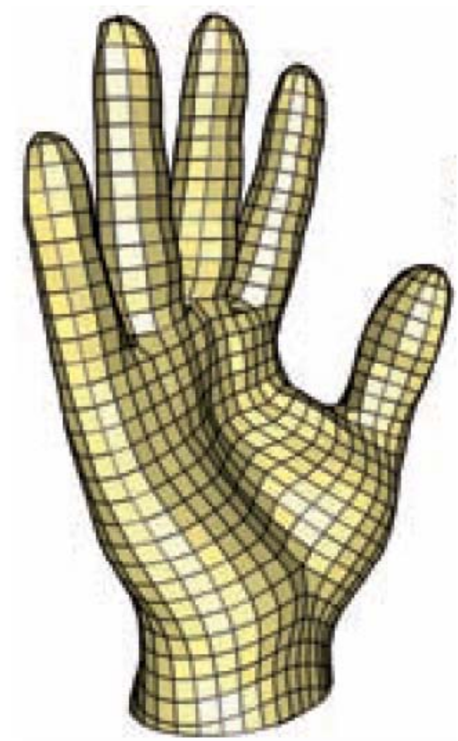
Quad Remeshing

Generate a quad (or quad-dominant) mesh that approximates the input

Input



Quad Dominant Mesh



Quad Mesh

Applications

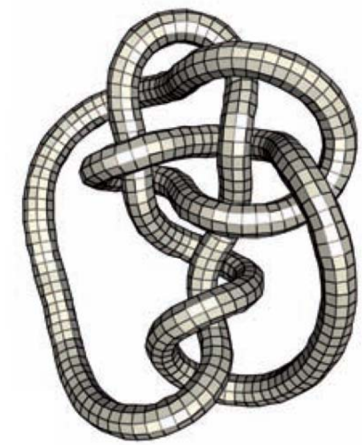
- Fitting B-spline surfaces
- Simulation
- Subdivision
- Modeling
- Architecture

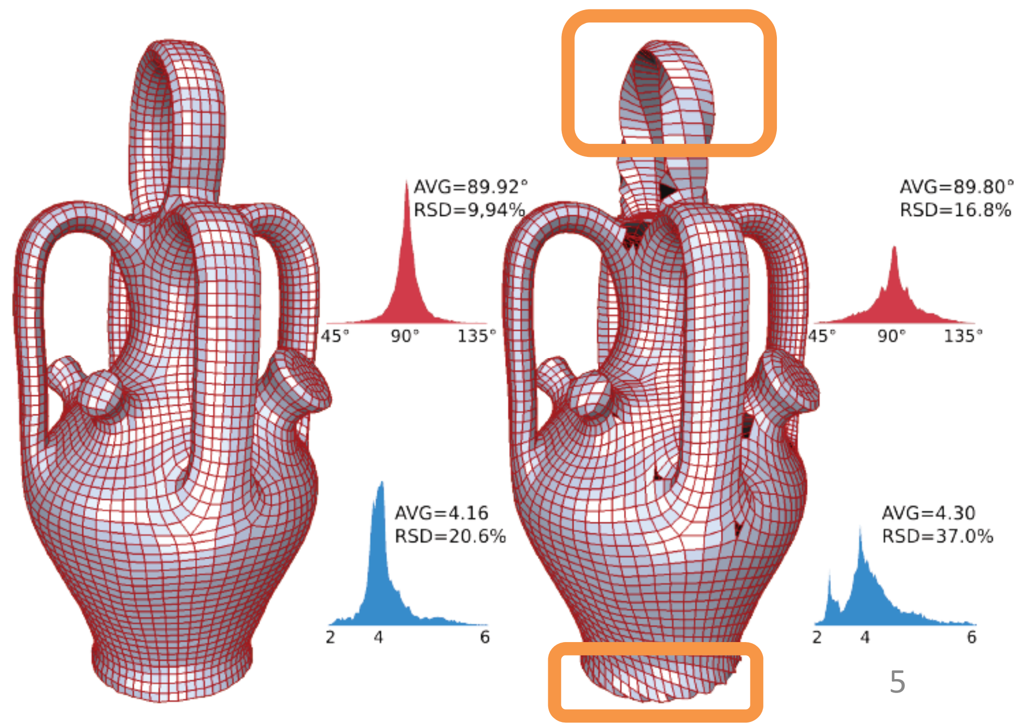
Requirements

- Lengths/angles distribution
- Orthogonality
- Alignment with curvature directions
- Regularity
- Planarity

Alignment with Curvature Directions

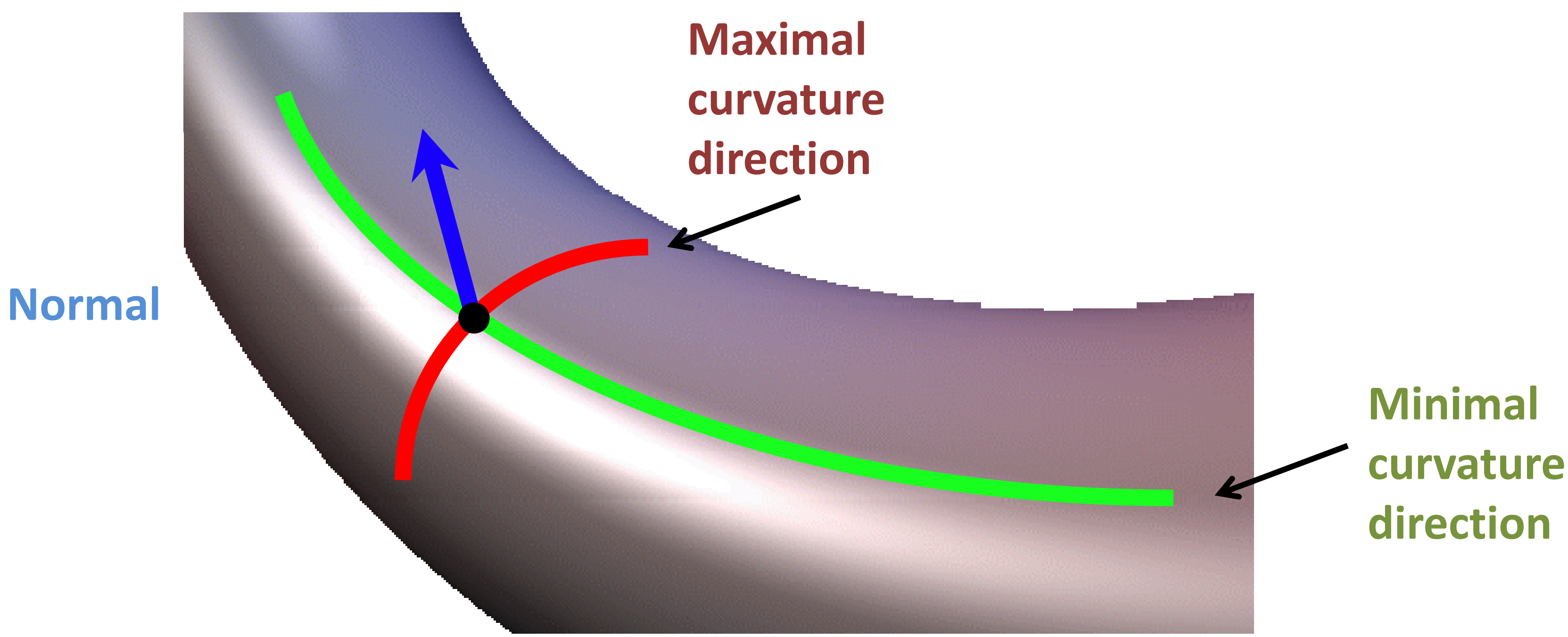
- More important than for triangular remeshing
  - Visible artifacts even for medium resolution
  - Aligned quads are closer to planar



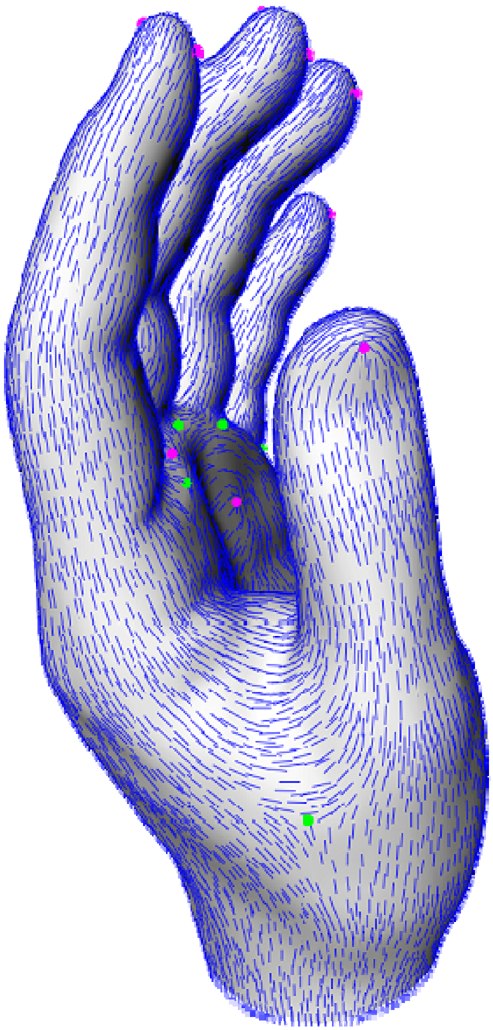


Curvature Directions

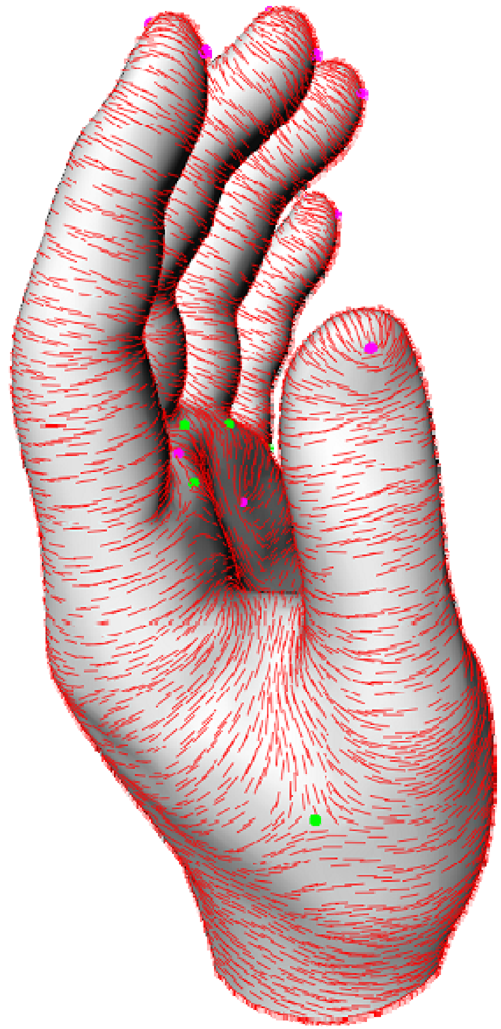
Second fundamental form defines local orthogonal frame



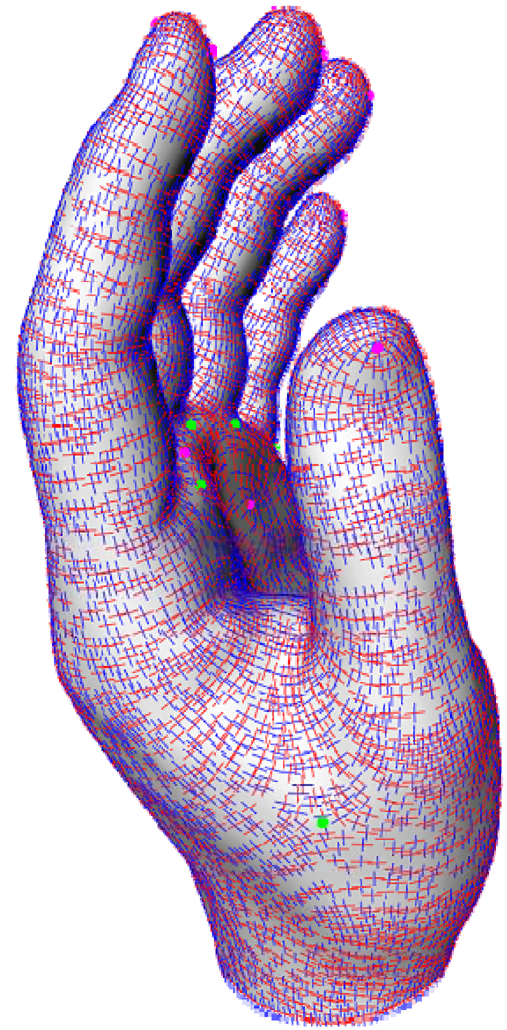
Cross Fields



Min curvature



Max curvature

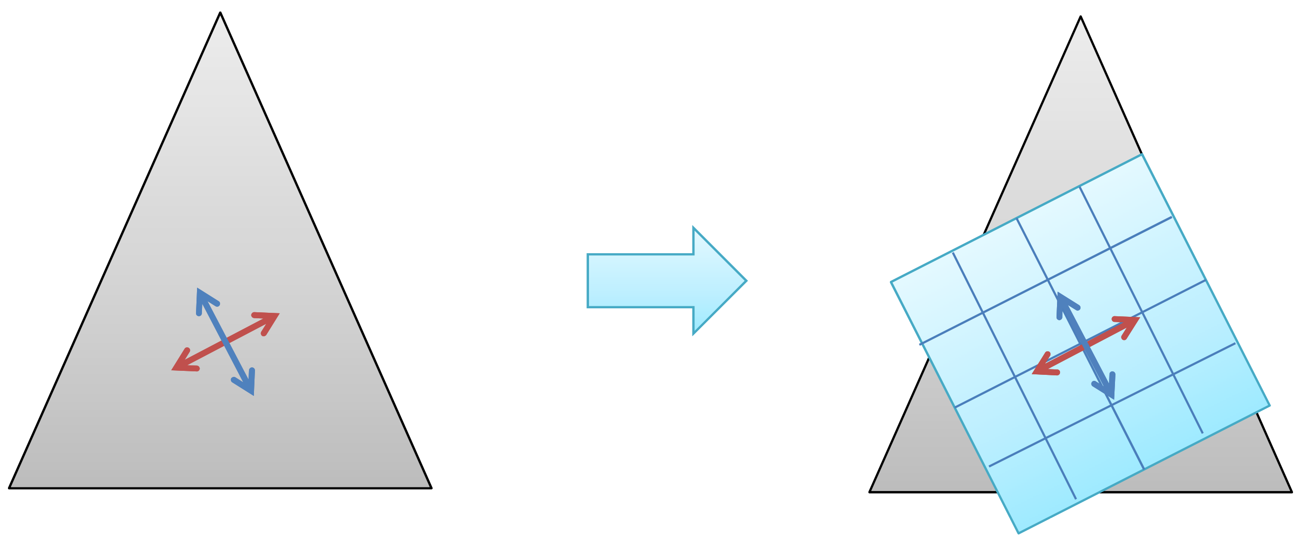


Cross Field

Cross Fields

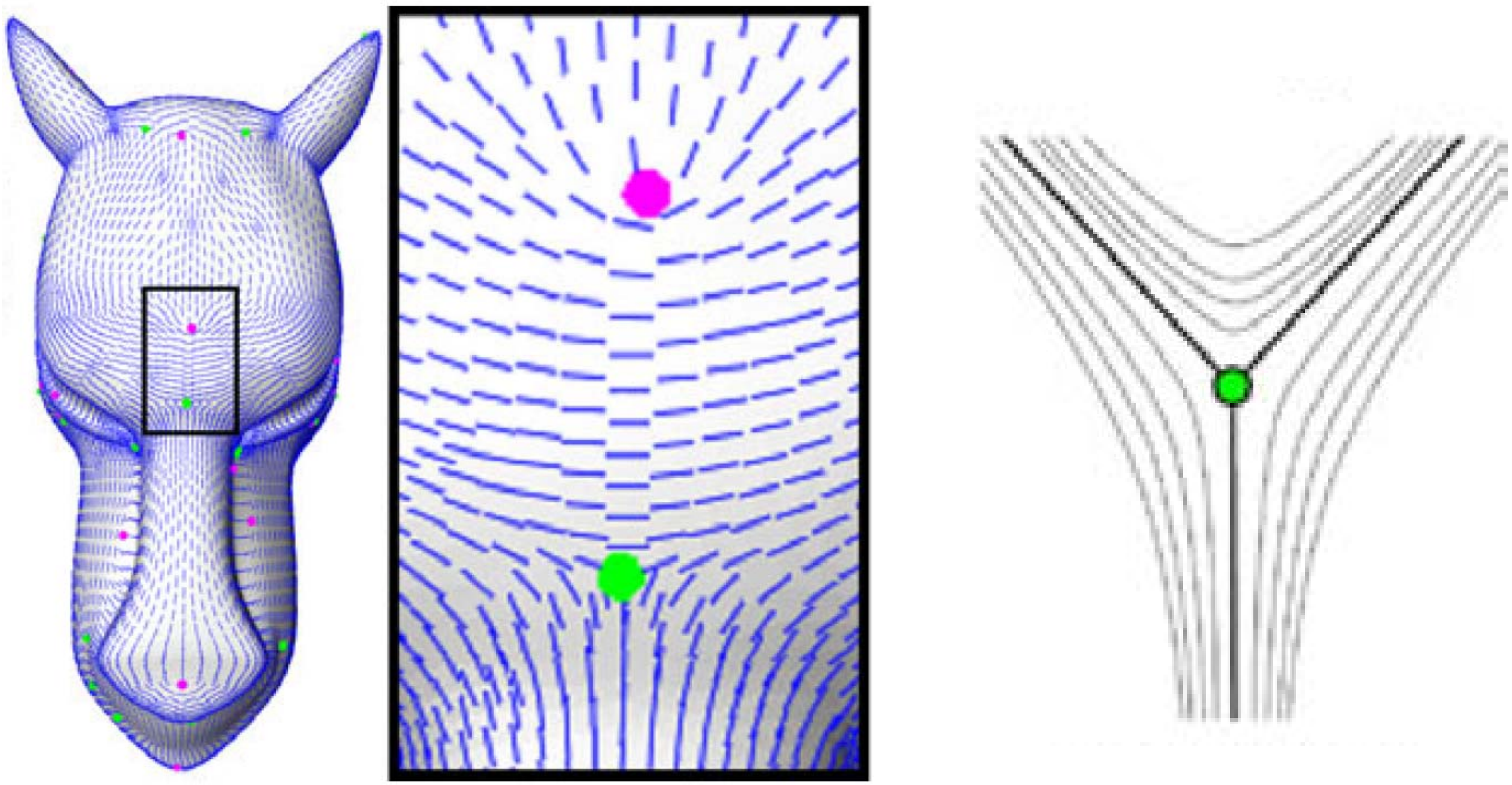
Two orthogonal directions per triangle specify **orientation** of quads

Do not specify **sizing**



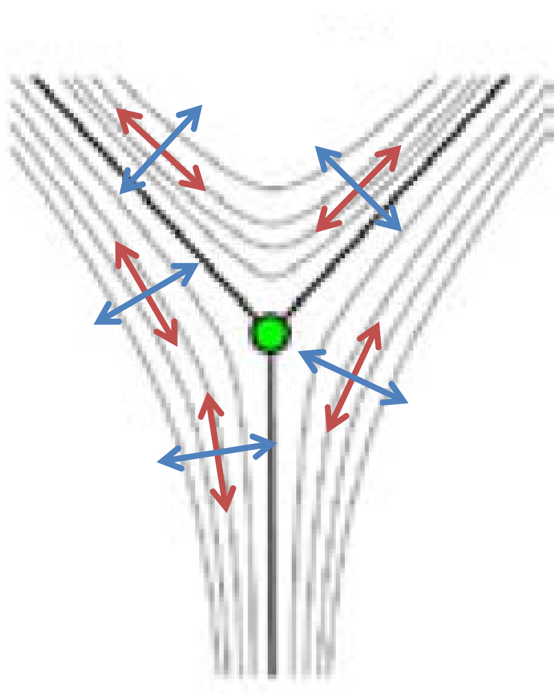
Umbilics

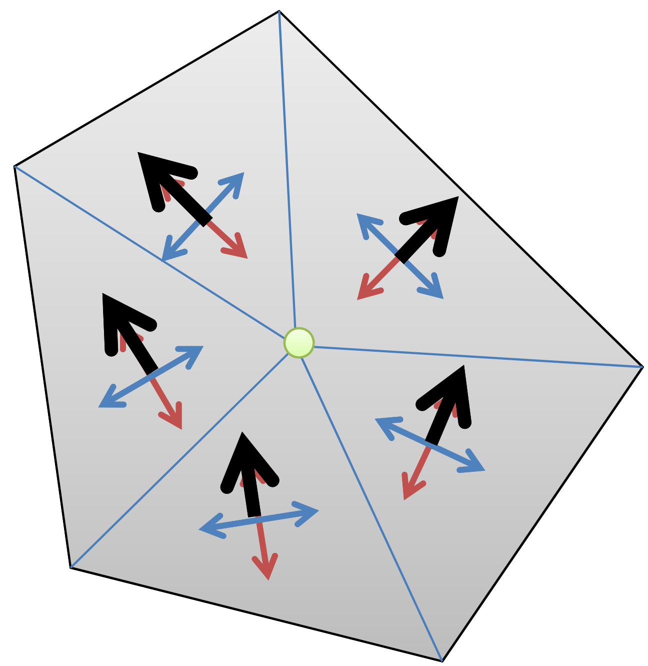
- Minimal curvature = maximal curvature
- No well defined directions locally



Cross Fields vs. Vector Fields

- Umbilics generate **singularities** in the cross field
- There is no consistent selection of 1 direction which gives a smooth vector field





Cross Fields vs. Vector Fields

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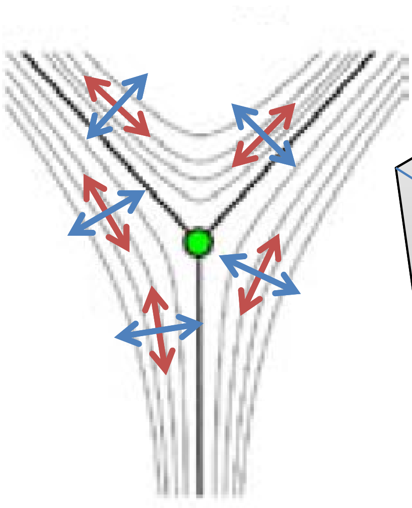
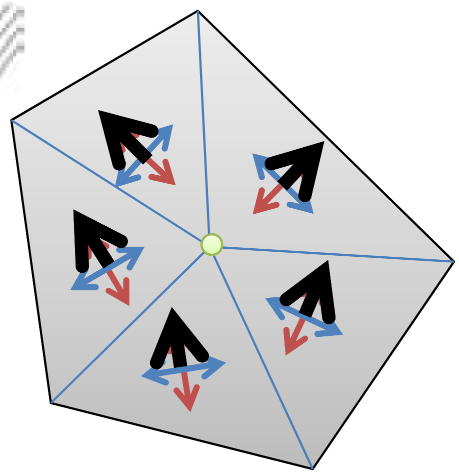
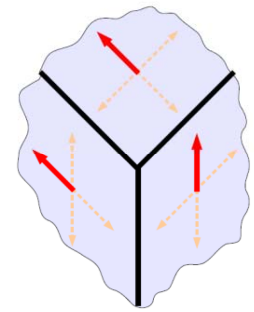
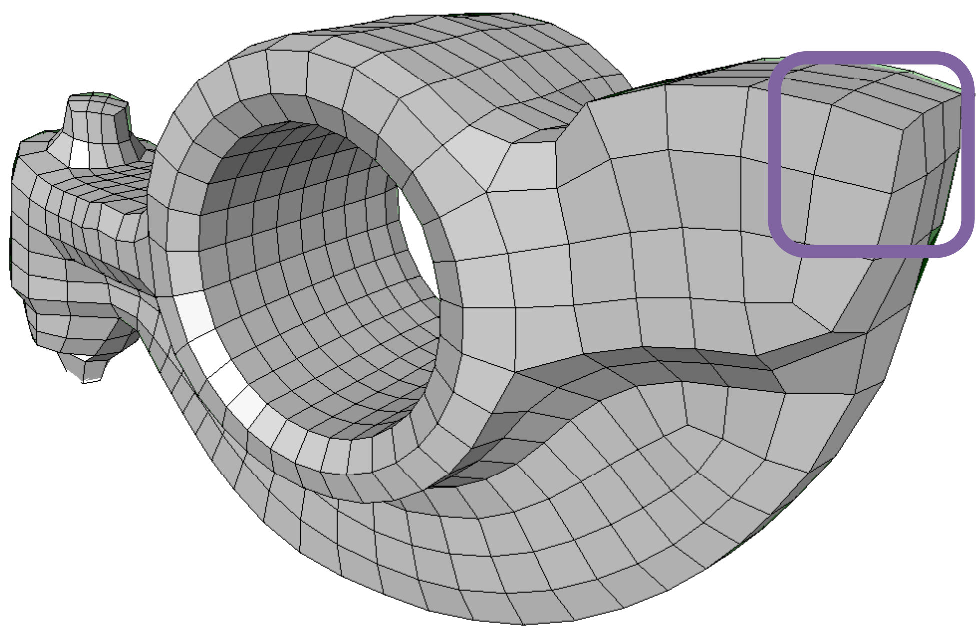
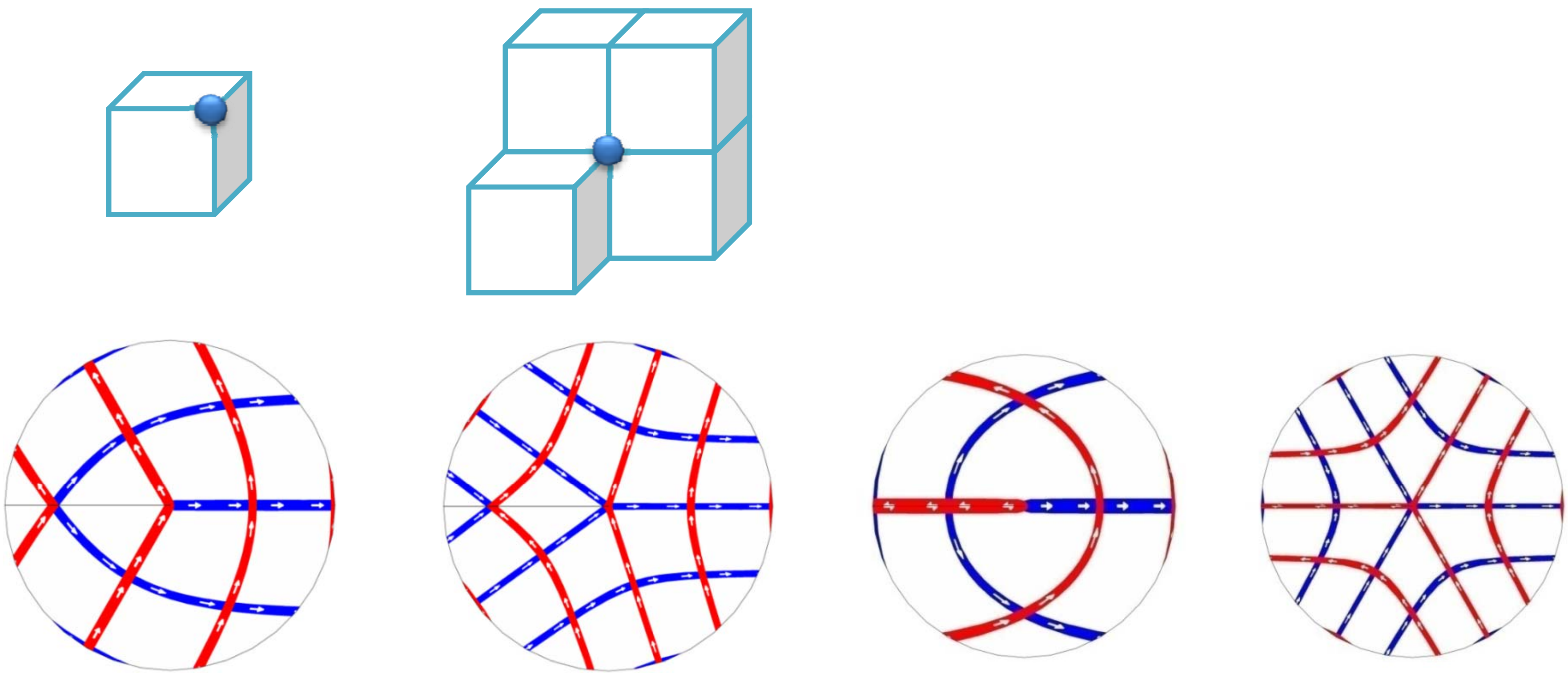






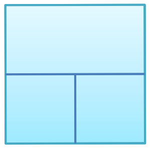


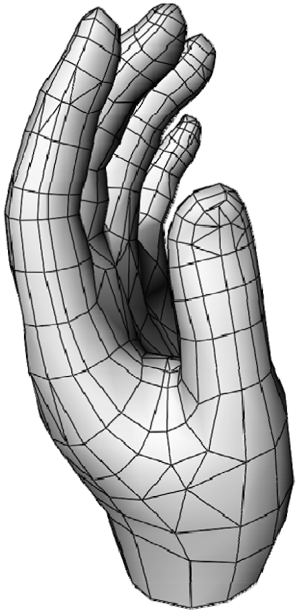
Singularities in the Wild

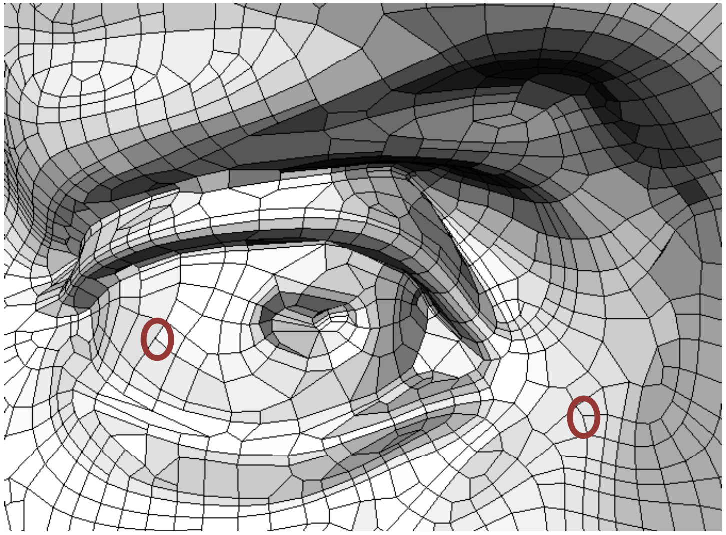


Index = 1/4 Index = -1/4 Index = 1/2 Index = -1/2

Methods

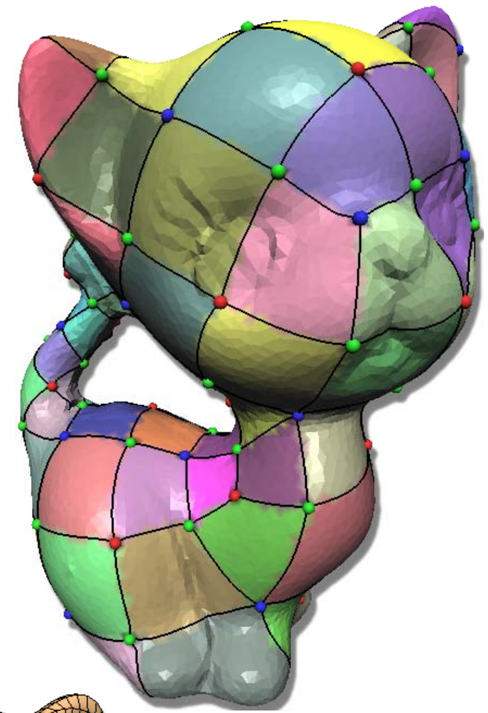
- Trace curves [Alliez '03, Marinov '04]
  - Generate curves tangent to cross field
  - Intersection of curves $\rightarrow$ vertices of quad mesh
  - T-Junctions, quad-dominant 

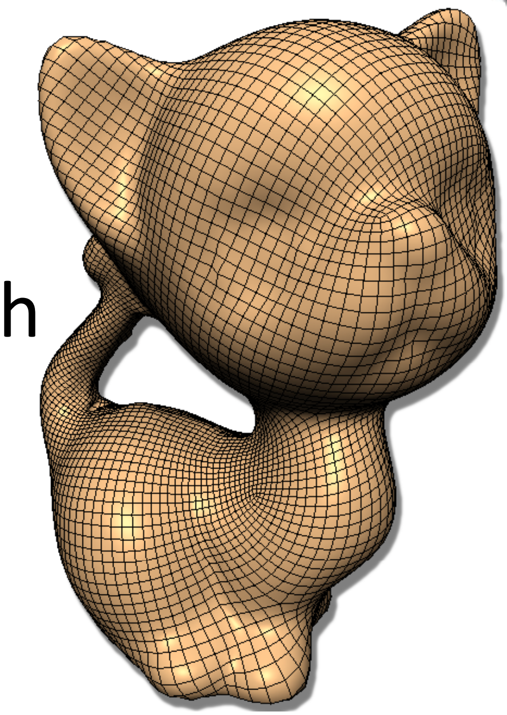




Methods

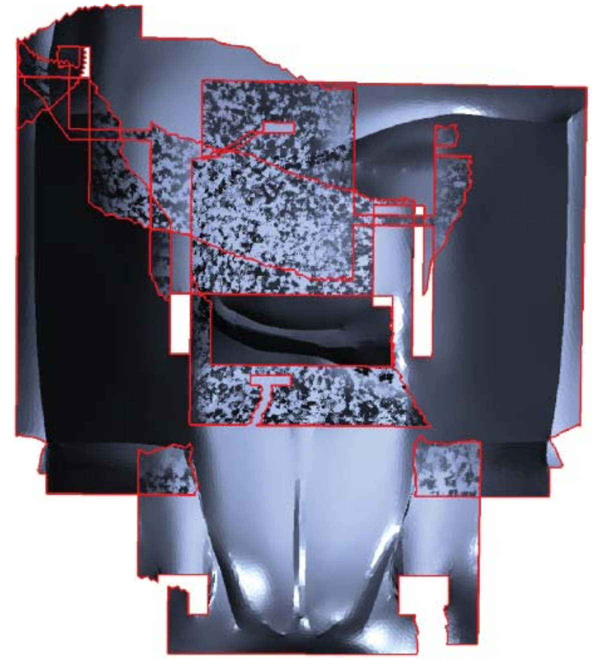
- Base mesh [Dong '06, Tong '06, Huang '08]
  - Construct coarse quad mesh
  - Refine it while preserving continuity
  - Hard to construct base mesh
    - harder than quad meshing

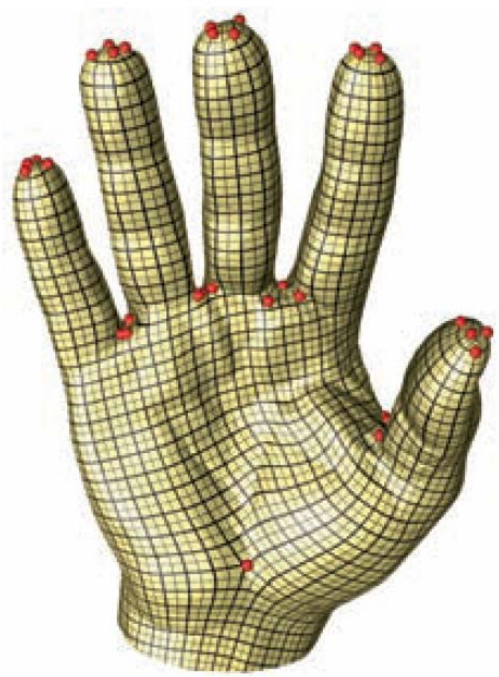


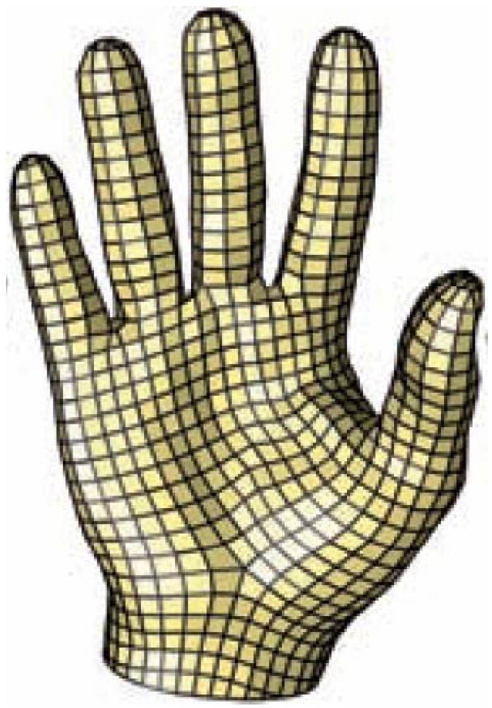


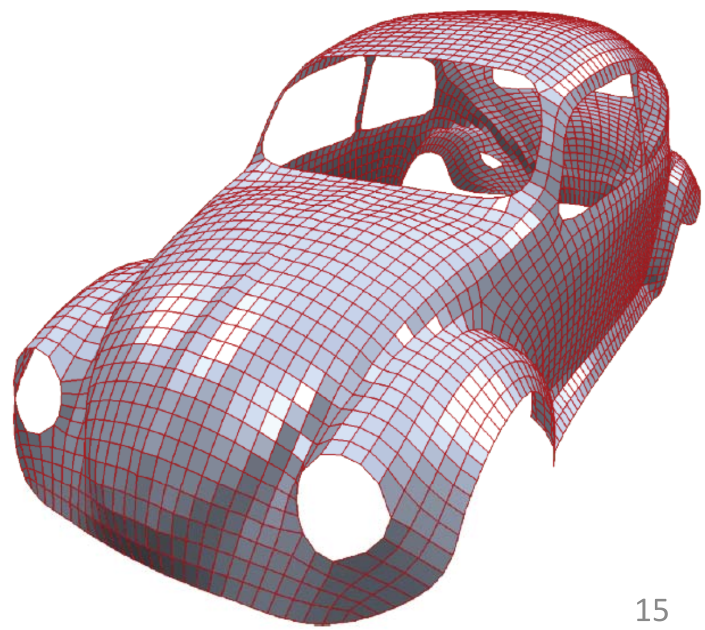
Methods

- Contouring [Ray '06, Kalberer '07, Zimmer '09]
  - Find global parameterization (u,v)
    - u and v can change roles!
  - Contour iso-u, iso-v lines



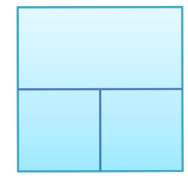






Why is Quad Remeshing Hard?

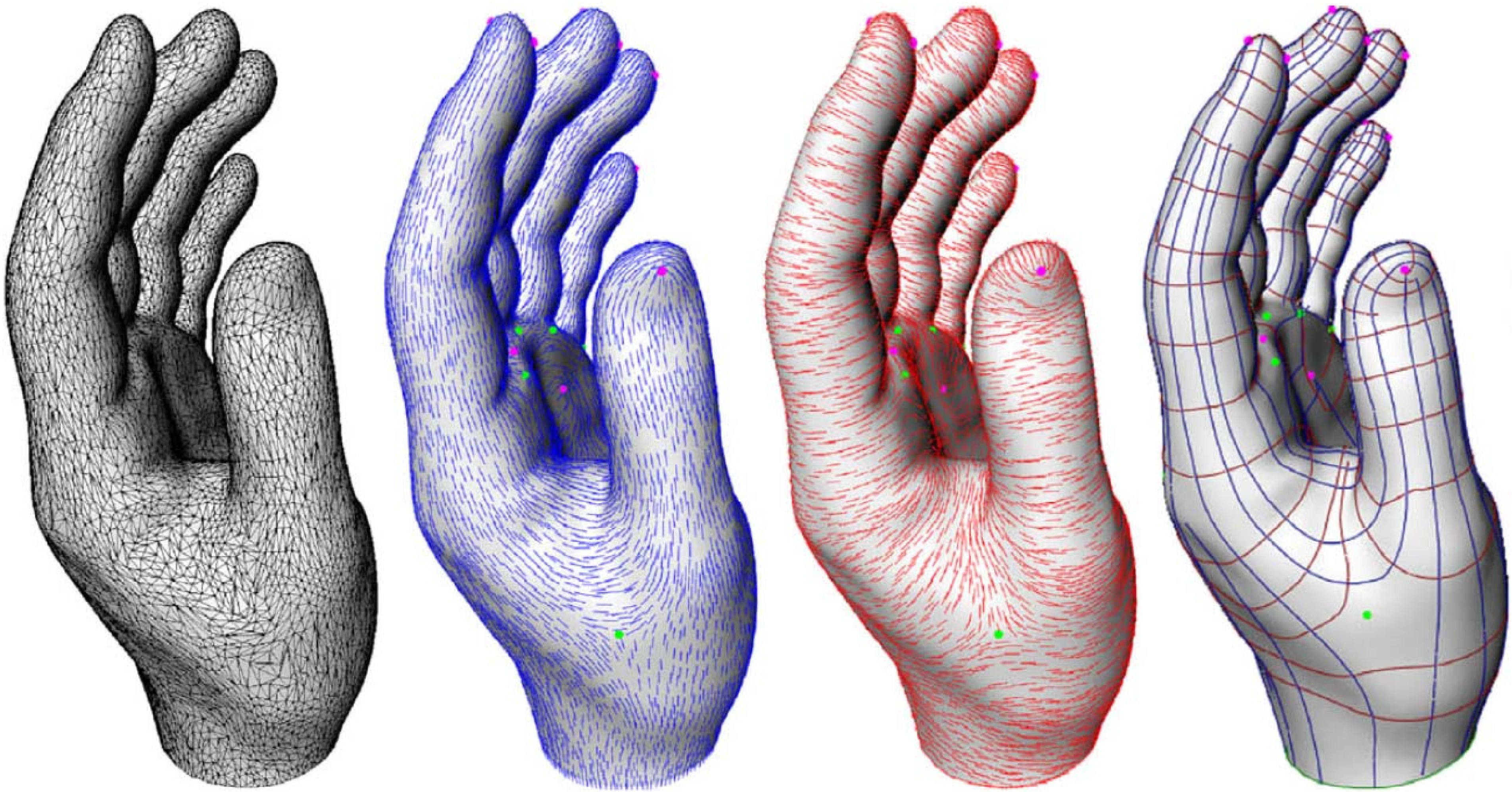
- Local methods generate T-Junctions



- Base mesh is too restrictive
  - Some good quad meshes do not have a base mesh
- Can't have both uniformity and alignment to curvature directions unless special surface
- **Locations and valences of singular vertices have a global effect**

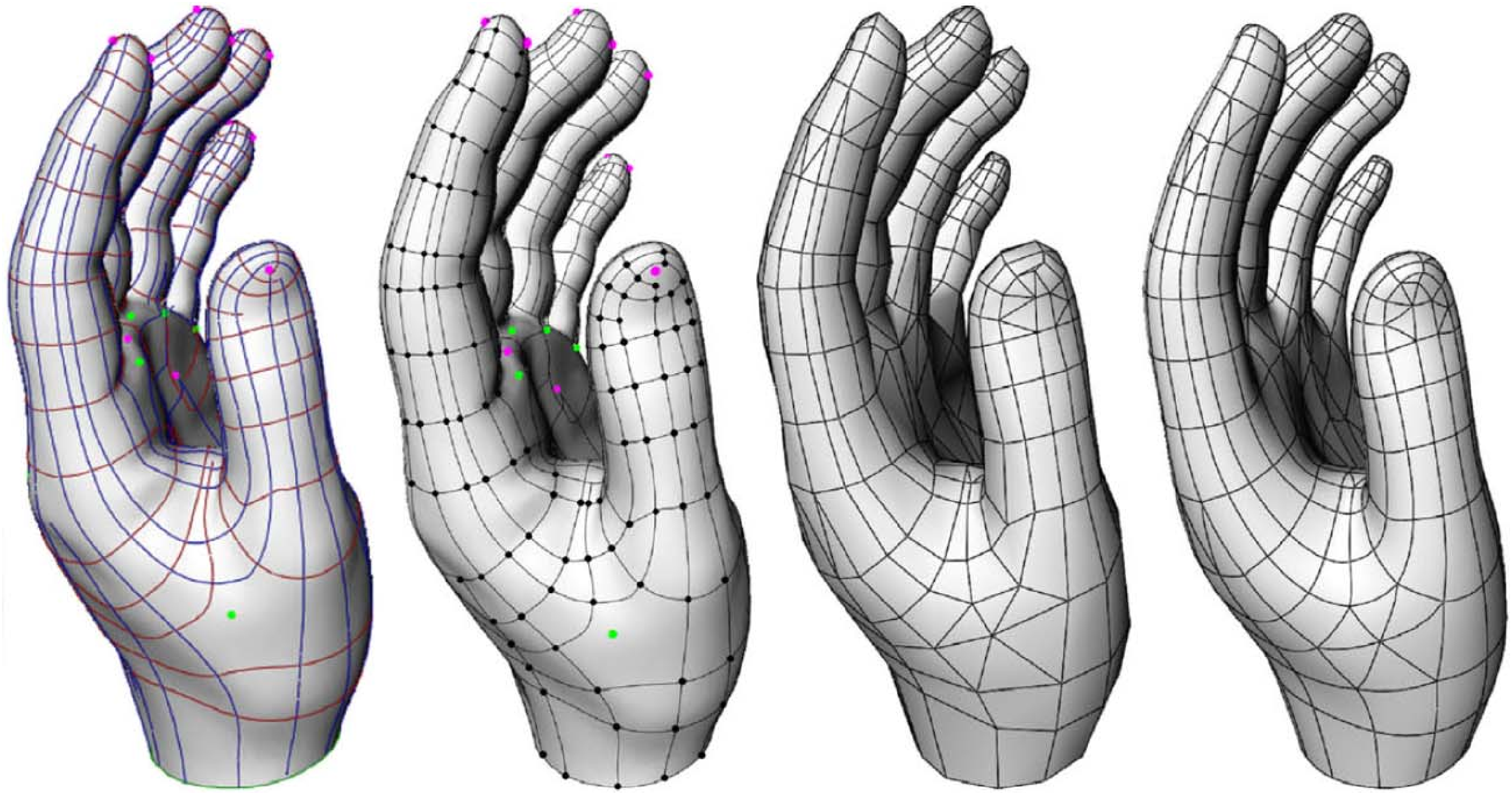
Anisotropic Polygonal Remeshing

[Alliez et al. ’03]



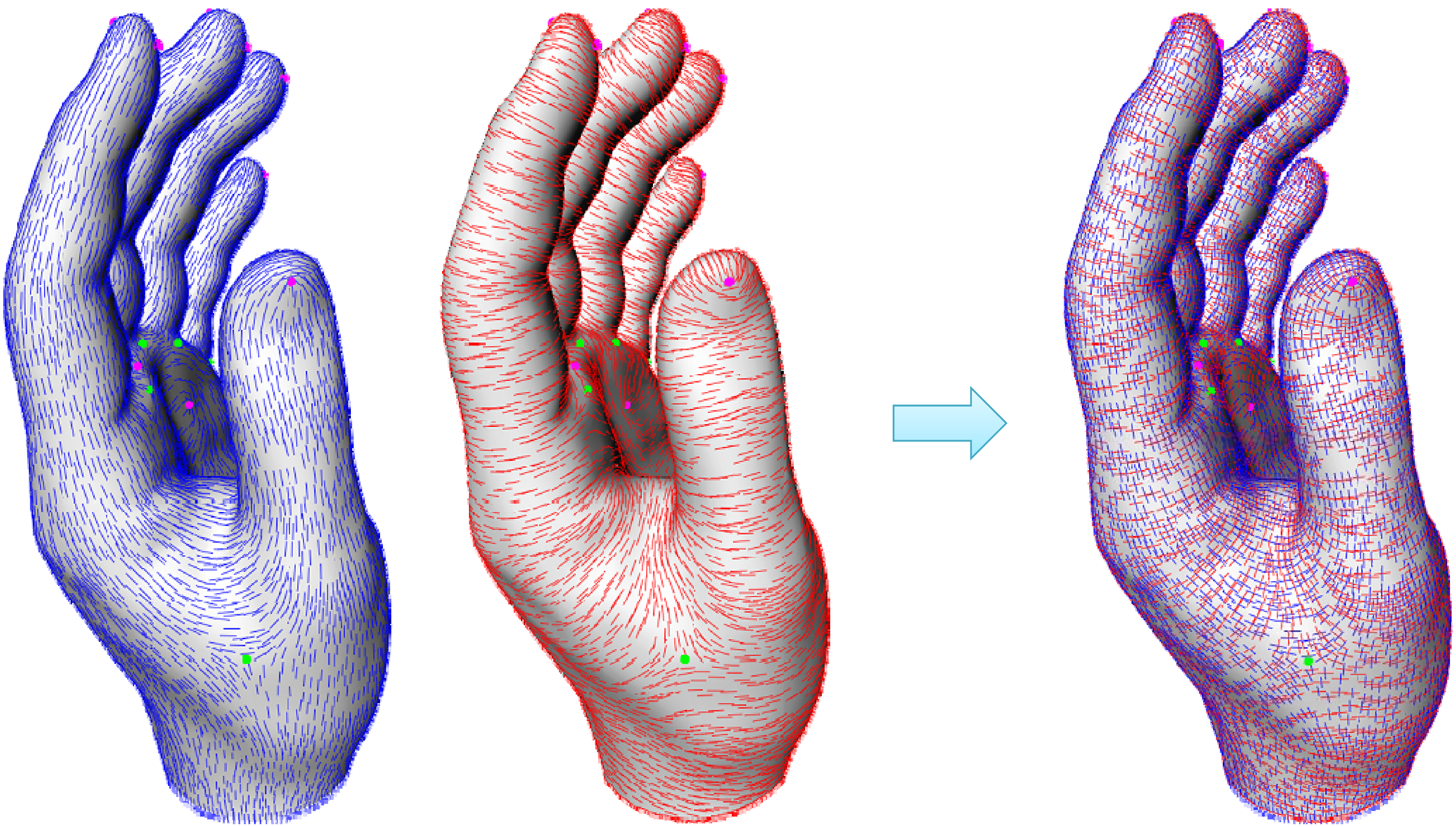
input mesh *direction fields* *sampling*

Anisotropic Polygonal Remeshing
[Alliez et al. ’03]



sampling *meshing* *output mesh* *after smoothing*

0. Compute Curvature Directions

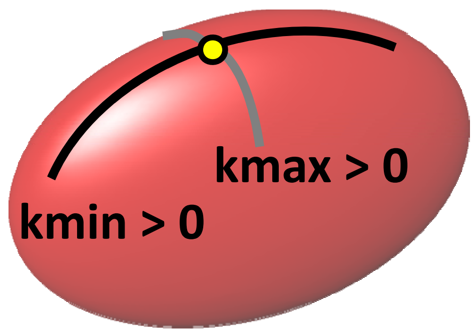


Min curvature Max curvature Cross Field

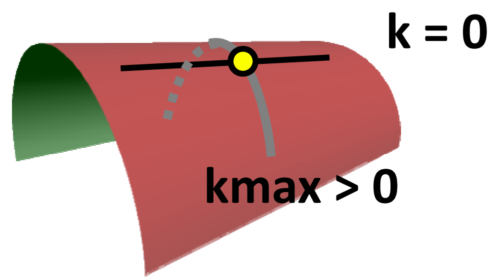
3D Curvature Tensor

Anisotropic

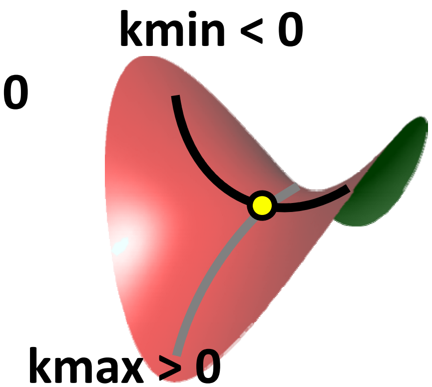
2 principal directions



elliptic

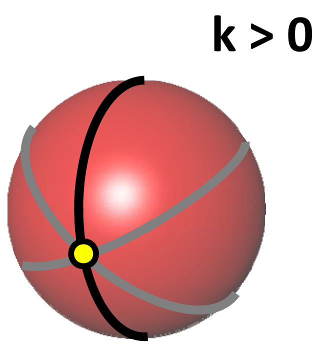


parabolic

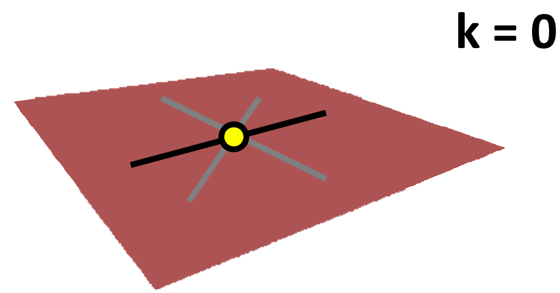


hyperbolic

Isotropic

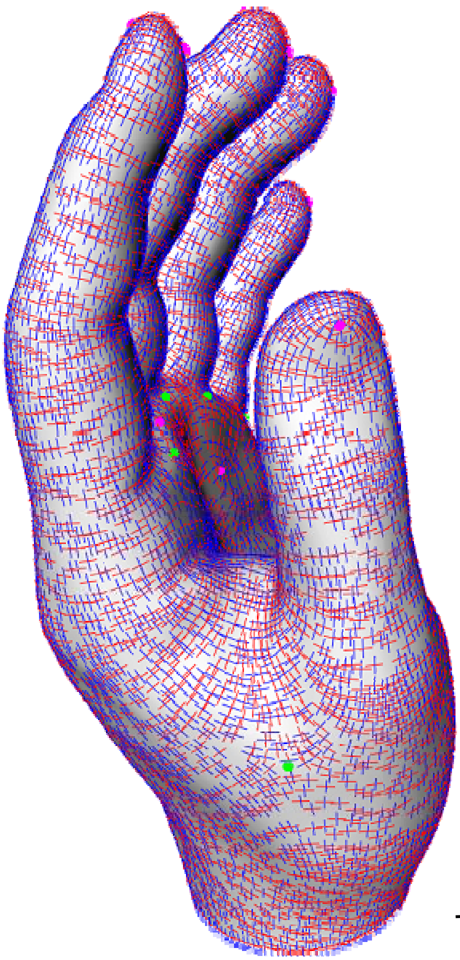


spherical

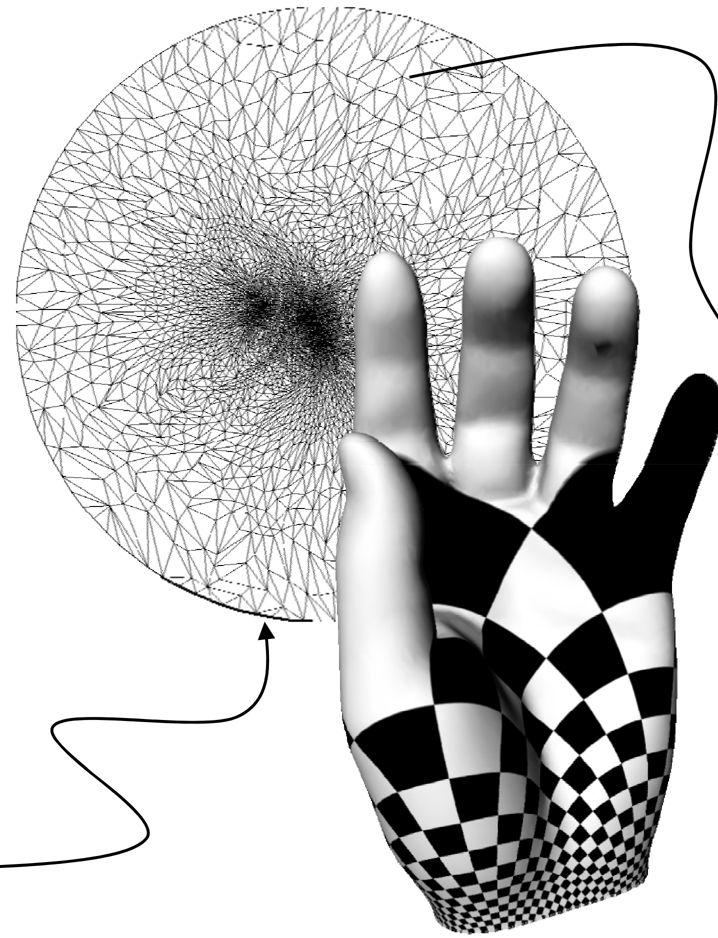


planar

1. Flatten to 2D

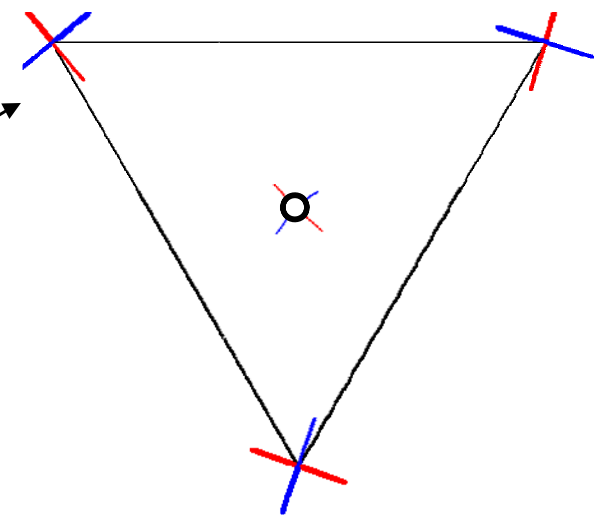


one 3D tensor
per vertex



discrete conformal
parameterization

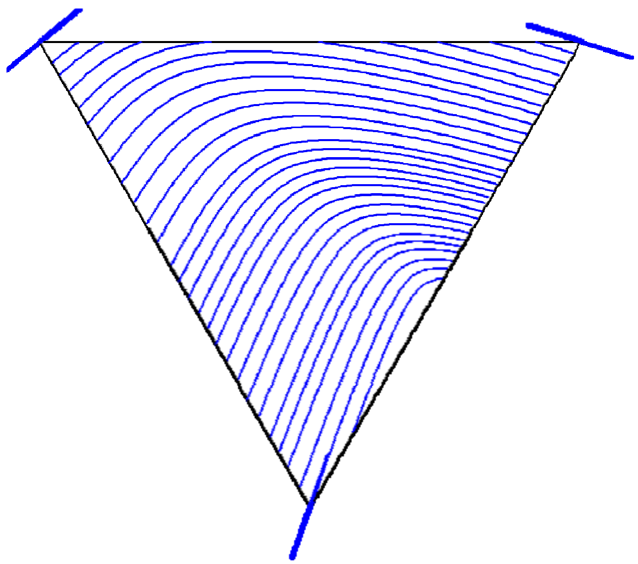
piecewise linear
interpolation of
2D tensors



2D tensor **field**
using barycentric
coordinates

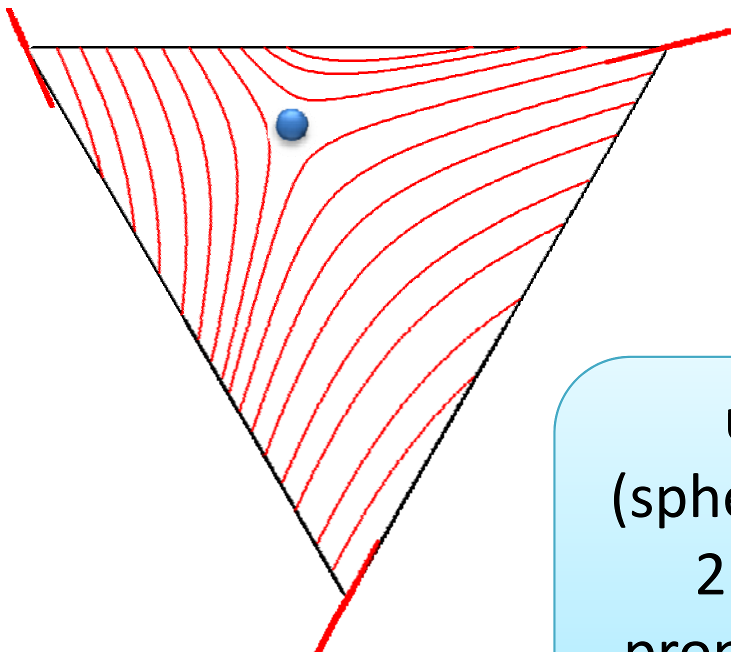
2. Find Singularities

- Regular case

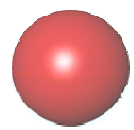


minor directions | major directions | Both directions

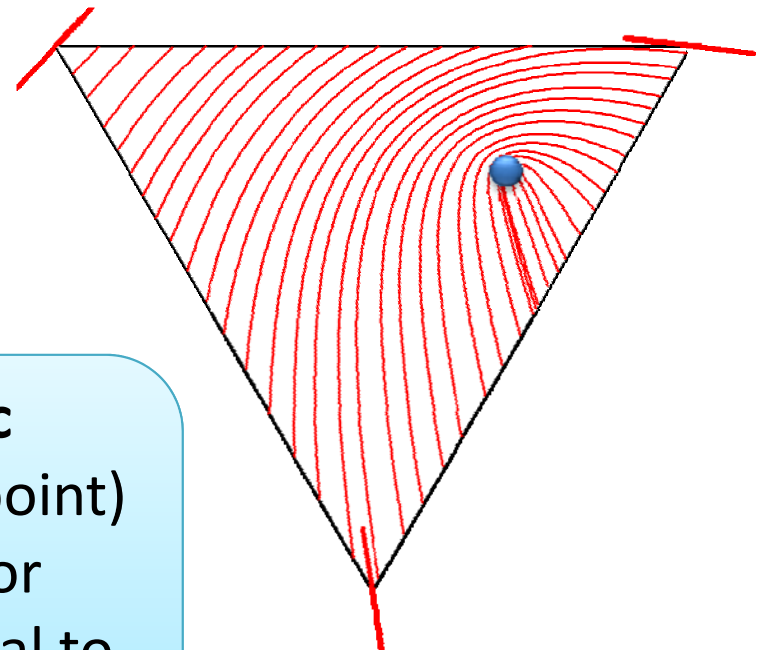
2. Find Singularities



trisector

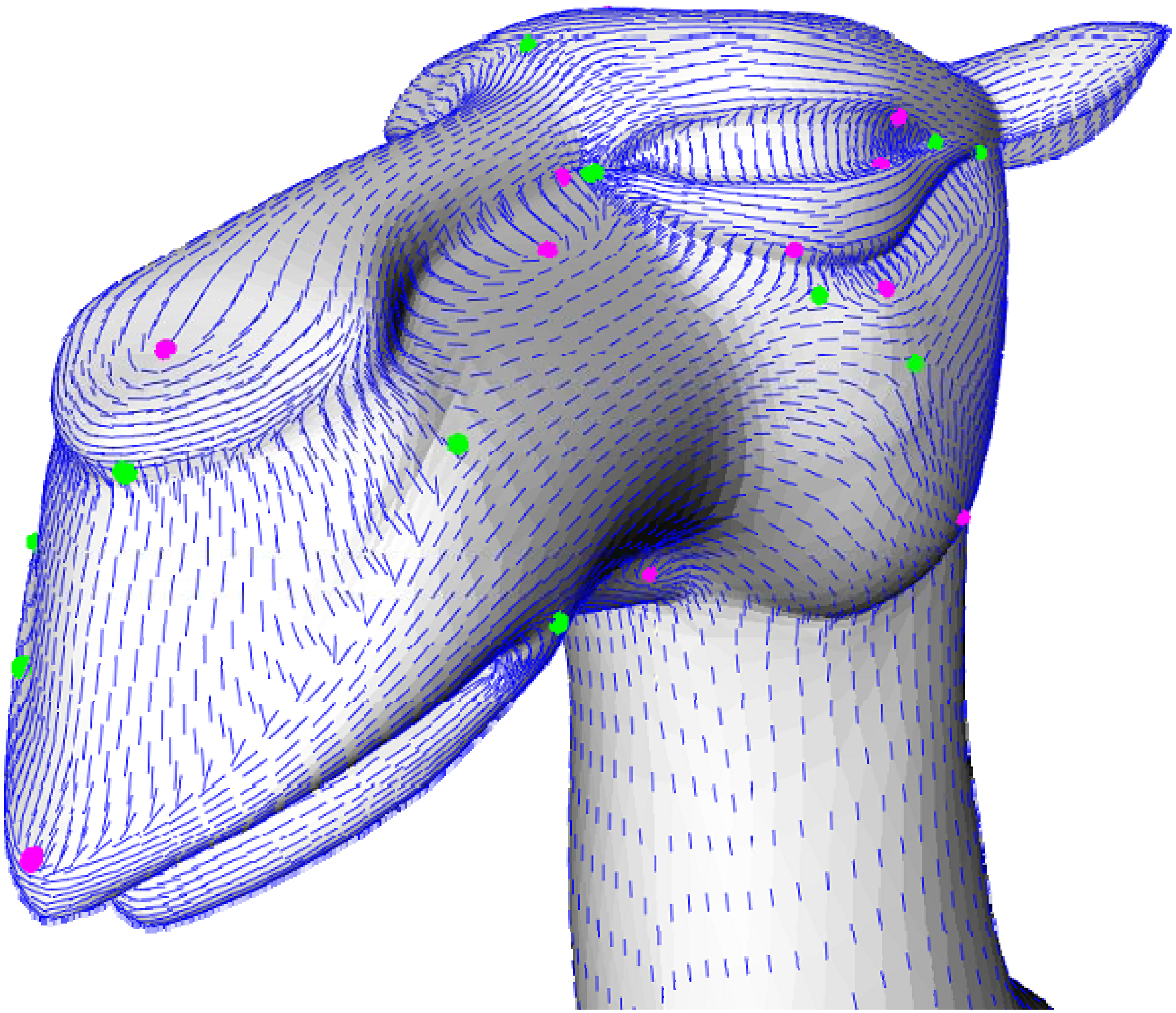


umbilic
(spherical point)
2D tensor
proportional to
identity

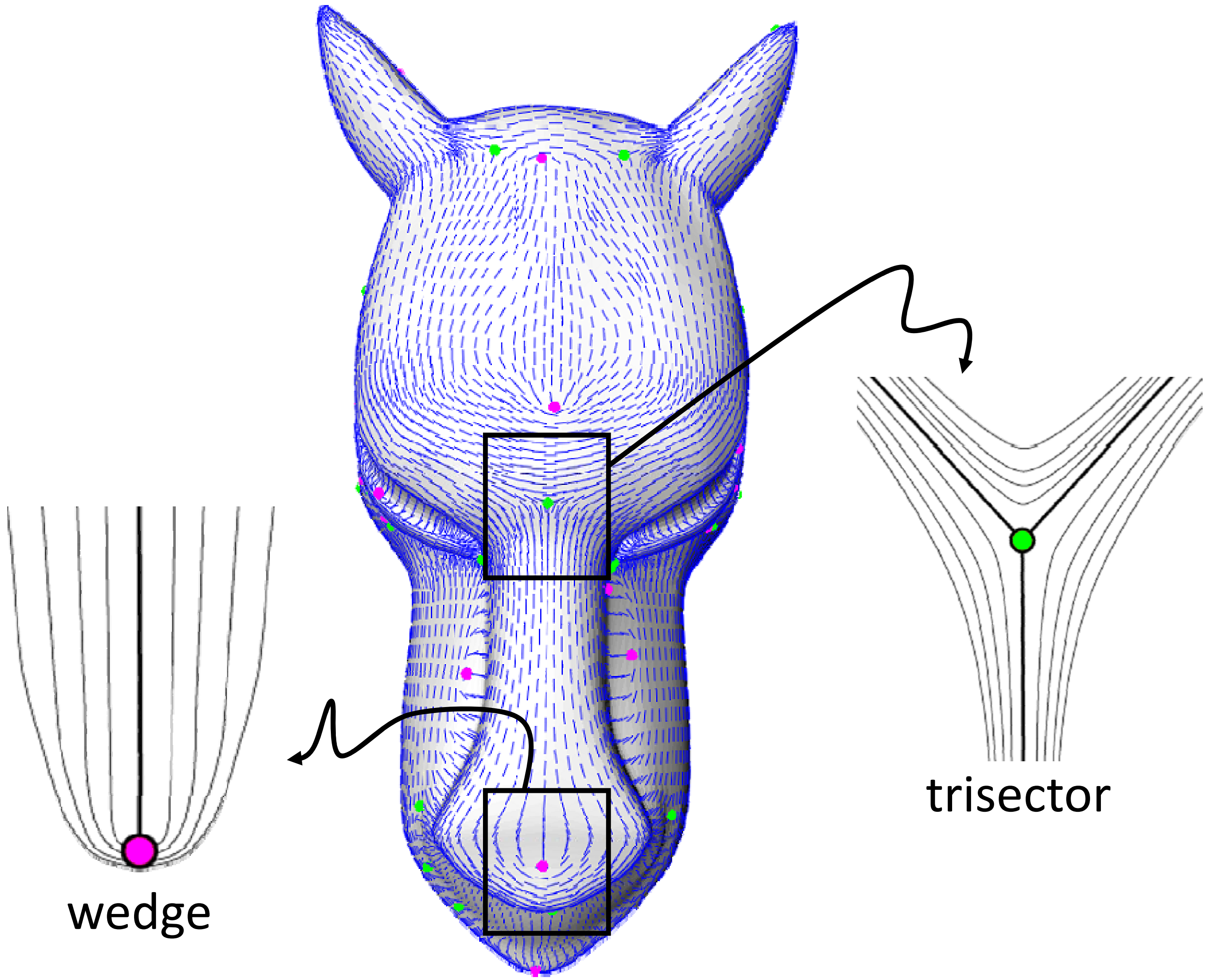


wedge

Umbilics



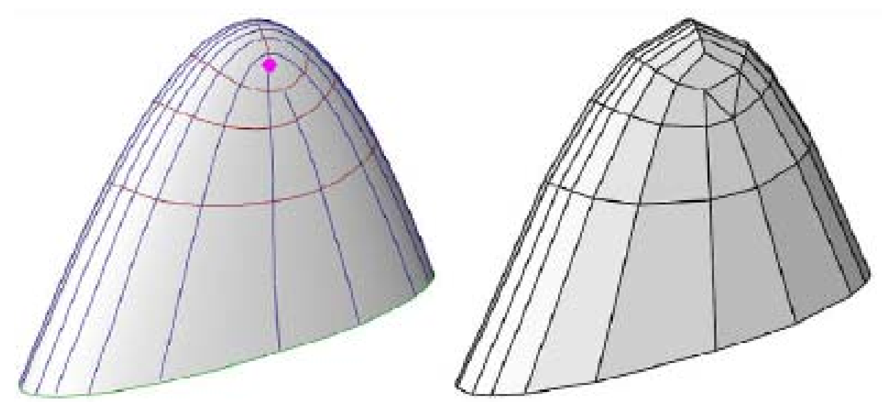
Umbilics



3. Trace Curvature Lines

- Numerical integration of curvature directions

- Need to take care of:
  - Choosing seed points
  - Spacing of lines
  - Numerical stability



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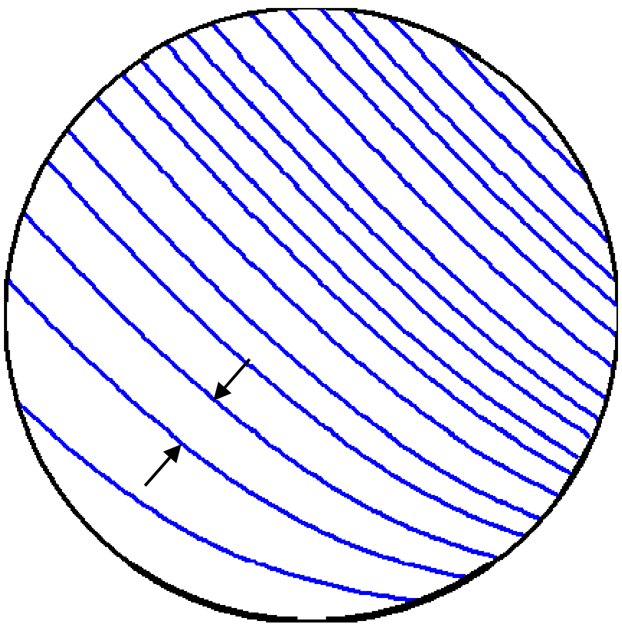
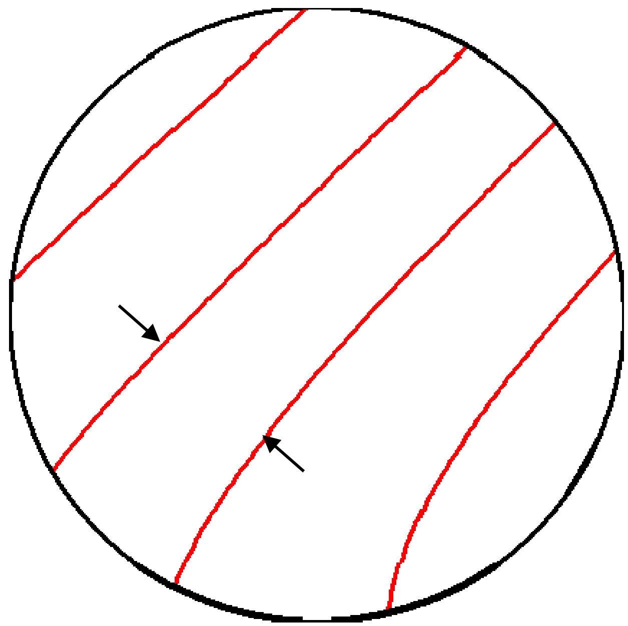
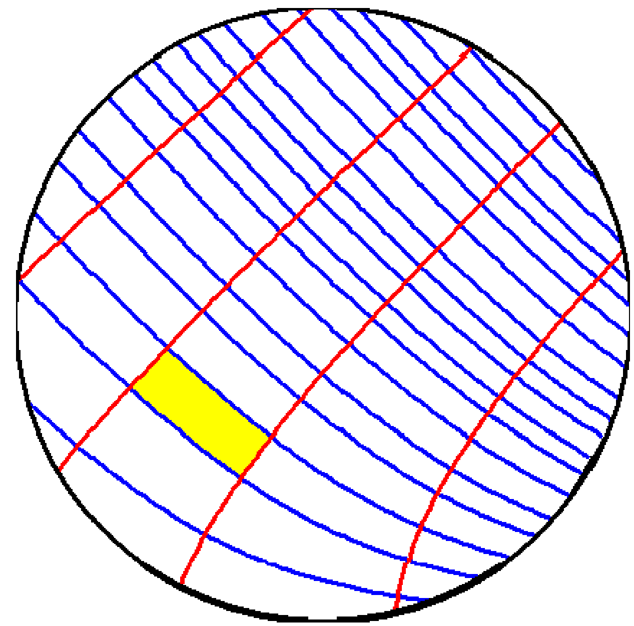


minor net

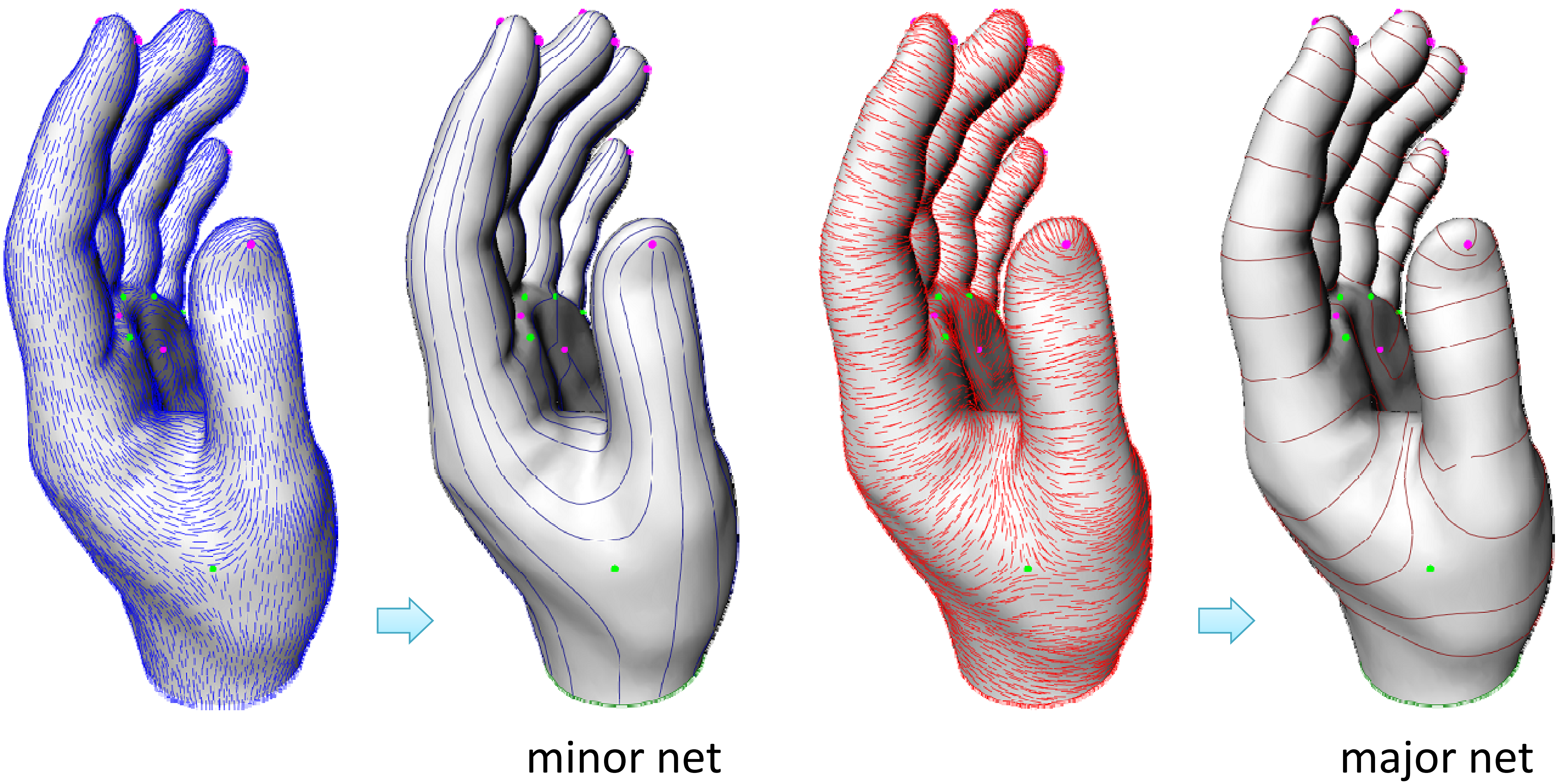


major net



overlay

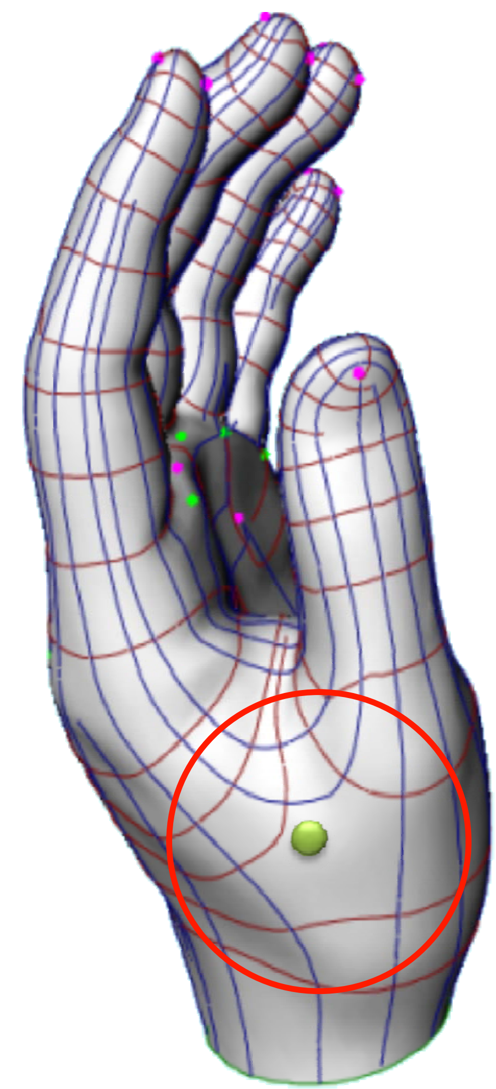
3. Trace Curvature Lines



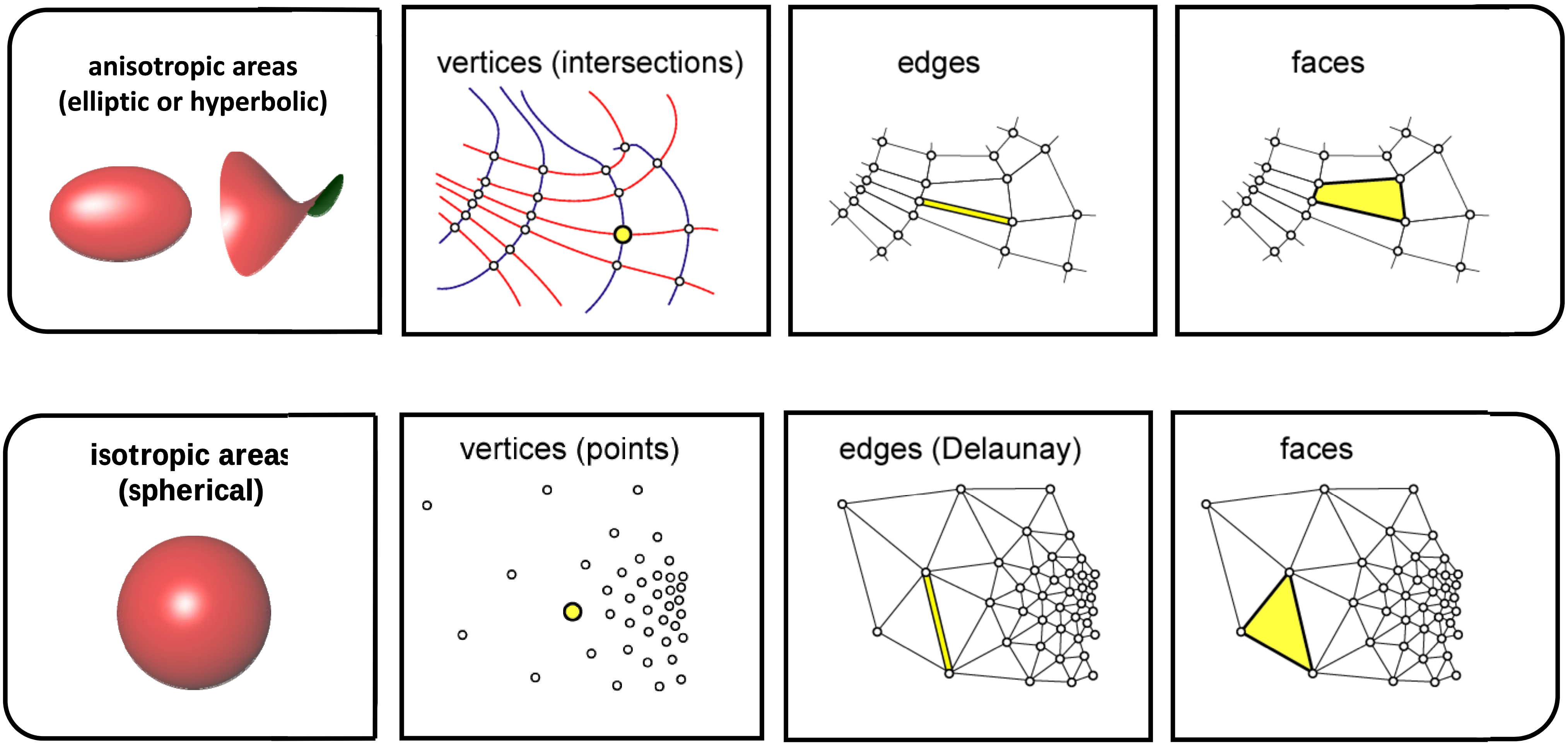
4. Overlay

- Overlay curvature lines in anisotropic regions

- Add umbilic points in isotropic regions

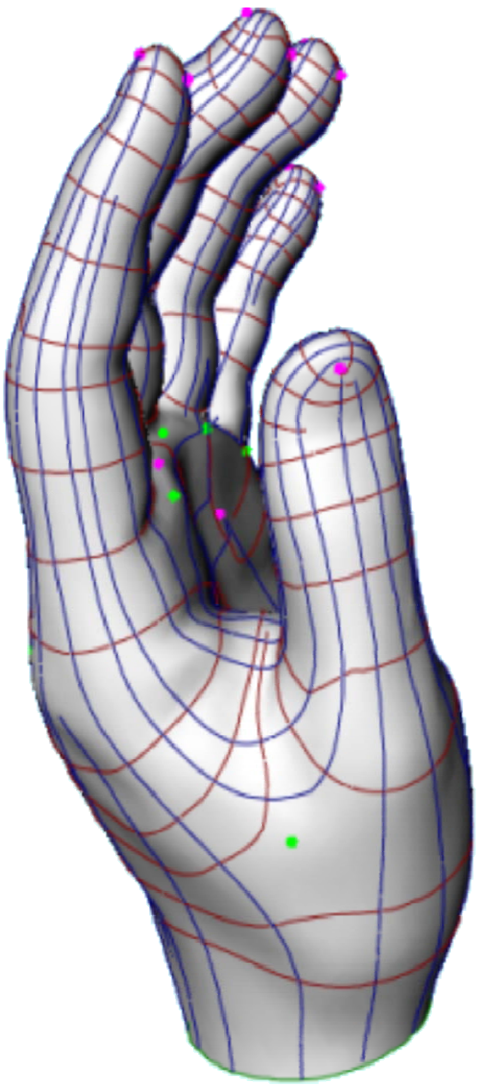


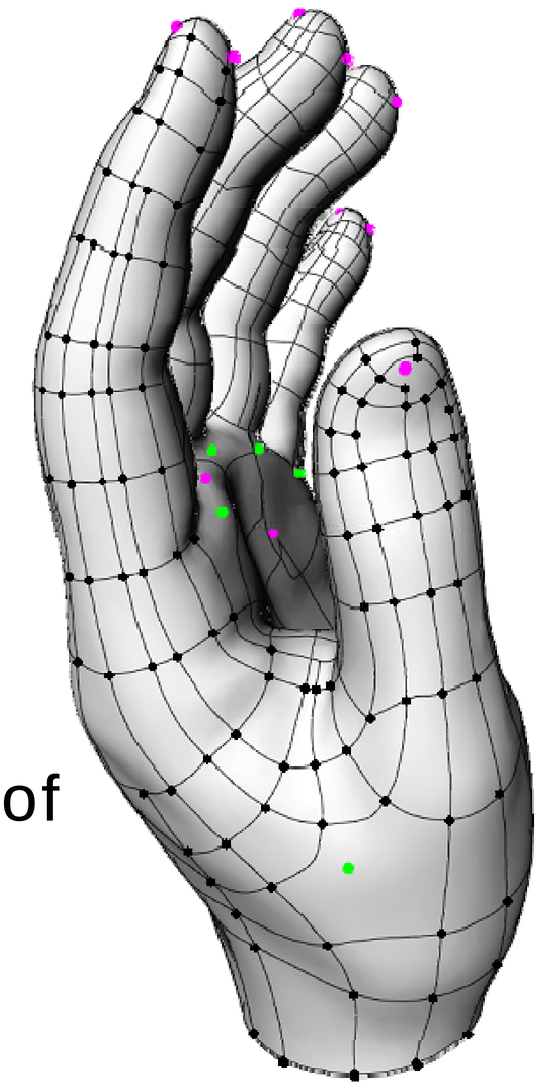
5. Meshing


anisotropic areas
(elliptic or hyperbolic)
vertices (intersections)
edges
faces
isotropic areas
(spherical)
vertices (points)
edges (Delaunay)
faces

5. Meshing

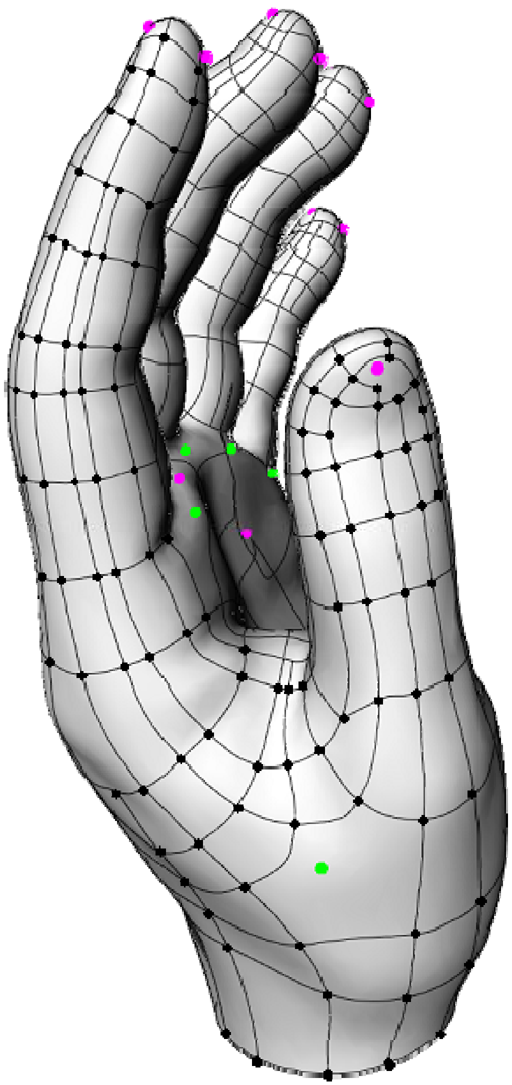
Vertices



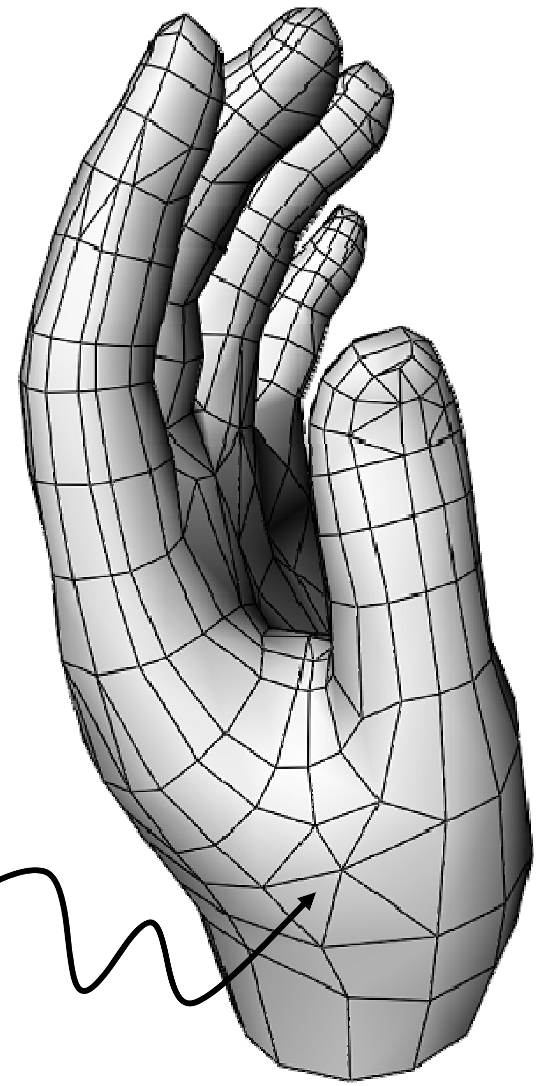


intersect lines of curvatures

5. Meshing
Edges

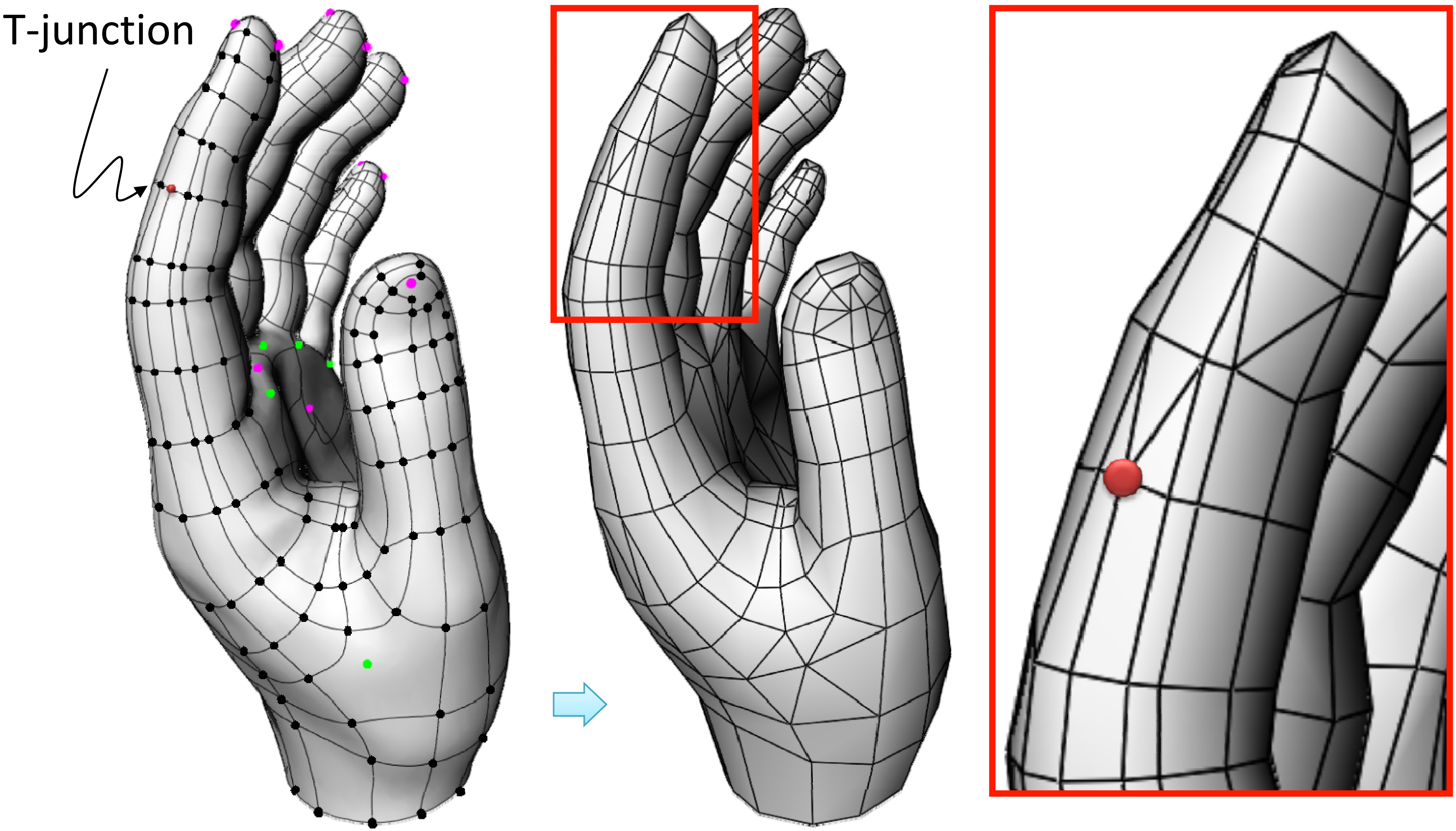


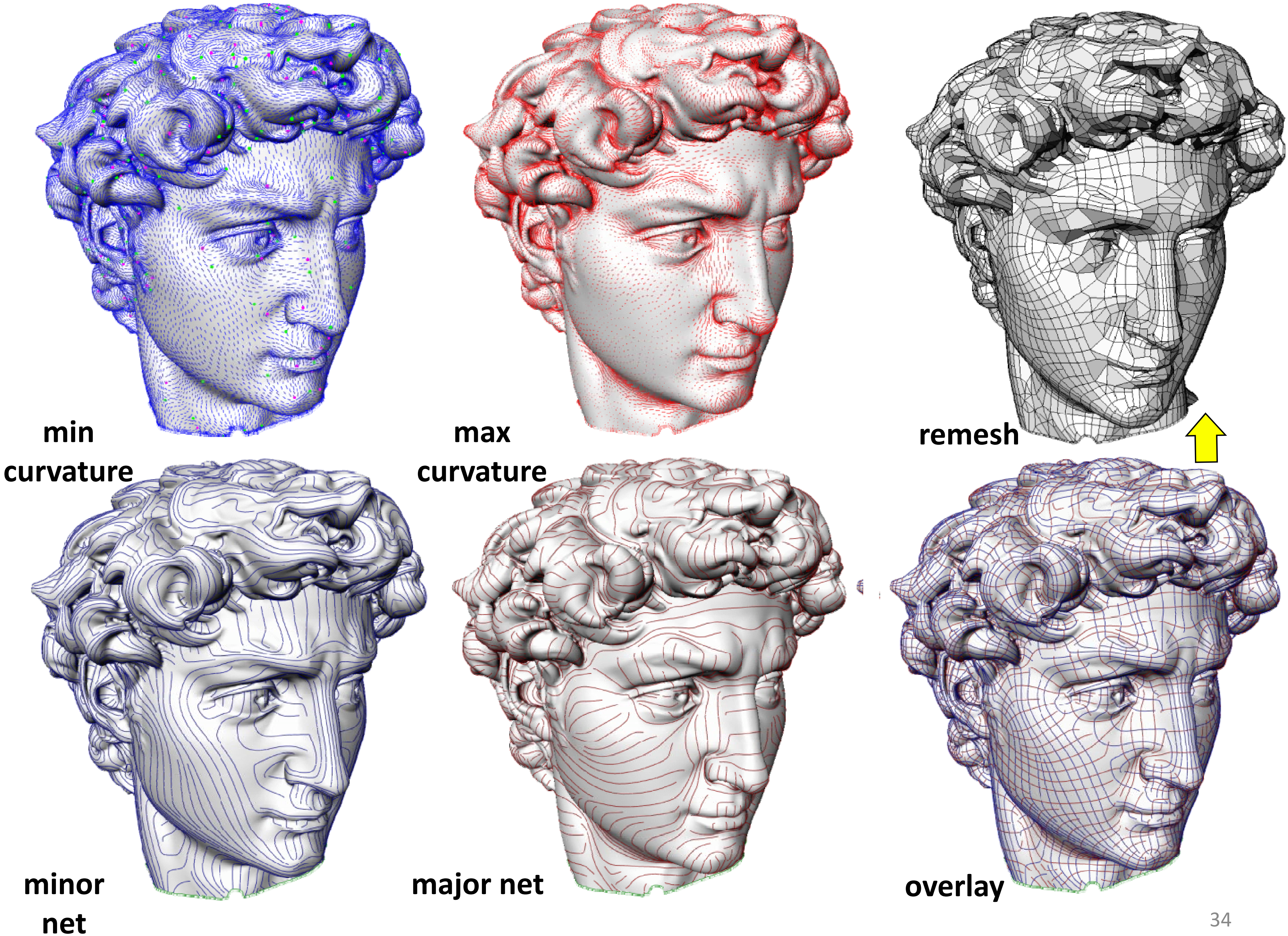
straighten lines of curvatures
+
Delaunay triangulation near umbilics



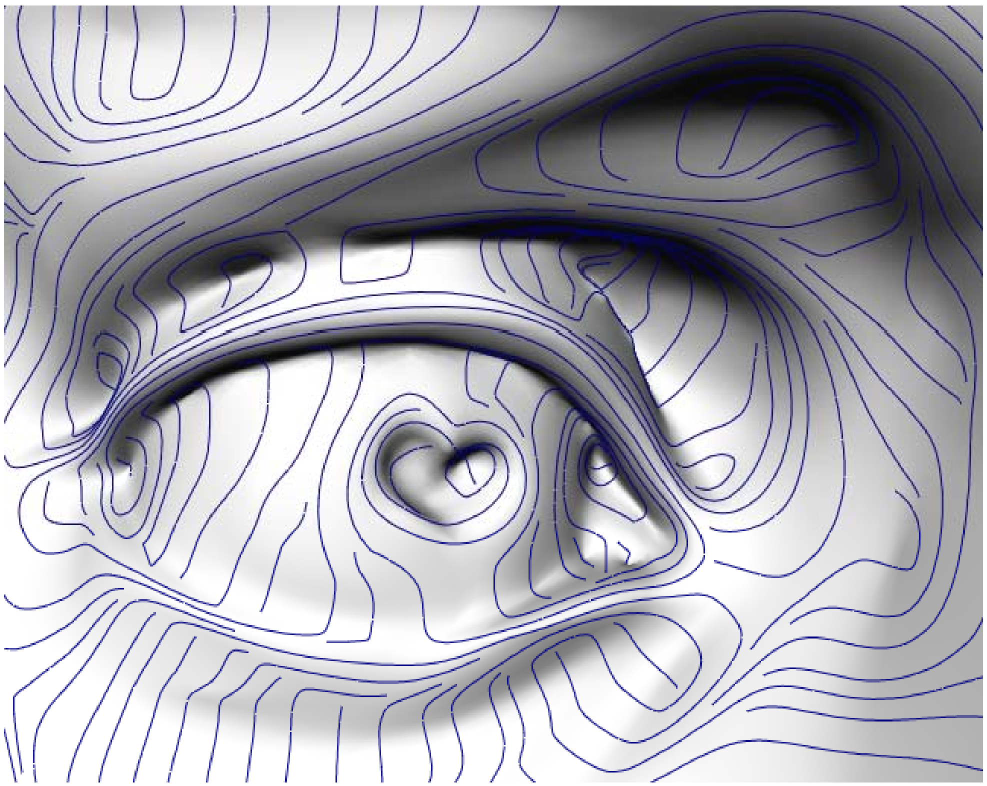
5. Meshing

Resolve T-Junctions

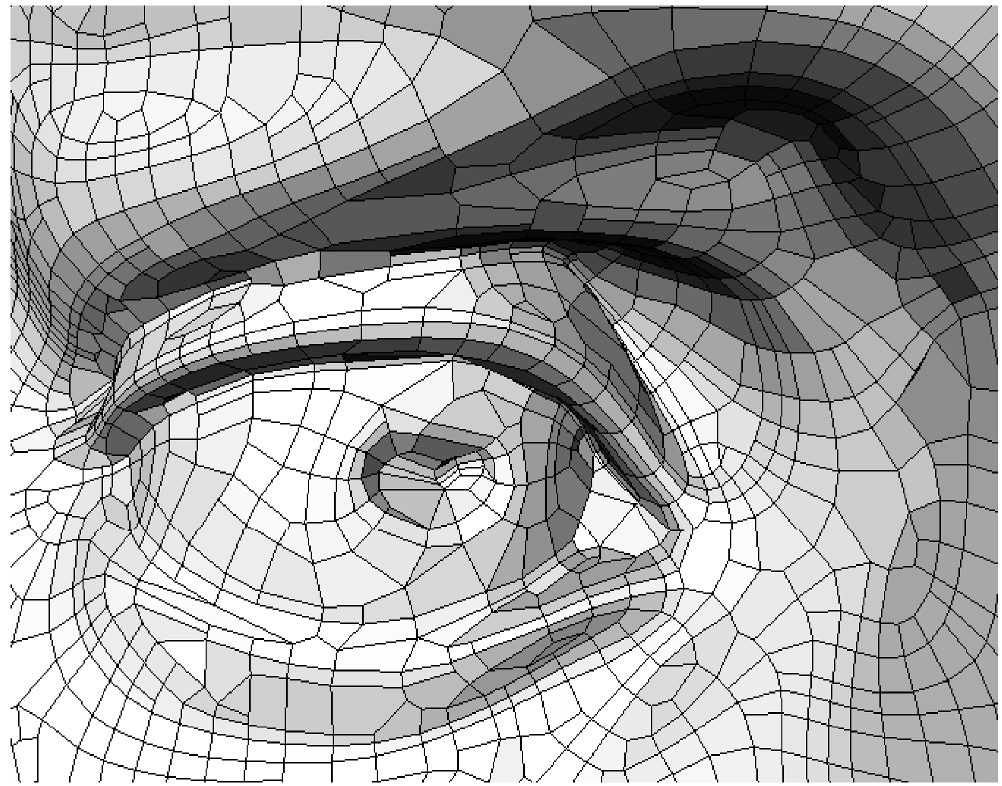



min curvature
max curvature
remesh
minor net
major net
overlay

Close-up



minor net

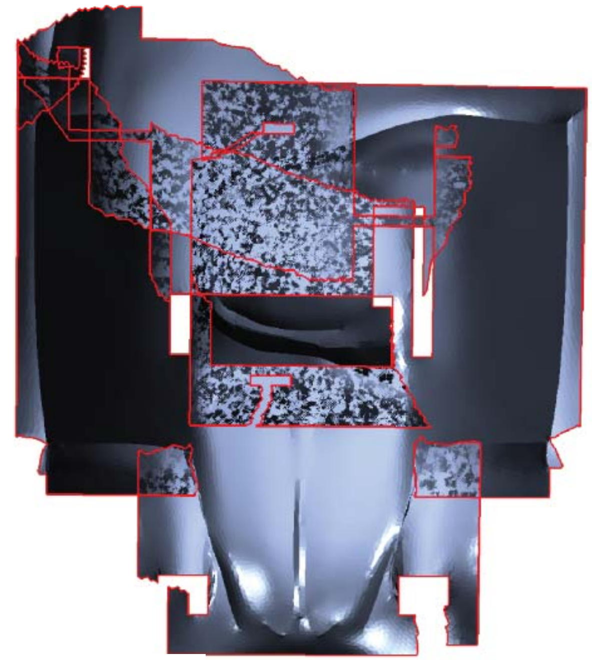


remesh

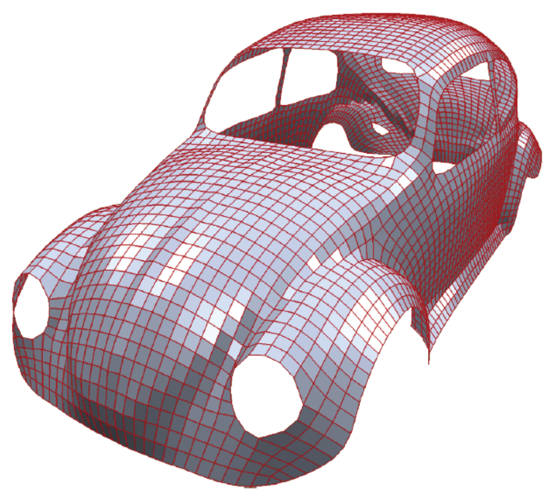
Limitations

- Global parameterization
  - Smart cutting required
  - Low distortion parameterization
  - Numerical issues
- Lines of curvature
  - Robust curvature tensor computation
  - Tensor smoothing required
  - Optimal placement of streamlines difficult
- Contains non-quad elements

Global Parameterization with Cone Points

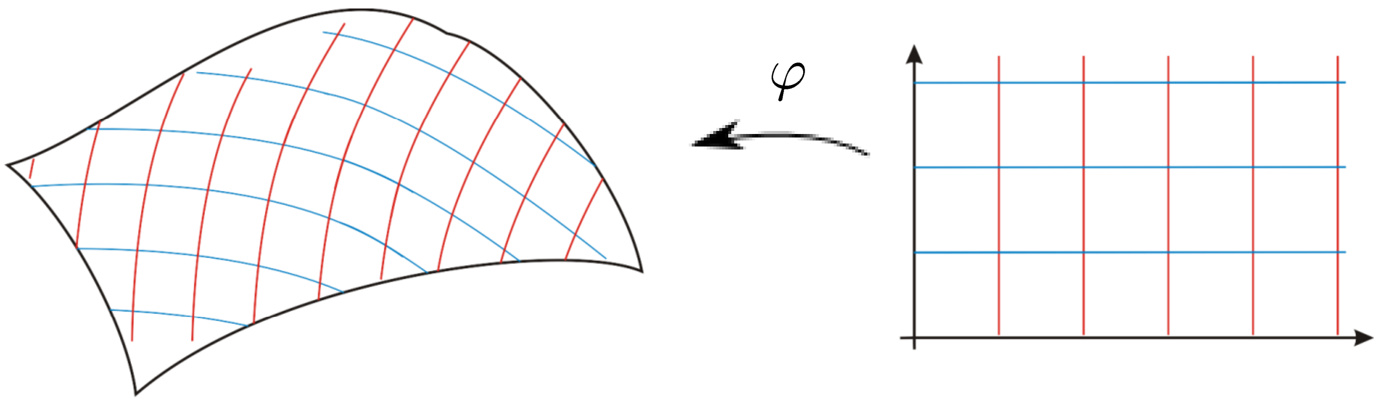


- Find global parameterization s.t. gradients align with input cross-field
  - Can be curvature directions, or any other
- Quad mesh easy to generate:
  - Take 2D grid covered by parameterization

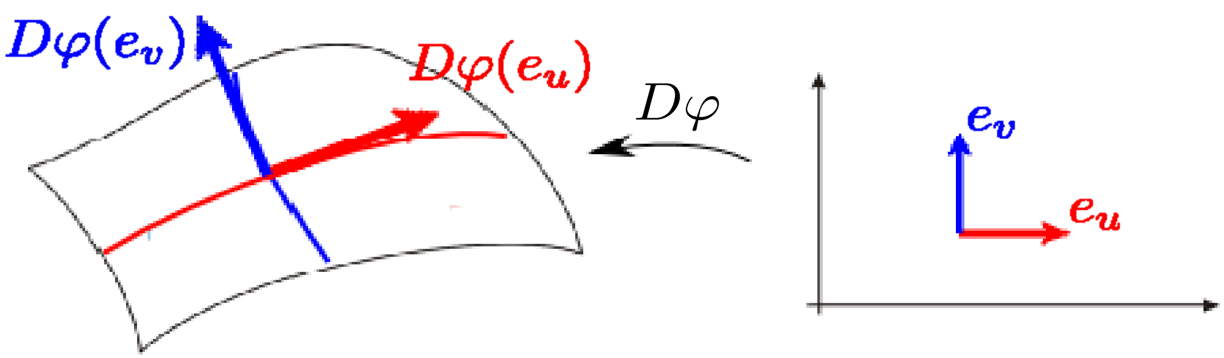


QuadCover [Kalberer ’07]

The parameter function …



… has two gradient vector fields:

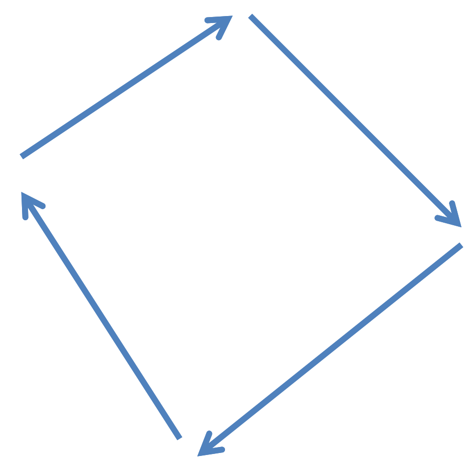


Integration of input fields yields parameterization.

Integrability

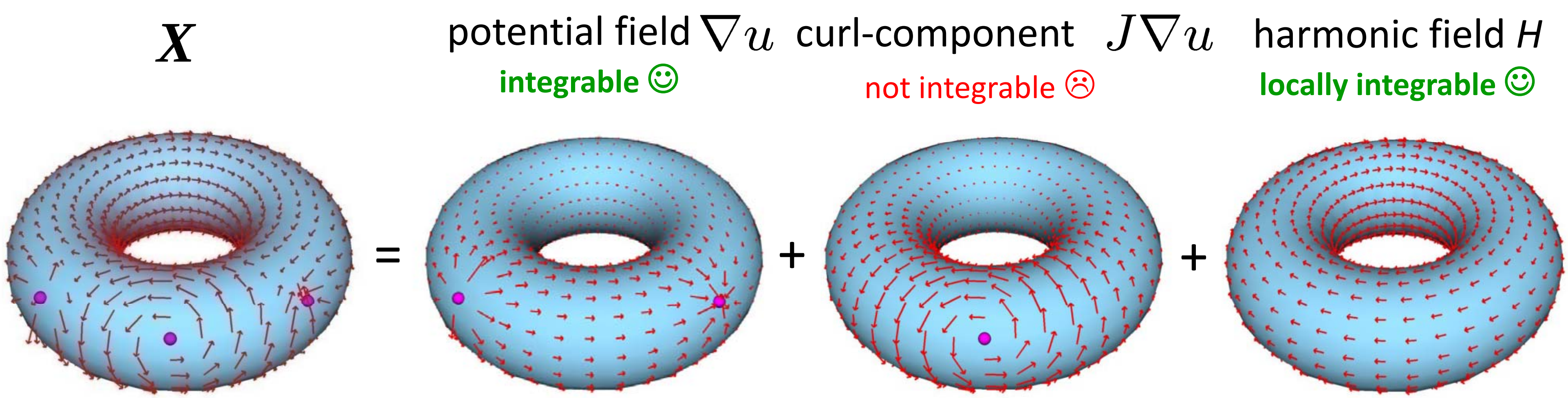
- Given f find g s.t. $f = \nabla g$
- Only possible if f is *"integrable"*

- A vector field U is ***locally integrable*** iff $\nabla \times U = 0$
  - – No closed local loops!



Hodge-Helmholtz Decomposition

The space of vector fields on any surface decomposes into:



Assure Local Integrability

Problem: cross field K is usually not locally integrable

Solution: (assume frame K splits into two vector fields)

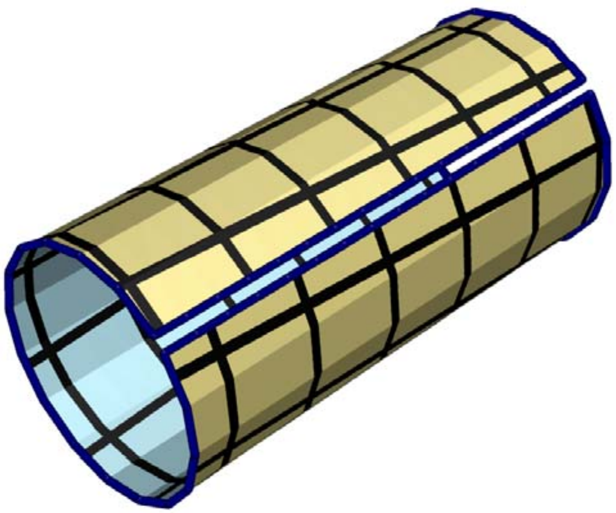
1. Compute Hodge-Helmholtz Decomposition K_1, K_2
2. Remove curl-component (non-integrable part) of K_1, K_2

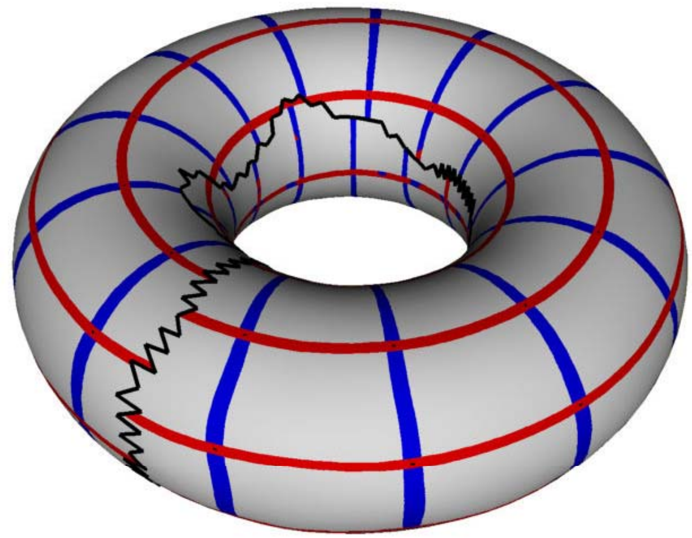
$$X_1 := K_1 - J\nabla v_1 = \nabla u_1 + \cancel{J\nabla v_1} + H_1$$
$$X_2 := K_2 - J\nabla v_2 = \nabla u_2 + \cancel{J\nabla v_2} + H_2$$

Result: new cross field $X = (X_1, X_2)$ is locally integrable

Global Integrability

Problem: mismatch of parameter lines around closed loops





Solution: mismatch of parameter lines around closed loops

1. Compute Homology generators (= basis of all closed loops)
2. Measure mismatch along Homology generator (next slide…)

Assure Global Integrability

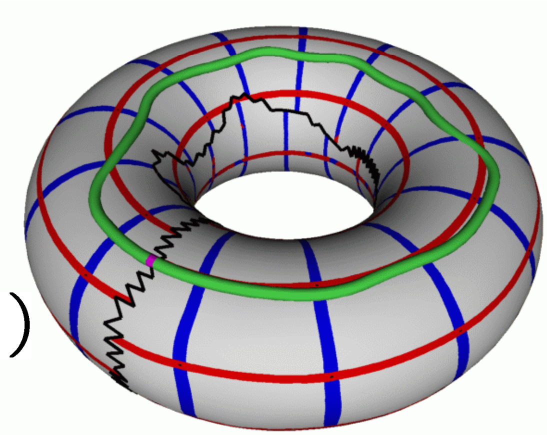
Solution: (… cont'ed)

2. Measure mismatch along Homology generator γ as curve integrals of both vector fields:

$$\int_{\gamma_p} X_1 ds \in \mathbb{R}, \quad \int_{\gamma_p} X_2 ds \in \mathbb{R}$$

3. Compute L_2-smallest harmonic vector fields H_1, H_2 s.t.

$$\int_{\gamma_p} (X_1 + H_1) ds \in \mathbb{Z}, \quad \int_{\gamma_p} (X_2 + H_2) ds \in \mathbb{Z}$$



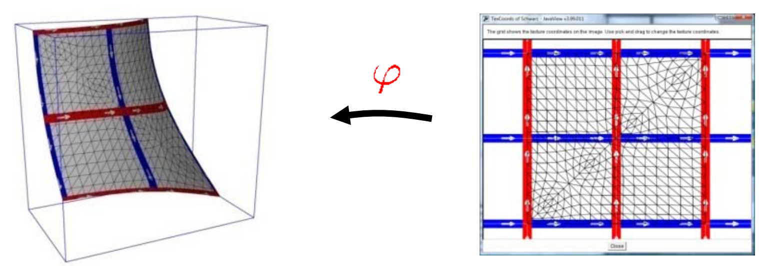
Result: new frame

is globally integra $X = (X_1 + H_1, X_2 + H_2)$

QuadCover Algorithm (unbranched)

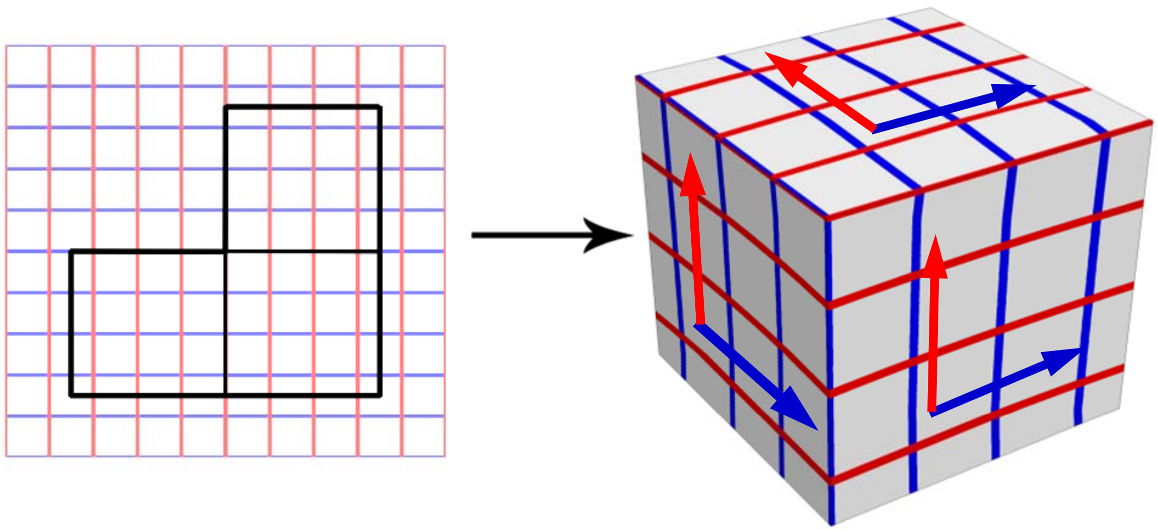
Given a simplicial surface M:

1. Generate a guiding frame field K
   (e.g. principal curvatures frames)
2. Assure local integrability of K via Hodge Decomp.
   (remove curl-component from K)
3. Assure global continuity of K along Homology gens.
   (add harmonic field to K s.t. all periods of K are integers)
4. Global integration of K on M
   gives parameterization



No Splitting of Parameter Lines

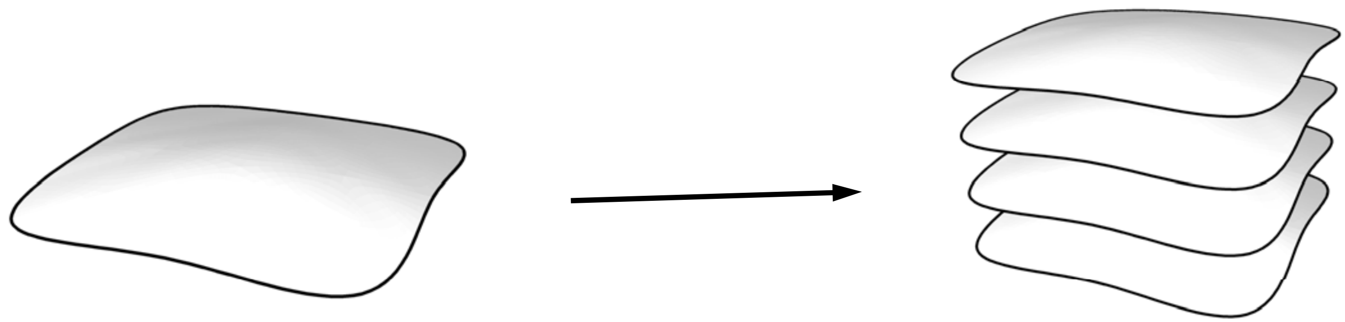
Warning: parameter lines do not split into red and blue lines !!!



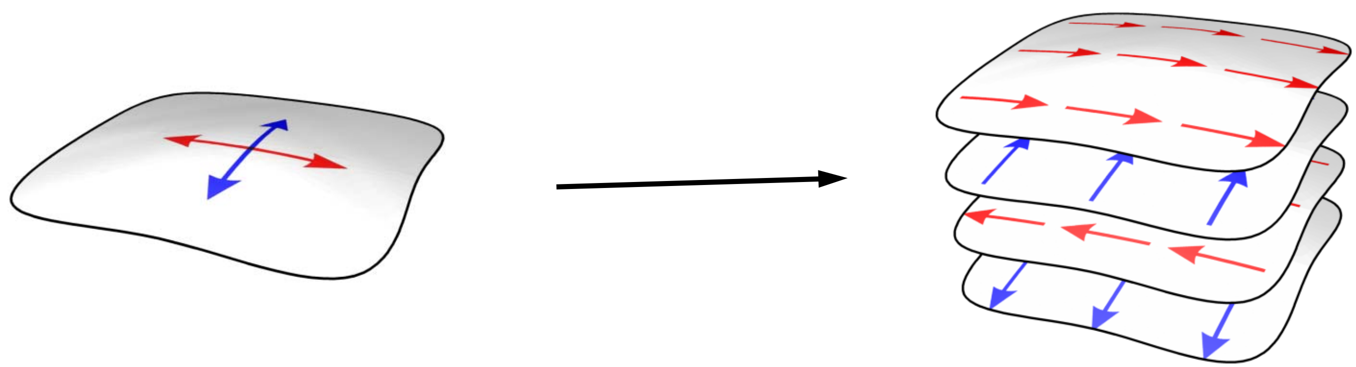
Consequence: a frame field does not globally split into four vector fields.

Construct a Branched Covering Surface

Step 1: Make four layers (copies) of the surface.

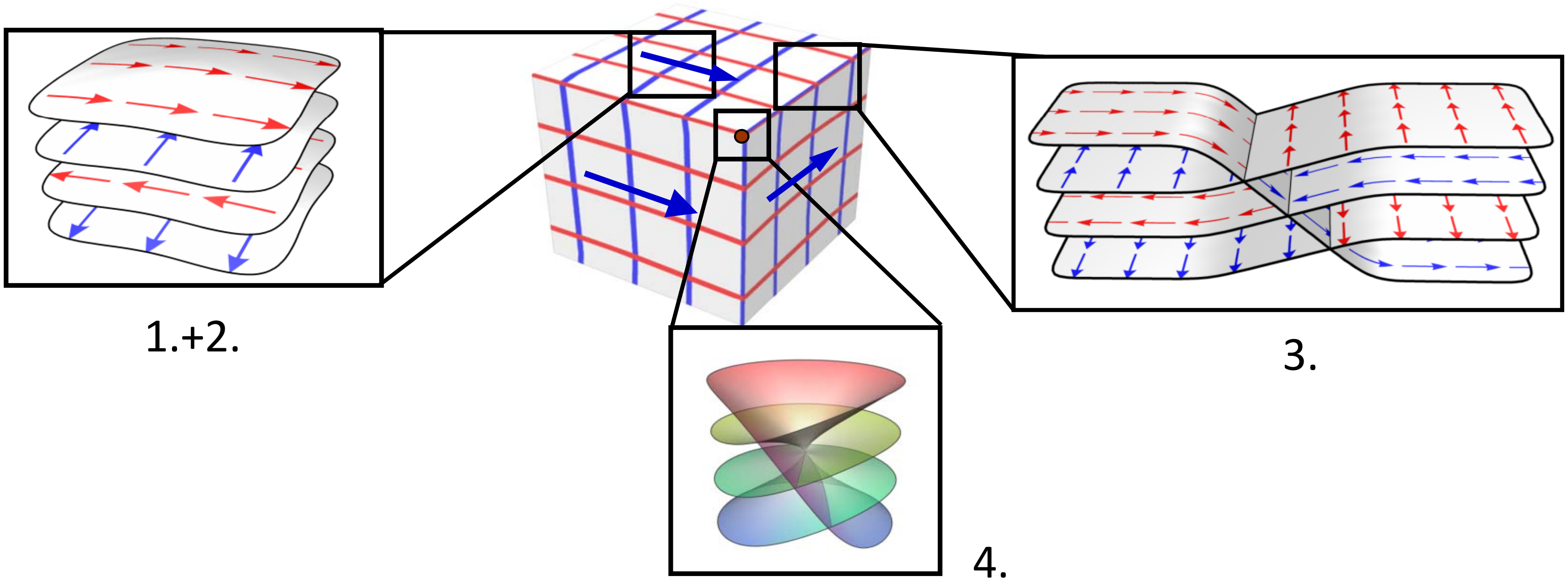


Step 2: Lift frame field to a vector field on each layer.



Construct a Branched Covering Surface

Step 3: Connect layers consistently with the vectors.

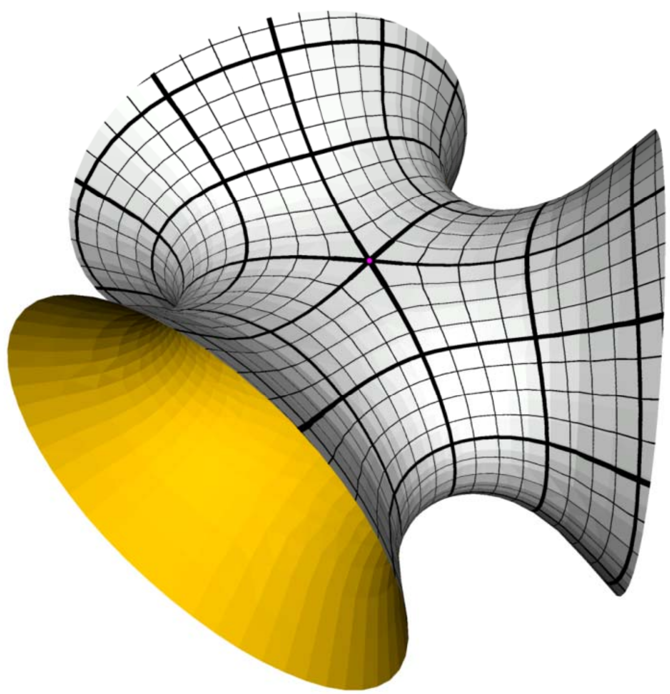


Result: The frame field simplifies to a vector field on the covering surface.

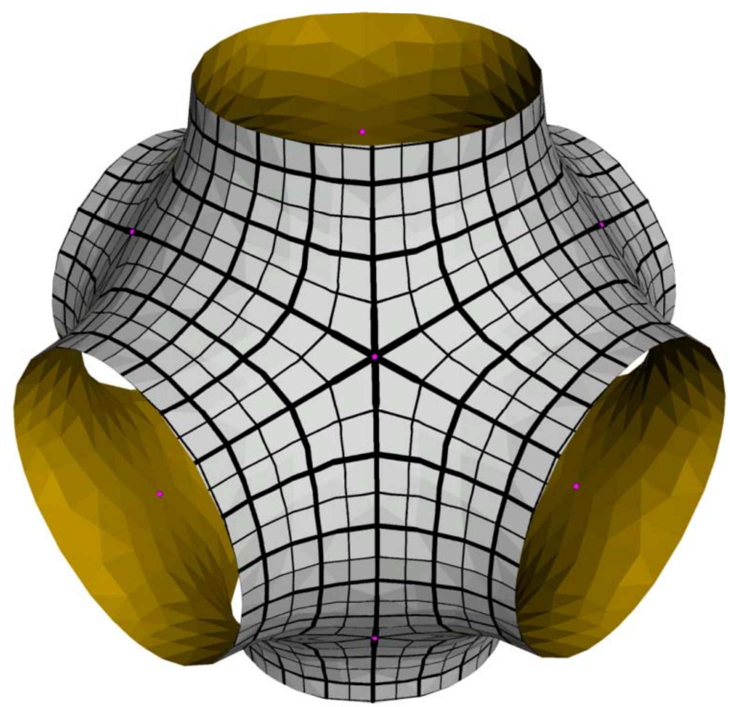
Examples

Minimal surfaces with isolated branch points

Index of each singularity = -1/2



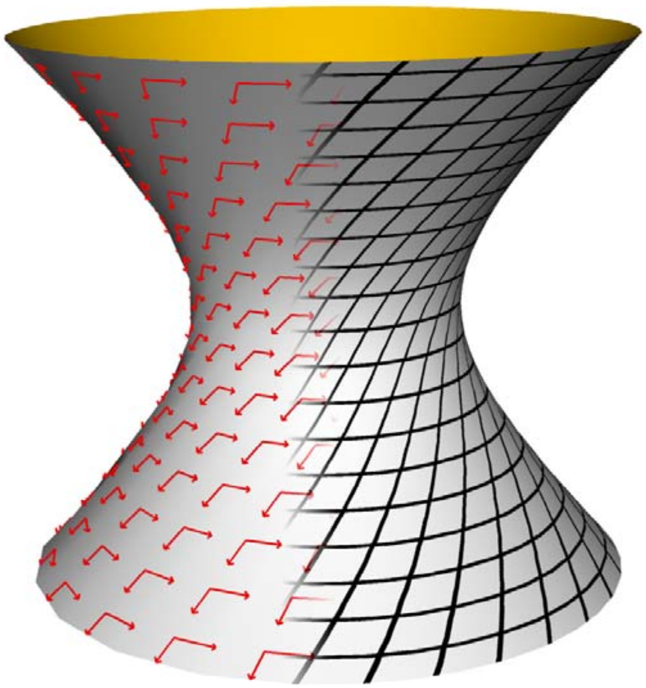
Trinoid



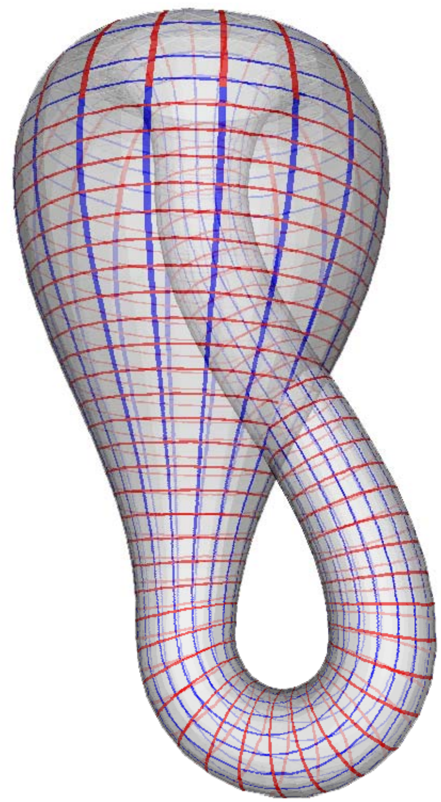
Schwarz-P Surface

Examples

Different Frame Fields



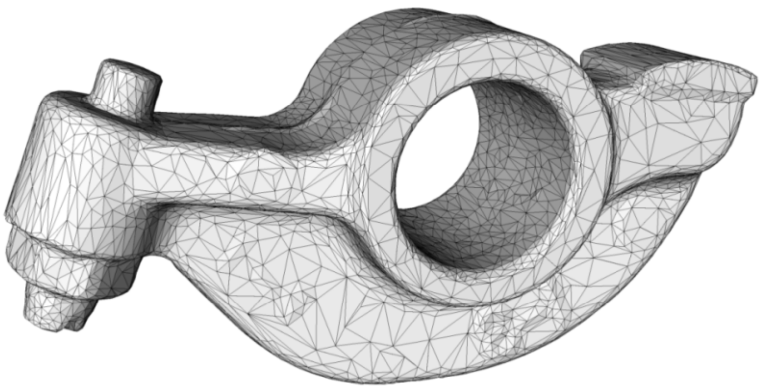
Non-orthogonal frame on hyperboloid

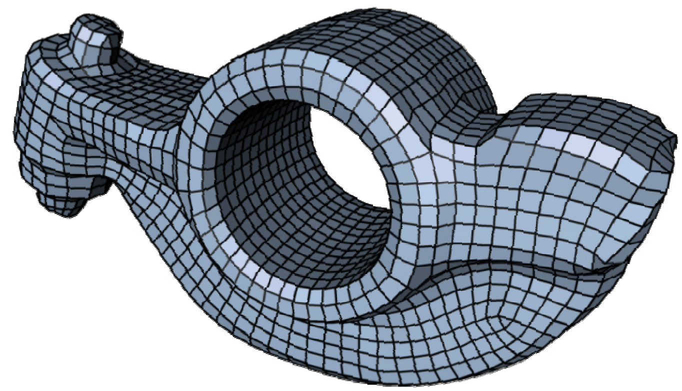


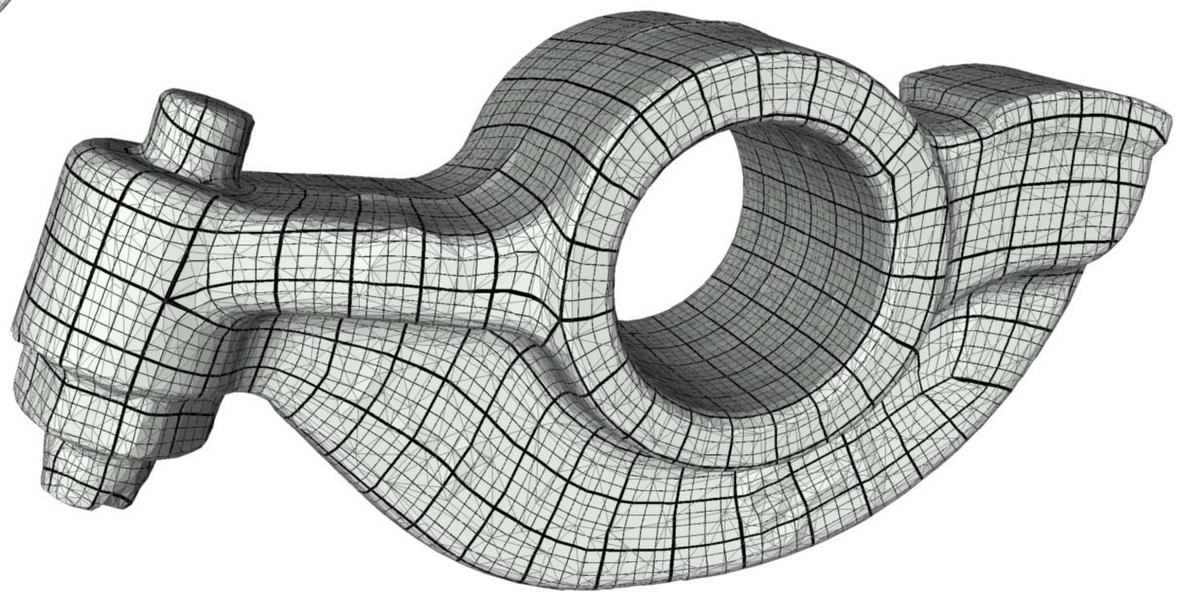
Non-orientable Klein bottle

Examples

Rocker arm test model







Mixed Integer Quadrangulation

[Zimmer '09]